

THERMODYNAMICS OF COUPLED CHANNEL SYSTEM

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25 SEP 2022

XV POLISH WORKSHOP ON RELATIVISTIC
HEAVY ION COLLISIONS

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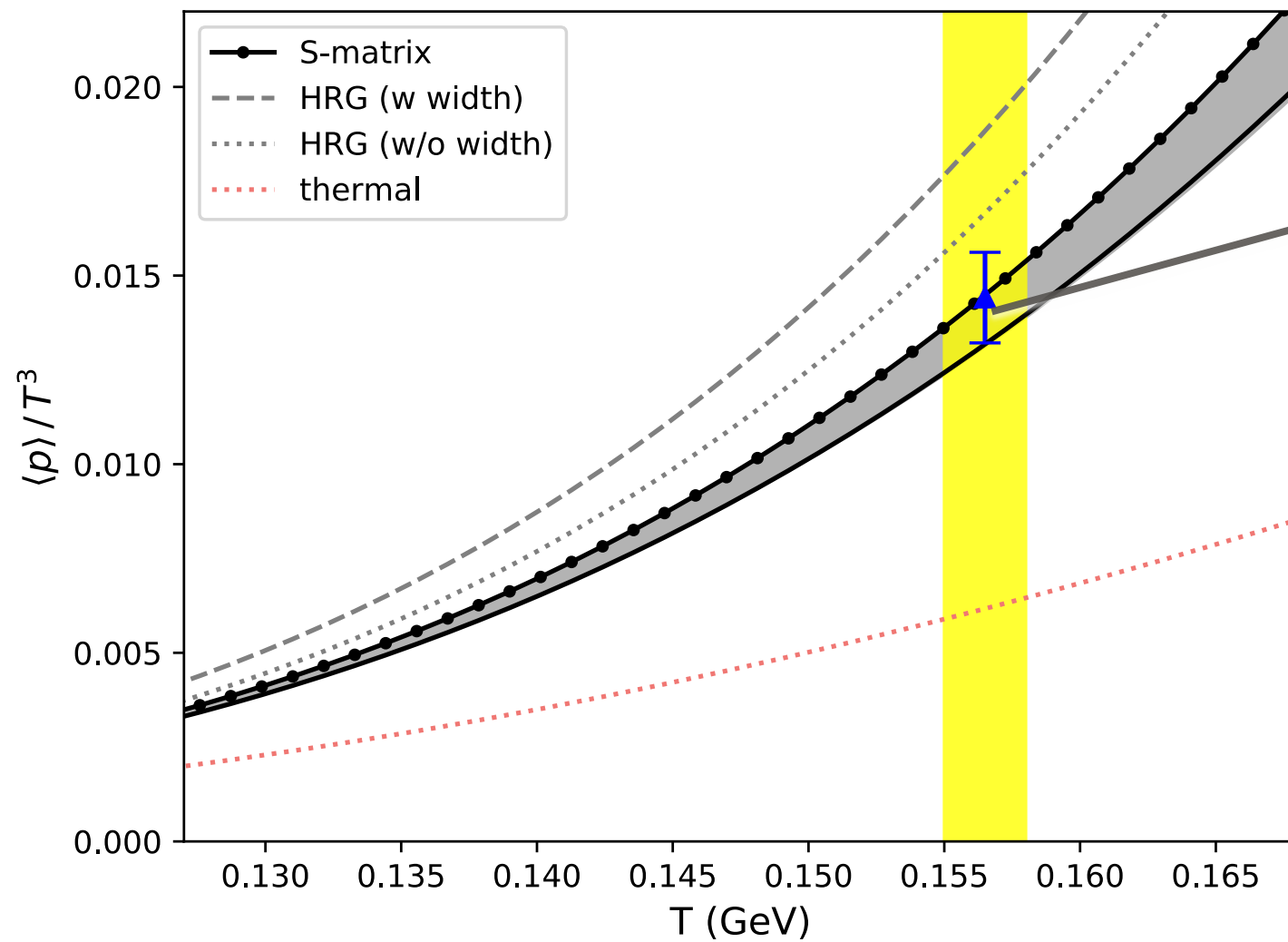
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CONCLUSIONS

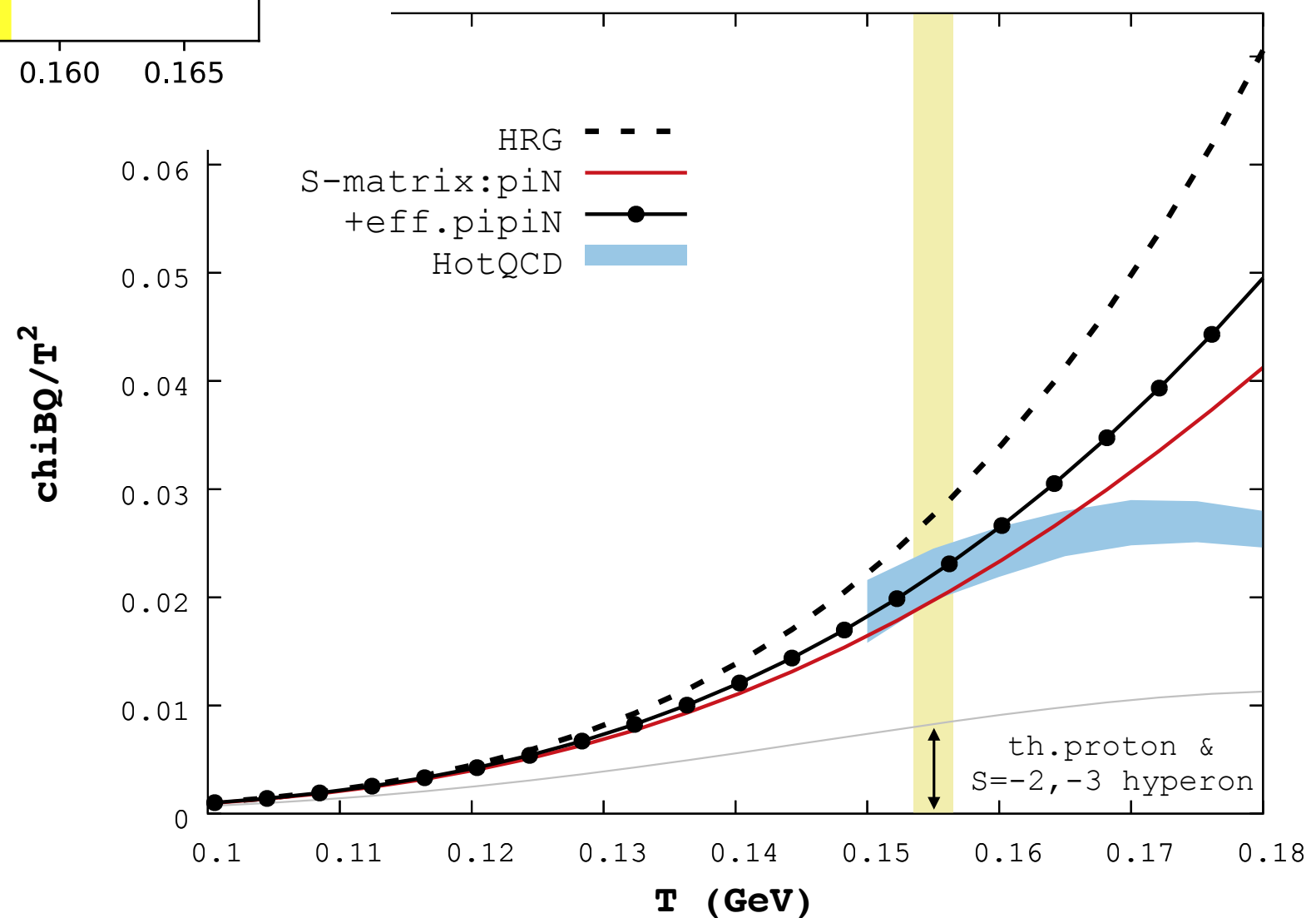


ALICE proton yield
Pb-Pb @ 2.76 TeV
thermal model est.

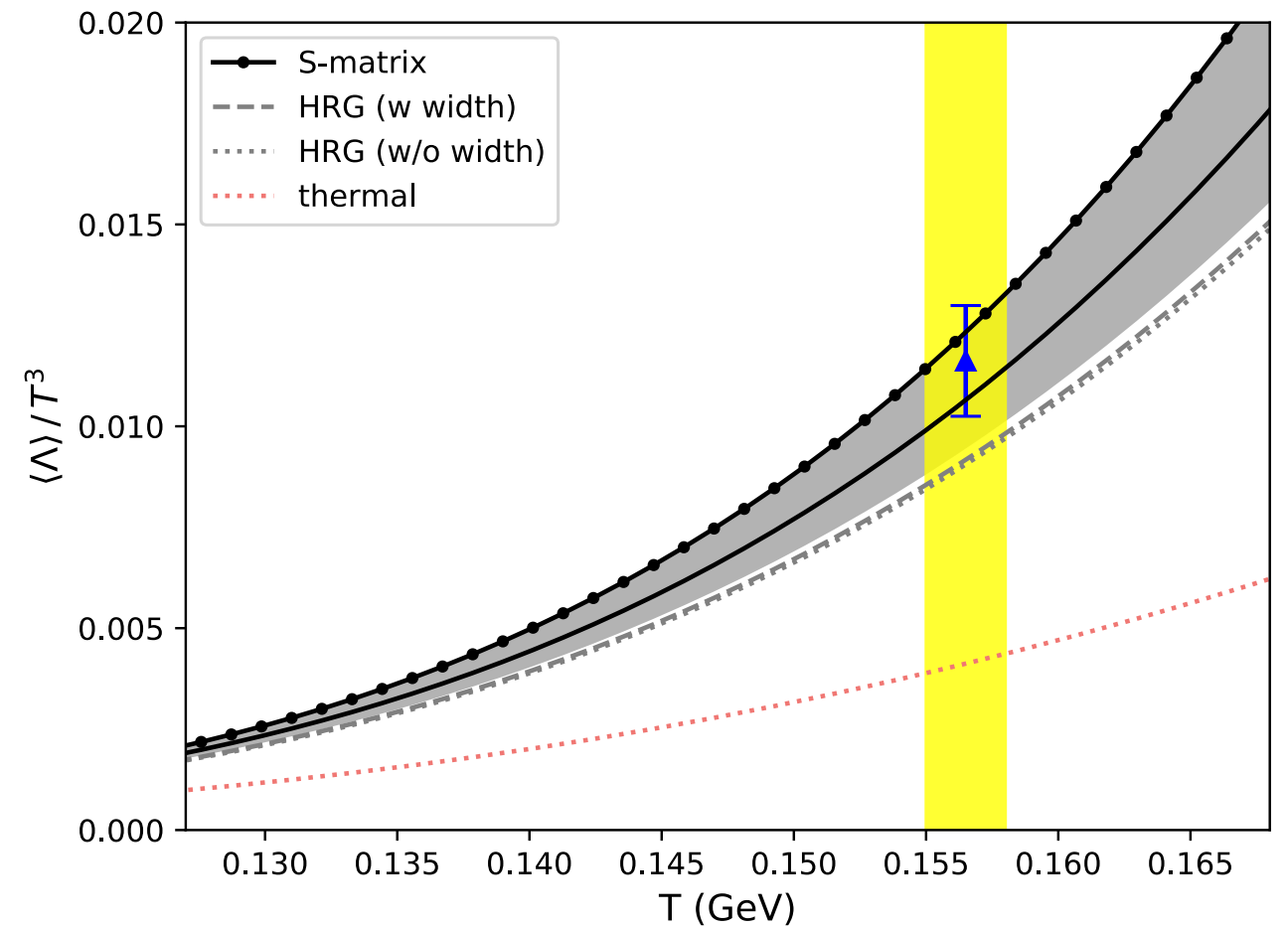
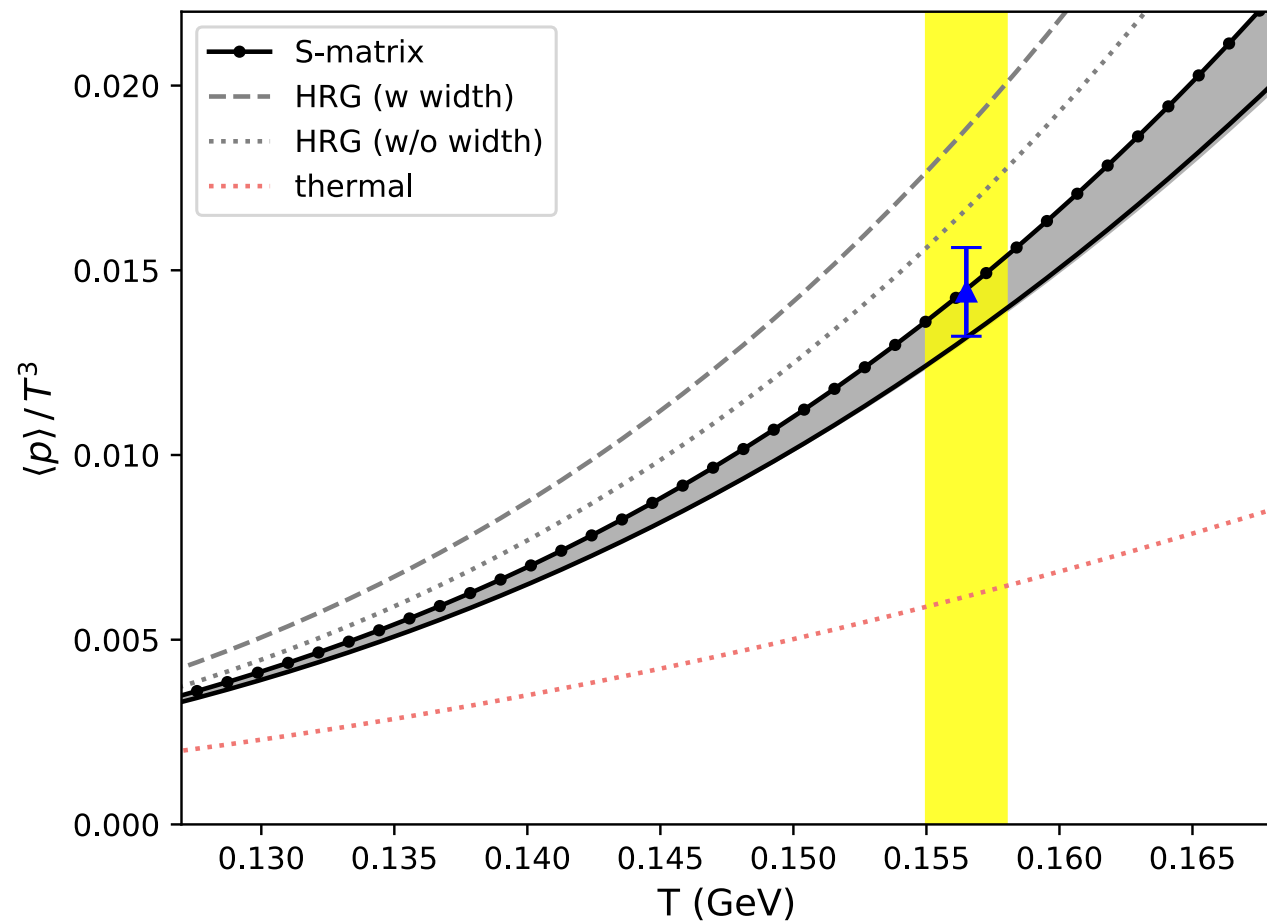
LQCD result on χ BQ

A. Bazavov, et al.,
 Phys. Rev. D 86 (2012) 034509.

see also
 Bellwied et al.
 Phys. Rev. D 101, 034506 (2020)



S-matrix VS HRG

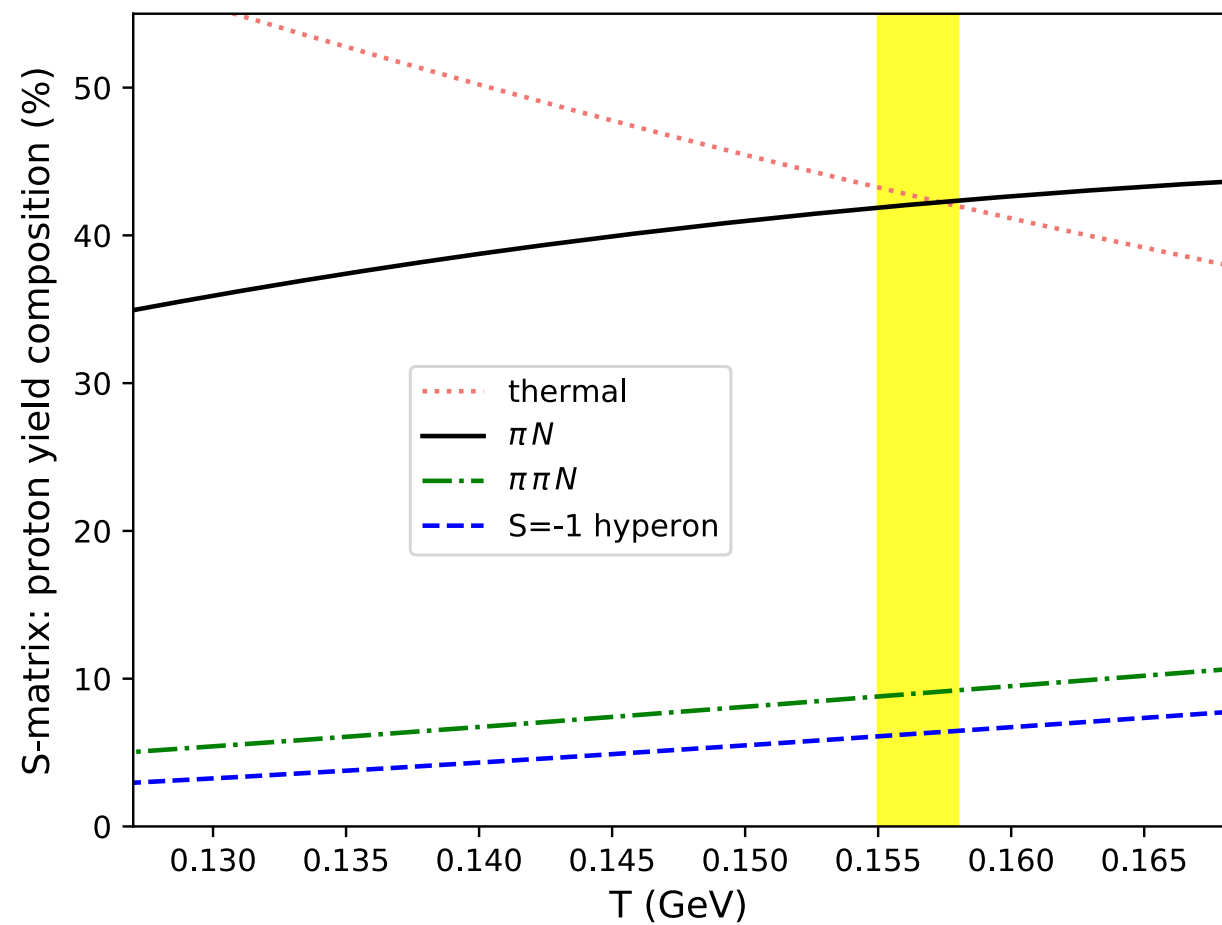


πN phase shifts
 $\pi \pi N$ BGs
 hyperons

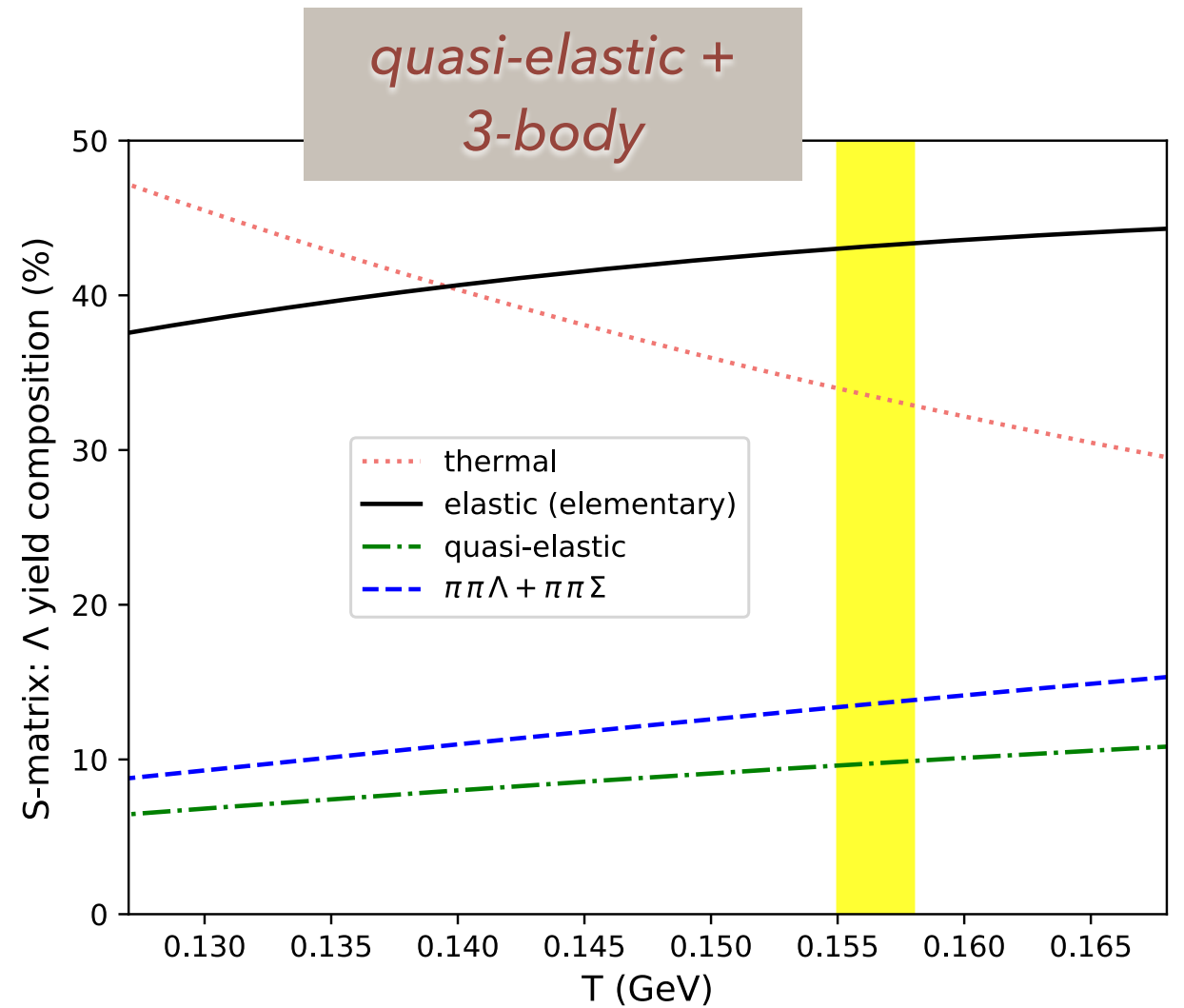
Coupled-Channel model:

$\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$
*extra hyperon states
 beyond PDG
 unitarity BGs*

consistent treatment of res and non-res. int.

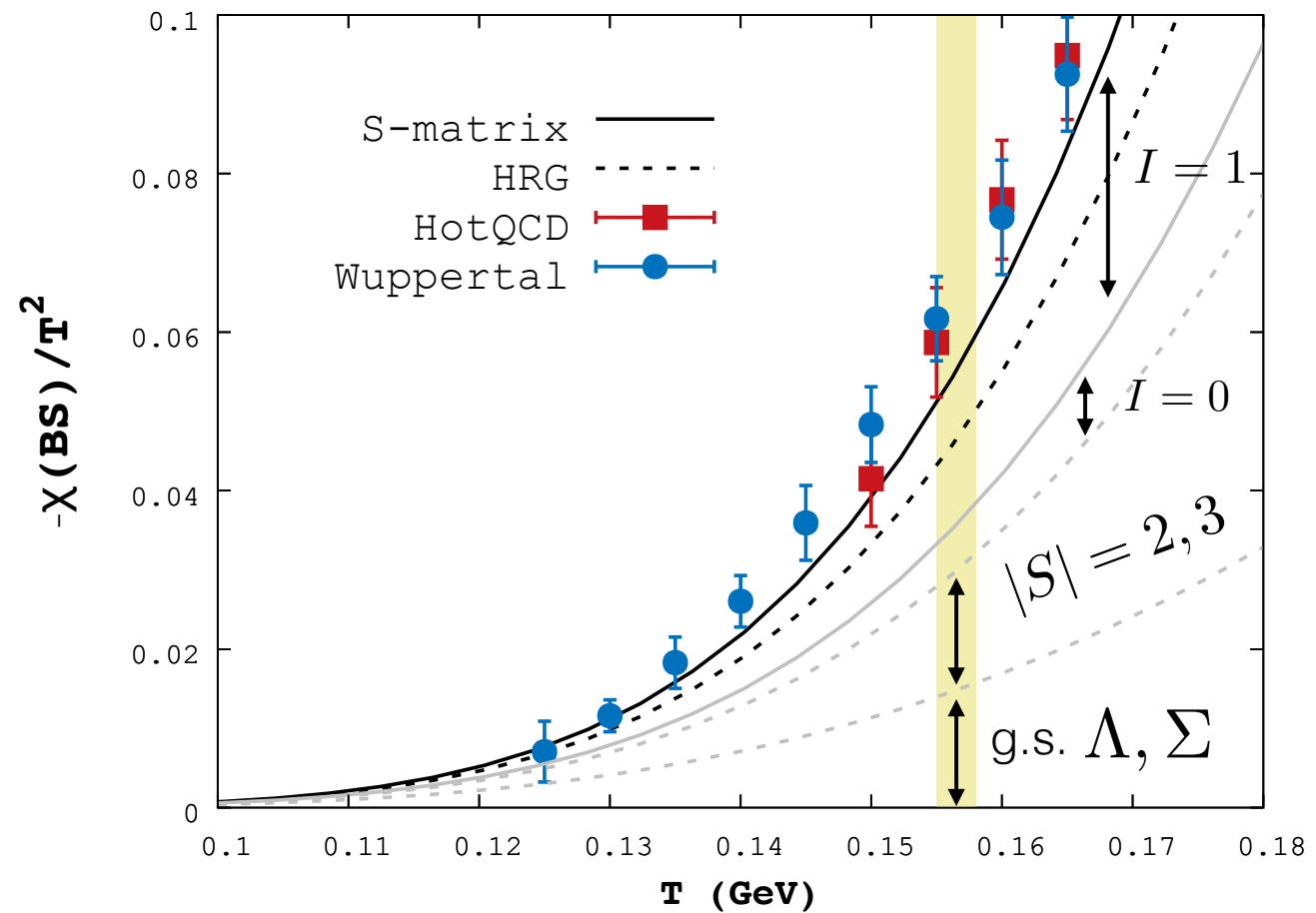
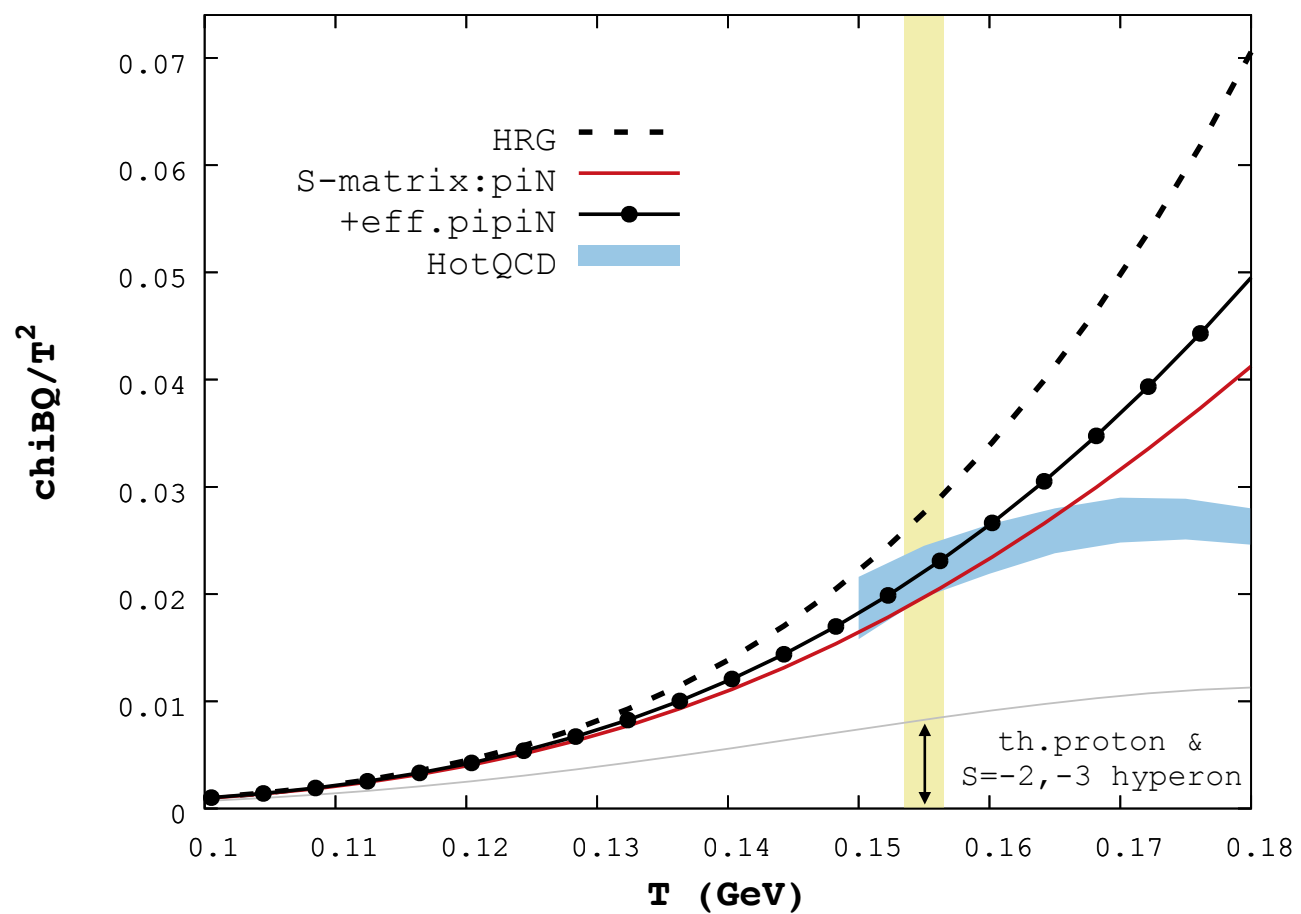
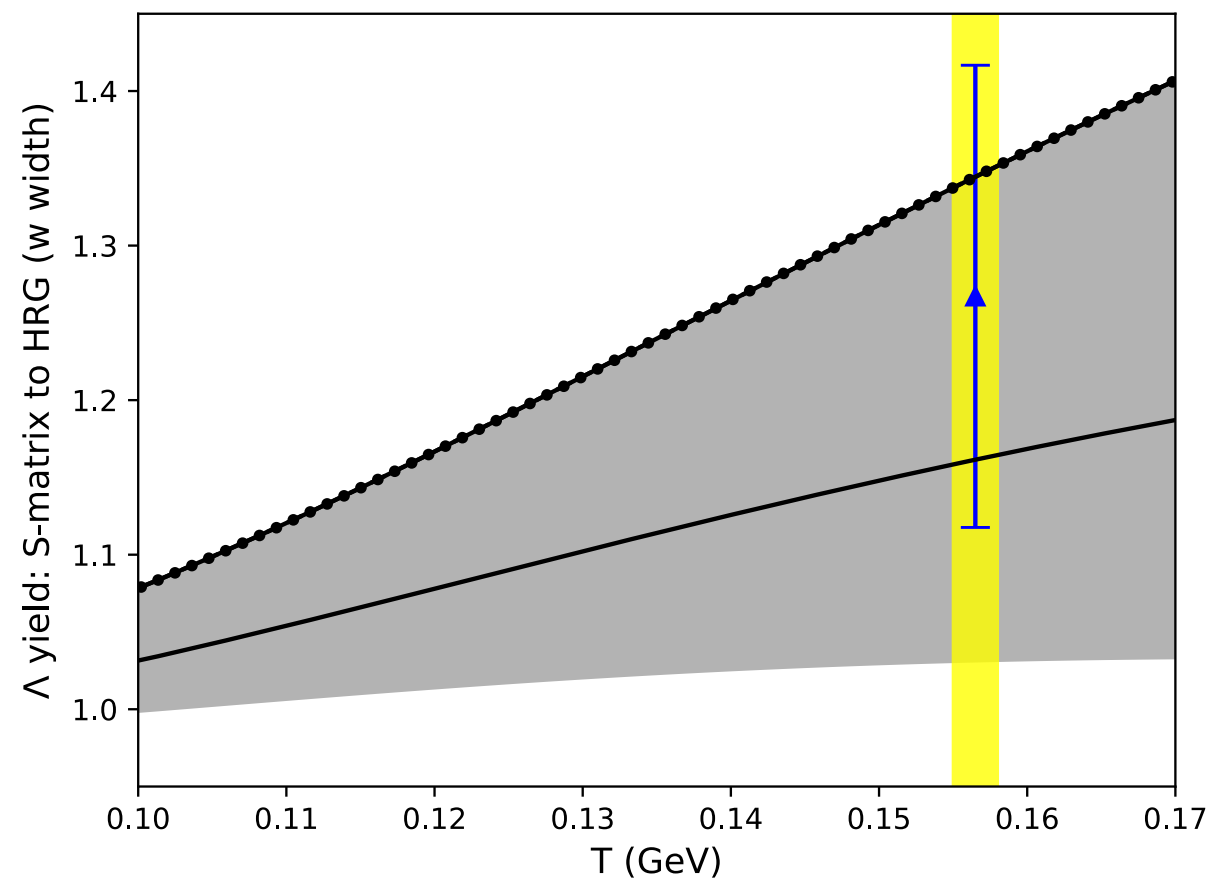
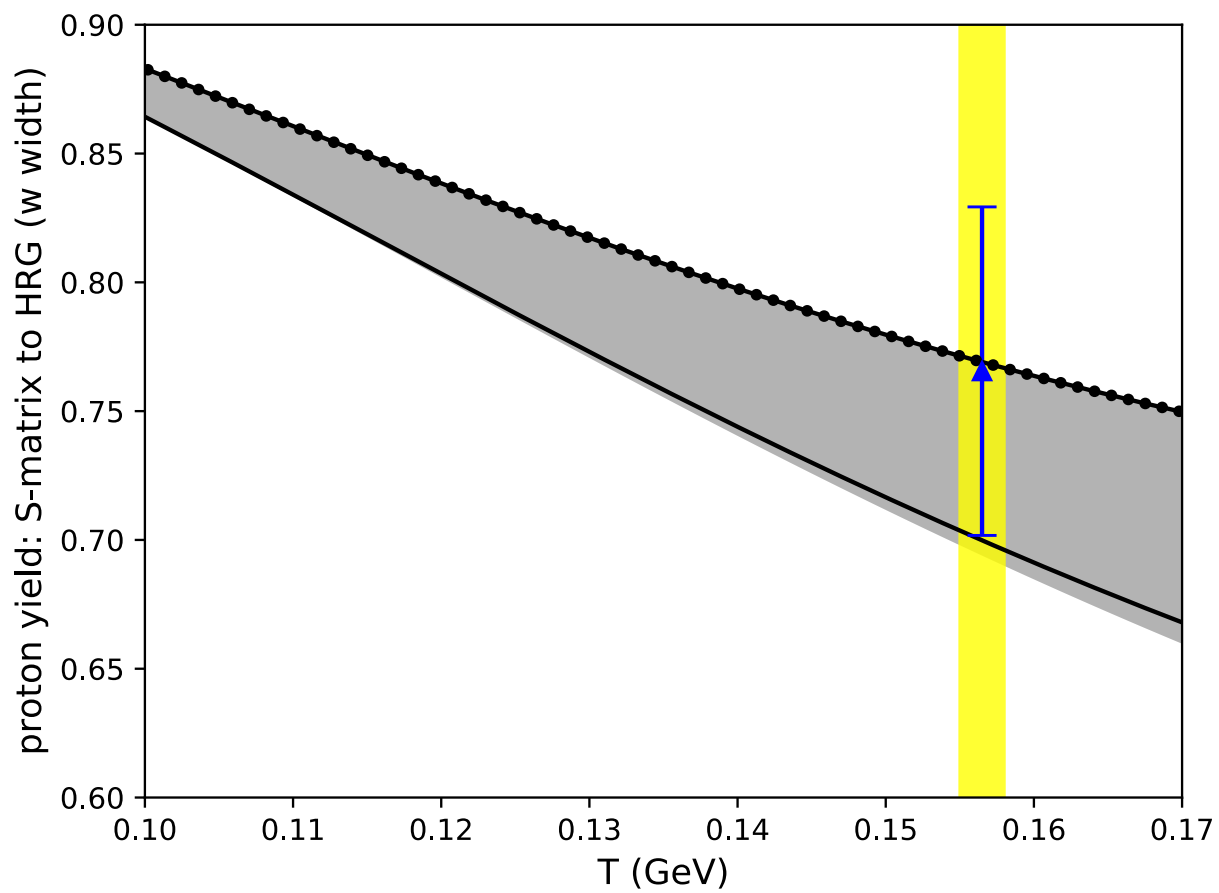


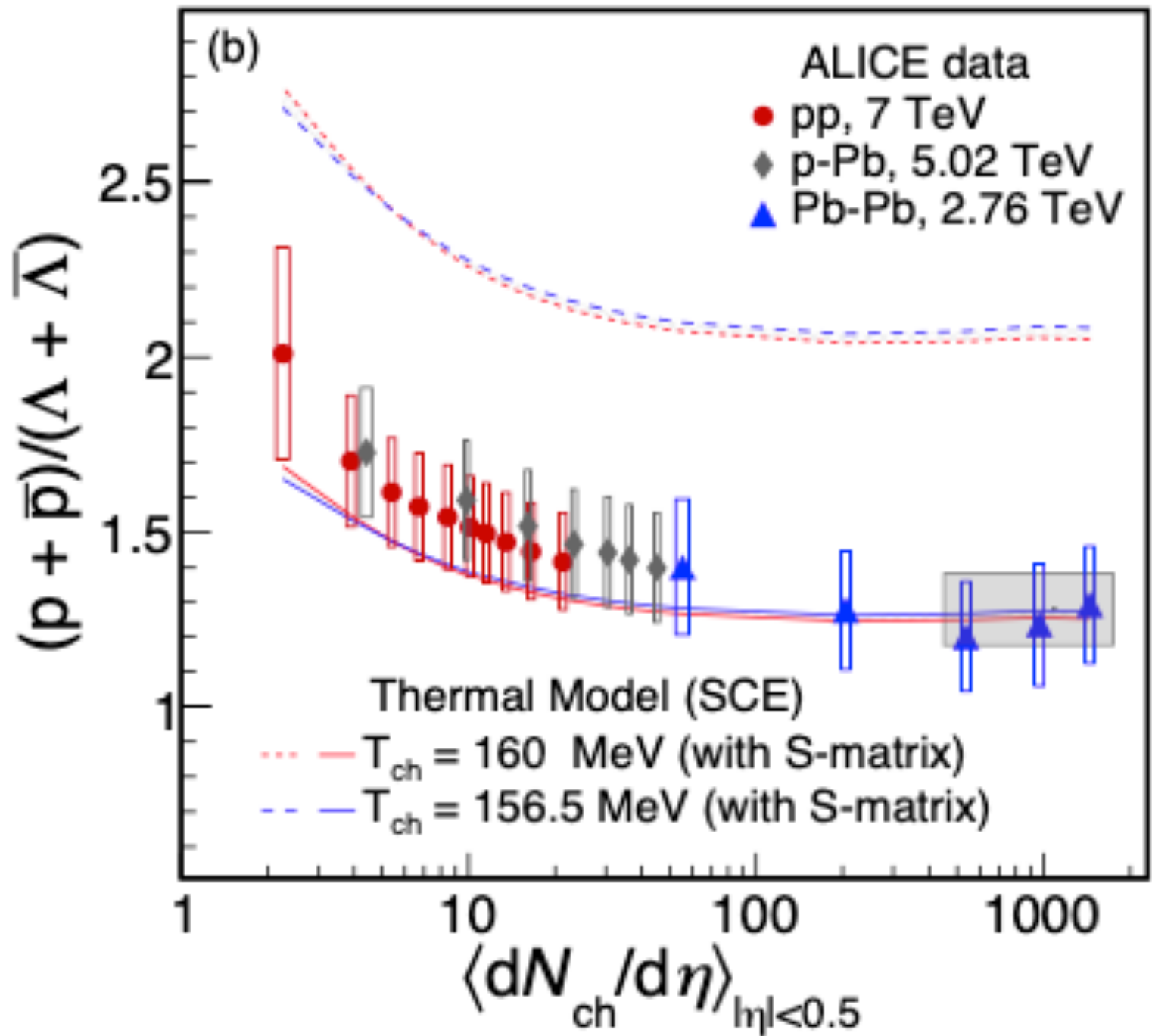
πN phase shifts
 $\pi\pi N$ BGs
 hyperons



Coupled-Channel system:
 $\bar{k}N, \pi\Lambda, \pi\Sigma, \dots$
extra hyperon states
beyond PDG
unitarity BGs

consistent treatment of res and non-res. int.





less protons

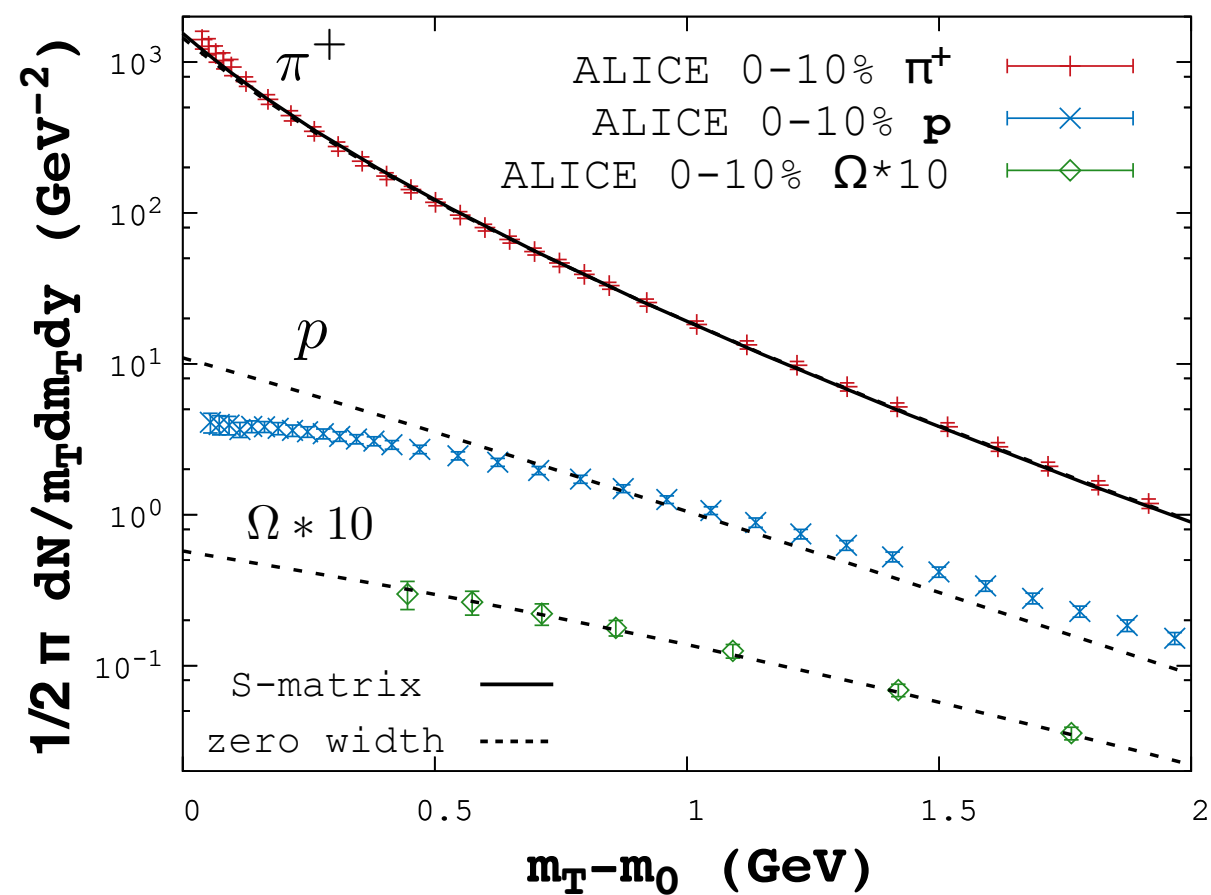
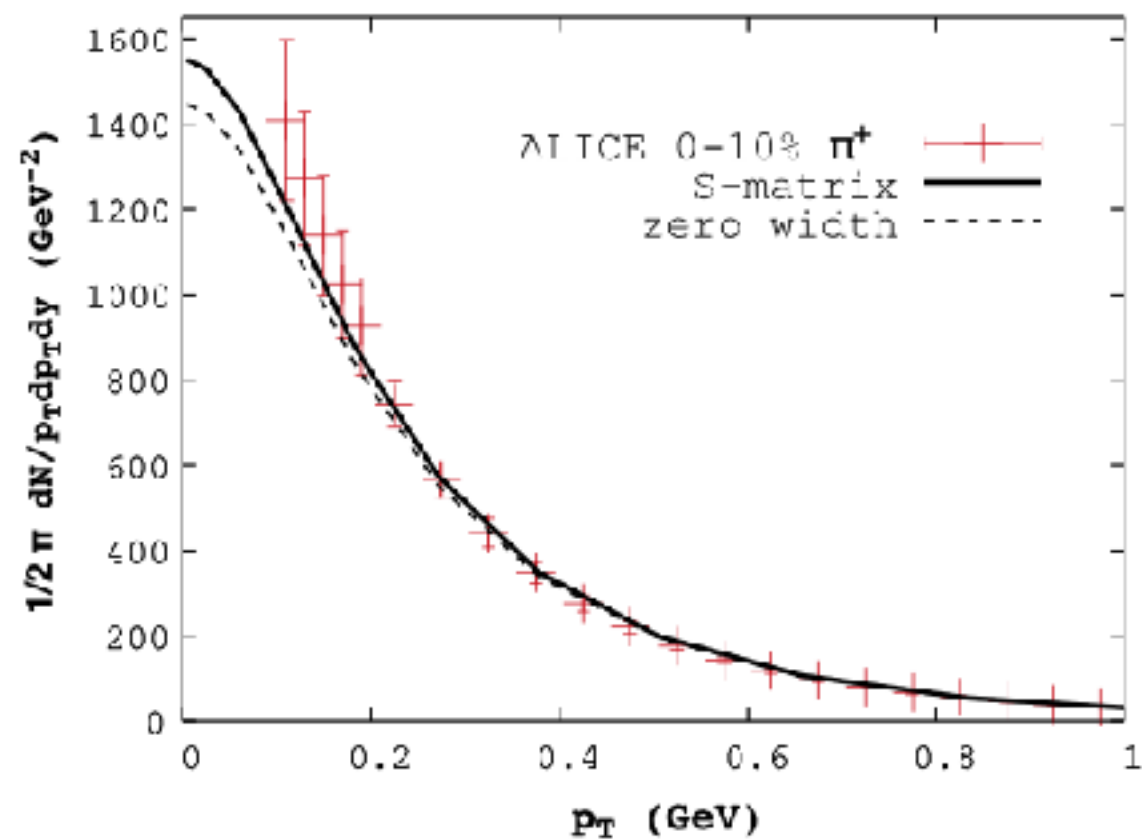
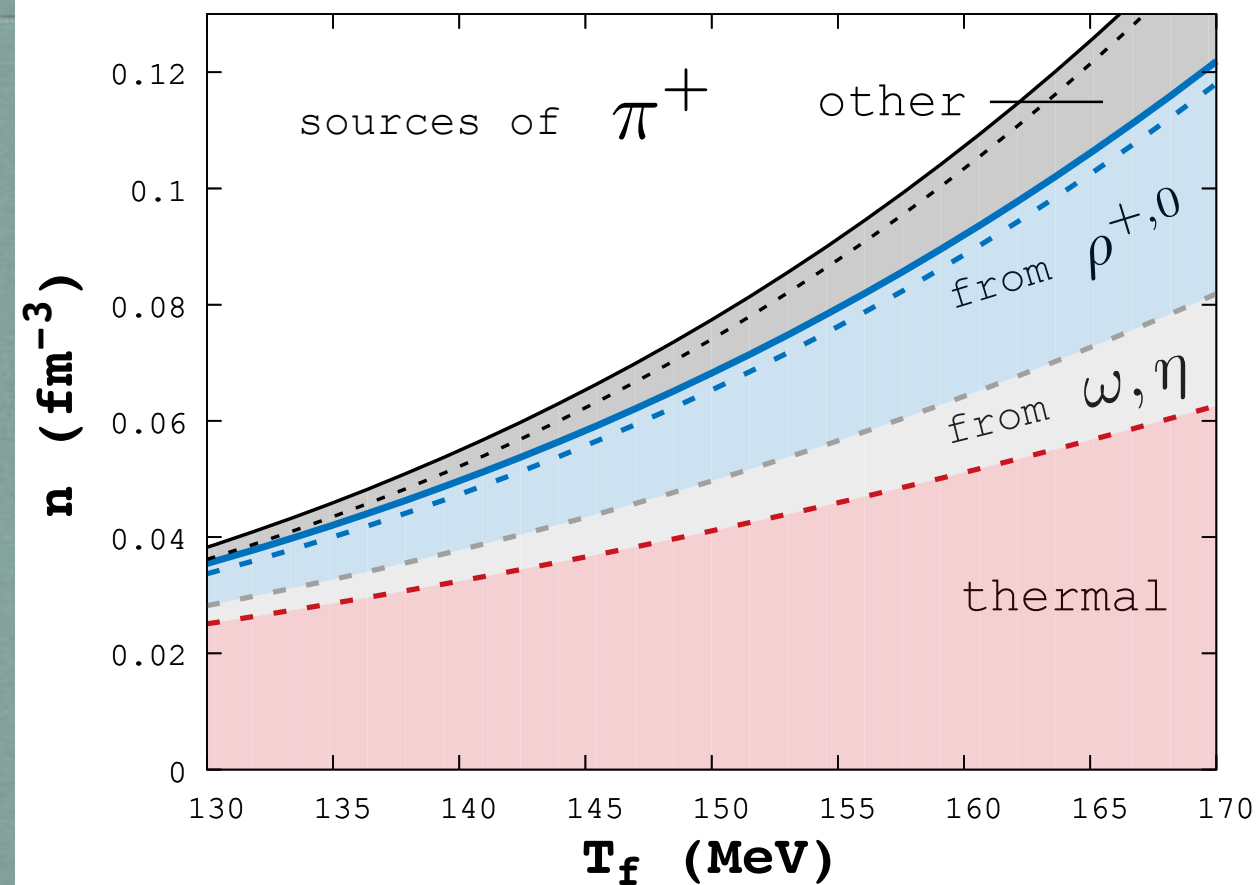
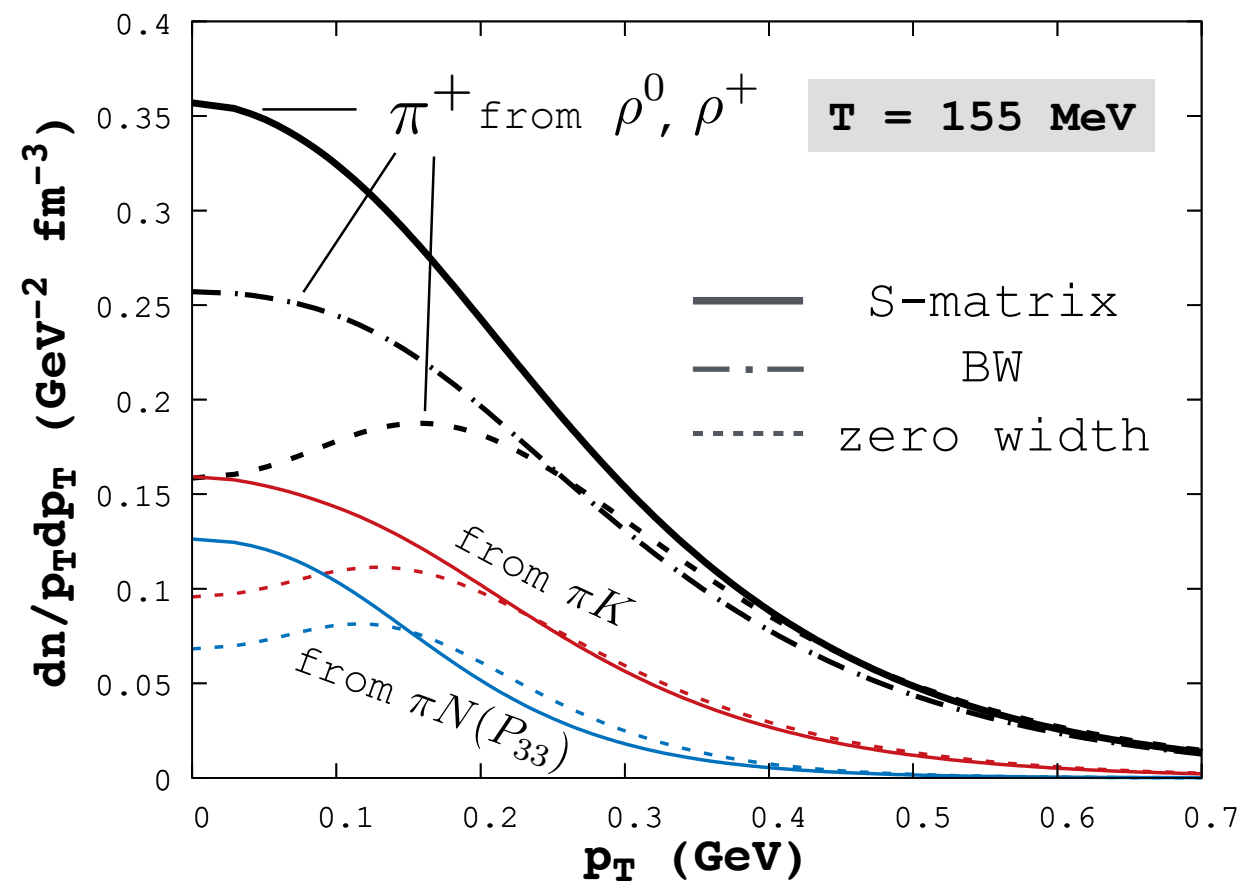
hrg

more lambdas

smat

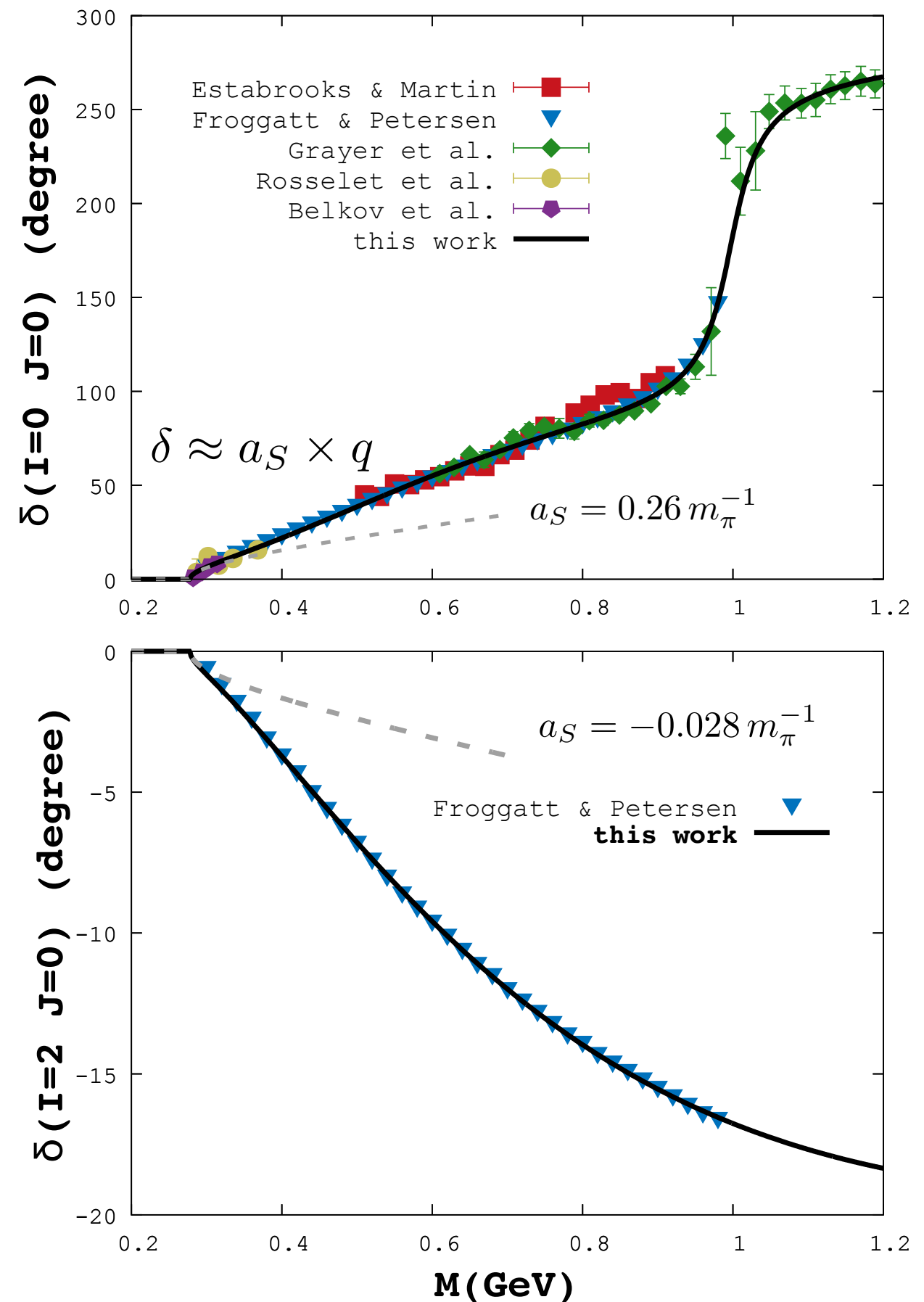
Phys. Rev. C 103, 014904 (2021).

Phys. Lett. B 792, 304 (2019).



phase shifts encode
hadronic interactions.

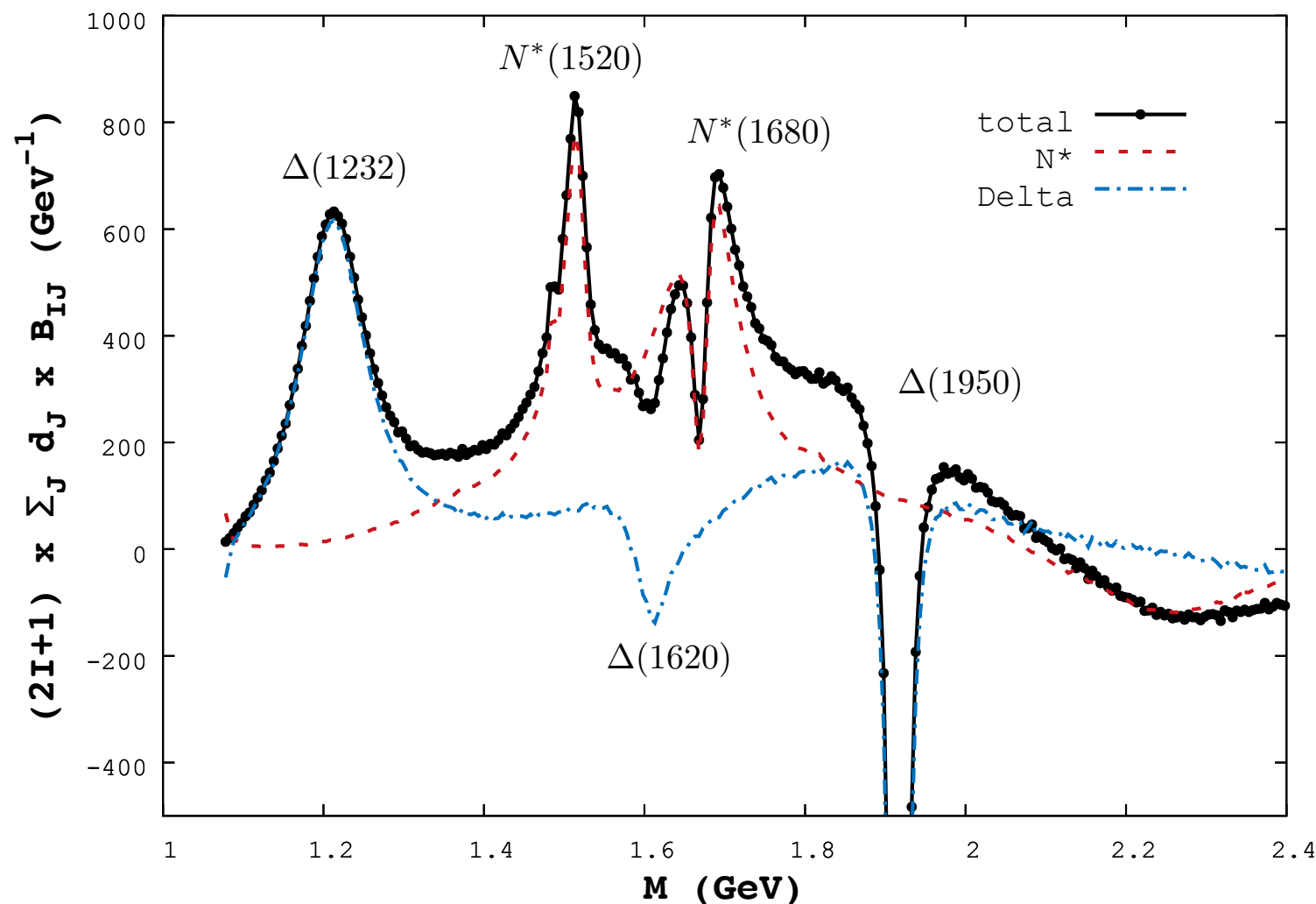
- positive:
 - attractive forces
 - resonances formation
- negative:
 - repulsive forces
 - hard-core
 - channel opening up
 - resonance not as strong



FACTS OF HADRON PHYSICS

- broad /overlapping resonances
- molecular states
- threshold effects /cusps

non-resonant interactions: +/-
 $\pi N \rightarrow \Delta \rightarrow \pi N$



BES III The neutral partner of the $Z_c(3900)$

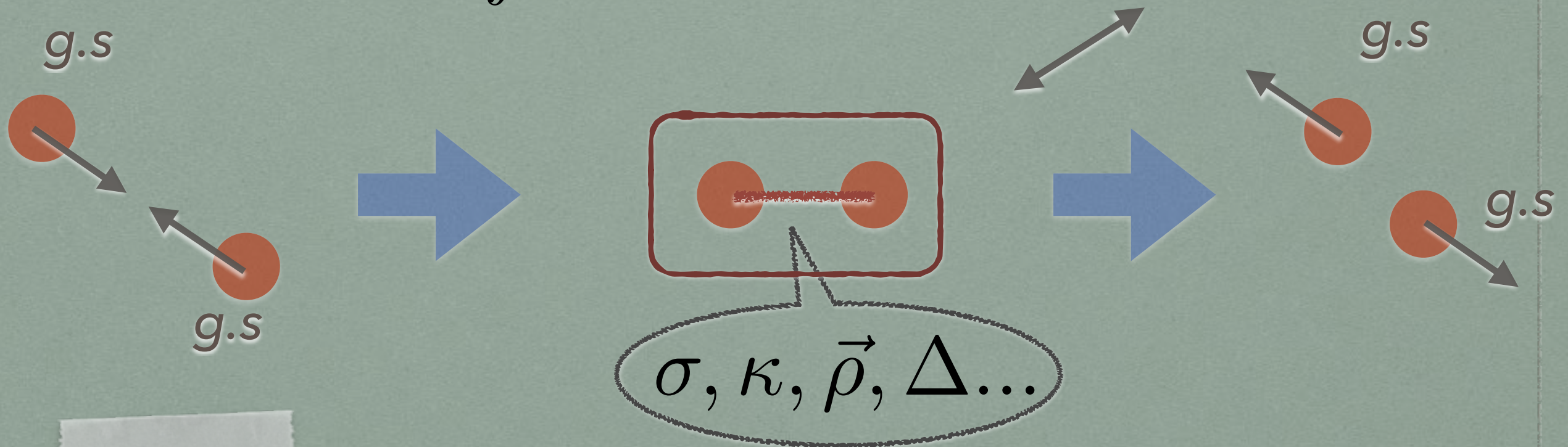
Observation of $Z_c(3900)^0 \rightarrow J/\psi \pi^0$
 in $e^+e^- \rightarrow J/\psi \pi^0 \pi^0$ GeV (2.8 fb⁻¹, 10.4σ)
 > confirms earlier evidence in CLEO-c data

"When I see a bird that walks like a duck and swims like a duck and quacks like a duck, I call that bird a duck."
 — James Whitcomb Riley
 Indiana Poet

PREPRINT: 1508.01170 [1508.01170]
 Frank Heesing | Reproduction of scientific data is permitted with BFRUPANDA, 14 | 06/29/2015

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E).$$



$\sigma, \kappa, \vec{\rho}, \Delta \dots$

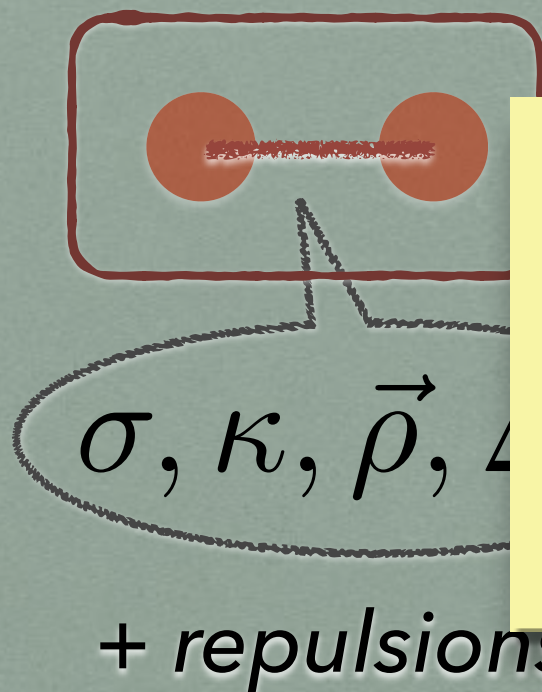
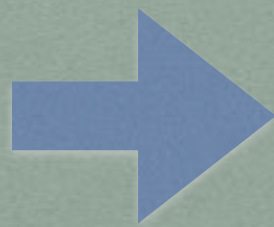
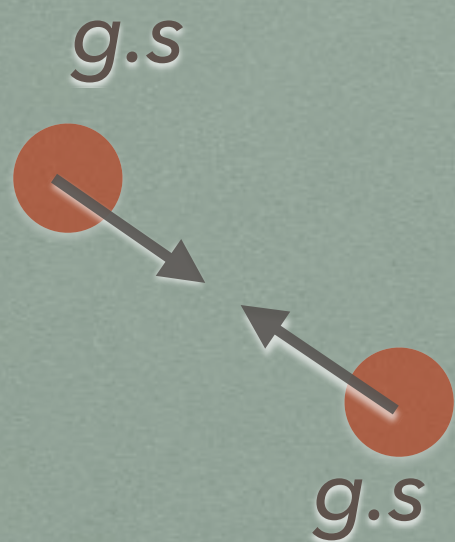
+ repulsions

$\delta \longrightarrow Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$

PWA
~~X~~
 S-matrix thermo.

S-MATRIX FORMULATION OF STATISTICAL MECHANICS

$$\Delta \ln Z = \int dE e^{-\beta E} \times \frac{1}{\pi} \frac{\partial}{\partial E} \text{tr} (\delta_E).$$



Elastic -> Inelastic
Beyond 2-body

PWA
X
S-matrix thermo.

$\delta \longrightarrow Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$

S-MATRIX FORMULATION OF THERMODYNAMICS

thermo-statistical

dynamical

$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \frac{\partial}{\partial E} S_E \right\}_c$$

$$b_{\pi\pi} \xi_\pi^2 + b_{\pi K} \xi_\pi \xi_K + b_{\pi N} \xi_\pi \xi_N + b_{\pi\eta}$$

$$b_{\pi\pi} = b_{\pi\pi}^{I=0} + 3 \times 3 \times b_{\pi\pi}^{I=1} + \dots$$

sum over eigenphases

$$Q = \frac{1}{2} \text{Im Tr} \ln S$$

$$= \sum_{\text{channels}} \lambda_i$$

EXERCISE: QM SCATTERING OPERATOR

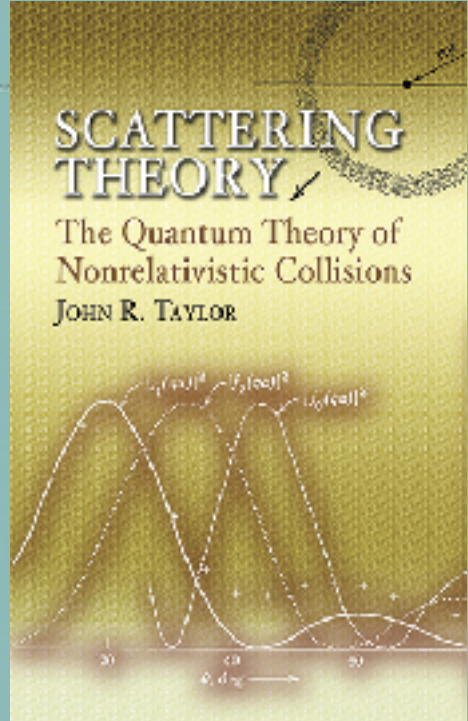
show that

$$\begin{aligned} S_E &= G_0^* G^{*-1} G G_0^{-1} \\ &= 1 - 2\pi i \times \delta(E - H_0) \times T_E \end{aligned}$$


$$G = \frac{1}{E - H + i\epsilon}$$

Verify
$$\Delta \ln Z = \int dE e^{-\beta E} \frac{1}{4\pi i} \text{tr} \left\{ S_E^{-1} \overleftrightarrow{\frac{\partial}{\partial E}} S_E \right\}_c$$

Alternative way to obtain the Beth-Uhlenbeck result!



From Hamiltonian to Scattering Matrix

$$\frac{1}{E - \mathcal{H}_0 \pm i\delta}$$


$$\begin{aligned}\tilde{S} &= (I - G_-^0 V) (I + G_+^0 T) \\ &= I - G_-^0 V + G_+^0 T - G_-^0 V G_+^0 T \\ &= I - G_-^0 V + G_+^0 V + G_+^0 V G_+^0 T - G_-^0 V G_+^0 T \\ &= I + (G_+^0 - G_-^0) V + (G_+^0 - G_-^0) V G_+^0 T \\ &= I + (G_+^0 - G_-^0) T \\ &\rightarrow I + 2i \operatorname{Im} (G_+^0) \times T. \quad \text{on-shell limit}\end{aligned}$$

**WHY IT IS NOT
A SUM OF BREIT-WIGNERS**

WHAT'S IN A NAME? THAT WHICH WE CALL A RESONANCES?

- A resonance is MORE than a MASS and a WIDTH

$f_0(500)$ [g]

$$J^G(J^{PC}) = 0^+(0^{++})$$

Mass (T-Matrix Pole \sqrt{s}) = (400–550)– i (200–350) MeV

Mass (Breit-Wigner) = (400–550) MeV

Full width (Breit-Wigner) = (400–700) MeV

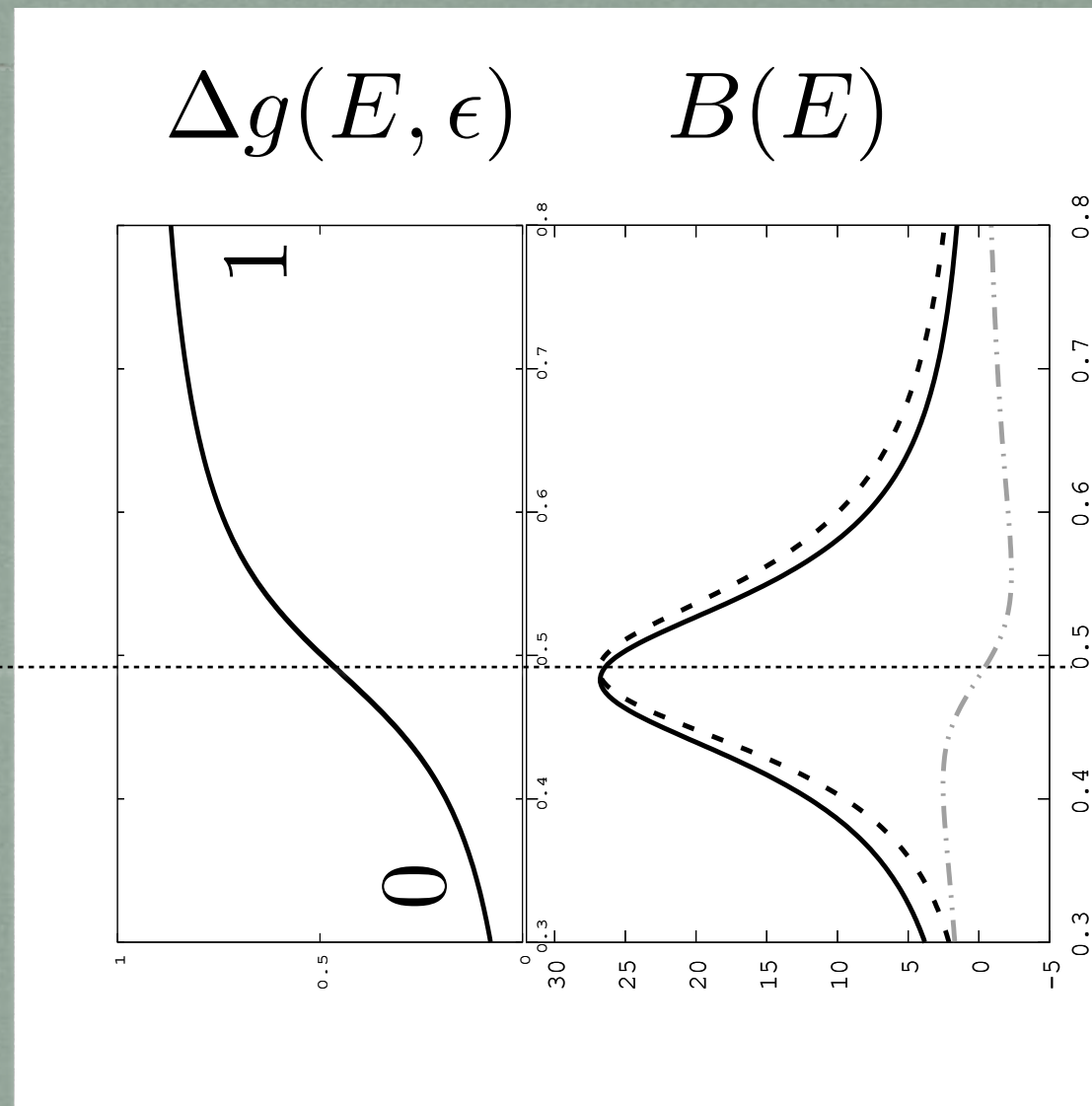
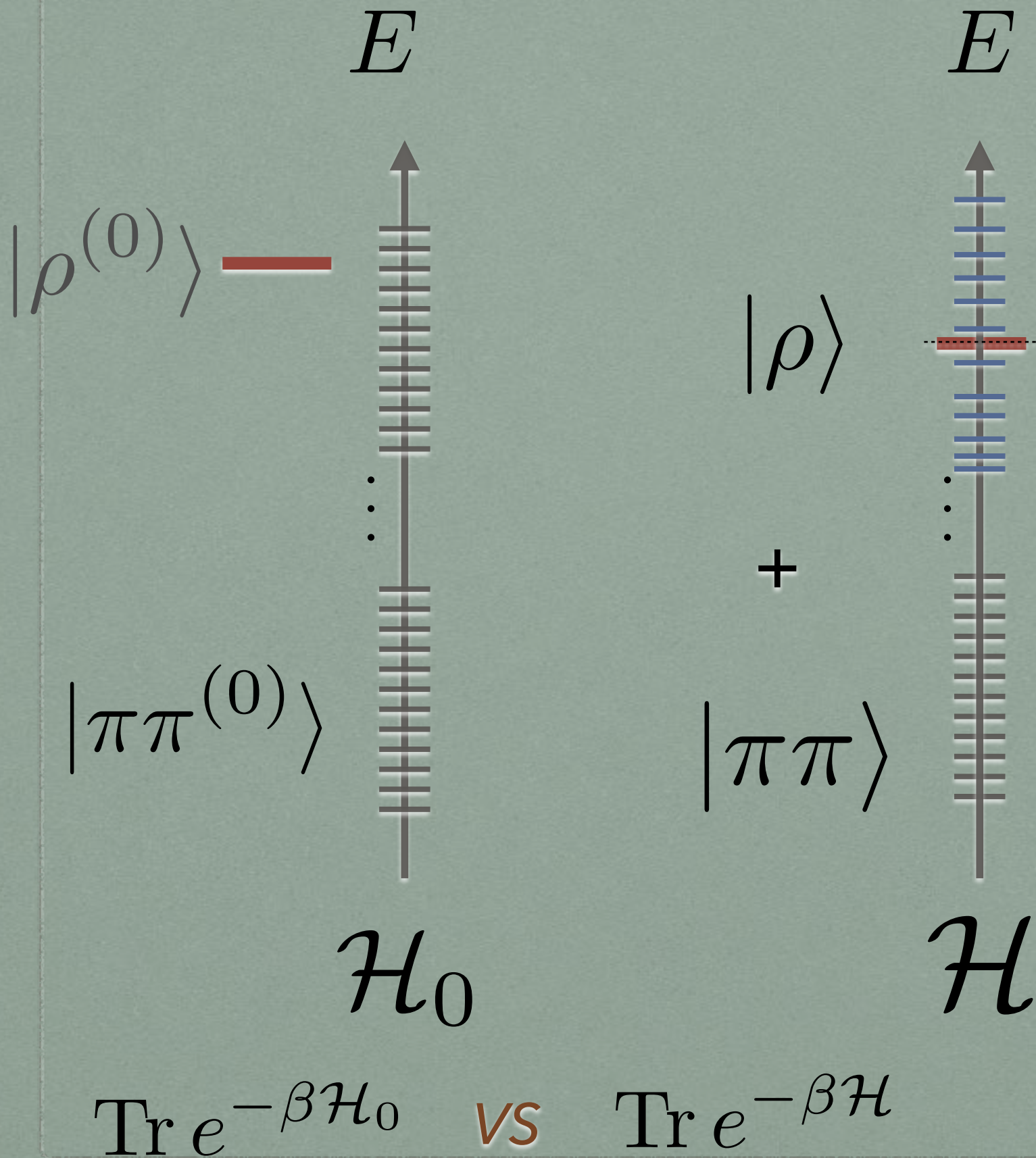
$\rho(770)$ [h]

$$J^G(J^{PC}) = 1^+(1^{--})$$

Mass $m = 775.26 \pm 0.25$ MeV

Full width $\Gamma = 149.1 \pm 0.8$ MeV

$\Gamma_{ee} = 7.04 \pm 0.06$ keV



$$g(E, \epsilon) = \sum_n \theta_\epsilon(E - E_n)$$

$$B(E) = 2\pi \frac{d}{dE} \Delta g(E, \epsilon)$$

$$= A_\rho + \Delta A_{\pi\pi}$$

PHYSICS OF B

$$\delta = -\text{Im Tr ln } G_{\rho}^{-1}$$

$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E} \ln G_{\rho}^{-1}$$

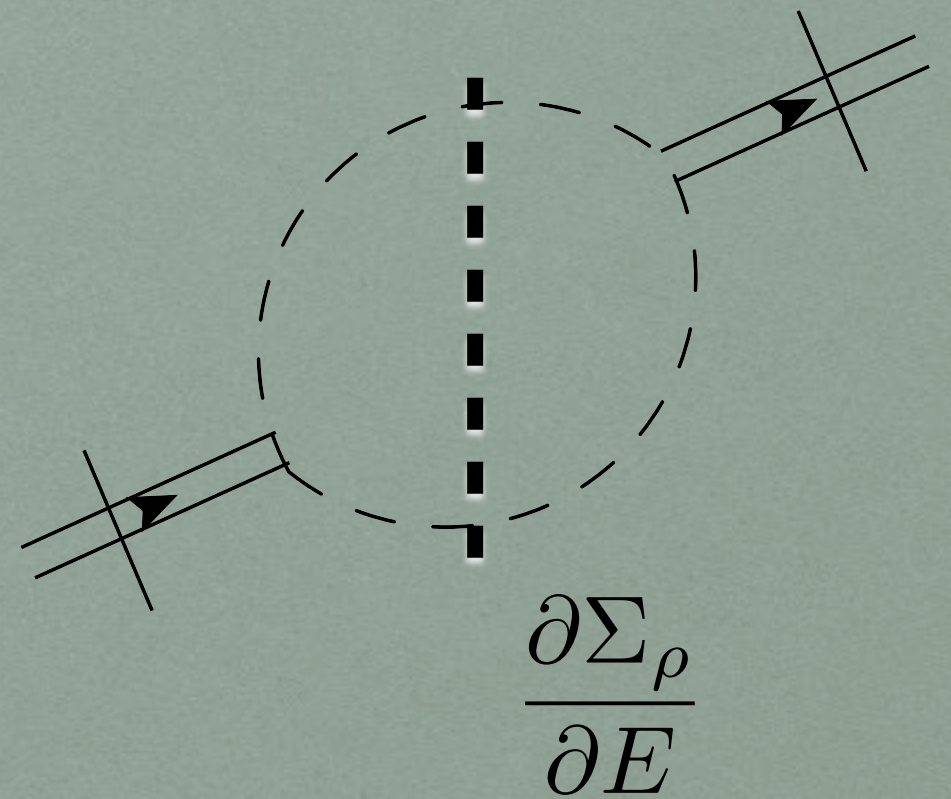
$$= -2 \text{Im}[G_{\rho}](2E) + 2 \text{Im} \left[\frac{\partial \Sigma_{\rho}}{\partial E} G_{\rho} \right]$$

$$= A_{\rho}(E) + \Delta A_{\pi\pi}$$

$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}}$$

physical interpretation:

contribution from correlated pi pi pair



pipi -> pipi

PHYSICS OF B

to rho or not to rho?
that's out of the question!

$$\delta = -\text{Im} T$$

$$B = 2 \frac{\partial}{\partial E} \delta$$

$$= -2 \text{Im} \frac{\partial}{\partial E}$$

$$= -2 \text{Im} [G$$

$$= A_\rho(E) + \Delta A_{\pi\pi}$$

resonance's picture:

$$B(E) = A_\rho(E) + \Delta A_{\pi\pi}$$

rho

scattering picture:

$$B_1 = \frac{\partial}{\partial E} \text{Tr} \hat{t}_{\text{re}}$$

pipi -> pipi

$$B_2 = \frac{1}{2} \text{Im} \text{Tr} \hat{t}^\dagger \overleftrightarrow{\partial}_E \hat{t}$$

$$-\frac{\partial}{\partial E} \int d\phi_E T_{\text{re}} \quad \text{pipi} \rightarrow \text{pipi}$$

$$\frac{\partial \Sigma_\rho}{\partial E}$$

DYNAMICAL GENERATION OF BS / RESONANCES

- dynamical generation of bound states / resonances:
 - f(980) close to $K \bar{K}$ threshold
 - f(500) dynamically generated
- coupling of open channels: $\pi\pi$, kk with a $|q\bar{q}\rangle$ state

what you give \Rightarrow what you get

1 in 5 out!

2 open channels

$$\frac{1}{E - \mathcal{H}_0} = \begin{bmatrix} \Pi_{\pi\pi}(E) & & \\ & \Pi_{K\bar{K}}(E) & \\ & & \frac{1}{E - m_{res}^0} \end{bmatrix}$$
$$V_{int} = \begin{bmatrix} g_{\pi\pi} & g_{\pi K} & g_{\pi R} \\ g_{\pi K} & g_{K K} & g_{K R} \\ g_{\pi R} & g_{K R} & \end{bmatrix}$$

$$G = G_0 + G_0 V_{int} G$$

TESTING THE ROBUSTNESS

$$Q(E) = \frac{1}{2} \text{Im Tr} \{ \ln S_E \}$$

*Getting
Effective DOS
on
REAL Energy*

effective DOS

$$B = 2 \frac{d}{dE} Q$$

what is being counted?

can it handle dynamically generated states?

TESTING THE ROBUSTNESS

$$Q(E) = \frac{1}{2} \text{Im Tr} \{ \ln S_E \}$$

effective DOS

$$B = 2 \frac{d}{dE} Q$$

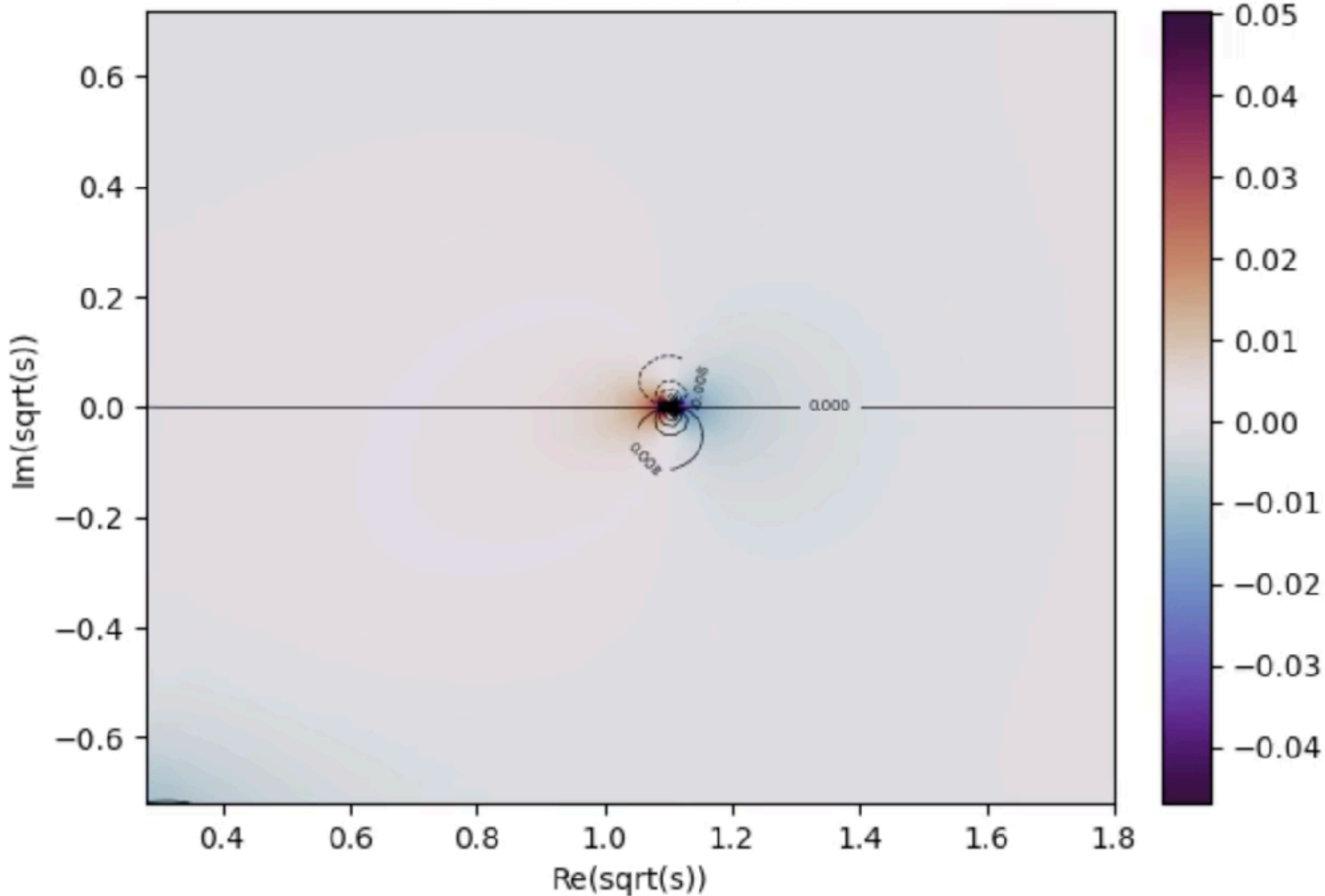
*Getting
Effective DOS
on
REAL Energy*

what is being counted?

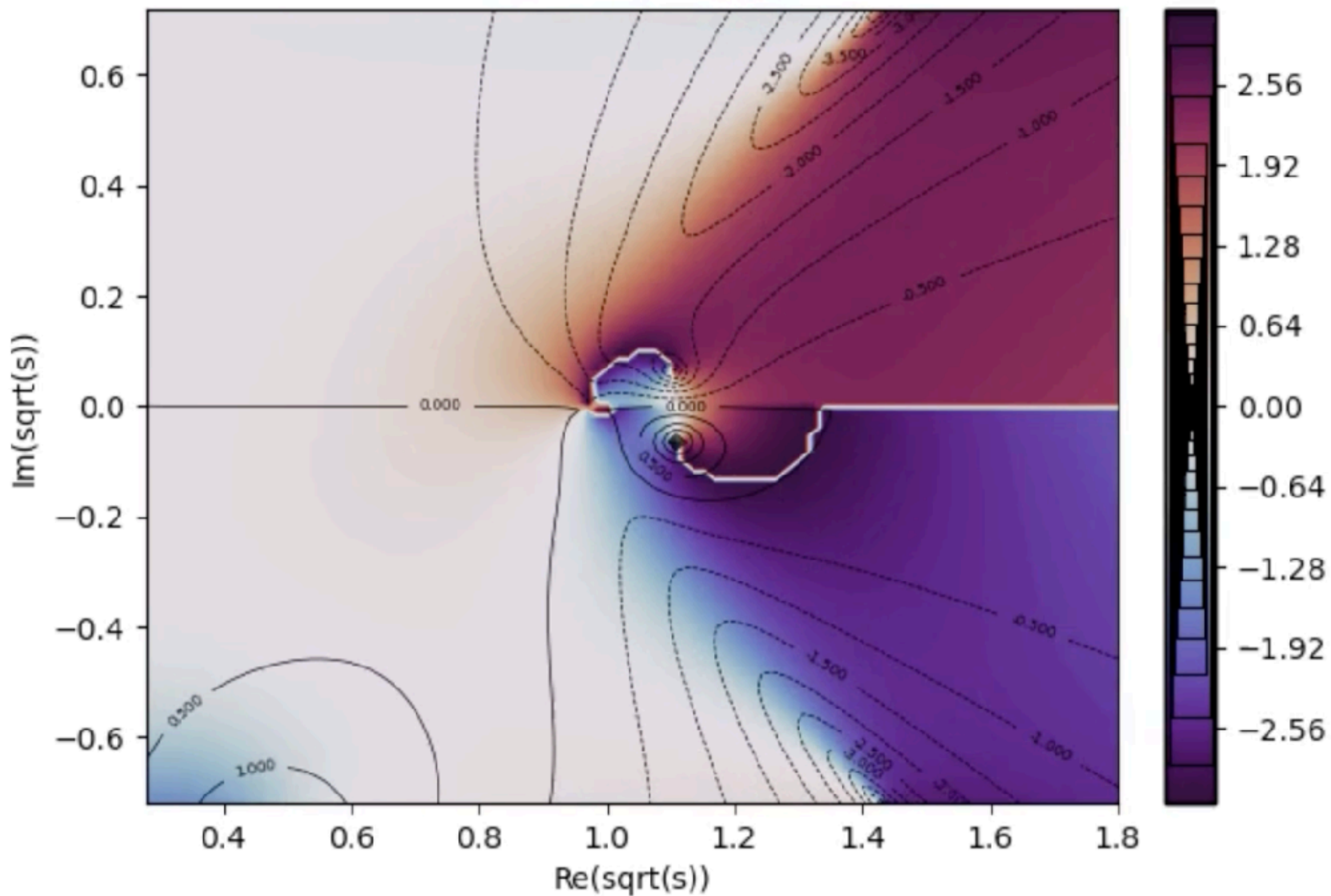
can it handle dynamically generated st



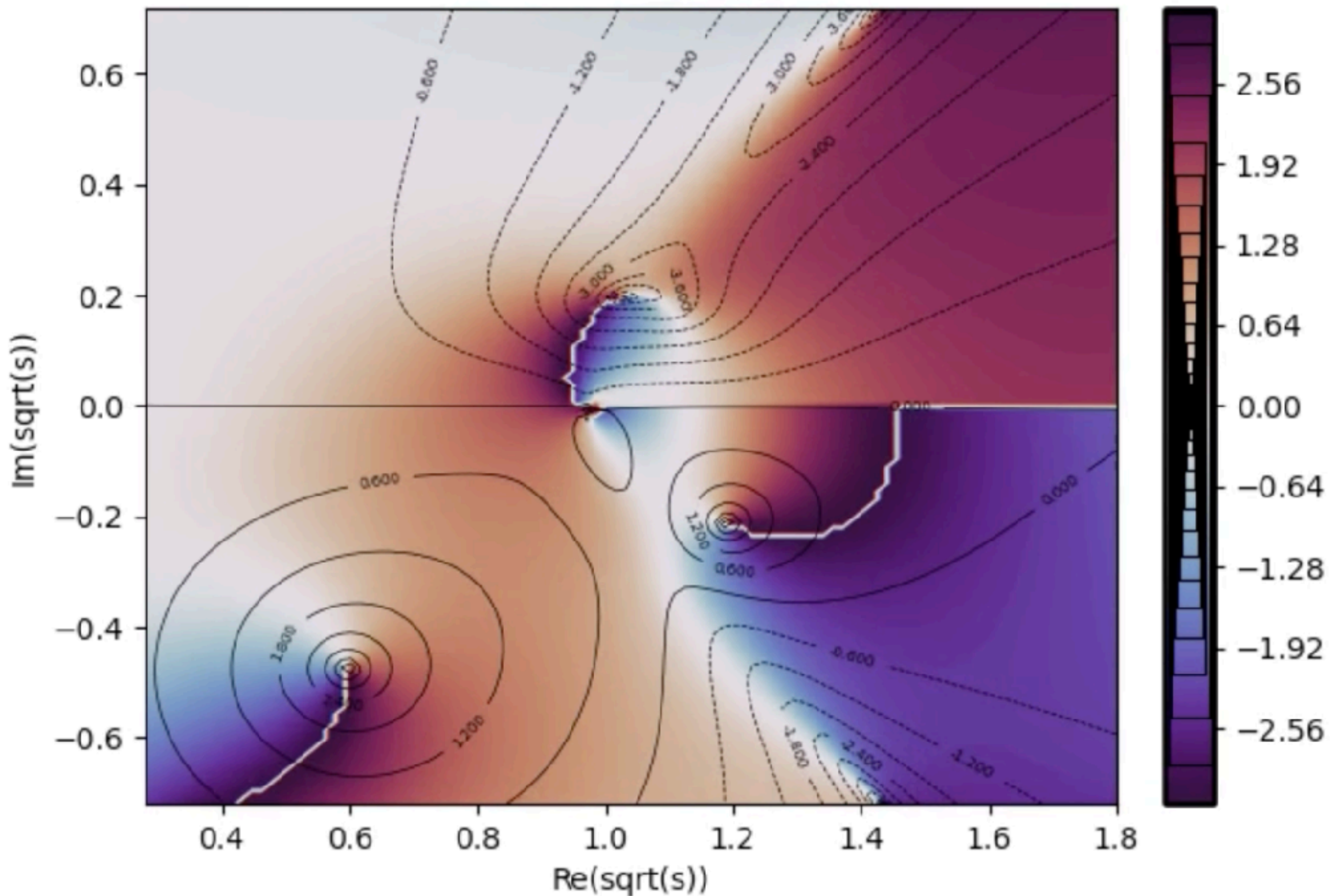
$(x,y)=(0.001, 0.001)$



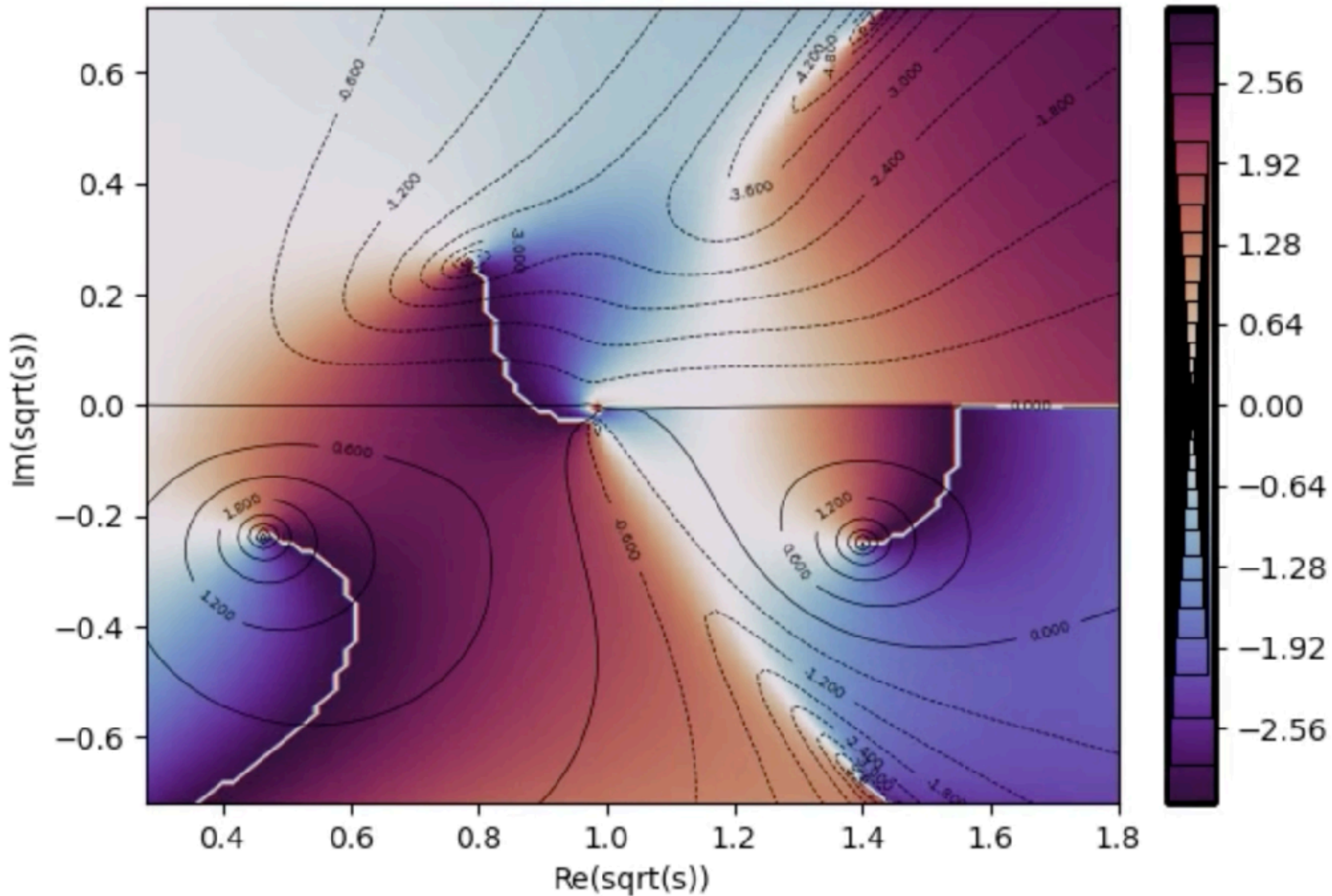
$(x,y)=(0.155, 1.0)$



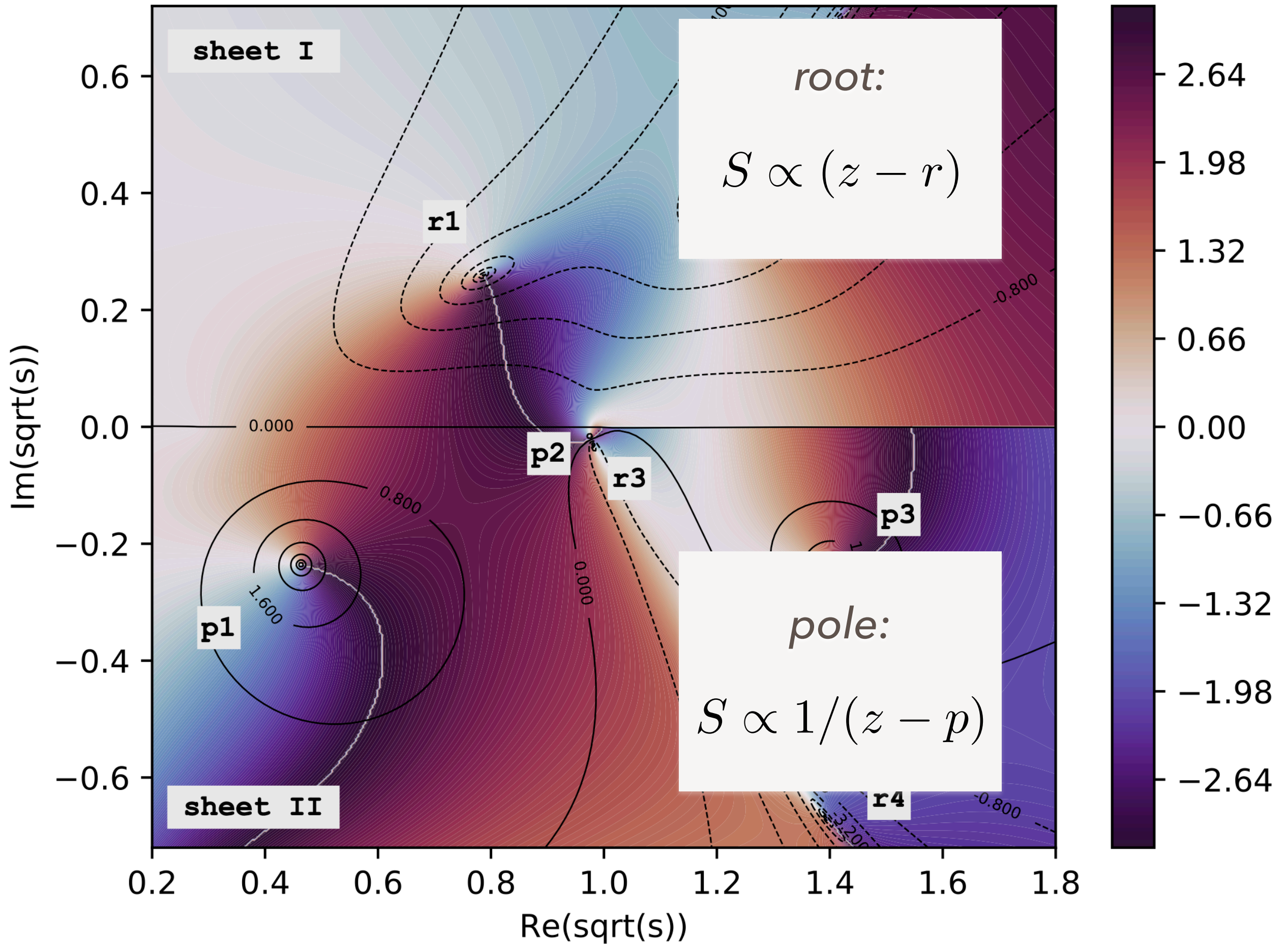
$(x,y)=(0.488, 1.0)$



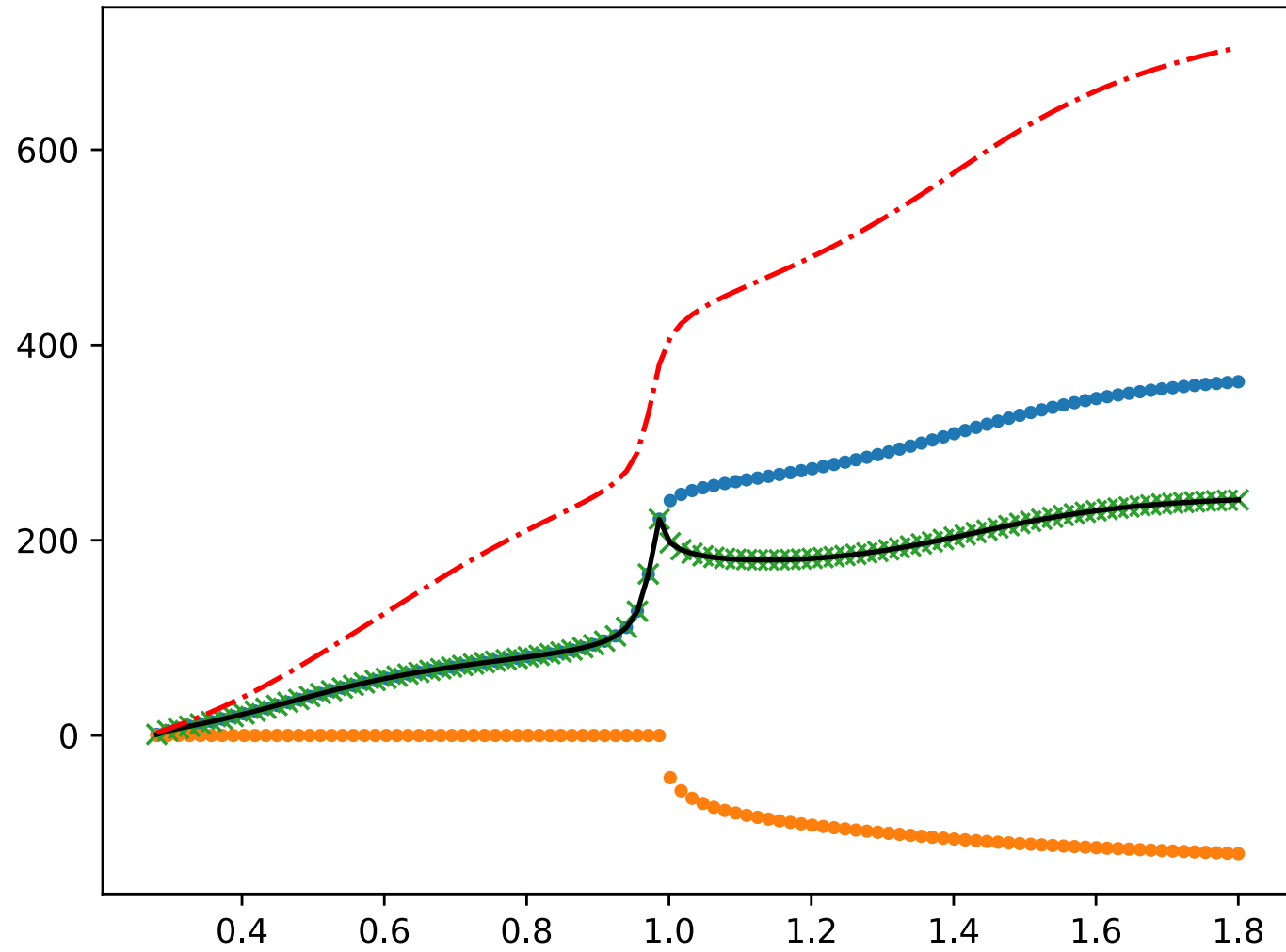
$(x,y)=(1.0, 1.0)$



detS(sqrt(s))

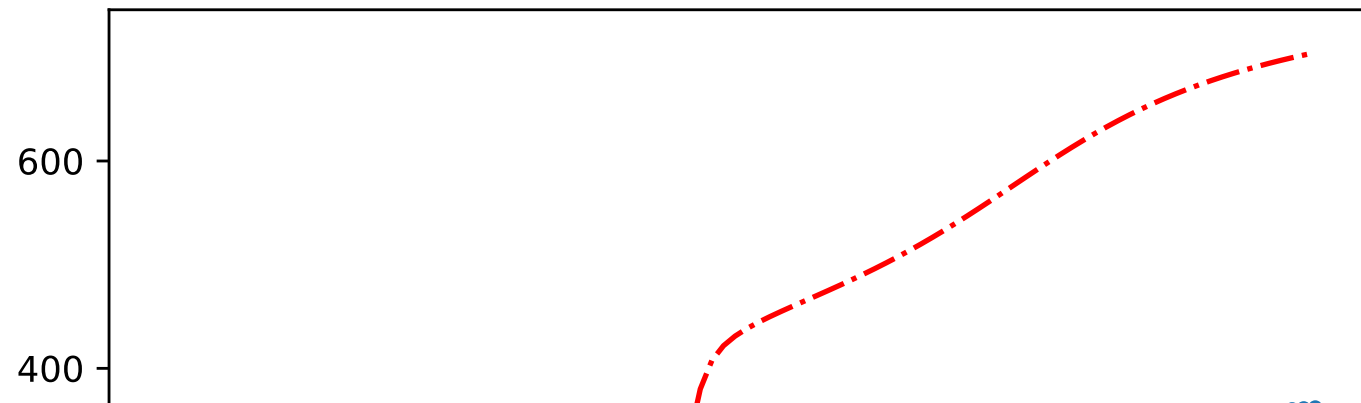


$x = 1.0, y = 1.0$

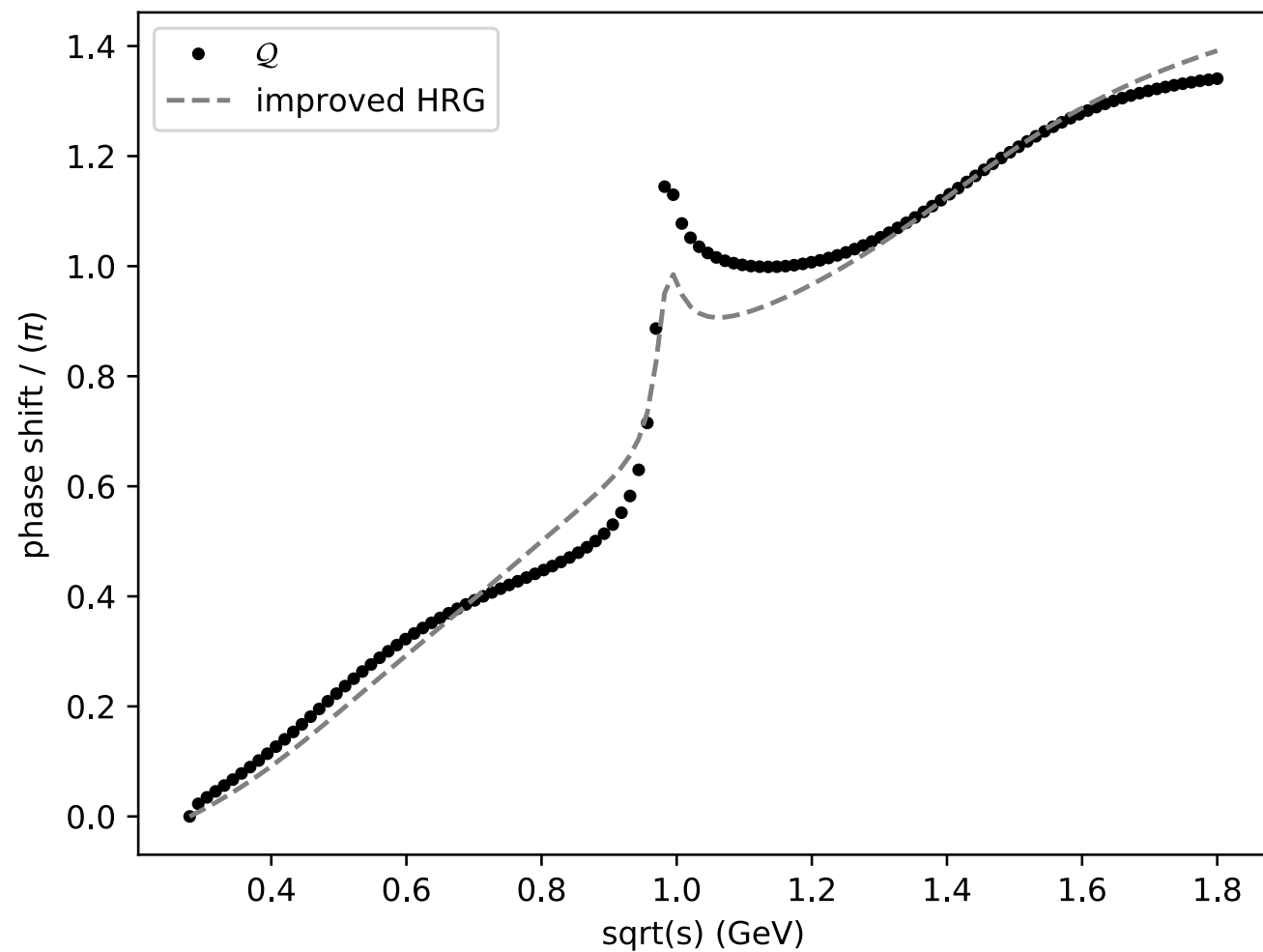


	$\text{Re } \sqrt{s}$	$\text{Im } \sqrt{s}$	sheet
p1	0.4637	-0.2357	II
p2	0.975	-0.0164	II
p3	1.401	-0.249	II
p4	0.6654	-0.2263	III
p5	1.4176	-0.2640	III
r1	0.787	+0.259	I
r2	1.410	+0.691	I
r3	0.981	-0.032	II
r4	1.393	-0.669	II
r5	0.918	+0.248	IV

$x = 1.0, y = 1.0$

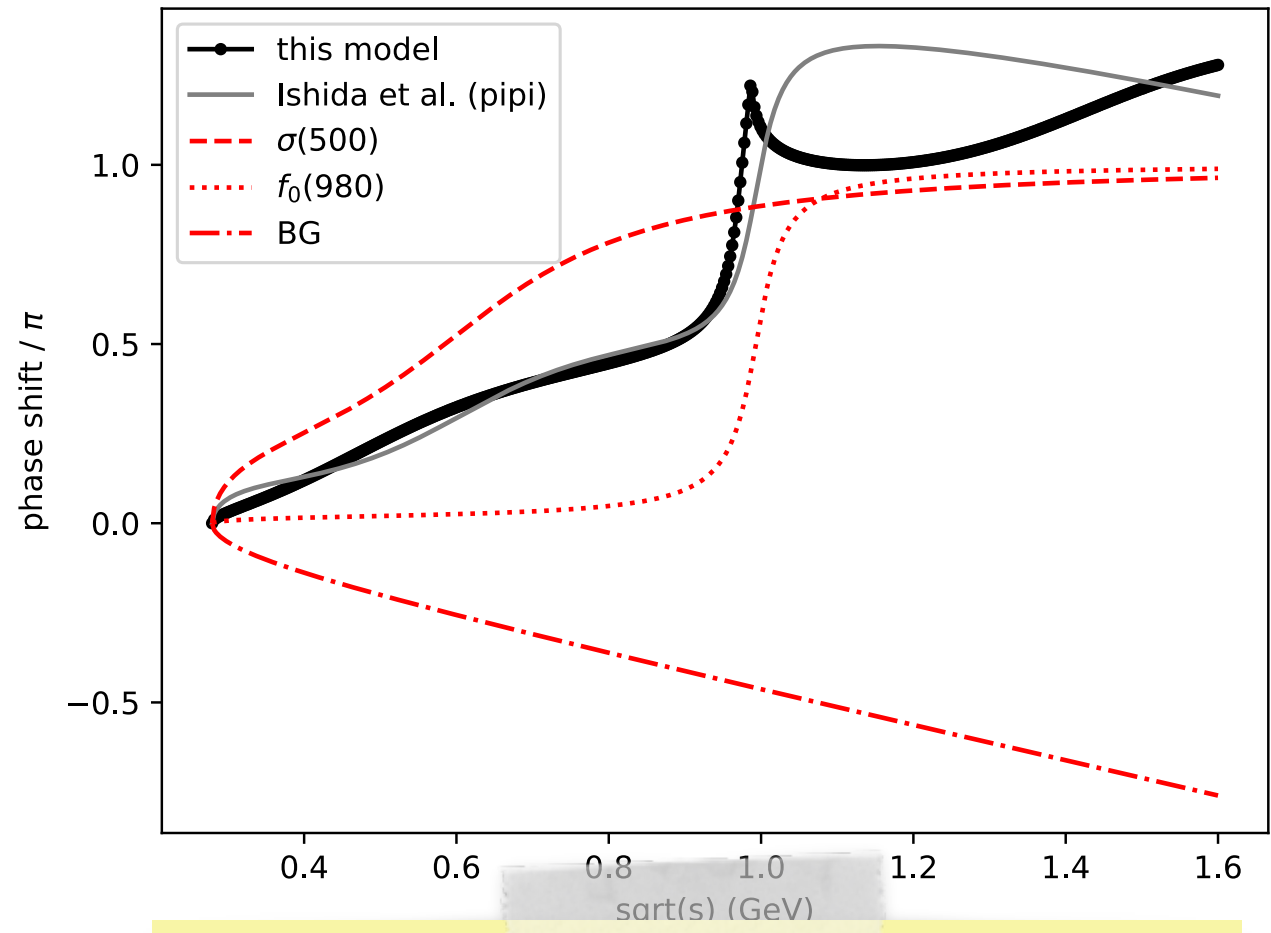


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p3	1.401	-0.249	II
p4	0.6654	-0.2263	III



repulsive corrections in
HRG-like scheme:
via roots

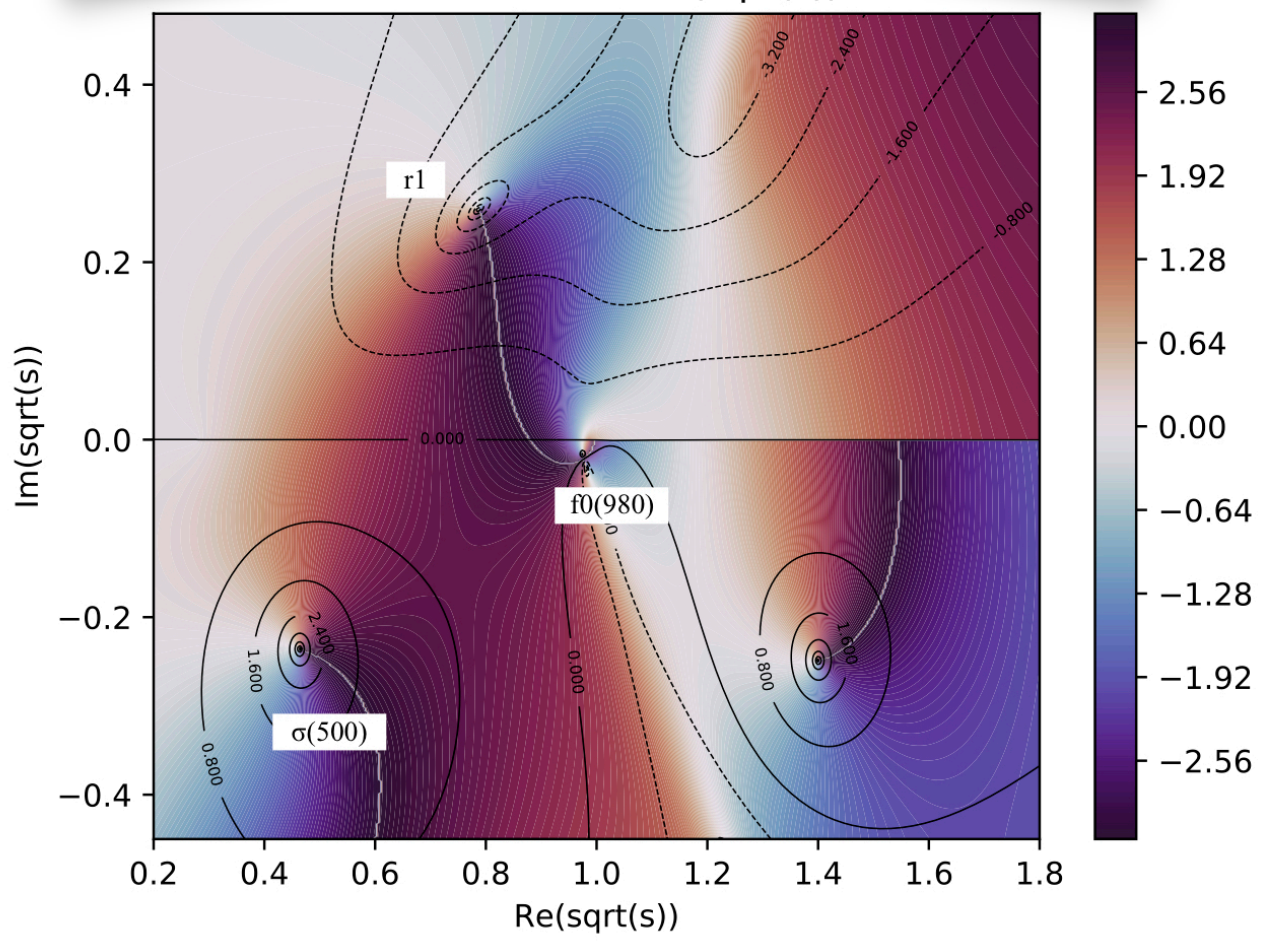
Re \sqrt{s}	Im \sqrt{s}	sheet
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non-Briet-Wigner
 \Rightarrow poles and roots
 distribution

subtractive corrections

sheet I and II: detS(sqrt(s))

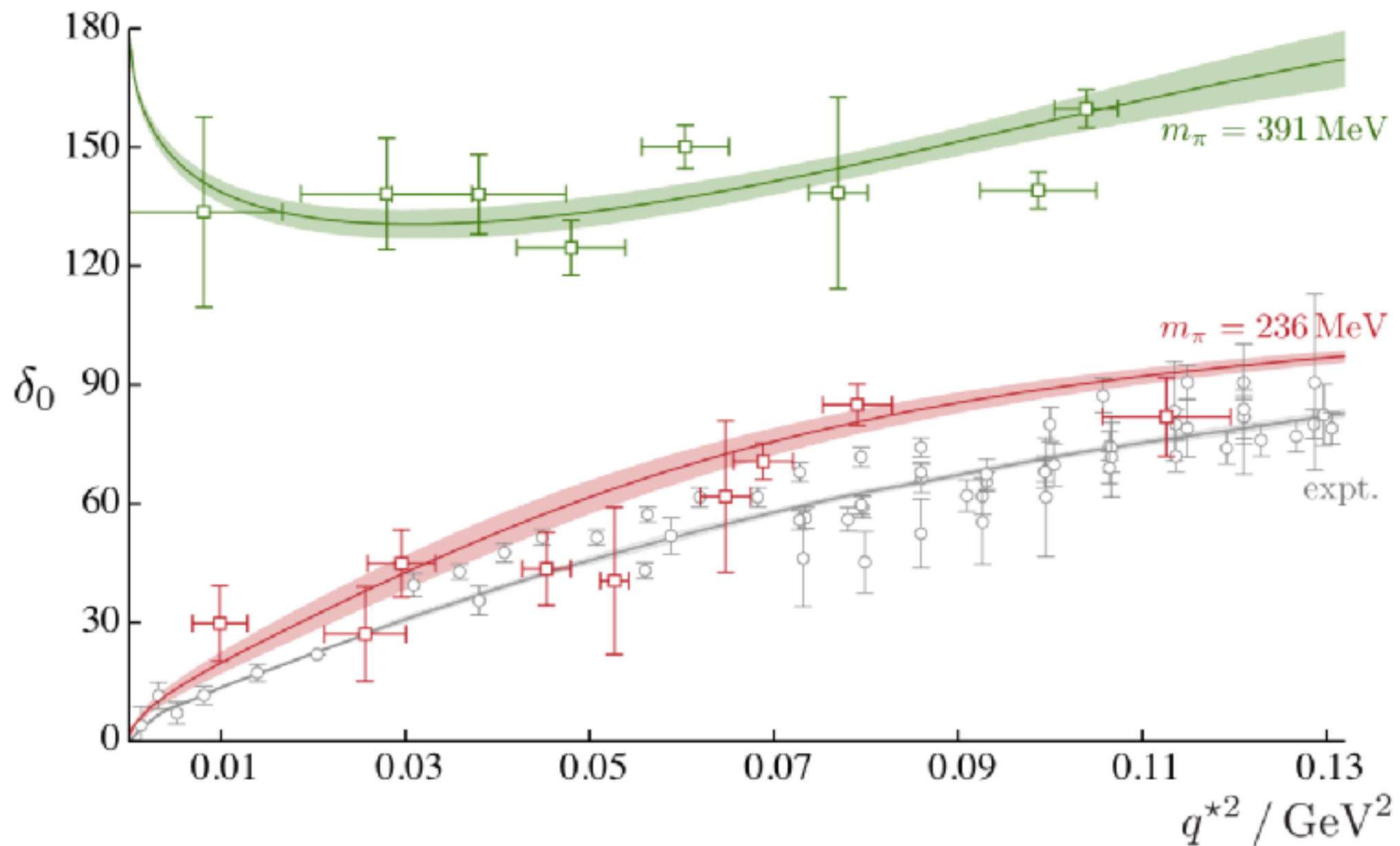


some resonances are more
 equal than others...
 (sheet structures)

$\Delta P/T^4$

T [GeV]

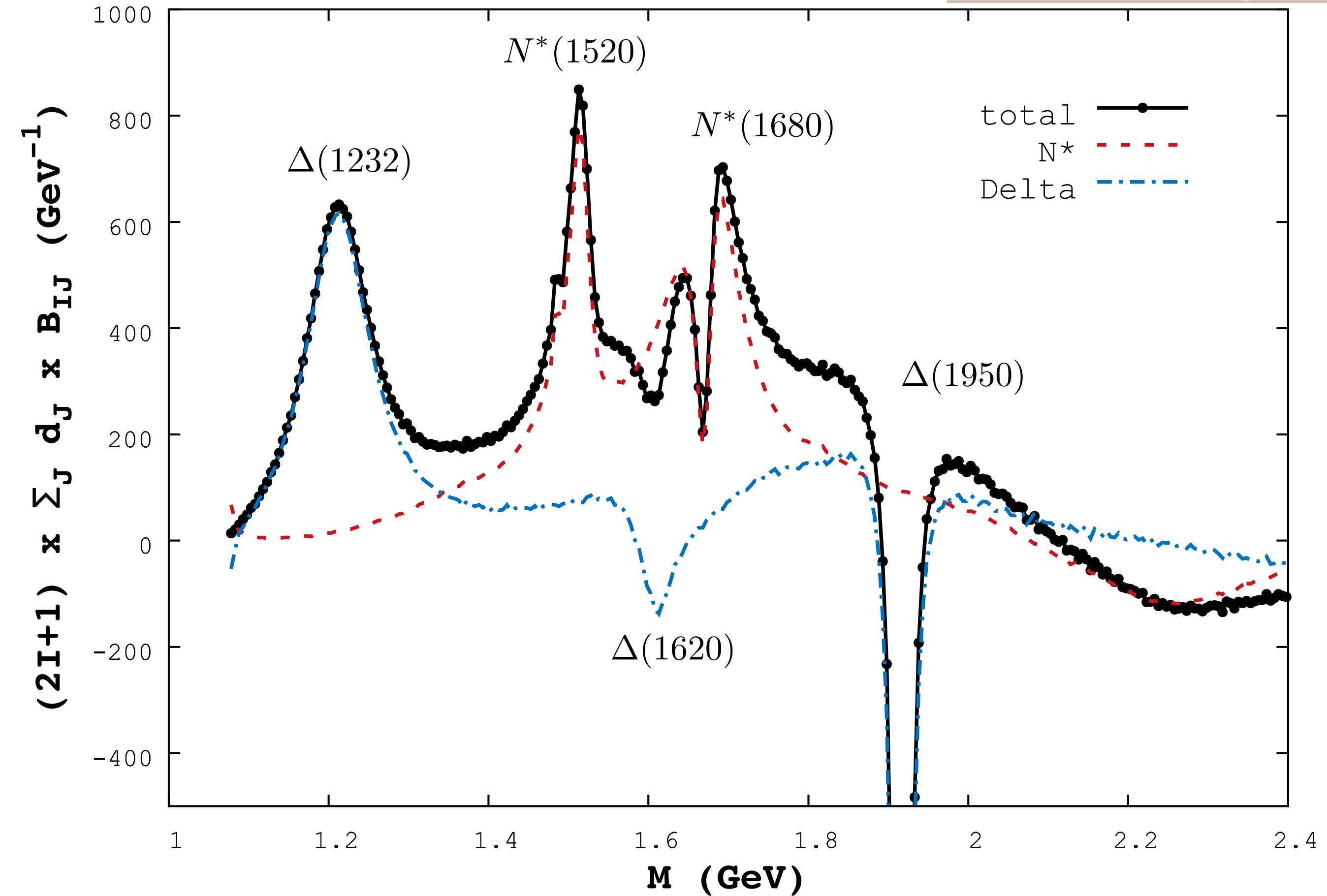
LATTICE COMPUTATIONS ON PHASE SHIFT



R. A. Briceño, J. J. Dudek and R. D. Young, arXiv:1706.06223 [hep-lat].

BARYON SPECTRUM IMPLEMENTATION STATUS

unflavored Baryons



PHASE SHIFT FROM PWA

Coupled Channels partial wave calculator for KN scattering

by the Joint Physics Analysis Center (JPAC)

Version: September 1, 2015

Authors:

Cesar Fernandez-Ramirez (Jefferson Lab)

Igor V. Danilkin (Jefferson Lab)

Vincent Mathieu (Indiana University)

Adam P. Szczepaniak (Indiana University and Jefferson Lab)

Citation: Fernandez-Ramirez et al., arxiv:1510.07065 [hep-ph]

First version: Cesar Fernandez-Ramirez (Jefferson Lab)

This version: Cesar Fernandez-Ramirez (Jefferson Lab)

Contact: cefera@gmail.com (Cesar Fernandez-Ramirez)

Disclaimers:

1 - This code follows the 'garbage in, garbage out' philosophy. If your parameters do not make sense, the output will not make sense either.

2 - You can use, share and modify this code under your own responsibility.

3 - This code is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the implied warranty of

MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.

4 - No PhD students or postdocs were severely damaged during the development of this project.

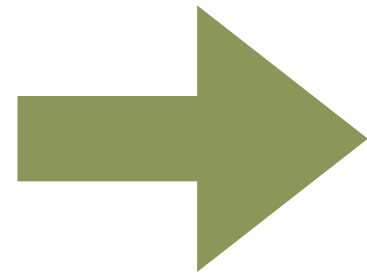
- 1 $\rightarrow \bar{K}N$,

- 2 $\rightarrow \pi\Sigma$,

- 3 $\rightarrow \pi\Lambda$,

- 4 $\rightarrow \eta\Lambda$,

- 5 $\rightarrow \eta\Sigma$,



elastic scatterings (elementary)

- 6 $\rightarrow \bar{K}_1N$,

- 7 $\rightarrow [\bar{K}_3N]_-$,

- 8 $\rightarrow [\bar{K}_3N]_+$,

- 9 $\rightarrow [\pi\Sigma^*]_-$,

- 10 $\rightarrow [\pi\Sigma^*]_+$,

- 11 $\rightarrow [\bar{K}\Delta]_-$,

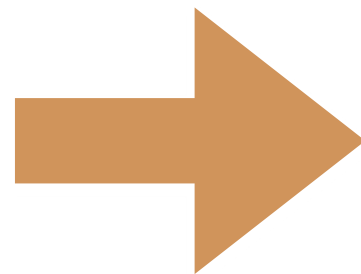
- 12 $\rightarrow [\bar{K}\Delta]_+$,

- 13 $\rightarrow [\pi\Lambda(1520)]_-$,

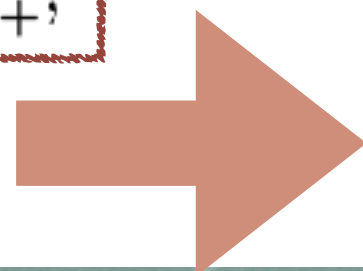
- 14 $\rightarrow [\pi\Lambda(1520)]_+$,

- 15 $\rightarrow \pi\pi\Lambda$,

- 16 $\rightarrow \pi\pi\Sigma$.

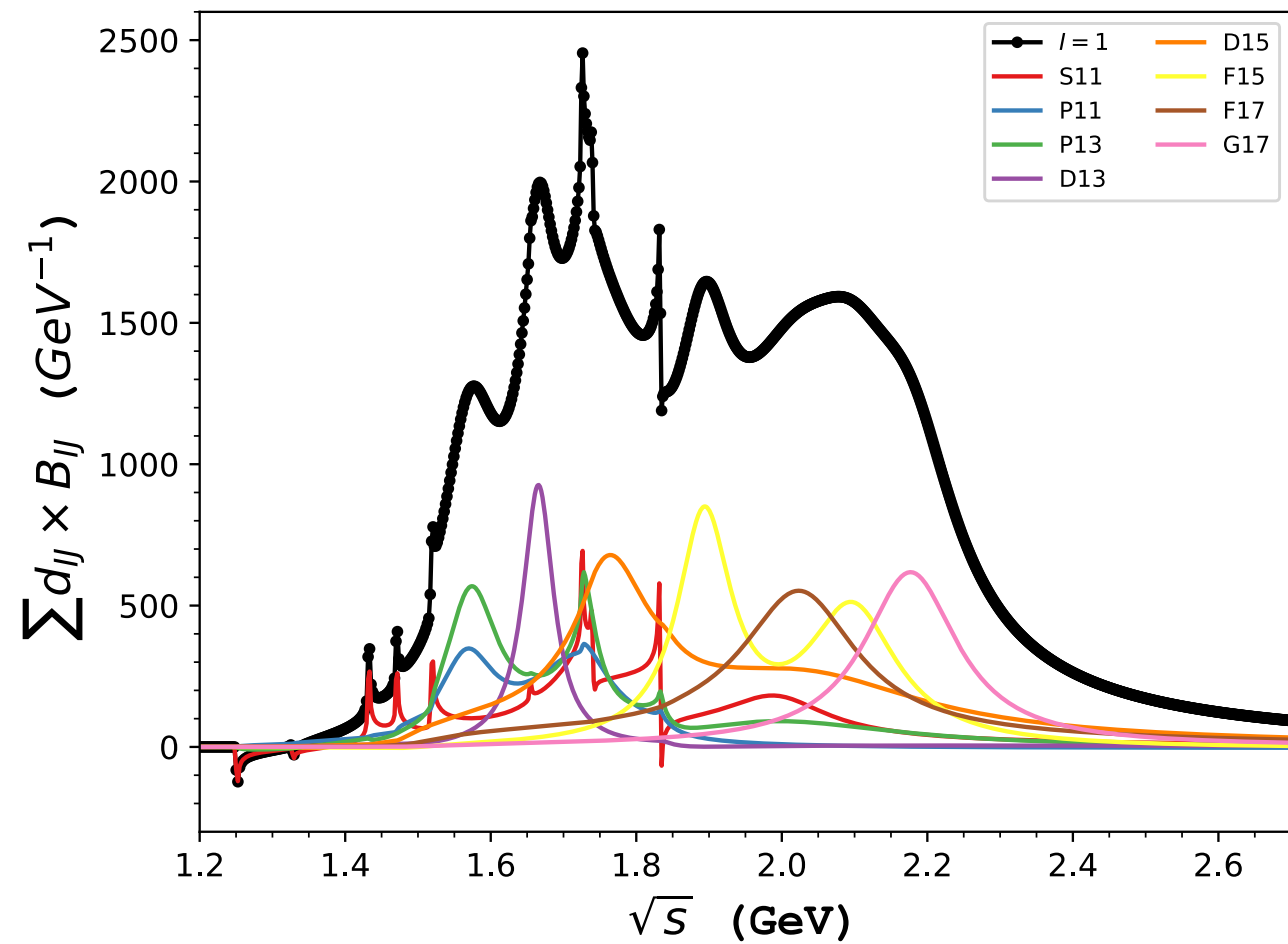
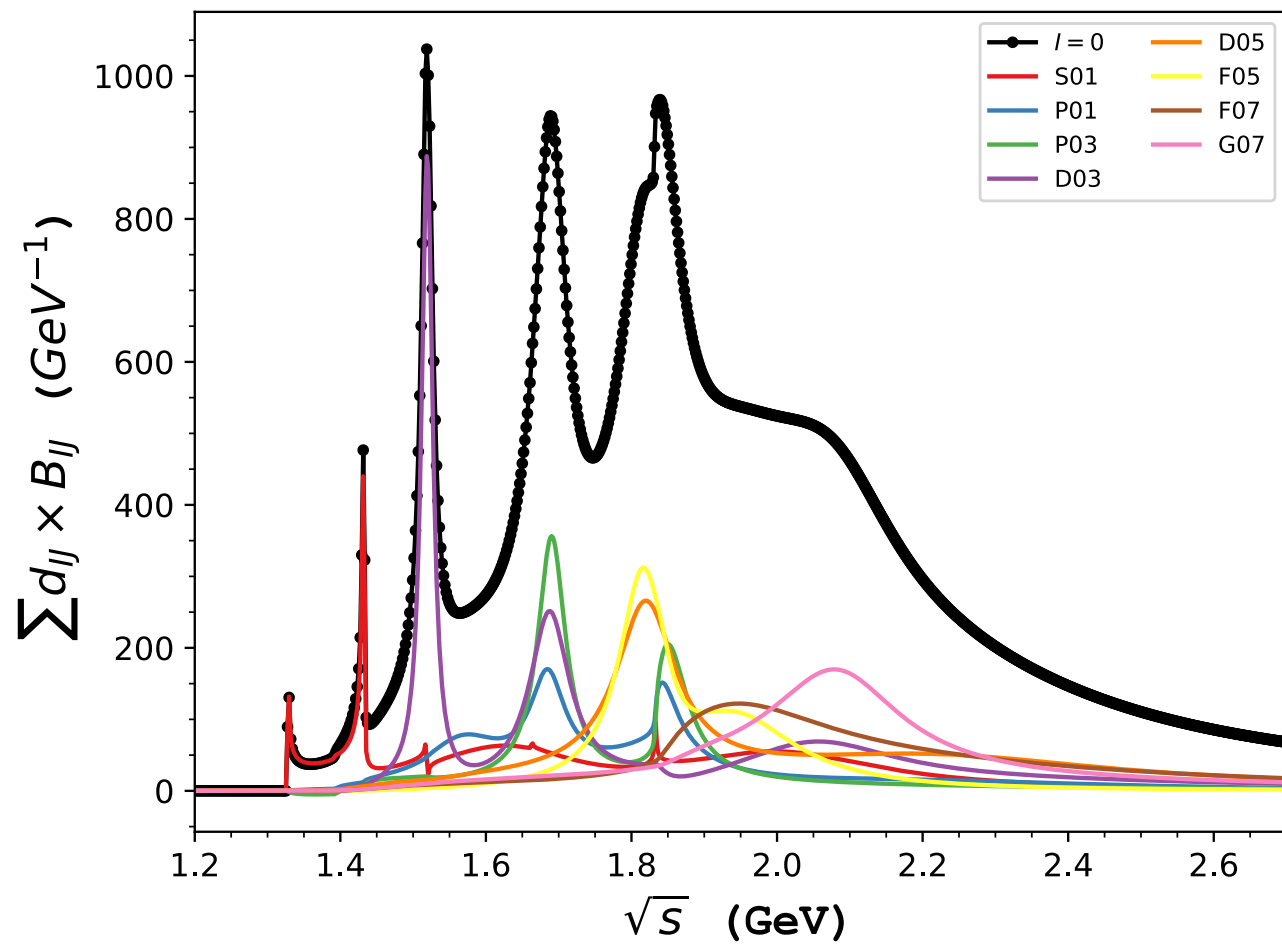


quasi elastic scatterings

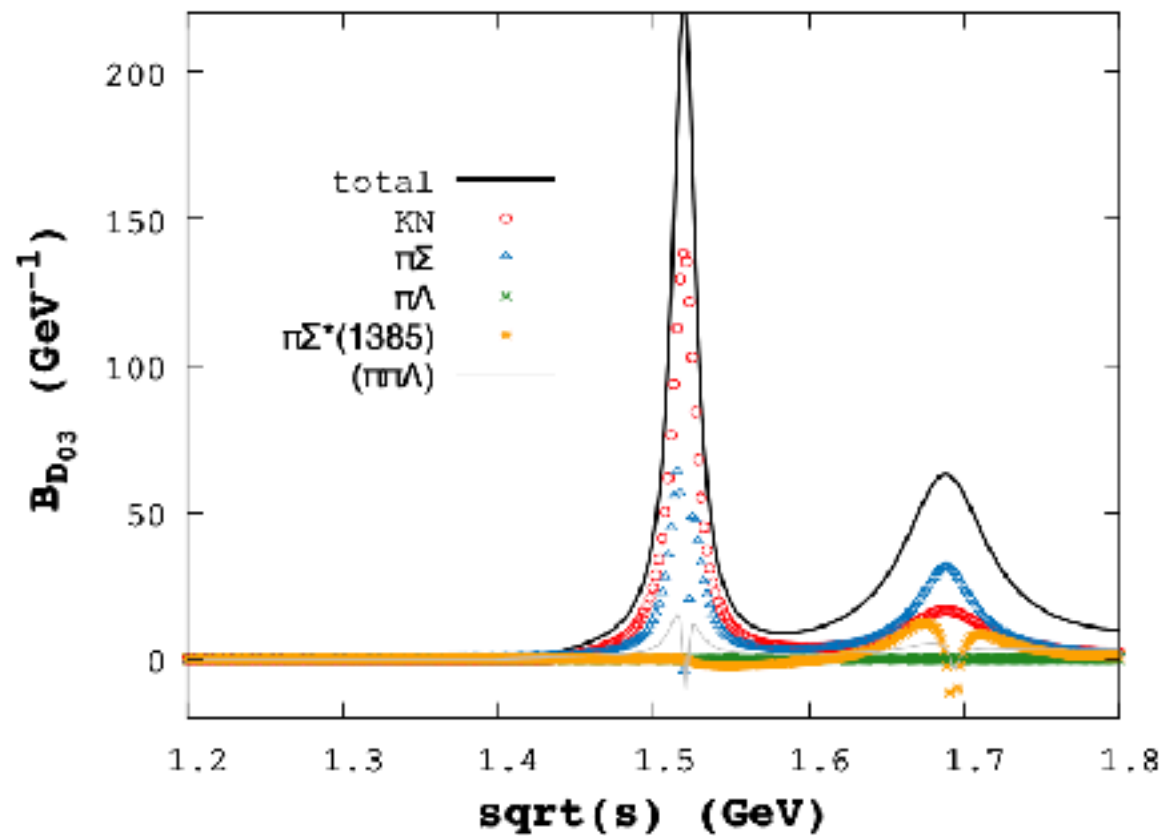


unitarity background

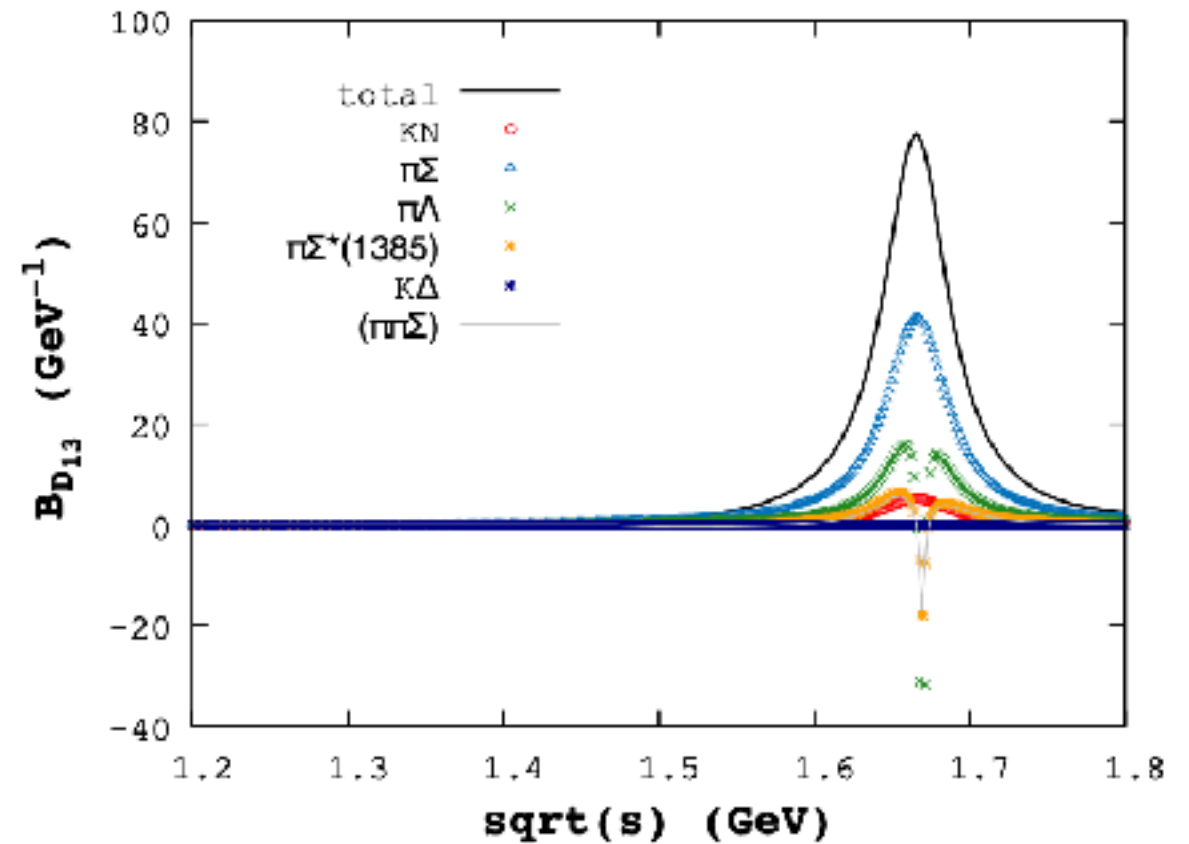
summing all channels...



1520, 1690



1670



$\Lambda(1520) 3/2^-$

$I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1519.5 \pm 1.0$ MeV [d]
 Full width $\Gamma = 15.6 \pm 1.0$ MeV [d]
 $p_{\text{beam}} = 0.36$ GeV/c $4\pi\chi^2 = 82.8$ mb

$\Lambda(1520)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	$45 \pm 1\%$	243
$\Sigma\pi$	$42 \pm 1\%$	268
$\Lambda\pi\pi$	$10 \pm 1\%$	259
$\Sigma\pi\pi$	$0.9 \pm 0.1\%$	169
$\Lambda\gamma$	$0.85 \pm 0.15\%$	350

$\Lambda(1690) 3/2^-$

$I(J^P) = 0(\frac{3}{2}^-)$

Mass $m = 1685$ to 1695 (≈ 1690) MeV
 Full width $\Gamma = 50$ to 70 (≈ 60) MeV
 $p_{\text{beam}} = 0.78$ GeV/c $4\pi\chi^2 = 26.1$ mb

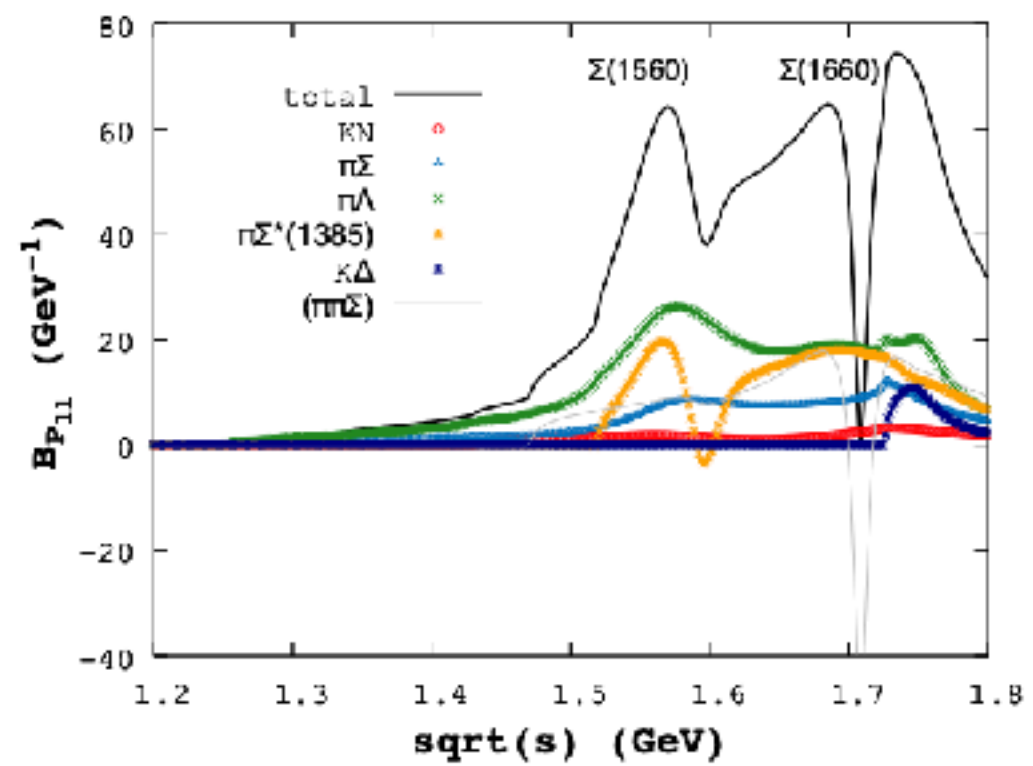
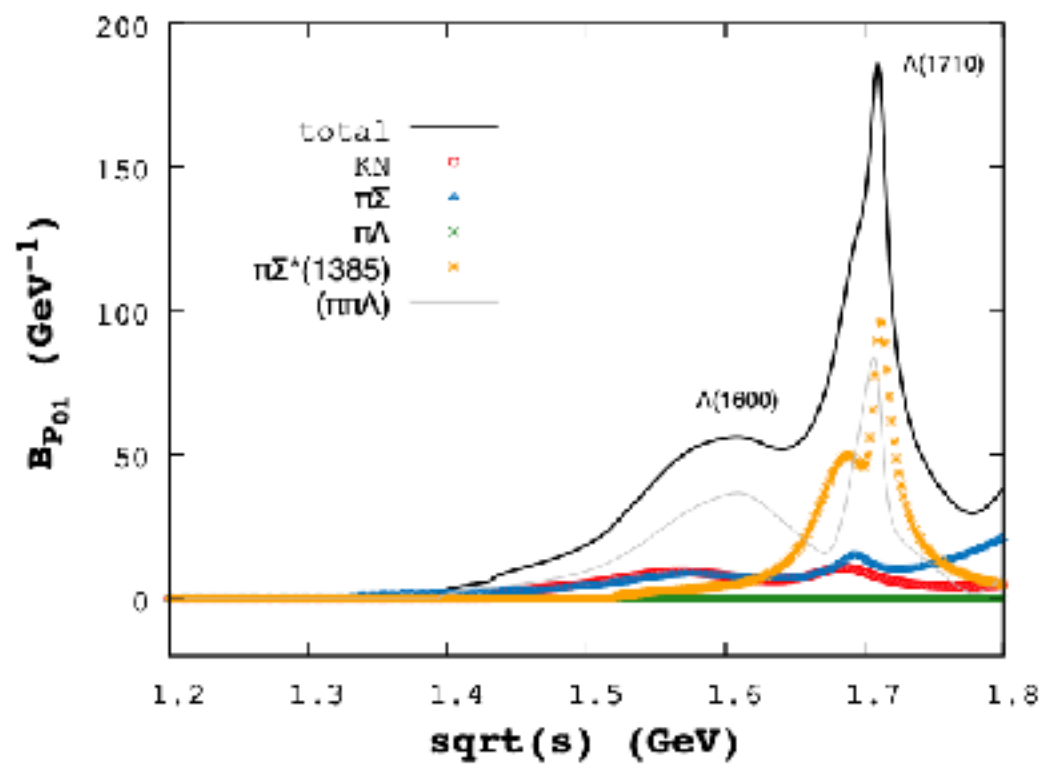
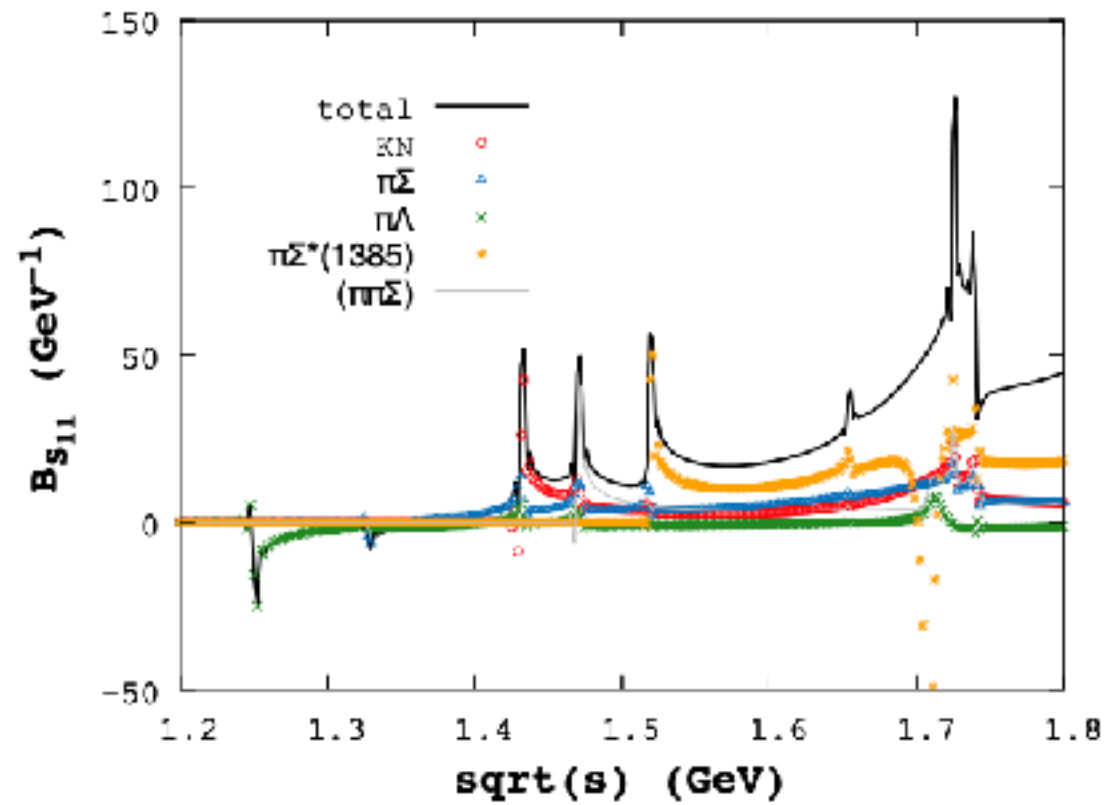
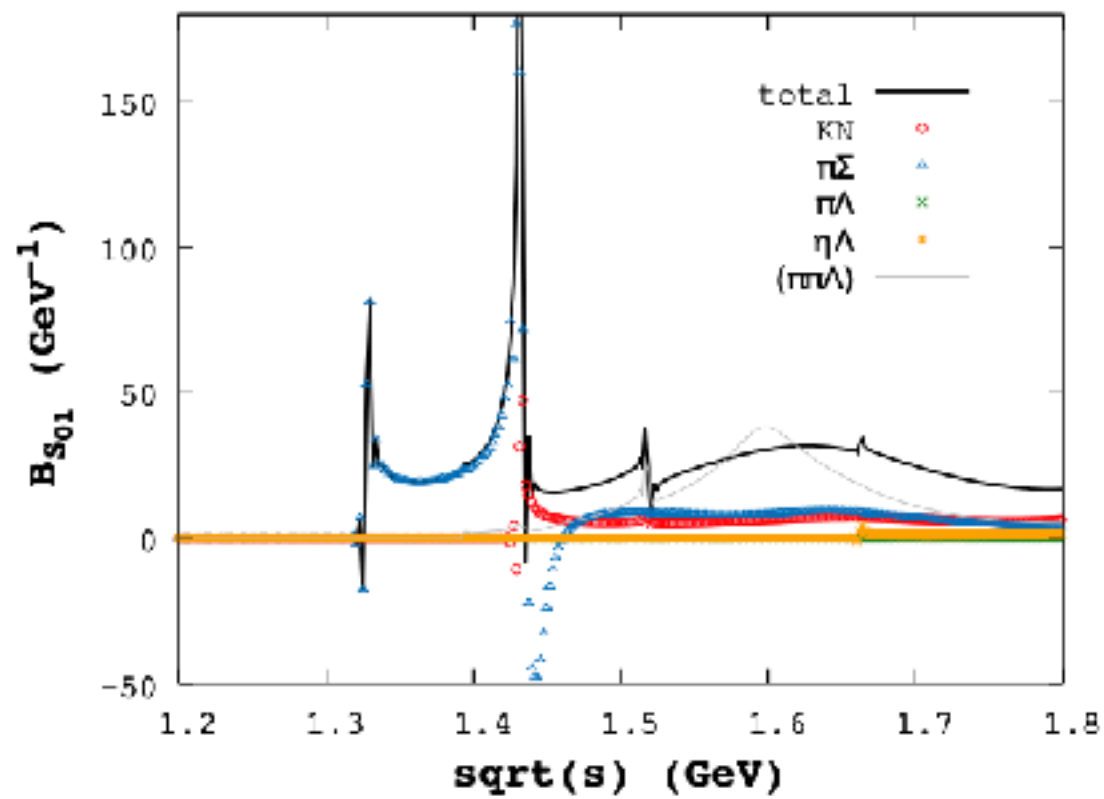
$\Lambda(1690)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	20–30 %	433
$\Sigma\pi$	20–40 %	410
$\Lambda\pi\pi$	~ 25 %	419
$\Sigma\pi\pi$	~ 20 %	358

$\Sigma(1670) 3/2^-$

$I(J^P) = 1(\frac{3}{2}^-)$

Mass $m = 1665$ to 1685 (≈ 1670) MeV
 Full width $\Gamma = 40$ to 80 (≈ 60) MeV
 $p_{\text{beam}} = 0.74$ GeV/c $4\pi\chi^2 = 28.5$ mb

$\Sigma(1670)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$N\bar{K}$	7–13 %	414
$\Lambda\pi$	5–15 %	448
$\Sigma\pi$	30–60 %	394



branching ratio?

STRANGENESS CONTENT IN A HADRON GAS

- K-N system requires a coupled channel analysis

$|\bar{K}N\rangle, |\pi\Sigma\rangle, |\pi\Lambda\rangle, |\eta\Lambda\rangle, \dots$ *16 basis states*

$$Q(M) \equiv \frac{1}{2} \text{Im} (\text{tr} \ln S)$$
$$= \frac{1}{2} \text{Im} (\ln \det [S])$$

$$= \delta_{\bar{K}N} + \delta_{\pi\Sigma} + \delta_{\pi\Lambda} + \dots$$

recipe to extract
eigenphases from PWA

COUPLED-CHANNEL PROBLEM

$$\{\gamma_1, \gamma_2, m_{\text{res}}\} \longleftrightarrow \{\delta_1, \delta_2, \eta\}$$

$$S = \begin{pmatrix} \eta e^{2i\delta_I} & i\sqrt{1-\eta^2} e^{i(\delta_I+\delta_{II})} \\ i\sqrt{1-\eta^2} e^{i(\delta_I+\delta_{II})} & \eta e^{2i\delta_{II}} \end{pmatrix}$$

$a_0(980)$ system

$$\begin{aligned} Q(M) &\equiv \frac{1}{2} \text{Im} (\text{tr} \ln S) \\ &= \frac{1}{2} \text{Im} (\ln \det [S]) \\ &= \delta_I + \delta_{II}. \end{aligned}$$

$$\begin{aligned} \pi\eta &\rightarrow \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \rightarrow \pi\eta \\ K\bar{K} &\rightarrow \begin{pmatrix} \pi\eta \\ K\bar{K} \end{pmatrix} \rightarrow K\bar{K} \end{aligned}$$

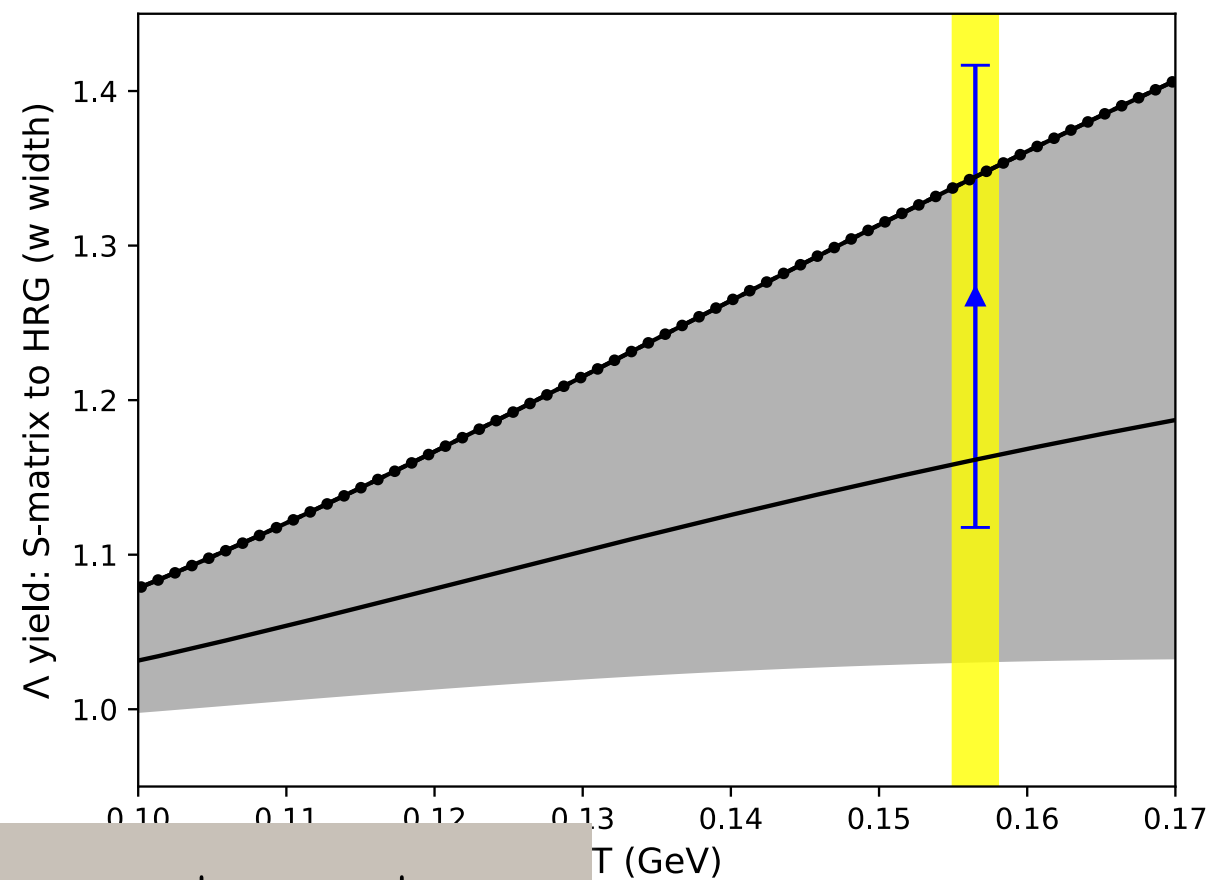
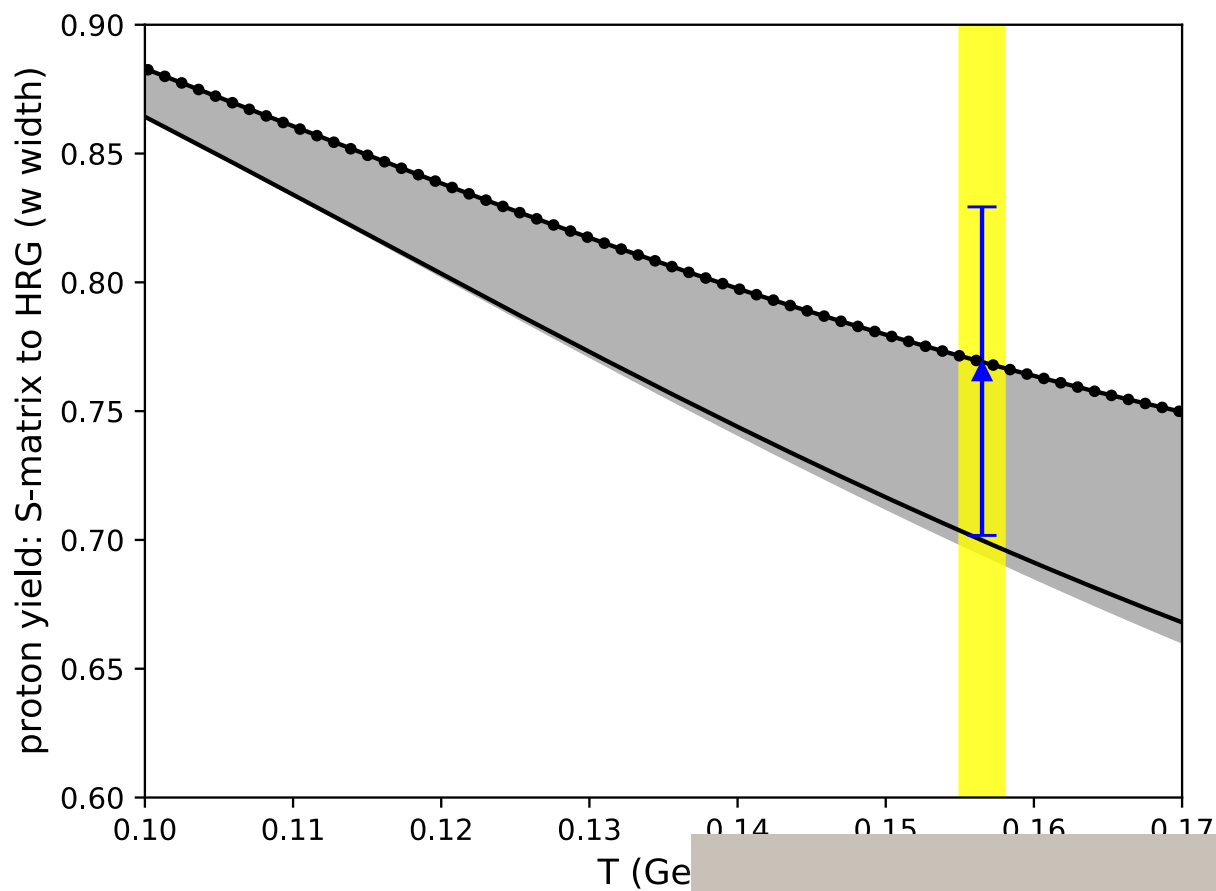
WIGNER, EISENBUD, SMITH, ...

$$S \rightarrow U^\dagger S_d U$$
$$S_d = \begin{pmatrix} e^{2i\delta_{\text{res}}(s)} & 0 \\ 0 & 1 \end{pmatrix},$$
$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$

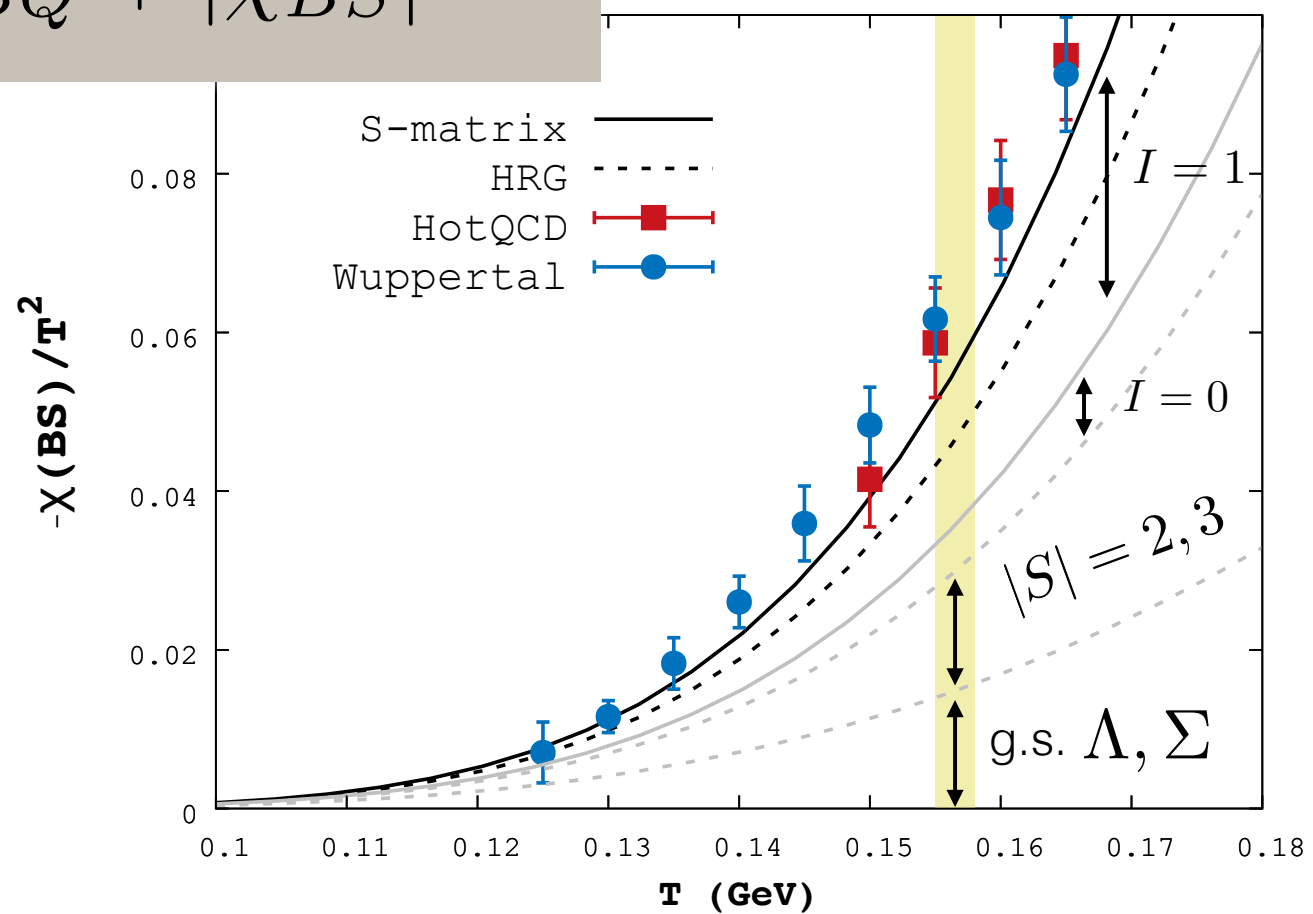
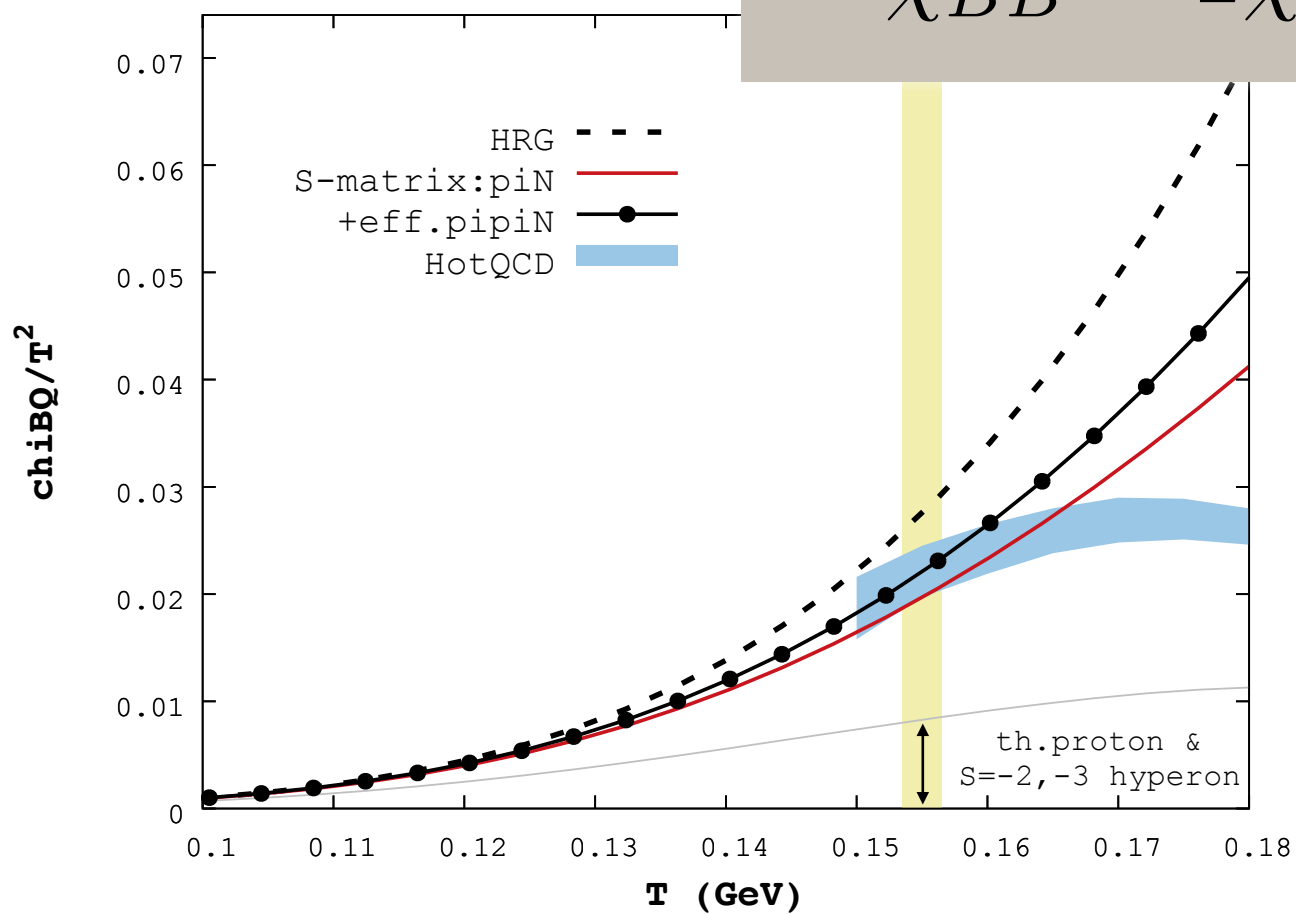
$$\text{BR}_a = \cos^2 \theta = \frac{g_a^2 \phi_a}{g_a^2 \phi_a + g_b^2 \phi_b},$$

$$\text{BR}_b = \sin^2 \theta = \frac{g_b^2 \phi_b}{g_a^2 \phi_a + g_b^2 \phi_b}.$$





$$\chi_{BB} = 2\chi_{BQ} + |\chi_{BS}|$$





Chew at his California home on July 2014

Born	June 5, 1924 Washington, D.C., United States
Died	April 12, 2019 (aged 94) Berkeley, California, United States
Nationality	American
Alma mater	University of Chicago
Known for	S-matrix theory , bootstrap theory , strong interactions , Chew–Frautschi plot
Awards	Hughes Prize (1962) Lawrence Prize (1969) Majorana Prize (2008)
	Scientific career
Fields	Theoretical physics
Institutions	University of Illinois UC Berkeley
Doctoral advisor	Enrico Fermi
Doctoral students	David Gross John H. Schwarz John R. Taylor

indecently optimistic...

S-Matrix Theory of Strong Interactions without Elementary Particles*†

GEOFFREY F. CHEW

Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California

1. INTRODUCTION

IN this paper I present an indecently optimistic view of strong interaction theory. My belief is that a major breakthrough has occurred and that within a relatively short period we are going to achieve a depth of understanding of strong interactions that a few years ago I, at least, did not expect to see within my lifetime. I know that few of you will be convinced by the arguments given here, but I would be masking my feelings if I were to employ a conventionally cautious attitude in this talk. I am bursting with excitement, as are a number of other theorists in this game.

tell me that this is a fetish, that field theory is an equally suitable language, but to me the basic strong-interaction concepts, simple and beautiful in a pure *S*-matrix approach, are weird, if not impossible, for field theory. It must be said, nevertheless, that my own awareness of these concepts was largely achieved through close collaboration with three great experts in field theory, M. L. Goldberger, Francis Low, and Stanley Mandelstam. Each of them has played a major role in the development of the strong interaction theory that I describe,¹ even though the language of my description may be repugnant to them. Murray Gell-Mann, also, although he has not actu-

PARTICLES AS S-MATRIX POLES; HADRON DEMOCRACY *

satisfy unitarity. There is no "reason" for any others. Similarly, as Feynman and Heisenberg have both emphasized, there is no reason why some particles should be on a different footing from others. The elementary particle concept is unnecessary, at least for baryons and mesons.

The second assumption may turn out to be closely related to the first, perhaps even a consequence, but

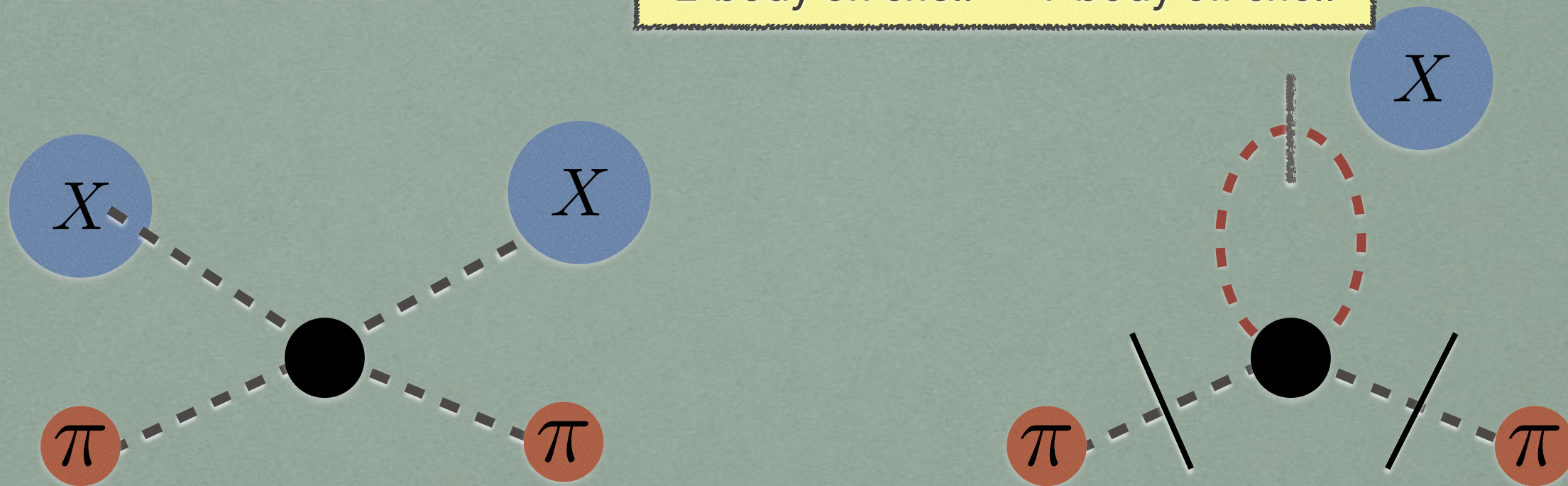
but some are more equal than the others?

GOING FURTHER WITH S-MATRIX

- Theoretical development:
N-body scattering: beyond $2 \rightarrow 2$; inelasticity
- In-medium effects with S-matrix
- Quark-Hadron Duality and QCD phase transition
- Equation of states for Dense Matter

IN-MEDIUM EFFECTS FROM S-MATRIX

2-body on-shell \rightarrow 1-body off-shell



$$\Delta P = \approx N_{\text{th}}^{\pi} N_{\text{th}}^N \times (-T_{\text{NR}}).$$

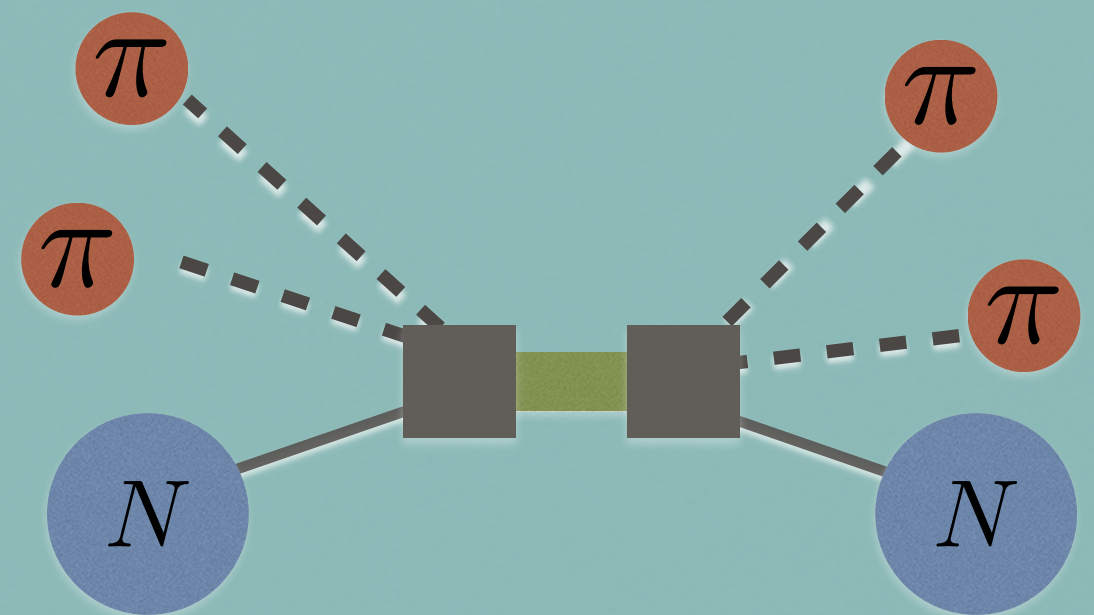
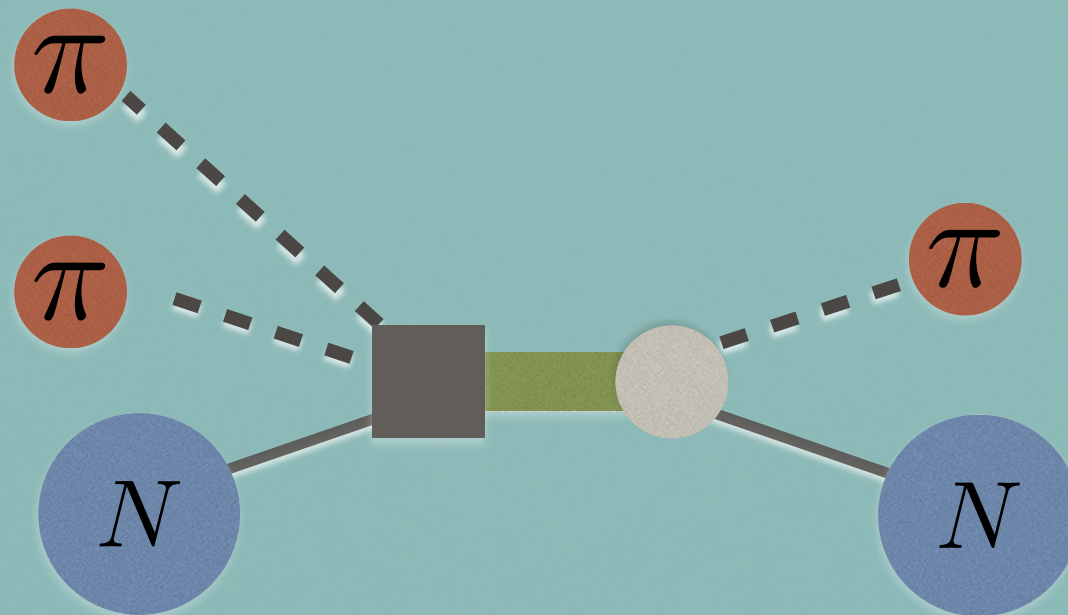
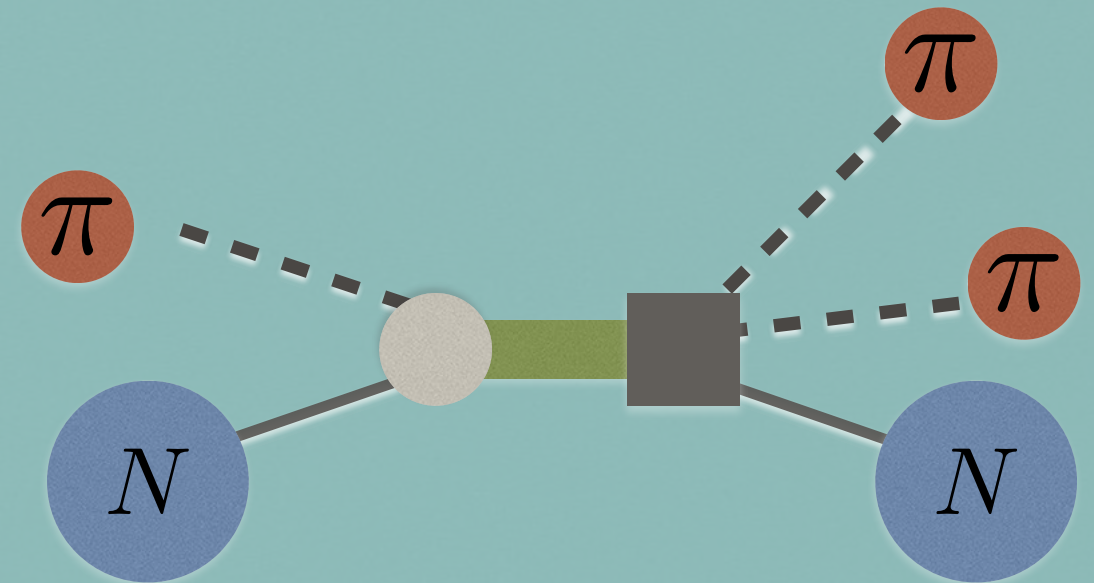
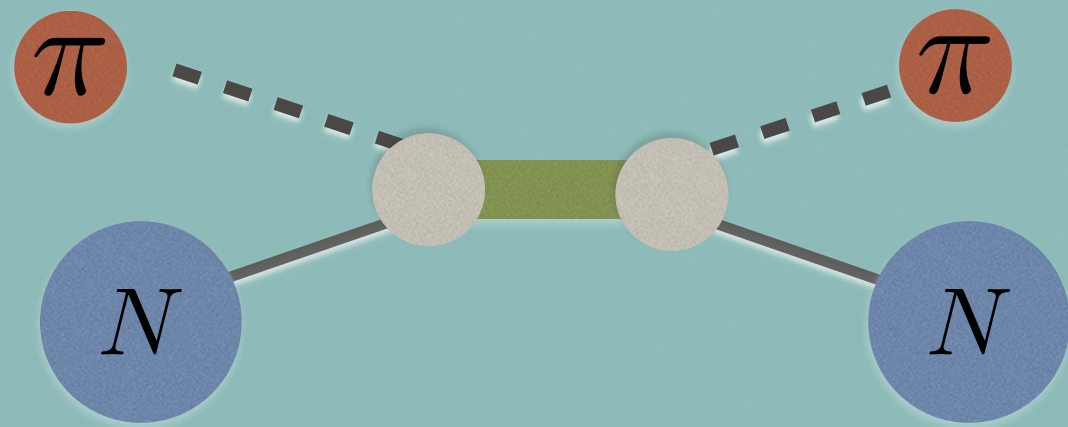
$$\Sigma_{\pi} \propto \int \frac{d^3q}{\omega_q} n_X T_{\pi X}(s)$$

forward amplitude

A. Schenk NPB 363 (1991)

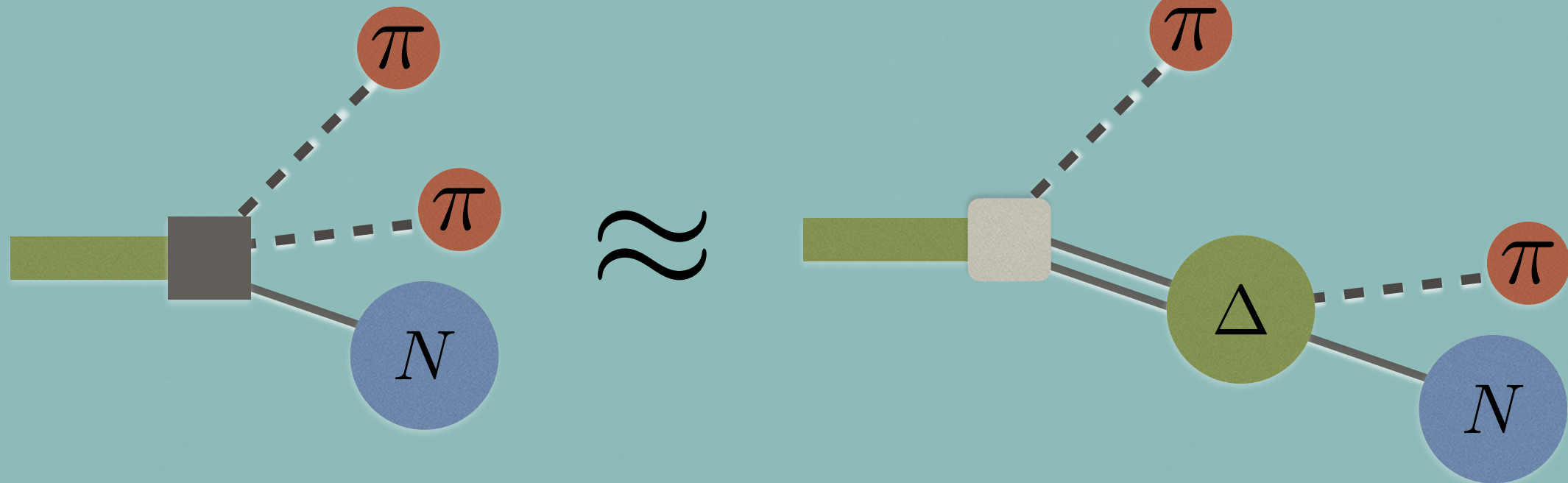
S. Jeon and P. J. Ellis PRD 58 045013 (1998)

ISOBAR MODEL

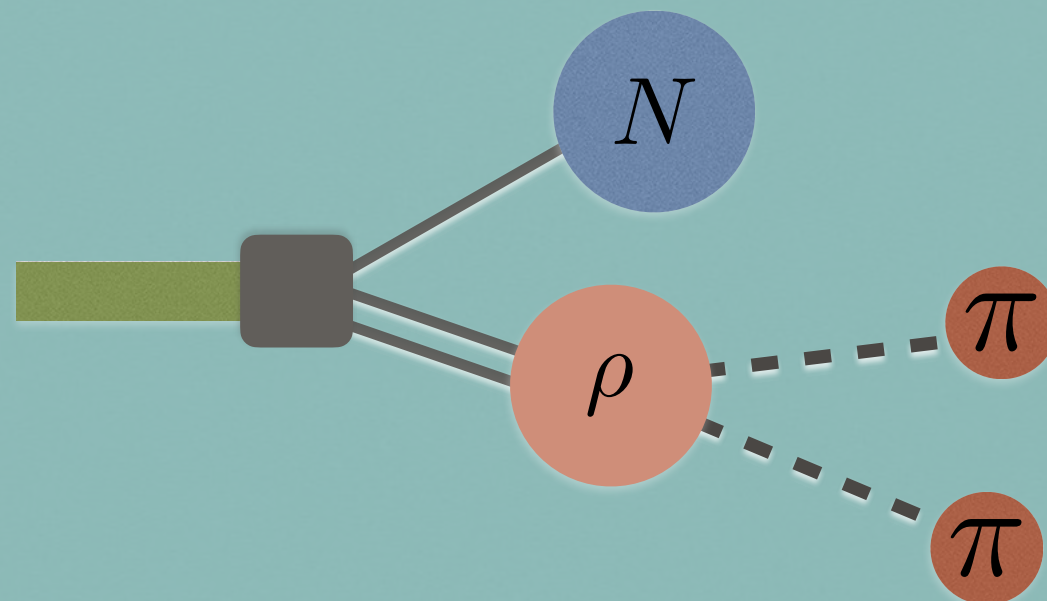


ISOBAR MODEL

sequential decay model



and / or



THANK YOU