$\mu^+\mu^-$ Production by Muons in Geant4

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Contents

Background

The muon is an important particle in particle physics experiments

Geant4 at present has four muon processes; e^+e^- pair production, ionization, bremsstrahlung and muon-nucleus interactions

At high energies, the cross section for $\mu^+\mu^-$ production is no longer negligible

With the high energies of the LHC and the FCC, there is a need to include the process in simulations

Theory of muon pair production

The most important aspect of the theory is the cross-section formula

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Kelner, Kokoulin and Petrukhin computed in their 2000 paper a cross section formula that considered the finite nuclear size as well as the screening effect

$$\sigma(E, v, \rho)\, dv\, d\rho = \frac{2}{3\pi}\,(Z\alpha\, r_\mu)^2\, \frac{1-v}{v}\, \Phi(v,\rho)\, \ln(X)\; dv\, d\rho\,. \quad (3)$$

Here $v = (E - E')/E = (E_+ + E_-)/E$ is the fraction of the energy transferred to the particles of the pair; E, E' are initial and final energy of the parent particle; E_+, E_- are energies of the produced particles; $\rho = (E_+ - E_-)/(E_+ + E_-)$ is the pair asymmetry parameter. Final energies are related with v, ρ by

$$E' = E\,(1 - v)\,, \quad E_\pm = Ev\,(1 \pm \rho)/2\,. \quad (4)$$

Kinematic region is defined by

$$2\mu/E \le v \le 1 - \mu/E\,, \quad |\rho| \le \rho_{\max} \equiv 1 - 2\mu/(vE)\,. \quad (5)$$

Function Φ may be expressed as

$$\Phi(v,\rho) = \Big((2+\rho^2)(1+\beta) + \xi\,(3+\rho^2)\Big)\ln\left(1 + \frac{1}{\xi}\right) - 1 - 3\,\rho^2 + \beta\,(1 - 2\,\rho^2) +$$

$$+ \Big((1+\rho^2)\left(1 + \frac{3}{2}\beta\right) - \frac{1}{\xi}\,(1 + 2\,\beta)(1-\rho^2)\Big)\ln(1+\xi)\,, \quad (6)$$

with

$$\xi = \frac{v^2(1-\rho^2)}{4\,(1-v)}\,, \quad \beta = \frac{v^2}{2\,(1-v)}\,. \quad (7)$$

Argument X of the logarithm in (3) is defined as follows. Let us denote as $U(E, v, \rho)$ the function

$$U(E,v,\rho) = \frac{0.65\,A^{-0.27}\,B\,Z^{-1/3}\,\mu/m}{1 + \dfrac{2\sqrt{e}\,\mu^2 B\,Z^{-1/3}(1+\xi)(1+Y)}{m\,E\,v\,(1-\rho^2)}}\,, \quad (8)$$

where $B = 183$, $e = 2.718\ldots$, A is atomic weight, $Y = 10\sqrt{\mu/E}$. Then

$$X = 1 + U(E, v, \rho) - U(E, v, \rho_{\max})\,, \quad (9)$$

with $\rho_{\max}$ defined by (5). The function U is chosen in such a way to reproduce the main logarithmic factor in the limiting cases of absence of screening and complete screening. Function Y and numerical constants serve to improve the description of the total cross section. Cross section $\sigma(E, v, \rho)$ is non-negative in the kinematic region (5) and comes to zero at $\rho = \pm\rho_{\max}$. Comparison with numerical integration shows that the accuracy of (3) is better than 10% for $E > 10$ GeV and final particle energies E', $E_+, E_- > 1$ GeV, the total cross section being reproduced better than 3% for $E > 30$ GeV.

Formulae (3)-(9) describe the distribution of final particles in $(v,\, \rho)$ variables. To obtain the distribution in the energies of the particles of the pair E_+, E_-, it is sufficient to use

$$\sigma(E, E_+, E_-)\, dE_+\, dE_- = \frac{2}{E^2 v}\,\sigma(E, v, \rho)\, dE_+\, dE_-\,. \quad (10)$$

Evaluation of cross-section

The cross-section formula mentioned in the previous slide is the second-derivative of the cross-section $d^2\sigma(E, v, \rho)$

What we need, however, is the value of the cross-section itself – this can be obtained by numerically integrating $d^2\sigma(E, v, \rho)$ in the kinematic region mentioned in the paper

To do this, we follow a process like what is mentioned in the Physics Reference Manual for electron-positron pair production – we integrate over ρ followed by v.

Comment on the Calculation of the Integral $\int d\rho$ in Eq.(155)

The integral $\int_0^{\rho_{max}} G(Z, E, v, \rho)\, d\rho$ is computed with the substitutions:

$$\begin{aligned} t &= \ln(1-\rho), \\ 1-\rho &= \exp(t), \\ 1+\rho &= 2-\exp(t), \\ 1-\rho^2 &= e^t\,(2-e^t). \end{aligned}$$

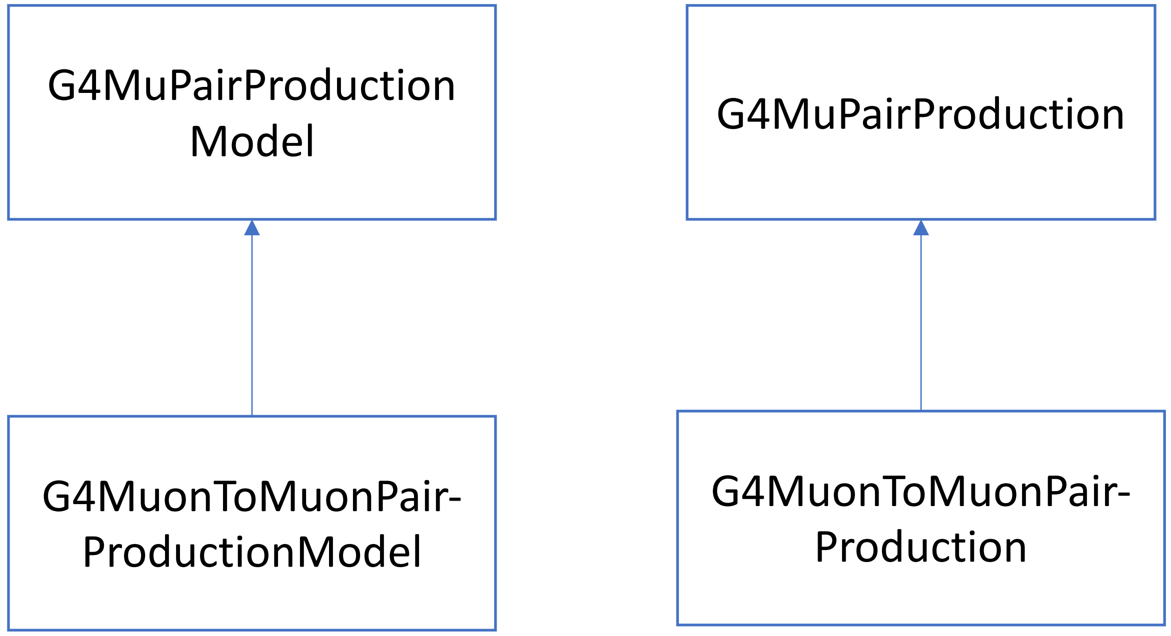
After that,

$$\int_0^{\rho_{max}} G(Z, E, v, \rho)\, d\rho = \int_{t_{min}}^{0} G(Z, E, v, \rho)\, e^t\, dt, \qquad (156)$$

Implementation

The class G4MuPairProduction(Model) was used as the base class for our new classes G4MuonToMuon-PairProduction(Model).

Two methods were overridden; ComputeD-MicroscopicCrossSection and Sample-Secondaries



Implementation

The method which dictates the final value of any of the quantities calculated (dE/dx, MuPairLoss, cross section) is ComputeDMicroscopicCrossSection

We use the formulae and kinematic region mentioned in earlier along with a weighted 8-point Simpson integral identical to the one used in the original electron-positron pair production

```cpp
G4double G4MuonToMuonPairProductionModel::ComputeDMicroscopicCrossSection(
                                            G4double tkin,
                                            G4double Z,
                                            G4double pairEnergy)
// Calculates the  differential (D) microscopic cross section
// using the cross section formula of Kelner, Kokoulin and Petrukhin (1999)
// Code written by Siddharth Yajaman (12/07/2022)
{
  if (pairEnergy <= 2. * particleMass)
    return 0.0;

  G4double totalEnergy  = tkin + particleMass;
  G4double residEnergy  = totalEnergy - pairEnergy;

  if (residEnergy <= particleMass)
    return 0.0;

  G4double a0 = 1.0 / (totalEnergy * residEnergy);
  G4double rhomax = 1.0 - 2*particleMass/pairEnergy;
  G4double tmnexp = 1. - rhomax;

  if(tmnexp >= 1.0) { return 0.0; }

  G4double tmn = G4Log(tmnexp);

  G4double z2 = Z*Z;
  G4double beta = 0.5*pairEnergy*pairEnergy*a0;
  G4double xi0 = 0.5*beta;

  // Gaussian integration in ln(1-ro) ( with 8 points)
  G4double rho[8];
  G4double rho2[8];
  G4double xi[8];
  G4double xi1[8];
  G4double xii[8];

  for (G4int i = 0; i < 8; ++i)
  {
    rho[i] = G4Exp(tmn*xgi[i]) - 1.0; // rho = -asymmetry
    rho2[i] = rho[i] * rho[i];
    xi[i] = xi0*(1.0-rho2[i]);
    xi1[i] = 1.0 + xi[i];
    xii[i] = 1.0 / xi[i];
  }

  G4double ximax = xi0*(1. - rhomax*rhomax);

  G4double Y = 10 * sqrt(particleMass/totalEnergy);
  G4double U[8];
```

```cpp
  for (G4int i = 0; i < 8; ++i)
  {
    U[i] = U_func(Z, rho2[i], xi[i], Y, pairEnergy);
  }

  G4double UMax = U_func(Z, rhomax*rhomax, ximax, Y, pairEnergy);

  G4double sum = 0.0;

  for (G4int i = 0; i < 8; ++i)
  {
    G4double X = 1 + U[i] - UMax;
    G4double lnX = G4Log(X);
    G4double phi = ((2 + rho2[i])*(1 + beta) + xi[i]*(3 + rho2[i]))*
                   G4Log(1 + xii[i]) - 1 - 3*rho2[i] + beta*(1 - 2*rho2[i])
                   + ((1 + rho2[i])*(1 + 1.5*beta) - xii[i]*(1 + 2*beta)
                   *(1 - rho2[i]))*G4Log(xi1[i]);
    sum += wgi[i]*(1.0 + rho[i])*phi*lnX;
  }

  return -tmn*sum*factorForCross*z2*residEnergy/(totalEnergy*pairEnergy);

}

G4double G4MuonToMuonPairProductionModel::U_func(G4double ZZ, G4double rho2,
                                                  G4double xi, G4double Y,
                                                  G4double pairEnergy,
                                                  const G4double B)
{
  G4int Z = G4lrint(ZZ);
  G4double A27 = nist->GetA27(Z);
  G4double Z13 = nist->GetZ13(Z);
  static const G4double sqe = std::sqrt(G4Exp(1.0));
  G4double res;
  res = (0.65 * B / (A27*Z13) * MUONELECTRONMASSRATIO)/(1 + (2*sqe
        *pow(particleMass, 2)*(B/Z13)*(1 + xi)*(1 + Y))
        /(CLHEP::electron_mass_c2*pairEnergy*(1 - rho2)));
  return res;
}
```

Changes to Geant4 example TestEm17

Since we added a new muon process, appropriate changes had to be made to TestEm17 which is an example focusing on muons

The local physics list PhysListEmStandard was modified to include the new process

MuCrossSections was modified to calculate the differential cross section of the new process

RunAction was modified to account for the new process and also generate some new histograms

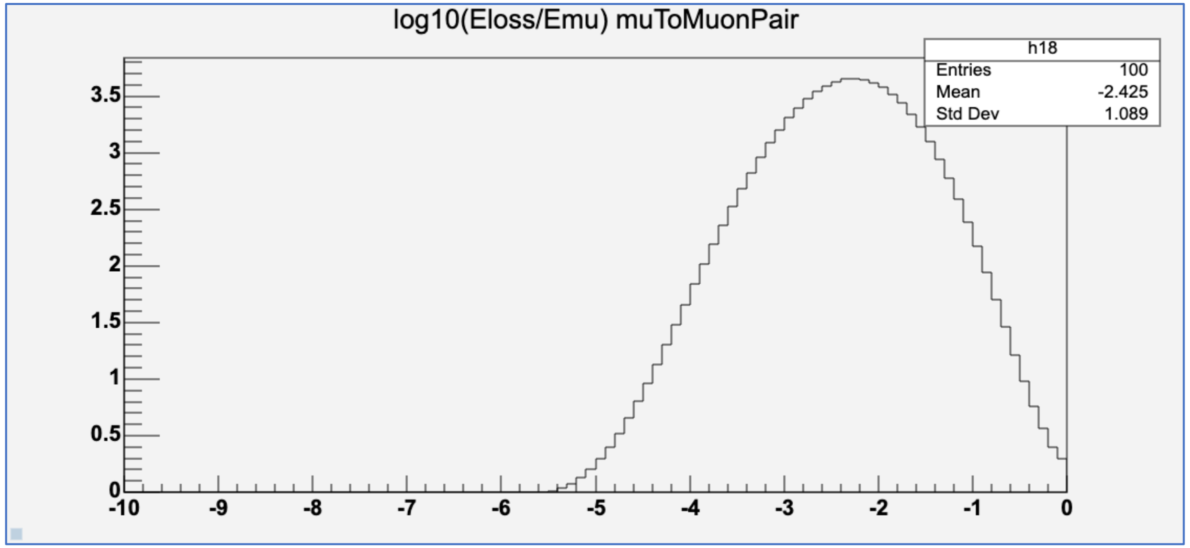
HistoManager was modified so that we could accommodate the new process and the new histograms from RunAction

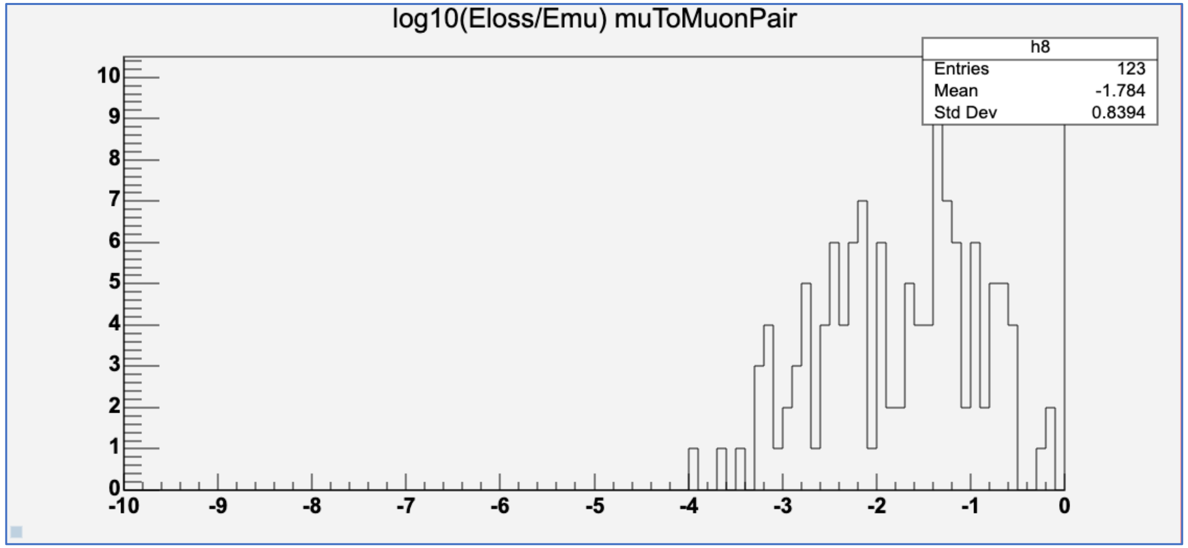
Results

```
Run Summary
  Number of events processed : 1000000
  User=41.840000s Real=41.920882s Sys=0.080000s [Cpu=100.0%]

 The run consists of 1000000 mu+ of 1e+02 TeV through 1 m   of Iron (density: 7.9 g/cm3 )

 Number of process calls --->   muPairProd : 10540368   muIoni : 47204146       muBrems : 75376
 Simulation: total CrossSection = 0.5782 /cm     MeanFreePath = 1.7295 cm         massicCrossSection = 0.073469 cm2/g
 Theory:     total CrossSection = 0.58768 /cm    MeanFreePath = 1.7016 cm         massicCrossSection = 0.074673 cm2/g
```

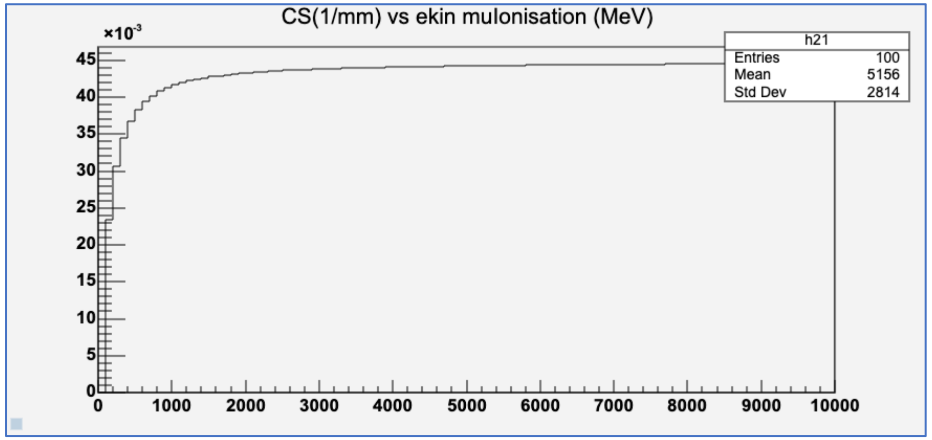
```
Run Summary
  Number of events processed : 1000000
  User=43.200000s Real=43.289958s Sys=0.080000s [Cpu=100.0%]

 The run consists of 1000000 mu+ of 1e+02 TeV through 1 m   of Iron (density: 7.9 g/cm3 )

 Number of process calls --->   muIoni : 47216477       muPairProd : 10540675   muBrems : 75473 muToMuonPairProd : 123
 Simulation: total CrossSection = 0.57833 /cm    MeanFreePath = 1.7291 cm        massicCrossSection = 0.073485 cm2/g
 Theory:     total CrossSection = 0.58768 /cm    MeanFreePath = 1.7016 cm        massicCrossSection = 0.074673 cm2/g
```

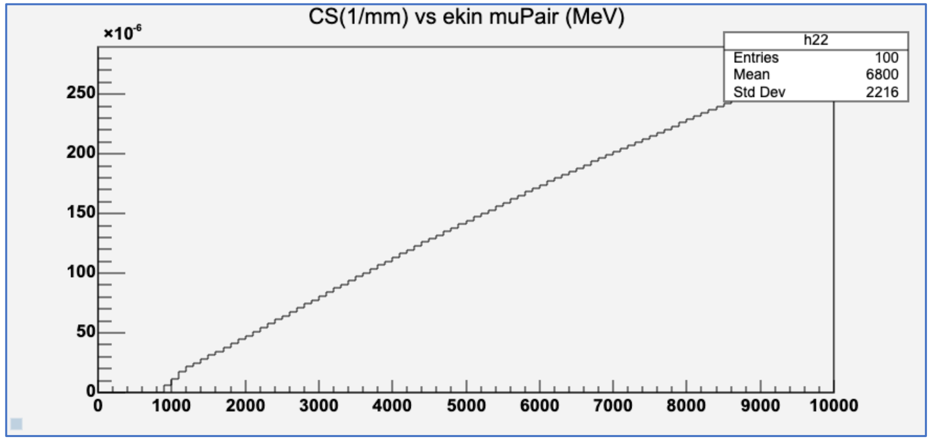
Energy transfer to lepton pair

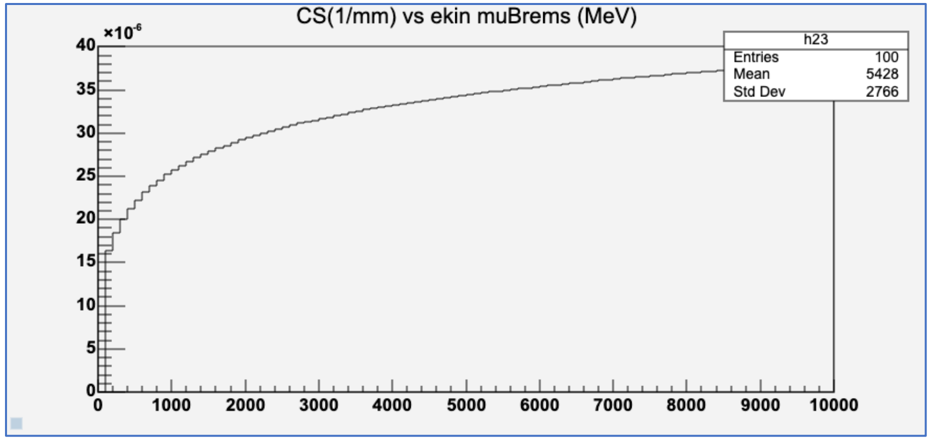


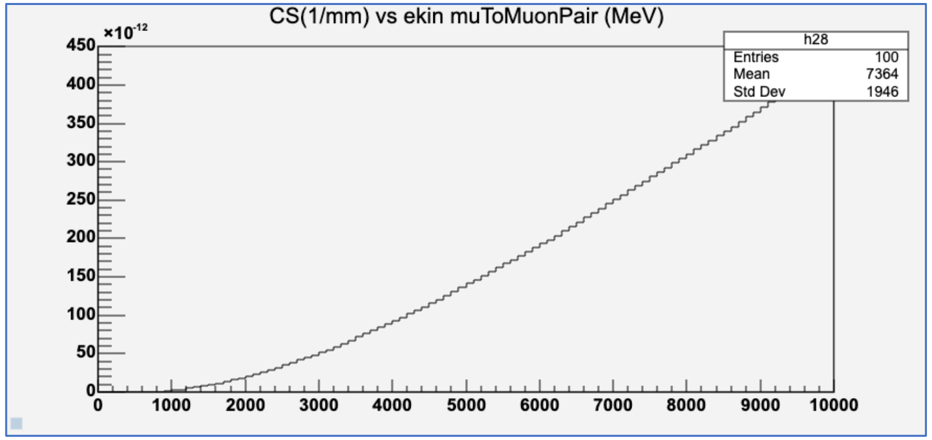


Cross section of muon processes









Conclusion

The process was successfully implemented in Geant4

It will be available in the next public release of Geant4

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