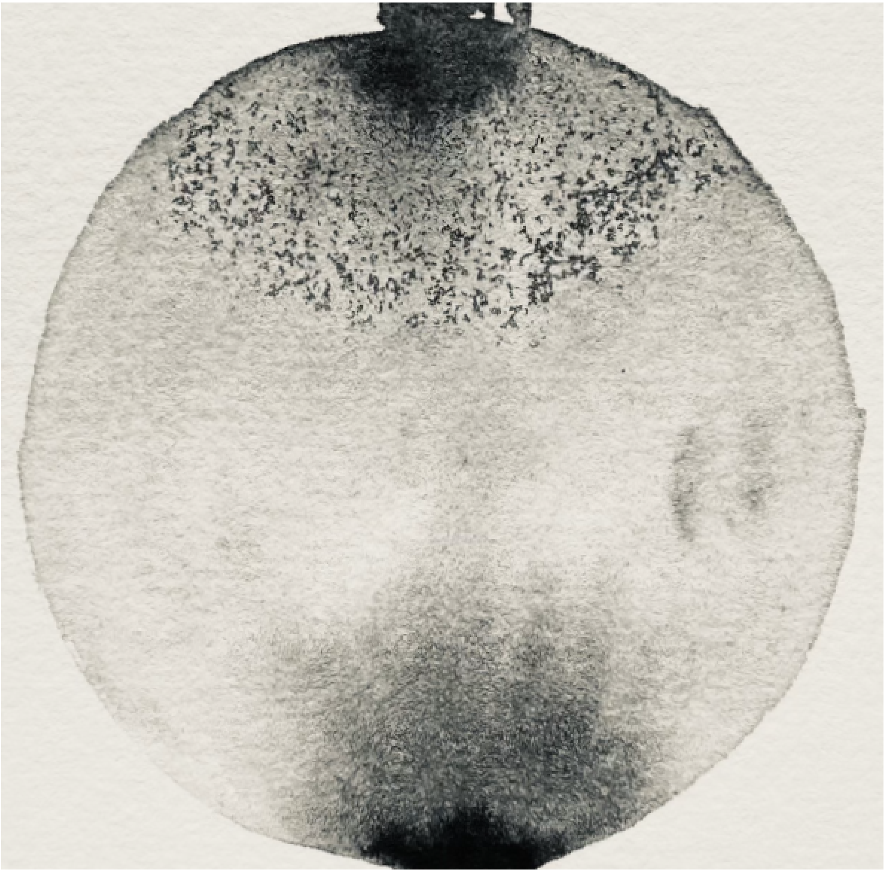
Concepts for unifying the descriptions of hadronic and quark matter



Plan:

1. Hagedorn – Rafelski 1980

2. String-Flip Model 1985

3. Switch functions, Interpolations 2005 – now

4. Generalized Beth-Uhlenbeck,
   Cluster virial & Φ-deriv. 2016 – now

5. Quo vadis ?

Hagedorn-Rafelski: From hadron gas to quark matter with statistical Bootstrap

Ref.TH.2947-CERN

FROM HADRON GAS TO QUARK MATTER 1 *)

R. Hagedorn
CERN -- Geneva

and

J. Rafelski
Institut für Theoretische Physik der Universität
Frankfurt

*) Invited lecture presented by R. H. at the "International Symposium on Statistical Mechanics of Quarks and Hadrons" University of Bielefeld, Germany, August 1980.

Ref.TH.2947-CERN
18 September 1980

Johann Rafelski (left) and Rolf Hagedorn (right) at the workshop on the occasion of Hagedorn's 75th birthday in Divonne (CH), 30.06.1994.

From the Springer book (open access):

Melting Hadrons, Boiling Quarks - From Hagedorn Temperature to Ultra-Relativistic Heavy-Ion Collisions at CERN

https://link.springer.com/book/10.1007/978-3-319-17545-4

Hagedorn-Rafelski: From hadron gas to quark matter with statistical Bootstrap

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Grand canonical partition function for the hadron resonance gas

$$Z(T,V,\lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int \sigma(p,V,b)\, e^{-\beta_\mu p^\mu} d^4p$$

Density of states (ansatz)

$$\sigma(p,V,b) = \sum_{N=0}^{\infty} \frac{1}{N!} \int \delta^4\Big(p - \sum_{i=1}^{N} p_i\Big) \sum_{\{b_i\}} \delta_K\Big(b - \sum_{i=1}^{N} b_i\Big) \prod_{i=1}^{N} \frac{2\Delta_\mu p^\mu}{(2\pi)^3} \tau(p_i^2, b_i)\, d^4p_i$$

unlimited creation & absorption

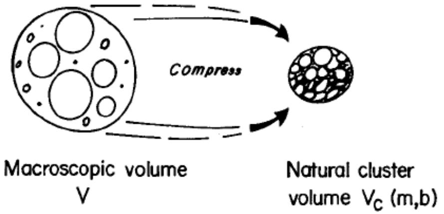
4-momentum conservation

baryon number conservation

chemical equilibrium of all constituents (clusters)

available volume

$$\Delta = V - \sum_{i=1}^{N} V_i$$



The bootstrap idea: a macroscopic system compressed to the "natural cluster volume" becomes itself almost a cluster consisting of clusters.

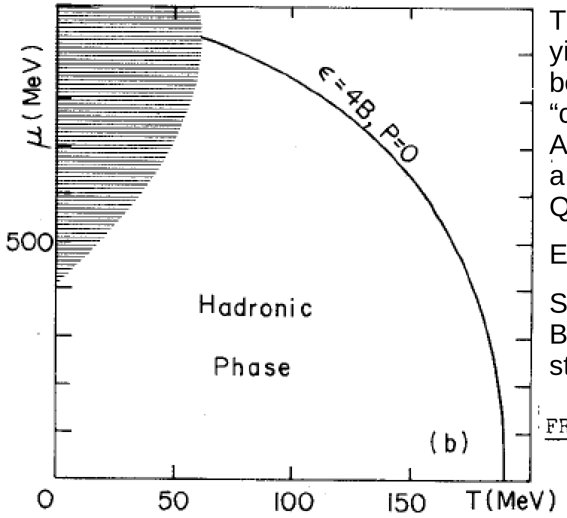
Hagedorn-Rafelski: From hadron gas to quark matter with statistical Bootstrap

Ansatz: cluster volume is 4-vector proportional to cluster 4-momentum

$$V_c^\mu(m,b) = A(m,b)\cdot\frac{p^\mu}{m} \xrightarrow[\text{rest frame}]{} \frac{m}{4B} \quad ; \quad m=\sqrt{p_\mu p^\mu}$$

Total volume: $$\langle V^\mu\rangle = \Delta^\mu + \frac{\langle p^\mu\rangle}{4B} \xrightarrow[\text{rest frame}]{} \Delta + \frac{\langle E\rangle}{4B}$$

$$\frac{\langle E\rangle}{\langle V\rangle} =: \mathcal{E}(\beta,\lambda) = \frac{\mathcal{E}_{pt}(\beta,\lambda)}{1+\mathcal{E}_{pt}(\beta,\lambda)/4B}$$, since $\langle E\rangle = \Delta\, \varepsilon_{pt}$



Transcendental bootstrap equation yields limiting line in the T-μ plane beyond which the hadronic world "ceases to exist":
All hadrons (clusters) dissolve into a giant cluster → "hadron liquid" = Quark-Gluon Plasma

Energy density on that curve = 4B

Shaded region: limitation because Bose-Einstein and Fermi-Dirac statistic not considered.

FROM HADRON GAS TO QUARK MATTER II *)

Ref.TH.2969-CERN
13 October 1980

Grand canonical partition function for the hadron resonance gas

$$Z(T,V,\lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int \sigma(p,V,b)\, e^{-\beta_\mu p^\mu} d^4p$$

Density of states (ansatz)

$$\sigma(p,V,b) = \sum_{N=0}^{\infty}\frac{1}{N!}\int \delta^4\Big(p-\sum_{i=1}^{N}p_i\Big)\sum_{\{b_i\}}\delta_K\Big(b-\sum_{i=1}^{N}b_i\Big)\prod_{i=1}^{N}\frac{2\Delta_\mu p^\mu}{(2\pi)^3}\,\tau(p_i^2,b_i)\,d^4p_i$$

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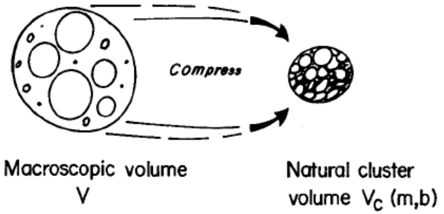


The bootstrap idea: a macroscopic system compressed to the "natural cluster volume" becomes itself almost a cluster consisting of clusters.

Concepts for unifying the descriptions of hadronic and quark matter

Statistical bootstrap is a concept that reproduces the exponentially rising density of states

Questions:

- Microphysical origin of this density of states?
  → multiplet structure, internal degrees oof freedom (spin, flavor, color) ...

- How to build hadrons as clusters from fundamental degrees of freedom (quarks & gluons)?
  → confining interaction (potential model, density functional, …)
  → dynamical chiral symmetry breaking

- Mechanism for the dissolution of hadronic clusters into the QGP beyond the limiting curve?
  → bound state dissociation (Mott effect) triggered by:
    → screening of the confining interaction
    → chiral symmetry restoration

String-Flip model: Quantum statistical unification of hadronic and quark matter

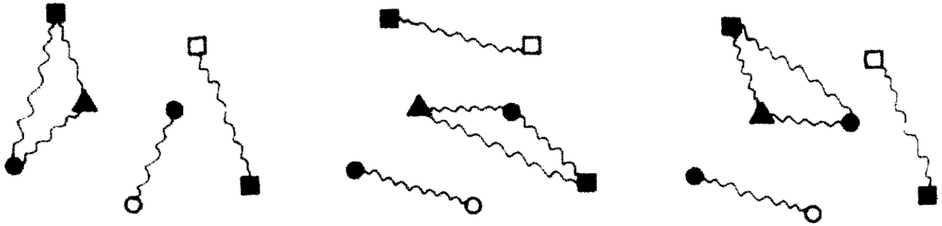
PHYSICAL REVIEW D VOLUME 34, NUMBER 11 1 DECEMBER 1986

Pauli quenching effects in a simple string model of quark/nuclear matter

G. Röpke and D. Blaschke
Department of Physics, Wilhelm-Pieck-Universtiy, 2500 Rostock, German Democratic Republic

H. Schulz
Central Institute for Nuclear Research, Rossendorf, 8051 Dresden, German Democratic Republic and The Niels Bohr Institute, 2100 Copenhagen, Denmark

(Received 16 December 1985)



Many string configurations saturate color in a given multi-quark system

Effective Hamiltonian

$$H^{\rm eff}=\sum_i (m_i+p_i^2/2m_i)+\sum_{i<j} V^{\rm eff}(r_{ij}) \qquad V^{\rm eff}(r_{ij})=V^{\rm conf}(r_{ij})c(r_{ij})$$

Saturation of color within nearest neighbors: $c(r)=\exp\left[-\frac{4\pi}{3}n_0 r^3\right]$

With the same confining potential one describes hadronic bound states and the effective interaction in the plasma (Hartree energy shift):

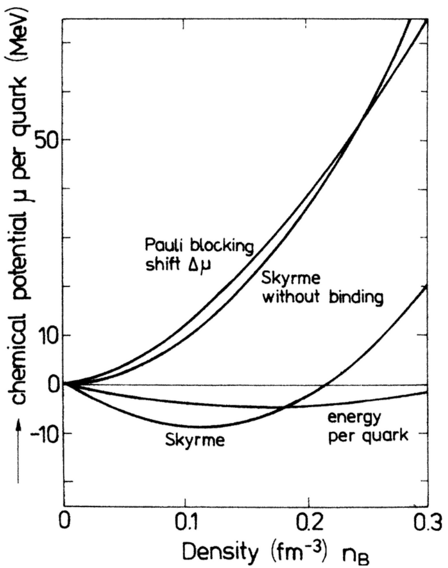
$$n(1)=f_{\alpha_1}[E(1)+\Delta(1)]+\sum_{23}\sum_{\nu P} f_3(E^0_{\nu P}+\Delta E^{\rm Pauli}_{\nu P})\,|\psi_{\nu P}(123)|^2$$

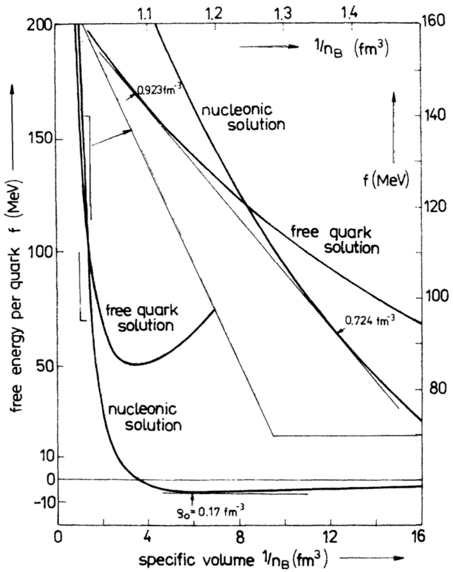
Cluster decomposition of the density → inversion → free energy density

$$f(n_B,T)-f(n_B^0,T)=\frac{1}{n_B}\int_{n_B^0}^{n_B}\mu(n_B',T)dn_B'$$

String-Flip model: Quantum statistical unification of hadronic and quark matter

Maxwell construction of a first-order phase transition (T=0)





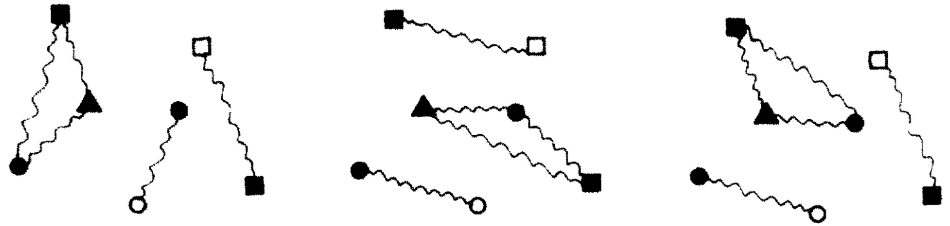
Density functional for the thermodynamic potential

$$f^h(n_B, T=0) = 313 + 24.75 n_B^{2/3} - 132.16 n_B + 301.32 n_B^2 + 15.65 n_B^{5/3}$$

hadronic phase

$$f^{\text{free}}(n_B, T=0) = 69.72\times 10^3 \frac{1}{m} n_B^{2/3} + 0.695 a n_B^{-2/3} - 212.03 \alpha_s n_B^{1/3} + 15.85\times 10^6 \alpha_s \frac{1}{m^2} n_B + 57.82\times 10^3 \frac{a}{m^2}\epsilon + C + m$$

quark phase



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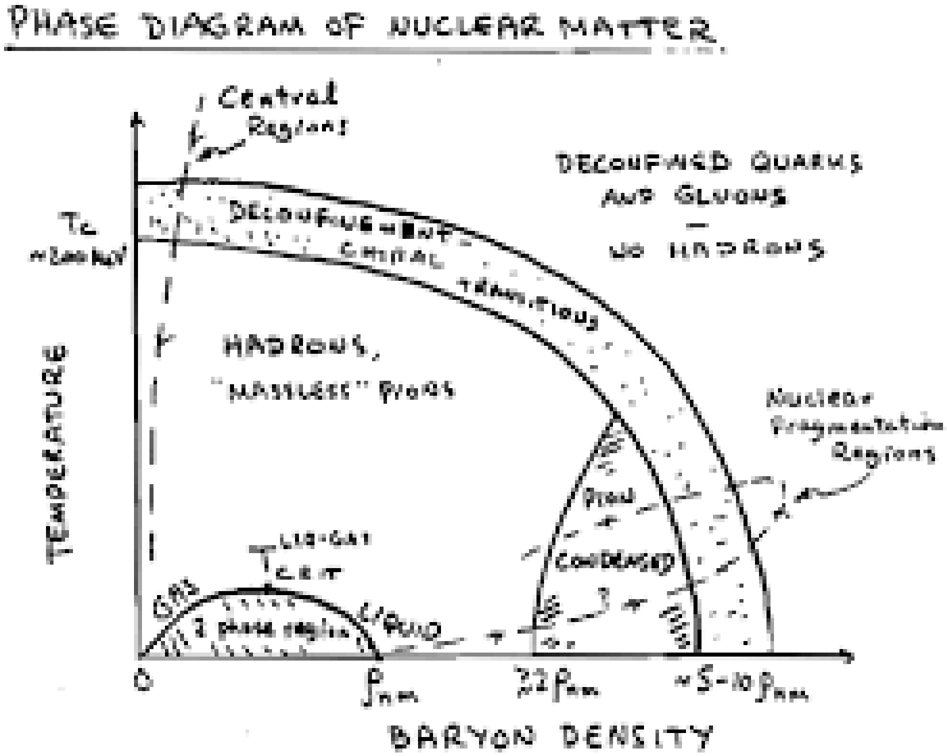
String-Flip model: Quantum statistical unification of hadronic and quark matter



Gordon Baym (1983)

Phase transition by Maxwell construction:
→ always first-order transition !!

Lattice QCD: Crossover !
→ CEP at T < 130 MeV or none?

Relativistic Density Functional appr. (Walecka-type models)
→ CEP for liquid-vapour transition
→ Cluster formation?
Typel et al. PRC 81 (2010) 015803
See talk by S. Liebing today

String-Flip model (II): Relativistic density functional + Path integral

PHYSICAL REVIEW D **105**, 114042 (2022)

Density functional approach to quark matter with confinement and color superconductivity

Oleksii Ivanytskyi* and David Blaschke†
Institute of Theoretical Physics, University of Wroclaw, Max Born Plac 9, 50-204 Wroclaw, Poland

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \hat{m})q - \mathcal{U} - G_V(\bar{q}\gamma_\mu q)^2 + G_D(\bar{q}i\gamma_5\tau_2\lambda_A q^c)(\bar{q}^c i\gamma_5\tau_2\lambda_A q),$$

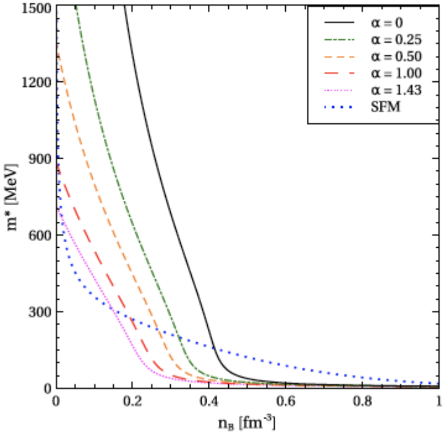
Gaussian expansion of the energy density functional:

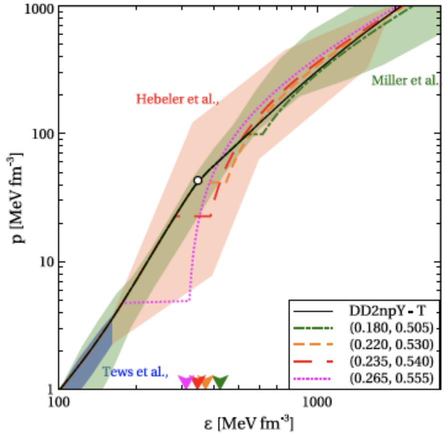
$$\mathcal{U}^{(2)} = \mathcal{U}_{\rm MF} + (\bar{q}q - \langle\bar{q}q\rangle)\Sigma_{\rm MF} - G_S(\bar{q}q - \langle\bar{q}q\rangle)^2 - G_{\rm PS}(\bar{q}i\gamma_5\vec{\tau}q)^2,$$

Defines selfenergies and effective, medium-dependent couplings

$$\Sigma_{\rm MF} = \frac{\partial\mathcal{U}_{\rm MF}}{\partial\langle\bar{q}q\rangle}, \qquad G_S = -\frac{1}{2}\frac{\partial^2\mathcal{U}_{\rm MF}}{\partial\langle\bar{q}q\rangle^2}, \quad G_{\rm PS} = -\frac{1}{6}\frac{\partial^2\mathcal{U}_{\rm MF}}{\partial\langle\bar{q}i\gamma_5\vec{\tau}q\rangle^2}.$$

Effective quark mass: $\hat{m}_0^* = \hat{m} + \frac{2}{3}D_0\alpha^{-2/3}\langle\bar{q}q\rangle_0^{-1/3},$





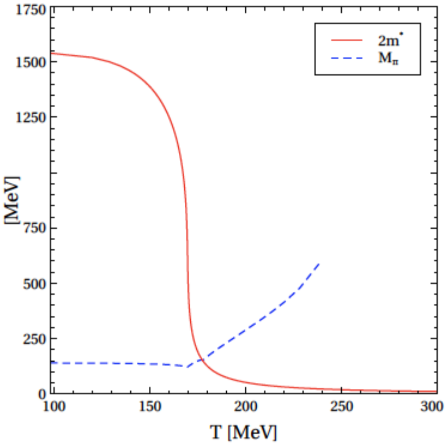
String-Flip model (II): Relativistic density functional + Path integral

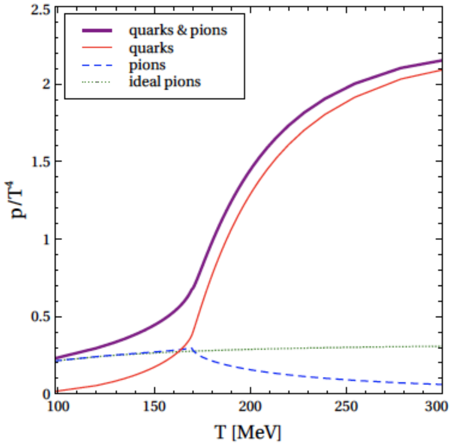
But where are the hadrons? … in the same formalism?

→ Go beyond the mean field, and quantize fluctuations!

$$\Omega = \Omega_{MF} - \frac{3\mathrm{Tr}\ln(\beta^2 D_\pi)}{2\beta V} = \Omega_{MF} + 3T\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{dp_0}{\pi}\ln(1-e^{\beta p_0})\frac{\partial\delta_\pi}{\partial p_0}$$

Beth-Uhlenbeck formula (phase shifts of qq interaction)
→ NJL model: D.B. et al. Annals Phys. 348 (2014) 228
→ Confining RDF: O.Ivanytskyi, D.B., K. Maslov (2022)





More mesons, baryons & multiquark states, comparison with lattice QCD thermodynamics → M. Cierniak's talk tomorrow

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Defines selfenergies and effective, medium-dependent couplings

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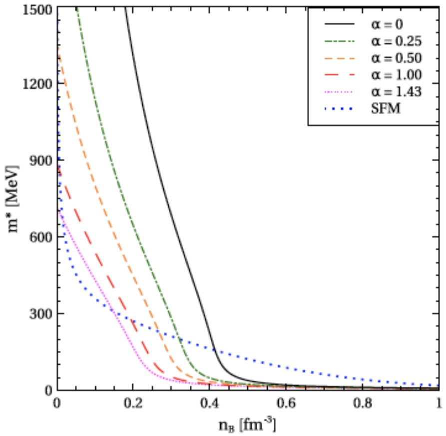

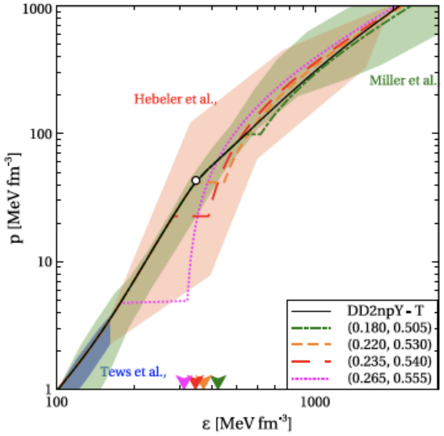

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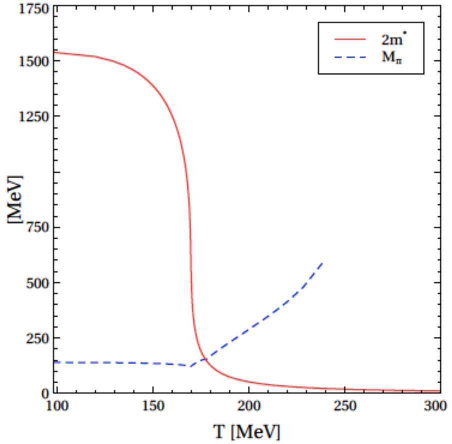
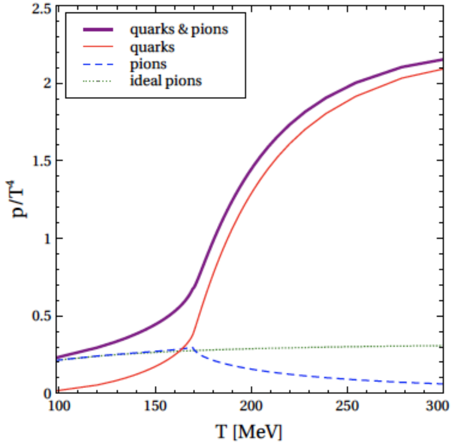
→ NJL model: D.B. et al. Annals Phys. 348 (2014) 228
→ Confining RDF: O.Ivanytskyi, D.B., K. Maslov (2022)
Mott dissociation of pions … and other hadrons!





More mesons, baryons & multiquark states, comparison with lattice QCD thermodynamics → M. Cierniak's talk tomorrow

Backreaction of hadrons on quark mean fields (selfenergies)?
→ Φ-derivable approach + cluster virial expansion

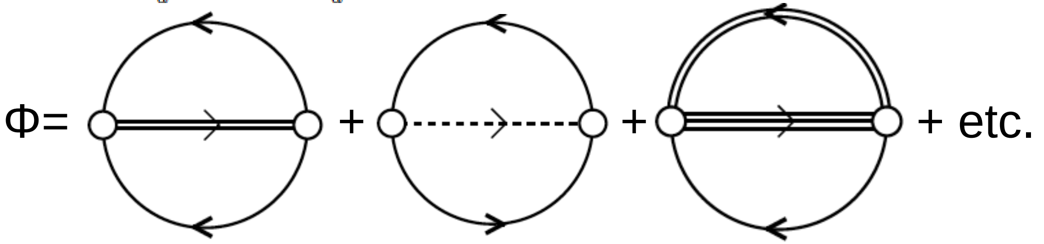
$$\Omega(T,\mu) = \sum_{a=1}^{N}\Omega_a(T,\mu) + \Phi\left[\{S_a\}\right] + \mathcal{U}(\phi,\bar{\phi}) + \mathcal{V}(\sigma)\ ,$$

$$\Omega_a(T,\mu) = c_a\left[\mathrm{Tr}\ln S_a^{-1} + \mathrm{Tr}(\Pi_a S_a)\right]\ ,$$

$$S_a^{-1} = S_a^{(0)^{-1}} - \Pi_a\ ,\ a = 2,\ldots,N, \qquad \Pi_a = \frac{\partial\Phi}{\partial S_a}\ .$$

$$n = -\frac{\partial\Omega}{\partial\mu} = \sum_a a\, n_a(T,\mu) = \sum_a a\, d_a\int\frac{d^3q}{(2\pi)^3}\left\{f_\phi^{(a),+} - \left[f_\phi^{(a),-}\right]^*\right\} 2\sin^2\delta_a(\omega,q)\frac{\partial\delta_a(\omega,q)}{\partial\omega}\ ,$$

$$s = -\frac{\partial\Omega}{\partial T} = \sum_a s_a(T,\mu) = \sum_a d_a\int\frac{d^3q}{(2\pi)^3}\left\{\sigma_\phi^{(a),+} + \left[\sigma_\phi^{(a),-}\right]^*\right\} 2\sin^2\delta_a(\omega,q)\frac{\partial\delta_a(\omega,q)}{\partial\omega}\ ,$$



First steps: D.B., A. Dubinin, L.Turko, APPBPS 10 (2017) 473
D.B., K. Devyatyarov, O. Kaczmarek, Symmetry 13 (2021) 514

Constructive approaches to the hadron-to-quark matter transition

PHYSICAL REVIEW C **90**, 024915 (2014)

Matching excluded-volume hadron-resonance gas models and perturbative QCD to lattice calculations

M. Albright, J. Kapusta, and C. Young
School of Physics & Astronomy, University of Minnesota, Minneapolis, Minnesota 55455, USA
(Received 1 May 2014; published 27 August 2014)

Strange Quark Matter Conference, Cracow 2011

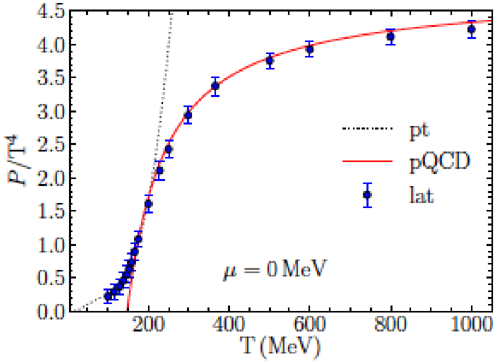
Matching pressure of hadronic $P_h(T,\mu)$ and quark-gluon $P_{qg}(T,\mu)$ matter with a switch function $S(T,\mu)$

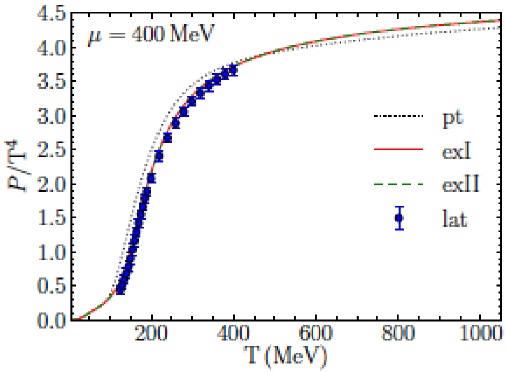
$$P(T,\mu) = S(T,\mu)P_{qg}(T,\mu) + [1 - S(T,\mu)]P_h(T,\mu)$$

$$S(T,\mu) = \exp\{-\theta(T,\mu)\}, \quad \theta(T,\mu) = \left[\left(\frac{T}{T_0}\right)^r + \left(\frac{\mu}{\mu_0}\right)^r\right]^{-1}$$

$P_h(T,\mu)$ is the hadron resonance gas pressure with excluded volume,

$P_{qg}(T,\mu)$ is the perturbative QCD pressure





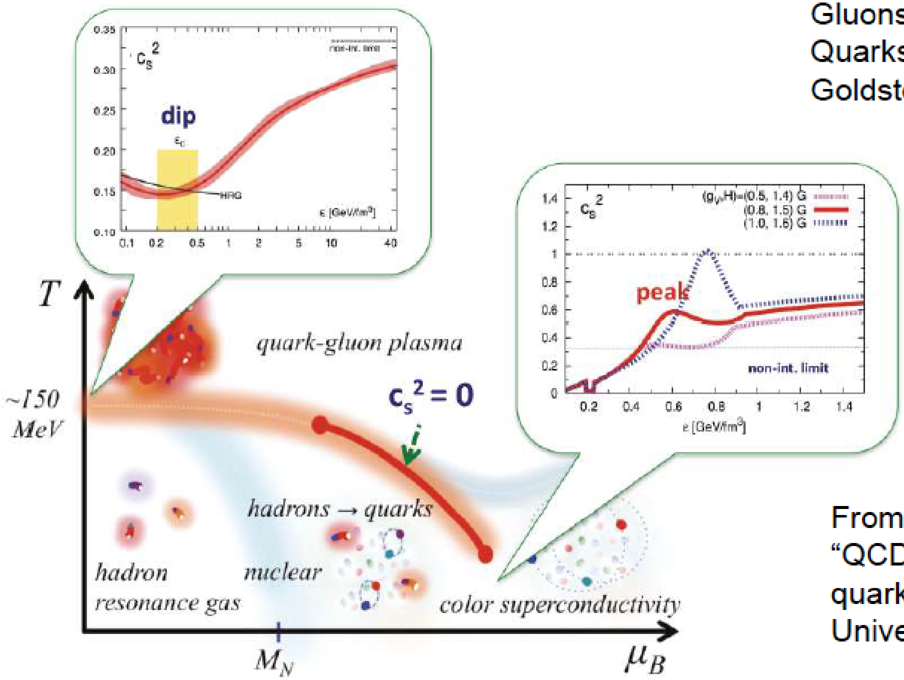
Constructive approaches to the hadron-to-quark matter transition

Many more approaches to construct hybrid EoS in the QCD phase diagram, e.g.,

- C. Nonaka and M. Asakawa, Phys. Rev. C71 (2005) 044904
- K. Masuda, T. Hatsuda and T. Takatsuka, PTEP 2013 (2013) 073D01
- P. Parotto et al., Phys. Rev. C 101 (2020) 034901
- M. Marczenko et al., Astron. & Astrophys. 643 (2020) A82
- J.I. Kapusta and T. Welle, arXiv:2205.12150 (2022)
- A. Sörensen et al., Phys. Rev. Lett. 127 (2021) 042303

...

Constructive approaches to the hadron-to-quark matter transition

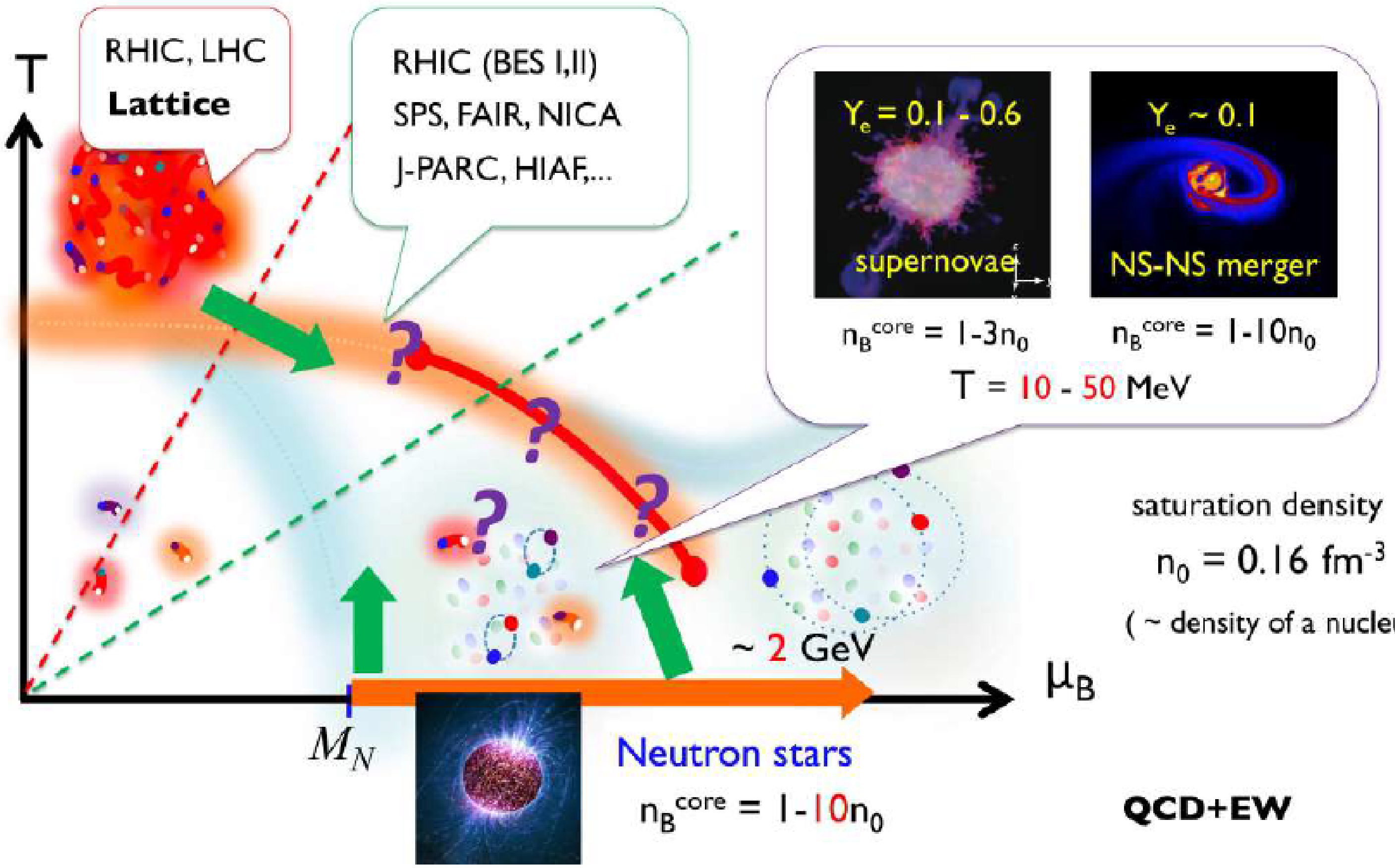


Gluons ↔ Vector mesons
Quarks ↔ Baryons
Goldstones ↔ Pseudoscalar mesons

From: T. Kojo,
“QCD equations of state in quark-hadron continuity”,
Universe 4 (2018) 42

T. Schaefer & F. Wilczek, Phys. Rev. Lett. 82 (1999) 3956
C. Wetterich, Phys. Lett. B 462 (1999) 164
T. Hatsuda, M. Tachibana, T. Yamamoto & G. Baym, Phys. Rev. Lett. 97 (2006) 122001

Constructive approaches to the hadron-to-quark matter transition



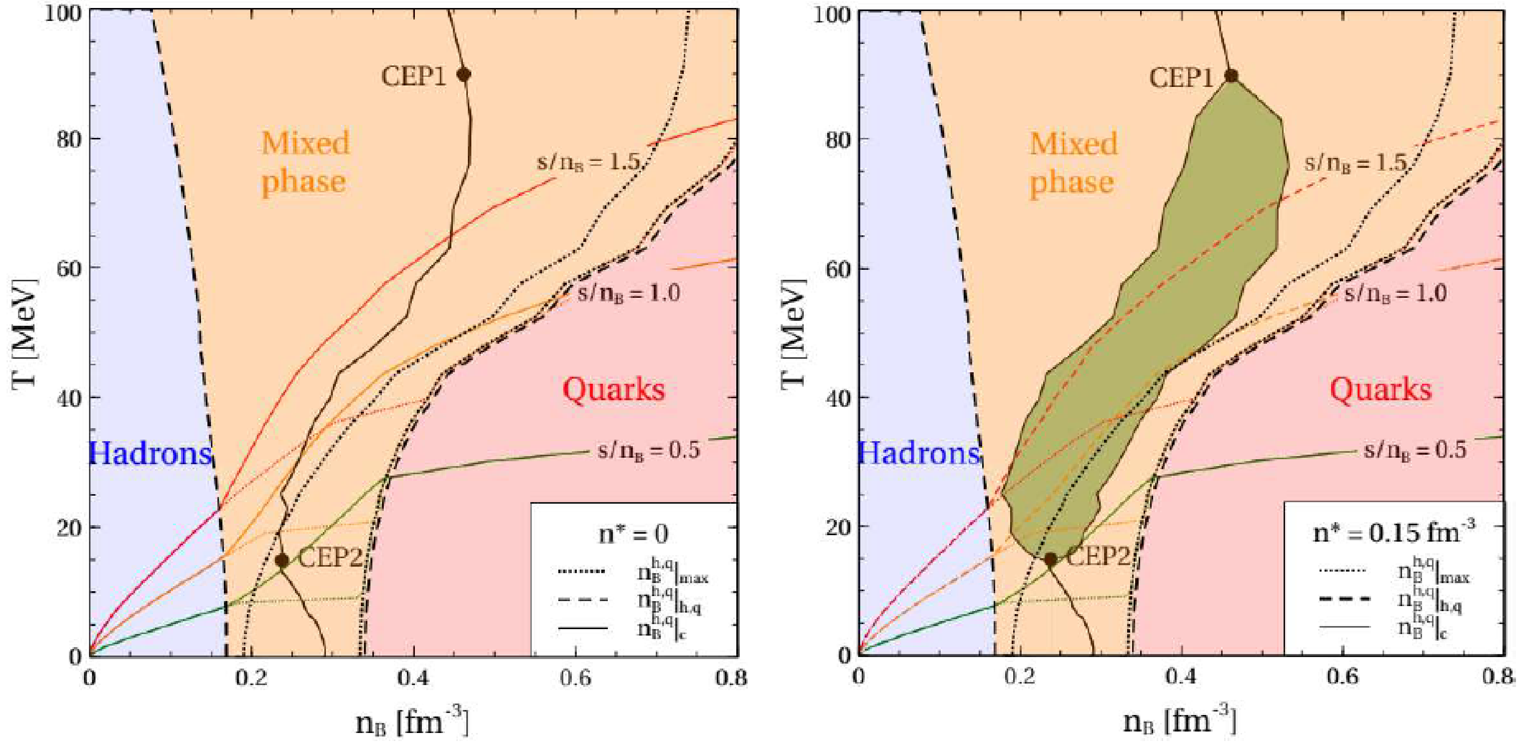
saturation density

$n_0 = 0.16\ fm^{-3}$

(~ density of a nucleus)

QCD+EW

Constructive approaches to the hadron-to-quark matter transition



O. Ivanytskyi & D. Blaschke, Eur. Phys. J. A 58 (2022) 152; arXiv:2205.03455 [nucl-th]
"A new class of hybrid EoS with multiple critical endpoints for simulations of supernovae, neutron stars and their mergers"

Concepts for unifying the descriptions of hadronic and quark matter

Summary:

- 42 years after the seminal paper by Hagedorn and Rafelski on the limiting line of the "hadronic world" in the Phase Diagram of strongly interacting matter (under extreme conditions) obtained from statistical bootstrap model, the quest continues for a unifying approach to quark-hadron matter.

- The Φ-derivable approach, using a set of 2PI "sunset" diagrams formed with generalized cluster Green's functions leads to a cluster virial expansion in the form of a generalized Beth-Uhlenbeck approach accounting for all binary collisions of quark clusters.
  It describes hadrons as strong correlations (bound and scattering states) and bound state dissolution as Mott dissociation.

- While such a general unifying approach is still under development, some constructive approaches to a hybrid equation of state are in use for phenomenological studies of the quark-hadron transition in heavy-ion collisions and in astrophysics