Formation of clusters and the chemical freeze-out in heavy-ion collisions

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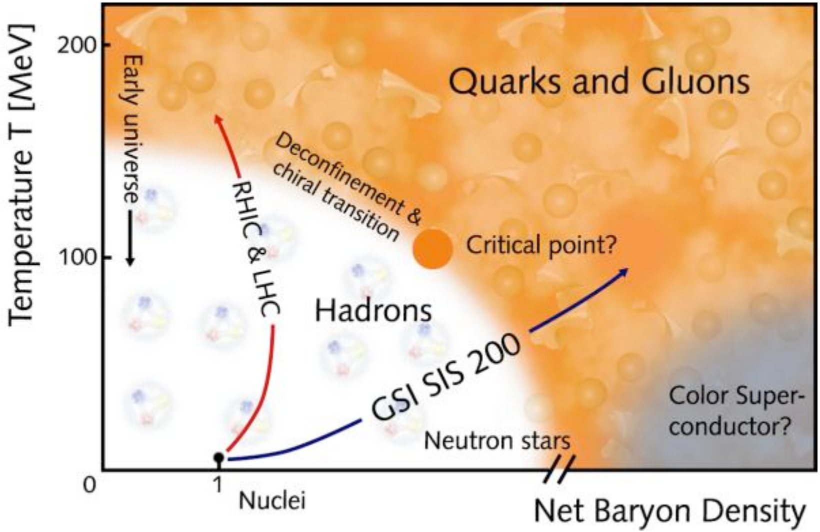


Temperature T [MeV]
200
100
0
1
Early universe
RHIC & LHC
Deconfinement & chiral transition
Quarks and Gluons
Critical point?
Hadrons
GSI SIS 200
Neutron stars
Color Super-conductor?
Nuclei
Net Baryon Density

experimental freeze-out curve

$$\frac{T_{cf}^{\mathrm{Cleymans}}}{\mathrm{GeV}} = 0.166 - 0.139\left(\frac{\mu_B}{\mathrm{GeV}}\right)^2 - 0.053\left(\frac{\mu_B}{\mathrm{GeV}}\right)^4 \quad (1)$$

$$\frac{\mu_B}{\mathrm{GeV}} = \frac{1.308}{1 + 0.273\sqrt{s_{NN}}/\mathrm{GeV}} \quad (2)$$

and $\sqrt{s_{NN}} = \sqrt{2m_N E_{\mathrm{lab}} + 2m_N^2}$, $m_N = 0.939$ GeV.

J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev. C **73** (2006), 034905

doi:10.1103/PhysRevC.73.034905

$$T_{cf}^{\mathrm{Andronic}} = \frac{158.4\mathrm{MeV}}{1 + \exp[2.60 - \ln(\sqrt{s_{NN}}/\mathrm{GeV})/0.45]} \quad (3)$$

$$\mu_B = \frac{1307.5\mathrm{MeV}}{1 + 0.288\sqrt{s_{NN}}/\mathrm{GeV}} \quad (4)$$

A. Andronic, P. Braun-Munzinger, K. Redlich and J. Stachel, Nature **561** (2018) no.7723, 321

doi:10.1038/s41586-018-0491-6

critical temperature on the lattice

$$T_c(\mu_X) = T_c(0)\left(1 - \kappa_2^X\left(\frac{\mu_X}{T_c(0)}\right)^2 - \kappa_4^X\left(\frac{\mu_X}{T_c(0)}\right)^4\right) \quad (5)$$

$T_c(0) =$156.5 MeV, $\kappa_2^X =$0.0124, $\kappa_4^X =$0.0004.

A. Bazavov *et al.* [HotQCD], Phys. Lett. B **795** (2019), 15, doi:10.1016/j.physletb.2019.05.013

mott lines

$$E_{A,\nu}(P) - E^0_{A,\nu}(P) = \Delta E^{\mathrm{SE}}_{A,\nu}(P) + \Delta E^{\mathrm{Pauli}}_{A,\nu}(P) + \Delta E^{\mathrm{Coulomb}}_{A,\nu}(P) \tag{6}$$

$$B^{\mathrm{bind}}_{A,\nu}(P; T, n_B, Y_p, T_{\mathrm{eff}}) = -[E_{A,\nu}(P; T, n_B, Y_p, T_{\mathrm{eff}}) - E^{\mathrm{cont}}_{A,\nu}(P; T, n_B, Y_p)] \tag{7}$$

$$E^{\mathrm{cont}}_{A,\nu}(P; T, n_B, Y_p) = NE_n(P/A; T, n_B, Y_p) + ZE_p(P/A; T, n_B, Y_p) \tag{8}$$

G. Röpke, Phys. Rev. C **92** (2015) no.5, 054001 doi:10.1103/PhysRevC.92.054001
G. Röpke *et al.*, Phys. Part. Nucl. Lett. **15** (2018) no.3, 225 doi:10.1134/S1547477118030159

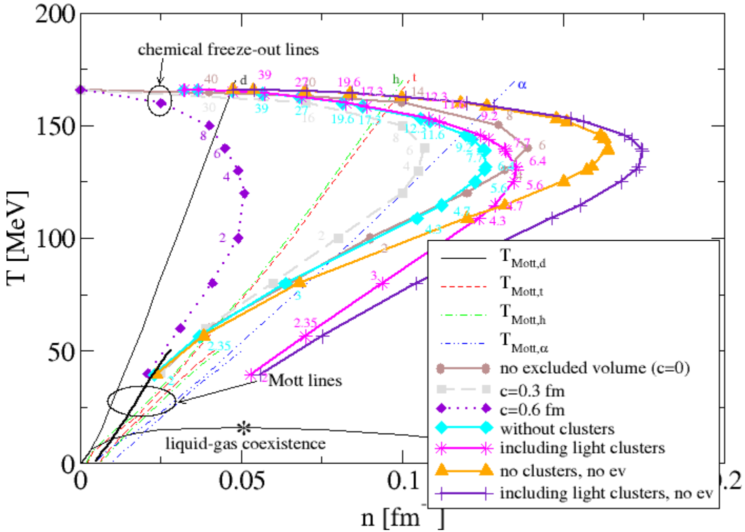
Pauli shift

$$\Delta E_\nu^{\mathrm{Pauli}}(P;T,n_B,Y_p) = c_\nu(P;T)\left\{1-\exp\left[-\frac{f_\nu(P;T,n_B)}{c_\nu(P;T)}y_\nu(Y_p)\,n_B - d_\nu(P;T,n_B)\,n_B^2\right]\right\} \qquad (9)$$

$$\begin{aligned} f_\nu(P;T,n_B) =& f_{\nu,1}\exp\left[-\frac{P^2/\hbar^2}{4(f_{\nu,4}^2/f_{\nu,3}^2)(1+T/f_{\nu,2})+u_\nu n_B}\right]\frac{1}{T^{1/2}}\frac{2f_{\nu,4}}{P/\hbar} \\ & \times \mathrm{Im}\left\{\exp\left[f_{\nu,3}^2(1+f_{\nu,2}/T)\left(1-i\frac{P/\hbar}{2f_{\nu,4}(1+T/f_{\nu,2})}\right)^2\right]\right. \\ & \left.\times \mathrm{erfc}\left[f_{\nu,3}(1+f_{\nu,2}/T)^{1/2}\left(1-i\frac{P/\hbar}{2f_{\nu,4}(1+T/f_{\nu,2})}\right)\right]\right\} \end{aligned}$$

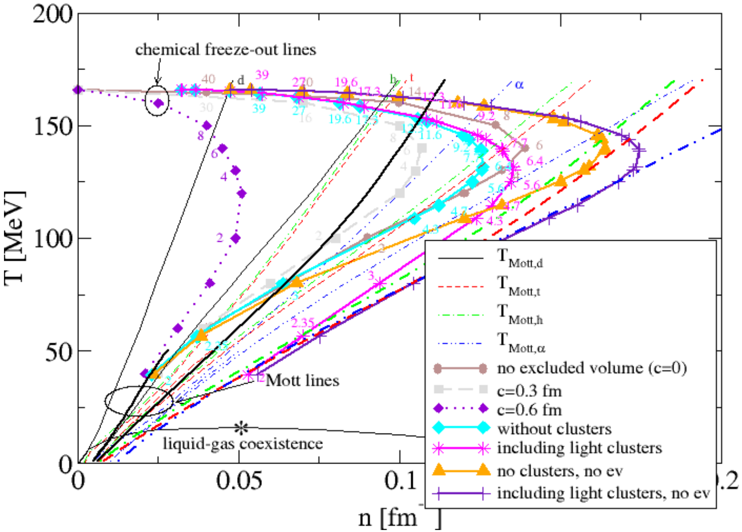
G. Röpke, Phys. Rev. C **92** (2015) no.5, 054001 doi:10.1103/PhysRevC.92.054001
G. Röpke *et al.*, Phys. Part. Nucl. Lett. **15** (2018) no.3, 225 doi:10.1134/S1547477118030159

Chemical freezeout



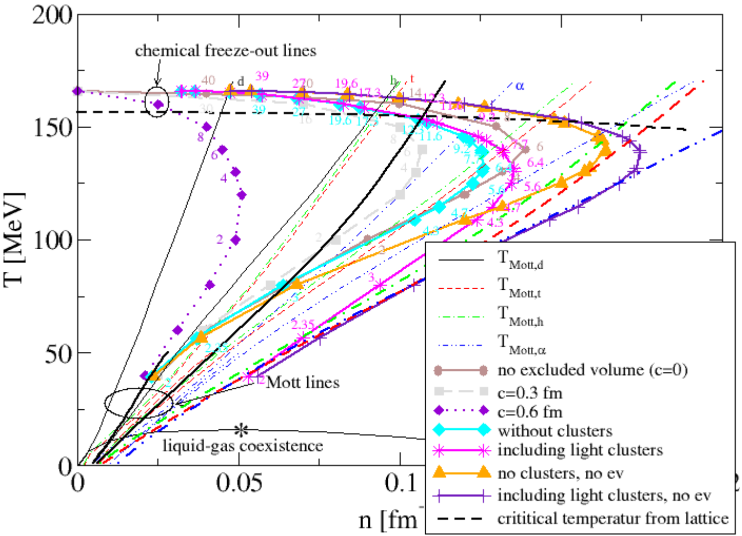
M. Albright, J. Kapusta and C. Young, Phys. Rev. C **90** (2014) no.2, 024915, doi:10.1103/PhysRevC.90.024915

Chemical freezeout



M. Albright, J. Kapusta and C. Young, Phys. Rev. C **90** (2014) no.2, 024915, doi:10.1103/PhysRevC.90.024915

Chemical freezeout



M. Albright, J. Kapusta and C. Young, Phys. Rev. C **90** (2014) no.2, 024915, doi:10.1103/PhysRevC.90.024915

Outlock

- recheck everything in the T-μ-plane
- Refit Cleymans using the lower Tc
- Fit a theoretical freeze-out line using the asymptotes
- compare to experimental data for clusters

Conclusions

- critical temperature can be a limiting line for the freeze-out at high temperature.
- at high densities formation of clusters has to be considered
- mott lines of light clusters can be a limiting line for freeze-out at low temperature.

Thank you for attention

Albright 2014

Matching excluded-volume hadron-resonance gas models and perturbative QCD to lattice calculations

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(Received 1 May 2014; published 27 August 2014)

We match three hadronic equations of state at low energy densities to a perturbatively computed equation of state of quarks and gluons at high energy densities. One of them includes all known hadrons treated as point particles, which approximates attractive interactions among hadrons. The other two include, in addition, repulsive interactions in the form of excluded volumes occupied by the hadrons. A switching function is employed to make the crossover transition from one phase to another without introducing a thermodynamic phase transition. A χ^2 fit to accurate lattice calculations with temperature $100 < T < 1000$ MeV determines the parameters. These parameters quantify the behavior of the QCD running gauge coupling and the hard core radius of protons and neutrons, which turns out to be 0.62 ± 0.04 fm. The most physically reasonable models include the excluded-volume effect. Not only do they include the effects of attractive and repulsive interactions among hadrons, but they also achieve better agreement with lattice QCD calculations of the equation of state. The equations of state constructed in this paper do not result in a phase transition, at least not for the temperatures and baryon chemical potentials investigated. It remains to be seen how well these equations of state will represent experimental data on high-energy heavy-ion collisions when implemented in hydrodynamic simulations.

DOI: 10.1103/PhysRevC.90.024915 PACS number(s): 25.75.−q, 25.70.−z, 21.65.Mn, 12.38.Gc

Switch function

$$P(T,\mu) = S(T,\mu)P_{qg}(T,\mu) + [1 - S(T,\mu)]P_h(T,\mu) \quad (10)$$

$$S(T,\mu) = exp[-\Theta(T,\mu)] \quad (11)$$

$$\Theta(T,\mu) = \left[\left(\frac{T}{T_0}\right)^r + \left(\frac{\mu}{\mu_0}\right)^r\right]^{-1} \quad (12)$$

This switching function contains two 3 parameters: r, T_0 and μ_0.

$$\mu_0 = 3\pi T_0 \quad (13)$$

That means in the end 2 free parameters.

Hadronic part

- mainly based on PDB 2014

$$P_\alpha(T,\mu) = (2s_\alpha + 1)\int \frac{d^3p}{(2\pi)^3}\frac{1}{\mathrm{e}^{\beta(E_\alpha(p)-n_b\mu_\alpha)} \pm 1} \tag{14}$$

excluded Volume

$$P_{ex}(T,\mu) = \frac{P_{PT}(T_*,\mu_*)}{1 - P_{PT}(T_*,\mu_*)/\epsilon_0} \quad (15)$$

Thereby T_* and μ_* are not independent but related by

$$\frac{T}{T_*} = \frac{\mu}{\mu_*}\,. \quad (16)$$

Using the equations 15 and 16 it remains one equation with one variable to determine the T_* and μ_* for a given pair of T and μ.

$$T = \frac{T_*}{1 - P_{PT}(T_*,\mu_*)/\epsilon_0} \quad (17)$$

Perturbative QCD

The pressure of QCD at finite temperatures and chemical potentials

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We compute the perturbative expansion of the pressure of hot QCD to order $g^6 \ln g$ in the presence of finite quark chemical potentials. In this process we evaluate all two- and three-loop vacuum diagrams of the theory at arbitrary T and μ and then use these results to analytically verify the outcome of an old order g^4 calculation of Freedman and McLerran for the zero-temperature pressure. The results for the pressure and the different quark number susceptibilities at high T are compared with recent lattice simulations showing excellent agreement especially for the chemical potential dependent part of the pressure.

Perturbative QCD

$$P = \frac{8\pi^2}{45} T^4 \left[f_0 + f_2 \left(\frac{\alpha_s}{\pi}\right) + f_3 \left(\frac{\alpha_s}{\pi}\right)^{\frac{3}{2}} + f_4 \left(\frac{\alpha_s}{\pi}\right)^2 + f_5 \left(\frac{\alpha_s}{\pi}\right)^{\frac{5}{2}} + f_6 \left(\frac{\alpha_s}{\pi}\right)^3 \right] \quad (18)$$

$$E_\tau(p;T,n_B,Y_p) = \sqrt{\left[m_\tau c^2 - S(T,n_B,Y_p)\right]^2 + \hbar^2 c^2 p^2} + V_\tau(T,n_B,Y_p) - m_\tau c^2 \tag{19}$$

$$\begin{aligned} E_{A,\nu}^{\mathrm{intr.}}(P;T,n_B,Y_p,T_{\mathrm{eff}}) &= E_{A,\nu}(P;T,n_B,Y_p,T_{\mathrm{eff}}) - E_{A,\nu}^{\mathrm{cont}}(P;T,n_B,Y_p) \\ &= E_{A,\nu}^0 + \Delta E_{A,\nu}^{\mathrm{SE,intr.}}(P;T,n_B,Y_p) + \Delta E_c^{\mathrm{Pauli}}(P;T_{\mathrm{eff}},n_B,Y_p)\,. \end{aligned} \tag{20}$$

scalar part

$$S(T, n_B, \delta) = \frac{s_1(T,\delta)\ n_B + s_2(T,\delta)\ n_B^2 + s_3(T,\delta)\ n_B^3}{1 + s_4(T,\delta)\ n_B + s_5(T,\delta)\ n_B^2} \tag{21}$$

with coefficients

$$\begin{aligned} s_i(T,\delta) &= s_{i,0}(\delta) + s_{i,1}(\delta)\ T + s_{i,2}(\delta)\ T^2, \\ s_{i,j}(\delta) &= s_{i,j,0} + s_{i,j,2}\ \delta^2 + s_{i,j,4}\ \delta^4; \end{aligned} \tag{22}$$

vector part

$$V_p(T, n_B, \delta) = \frac{v_1(T,\delta)\ n_B + v_2(T,\delta)\ n_B^2 + v_3(T,\delta)\ n_B^3}{1 + v_4(T,\delta)\ n_B + v_5(T,\delta)\ n_B^2} \tag{23}$$

with coefficients

$$\begin{aligned} v_i(T,\delta) &= v_{i,0}(\delta) + v_{i,1}(\delta)\ T + v_{i,2}(\delta)\ T^2, \\ v_{i,j,k}(\delta) &= v_{i,j,0} + v_{i,j,1}\ \delta + v_{i,j,2}\ \delta^2 + v_{i,j,3}\ \delta^3 + v_{i,j,4}\ \delta^4 \end{aligned} \tag{24}$$

$$c_\nu(P;T) = c_\nu(0;T) = c_{\nu,0} + \frac{c_{\nu,1}}{(T - c_{\nu,2})^2 + c_{\nu,3}} \tag{25}$$

$$d_\nu(P;T,n_B) = d_\nu(0;T,n_B)\,\exp\left[-\frac{P^2/\hbar^2}{w_\nu T n_B}\right], \tag{26}$$

$$d_\nu(0;T,n_B) = \frac{d_{\nu,1}}{(T - d_{\nu,2})^2 + d_{\nu,3}}, \tag{27}$$