Pressure distribution inside nucleons in a Tsallis-MIT bag model

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G. Herrera Corral

Cinvestav

Overview

A. Introduction and motivation

B. Quick overview of the MIT bag Model

C. Non extensive Tsallis statistics in the MIT bag Model

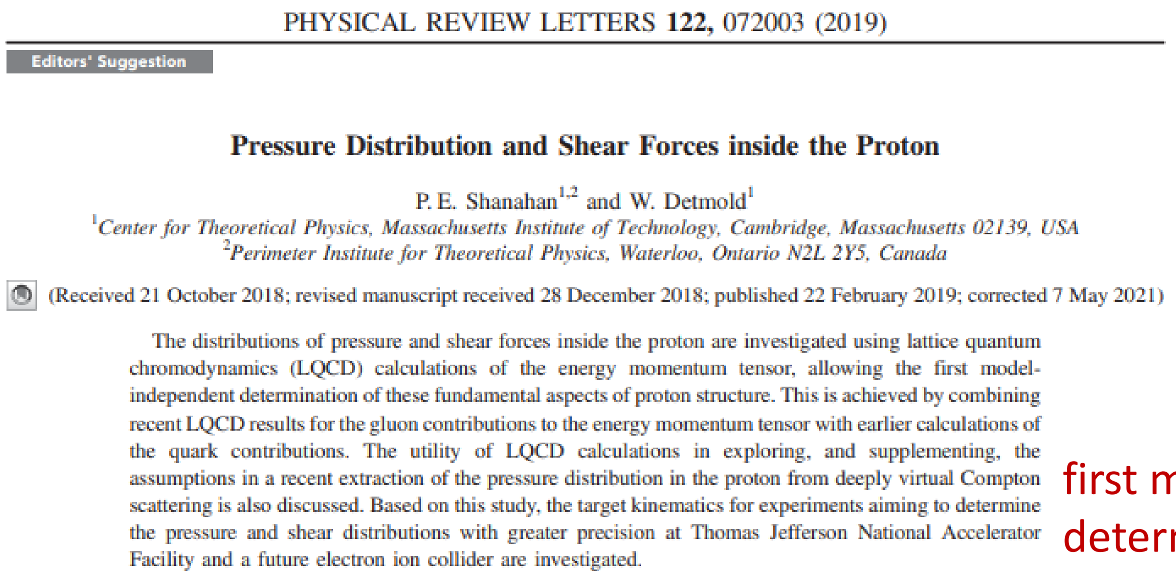
D. Pressure in the T - MIT bag Model

Introduction and motivation

A detailed description of hadron structure is needed to disentangle the basic mechanisms behind strong interaction

Recent Lattice QCD calculations ...

Lattice QCD


PHYSICAL REVIEW LETTERS 122, 072003 (2019)

Editors' Suggestion

Pressure Distribution and Shear Forces inside the Proton

P. E. Shanahan[1,2] and W. Detmold[1]
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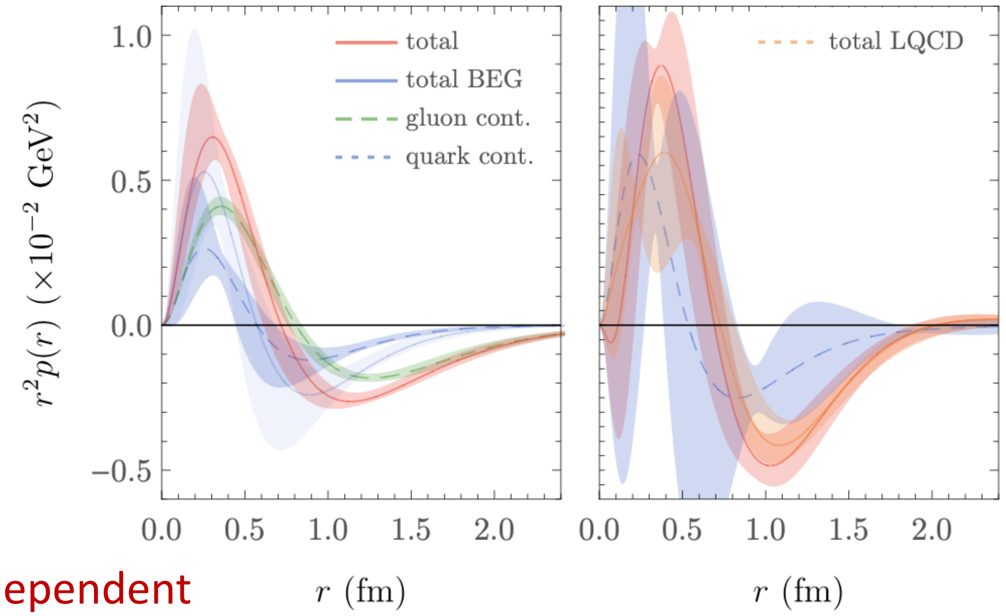
The distributions of pressure and shear forces inside the proton are investigated using lattice quantum chromodynamics (LQCD) calculations of the energy momentum tensor, allowing the first model-independent determination of these fundamental aspects of proton structure. This is achieved by combining recent LQCD results for the gluon contributions to the energy momentum tensor with earlier calculations of the quark contributions. The utility of LQCD calculations in exploring, and supplementing, the assumptions in a recent extraction of the pressure distribution in the proton from deeply virtual Compton scattering is also discussed. Based on this study, the target kinematics for experiments aiming to determine the pressure and shear distributions with greater precision at Thomas Jefferson National Accelerator Facility and a future electron ion collider are investigated.



first model independent determination of pressure

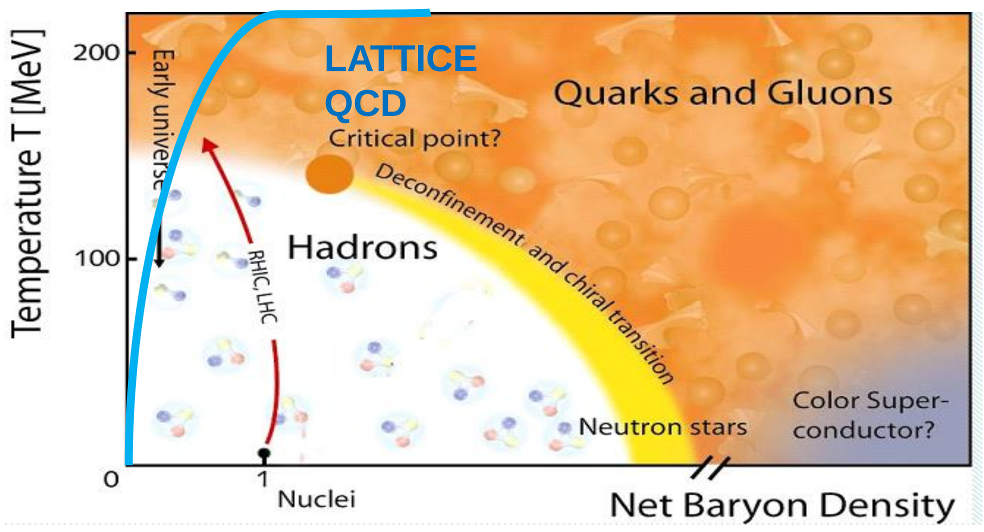
But:

Finite baryon density is finite in experiment
Non-zero μ (chemical potential) lead to non-Hermitian Dirac operator

sign problem

MC calculations are not applicable anymore
important constraint in the theoretical characterization of the physical states under different conditions of temperature and pressure.

The Phase Diagram of QCD Matter



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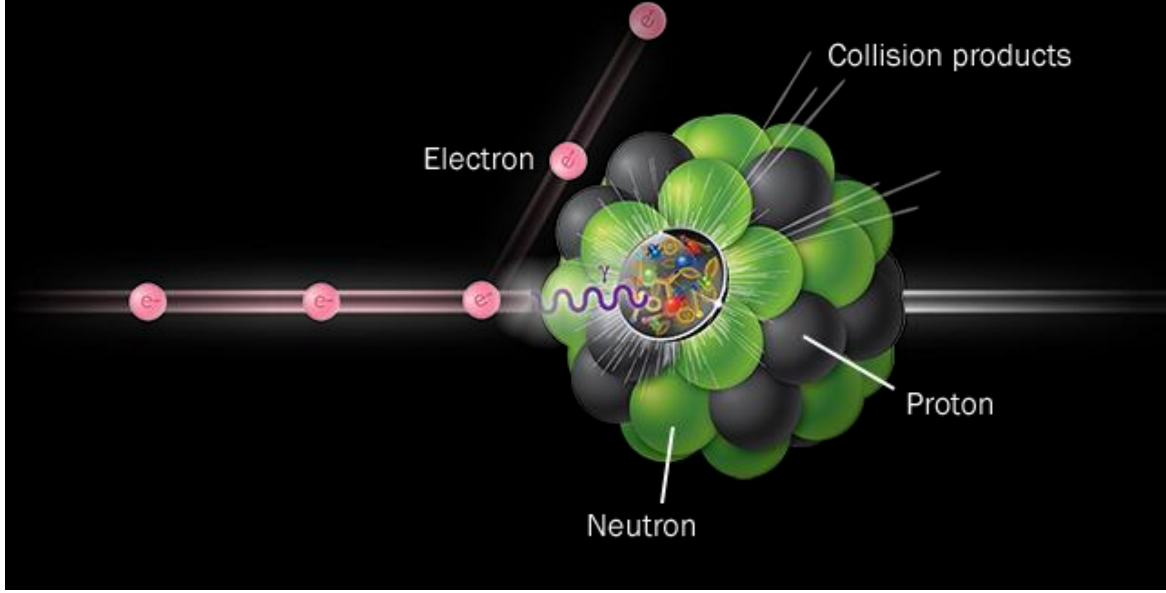
Recent Lattice QCD calculations …

Future projects aiming to a better understanding of the 3D structure of protons e.g. Electron Ion Collider - EIC

EIC science



The Electron-Ion Collider (EIC), a powerful new facility to be built in the United States at the U.S. Department of Energy's Brookhaven National Laboratory in collaboration with Thomas Jefferson National Accelerator Facility

Precision 3D imaging of the internal structure of protons and atomic nuclei.

High-energy electrons will interact with the internal microcosm to reveal unprecedented details—zooming in beyond the simplistic structure of three valence quarks bound by a mysterious force.

We know that gluons multiply and play a significant role in establishing key properties of protons and nuclear matter.

EIC will reveal features of this "ocean" of gluons and the "sea" of quark-antiquark pairs that form when gluons interact—allowing the mapping the distribution and movement within protons and nuclei.

Construction is expected to start around 2024, with operations beginning in the early 2030s

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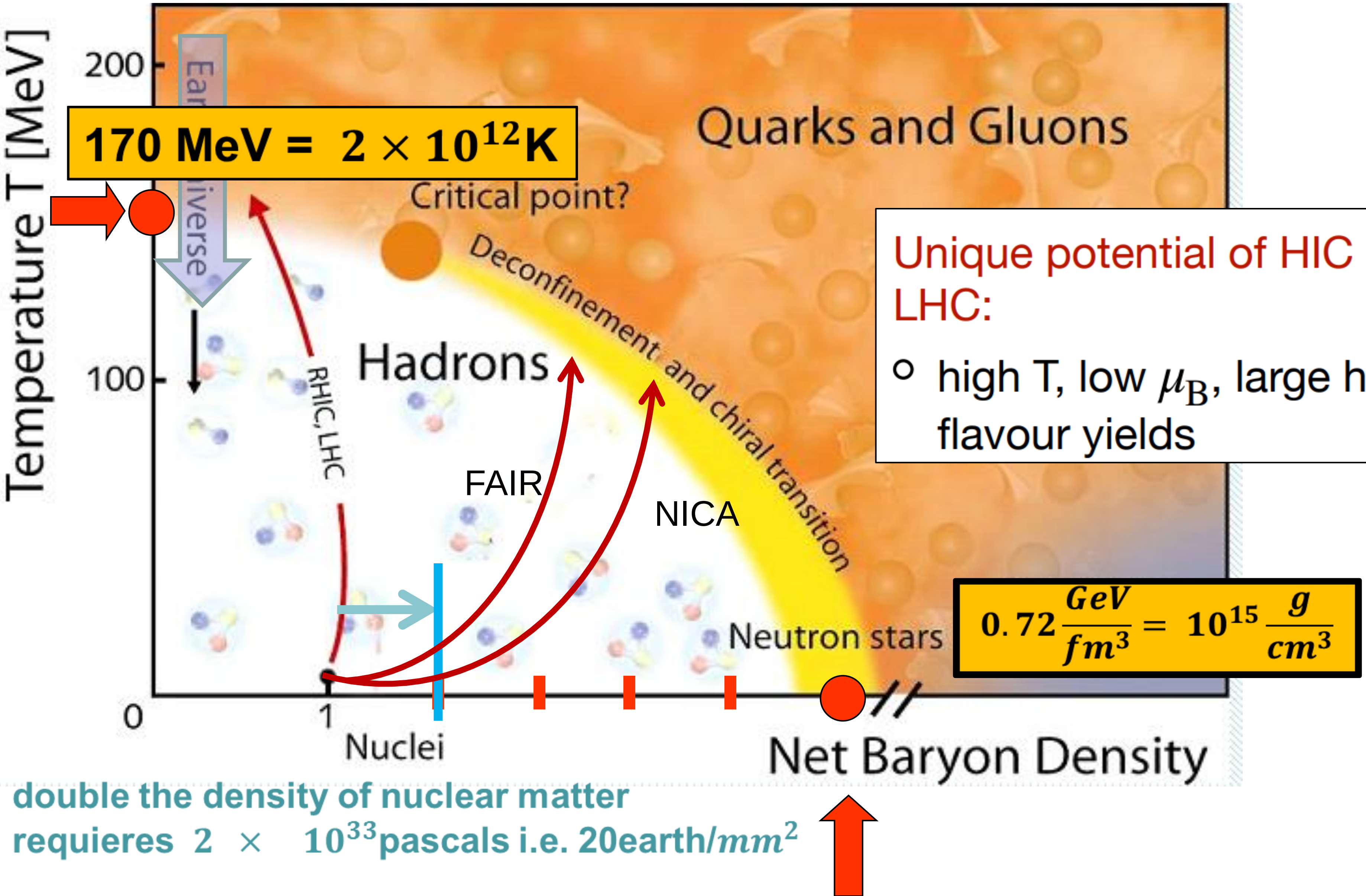
Modeling of heavy astrophysical objects, matter under extreme conditions

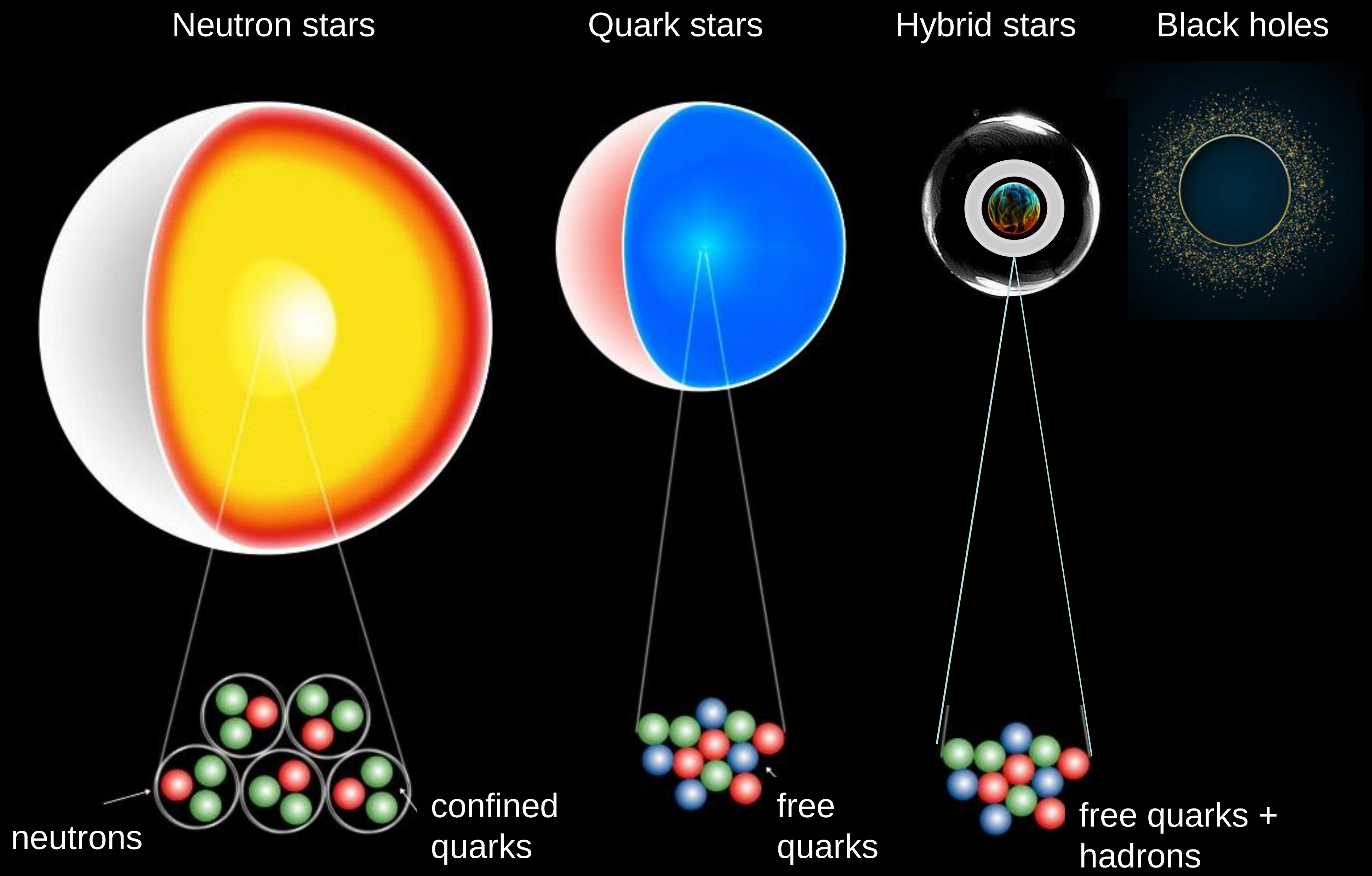
The Phase Diagram of QCD Matter



Unique potential of HIC at the LHC:

- high T, low μ_B, large heavy-flavour yields

double the density of nuclear matter requieres 2×10^{33}pascals i.e. 20earth/mm^2



Neutron stars
Quark stars
Hybrid stars
Black holes
neutrons
confined
quarks
free
quarks
free quarks +
hadrons

Introduction and motivation

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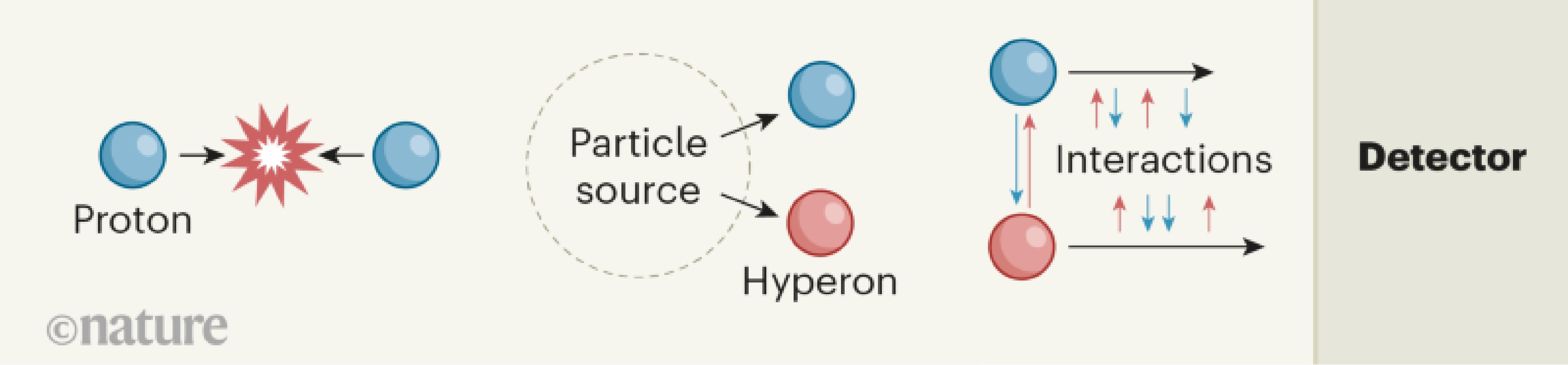
Modeling of heavy astrophysical objects, matter under extreme conditions

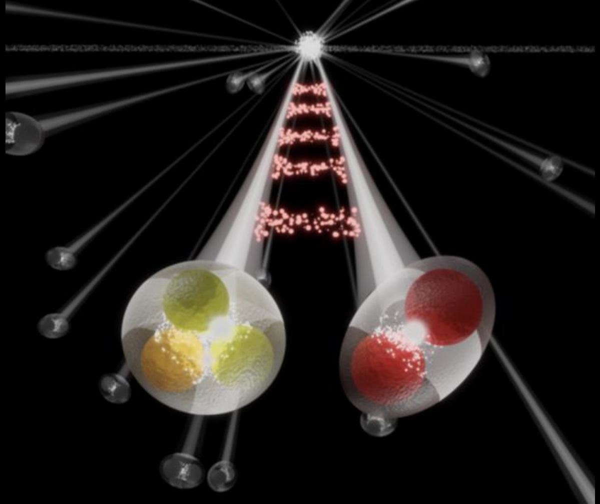
Recent measurements by the ALICE collaboration of final state interactions between hyperons and proton - neutrons

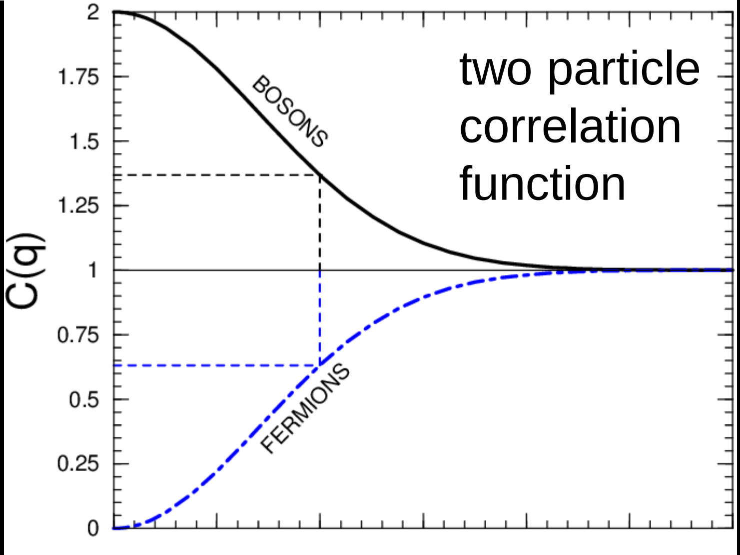
ALICE

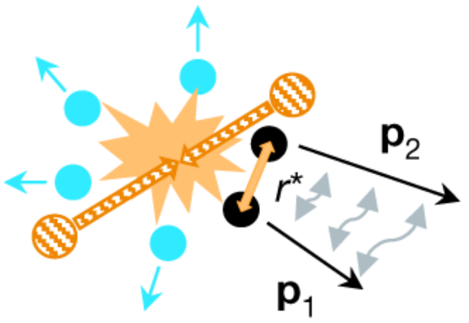
proton proton collisions
$\sqrt{s}$ = 13 TeV



FINAL STATE INTERACTION







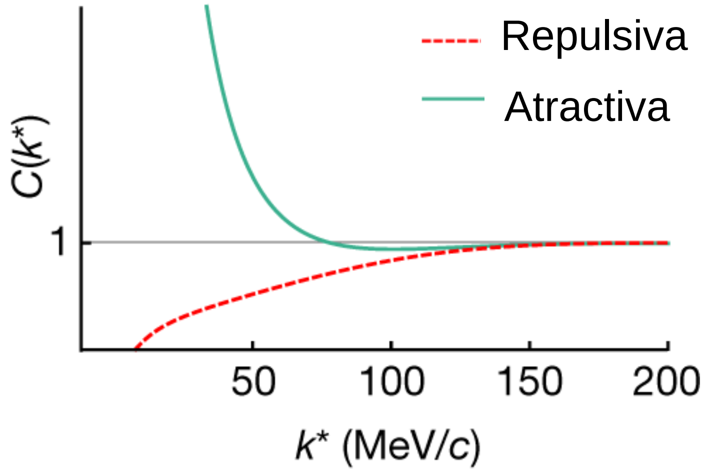
fuente de emisión

Ecuación de Schrödinger

↓

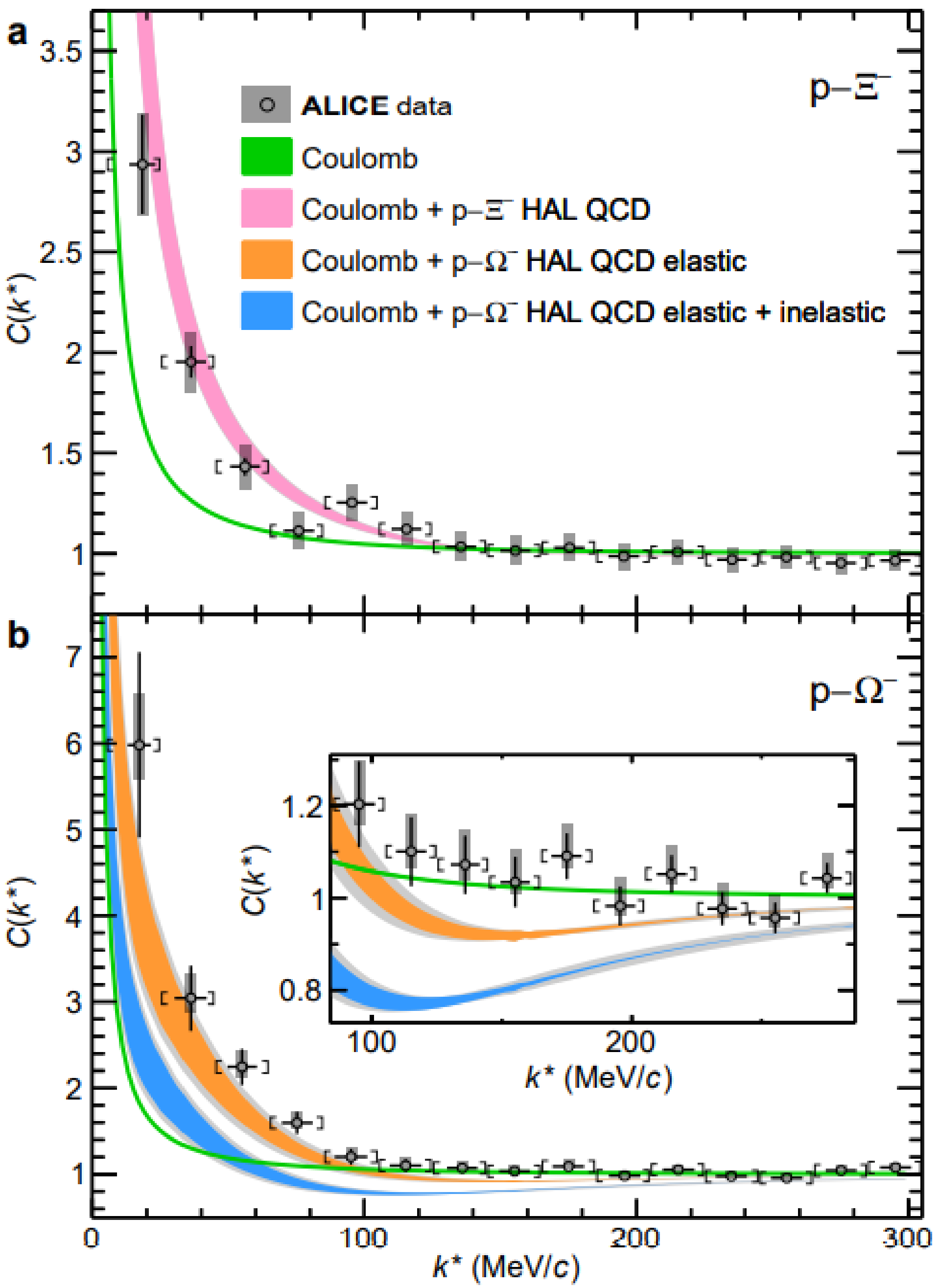
función de onda de dos partículas

$|\psi(\mathbf{k}^*, \mathbf{r}^*)|$

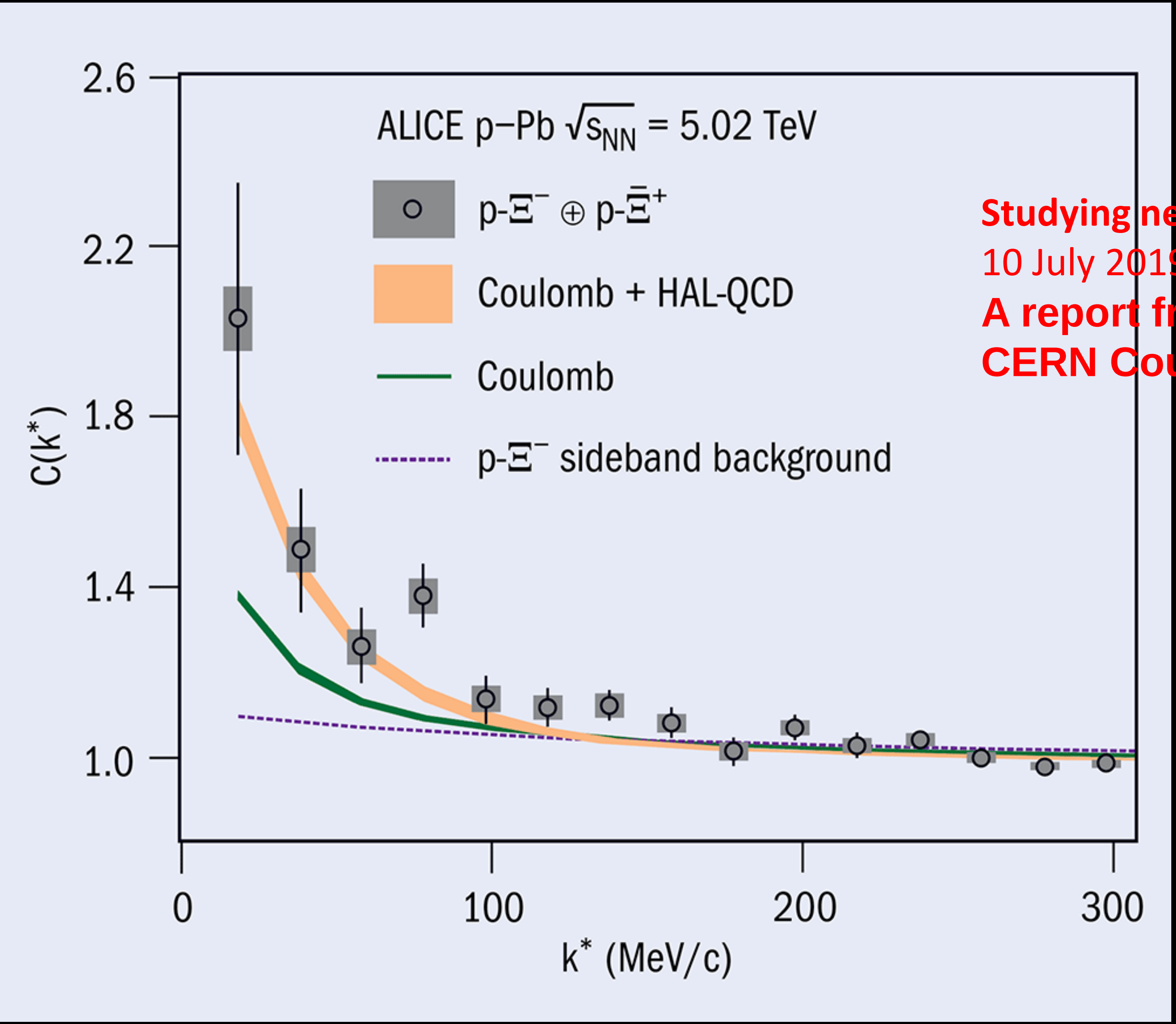


función de correlación

$$C(k^*) = \int S(r^*) |\psi(\mathbf{k}^*, \mathbf{r}^*)|^2 d^3r^* = \xi(k^*) \frac{N_{same}(k^*)}{N_{mixed}(k^*)}$$



a
p–Ξ⁻
ALICE data
Coulomb
Coulomb + p–Ξ⁻ HAL QCD
Coulomb + p–Ω⁻ HAL QCD elastic
Coulomb + p–Ω⁻ HAL QCD elastic + inelastic
C(k*)
b
p–Ω⁻
C(k*)
k* (MeV/c)
C(k*)
k* (MeV/c)



Studying neutron stars in the laboratory

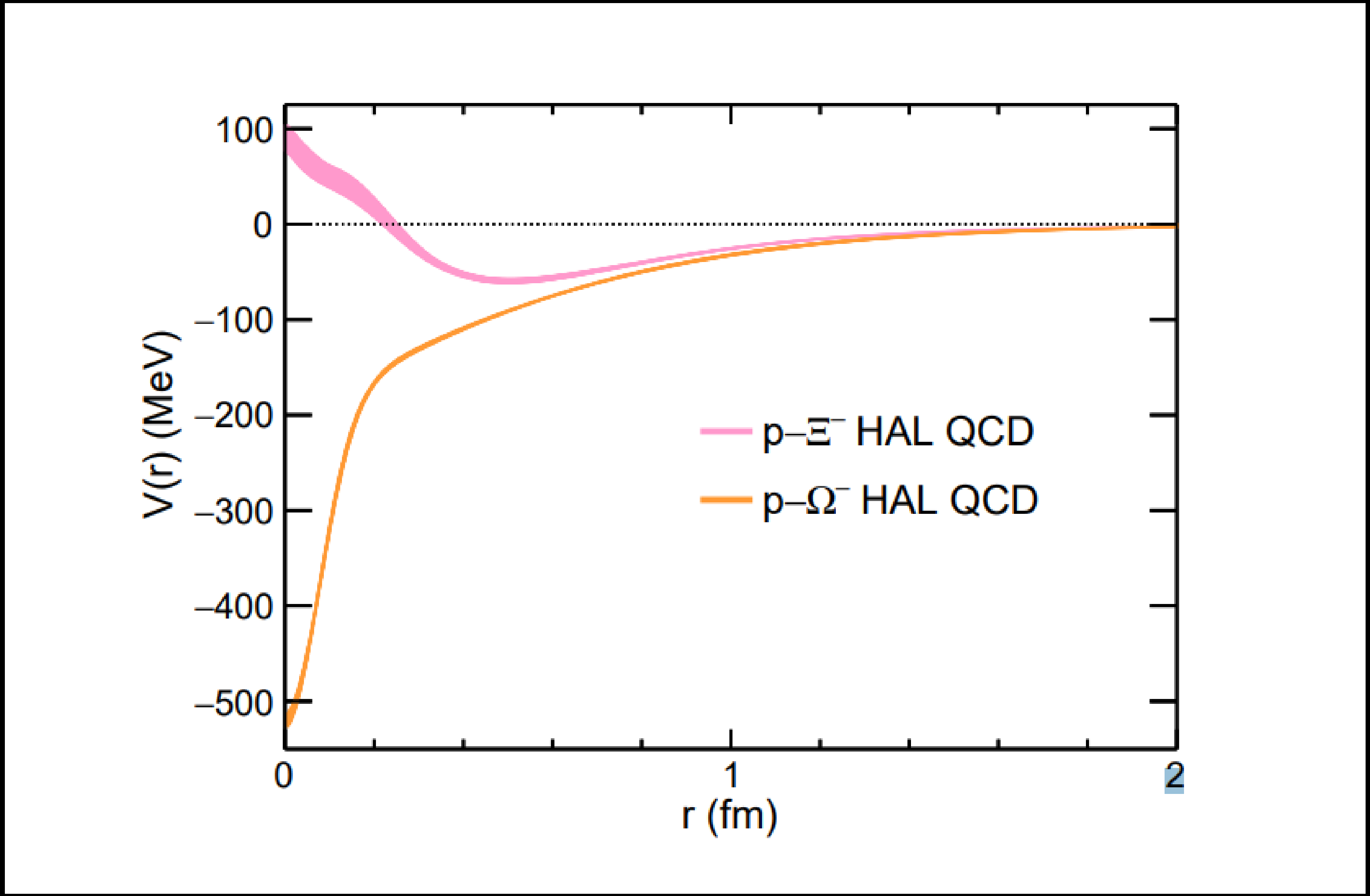
10 July 2019

A report from the ALICE experiment

CERN Courier

ALICE Collaboration 2019
arXiv:1904.12198.

p-Ξ⁻ correlation as a function of relative momenta between proton and Ξ⁻



Potential p–Ξ and p–Ω
HAL QCD collaboration

HAL (**H**adrons to **A**tomic nuclei from **L**attice) QCD Collaboration

Overview

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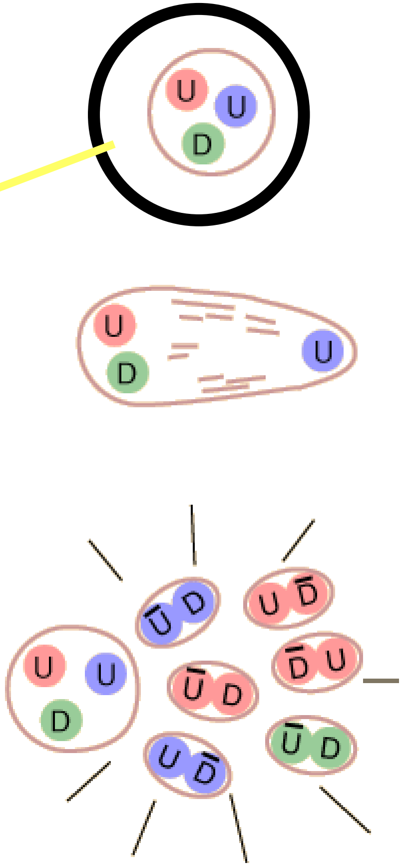
Quick overview of the MIT bag model

What´s in model´s bag

nothing

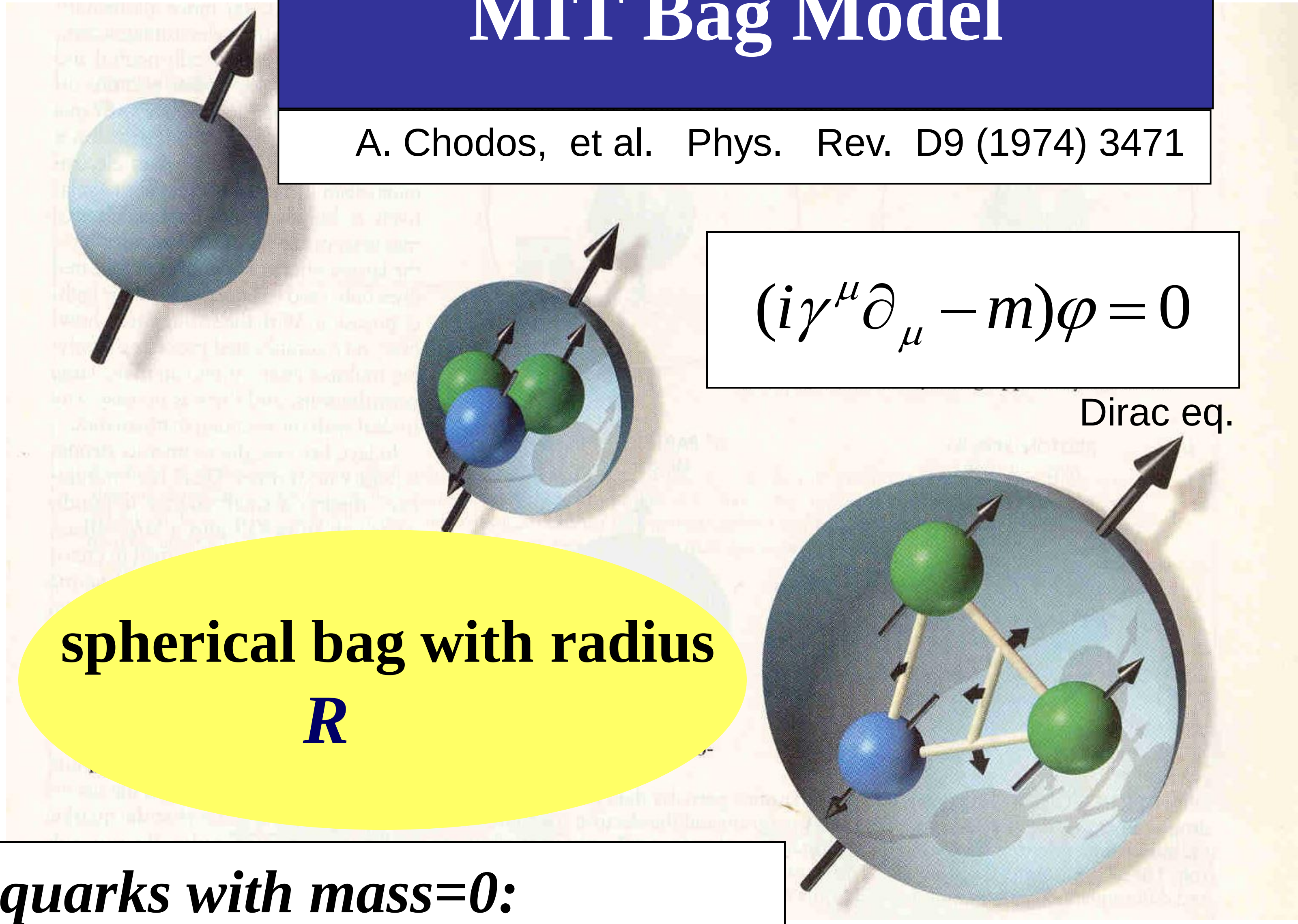
What´s in a bag´s model



massles quarks

MIT Bag Model

A. Chodos, et al. Phys. Rev. D9 (1974) 3471



$$(i\gamma^{\mu}\partial_{\mu} - m)\varphi = 0$$

Dirac eq.

spherical bag with radius ***R***

quarks with mass=0:

$$\gamma p \varphi = 0$$

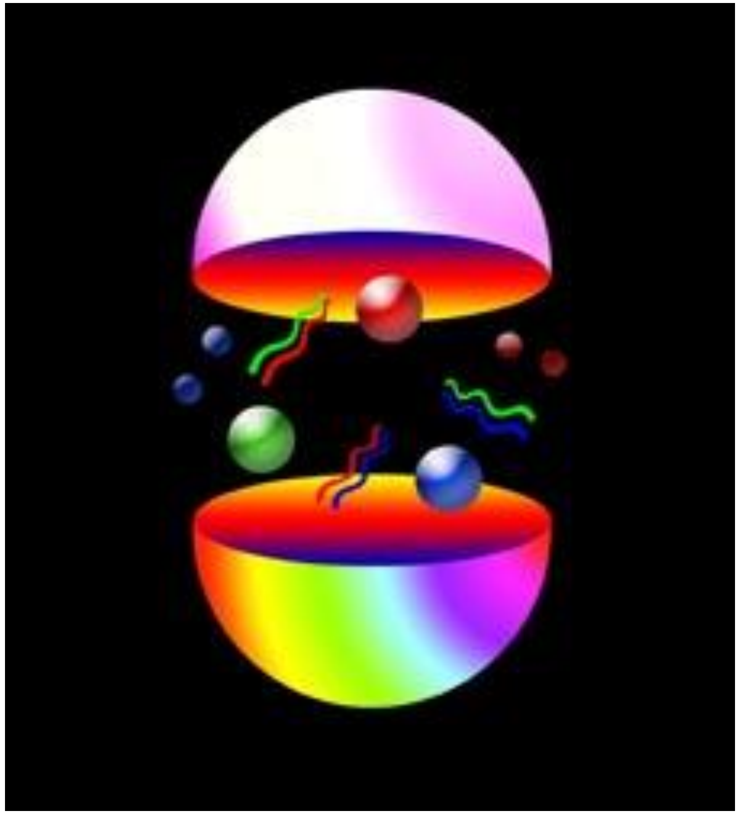
MIT bag model

$$i\partial_\mu = (p^0, \mathbf{p})$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0, \quad m = 0$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix}$$

Using the Dirac representation of the γ matrices



$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

and a two component spinor

MIT Bag Model

$$(\gamma^0 p^0 - \vec{\gamma}\cdot\vec{p})\varphi = 0$$

$$\begin{pmatrix} p^0 & -\vec{\sigma}\cdot\vec{p} \\ \vec{\sigma}\cdot\vec{p} & -p^0 \end{pmatrix}\begin{pmatrix} \varphi_+ \\ \varphi_- \end{pmatrix} = 0$$

$$p^0\varphi_+ - \vec{\sigma}\cdot\vec{p}\varphi_- = 0 \qquad \vec{\sigma}\cdot\vec{p}\varphi_+ - p_0\varphi_- = 0$$

$$(\vec{p}^{\,2} - (p^0)^2)\varphi_+ = 0$$

$$\varphi_+(\vec{r},t) = Ne^{-ip^0t} j_0(p^0 r)\chi_+$$
$$\varphi_-(\vec{r},t) = Ne^{-ip^0t}\vec{\sigma}\cdot\hat{r} j_1(p^0 r)\chi_-$$

lowest energy solution

Spherical Bessel function $j_0 \quad j_1$

Confinement ⇒ ***current flux through the surface of the sphere*** = 0

i.e. the normal component of $J_\mu = \bar{\varphi}\gamma_\mu\varphi$

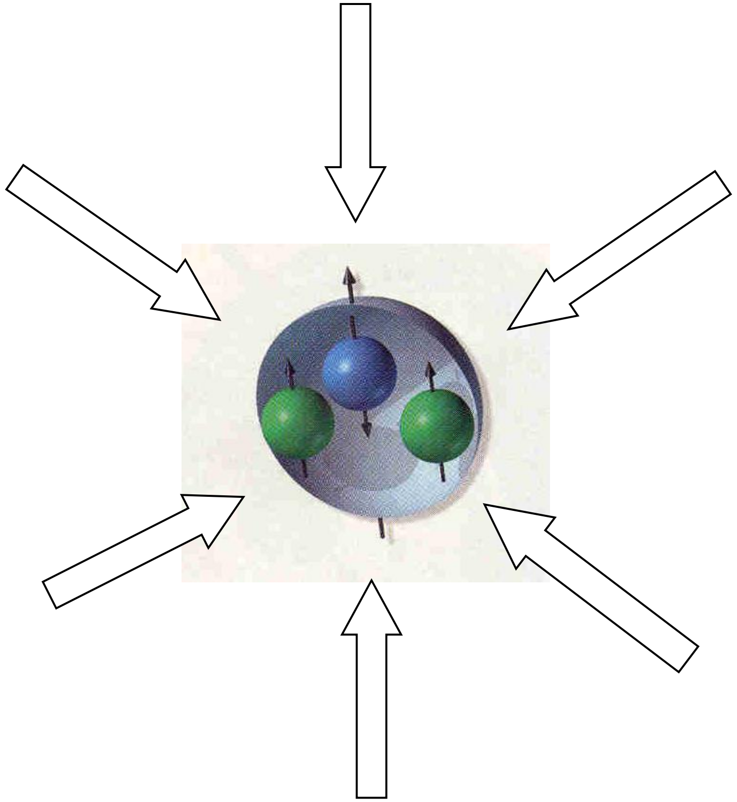
$$n^\mu \bar{\varphi}\gamma_\mu\varphi = 0 \quad \Rightarrow \quad \bar{\varphi}\varphi = 0$$

$$\bar{\varphi}\varphi\Big|_{r=R} = \left[j_0(p^0R)\right]^2 - \vec{\sigma}\cdot\hat{r}\,\sigma\cdot\hat{r}\left[j_1(p^0R)\right]^2 = 0$$

$$\left[j_0(p^0R)\right]^2 - \left[j_1(p^0R)\right]^2 = 0$$

$$p^0R = 2.04$$

energy of N quarks: $E = \dfrac{2.04N}{R}$



Bag pressure

B

energy of N quarks in the bag with presure B:

$$E = \frac{2.04N}{R} + \frac{4\pi}{3} R^3 B$$

Equilibrium

$$\frac{\partial E}{\partial R} = 0 \quad \Rightarrow \quad 4\pi R^2 B - \frac{2.04N}{R^2} = 0$$

$$\therefore R = \left(\frac{2.04N}{4\pi B} \right)^{1/4}$$

for a proton $R = 0.8\, fm$ ***with three quarks***

$$B^{1/4} = \left(\frac{2.04N}{4\pi} \right)^{1/4} \frac{1}{R}$$

$$\hbar c = 197 MeV - fm$$

$$B^{1/4} = 1.044 \times 197.3$$

$$B^{1/4} = 206 MeV$$

Equilibrium $\frac{\partial E}{\partial R} = 0 \Rightarrow 4\pi R^2 B - \frac{2.04N}{R^2} = 0$

$$\therefore R = \left(\frac{2.04N}{4\pi B}\right)^{1/4}$$

for a proton $R = 0.8\, fm$ ***with three quarks***

$$B^{1/4} = \left(\frac{2.04N}{4\pi}\right)^{1/4} \frac{1}{R} \qquad \hbar c = 197 MeV - fm$$

$$B^{1/4} = 1.044 \times 197.3$$

$$B^{1/4} = 206 MeV$$

A non correlated estimate of pressure from the subsystems in the proton

The phase space volume of quarks in a spatial volume ***V*** with momentum ***p*** in the interval ***dp*** is

$$4\pi p^2 V dp$$

since each state occupies a phase space volume of $(2\pi\hbar)^3$ the number of states characterized by a momentum ***p*** in the interval ***dp*** is $4\pi p^2 dpV/(2\pi)^3$

The ocupation probability is given by the Fermi –Dirac distribution

The number of quarks in a volume ***V*** with momentum ***p*** within the interval ***dp*** is

$$dN_q = \frac{g_q V 4\pi p^2 dp}{(2\pi)^3}\left[\frac{1}{1+e^{(p-\mu_q)/T}}\right]$$

chemical potential

quark degeneracy

Fermi-Dirac distribution

The presence of anti-quarks corresponds to the absence of quarks in the negative energy states

$$n_q = \frac{g_q}{(2\pi)^3}\int_{-\infty}^{0}\left[1-\frac{1}{1+e^{(p_0-\mu_q)/T}}\right]4\pi p_0^2 dp_0$$

$$n_q = \frac{g_q}{(2\pi)^3} \int_0^\infty \frac{1}{1+e^{(p_0+\mu_q)/T}} 4\pi p_0^2 dp_0$$

In our case the number of quarks is the same as the antiquarks, $\mu_q = 0$

Relativistic massless quark gas ➡ $E^2 = p^2 + m^2$ (with m^2 struck out)

The energy of a quark in a system of Volume **V** and temperature **T**

mean value

$$E_q = \frac{g_q V}{2\pi^2} \int_0^\infty \frac{p^3 dp}{1+e^{p/T}}$$

pressure due to quarks

$$E_q = \frac{g_q V}{2\pi^2} T^4 \int_0^\infty \frac{z^3 dz}{1+e^z}$$

$$E_q = \frac{g_q V}{2\pi^2} T^4 \Gamma(4) \sum_{n=0}^\infty (-1)^n \frac{1}{(n+1)^4}$$

The energy of the system due to quarks is therefore

$$E_q = \frac{7}{8} g_q V \frac{\pi^2}{30} T^4$$

for massles fermions and bosons pressure and energy are related:

$$P = \frac{1}{3} \frac{E}{V}$$

so that, pressure due to quarks

$$P_q = \frac{7}{8} g_q V \frac{\pi^2}{90} T^4$$

anti-quarks

$$P_{\bar{q}} = \frac{7}{8} g_{\bar{q}} V \frac{\pi^2}{90} T^4$$

the pressure due to gluons

$$E_g = \frac{Vg_g}{2\pi^2}\int_0^\infty p^3 dp\left[\frac{1}{e^{p/T}-1}\right]$$

gluon degeneracy

Bose-Einstein distribution

pressure due to gluons

$$E_g = g_g V \frac{\pi^2}{30} T^4$$

and

$$P_g = g_g V \frac{\pi^2}{90} T^4$$

so that, the total pressure is $P = P_q + P_{\bar{q}} + P_g$

High Temperature

Consider a quark gluon system in thermal equilibrium
Quarks and Gluons are non interacting and massles

$$P = [g_g + \frac{7}{8}(g_q + g_q)]\frac{\pi^2}{90}T^4$$

degeneracy number of gluons quarks anti-quarks

$$g_g = 8 \times 2$$

gluons polarization

$$g_q = g_q = N_{colors} \times N_{spin} \times N_{flavor}$$

3 2 2 or 3

$P = 37\frac{\pi^2}{90}T^4$	$\varepsilon = 37\frac{\pi^2}{30}T^4$

We have therefore an energy density of $2.54 \; {}^{GeV}\!/\!_{fm^3}$

at $T = 200 \; MeV$

$$T_c = \left(\frac{90}{37\pi^2} \right)^{1/4} B^{1/4}$$

$$T_c = \left(\frac{90}{37\pi^2} \right)^{1/4} 206$$

$$T_c = 145 \; MeV$$

If quark matter is heated to a

$$T > T_c$$

the quark gluon matter in the bag will have pressure greater that the bag-pressure

→

bag breaks deconfined QGP

High Baryon Density

deconfinement may happen even at T = 0 → at which baryon density ?

The number of states in a volume ***V*** with momentum ***p*** in the interval ***dp***

$$\frac{g_q V}{(2\pi)^3} 4\pi p^2 dp$$

The number of quarks is

$$N_q = \frac{g_q V}{(2\pi)^3} \int_0^{\mu_q} 4\pi p^2 dp = \frac{g_q V}{6\pi^2} \mu_q^3$$

The number density of quarks is

$$n_q = \frac{N_q}{V} = \frac{g_q}{6\pi^2} \mu_q^3$$

The energy of the quark gas in a volume ***V***

$$E_q = \frac{g_q V}{(2\pi)^3} \int_0^{\mu_q} 4\pi p^3 dp = \frac{V g_q}{8\pi^2} \mu_q^4$$

energy density

$$\varepsilon = \frac{E_q}{V} = \frac{g_q}{8\pi^2} \mu_q^4$$

from the relation between the pressure and the energy density

$$P_q = \frac{1}{3} \frac{E}{V} = \frac{g_q}{24\pi^2} \mu_q^4$$

Change of state ⟶ $P_q = B$ ⇨ $\mu_q = \left(\frac{24\pi^2}{g_q} B \right)^{1/4}$

this corresponds to a CRITICAL quark number density

$$n_q(QGP) = 4\left(\frac{g_q}{24\pi^2}\right)^{1/4} B^{3/4}$$

i.e. **Critical Baryon Number**

$$n_q(QGP) = \frac{4}{3}\left(\frac{g_q}{24\pi^2}\right)^{1/4} B^{3/4}$$

Let's take ordinary nuclear matter as composed only of ***u*** and ***d*** quarks

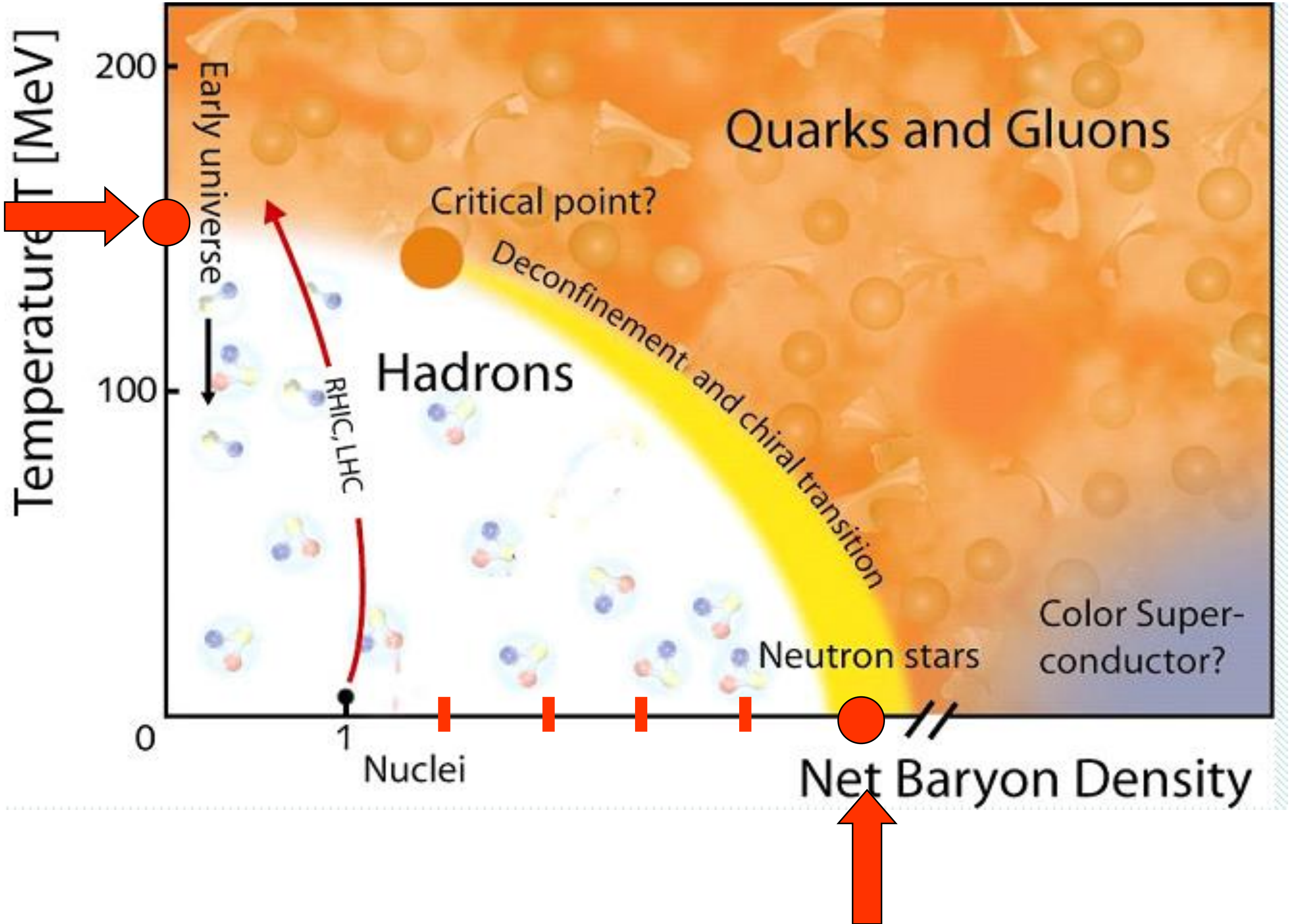
$$g_q = 3_{colors} \times 2_{spin} \times 2_{flavor}$$

For a Bag pressure $B^{1/4} = 206\ MeV$

The critical baryon number density $= 0.72 / fm^3$

at T=0

Phase Diagram of QCD Matter



Tsallis statistics

Journal of Statistical Physics 52(1988)

Possible Generalization of Boltzmann–Gibbs Statistics

Constantino Tsallis[1]

Received November 12, 1987; revision received March 8, 1988

With the use of a quantity normally scaled in multifractals, a generalized form is postulated for entropy, namely $S_q \equiv k[1-\sum_{i=1}^{W} p_i^q]/(q-1)$, where $q \in \mathbb{R}$ characterizes the generalization and $\{p_i\}$ are the probabilities associated with W (microscopic) configurations ($W \in \mathbb{N}$). The main properties associated with this entropy are established, particularly those corresponding to the microcanonical and canonical ensembles. The Boltzmann–Gibbs statistics is recovered as the $q \to 1$ limit.

correlated systems

$$S_q \equiv k \frac{1-\sum_{i=1}^{W} p_i^q}{q-1} \quad (q \in \mathbb{R})$$

$$S_1 \equiv \lim_{q\to 1} S_q = k \lim_{q \to 1} \frac{1-\sum_{i=1}^{W} p_i e^{(q-1)\ln p_i}}{q-1}$$

$$= -k \sum_{i=1}^{W} p_i \ln p_i$$

Additivity

$$\sum_{i,j}^{W_A, W_B} (p_{ij}^{A\cup B})^q = \left[\sum_{i=1}^{W_A} (p_i^A)^q\right] \left[\sum_{i=1}^{W_B} (p_i^B)^q\right]$$

$$\frac{S_q(A+B)}{k} = \frac{S_q(A)}{k} + \frac{S_q(B)}{k} + (1-q)\frac{S_q(A)}{k}\frac{S_q(B)}{k}$$

Successfully applied to high energy physics

Journal of Statistical Physics 52(1988)

Nonextensive statistical mechanics: Applications to high energy physics

Constantino Tsallis[1,2 a]

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Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil
[2] Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA

Abstract. Nonextensive statistical mechanics was proposed in 1988 on the basis of the nonadditive entropy $S_q = k[1 - \sum_i p_i^q]/(q-1)$ $(q \in \mathcal{R})$ which generalizes that of Boltzmann-Gibbs $S_{BG} = S_1 = -k \sum_i p_i \ln p_i$. This theory extends the applicability of standard statistical mechanics in order to also cover a wide class of anomalous systems which violate usual requirements such as ergodicity. Along the last two decades, a variety of applications have emerged in natural, artificial and social systems, including high energy phenomena. A brief review of the latter will be presented here, emphasizing some open issues.

EPJ Web of Conferences
Volume 13, 2011
HCBM 2010 – International Workshop on Hot and Cold Baryonic Matter

Tsallis in high energy physics

Fitting parameter

Hadronization temperature

$$\mu = \sqrt{p_T^2 + m^2}$$

Hagedorn theory

$$\sqrt{s} < 10 \text{ Gev}$$

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} = c\, p_T \int_0^\infty dp_l \, exp\left(-\frac{1}{T_0}\sqrt{p_l^2 + \mu^2}\right)$$

$$\left.\begin{array}{l} p_T \gg T_0 \\ p_T \gg m \end{array}\right\} \quad \frac{1}{\sigma}\frac{d\sigma}{dp_T} \sim p_T^{3/2}\, e^{-p_T/T_0}$$

Exponential asymptotic behavior

T_0 should not depend on the center of mass energy $\sqrt{s}$
c depends on the average multiplicity of the events → $\sqrt{s}$

same shape for all energies spectrum

Tsallis distribution

First time in HEP

Deviations from exponential decay were observed → Polynomial decay (Higher P_T)

$\sqrt{s} > 10$ Gev

Non-extensivity Parameter

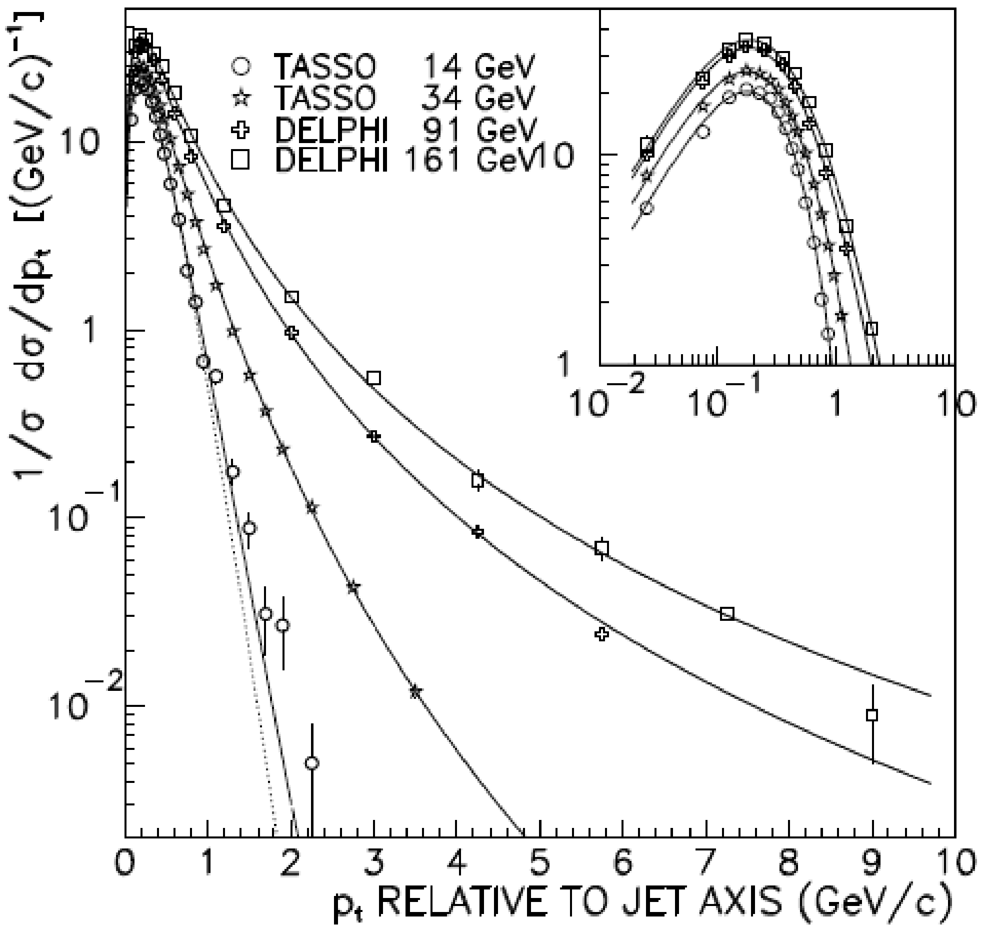
I. Bediaga et al (2000)

$e^+e^- \to \text{hadrons}$

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} = c\, p_T \int_0^\infty dp_l \left[1 - \frac{1-q}{T_0}\sqrt{p_l^2+\mu^2}\right]^{\frac{q}{1-q}}$$

I. Bediaga, E. M. Curado and J. M. de Miranda, Physica A 286, 156 (2000).

Findings $e^+e^- \rightarrow \text{hadrons}$



Transverse momentum distribution of charged hadrons in the e+e- interaction

The Hagedorn predicted exponential behavior is shown by the dotted lined.

The deviation of the exponential behavior increases when the energy increases.

The continuous lines are the Tsallis fits.

Good description of the transverse momentum.

TASSO, Z. Phys. C 22, 307-340 (1984).

DELPHI data get from: PhD Thesis from Oliver Passon, 1997, Wuppertal University-Germany

The power law associated with Tsallis statistics (Tsallis distributions) is being widely used in pp collisions

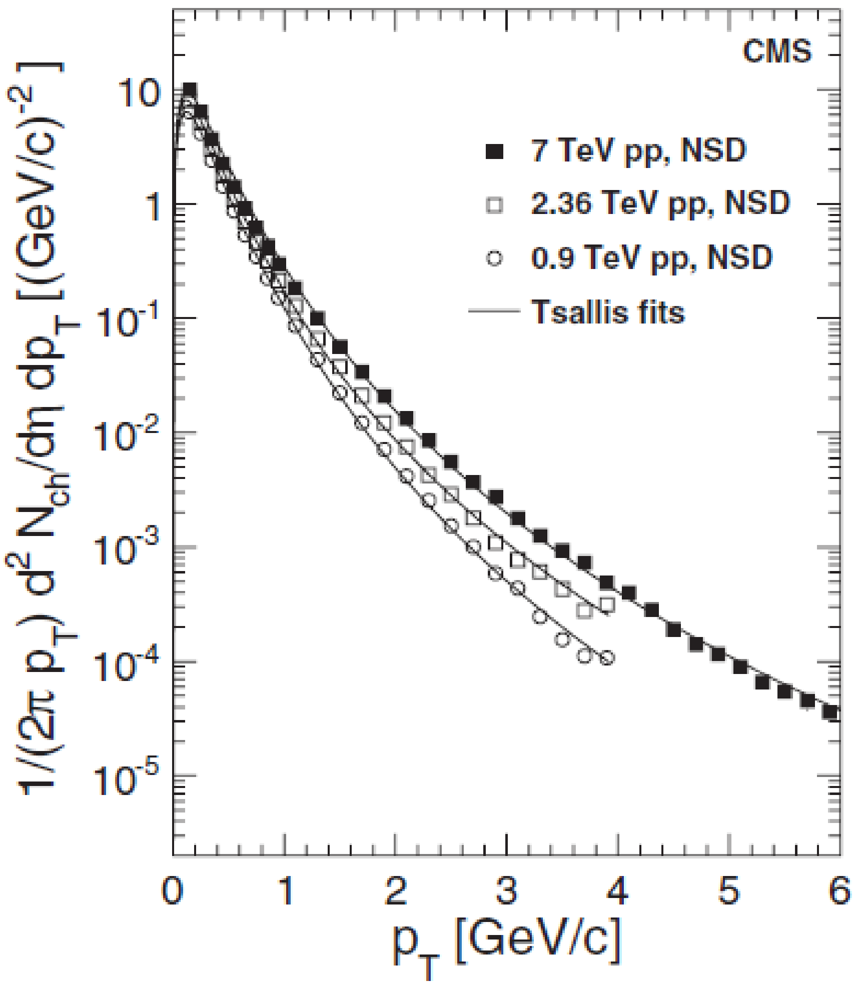
CMS

$$E\frac{d^3N_{\rm ch}}{dp^3} = \frac{1}{2\pi p_T}\,\frac{E}{p}\,\frac{d^2N_{\rm ch}}{d\eta\,dp_T} = C\frac{dN_{\rm ch}}{dy}\left(1+\frac{E_T}{nT}\right)^{-n},$$

$$E\,\frac{d^3N_{\rm ch}}{dp^3} \sim \frac{dN_{\rm ch}}{d\eta}\left(1+\frac{(q-1)\,E_\perp}{T}\right)^{-\frac{1}{q-1}}$$

q-exponential function

Fitting parameters: T, q



Charged-hadron yield as a function of pT in pp collisions

V. Khachatryan et al. (CMS Collaboration) Phys. Rev. Lett. 105, 022002 (2010)

Other references

pp collisions

L. Marques, J. Cleymans, and A. Deppman Phys. Rev. D **91**, 054025 (2015)

Transverse momentum and rapidity distributions for different identified particles.
PHENIX, ALICE, CMS (0.2 → 7 Tev)

T. Bhattacharyya, J. Cleymans, L. Marques, S. Mogliacci and M. W. Paradza, J. Phys. G 45 no.5, 055001 (2018)

Transverse momentum for identified particles (pions, kaons and protons).
ALICE (0.9, 2.76 and 7 Tev)

A. Khuntia, S. Tripathy, R. Sahoo and J. Cleymans. Eur. Phys. J. A (2017) 53: 103.

Transverse momentum for strange and mutil-strange particles.
LHC at 7 Tev

Heavy ions collisions

Tsallis distributions with some modifications to include the transverse flow

K. Saraswat, P. Shukla, V. Singh. J. Phys. Commun. 2 (2018) 035003

Transverse momentum of charged hadrons. LHC at 13 Tev

Transverse momentum of hadrons in pPb and PbPb collisions (different centralities) . CMS at 5.02 Tev.

K. Saraswat, P. Shukla, V. Kumar and V. Singh, Eur. Phys. J. A 53 (2017) 84.

Transverse momentum of the strange hadrons in pPb (5.02 Tev) and PbPb (2.76 Tev) collisions at CMS

all this may indicate a universal underlying mechanism in the hadron structure

all this may indicate a universal underlying mechanism in the hadron structure

Tsallis MIT bag model

- We present a phenomenological framework based on the MIT bag model to estimate the pressure due to the gluons as well as the total pressure inside nucleons. To do so, a non-extensive Tsallis statistics of quarks and gluons is implemented

T-MIT **-** *Tsallis MIT Bag Model*

Pressure in the MIT bag model with a non extensive Tsallis statistics

- Quarks and gluones are two correlated systems in the hadron structure.

The correlation is represented effectively by the q Tsallis parameter. The non extensive entropy for the hadron is then given by

$$S_q = S_Q + S_G + (1-q)S_Q S_G$$

- where S_Q y S_G están dados por

$$S = \frac{1}{T}\left(E + PV - \sum_i \mu_i N_i\right)$$

$$S_Q = g_Q \left[\frac{7\pi^2}{90} + \frac{1}{6}\left(\frac{\mu}{T}\right)^2\right] VT^3$$

$$S_G = 4g_G \left[\frac{\pi^2}{90}\right] VT^3$$

One may obtain entropy from the Maxwell relations:

$$\left.\frac{\partial S_q}{\partial V}\right|_{V,\,\mu} = \left.\frac{\partial P_q}{\partial T}\right|_{V,\,\mu}$$

$$\left.\frac{\partial S_q}{\partial V}\right|_{V,\,\mu} = [7g_Q+4g_G]\frac{\pi^2}{90}T^3+\frac{1}{6}g_Q\left(\frac{\mu}{T}\right)^2T^3+\frac{8\pi^2}{90}g_Qg_G(1-q)\left[\frac{7\pi^2}{90}+\frac{1}{6}\left(\frac{\mu}{T}\right)^2\right]VT^6$$

And then integrating this expresión with respect to Temperature and requiring that $q \to 1$, the Tsallis pressure is simply $P_q = P_Q + P_G$

$$P_q = \left[\frac{7}{4}g_Q + g_G\right]\frac{\pi^2}{90}T^4+\frac{1}{3}g_Q\left[\frac{1}{4}\left(\frac{\mu}{T}\right)^2+\frac{1}{8\pi^2}\left(\frac{\mu}{T}\right)^4\right]T^4+\frac{8\pi^2}{90}g_Qg_G(1-q)\left[\frac{\pi^2}{90}+\frac{1}{30}\left(\frac{\mu}{T}\right)^2\right]VT^7$$

$$P_{\text{total}}(r) = P_q(r) - B(r)$$

Parameters for the proton in a Bag Model

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Some characteristic parameters of proton from the bag model

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A B S T R A C T

We treat the mass of a proton as the total static energy which can be separated into two parts that come from the contribution of quarks and gluons respectively. We adopt the essential of the bag model of hadron to discuss the structure of a proton and find that the calculated temperature, proton radius, the bag constant are acceptable if a proton is a thermal equilibrium system of quarks and gluons.

$$(i\gamma^{\mu}\partial_{\mu} - m)\psi = 0, m = 0$$

$$\overline{\psi}\psi|_{r=R} = [j_0(p^0R)]^2 - \sigma\cdot\hat{\mathbf{r}}\sigma\cdot\hat{\mathbf{r}}[j_1(p^0R)]^2 = 0$$

$$p_m^0 = \frac{2.04}{R}$$

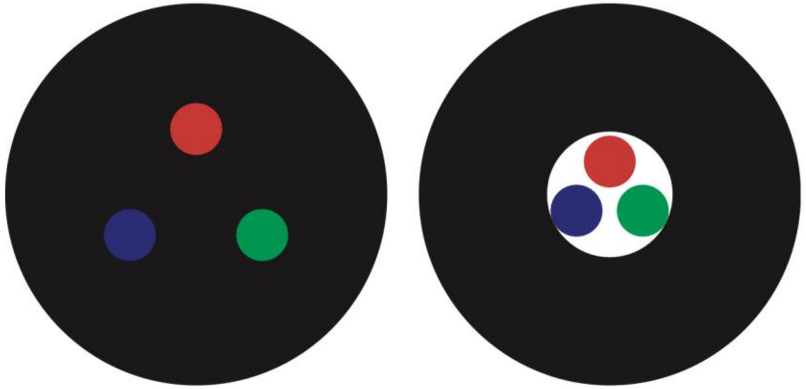


Fig. 2. Two scenarios on the structures of a nucleon. In the left one the space inside is full of gluons and three quarks can swim in the gluon sea, while the right one is another version in which the quarks are enclosed by the gluons.

Temperature as a function of radii

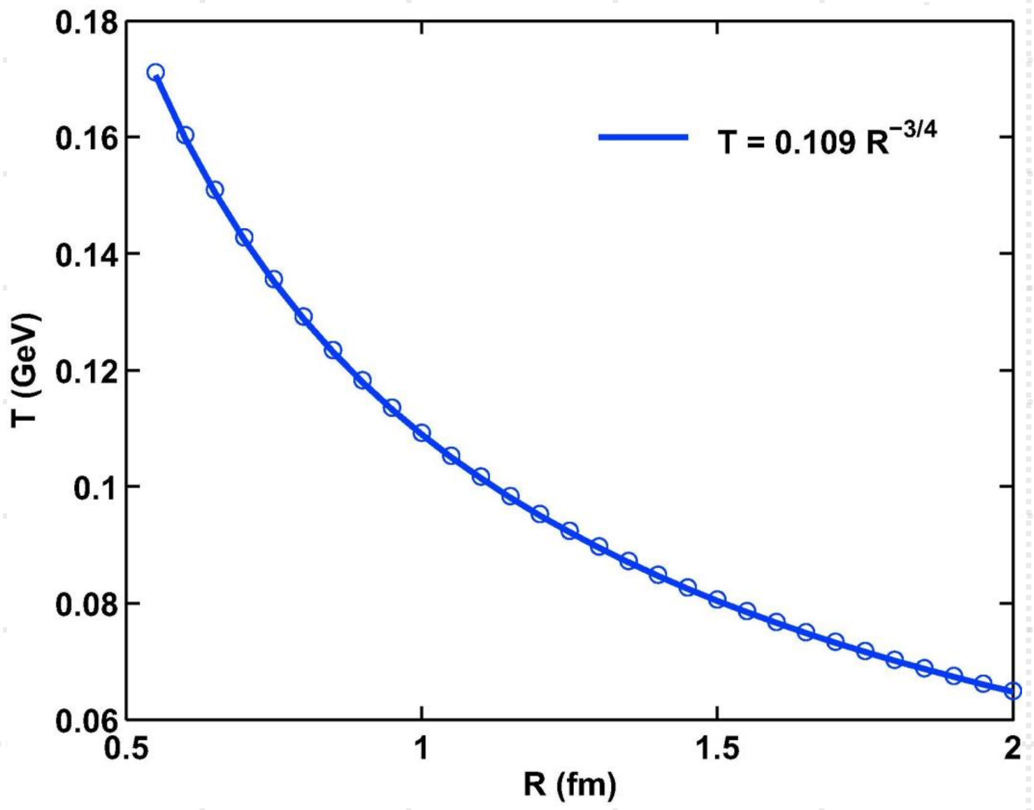
- By considering the mass of the nucleon as the totalenergy

$$E_t = M$$

- Taking the proton as a system in termic equilibrium

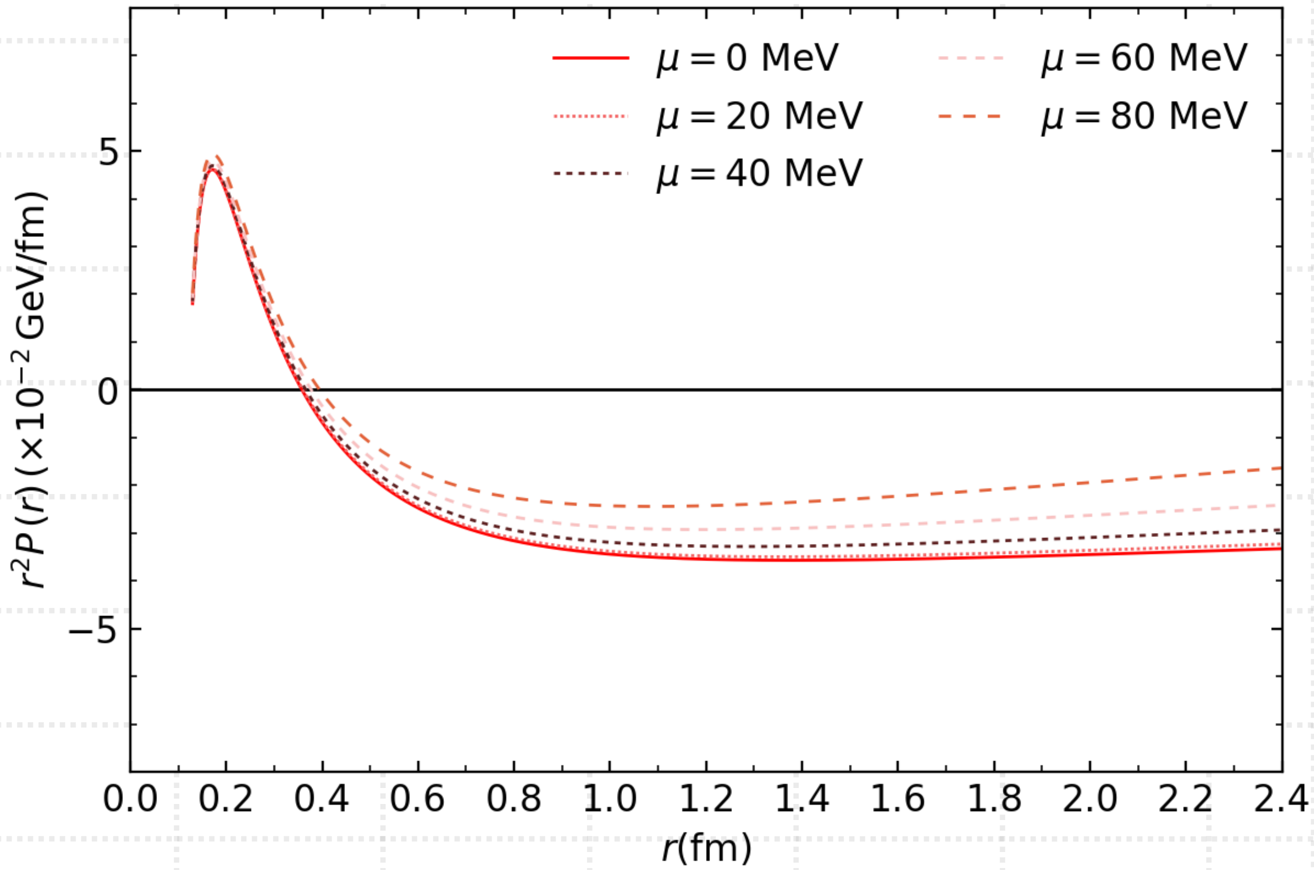
- Neglecting the chemical potential $\mu_q = 0$

$$T = 109 r^{-\frac{3}{4}}$$



Pressure distribution in the T-MIT bag model

Tsallis parameter: q=1.002



The pressure of quarks

NATURE | VOL 557 | 17 MAY 2018 LETTER RESEARCH

LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]

The proton, one of the components of atomic nuclei, is composed of fundamental particles called quarks and gluons. Gluons are the carriers of the force that binds quarks together, and free quarks are never found in isolation—that is, they are confined within the composite particles in which they reside. The origin of quark confinement is one of the most important questions in modern particle and nuclear physics because confinement is at the core of what makes the proton a stable particle and thus provides stability to the Universe. The internal quark structure of the proton is revealed by deeply virtual Compton scattering[1,2], a process in which electrons are scattered off quarks inside the protons, which subsequently emit high-energy photons, which are detected in coincidence with the scattered electrons and recoil protons. Here we report a measurement of the pressure distribution experienced by the quarks in the proton. We find a strong repulsive pressure near the centre of the proton (up to 0.6 femtometres) and a binding pressure at greater distances. The average peak pressure near the centre is about 10^{35} pascals, which exceeds the pressure estimated for the most densely packed known objects in the Universe, neutron stars[3]. This work opens up a new area of research on the fundamental gravitational properties of protons, neutrons and nuclei, which can provide access to their physical radii, the internal shear forces acting on the quarks and their pressure distributions.

(2) We then define the complex CFF, $\mathcal{H}$, which is directly related to the experimental observables describing the DVCS process, that is, the differential cross-section and the beam-spin asymmetry.
(3) The real and imaginary parts of $\mathcal{H}$ can be related through a dispersion relation[14–16] at fixed t, where the term $D(t)$, or D-term, appears as a subtraction term[17].
(4) We derive $d_1(t)$ from the expansion of $D(t)$ in the Gegenbauer polynomials of ξ, the momentum transfer to the struck quark.
(5) We apply fits to the data and extract $D(t)$ and $d_1(t)$.
(6) Then, we determine the pressure distribution from the relation between $d_1(t)$ and the pressure $p(r)$, where r is the radial distance from the proton's centre, through the Bessel integral.

The sum rules that relate the second Mellin moments of the chiral-even GPDs to the GFFs are[1]:

$$\int x[H(x,\xi,t)+E(x,\xi,t)]\,dx = 2J(t)$$

$$\int xH(x,\xi,t)dx = M_2(t)+\frac{4}{5}\xi^2 d_1(t)$$

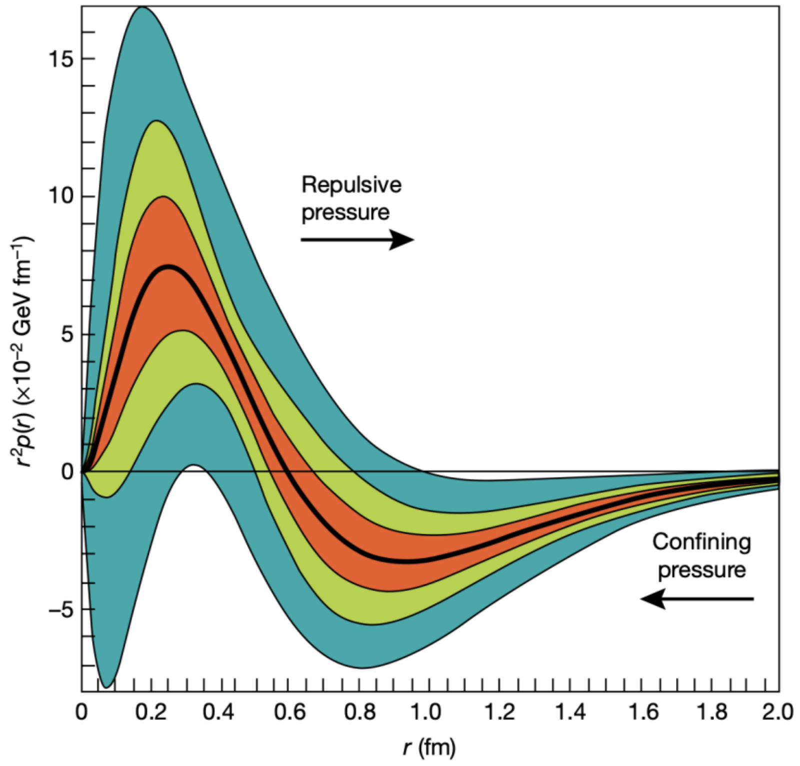
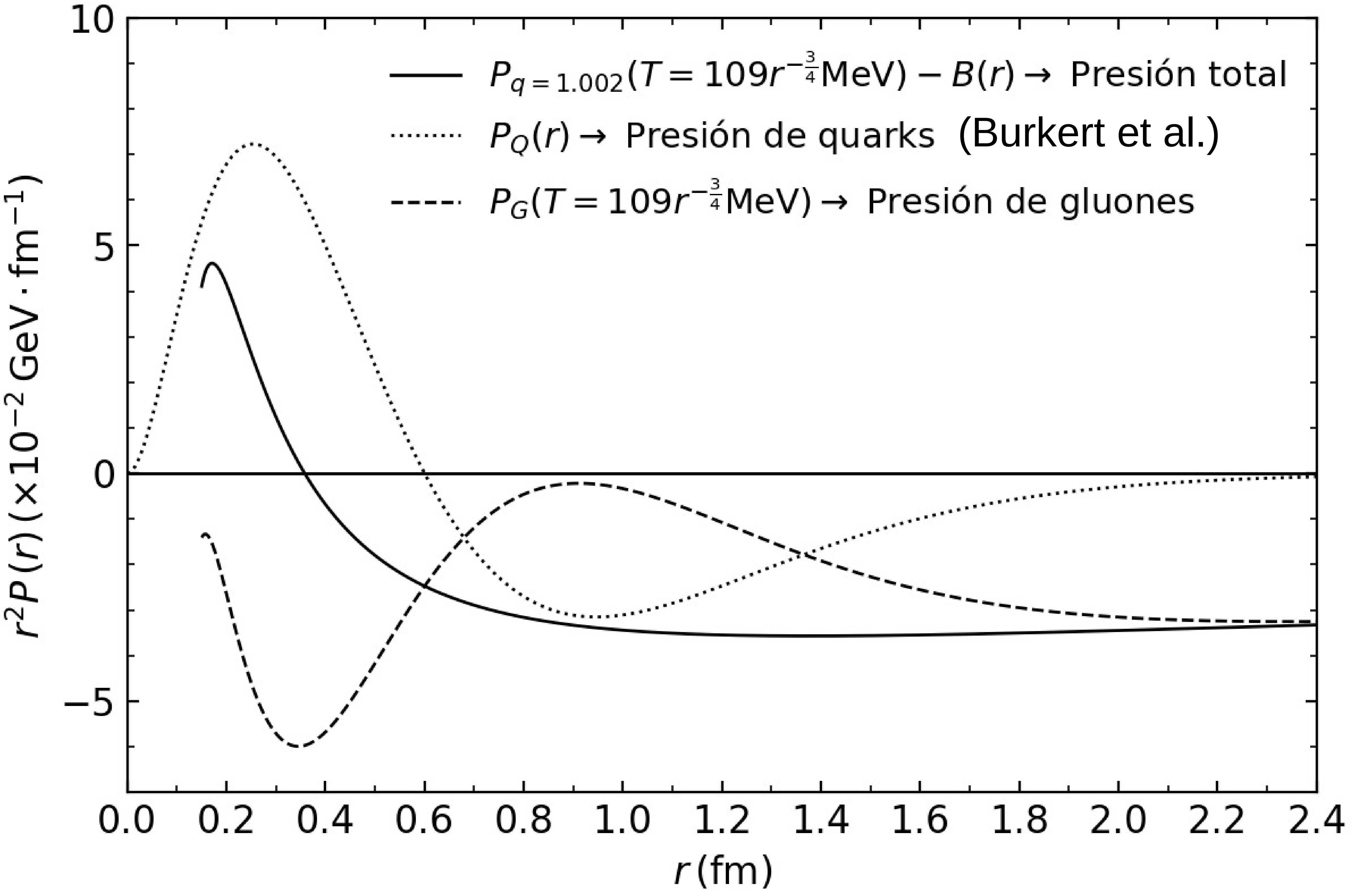
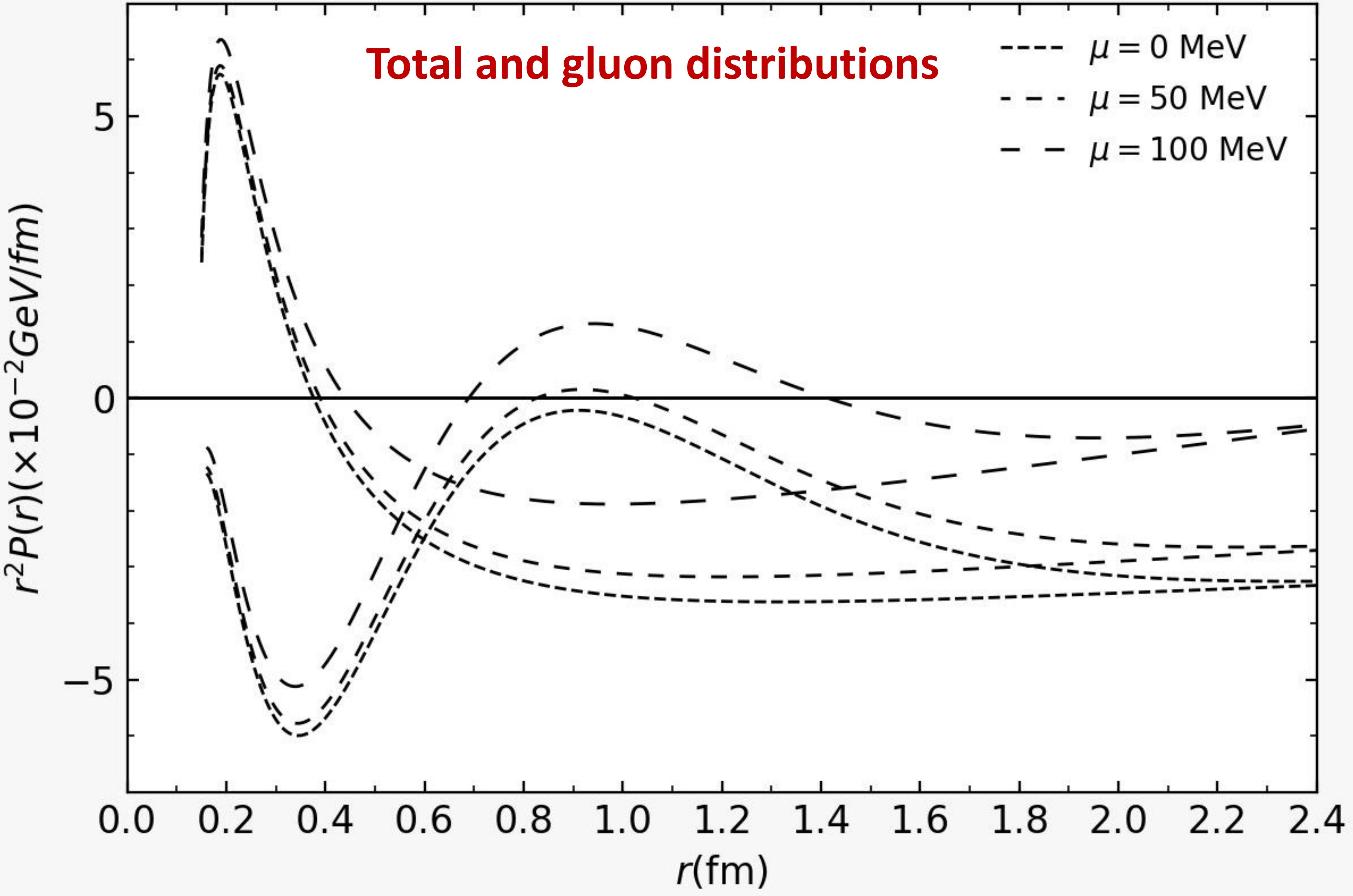


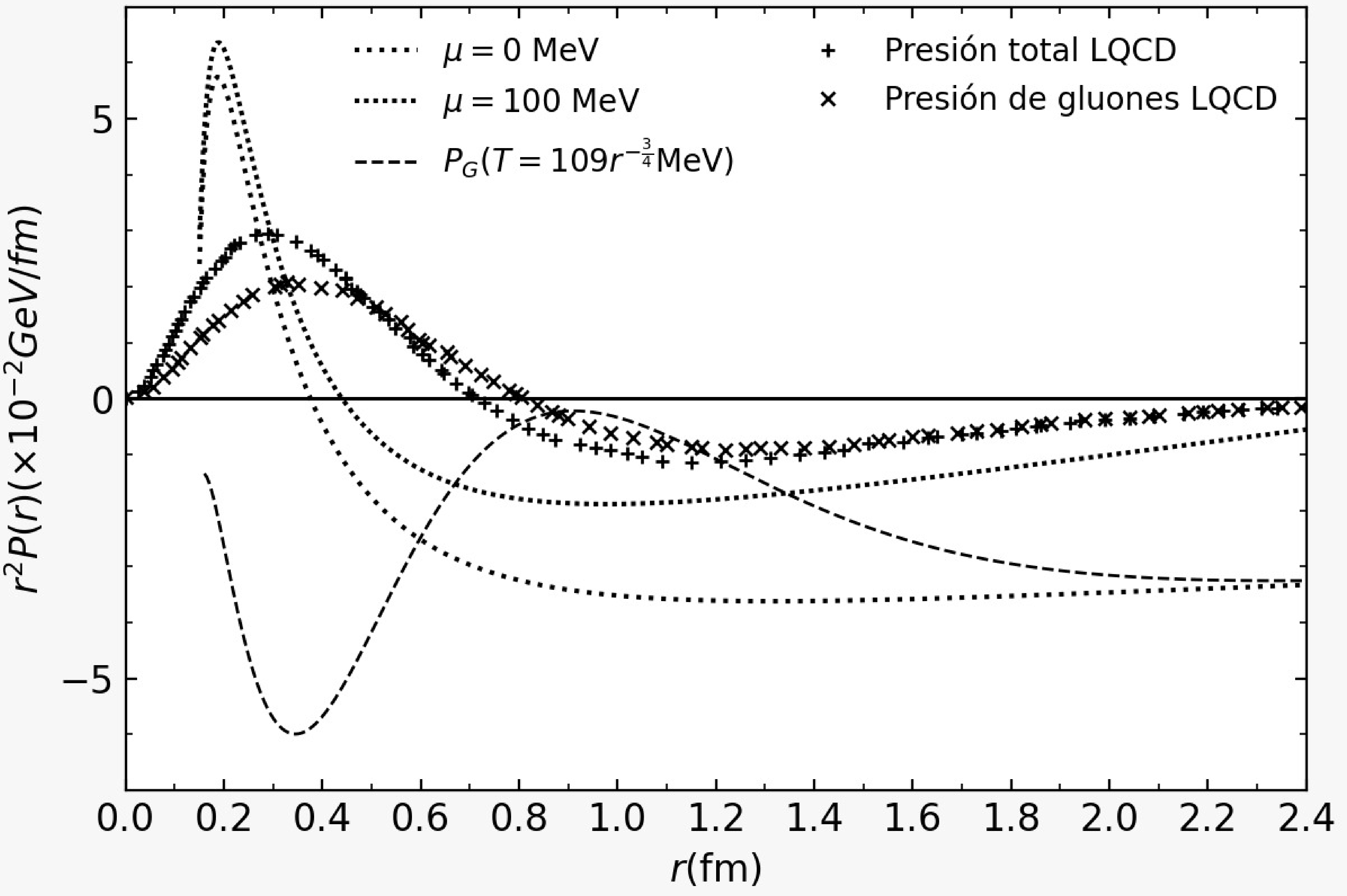
Fig. 1 | Radial pressure distribution in the proton. The graph shows the pressure distribution $r^2p(r)$ that results from the interactions of the quarks in the proton versus the radial distance r from the centre of the proton. The thick black line corresponds to the pressure extracted from the D-term parameters fitted to published data[22] measured at 6 GeV. The corresponding estimated uncertainties are displayed as the light-green shaded area shown. The blue area represents the uncertainties from all the data that were available before the 6-GeV experiment, and the red shaded area shows projected results from future experiments at 12 GeV that will be performed with the upgraded experimental apparatus[30]. Uncertainties represent one standard deviation.



$P_{q=1.002}(T = 109r^{-\frac{3}{4}}\text{MeV}) - B(r) \rightarrow$ Presión total
$P_Q(r) \rightarrow$ Presión de quarks (Burkert et al.)
$P_G(T = 109r^{-\frac{3}{4}}\text{MeV}) \rightarrow$ Presión de gluones
$r^2 P(r)\,(\times 10^{-2}\,\text{GeV}\cdot\text{fm}^{-1})$
$r\,(\text{fm})$
10
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4



Total and gluon distributions
μ = 0 MeV
μ = 50 MeV
μ = 100 MeV
5
0
−5
$r^2P(r)(\times10^{-2}GeV/fm)$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
r(fm)



$\mu = 0$ MeV
$\mu = 100$ MeV
$P_G(T = 109r^{-\frac{3}{4}}$MeV)
Presión total LQCD
Presión de gluones LQCD
$r^2P(r)(\times 10^{-2}GeV/fm)$
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
r(fm)

Conclusions

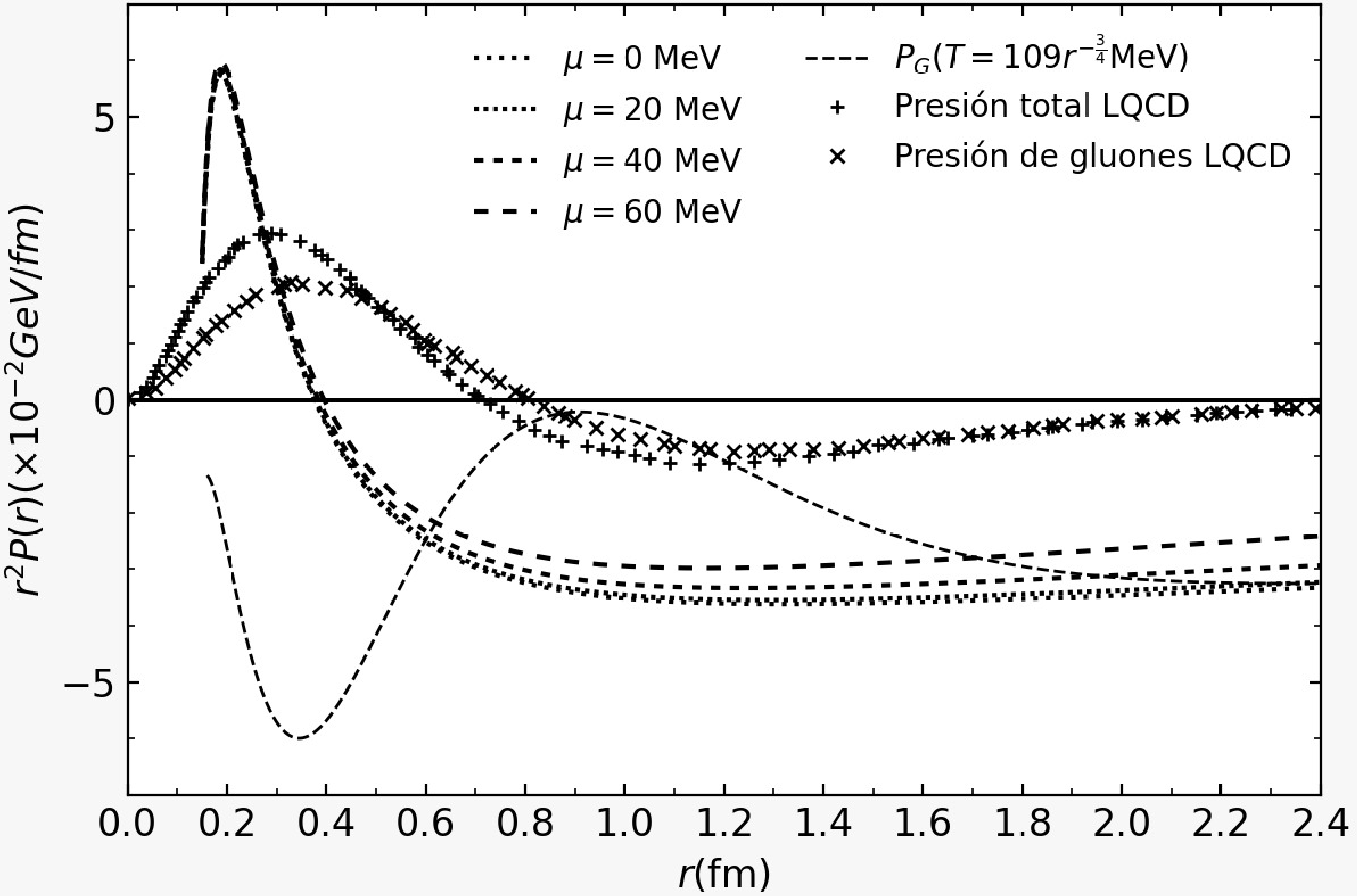
A. A Tsallis Statistics treatment of the interacting subsystems in a hadron has been implemented

B. In the new frame one may obtain the QCD phase diagram, masses and the pressure inside hadrons … as well as an interpretation of the “q” Tsallis parameter

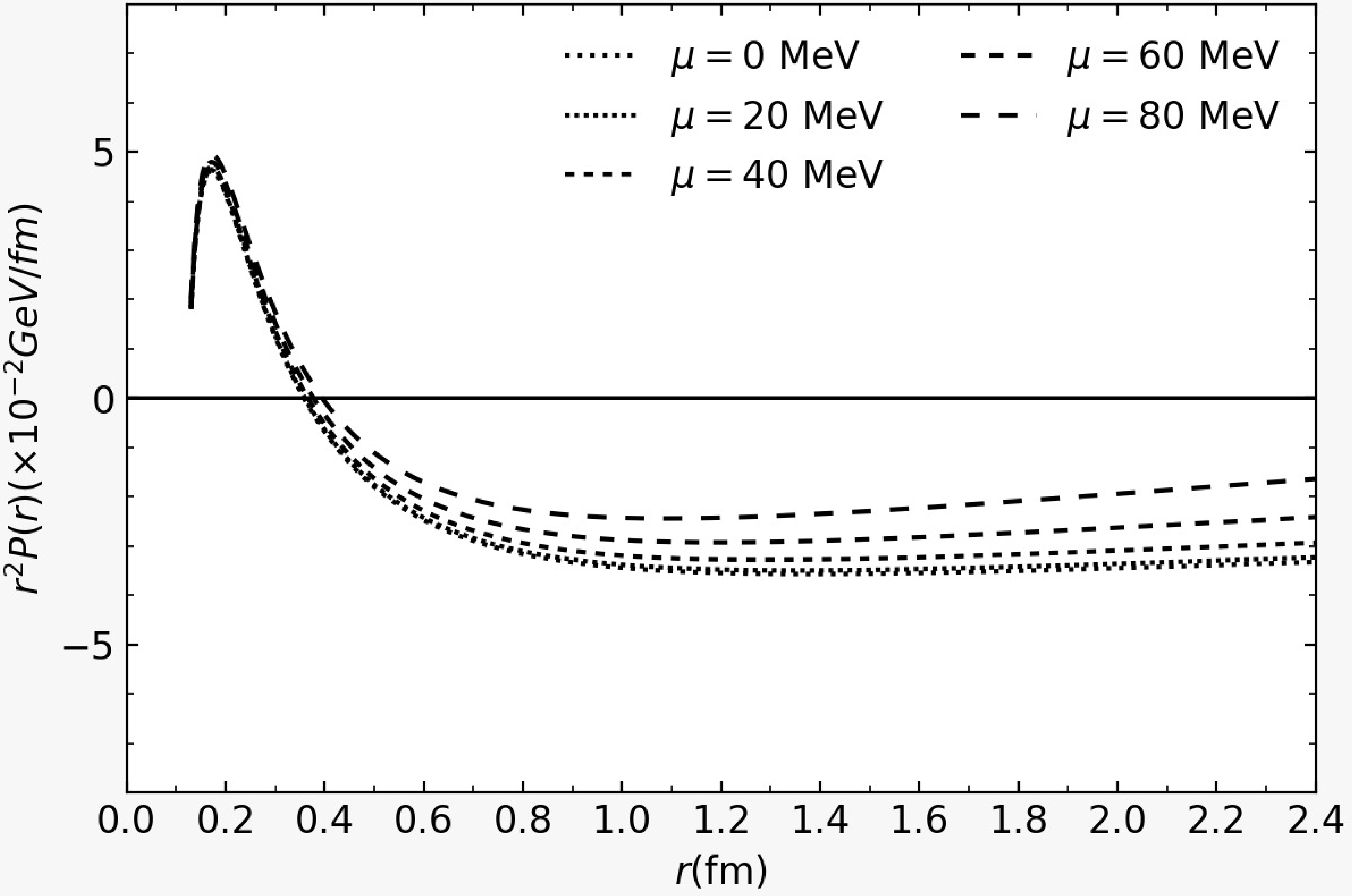
C. A quick calculation gives a total pressure which describes general aspects of the one obtained with LQCD calculations

D. The gluon pressure may be obtained using previous results together with the T - MIT bag Model

Backup



$\mu = 0$ MeV
$\mu = 20$ MeV
$\mu = 40$ MeV
$\mu = 60$ MeV
$P_G(T = 109r^{-\frac{3}{4}}\text{MeV})$
Presión total LQCD
Presión de gluones LQCD
$r^2P(r)(\times 10^{-2}GeV/fm)$
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
r(fm)



$\mu = 0$ MeV
$\mu = 20$ MeV
$\mu = 40$ MeV
$\mu = 60$ MeV
$\mu = 80$ MeV
$r^2P(r)(\times10^{-2}GeV/fm)$
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
r(fm)

Bag pressure

$$E_q = \frac{(g_q + g_{\bar{q}})V}{2\pi^2\hbar^3} \int_0^{p_m^0} \frac{p^3\,dp}{1 + e^{p/T(R)}}$$

$$B = \frac{E_t - E_q}{V}$$

$$B^{\frac{1}{4}} = 170 r^{-0.65}$$