# Standard Model PDFs for the Muon Collider


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LISHEP2023
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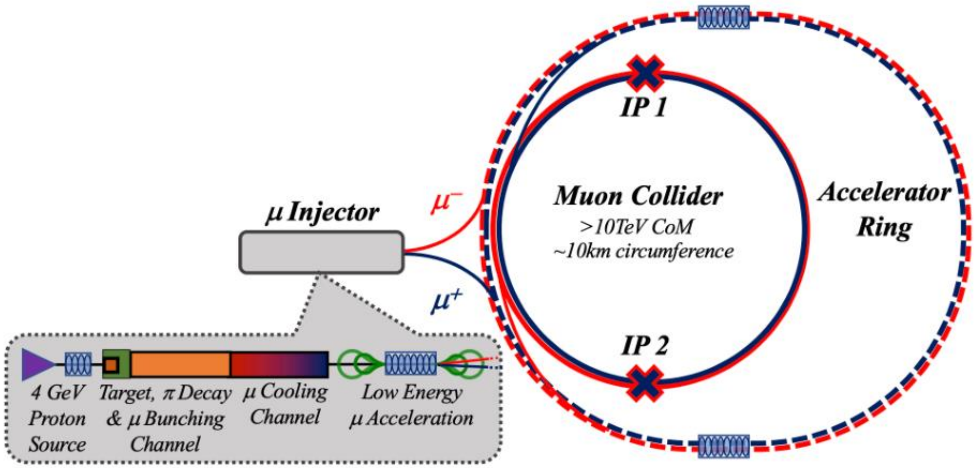
[Garosi, Marzocca, ST] TBA

# High Energy Muon Collider




- A Muon Collider (MuC) collaboration has been created at CERN.
- EU Design Study for a MuC has been approved.

- There could be a staged development, with a 3 TeV first phase and a 10 TeV later. Several components could be re-used.
- Collider Rings: **3 TeV ~ 4.5 km** circumference
  **10 TeV ~ 10 km** circumference

[Snomass reports]
2203.08033, 2203.07224,
2203.07256, 2203.07261



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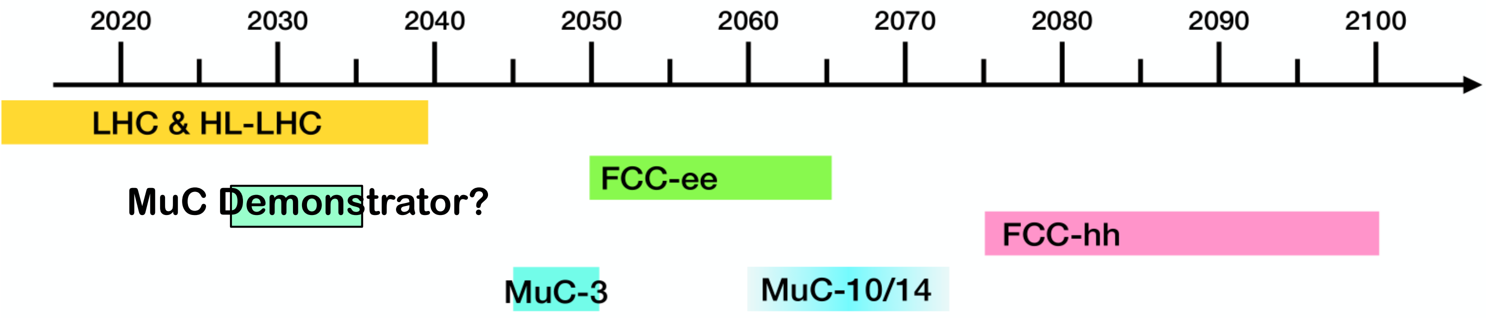
# Timelines

- Starting now, a 3 TeV MuC could start physics in ~2045.

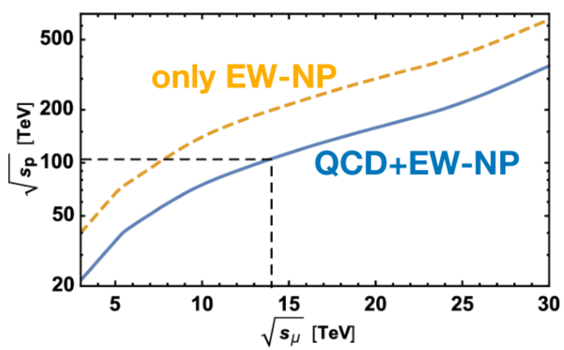



MuC R&D: Lots of progress on all fronts, no show-stoppers so far

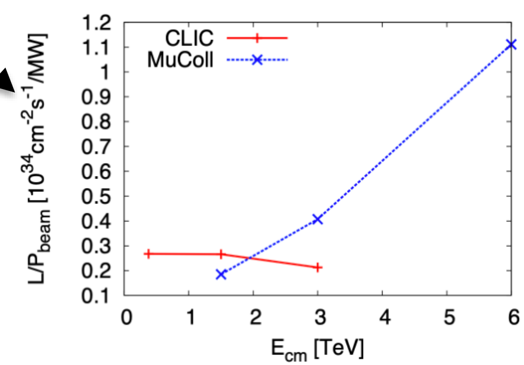
- A MuC could run in parallel with an $e^+e^-$ Higgs factory (FCC-ee, ILC)

# Why Muon Colliders?

- Muon colliders combine the advantages of both proton-proton (discovery) and electron-positron colliders (precision):
  - ❑ high energy reach (not limited by synchrotron radiation)
  - ❑ high precision measurements (low QCD background & clean initial state)
  - ❑ Luminosity / Beam power increases with energy.
  - ❑ all beam energy available in $\mu^+\mu^-$ collisions.




See also talk from S. R. Dasu




[David Shulte, CGI talk https://youtu.be/17JoTcuIs6k]

# The muon collider is a weak boson collider!

- At zeroth order in perturbation theory the muon carries all the momentum of the beam.
- At high energies, collinear radiation emitted by splitting of the initial state must be taken into account.
- For example, well above the NP scale $m_X$, we expect the **VBF** to become an important production channel:

$$\frac{\sigma_{\rm VBF}^{\rm BSM}}{\sigma_{\rm ann}^{\rm BSM}} \propto \alpha_W^2 \frac{s}{m_X^2} \log^2 \frac{s}{m_V^2} \log \frac{s}{m_X^2}$$

[The Muon Smasher's Guide] 2103.14043

The MuC overqualifies as a Higgs factory!

The muon beam includes all other SM particles (including quarks and gluons)!!

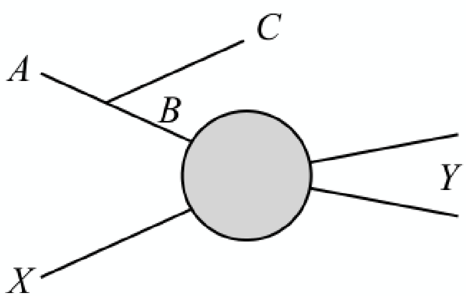
# Muon PDFs and DGLAP equations

- The initial muon state can be treated in the same way as a proton, using generalized parton distribution functions (PDFs) $f_A(x, Q)$

$$\sigma(\mu + X \to Y) = \sum_A \int_0^1 dx\, f_A(x) \sigma_x(A + X \to Y)$$

[Han et al]
2007.14300,
2103.09844

- Strongly-ordered multiple splittings can be resummed, obtaining **DGLAP evolution** for PDFs of a lepton (which can be solved perturbatively!)




$$Q^2 \frac{df_B(x, Q^2)}{dQ^2} = P_B^v f_B(x, Q^2) + \sum_{A,C} \frac{\alpha_{ABC}(Q)}{2\pi} \int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

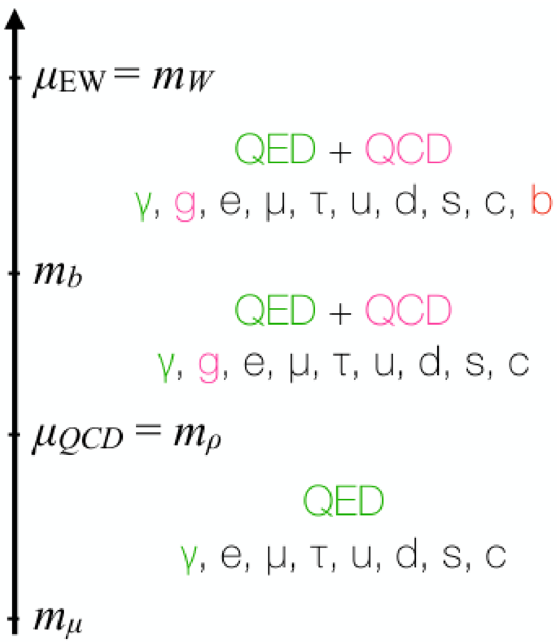
virtual corrections

splitting functions

# Evolution below the EW scale

- At LO the **boundary conditions** are $f_\mu(x, m_\mu) = \delta(1-x)$, $f_{i\neq\mu}(x, m_\mu) = 0$.

[Fixione] 1909.03886




- Solving the DGLAP equations accounts for resummation of the Leading Logs (LL):

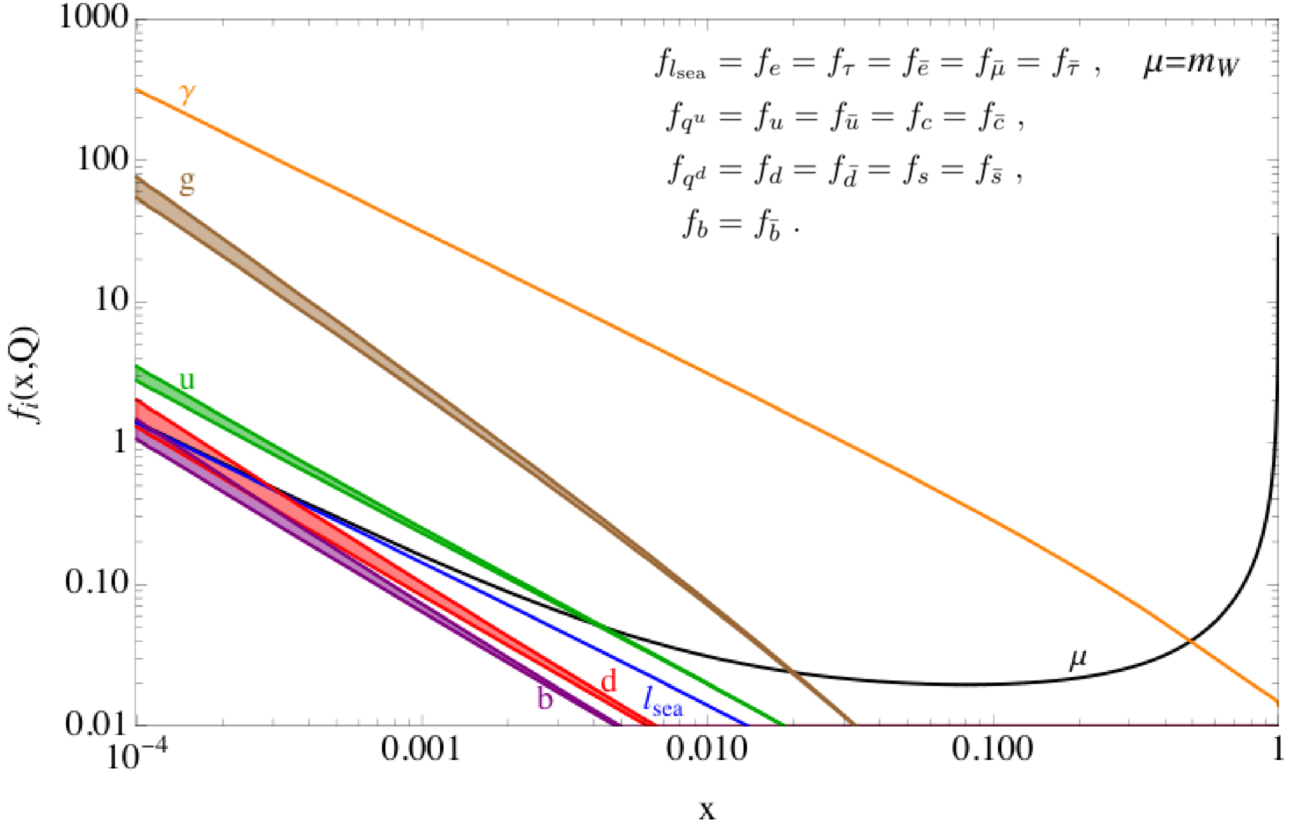
$$(\alpha \log \mu / m_\mu)^n$$

The procedure is necessary for collinear QCD.

- Numerical procedure:

1. Discretization in a grid of size $N_x$ in x.
2. Integrate using the rectangular method.
3. Solve numerically the differential equation system using the Runge-Kutta algorithm
4. Impose momentum conservation in each step.

# PDFs at the EW threshold



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# Evolution above the EW scale

- The **full unbroken SM** interactions must be considered.
- We work in the mass eigenstates basis and in the *Goldstone Equivalent Gauge*. The same numerical method used below the EW scale is employed.
- EWSB modifies the DGLAP equations as follows

[Chen et al]
1611.00788

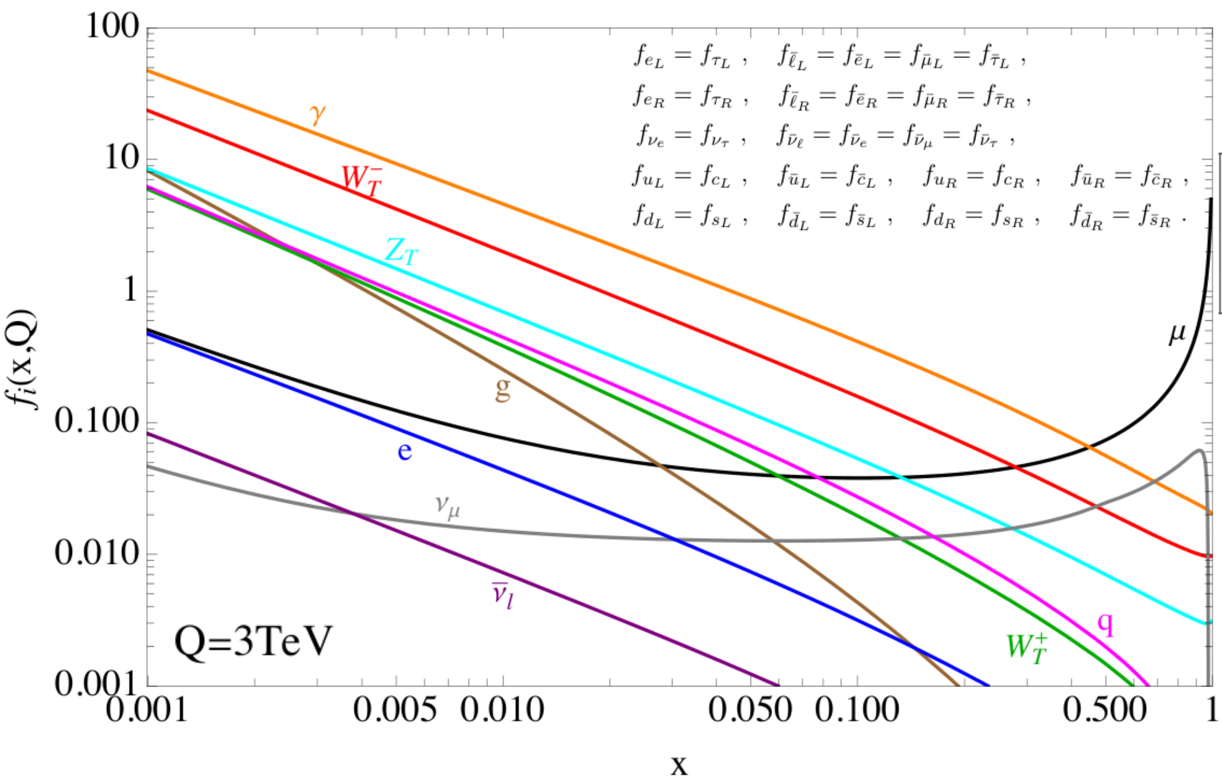
$$Q^2 \frac{df_B(x,Q^2)}{dQ^2} = P_B^v f_B(x,Q^2) + \sum_{A,C} \frac{\alpha_{ABC}}{2\pi} \widetilde{P}_{BA}^C \otimes f_A + \frac{v^2}{16\pi^2 Q^2} \sum_{A,C} \widetilde{U}_{BA}^C \otimes f_A$$

i. **Massive propagators**: $\widetilde{P}_{BA}^C(z,p_T^2) = \left(\frac{p_T^2}{\widetilde{p}_T^2}\right)^2 P_{BA}^C(z)$ , $\widetilde{p}_T^2 \equiv \bar{z}(m_B^2 - q^2)$

ii. **Ultra-collinear splittings**: $|\mathcal{M}_{A\to B+C}|^2 \equiv 8\pi\alpha_{ABC} \frac{p_T^2}{z\bar{z}} P_{BA}^C(z) + \frac{v^2}{z\bar{z}} U_{BA}^C(z)$

e.g. for $f_L^{(1)} \to W_L f_L^{(2)}$: $U_{W_L f_L^{(1)}}^{f_L^{(2)}}(z) = \left(y_{f_1}^2 z\bar{z} - y_{f_2}^2 z - g_2^2 \bar{z}\right)^2 \frac{1}{2z}$

# PDFs above the EW scale




- The DGLAP system contains **42 independent PDFs**.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $\mu_L$ | $\mu_R$ | $e_L$ | $e_R$ | $\nu_\mu$ | $\nu_e$ | $\bar{\ell}_L$ | $\bar{\ell}_R$ | $\bar{\nu}_\ell$ |
| $u_L$ | $d_L$ | $u_R$ | $d_R$ | $t_L$ | $t_R$ | $b_L$ | $b_R$ | + h.c. |
| $\gamma_\pm$ | $Z_\pm$ | $Z\gamma_\pm$ | $W^\pm_\pm$ | $G_\pm$ | | | | |
| $h$ | $Z_L$ | $hZ_L$ | $W^\pm_L$ | | | | | |

- The photon-$Z_T$ boson interference is described by a mixed $Z_T/\gamma$ PDF. Similarly for $Z_L$ and $H$.

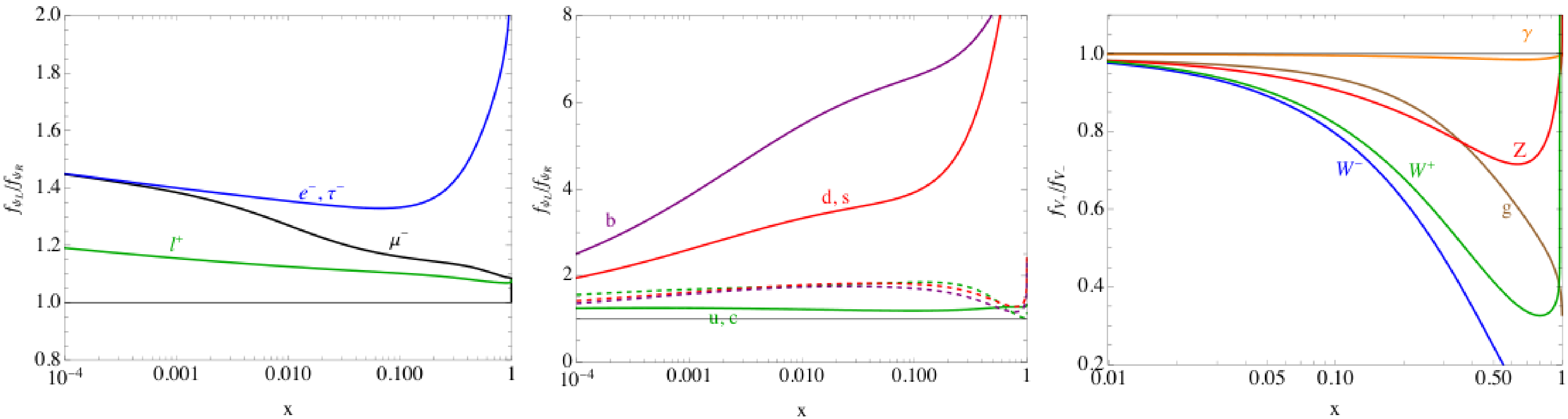
[Ciafaloni et al]
hep-ph/0505047

- We provide the PDFs both without and with the top.

# Polarizations

[Bauer et al]
1808.08831

- Due to the chiral nature of $SU(2)_L$, PDFs become polarized.
- Splitting functions depend on the helicity of the states. E.g. in case of $W$-PDF, coupled to $\mu_L$, the PDF or RH $W$ goes to zero for $x \to 1$ faster than LH $W$, since $P_{V_+ f_L}(z) \;=\; (1-z)/z$ while $P_{V_- f_L}(z) \;=\; 1/z$.



# EW Sudakov double logs

[Ciafaloni et al] hep-ph/0001142

- The *Bloch-Nordsieck theorem* is violated for non-abelian gauge theories.
- The EW **Sudakov double logs** arises as a non-cancellation of the IR soft divergences ($z \to 1$) between real emission and virtual corrections in isospin flipping transitions (e.g. $\mu_L \leftrightarrow \nu_\mu$ with $W^\pm$ emission). For these splititngs we introduce the explicit IR cut-off $z_{\max}^{ABC}(Q) = 1 - Q_{\mathrm{EW}}/Q$

[Bauer et al] 1703.08562

$$\frac{\alpha_{ABC}(Q)}{2\pi}\int_x^1 \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right) \quad \to \quad \frac{\alpha_{ABC}(Q)}{2\pi}\int_x^{z_{\max}^{ABC}(Q)} \frac{dz}{z} P_{BA}^C(z) f_A\left(\frac{x}{z}, Q^2\right)$$

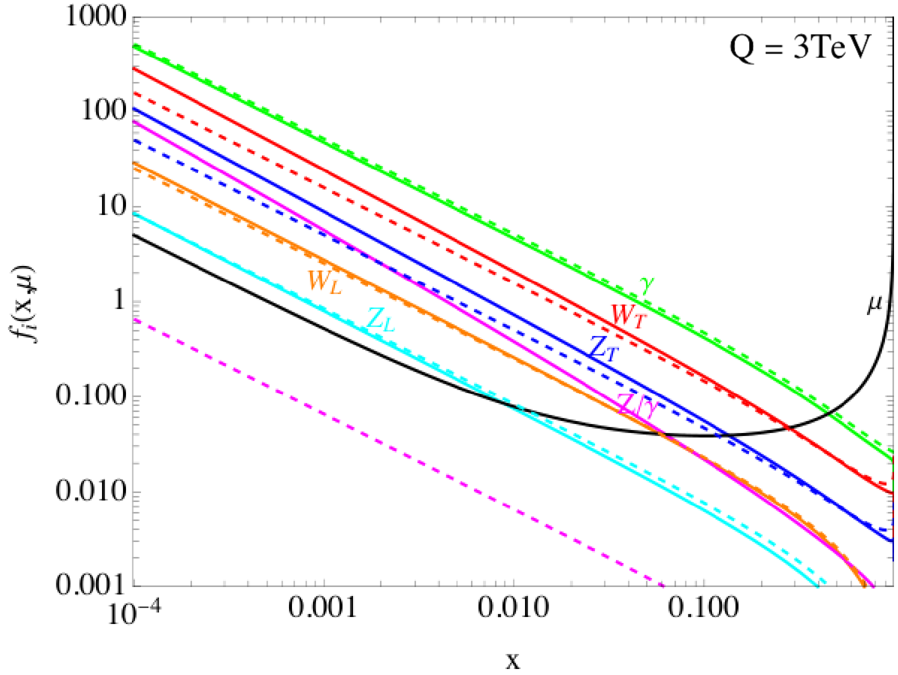
The virtual corrections are modified accordingly.

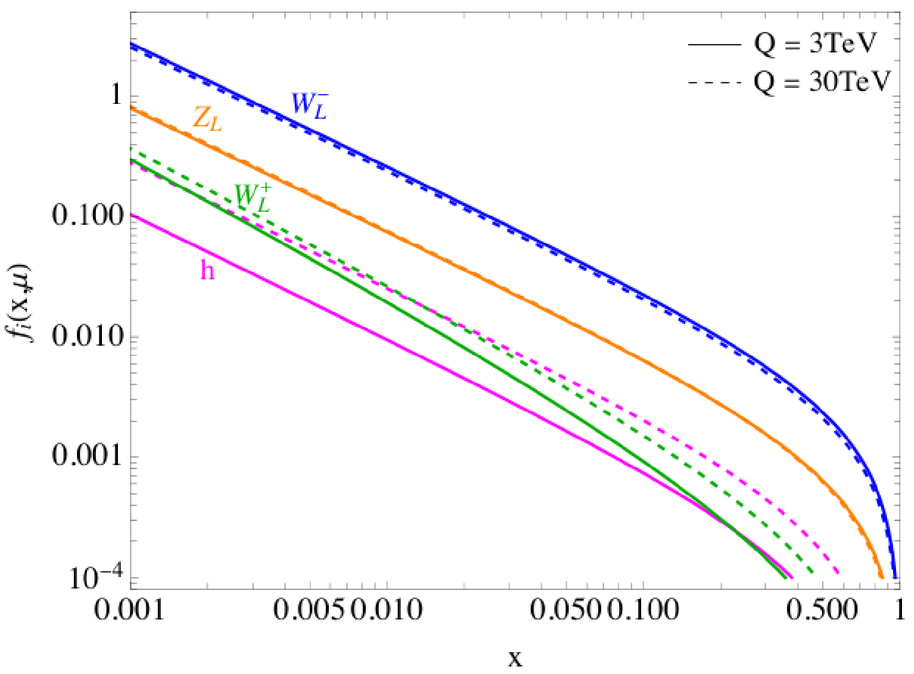
- The physical effect of these double logs is to restore $SU(2)_L$ invariance at high scales.

# Effective Vector Boson Approximation (EVA)

- The case of collinear photon emission from an electron gives the *Equivalent Photon Approximation* (EPA): $f_\gamma^{\mathrm{EPA}}(x) = \frac{\alpha_\gamma}{2\pi} P_{\gamma e}(x) \log \frac{E^2}{m_e^2}$
- The EPA has been generalized to describe EW gauge bosons in high-energy collisions, in what is now known as **EVA**.
- Solving the DGLAP equations iteratively at LO we recover the EVA.
- However, in case of transverse gauge bosons PDFs we notice significant discrepancies from the numerical LL result mainly in the transverse gauge bosons PDFs. They can be traced back to reasons such as:
  - × The $V \to VV$ is not incorporated in EVA in LO.
  - × In EVA the initial state is assumed to be unpolarized.

# PDFs above the EW scale (gauge bosons & scalars)



# Conclusions

- The near-term future of particle physics will be charted by precision measurements. The long-term future of the field crucially depends on the decisions we make today about the next generation of high-energy colliders.
- The two most prominent options on the table are the FCC-hh and a multi-TeV MuC.

Note: MuC3 could start ~30 years before FCC-hh!

- In this work, we derive the SM PDFs for lepton colliders. We show that the EVA, on which current estimates of cross-sections are based, is not always an adequate approximation.
- We aim at making our result public in a **LHAPDF-type** format, which should be extended to include helicity. Ultimately, an implementation of our results (e.g. in **MadGraph)** is important for any SM or BSM research in the MuC.

# Thank you!!!!


International
MUON Collider
Collaboration

# Backup slides

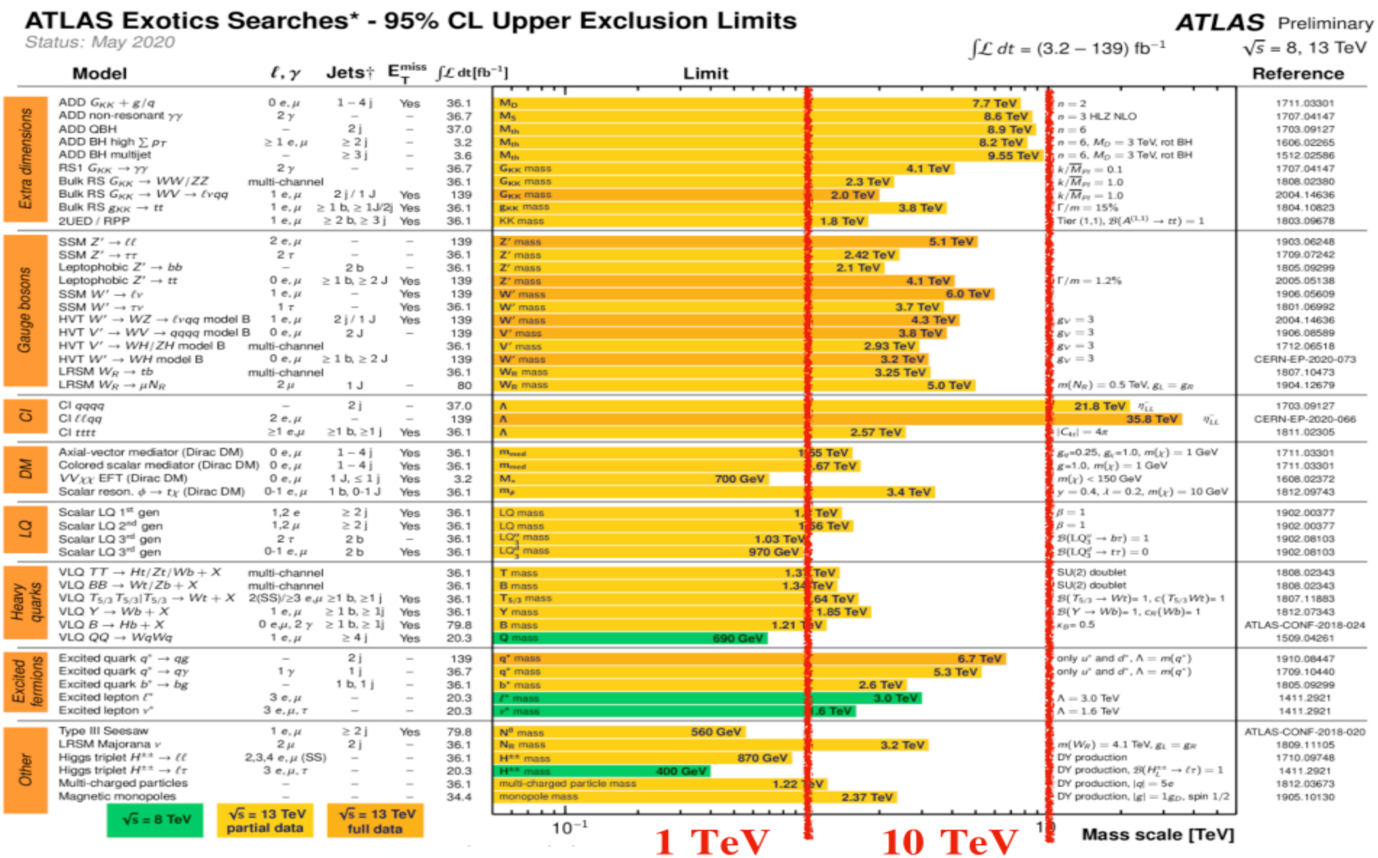
# LHC: the past and the future

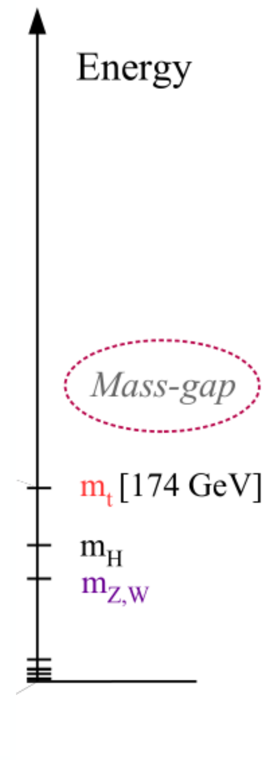
[Azatov, Garosi, Greljo, Marzocca, Salko, ST] 2205.13552

- LHC has already provided ground-breaking results:
  - ✓ completion of the SM spectrum (Higgs boson discovery)
  - ✓ exquisite precise measurements of a huge number of other SM processes
  - ✓ fundamentally challenged our New Physics expectations at the EW scale


- We are moving towards the HL-phase and there is still lots of data to collect!

# Still no direct evidence for NP!

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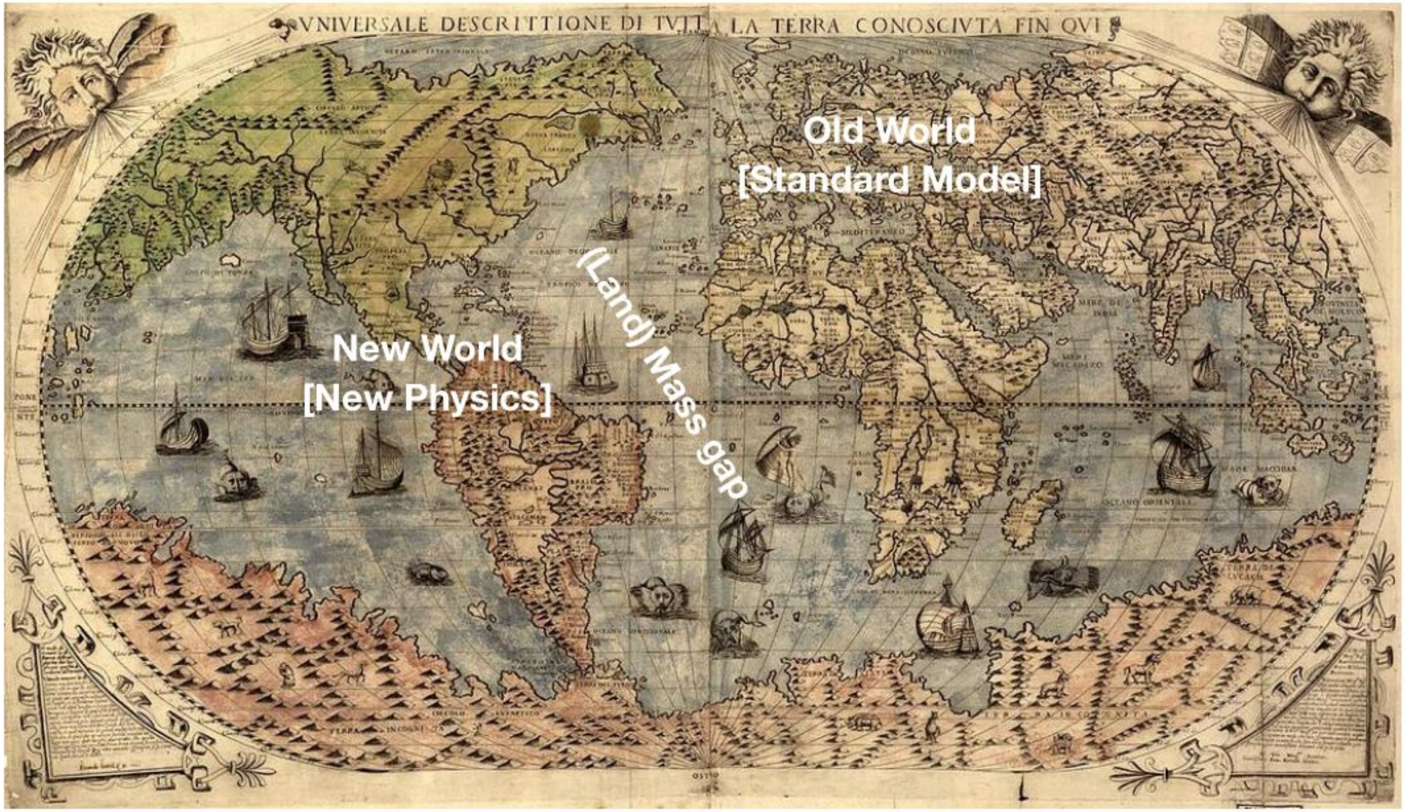
# The search for Terra Incognita




[J. Fuentes-Martin]

# New Physics Quest: two avenues

**High-energy frontier (ATLAS, CMS & future colliders):** Direct discovery of NP, but the mass gap should not be too large

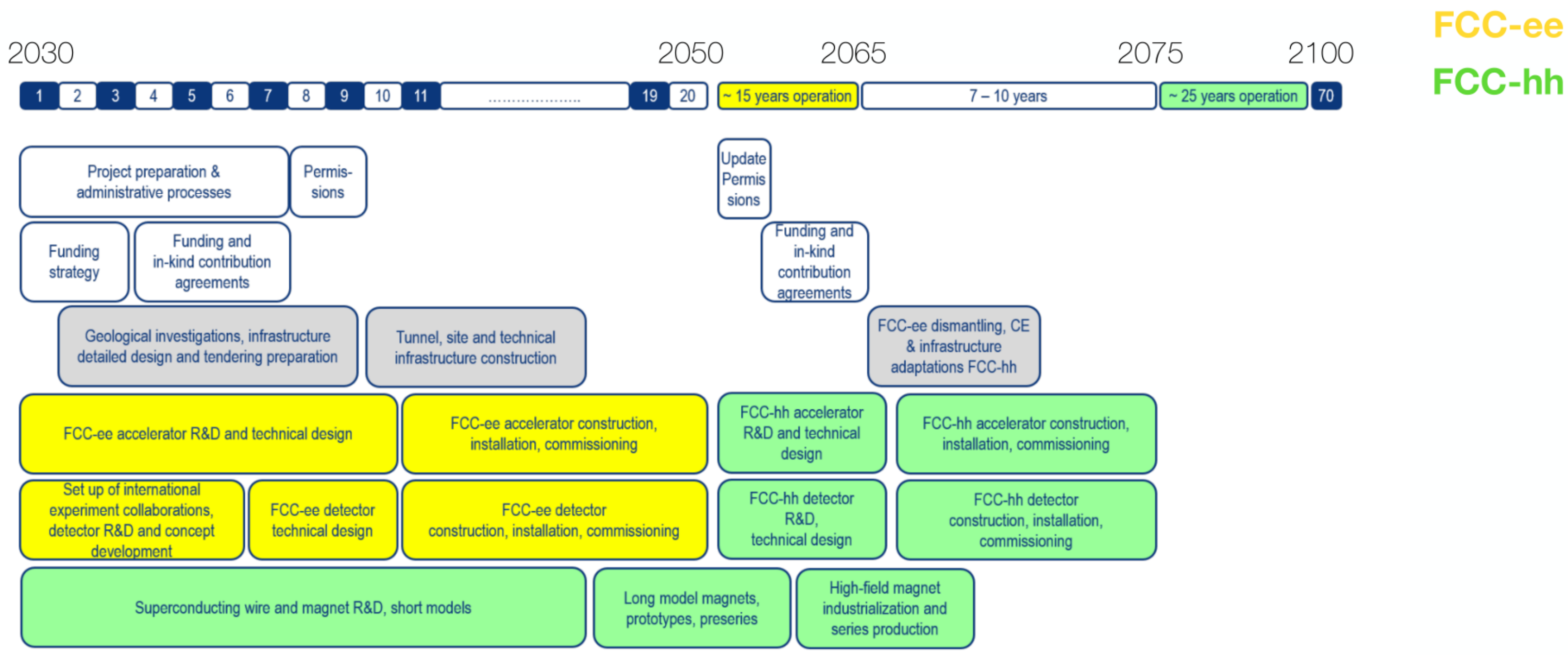
**Precision frontier (COMET, mu3e, LHCb, Belle II,…):** Indirect NP evidence in low-energy probles, breaking of (approximate) SM symmetries

# New interactions within reach

- Which future collider would offer best sensitivity reach for **tree-level** heavy NP mediators?

| **Collider** | **C.o.m. Energy** | **Luminosity** | **Label** |
|---|---|---|---|
| LHC Run-2 | 13 TeV | 140 $\text{fb}^{-1}$ | LHC |
| HL-LHC | 14 TeV | 6 $\text{ab}^{-1}$ | HL-LHC |
| FCC-hh | 100 TeV | 30 $\text{ab}^{-1}$ | FCC-hh |
| Muon Collider | 3 TeV | 1 $\text{ab}^{-1}$ | MuC3 |
| Muon Collider | 10 TeV | 10 $\text{ab}^{-1}$ | MuC10 |
| Muon Collider | 14 TeV | 20 $\text{ab}^{-1}$ | MuC14 |

# FCC timeline



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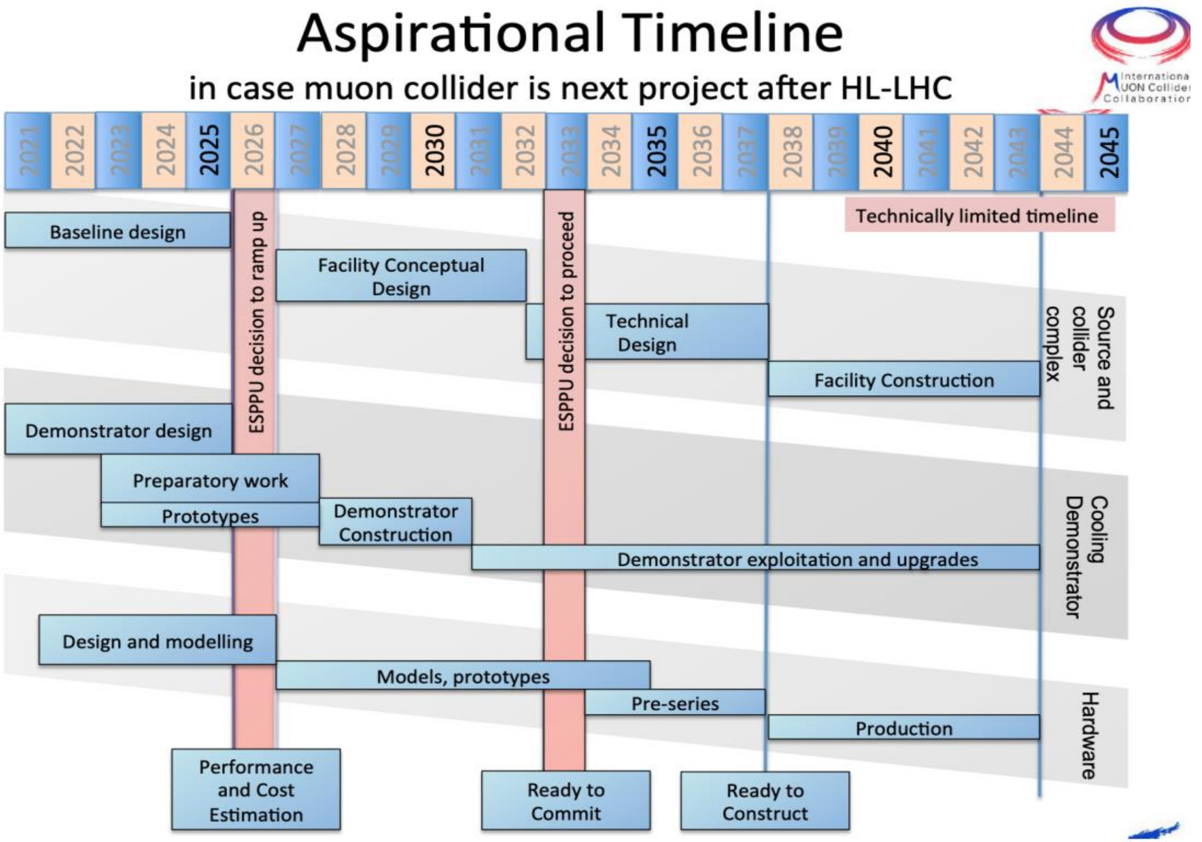
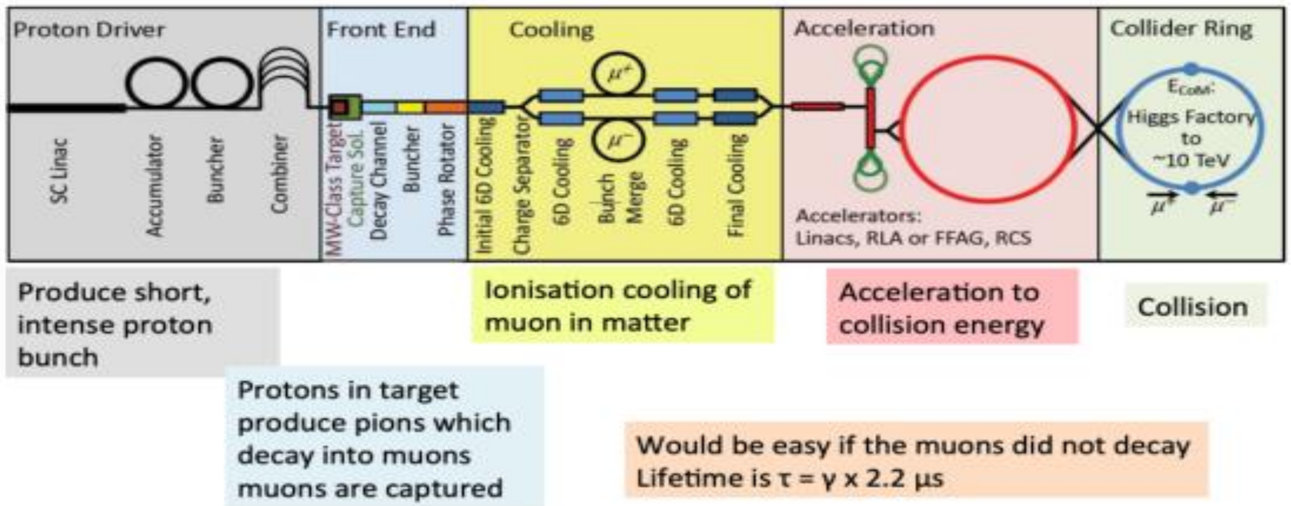
# MuC timeline

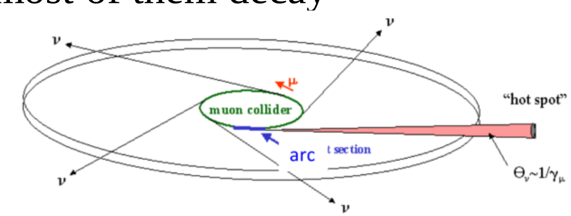


# High Energy Muon Collider (design)




➢ **Key Challenges** / **Opportunities for progress :**

- ❑ $\mu^+\mu^-$ must be cooled and accelerated before most of them decay
- ❑ Intense and collimated high-energy beam of neutrinos induces potential radiation risk.



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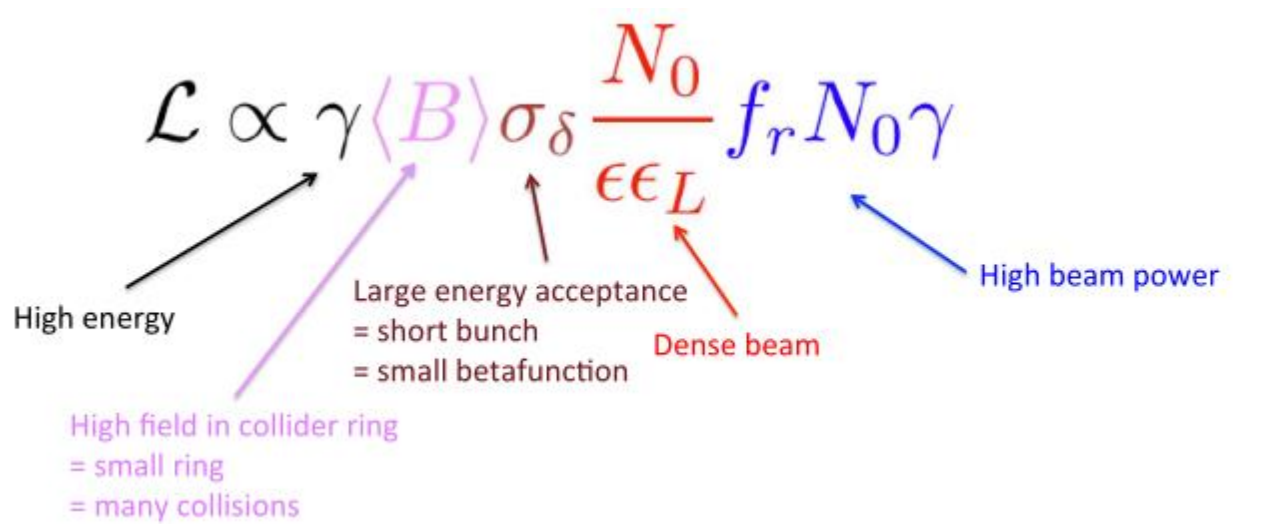
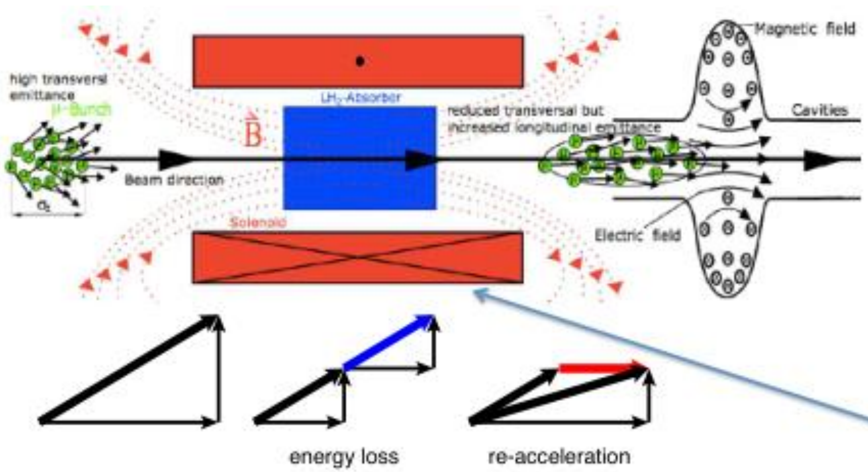
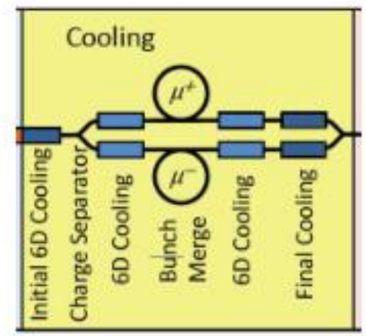
# MuC Luminosity Scaling

- Assumes no emittance growth after source and no technical limitation.
- Applies to MAP scheme



# Final cooling in MuC




High field solenoids minimise beta-function and impact of multiple scattering

Energy loss = cooling

Multiple scattering = heating

$$\frac{d\epsilon_\perp}{ds} = -\frac{1}{(v/c)^2}\frac{dE}{ds}\frac{\epsilon_\perp}{E} + \frac{1}{2}\frac{1}{(v/c)^3}\left(\frac{14\,\mathrm{MeV}}{E}\right)^2\frac{\beta\gamma}{L_R}$$

# DGLAP equations below the EW scale

$$
\begin{aligned}
\frac{df_l}{dt} &= \frac{\alpha_\gamma(t)}{2\pi}\left[\left(P_f^v + P_{ff}^V\right)\otimes f_l + P_{fV}^f \otimes f_\gamma\right] \ , \\
\frac{df_{q^u}}{dt} &= \frac{\alpha_\gamma(t)}{2\pi} Q_u^2 \left[\left(P_f^v + P_{ff}^V\right)\otimes f_{q^u} + N_c P_{fV}^f \otimes f_\gamma\right] \\
&\quad + \frac{\alpha_3(t)}{2\pi}\left[C_F\left(P_f^v + P_{ff}^V\right)\otimes f_{q^u} + T_F P_{fV}^f \otimes f_g\right] \ , \\
\frac{df_{q^d,b}}{dt} &= \frac{\alpha_\gamma(t)}{2\pi} Q_d^2 \left[\left(P_f^v + P_{ff}^V\right)\otimes f_{q^d,b} + N_c P_{fV}^f \otimes f_\gamma\right] \\
&\quad + \frac{\alpha_3(t)}{2\pi}\left[C_F\left(P_f^v + P_{ff}^V\right)\otimes f_{q^d,b} + T_F P_{fV}^f \otimes f_g\right] \ , \\
\frac{df_\gamma}{dt} &= \frac{\alpha_\gamma(t)}{2\pi}\left[P_\gamma^v f_\gamma + \sum_f Q_f^2 P_{Vf}^f \otimes \left(f_f + f_{\bar f}\right)\right] \ , \\
\frac{df_g}{dt} &= \frac{\alpha_3(t)}{2\pi}\left[C_A\left(P_g^v + P_{VV}\right)\otimes f_g + C_F P_{Vf}^f \otimes \sum_q \left(f_q + f_{\bar q}\right)\right]
\end{aligned}
$$



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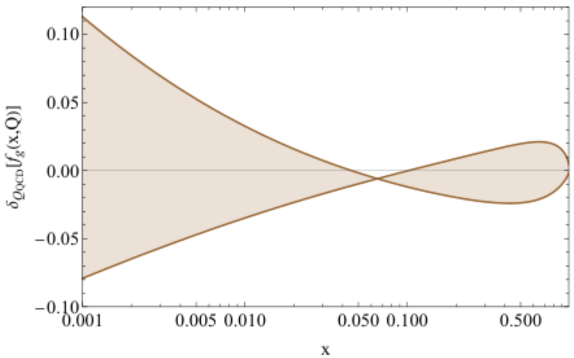
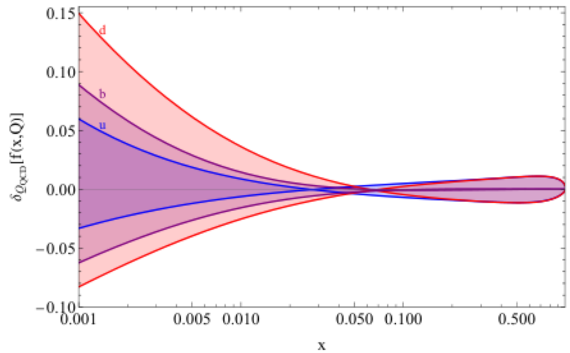
# Uncertainties

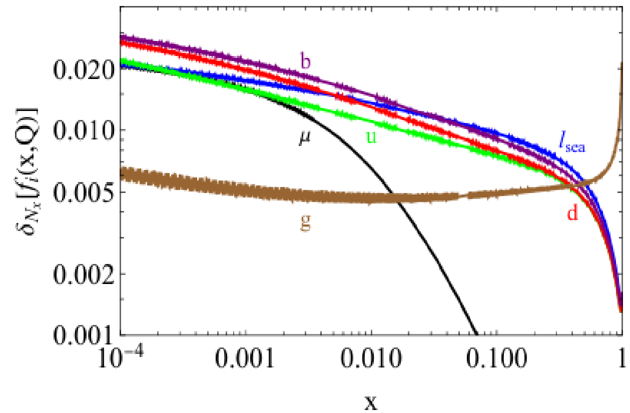
1. Due to the choice of the QCD scale ($\mu_{QCD} = [0.5 - 1]$ GeV):



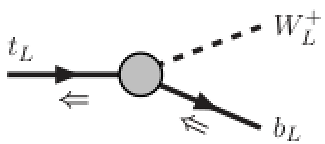

$$\delta_{Q_{QCD}}\left[f_A(x,Q)\right] = \frac{f_A(x,Q)|_{Q_{QCD}} - f_A(x,Q)|_{0.7\text{GeV}}}{f_A(x,Q)|_{0.7\text{GeV}}}$$

2. Due to the discretization ($N_x = [600{,}1000]$):



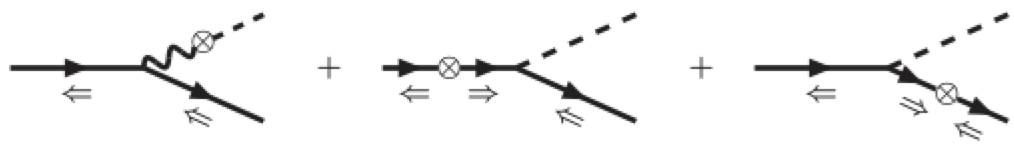
# Ultra-collinear splittings in GEG

**GEG** (hybrid of Coulomb & ligh-cone): $\mathcal{L}_{\rm fix} = -\frac{1}{2\xi}\big(n(k)\cdot W(k)\big)\big(n(k)\cdot W(-k)\big)$ → $\xi \to 0$




$$i\mathcal{M}(t_s \to W_L^+ b_s) = i\mathcal{M}(t_s \to \phi^+ b_s) + i\mathcal{M}(t_s \to W_n^+ b_s)$$

$$W^\mu(k) = W_T(k)\,\epsilon_T^\mu(k) + W_n(k)\,\epsilon_n^\mu(k) + W_{\bar{n}}(k)\,\epsilon_{\bar{n}}^\mu(k)$$

= 

$$\epsilon_n^\mu(k) \equiv \frac{-\sqrt{|k^2|}}{n(k)\cdot k}\, n^\mu(k)$$

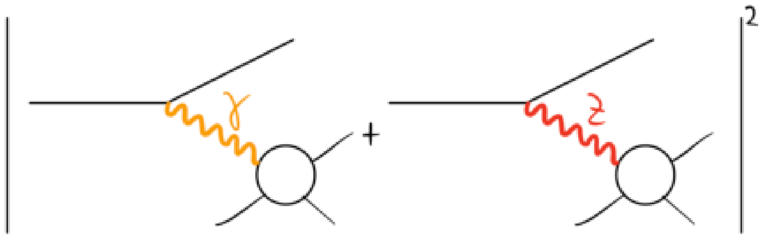
$$\begin{aligned} i\mathcal{M}(t_L \to \phi^+ b_L) &= i\,\bar{u}(b_L)(y_t P_R - y_b P_L)u(t_L) \\ &\simeq i\,y_t\sqrt{2E_b}\frac{m_t}{\sqrt{2E_t}} - i\,y_b\frac{m_b}{\sqrt{2E_b}}\sqrt{2E_t} \\ &\simeq i\,v\left(\frac{y_t^2}{\sqrt{2}}\sqrt{\bar{z}} - \frac{y_b^2}{\sqrt{2}}\frac{1}{\sqrt{\bar{z}}}\right) \end{aligned}$$

$$\begin{aligned} i\mathcal{M}(t_L \to W_n^+ b_L) &= i\,\frac{g_2}{\sqrt{2}}\bar{u}(b_L)(\not{\epsilon}_n(W)P_L)u(t_L) \\ &\simeq i\,\frac{g_2}{\sqrt{2}}\cdot 2\sqrt{2E_b}\left(-\frac{m_W}{2E_W}\right)\sqrt{2E_t} \\ &= -i\,v\frac{g_2^2}{\sqrt{2}}\frac{\sqrt{\bar{z}}}{z} \end{aligned}$$

[Chen et al]
1611.00788

$$U^{b_L}_{W_L t_L}(z) = \left(y_t^2 z\bar{z} - y_b^2 z - g_2^2\bar{z}\right)^2 \frac{1}{2z}$$

# Photon – Z mixing




➢ The splitting function must be generalised to a splitting matrix. The rate is computed by tracing against the matrix of the hard scattering process.

$$\left[\frac{d\mathcal{P}_{A\to B+C}}{dz\, dk_T^2}\right]_{ij} \simeq \frac{1}{16\pi^2}\,\frac{1}{z\bar{z}}\,\mathcal{M}_k^{(\mathrm{split})*}\mathcal{D}_{ki}^{*}\mathcal{D}_{jl}\mathcal{M}_l^{(\mathrm{split})}$$

➢ The propagators are diagonal in the mass basis:

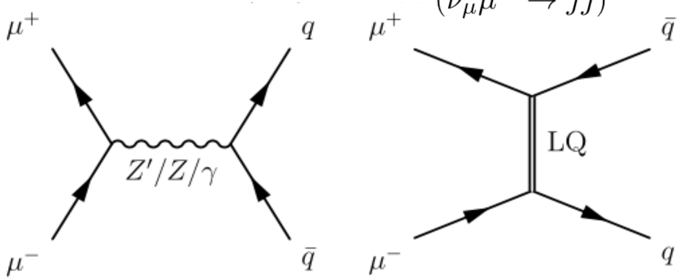
[Ciafaloni et al] hep-ph/0505047, hep-ph/0505047
[Chen et al] 1611.00788

$$\mathcal{D}_{\gamma\gamma} = \frac{i}{q^2}\,,\quad \mathcal{D}_{ZZ} = \frac{i}{q^2 - m_Z^2}\,,\quad \mathcal{D}_{\gamma Z} = \mathcal{D}_{Z\gamma} = 0$$

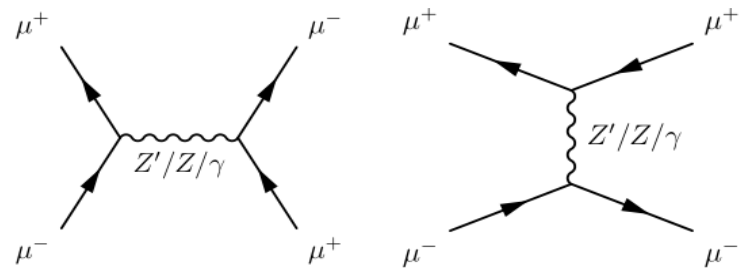
$$\mathcal{D}_{hh} = \frac{i}{q^2 - m_h^2}\,,\quad \mathcal{D}_{Z_L Z_L} = \frac{i}{q^2 - m_Z^2}\,,\quad \mathcal{D}_{hZ_L} = \mathcal{D}_{Z_L h} = 0$$

# Signatures at a muon collider (channels)

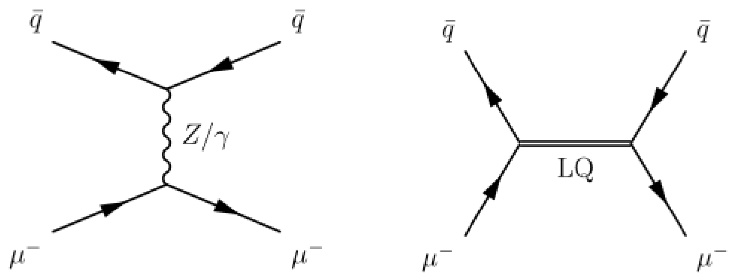
**i) Inverted Drell-Yan:** $\mu\bar{\mu} \to jj$
$(\nu_\mu \mu^+ \to jj)$



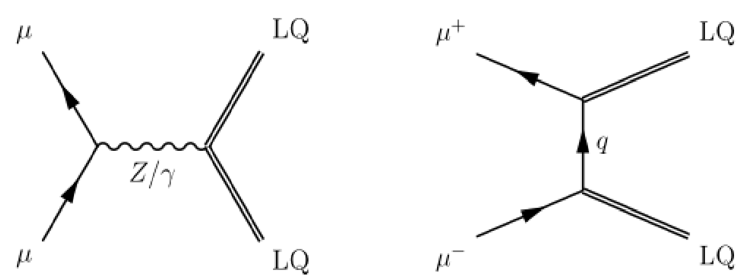

**ii) Di-Muon (Tau):** $\mu\bar{\mu} \to \mu^+\mu^-$




**iii) Mono-lepton plus jet:** $\mu\bar{\mu} \to \mu^- j$



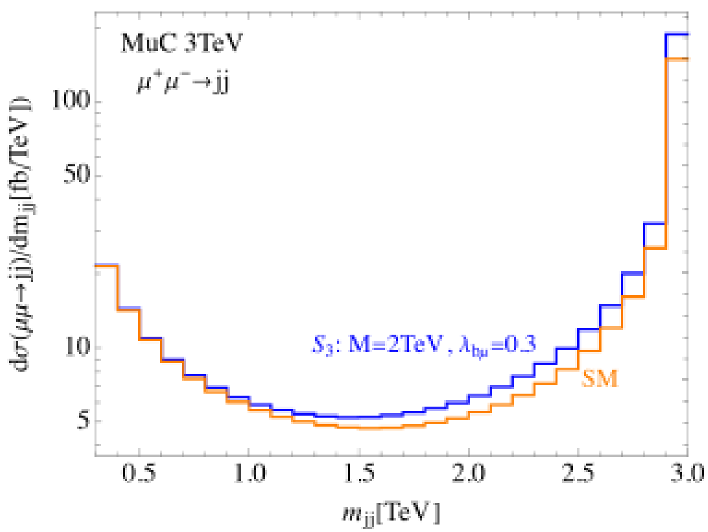

**iv) LQ pair production:** $\mu\bar{\mu} \to \mathrm{LQ}\overline{\mathrm{LQ}}$

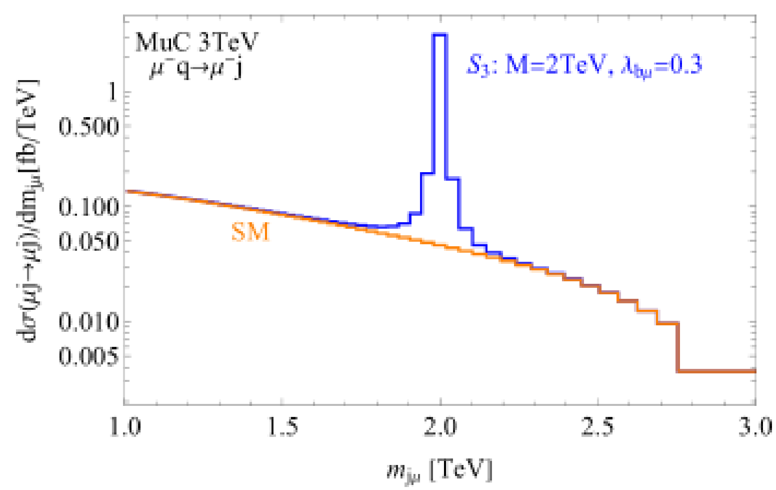


# Signatures at a muon collider (sensitivity)

- Due to the luminosities of the valence partons, if the $M_{\rm NP} < \sqrt{s}$ below the collider energy, the effect is visible both at the shape of the cross-section (resonance peak or *t(u)*-channel exchange) as well as the very precise measurement in the last invariant mass bin.
  For $M_{\rm NP} > \sqrt{s}$, the sensitivity arises from the latter strategy.

[Azatov et al]
2205.13552



# $Z'$ gauge bosons

We consider models in which the dominant quark coupling is to heavy flavours. There are two qualitatively different scenarios:

*1)* $g_{sb} \ll g_{bb} \sim g_{\mu\mu}$ realized by gauging $U(1)_{B_3-L_\mu}$:

$$\mathcal{L}^{\rm int}_{Z'_{B_3-L_\mu}} = -\, g_{Z'} Z'_\alpha \left[\frac{1}{3}\bar{Q}^3_L\gamma^\alpha Q^3_L + \frac{1}{3}\bar{b}_R\gamma^\alpha b_R + \frac{1}{3}\bar{t}_R\gamma^\alpha t_R - \bar{L}^2_L\gamma^\alpha L^2_L - \bar{\mu}_R\gamma^\alpha\mu_R + \right.$$
$$\left. + \left(\frac{1}{3}\epsilon_{sb}\bar{Q}^2_L\gamma^\alpha Q^3_L + \text{h.c.}\right) + \mathcal{O}(\epsilon^2_{sb})\right] \,,$$

approximate $U(2)^3$

[Greljo et al]
2107.07518

*2)* $g_{sb} \sim g_{bb} \ll g_{\mu\mu}$ realized by gauging $U(1)_{L_\mu-L_\tau}$ :

$$\mathcal{L}^{\rm int}_{Z'_{L_\mu-L_\tau}} = -\, g_{Z'} Z'_\alpha \left[\bar{L}^2_L\gamma^\alpha L^2_L + \bar{\mu}_R\gamma^\alpha\mu_R - \bar{L}^3_L\gamma^\alpha L^3_L - \bar{\tau}_R\gamma^\alpha\tau_R + \right.$$
$$\left. + |\epsilon_b|^2\bar{Q}^3_L\gamma^\alpha Q^3_L + |\epsilon_s|^2\bar{Q}^2_L\gamma^\alpha Q^2_L + (\epsilon_b\epsilon_s^*\bar{Q}^2_L\gamma^\alpha Q^3_L + \text{h.c.}) + \ldots\right] \,.$$

quark-phobic (couplings generated via mixing with heavy VLQs)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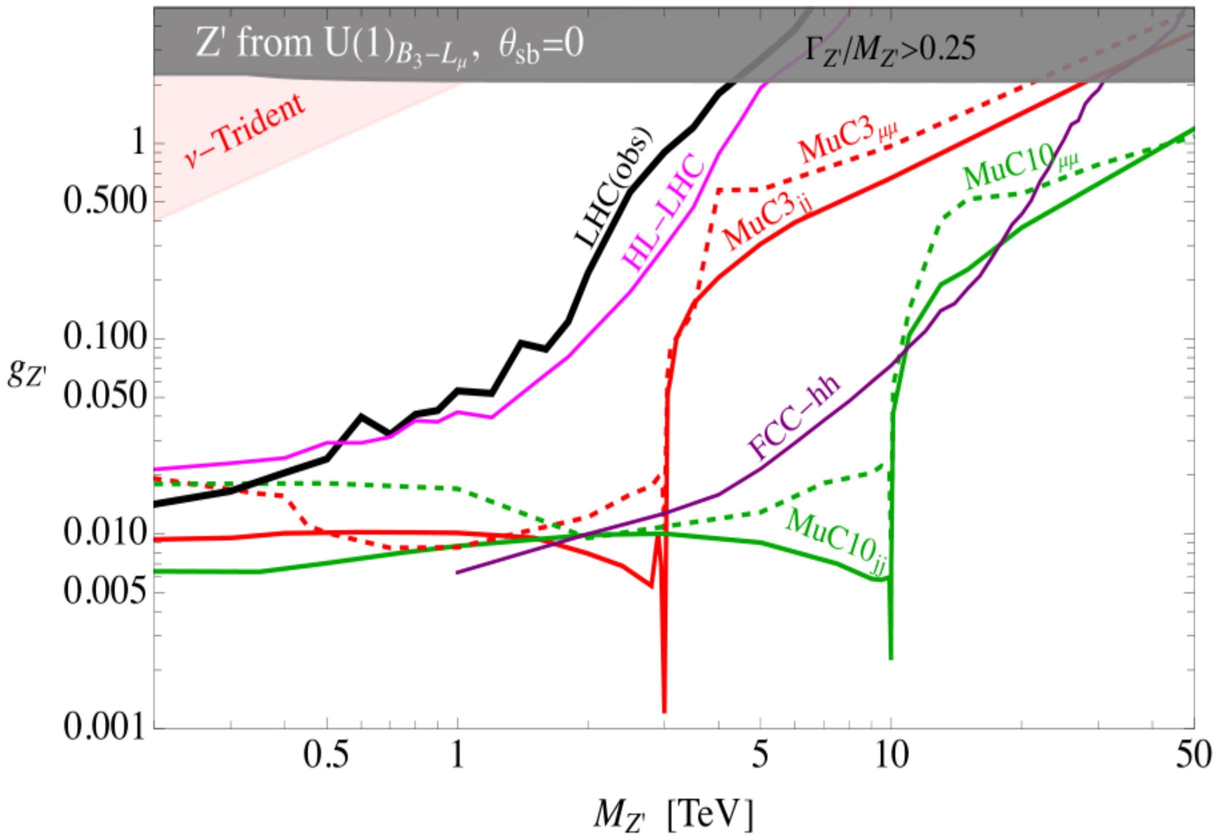
# Z' gauge bosons ($U(1)_{B_3-L_\mu}$, no mixing)



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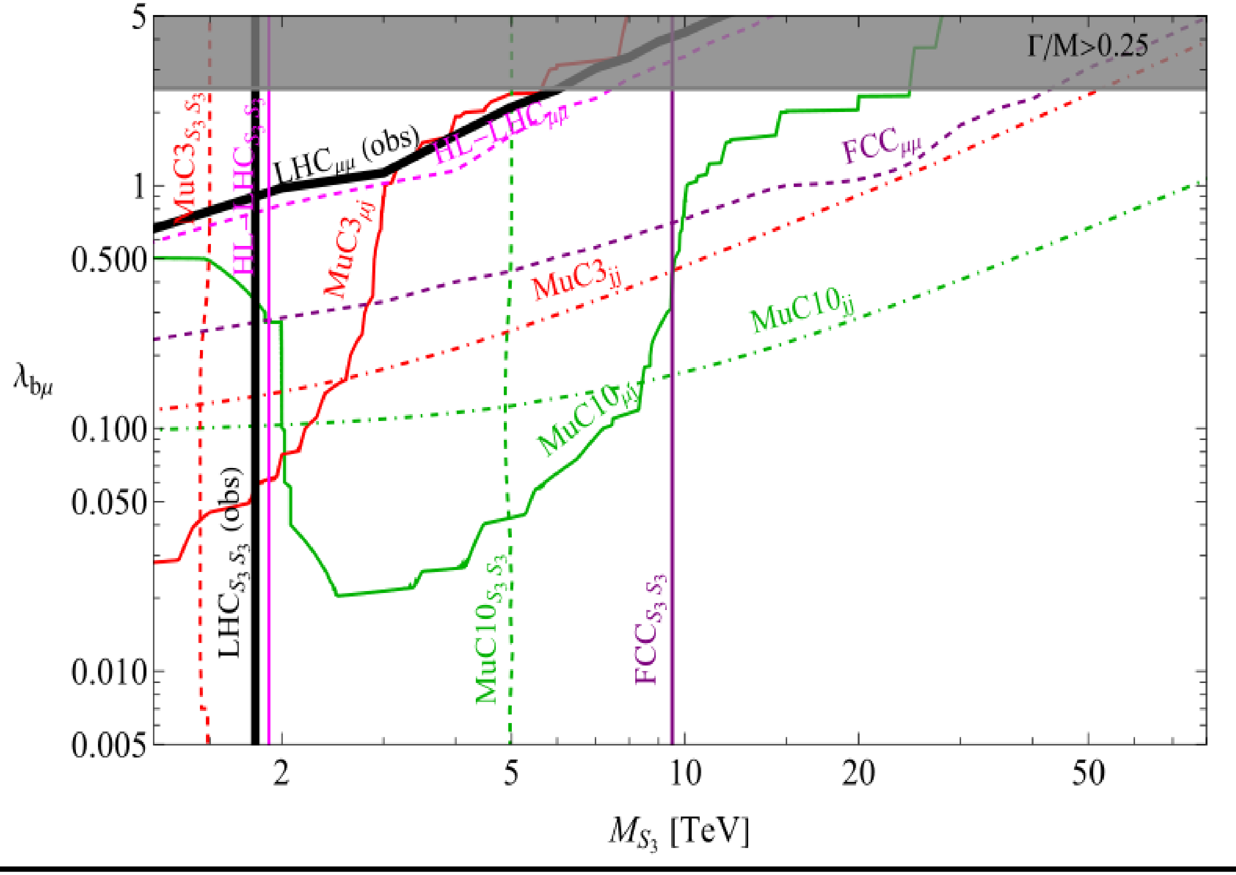
# $S_3$ leptoquark ($U(2)^3$ - symmetric)

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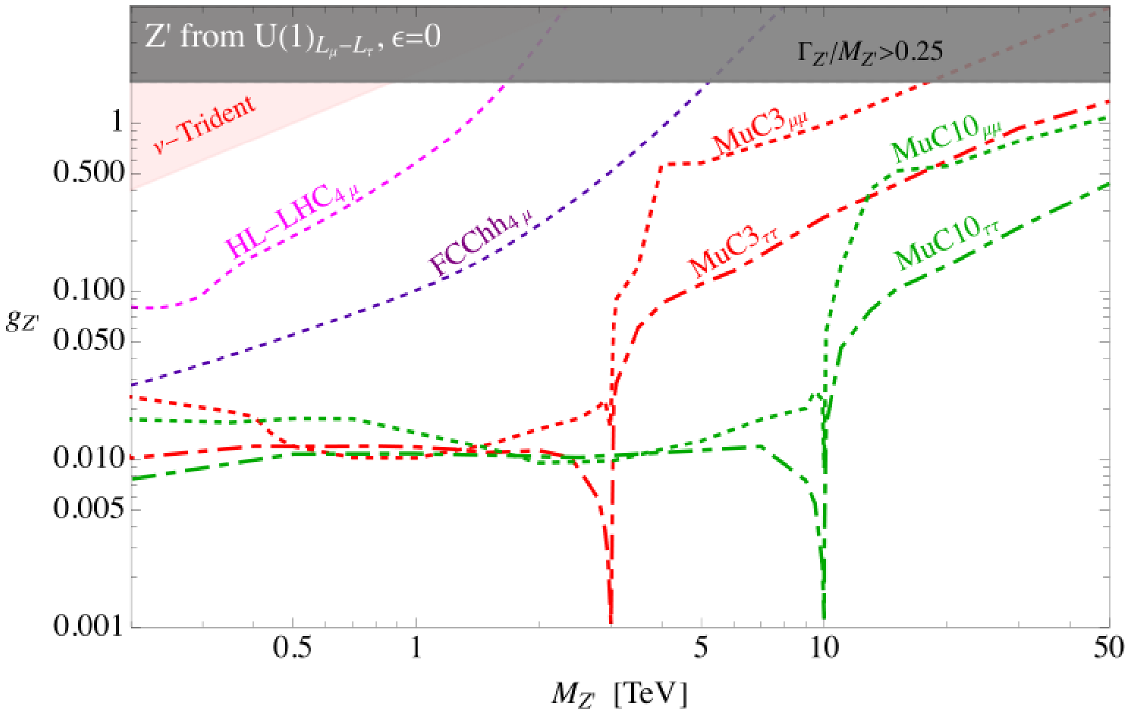
# $Z'$ gauge bosons ($U(1)_{L_\mu - L_\tau}$)

**Quark-phobic scenario:**



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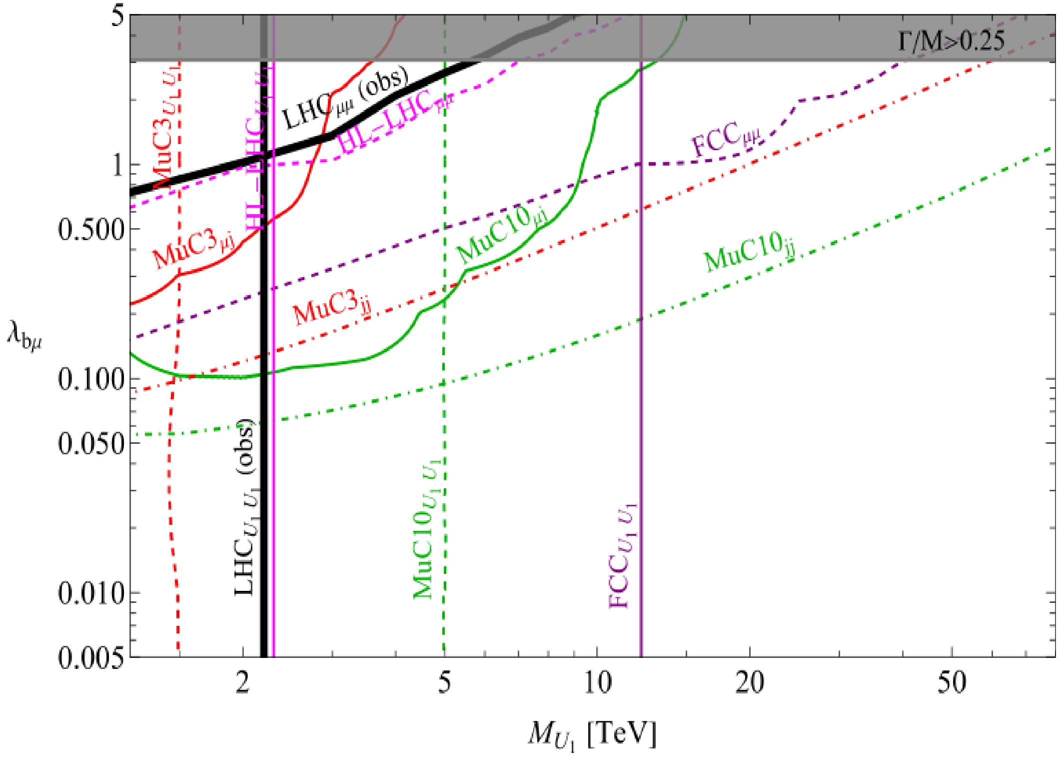
# $U_1$ leptoquark

## $U(2)^3$ - symmetric



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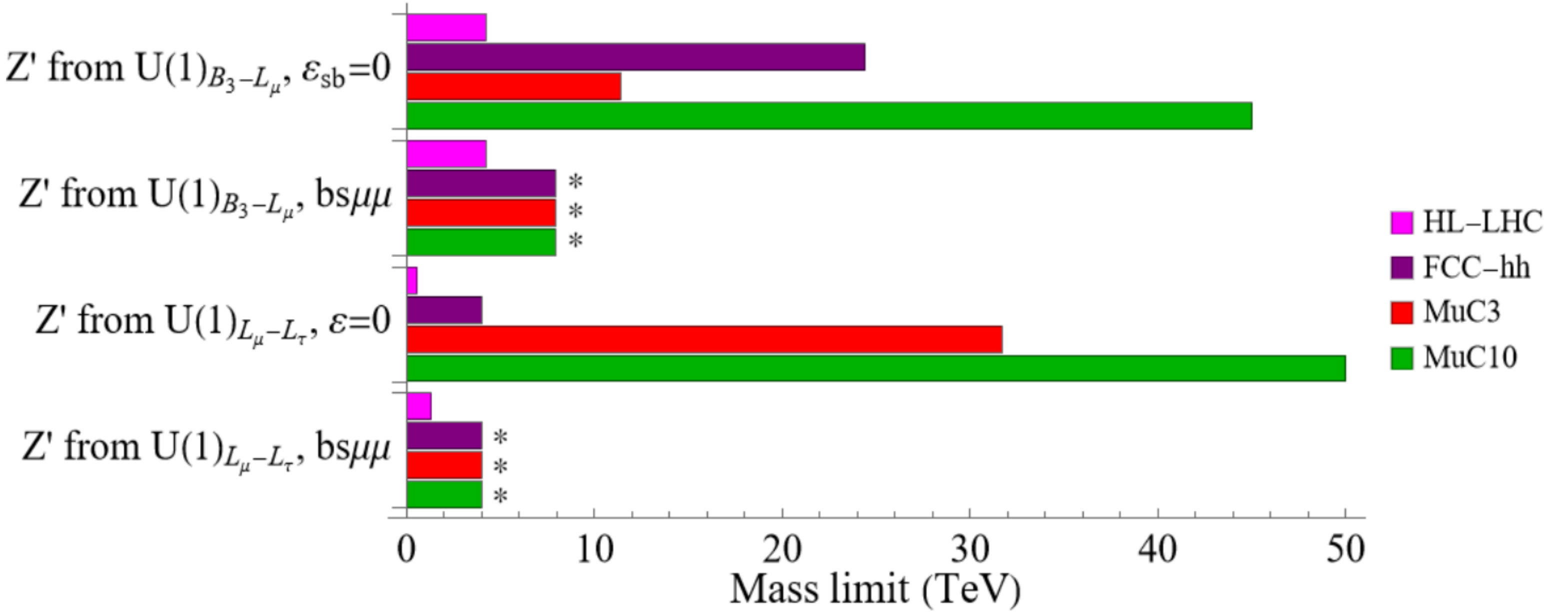
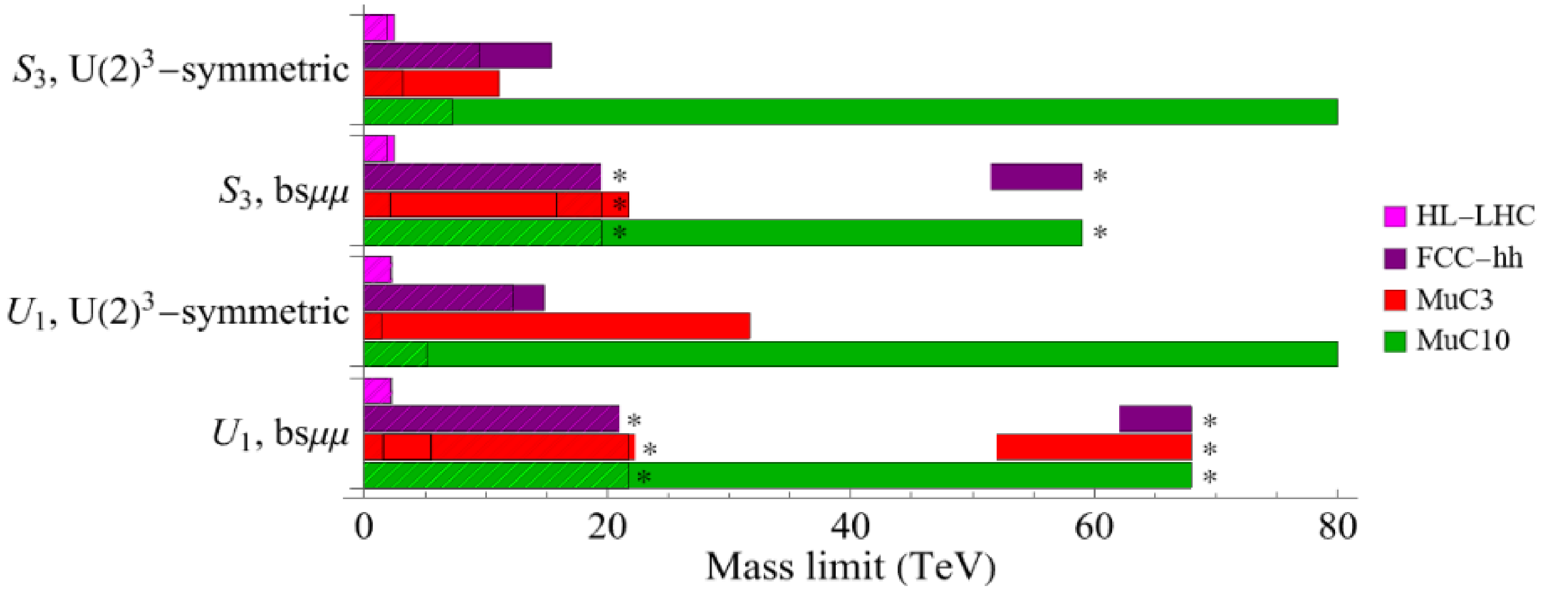
# Z' gauge bosons (prospects)



# Leptoquarks (prospects)



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