DMNet

Accurate gamma-ray spectra for heavy (electro-)weakly interacting dark matter annihilation

DMNet International Symposium “Direct and Indirect Detection of Dark Matter” — Heidelberg 2022

Martin Vollmann — 14.9.2022

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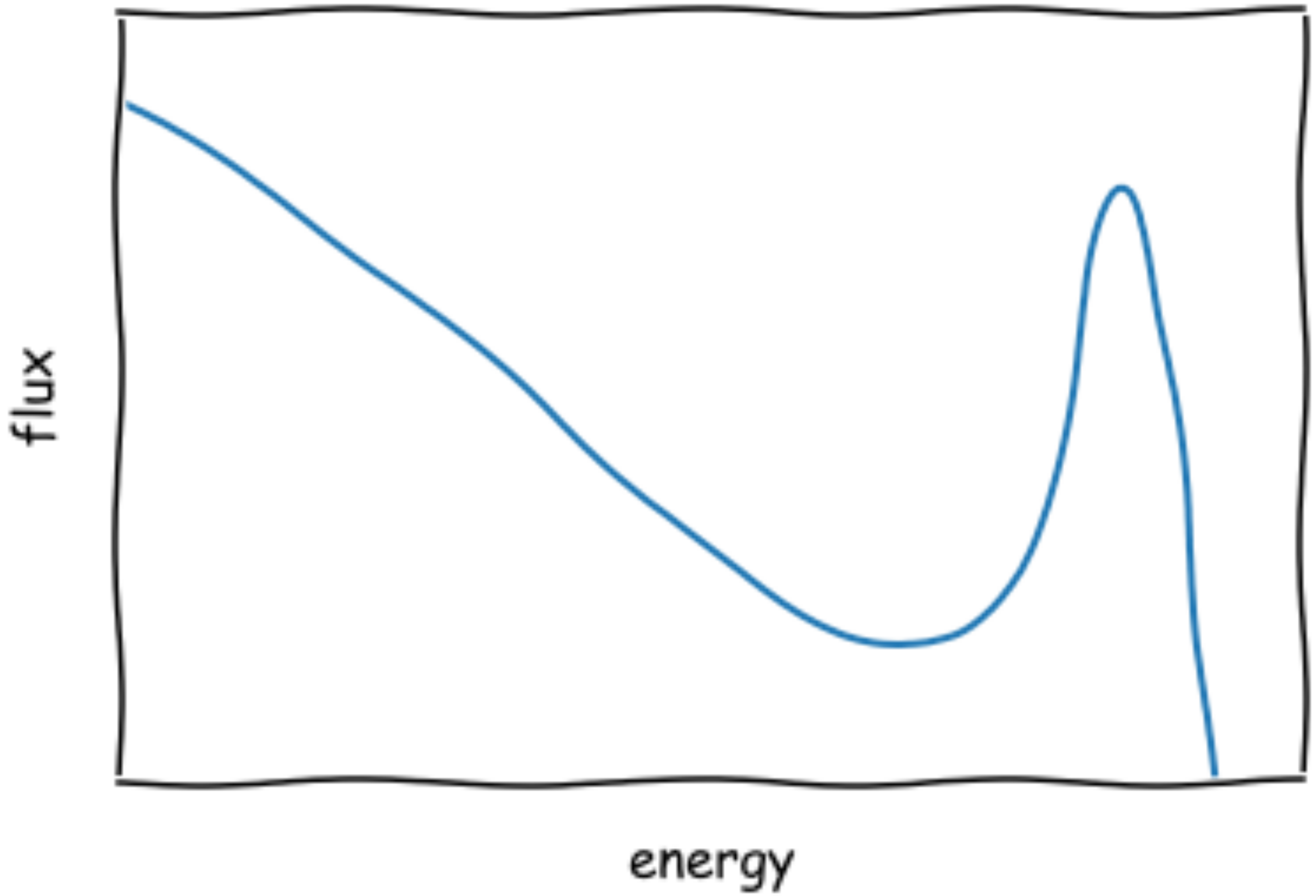
Gamma rays from DM annihilation

e. g. in the Milky Way



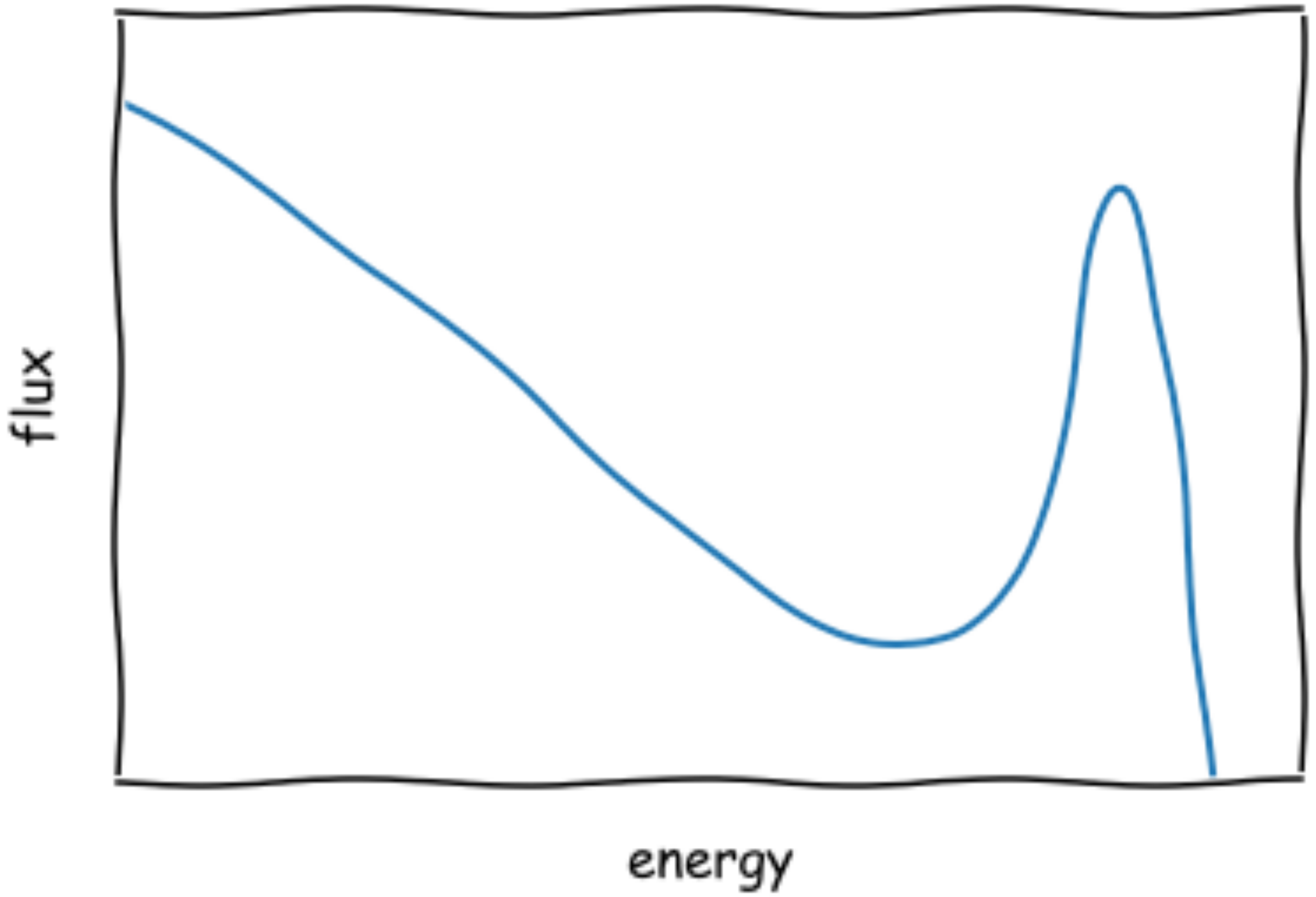
γs from DM ann.

Promising avenue

- *Bumpy* endpoint (spectral line)

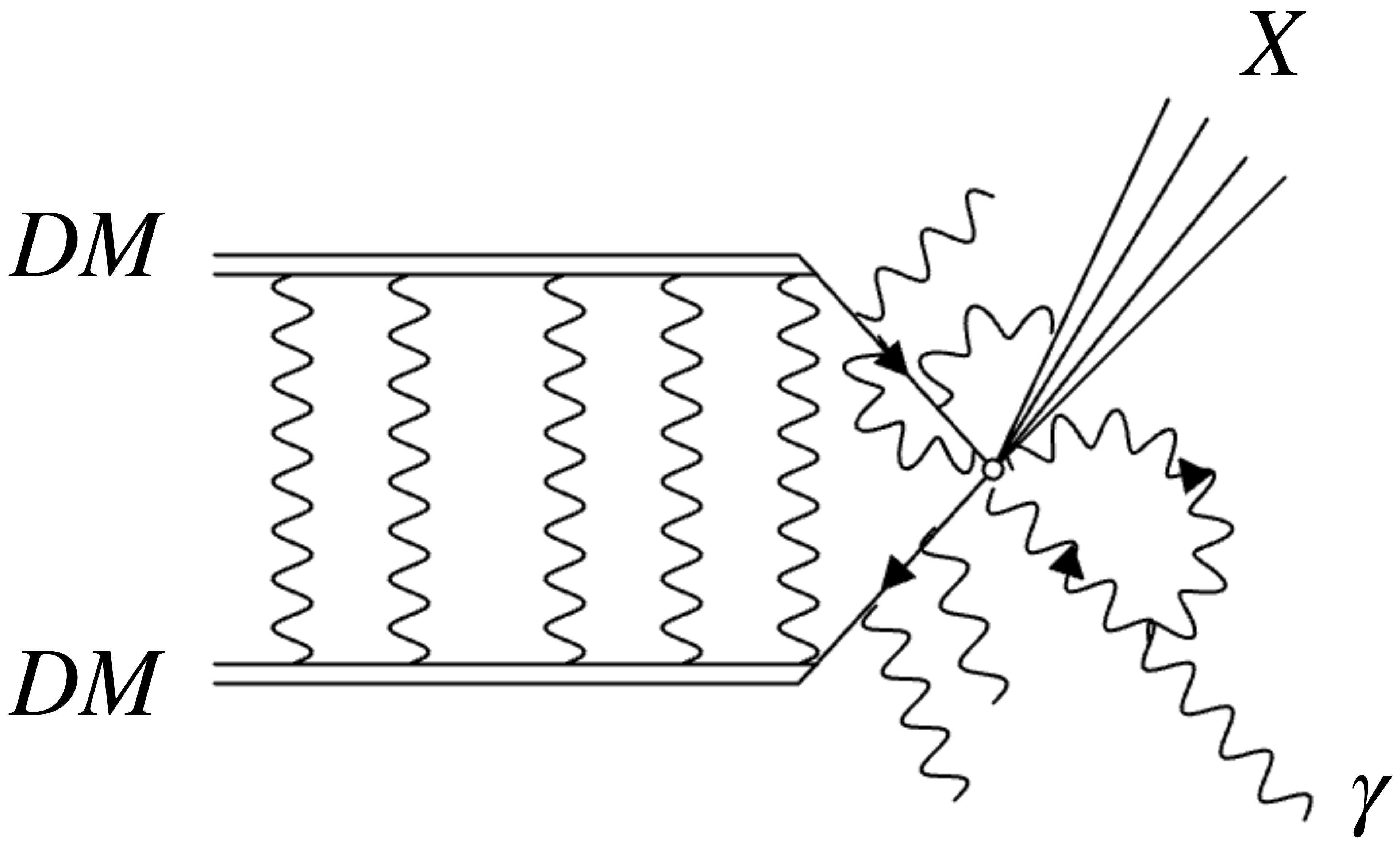
  $\Rightarrow$ **smoking gun**

- Theoretically and experimentally challenging, though …
  - Focus on the theoretical (particle-physics) challenges



Gamma rays from heavy DM annihilation

Theoretical challenges



γs from DM ann.

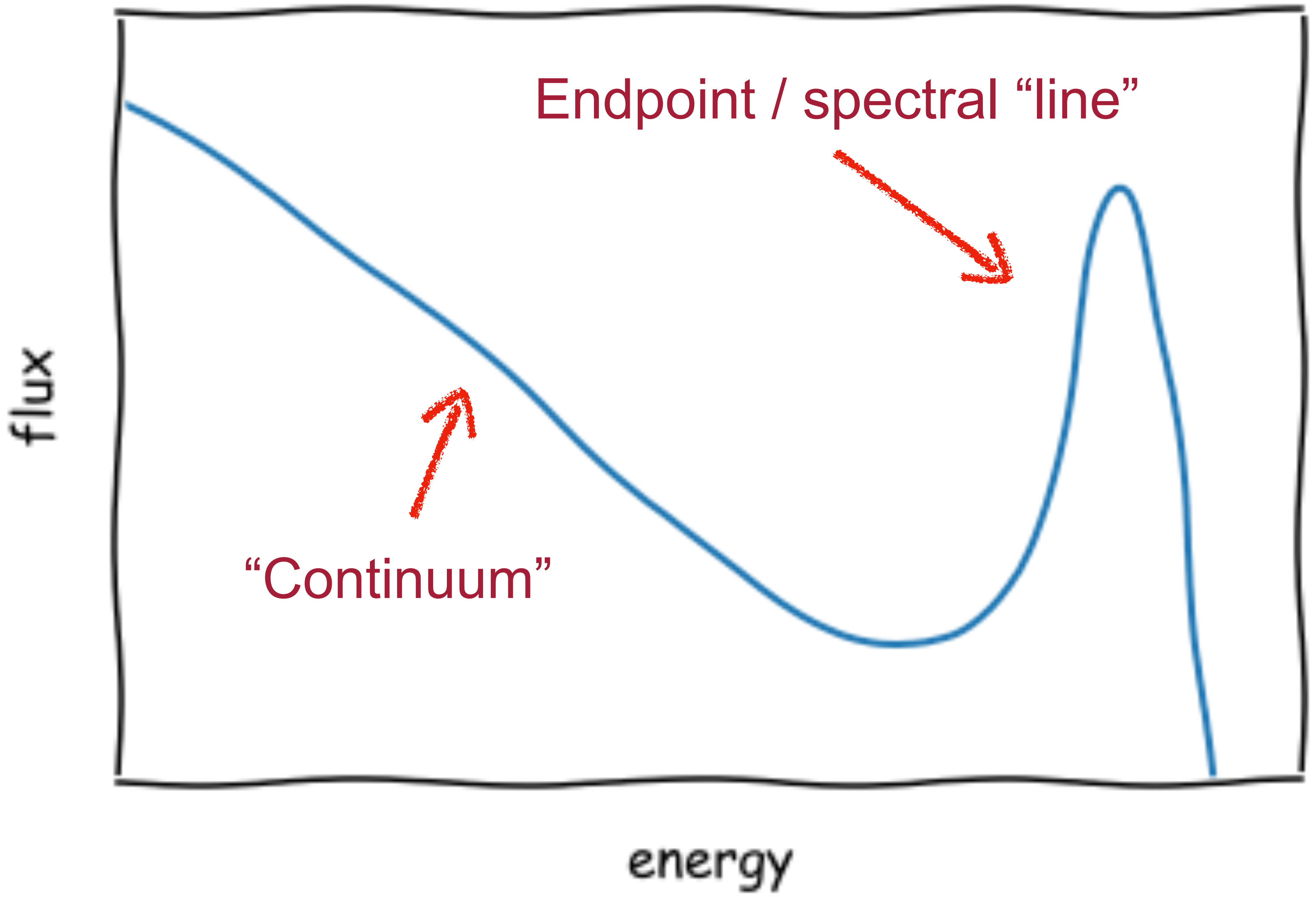
Theoretical challenges

- Continuum

  Fixed-order + Parton showers + Non-relativistic effective field theory (NREFT)

- Endpoint

  NREFT and soft-collinear effective theory (SCET)

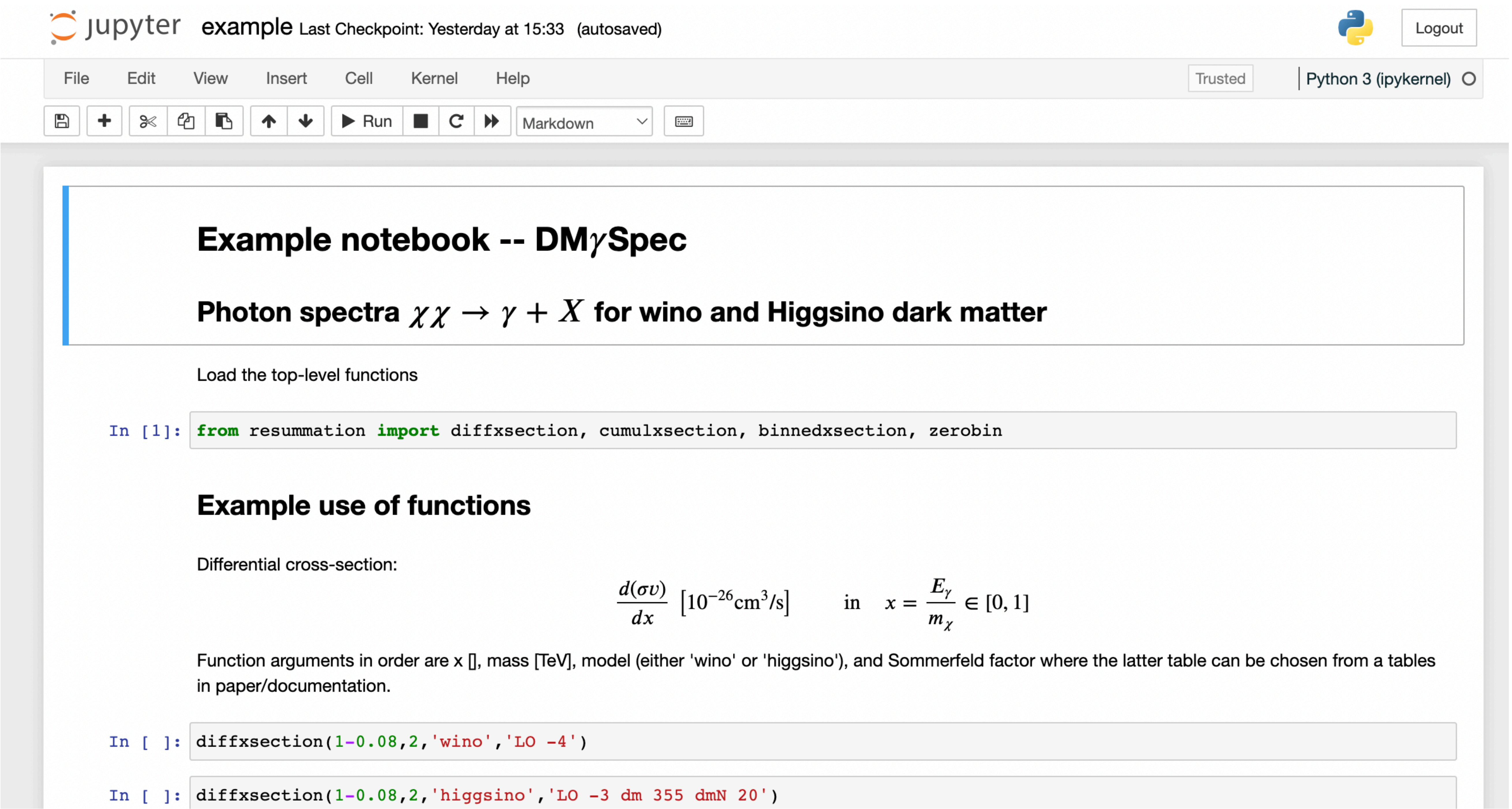


DMγSpec

Resum and plot your spectrum!

Open source python-based tool to compute and plot fully resummed γ-ray spectra from wino/higgsino ann.

- Floating wino/higgsino masses for e.g. parameter scans
- Variable-width Gaussian Instrument-Response Function (IRF) built in
  - Convolution with generic IRFs straightforward
- $\mathcal{O}(1\%)$ accuracy



Example notebook -- DMγSpec

Photon spectra $\chi\chi \to \gamma + X$ for wino and Higgsino dark matter

Load the top-level functions

```
In [1]: from resummation import diffxsection, cumulxsection, binnedxsection, zerobin
```

Example use of functions

Differential cross-section:

$$\frac{d(\sigma v)}{dx}\ [10^{-26}\mathrm{cm}^3/\mathrm{s}] \qquad \text{in} \quad x = \frac{E_\gamma}{m_\chi} \in [0,1]$$

Function arguments in order are x [], mass [TeV], model (either 'wino' or 'higgsino'), and Sommerfeld factor where the latter table can be chosen from a tables in paper/documentation.

```
In [ ]: diffxsection(1-0.08,2,'wino','LO -4')
```

```
In [ ]: diffxsection(1-0.08,2,'higgsino','LO -3 dm 355 dmN 20')
```

See https://dmyspec.hepforge.org/

Gamma rays from heavy DM annihilation

Literature

Mainly…

Matching resummed endpoint and continuum γ-ray spectra from dark-matter annihilation
Beneke, Vollmann, Urban — 2022
arXiv:2203.01692

Resummed photon spectrum from dark matter annihilation for intermediate and narrow energy resolution
Beneke, Broggio, Hasner, Vollmann, Urban — 2019 (~100 pages)
arXiv:1903.08702

Precise yield of high-energy photons from Higgsino dark matter annihilation
Beneke, Hasner, Vollmann, Urban — 2019
arXiv:1912.02034

But also,

Energetic γ-rays from TeV scale dark matter annihilation resummed
Beneke, Broggio, Hasner, Vollmann — 2018
arXiv:1805.07367

NLO electroweak potentials for minimal dark matter and beyond
Urban — 2021
arXiv:2108.07285

Wino potential and Sommerfeld effect at NLO
Beneke, Szafron, Urban — 2019
arXiv:1909.04584

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Motivation

Phenomenology

Sommerfeld enhancement

Sudakov logs

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Motivation

Dark Matter exists

Electroweak interactions exist

A 50yrs-old story of wimps

Coming to an end with Cherenkov telescopes?

- Freeze-out mechanism / **WIMP** miracle
  - Electroweak sector ⇔ dark matter

Supersymmetry

  - Pure “wino”/ “higgsino” ➠ **minimal BSM content**
- **Cherenkov Telescopes** can search for TeV-scale spectral lines
  - **Sommerfeld** effect: enhancements by several orders of magnitude
  - Besides the Sommerfeld effect, large EW effects at the endpoint (**Sudakov double logarithms**) have to be understood and resummed
- **Crucial** to have a **reliable computation** of the **full** continuum+line **spectrum**

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Dark Matter annihilation

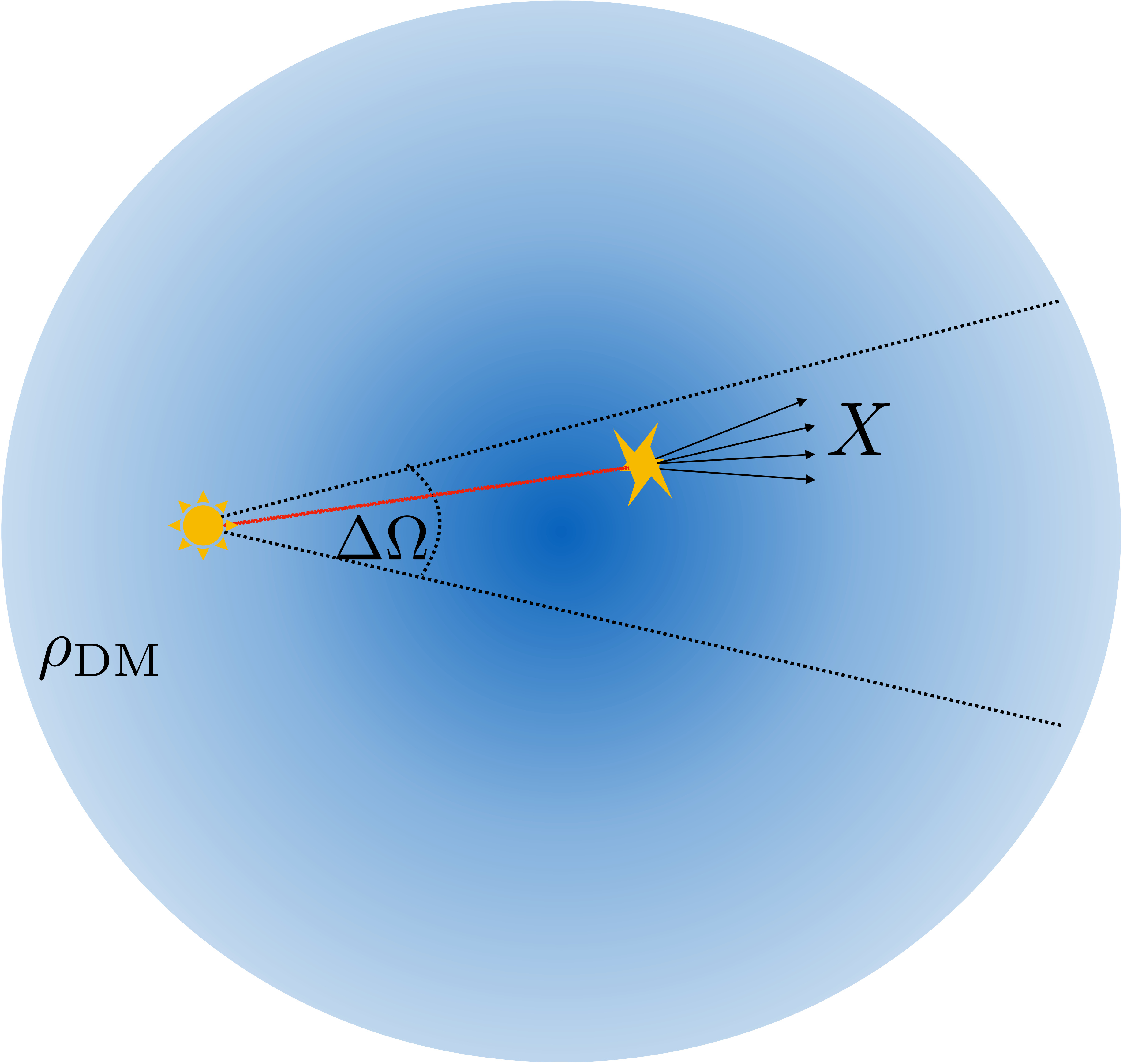
Prompt gamma rays

- “Count” the number of rays subtended in $\Delta\Omega$

$$\Phi_\gamma = \int_{\Delta\Omega} \mathrm{d}\Omega \, I_\gamma \, , \quad I_\gamma = \int_{\mathrm{l.o.s.}} \mathrm{d}s \, \frac{1}{4\pi} S_\gamma$$

- Rate sensitive to the (unknown) number density of DM particles
  - DM mass density ρ (if uncertain) is the available quantity

$$S_\gamma = \frac{1}{2} n_\chi^2 \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma} = \frac{1}{2\, m_\chi^2} \rho_{\mathrm{DM}}^2 \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma}$$



Dark Matter annihilation

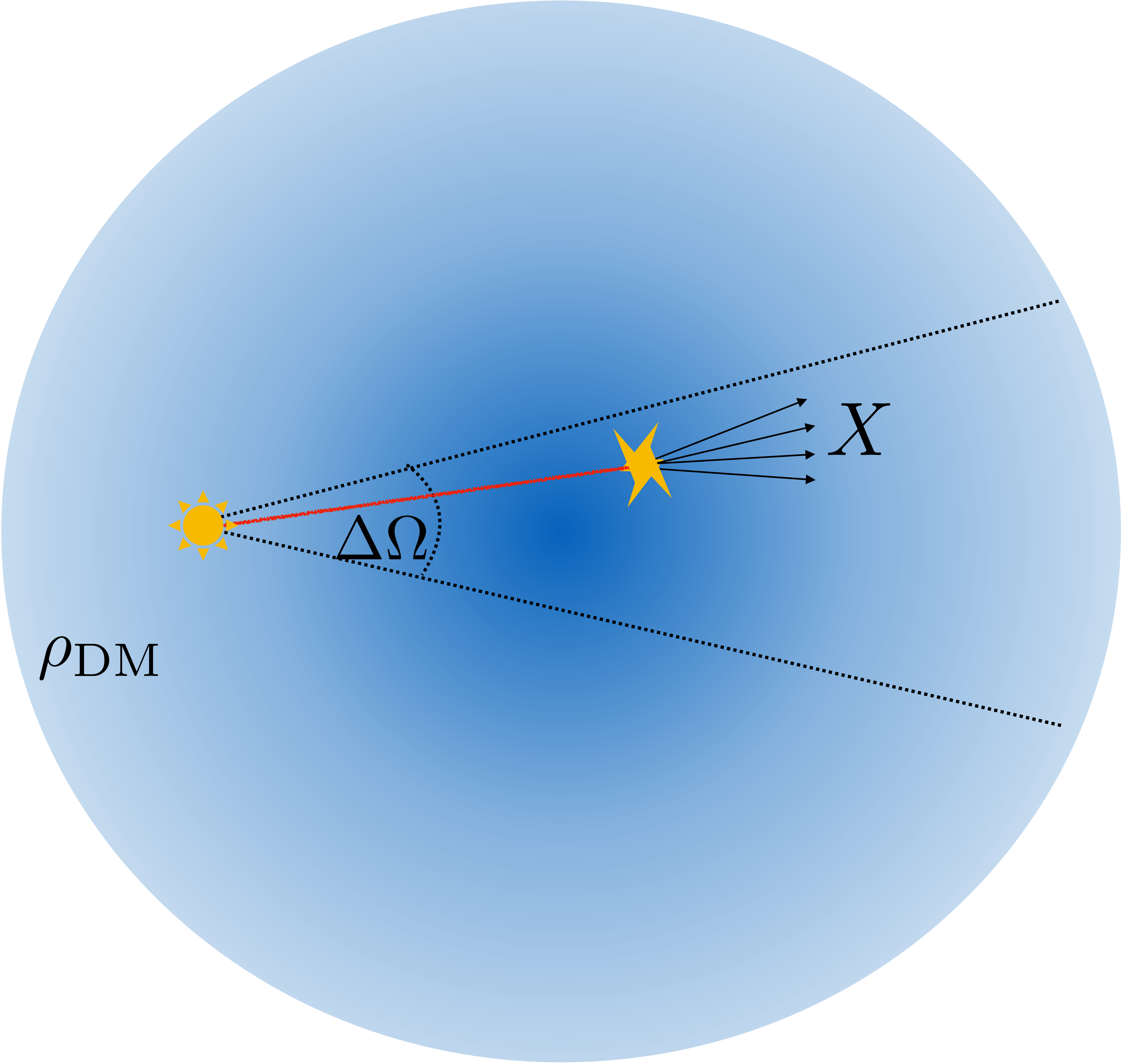
Astrophysics factored out

- Putting things together:

$$\Phi_\gamma = \frac{1}{8\pi m_\chi^2} \times J \times \frac{\mathrm{d}\langle\sigma v\rangle}{\mathrm{d}E_\gamma},$$

where the “J” factor is defined as

$$J = \int \mathrm{d}\Omega \int_{l.o.s.} \mathrm{d}s\, \rho_{\mathrm{DM}}^2$$



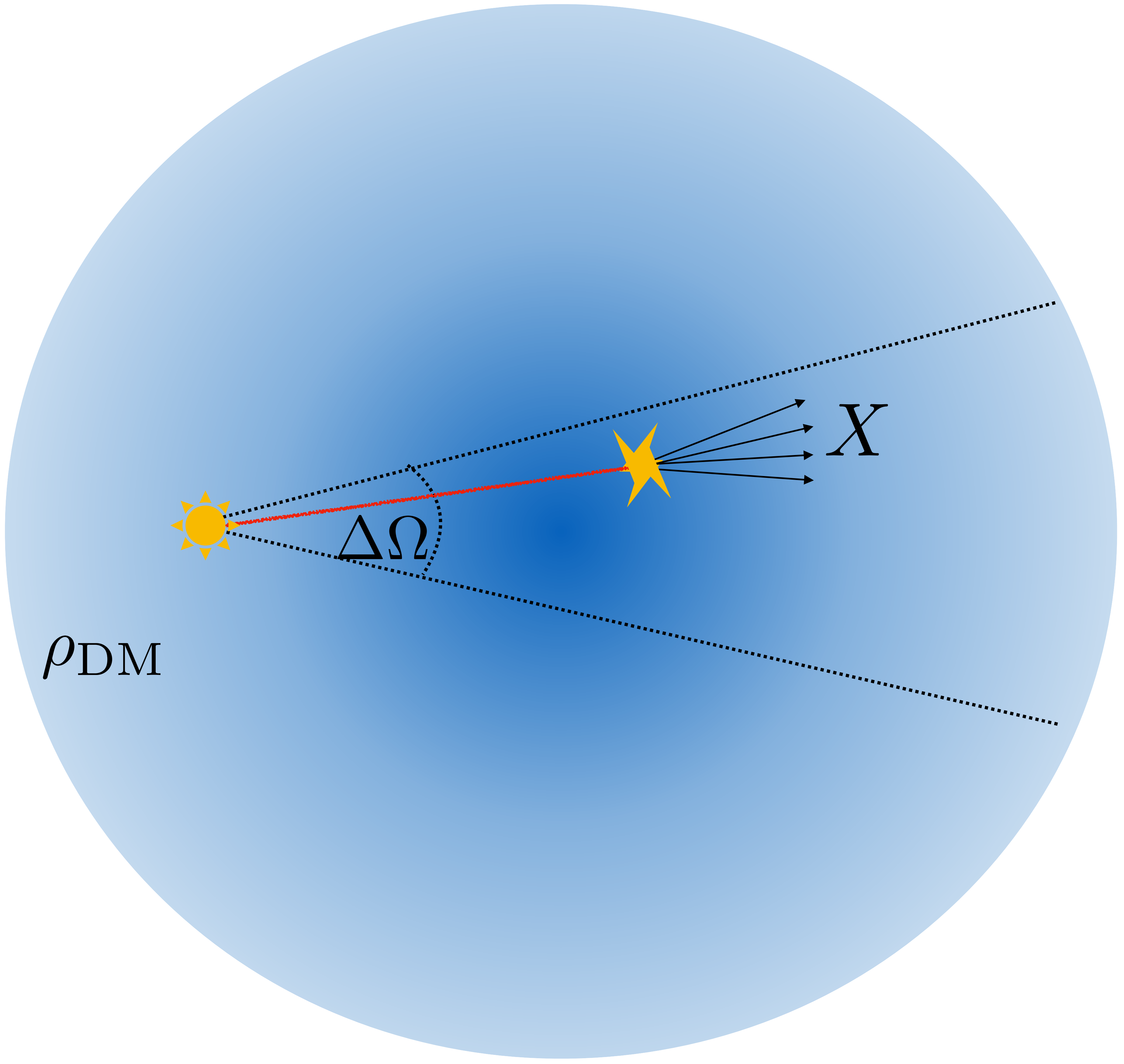
Dark Matter annihilation

Prompt gamma rays

- γ-ray flux via dark-matter annihilation

$$\Phi_\gamma = \frac{1}{8\pi m_\chi^2} \times J \times \boxed{\frac{\mathrm{d}\langle \sigma v \rangle}{\mathrm{d}E_\gamma}}$$

➡ Focus on a particle physics problem!



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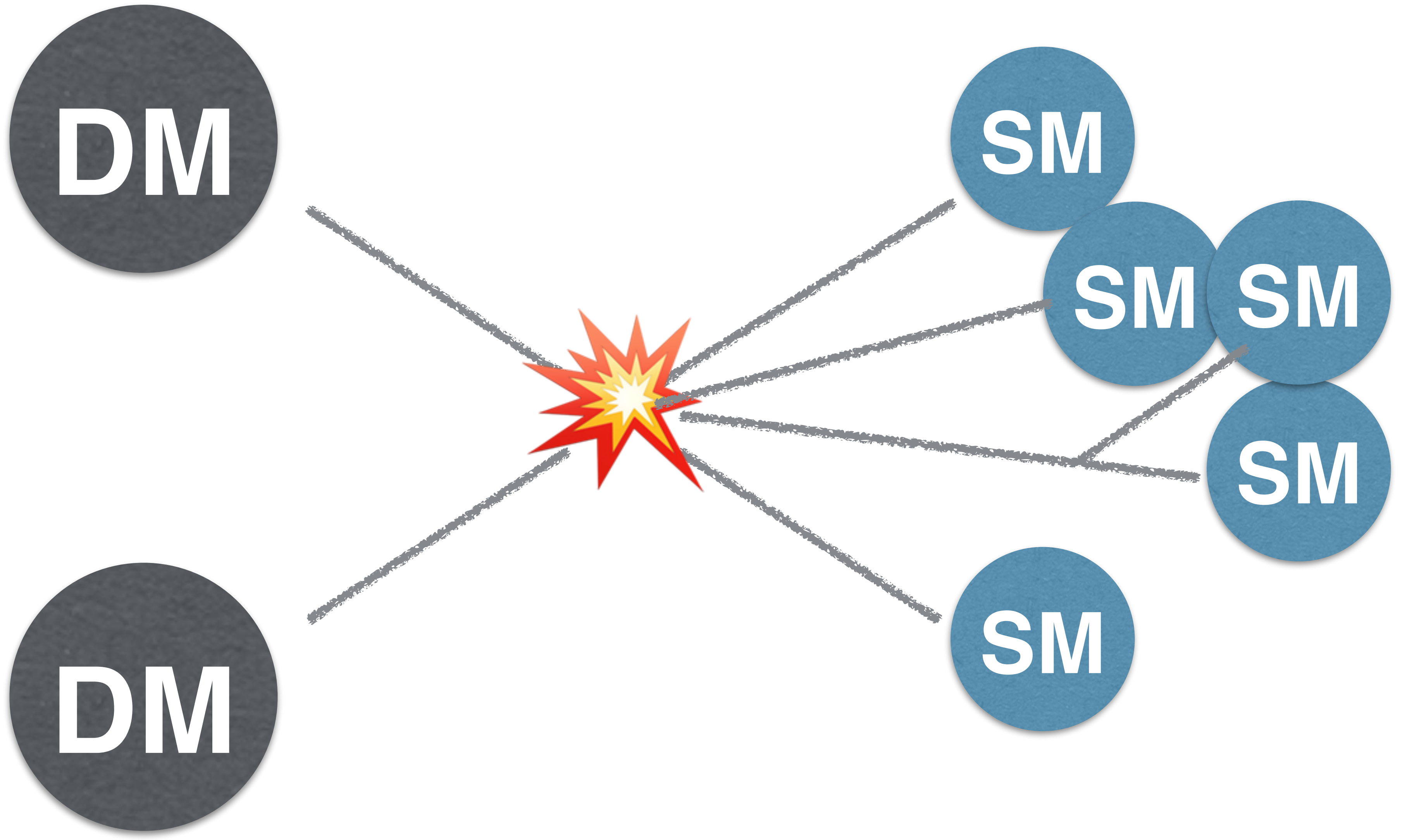
Dark Matter annihilation

Prompt gamma rays

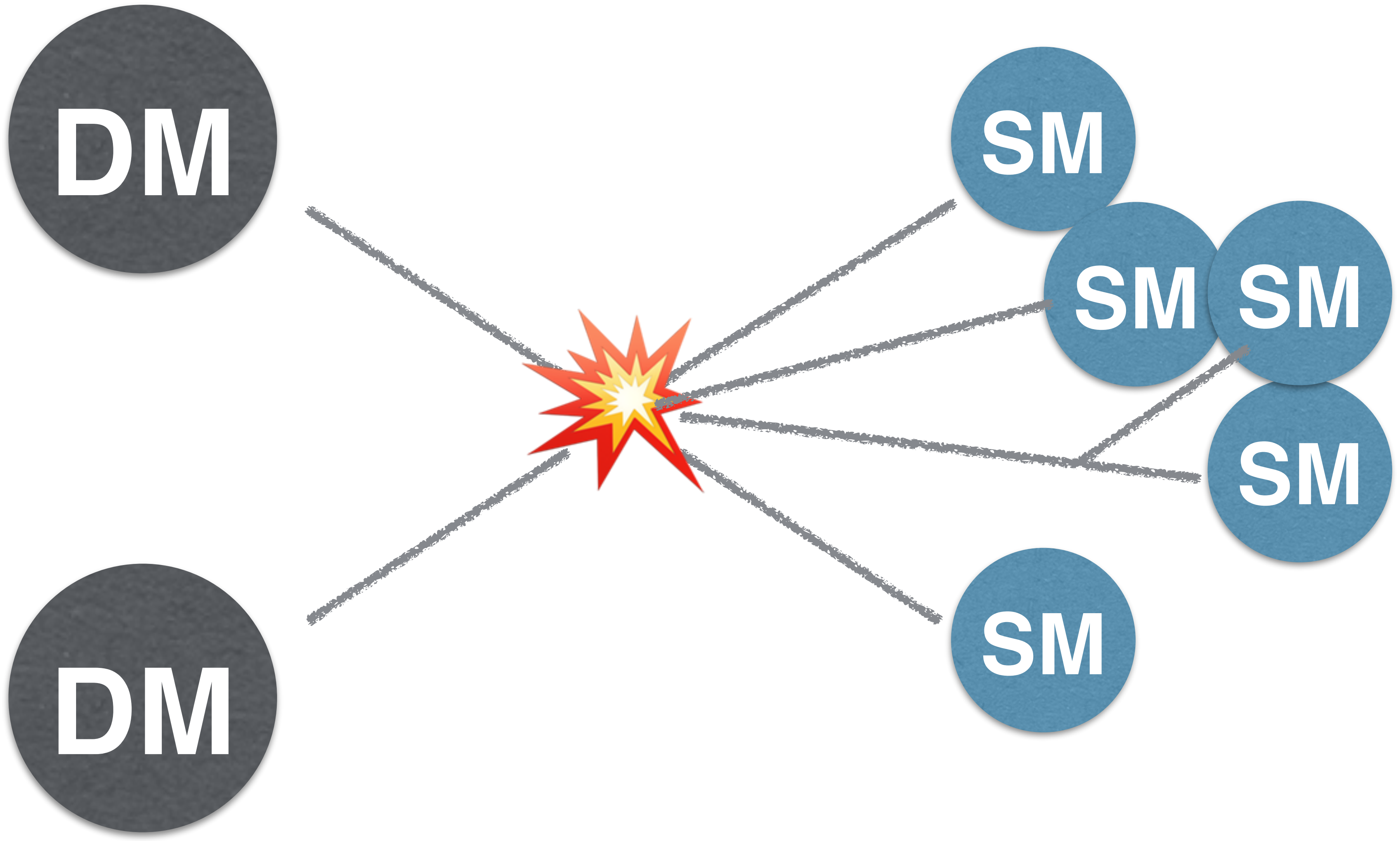


Dark Matter annihilation

Prompt gamma rays

- Simple kinematics (The dark matter is cold ➝ non relativistic)
  - Lab frame ≃ CoM frame

$$\sqrt{s} = 2\,m_\chi + \mathcal{O}(m_\chi v^2)$$



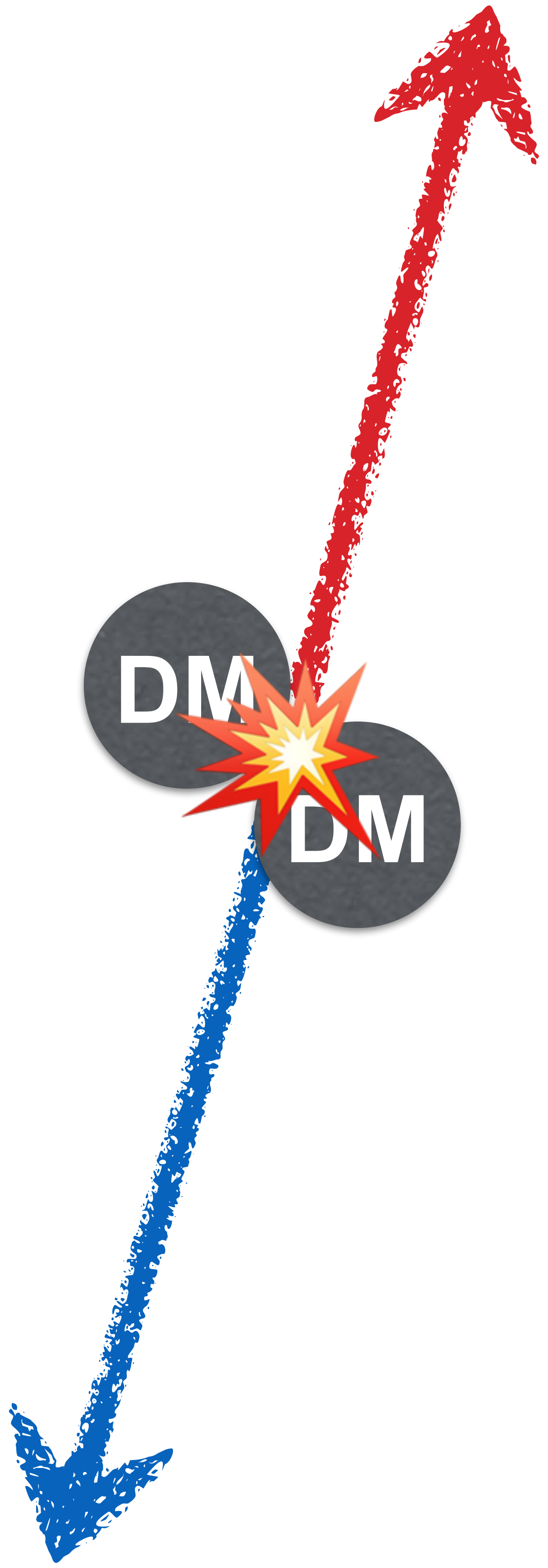
Dark Matter annihilation

Endpoint spectrum

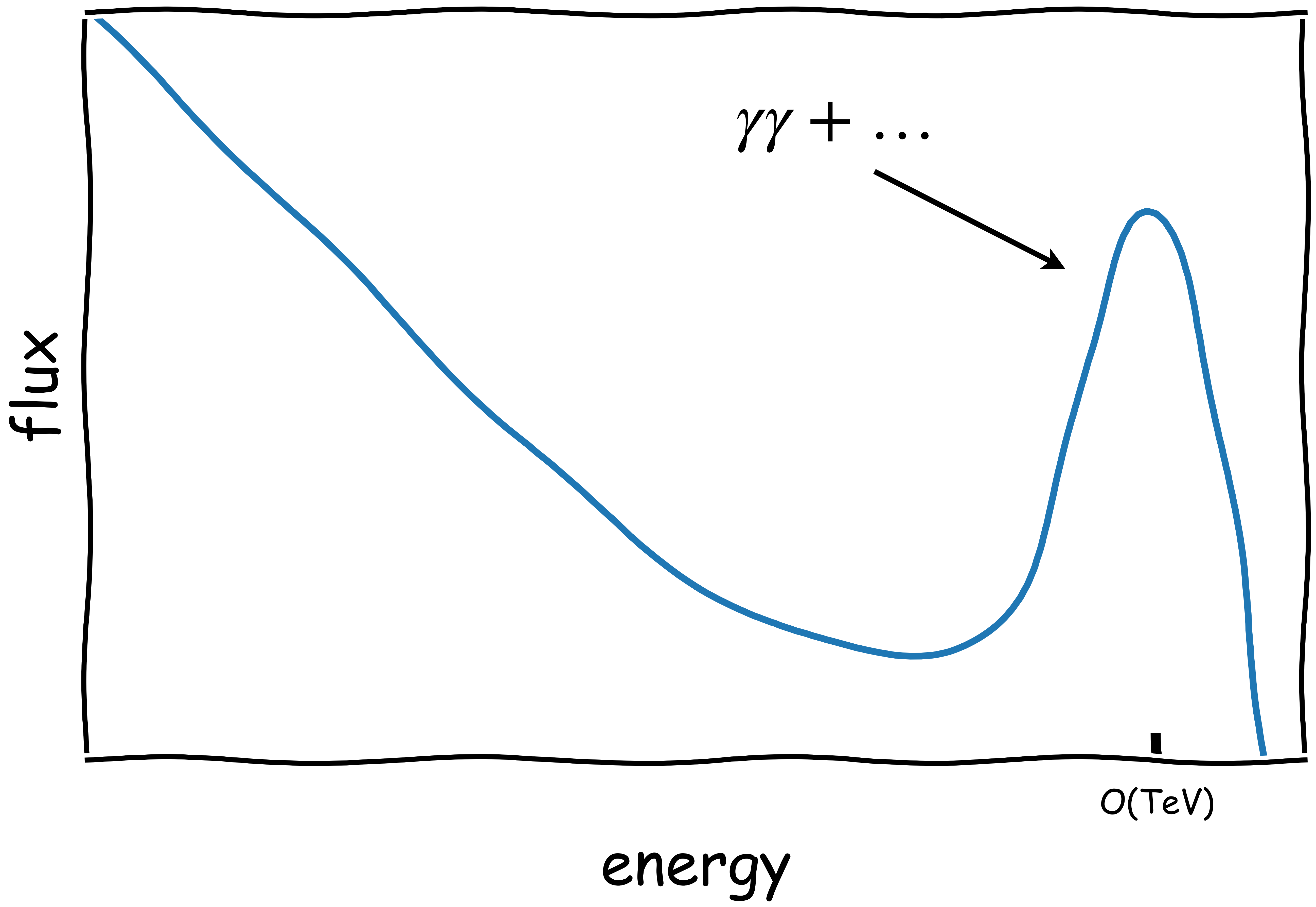
- Consider the fully-exclusive process $\chi_0\chi_0 \to \gamma\gamma$

$$E_\gamma = m_\chi$$

  - Back-to-back monochromatic TeV-scale photons



Quasi-monochromatic spectral line



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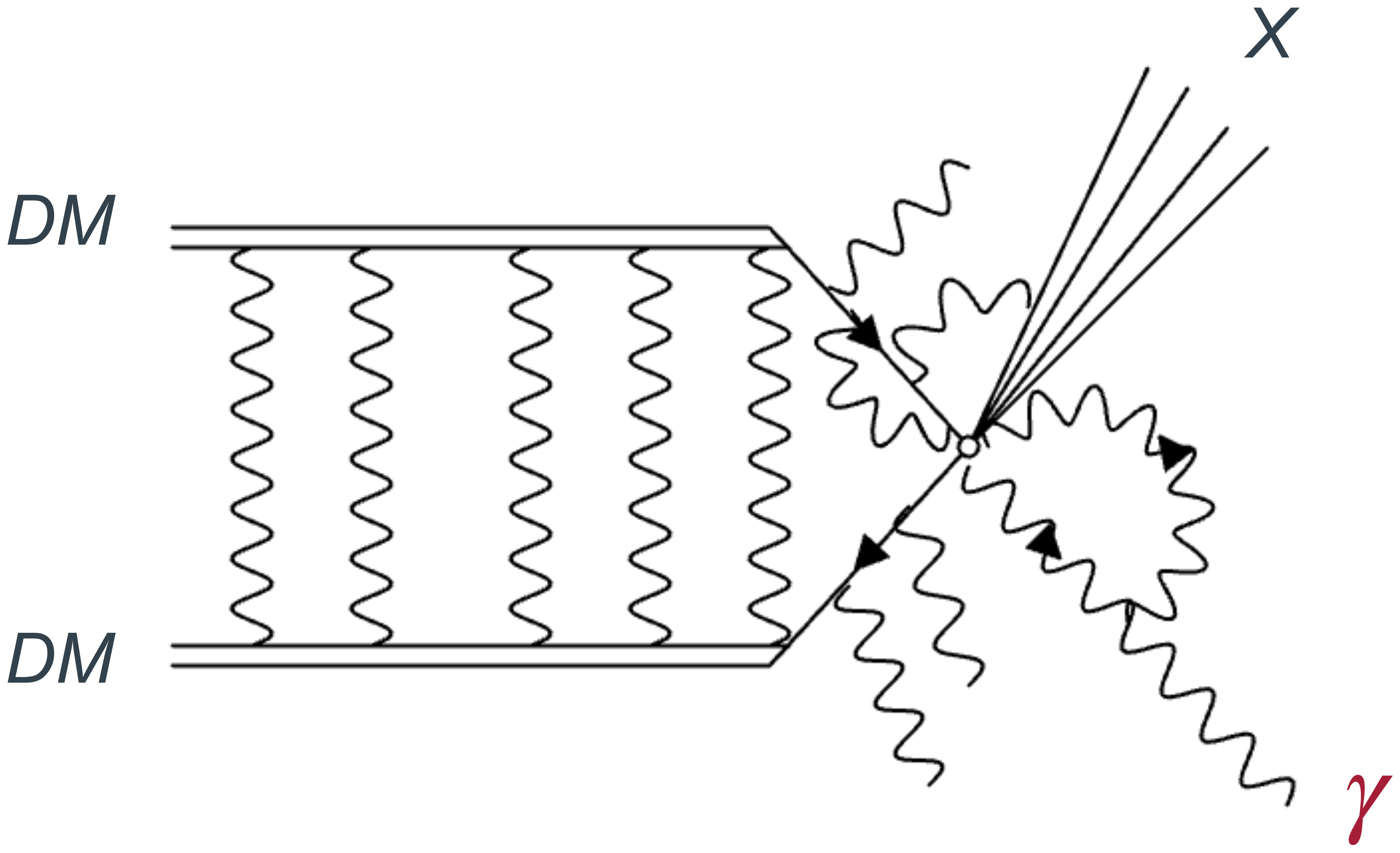
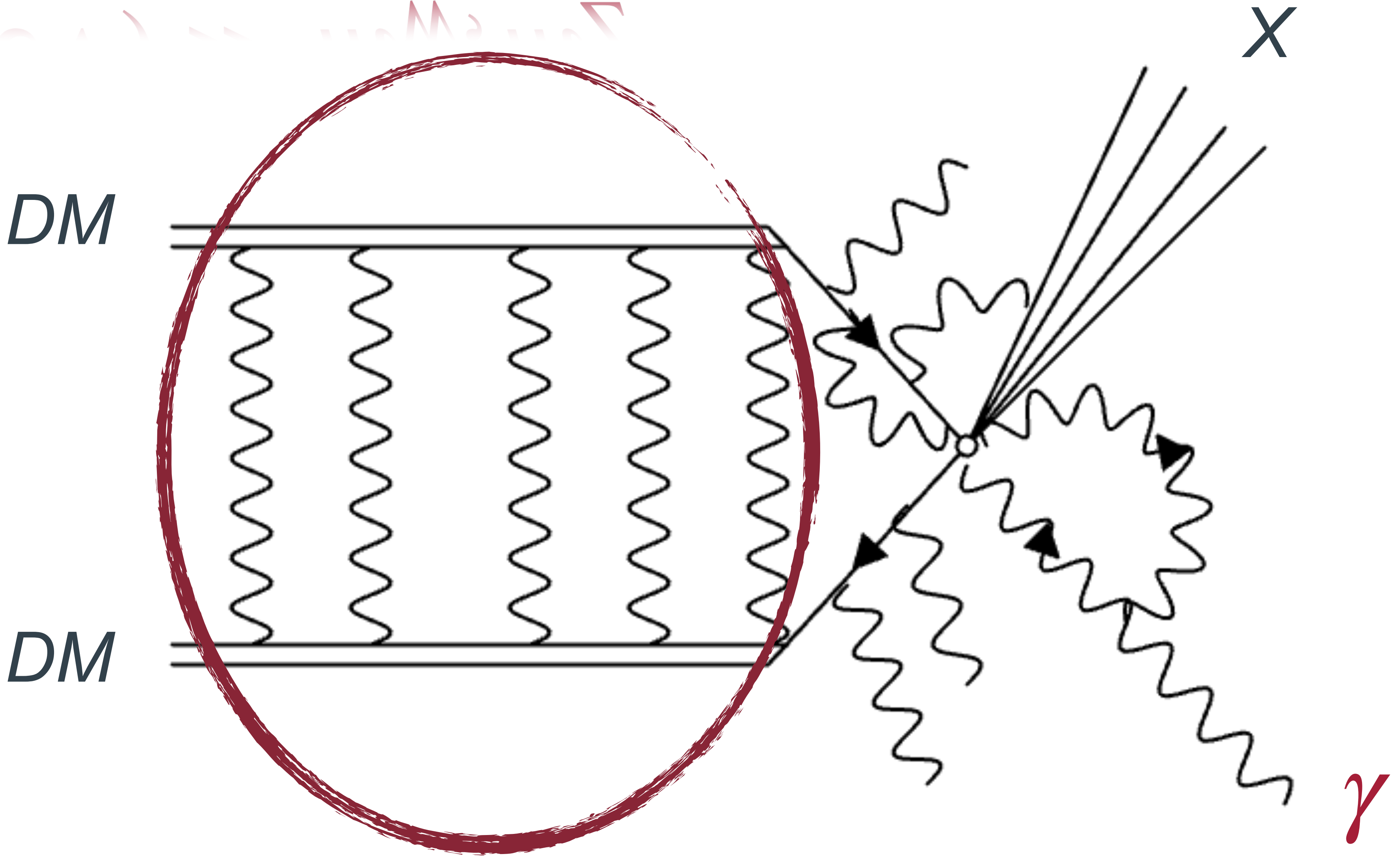
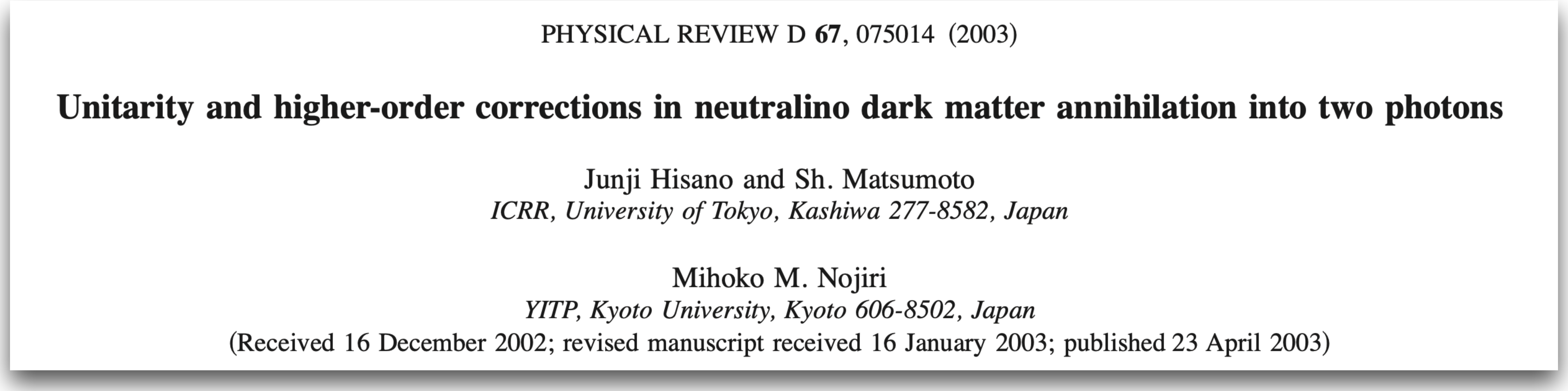

X
DM
DM
γ

$$m_{DM} \sim \mathcal{O}(\text{TeV}) \gg m_W, m_Z$$



Sommerfeld enhancement



PHYSICAL REVIEW D **67**, 075014 (2003)

Unitarity and higher-order corrections in neutralino dark matter annihilation into two photons

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YITP, Kyoto University, Kyoto 606-8502, Japan
(Received 16 December 2002; revised manuscript received 16 January 2003; published 23 April 2003)

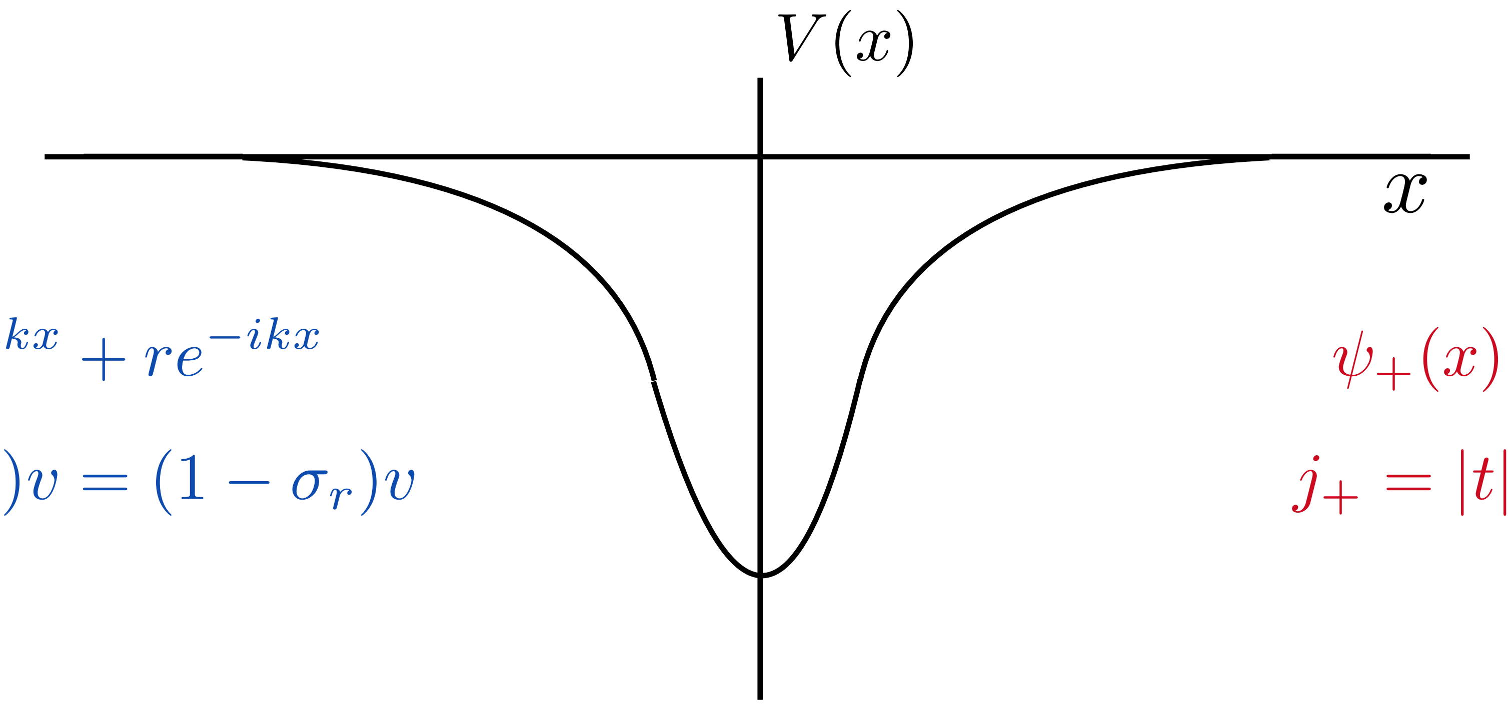
Sommerfeld enhancement

Sommerfeld enhancement

The wave function of a two-wimp system

$$\left(-\frac{1}{m_\chi}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V(x)\qquad\qquad\qquad\qquad\right)\psi(x)=E\psi(x)$$

$$j(x)=\frac{i}{m_\chi}\left[\psi(x)\psi'^*(x)-\psi^*(x)\psi'(x)\right]$$ = const.



$\psi_-(x)=e^{ikx}+re^{-ikx}$

$j_-=(1-|r|^2)v=(1-\sigma_r)v$

$\psi_+(x)=te^{ikx}$

$j_+=|t|^2v=\sigma_t v$

Unitarity: $j_-=j_+\quad\rightarrow\quad\sigma_r+\sigma_t=1$ (No absorption)

Sommerfeld enhancement

The wave function of a two-wimp system

$$\left(-\frac{1}{m_\chi}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+V(x)+\frac{i}{2}\sigma_a^{(0)}v\delta(x)\right)\psi(x)=E\psi(x)$$

Unitarity-violating term $\rightarrow\ j_+ = j_- + |\psi(0)|^2\sigma_a v$

$$\sigma_r+\sigma_t+\boldsymbol{\sigma_a}=1$$

$$\sigma_a=\boldsymbol{|\psi(0)|^2}\sigma_a^{(0)}$$

Resummed cross section = Sommerfeld factor (“long” range NR physics) X QFT cross section (short range physics)

Sommerfeld enhancement

Concrete example: pure wino

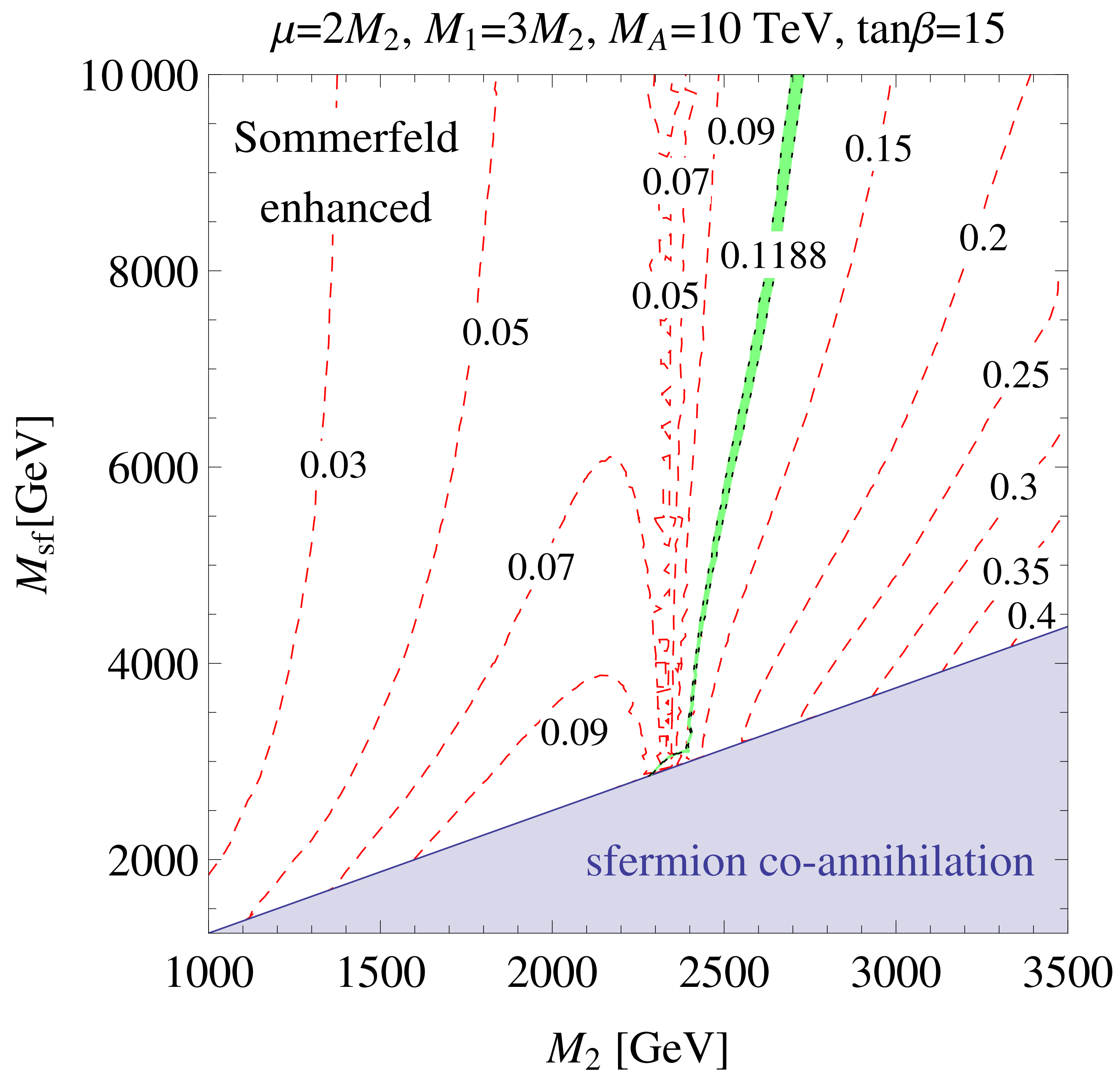
SM + Majorana SU(2) triplet

$$\delta\mathcal{L}_{\text{Wino}} = \frac{1}{2}\bar{\chi}(i\gamma^{\mu}\mathrm{D}_{\mu} - m_{\chi})\chi$$

$\xrightarrow{\cancel{\text{EW}}}$

Q=0 *Majorana* DM
Q=1 *Dirac* chargino

- $m_{\chi^+} - m_{\chi^0} \simeq 164\text{MeV}$
- DM stable through a $\mathbb{Z}_2$ symmetry
- Suitable WIMP for $m_{\chi^0} \simeq 3\text{TeV}$
- Super-partner of the SU(2) gauge bosons in SUSY



Sommerfeld enhancement

Concrete example: pure wino

$$\frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma} = 2\sum_{I,J} S_{IJ}\frac{\mathrm{d}(\sigma v)_{IJ}}{\mathrm{d}E_\gamma}$$

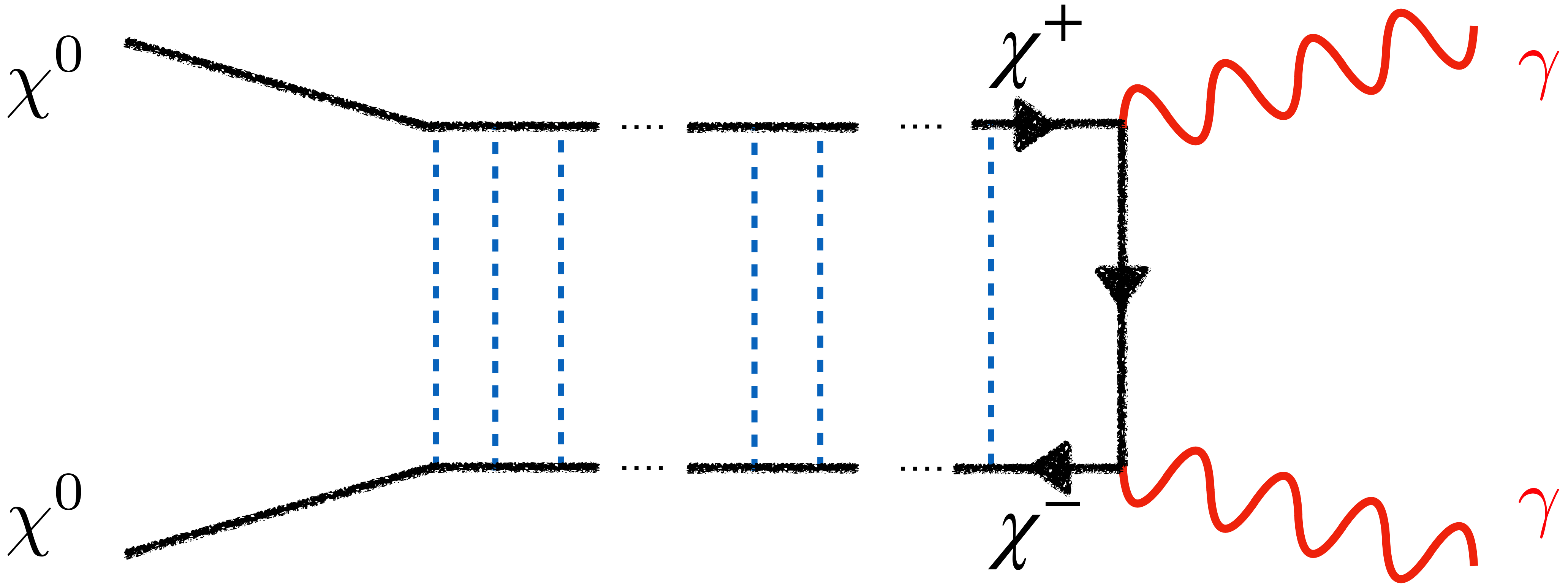
Sommerfeld **matrix**
I, *J* = (χ0χ0) or (χ+χ-)

$$V(r) = \begin{pmatrix} 0 & -\sqrt{2}\alpha_2\frac{\mathrm{e}^{-m_W r}}{r} \\ -\sqrt{2}\alpha_2\frac{\mathrm{e}^{-m_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2\frac{\mathrm{e}^{-m_Z r}}{r} \end{pmatrix}$$

Sommerfeld enhancement

Concrete example: pure wino

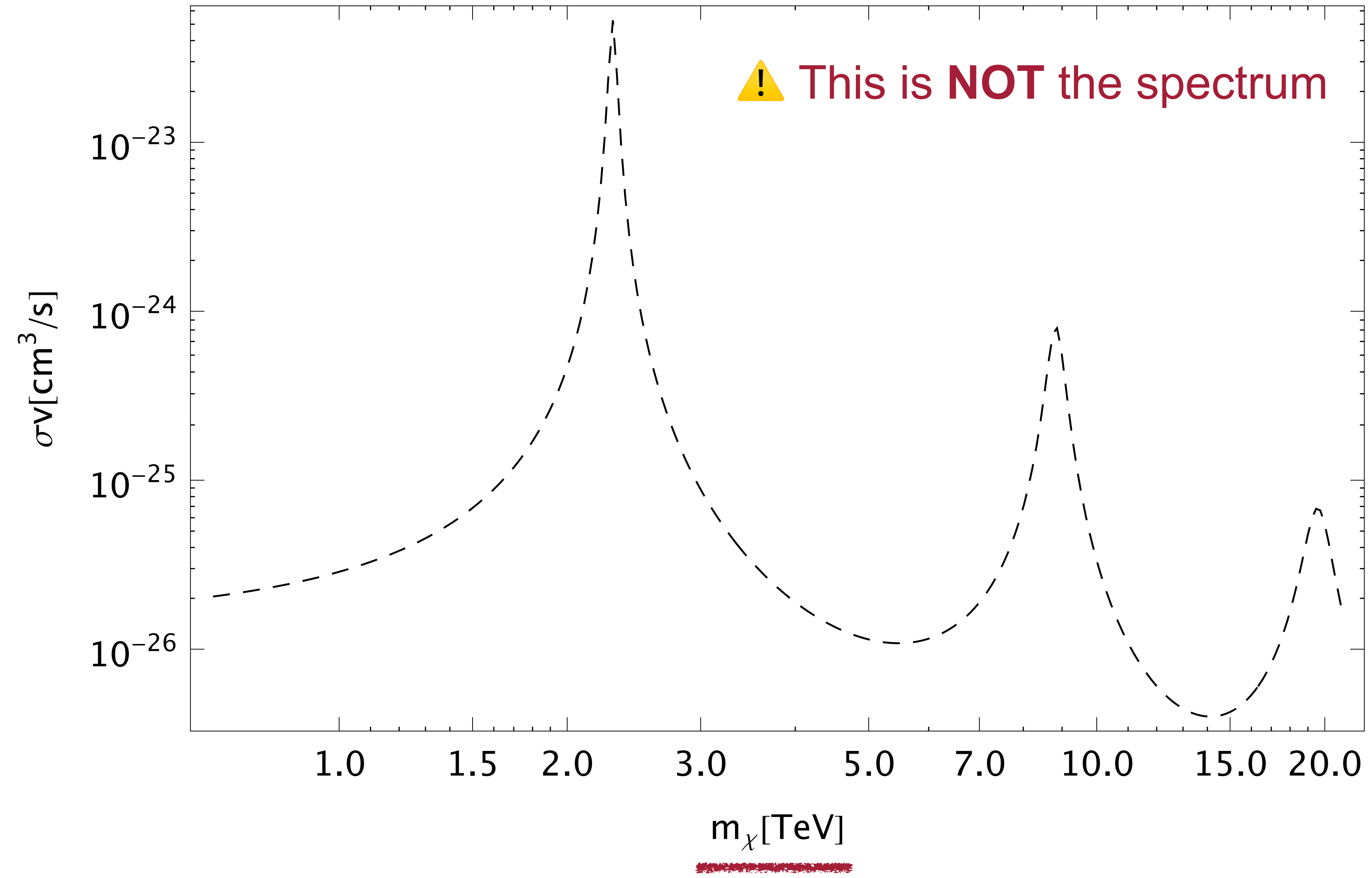
$$\frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma} = 2\sum_{I,J} S_{IJ}\frac{\mathrm{d}(\sigma v)_{IJ}}{\mathrm{d}E_\gamma}$$



Sommerfeld enhancement

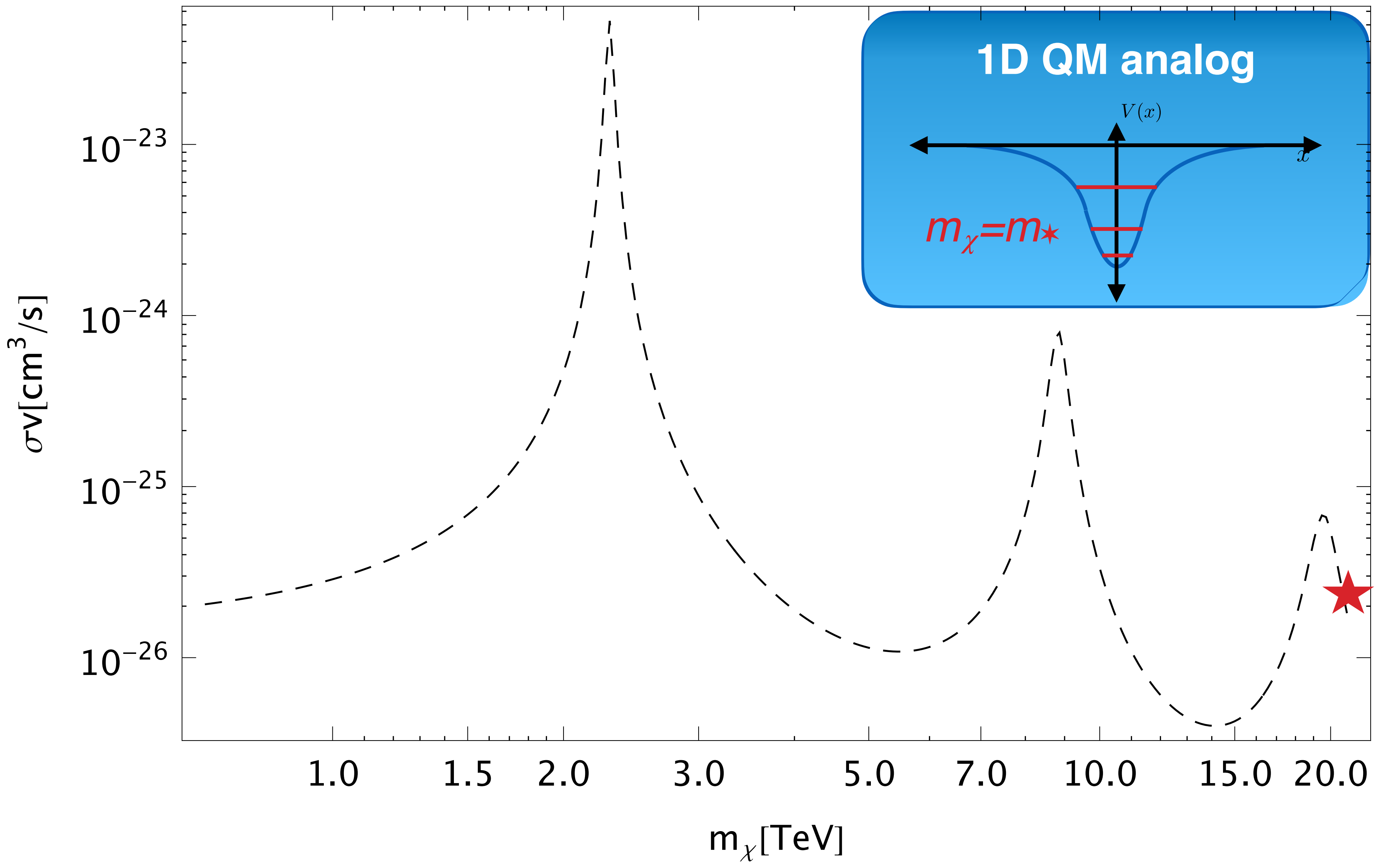
“Explosive” Dark Matter annihilation

$$\sigma v\Big|_{2\gamma+\gamma Z} = 2\times S_{(+-)(+-)}\times(\sigma v)^{\text{tree}}_{(+-)(+-)}$$



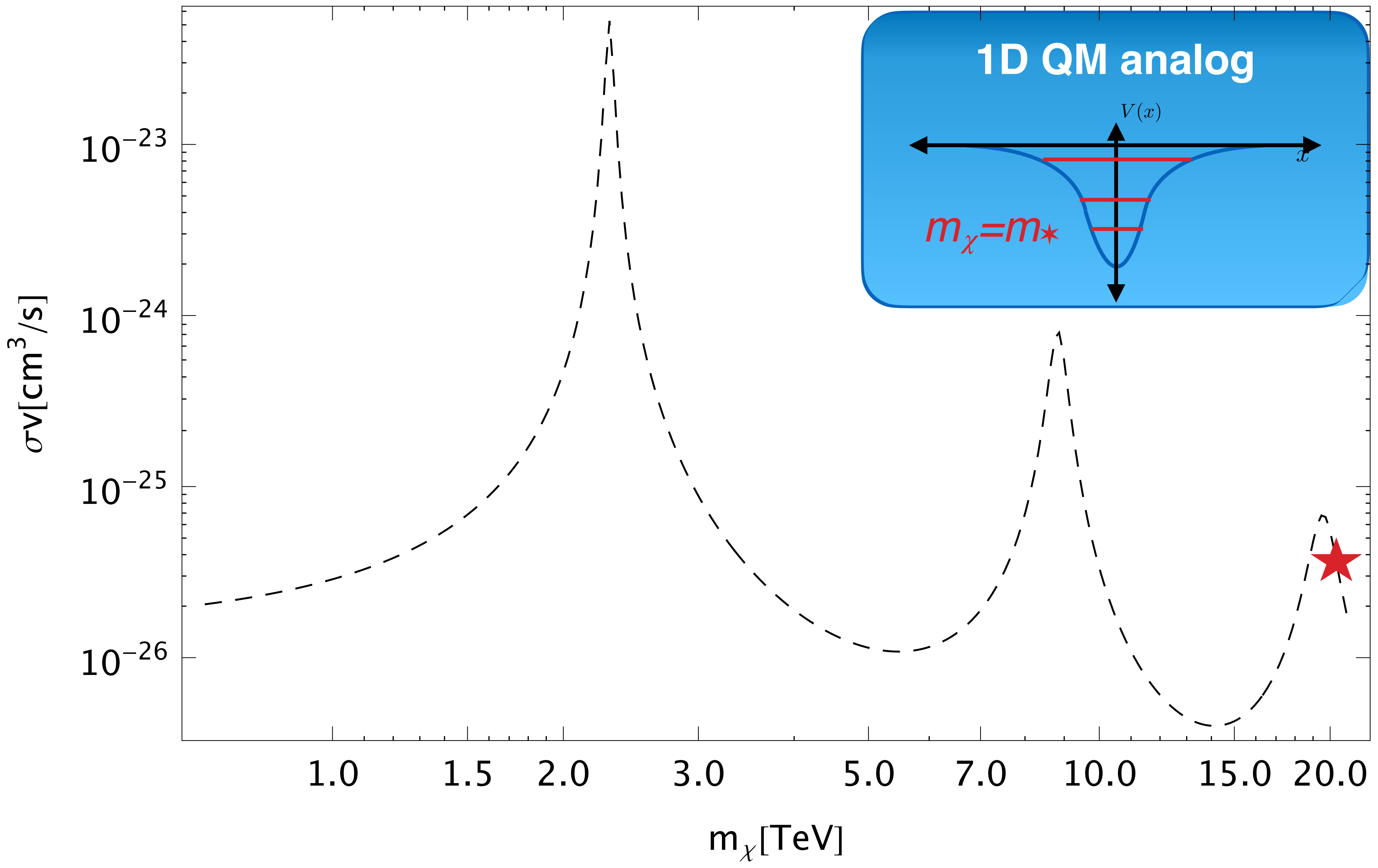
Sommerfeld enhancement

Relationship with bound states



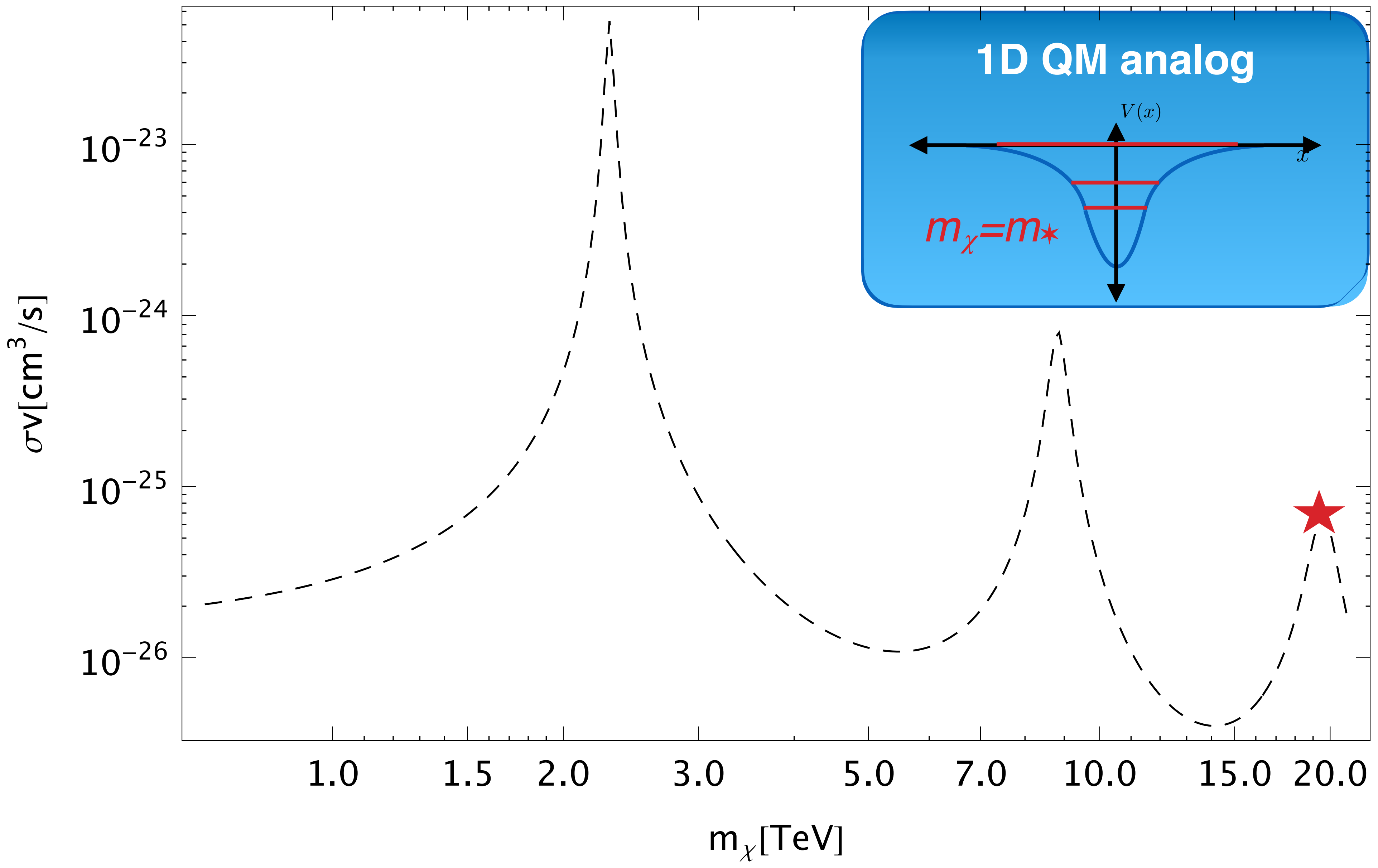
Sommerfeld enhancement

Bound states? … Not quite



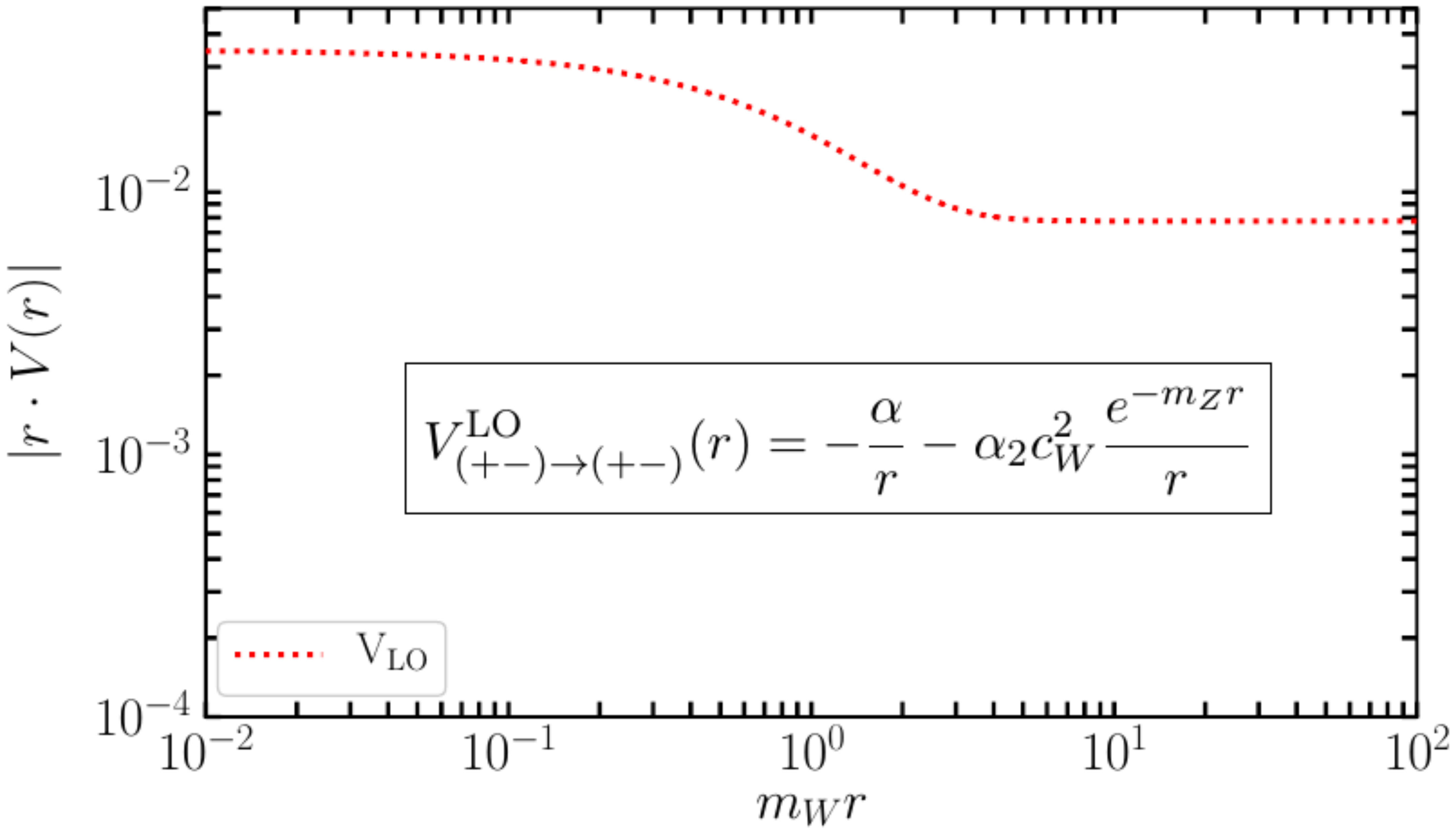
Sommerfeld enhancement

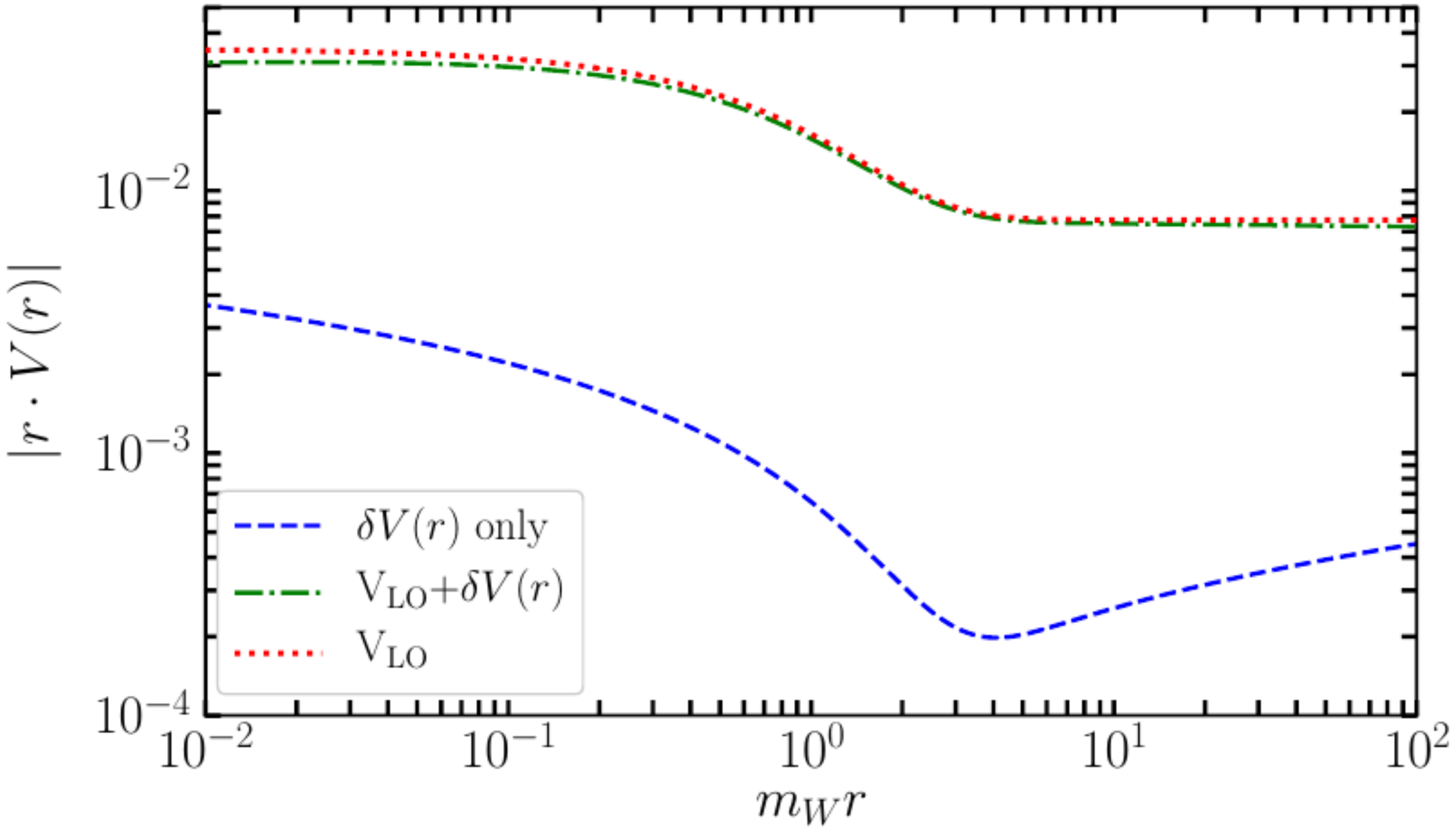
Bound states? … Not quite



Sommerfeld enhancement

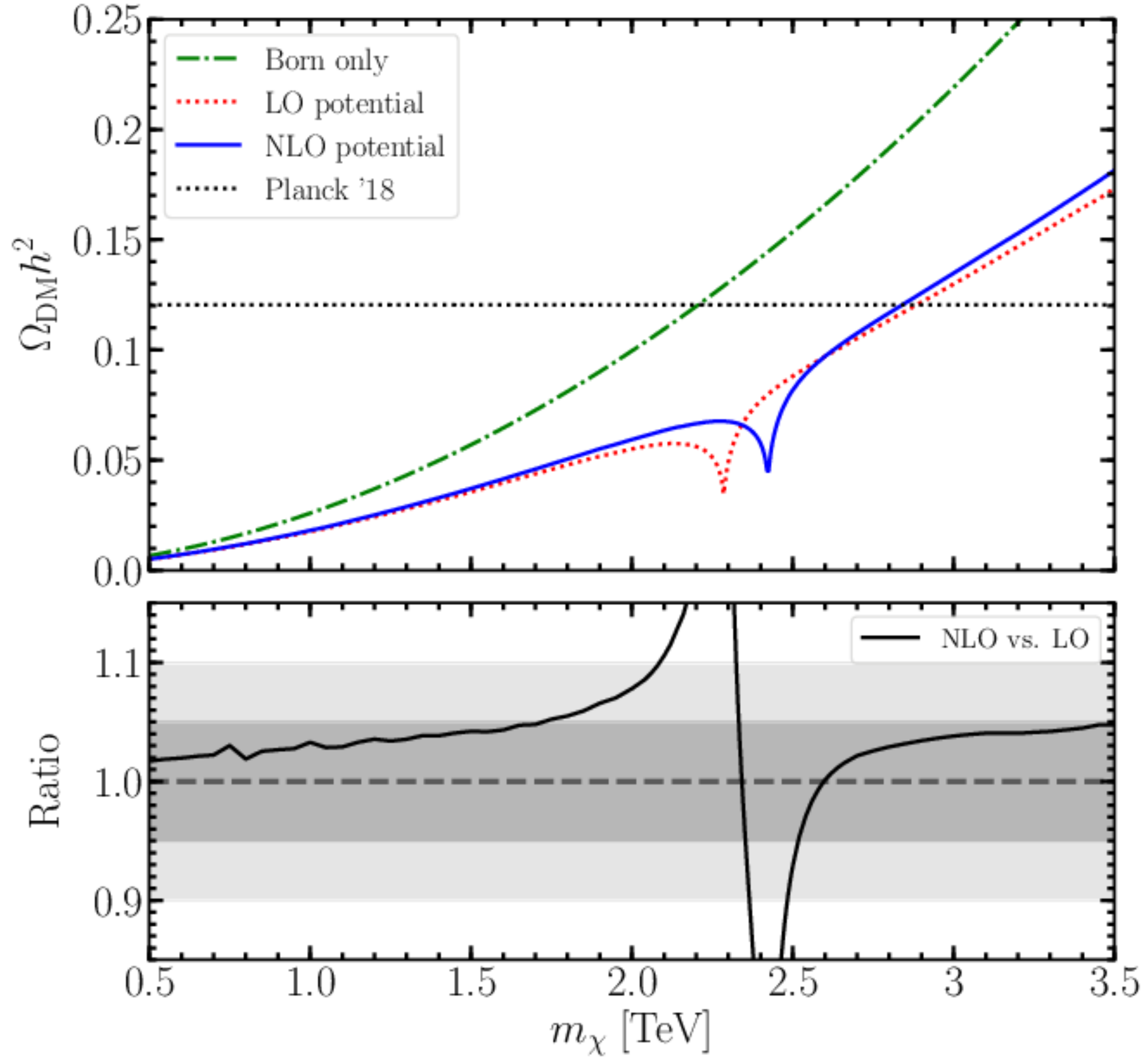
NLO potential — Beneke, Szafron, Urban (arXiv:1909.04584)





Sommerfeld enhancement

NLO potential — Beneke, Szafron, Urban (arXiv:2009.00640)



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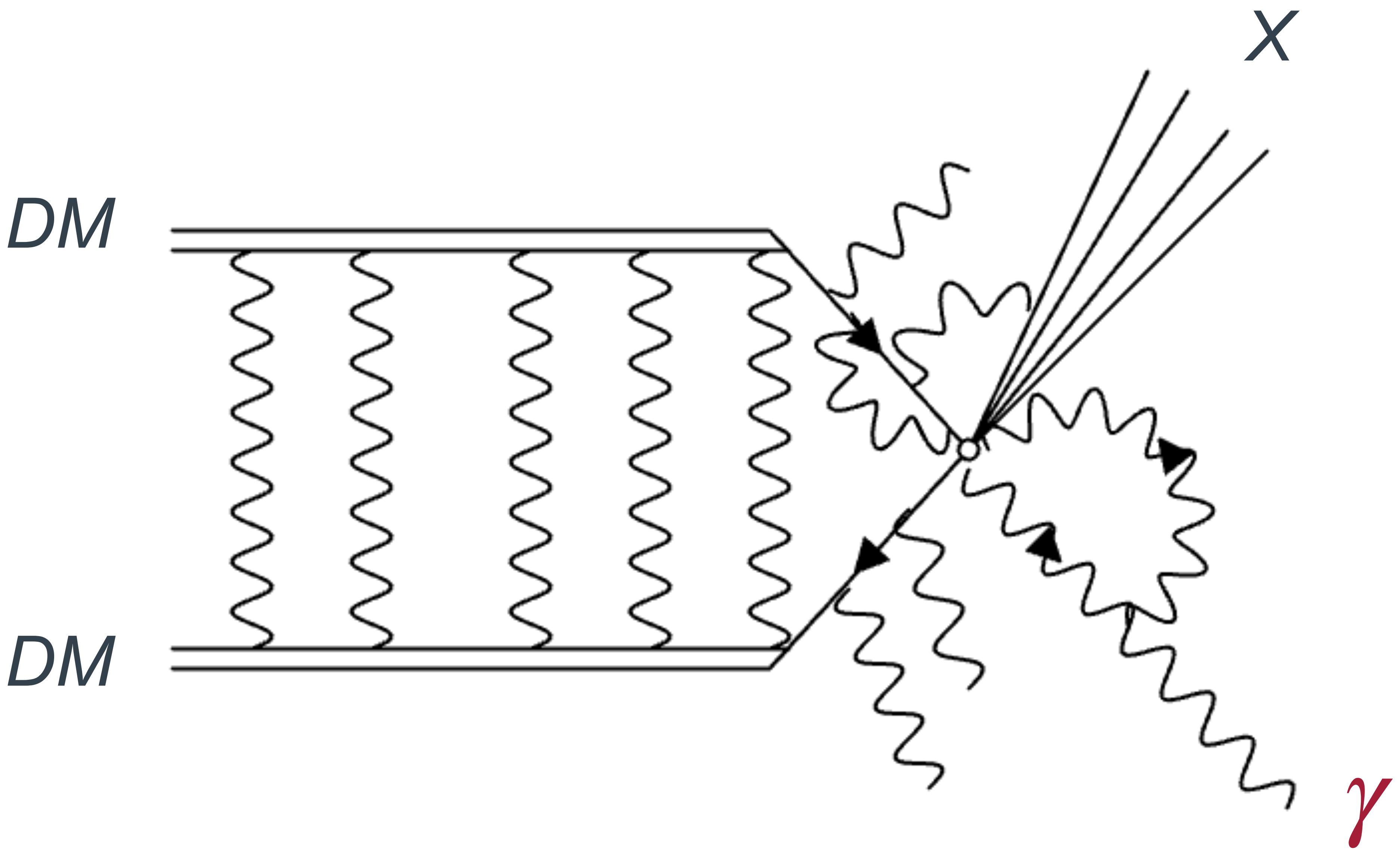
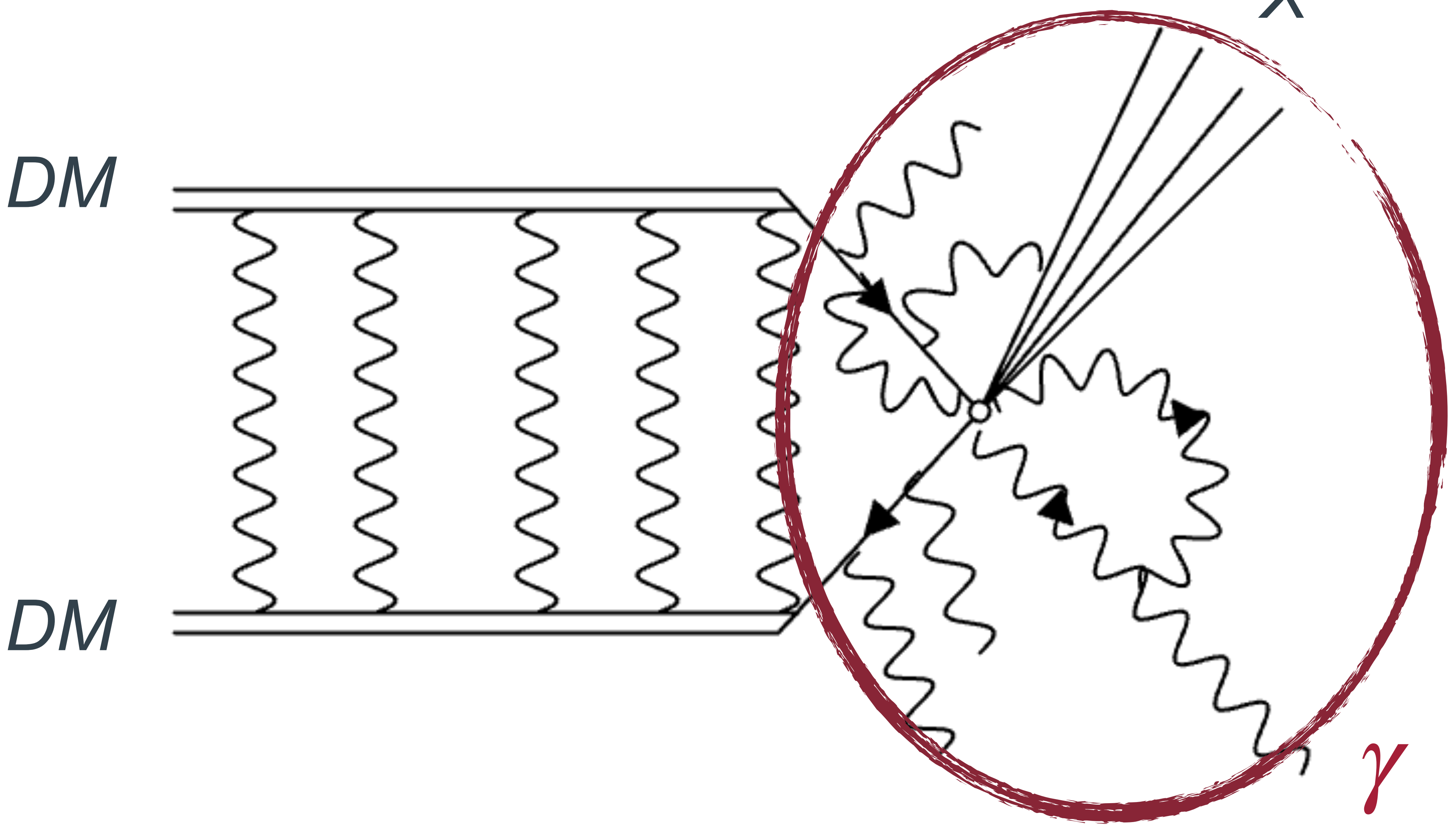

X
DM
DM
γ

$$m_X^2 \ll s = 4\, m_{\mathrm{DM}}^2$$

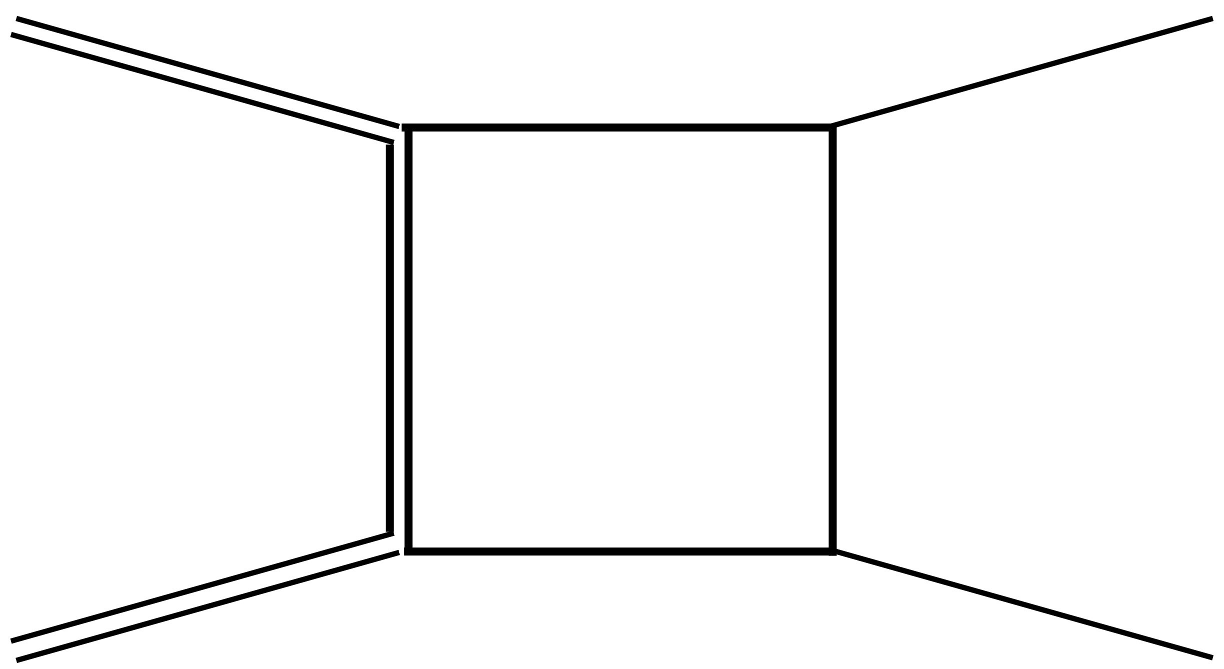


$$\propto \alpha_{EW} \log \frac{s}{m_X^2} \log \frac{m_\chi^2}{m_W^2}$$

Sudakov double logs

Soft collinear effective field theory (SCET)

Method of regions



$p_1 = m_\chi(1, \overrightarrow{0})$

$k_3 = m_\chi(1, \hat{n}) \equiv m_\chi n$

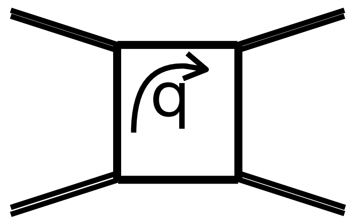
$p_2 = m_\chi(1, \overrightarrow{0})$

$k_4 = m_\chi(1, -\hat{n}) \equiv m_\chi \bar{n}$

$$I_{\text{example}} = \quad = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{1}{(q + k_3 - p_1)^2 - m_\chi^2} \frac{1}{(q + k_3)^2 - m_W^2} \frac{1}{q^2 - m_W^2} \frac{1}{(q - k_4)^2 - m_W^2}$$

SCET for indirect DM detection

Method of regions

$I_{\text{ex.}} =$ 

$$= \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q+k_3-p_1)^2 - m_\chi^2} \frac{1}{(q+k_3)^2 - m_W^2} \frac{1}{q^2 - m_W^2} \frac{1}{(q-k_4)^2 - m_W^2}$$

Light-cone coordinates $q = q_c n + q_{\bar{c}} \bar{n} + q_\perp \longrightarrow (q_c, q_{\bar{c}}, q_\perp)$

Expand propagators in according to 4 different momentum scalings

$$q_h \sim m_\chi (1,1,1) \qquad q_s \sim m_W (1,1,1)$$

$$q_c \sim \left(\frac{m_W^2}{m_\chi}, m_\chi, m_W \right) \qquad q_{\bar{c}} \sim \left(m_\chi, \frac{m_W^2}{m_\chi}, m_W \right)$$

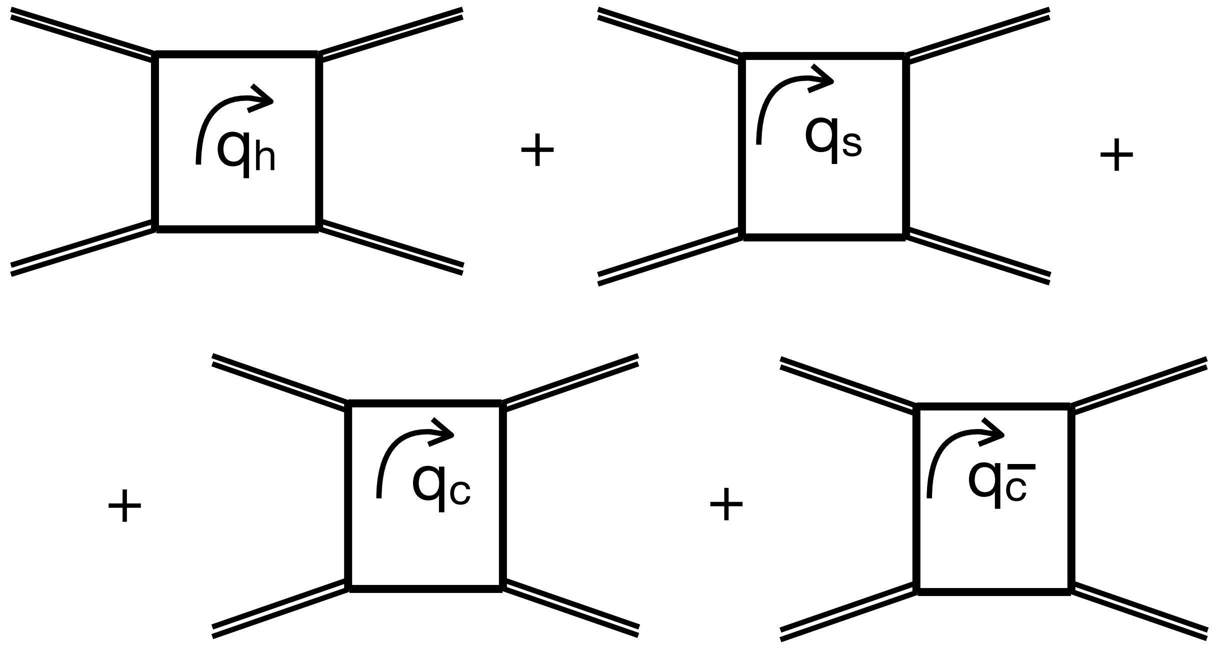
For example: $I_h = \int \frac{d^D q}{(2\pi)^D} \frac{1}{(q+k_3-p_1)^2 - m_\chi^2} \frac{1}{(q+k_3)^2} \frac{1}{q^2} \frac{1}{(q-k_4)^2}$

SCET for indirect DM detection

Method of regions

Let the *magic* happen:

$I_{\text{ex.}} =$

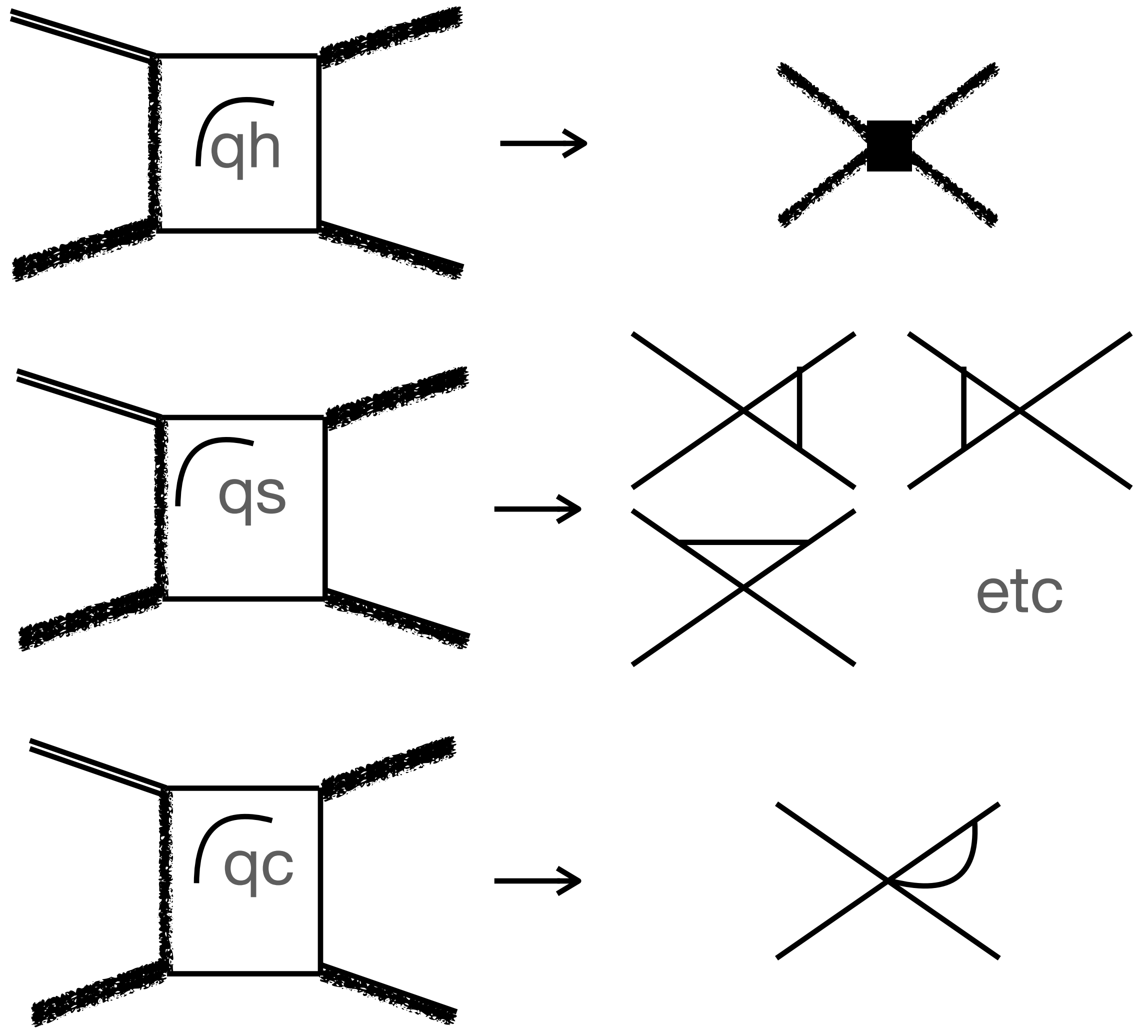


+ power corrections

SCET for indirect DM detection

Method of regions

Interpret each expansion as a Feynman diagram of the SCET



Factorization (after including all diagrams)

Wilson coefficients	only depend on the hard scale m_χ
Soft functions	only depend on the soft scale m_W
Jet functions	only depends on the typical jet scale ⚠

SCET for indirect DM detection

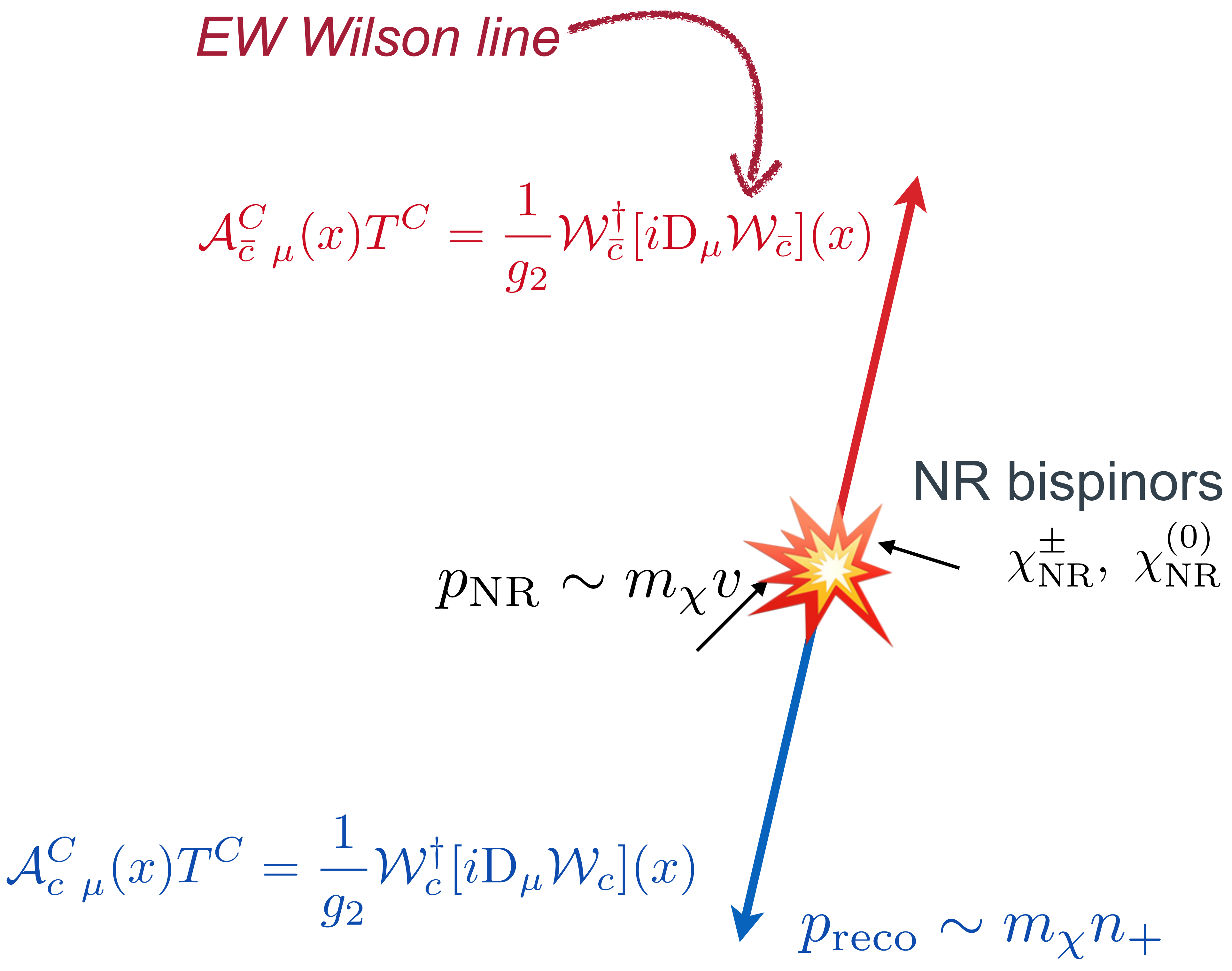
Concretely …

$$\mathcal{L}_{\text{NRDM}\times\text{SCET}} = \mathcal{L}_{\text{NRDM}} + \mathcal{L}_{\text{SCET}} + \frac{1}{2m_\chi}\sum_{i=1}^{2}\int \text{dsdt}\ \hat{C}_i(t,s,\mu)\mathcal{O}_i$$

Two-dimensional operator basis (for the $\chi\chi \to \gamma$+X process)

$$\mathcal{O}_1 = \chi^{c\dagger}_{\text{NR}}\chi_{\text{NR}}\varepsilon^{\mu\nu}_{\perp}\mathcal{A}^{C}_{\perp\ c,\ \mu}(sn_+)\mathcal{A}^{C}_{\perp\ \bar{c},\ \nu}(tn_-)$$

$$\mathcal{O}_2 = \chi^{c\dagger}_{\text{NR}}\{T^C, T^D\}\chi_{\text{NR}}\varepsilon^{\mu\nu}_{\perp}\mathcal{A}^{C}_{\perp\ c,\ \mu}(sn_+)\mathcal{A}^{D}_{\perp\ \bar{c},\ \nu}(tn_-)$$



EW Wilson line

$$\mathcal{A}^{C}_{\bar{c}\ \mu}(x)T^C = \frac{1}{g_2}\mathcal{W}^{\dagger}_{\bar{c}}[i\text{D}_\mu\mathcal{W}_{\bar{c}}](x)$$

NR bispinors

$\chi^{\pm}_{\text{NR}},\ \chi^{(0)}_{\text{NR}}$

$p_{\text{NR}} \sim m_\chi v$

$$\mathcal{A}^{C}_{c\ \mu}(x)T^C = \frac{1}{g_2}\mathcal{W}^{\dagger}_{c}[i\text{D}_\mu\mathcal{W}_{c}](x)$$

$p_{\text{reco}} \sim m_\chi n_+$

Fully resummed result

NREFT × SCET for indirect dark-matter detection

$$\frac{\mathrm{d}}{\mathrm{d}E_\gamma}[\sigma v] = |\psi(0)|^2 \times |C|^2(\mu) \times Z_\gamma(\mu,\nu) \times J(\mu,\nu) \otimes W(\mu,\nu)$$

Prescription for resummation

SOLVE:

1. a suitable Schrödinger equation
2. renormalization group equations (in μ and ν) for every term in the factorization formula

Factorization formulas (Sudakov-log resumm.)

Regime ‘int’

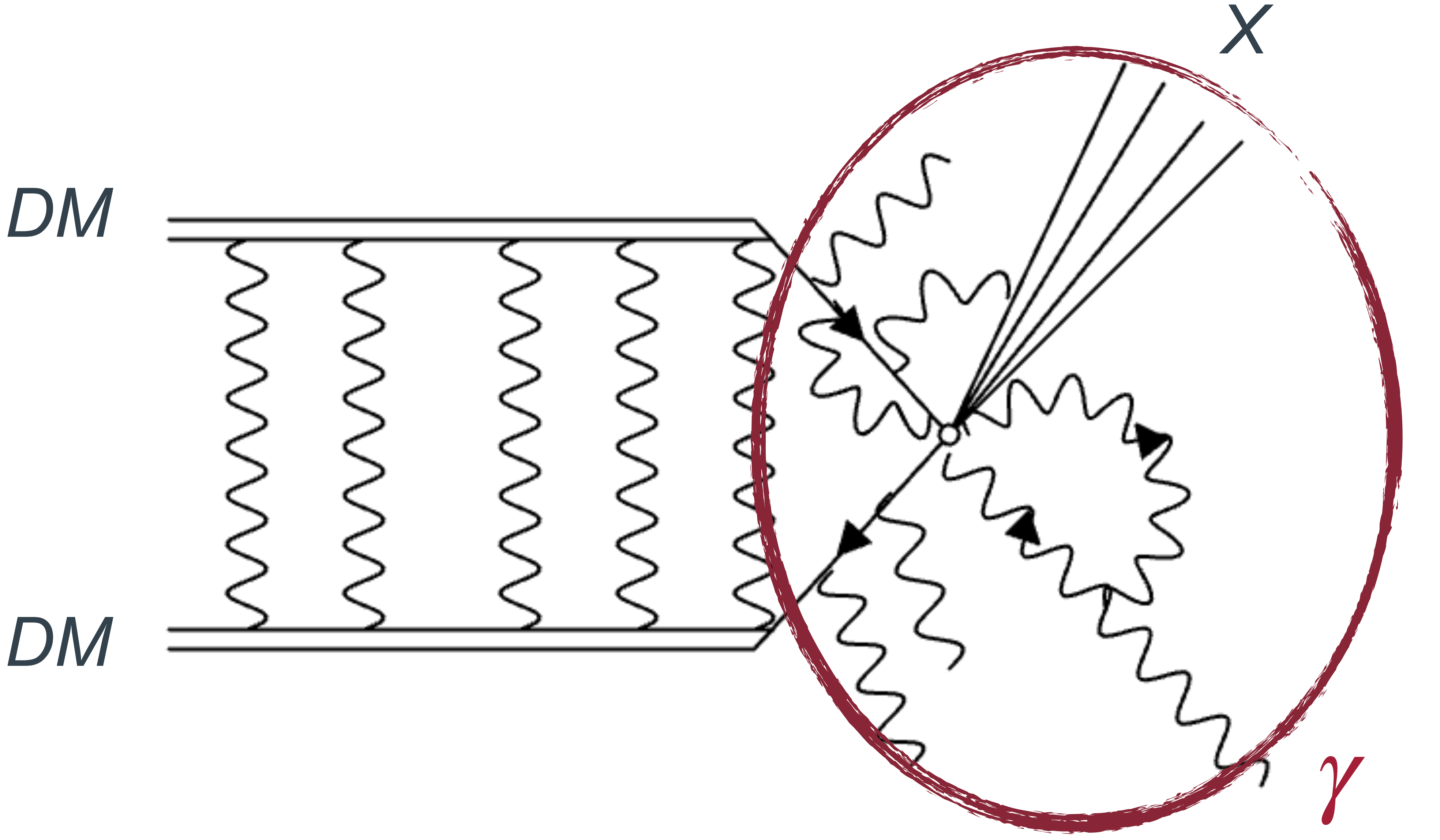
$$\Gamma^{\mathrm{higgsino}}_{IJ}(E_\gamma) = \frac{1}{(\sqrt{2})^{n_{id}}}\frac{1}{4}\frac{2}{\pi m_\chi}\sum_{i,j} C_i(\mu)C_j^*(\mu) \times Z_\gamma^{WY}(\mu,\nu) \times \int \mathrm{d}\omega \left(J^{\mathrm{SU(2)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, W^{\mathrm{SU(2)},ij}_{IJ,WY}(\omega,\mu,\nu) + \right.$$

$$\left. + J^{\mathrm{U(1)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, W^{\mathrm{U(1)},ij}_{IJ,WY}(\omega,\mu,\nu) \right)$$

……

$$\Gamma^{\mathrm{wino}}_{IJ}(E_\gamma) = \frac{1}{(\sqrt{2})^{n_{id}}}\frac{1}{4}\frac{2}{\pi m_\chi}\sum_{i,j} C_i(\mu)C_j^*(\mu) \times Z_\gamma^{33}(\mu,\nu) \times \int \mathrm{d}\omega J^{\mathrm{SU(2)}}(4m_\chi(m_\chi - E_\gamma - \omega/2),\mu)\, \tilde{W}^{ij}_{IJ}(\omega,\mu,\nu)$$

$m_X^2 \ll s = 4\, m_{\mathrm{DM}}^2$



Sudakov double logs

Endpoint spectrum

Validity regimes and accuracies of several existing calculations

- Line only: $m_X^2 = 0$
  - Ovanesyan, Rodd, Slatyer, Stewart — 1612.04814 — NLL’ for wino
- Narrow ‘nrw’: $4m_\chi^2 \gg m_X^2 \sim m_W^2$ (or $1 \gg 1-x \sim m_W^2/m_\chi^2$)
  - Beneke, Broggio, Hasner, MV — 1805.07367 — NLL’ for wino
  - Beneke, Hasner, MV, Urban — 1912.02034 — NLL’ for higgsino
- Intermediate ‘int’: $4m_\chi^2 \gg m_X^2 \sim 2\,m_\chi m_W$ (or $1-x \sim m_W/m_\chi$)
  - Beneke, Broggio, Hasner, MV, Urban — 1903.08702 — NLL’ for wino
  - Beneke, Hasner, MV, Urban — 1912.02034 — NLL’ for higgsino
- Wide: $4m_\chi^2 \gg m_X^2 \gg m_\chi m_W$ (or $1 \gg 1-x \gg m_W/m_\chi$)
  - Baumgart, Cohen, Moulin, Moult, Rinchiuso, Rodd, Slatyer, Stewart, Vaidya — 1808.08956 — NLL for wino

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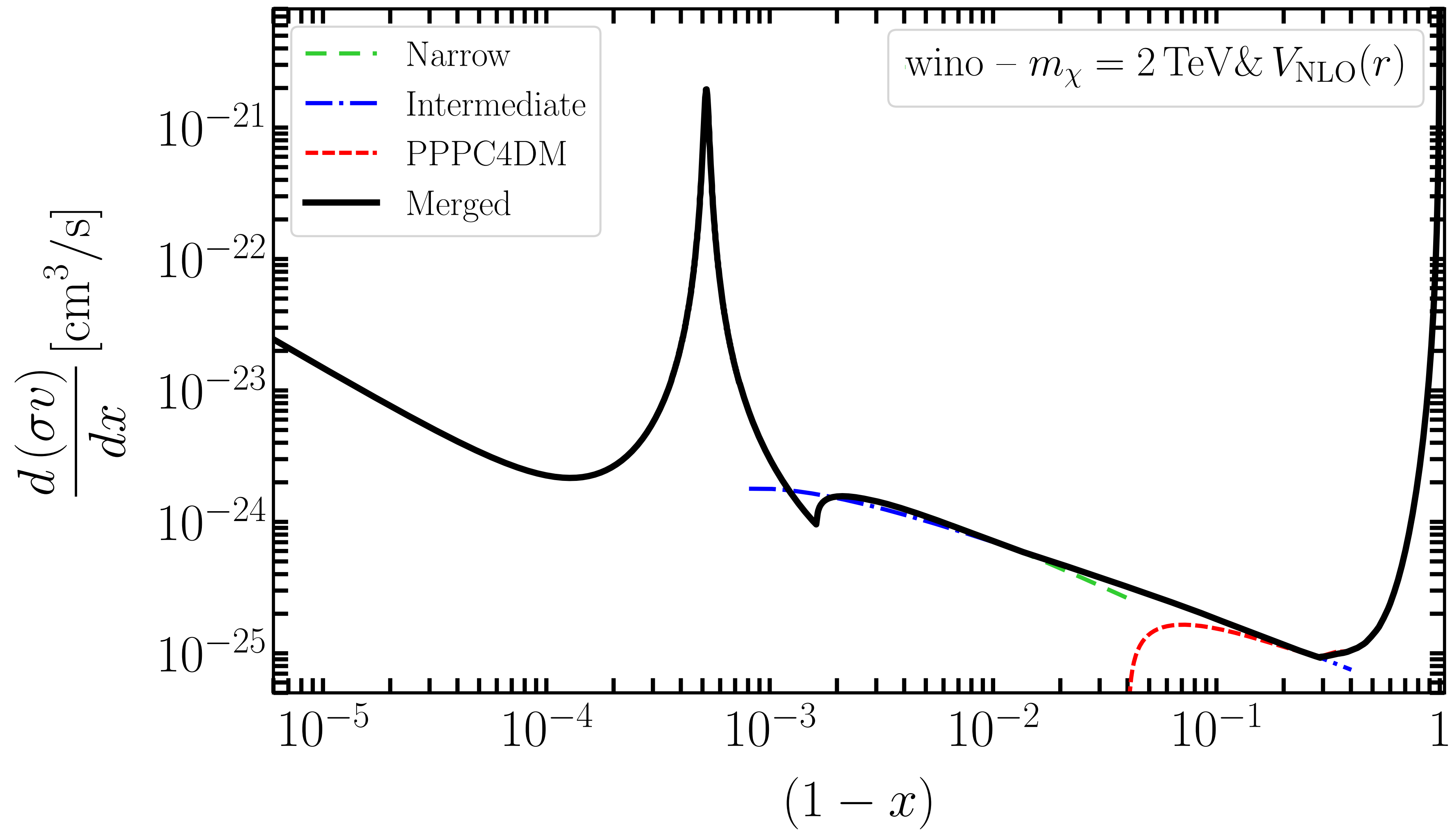
- Sudakov logs
- **Results**
- Conclusions

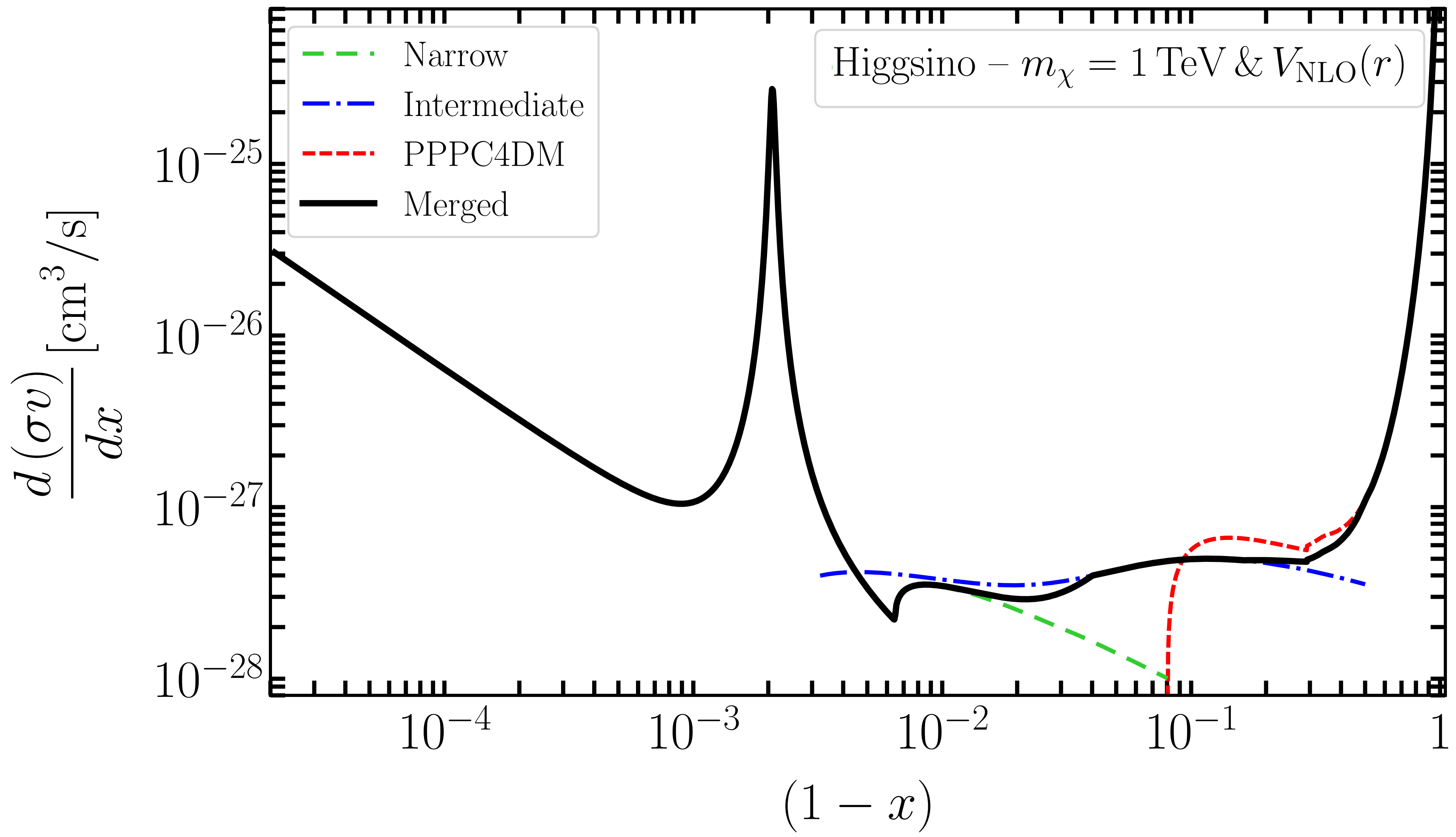
Results

Numerical results with DMγSpec

Resummed wino and higgsino endpoint spectra + parton showers

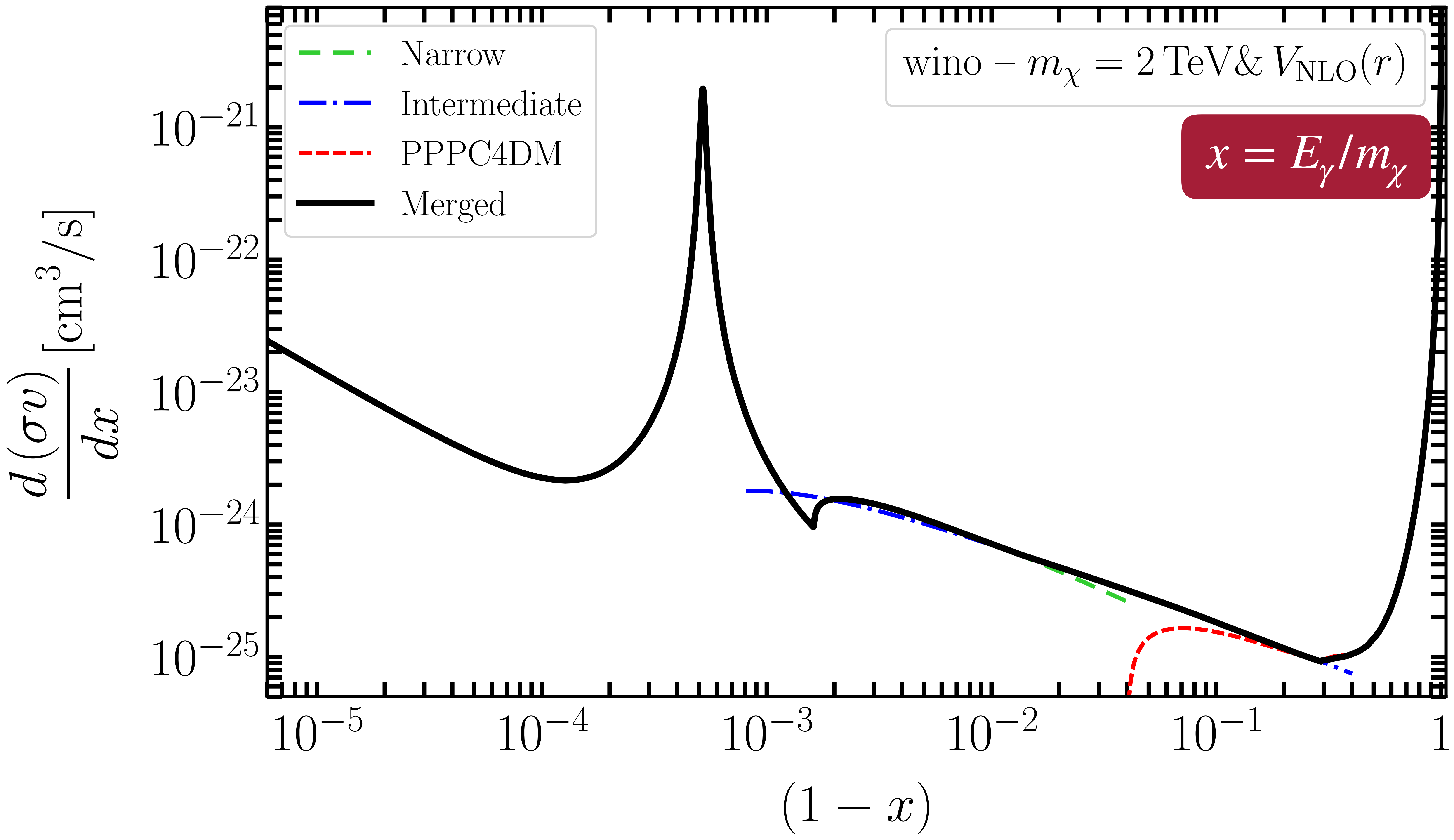
$x = E_\gamma / m_\chi$





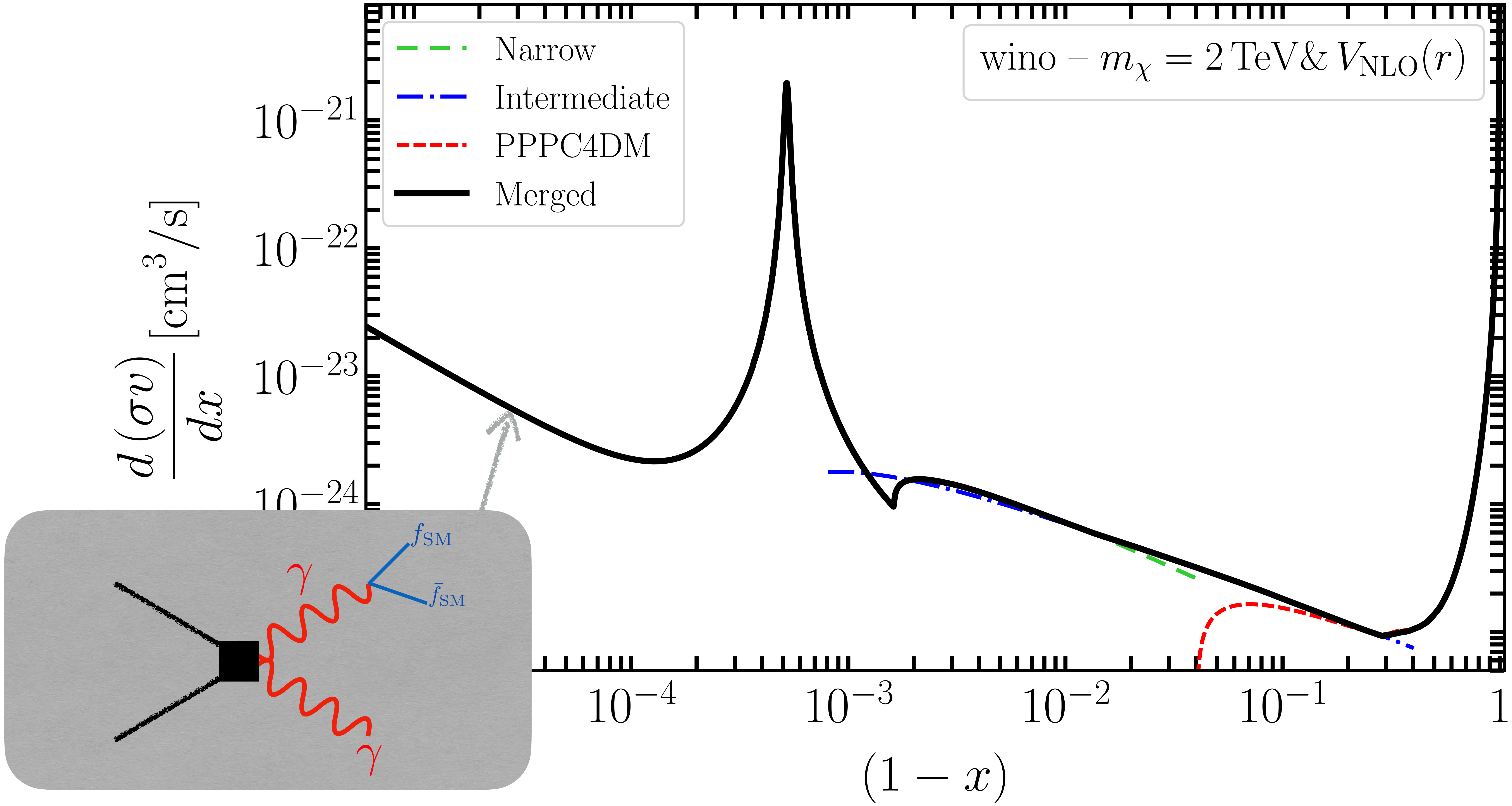
Numerical results with DMγSpec

Resummed wino and higgsino endpoint spectra + parton showers



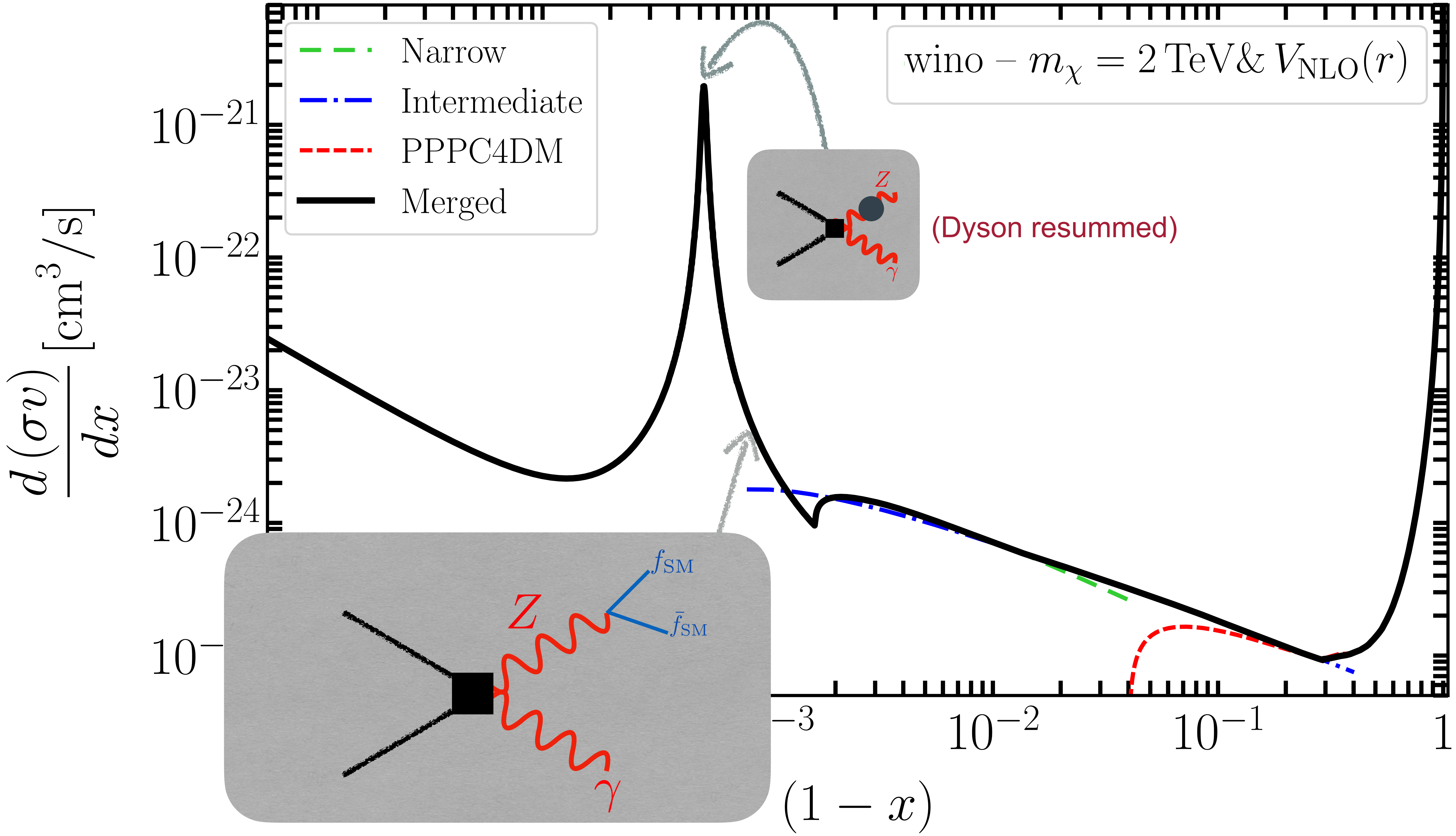
Numerical results with DMγSpec

Resummed wino and higgsino endpoint spectra + parton showers



Numerical results with DMγSpec

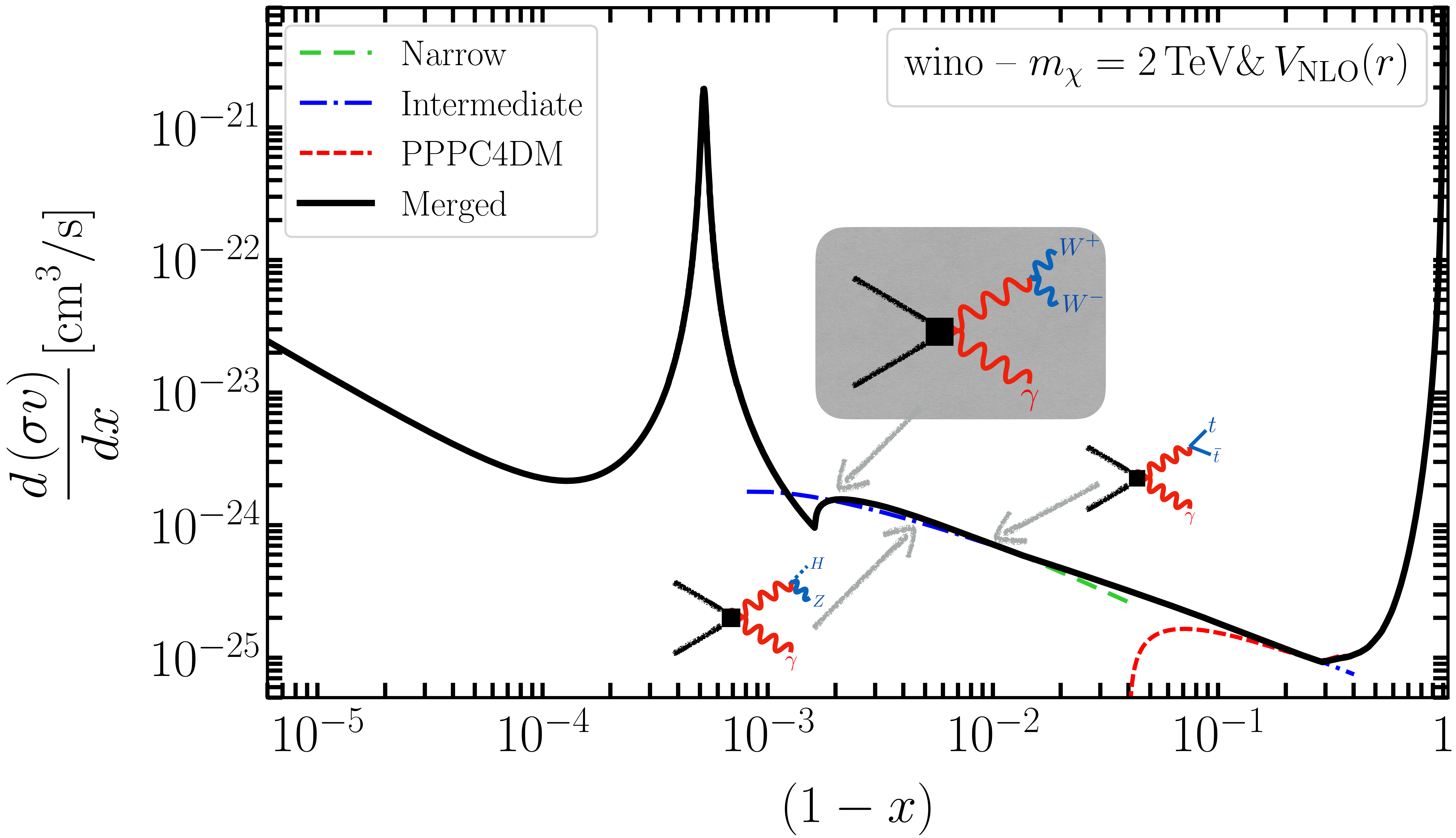
Resummed wino and higgsino endpoint spectra + parton showers



(Dyson resummed)

Numerical results with DMγSpec

Resummed wino and higgsino endpoint spectra + parton showers



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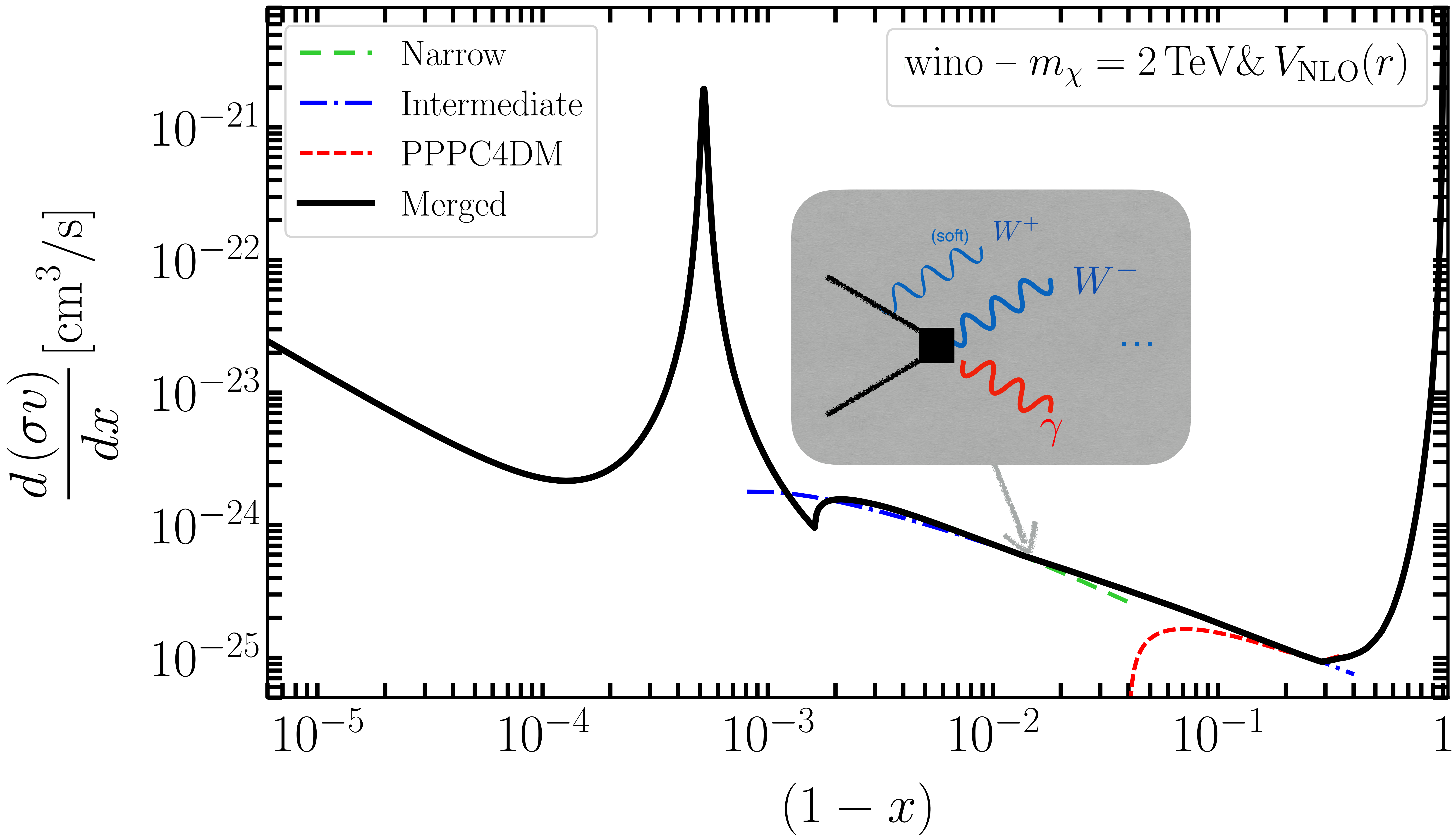
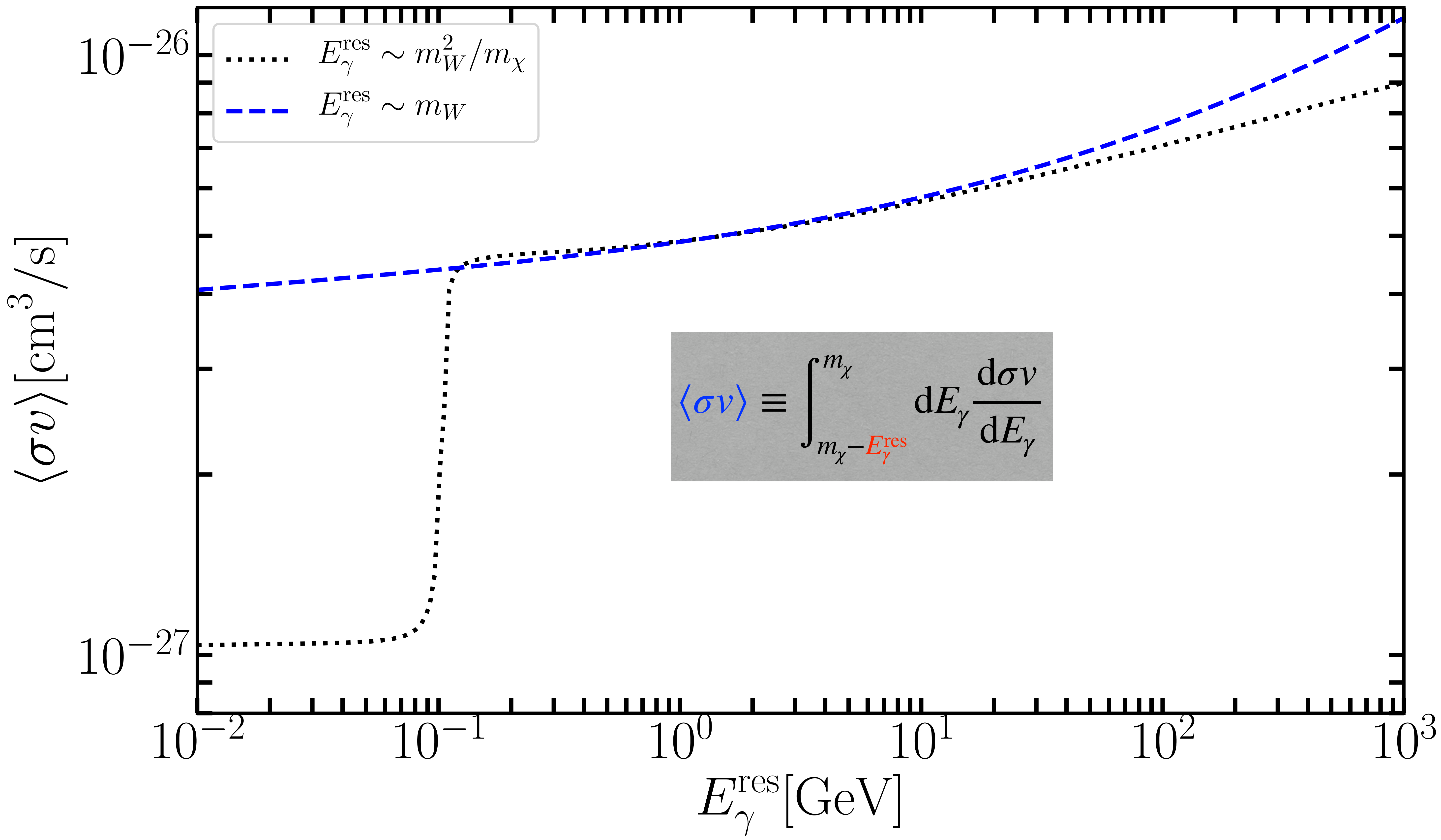
Numerical results with DMγSpec

Resummed wino and higgsino endpoint spectra + parton showers



Numerical results with DMγSpec

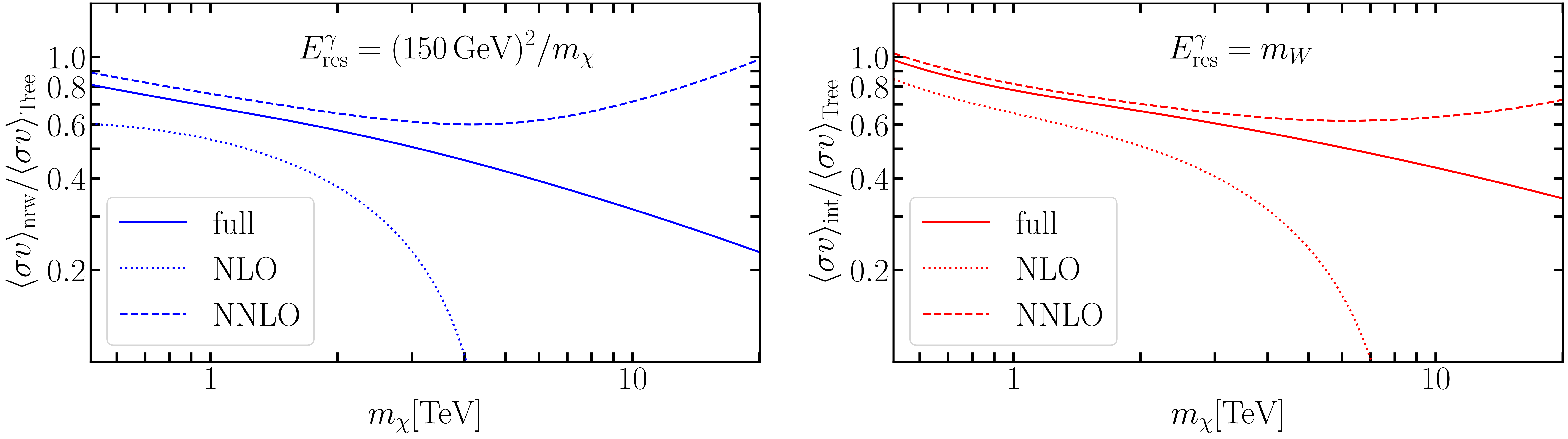
Cumulative cross sections (effect of the $\gamma\gamma$ line)



$$\langle \sigma v \rangle \equiv \int_{m_\chi - E_\gamma^{\rm res}}^{m_\chi} \mathrm{d}E_\gamma \frac{\mathrm{d}\sigma v}{\mathrm{d}E_\gamma}$$

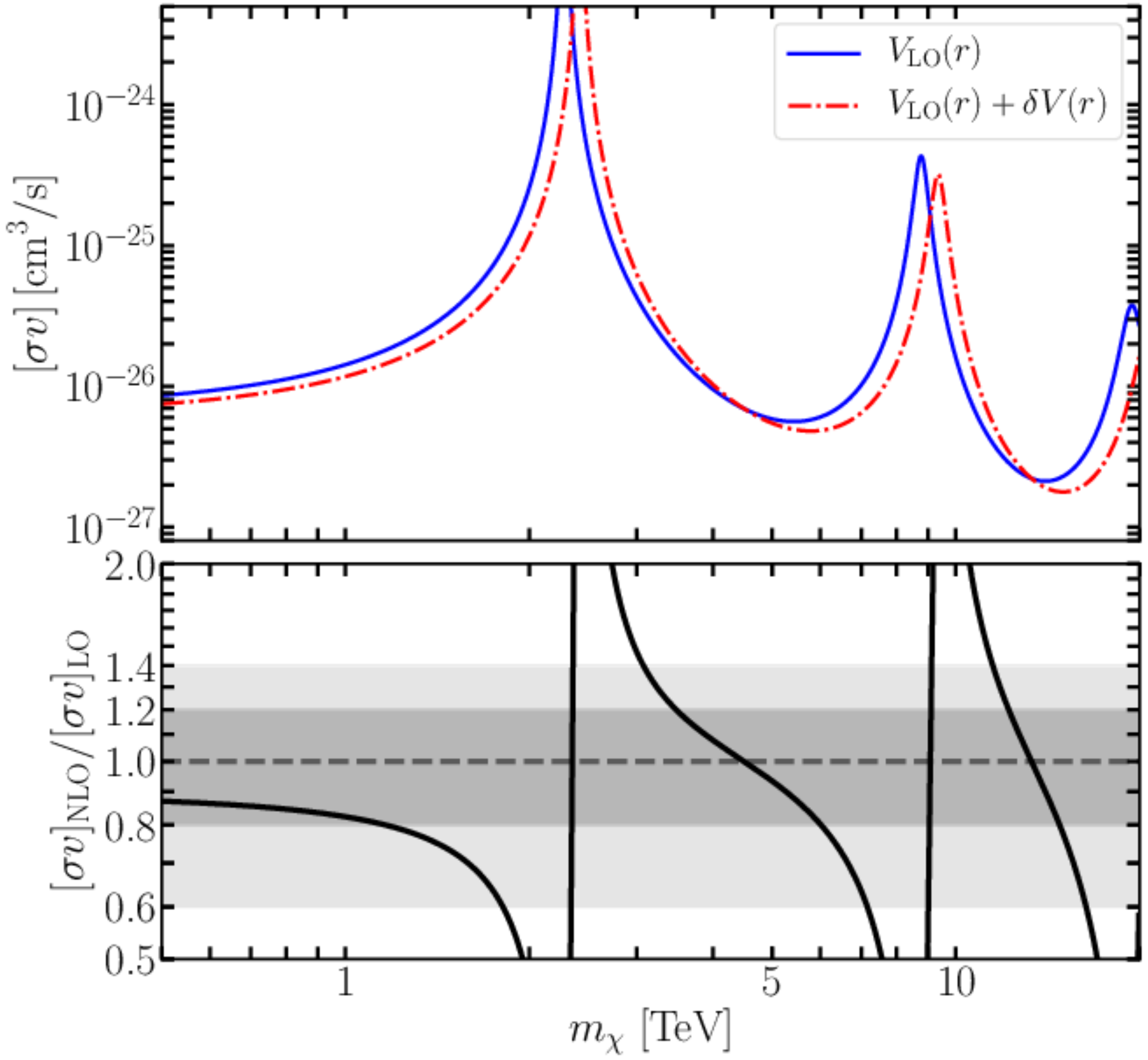
Fixed-order cross sections

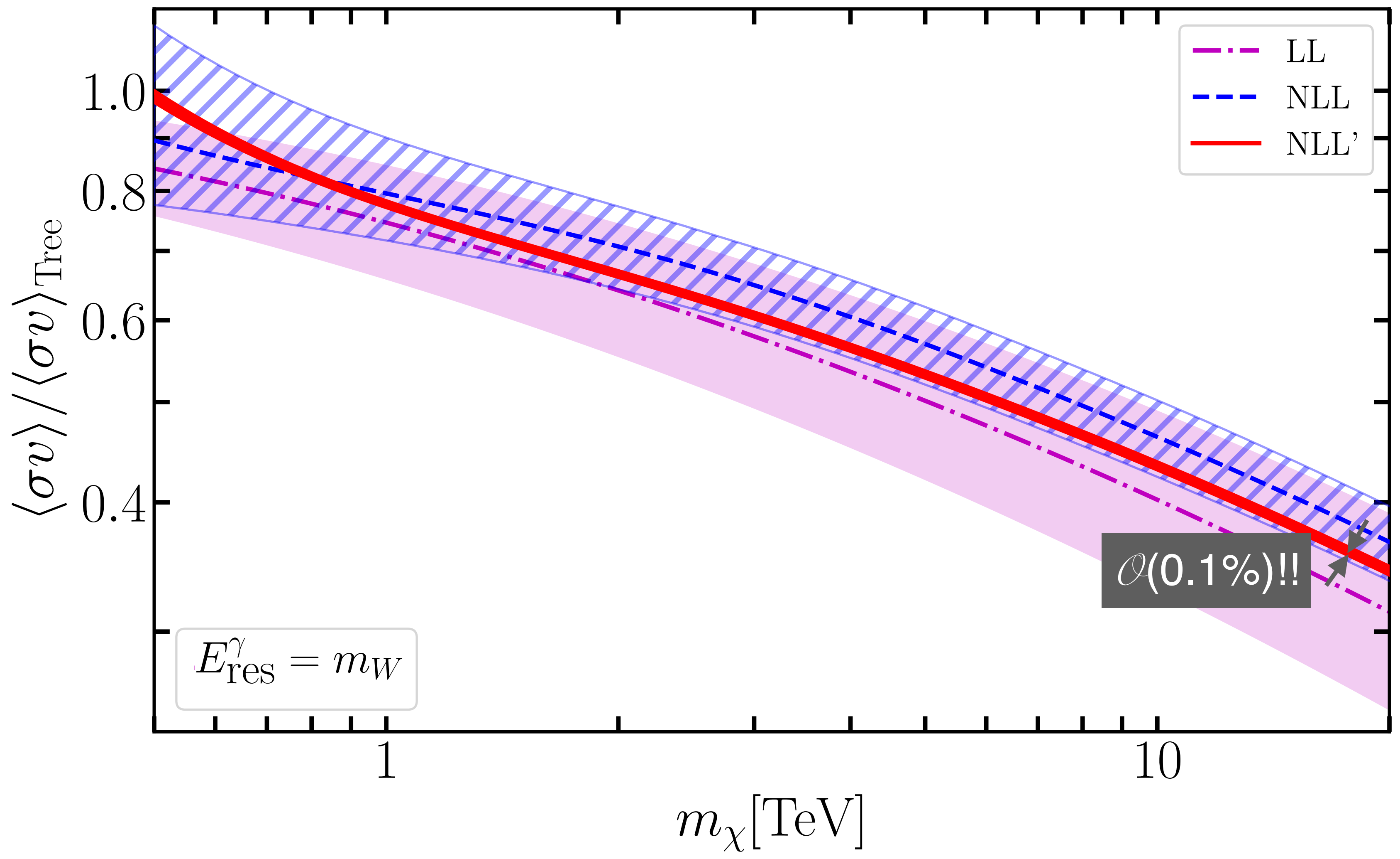
Breakdown of the perturbative expansion (after Sommerfeld resummation)



Numerical results

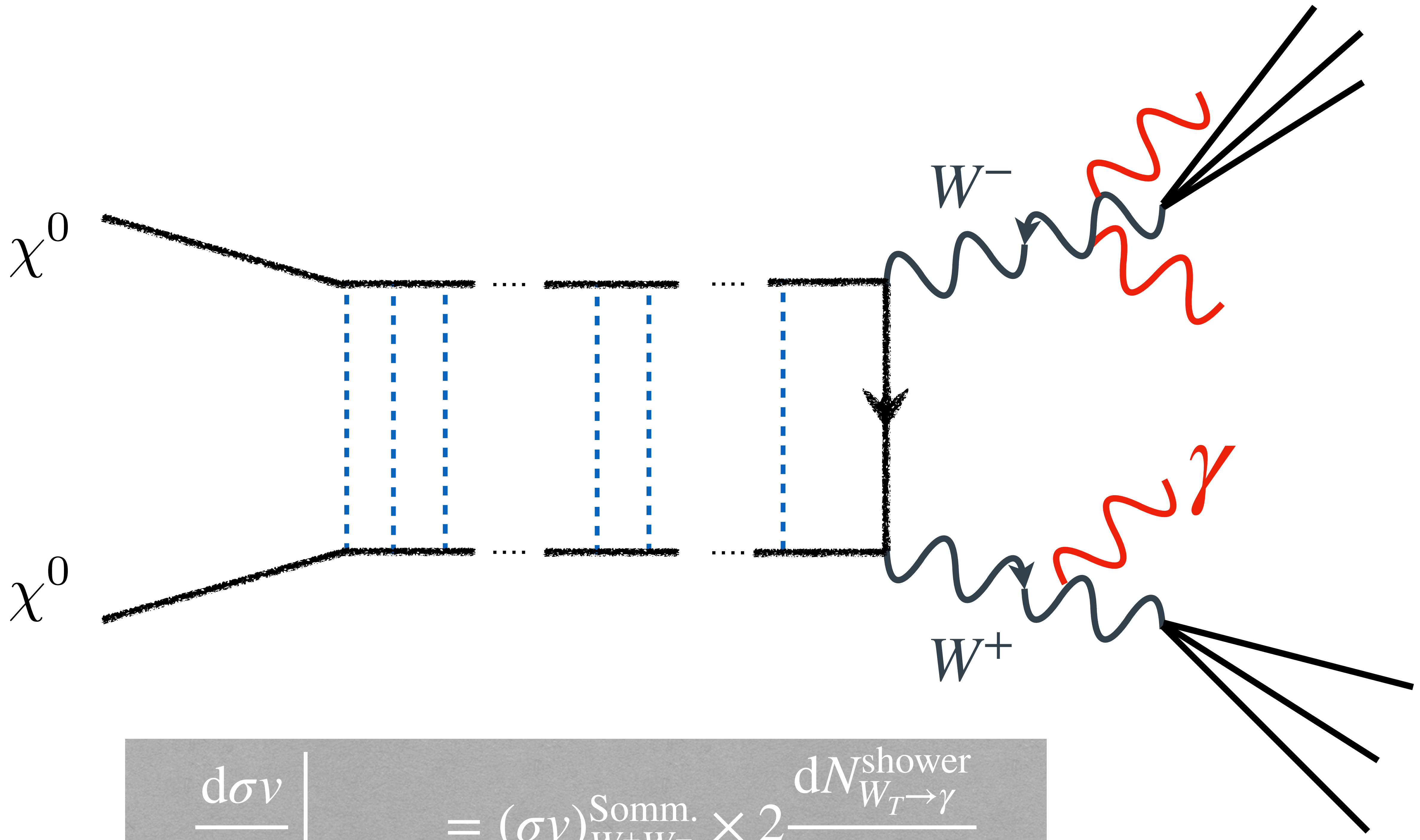
Sommerfeld resonance shift and Sudakov suppression



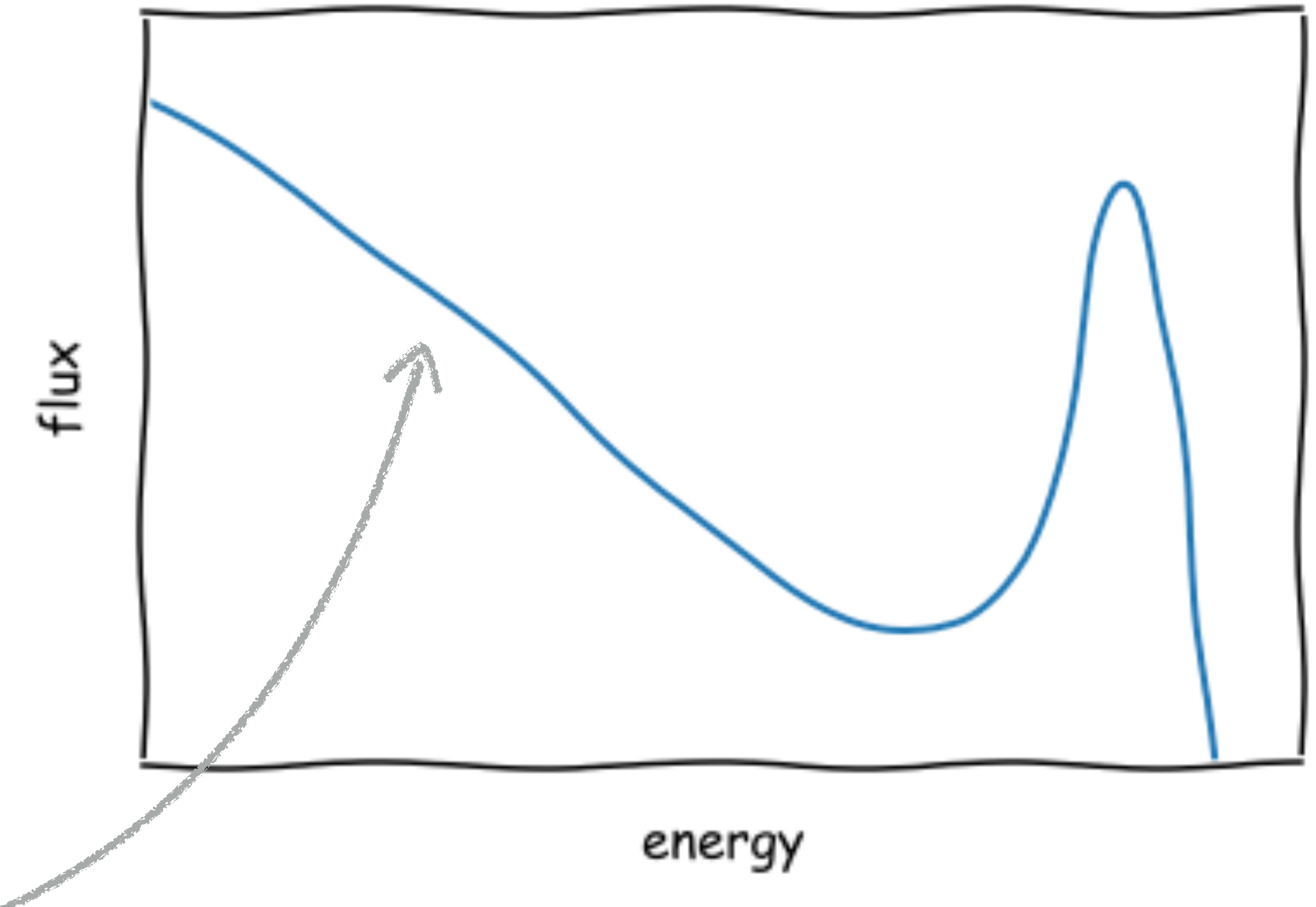


$\mathcal{O}$(0.1%)!!

Matching with the continuum (parton showers)



$$\left.\frac{\mathrm{d}\sigma v}{\mathrm{d}x}\right|_{\text{shower}} = (\sigma v)^{\text{Somm.}}_{W^+W^-} \times 2\frac{\mathrm{d}N^{\text{shower}}_{W_T\to\gamma}}{\mathrm{d}x}$$



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Matching with the continuum (parton showers)

Parton shower

$$\left.\frac{\mathrm{d}\sigma v}{\mathrm{d}x}\right|_{\text{shower}} = (\sigma v)^{\text{Somm.}}_{W^+W^-} \times 2\frac{\mathrm{d}N^{\text{shower}}_{W_T\to\gamma}}{\mathrm{d}x}$$

Factorization formula ‘int’

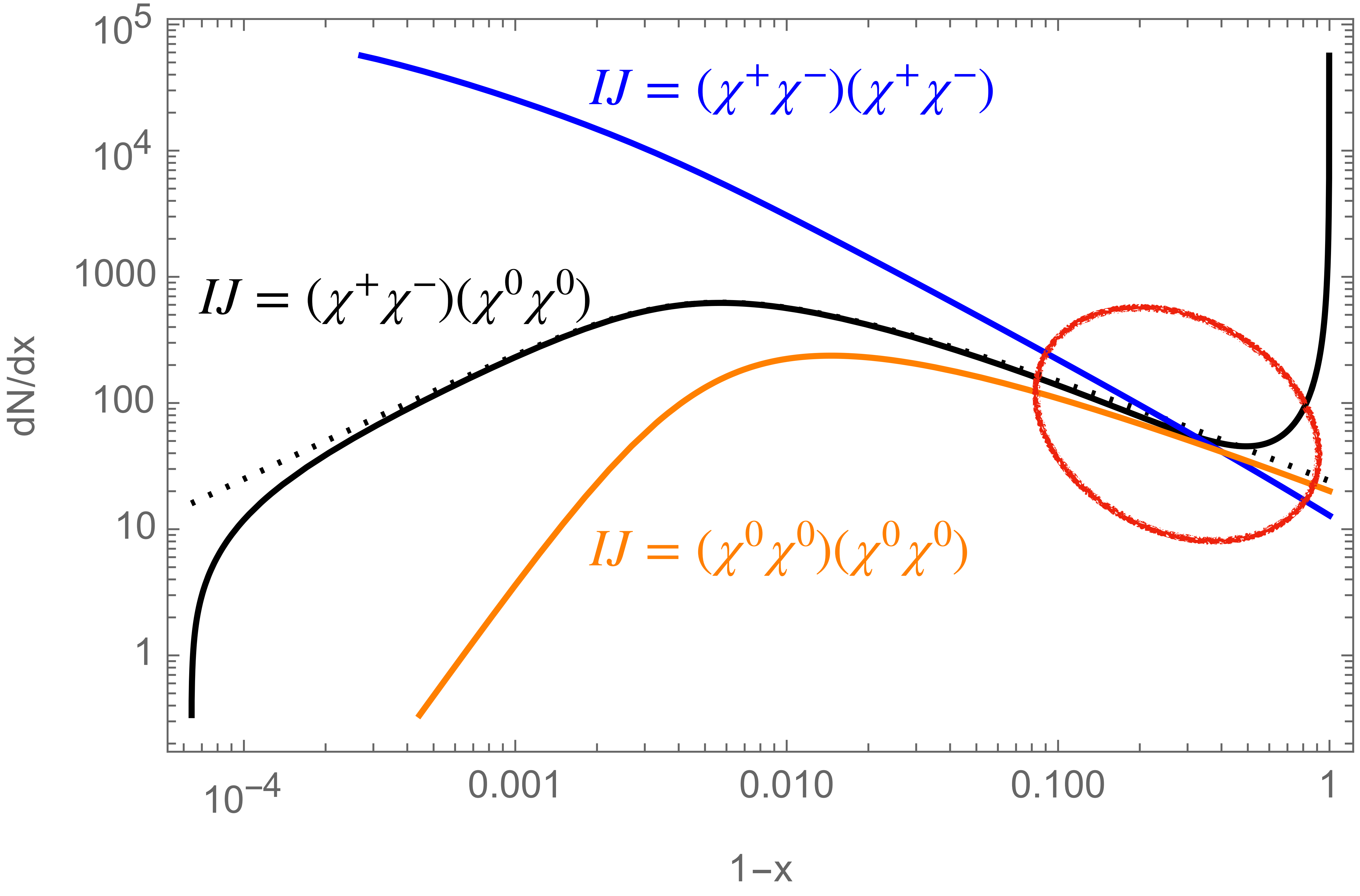
$$\left.\frac{\mathrm{d}\sigma v}{\mathrm{d}x}\right|_{\text{fact}} = 2\sum S_{IJ}\Gamma_{IJ}(x)$$

$$\Gamma^{\text{LO}}_{IJ}(x) \to (\sigma v)^{W^+W^-}_{IJ} \times 2\frac{\mathrm{d}N^{\text{LO}}_{W_T\to\gamma}}{\mathrm{d}x}$$

$$2\frac{\mathrm{d}N^{\text{LO}}_{W_T\to\gamma}}{\mathrm{d}x} = \frac{4\alpha_{\text{em}}}{\pi}\frac{1}{1-x}\log\frac{4m_\chi^2}{m_W^2} + \ldots$$

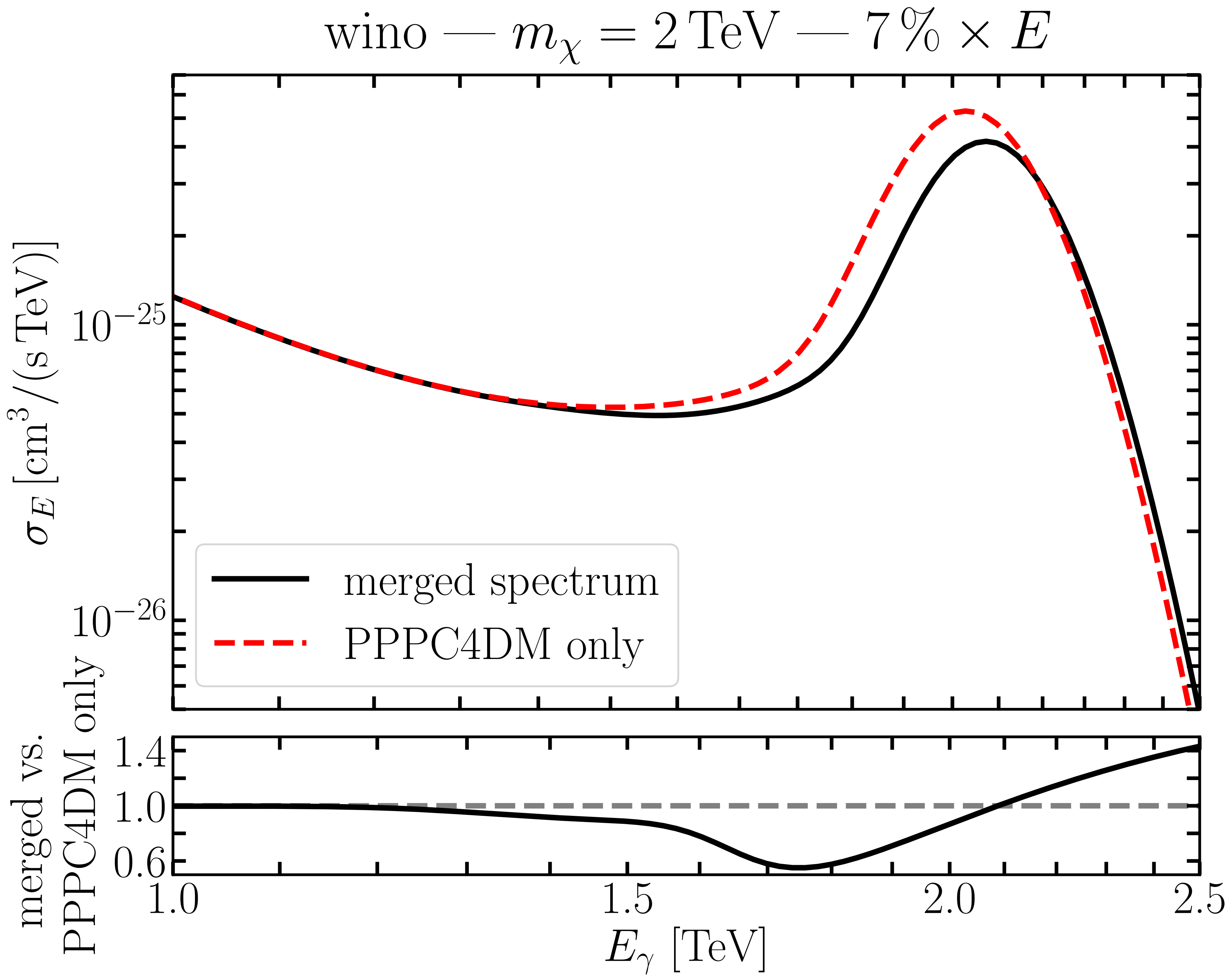
$$(\sigma v)^{\text{Somm.}}_{W^+W^-} = 2\sum S_{IJ}(\sigma v)^{W^+W^-}_{IJ}$$

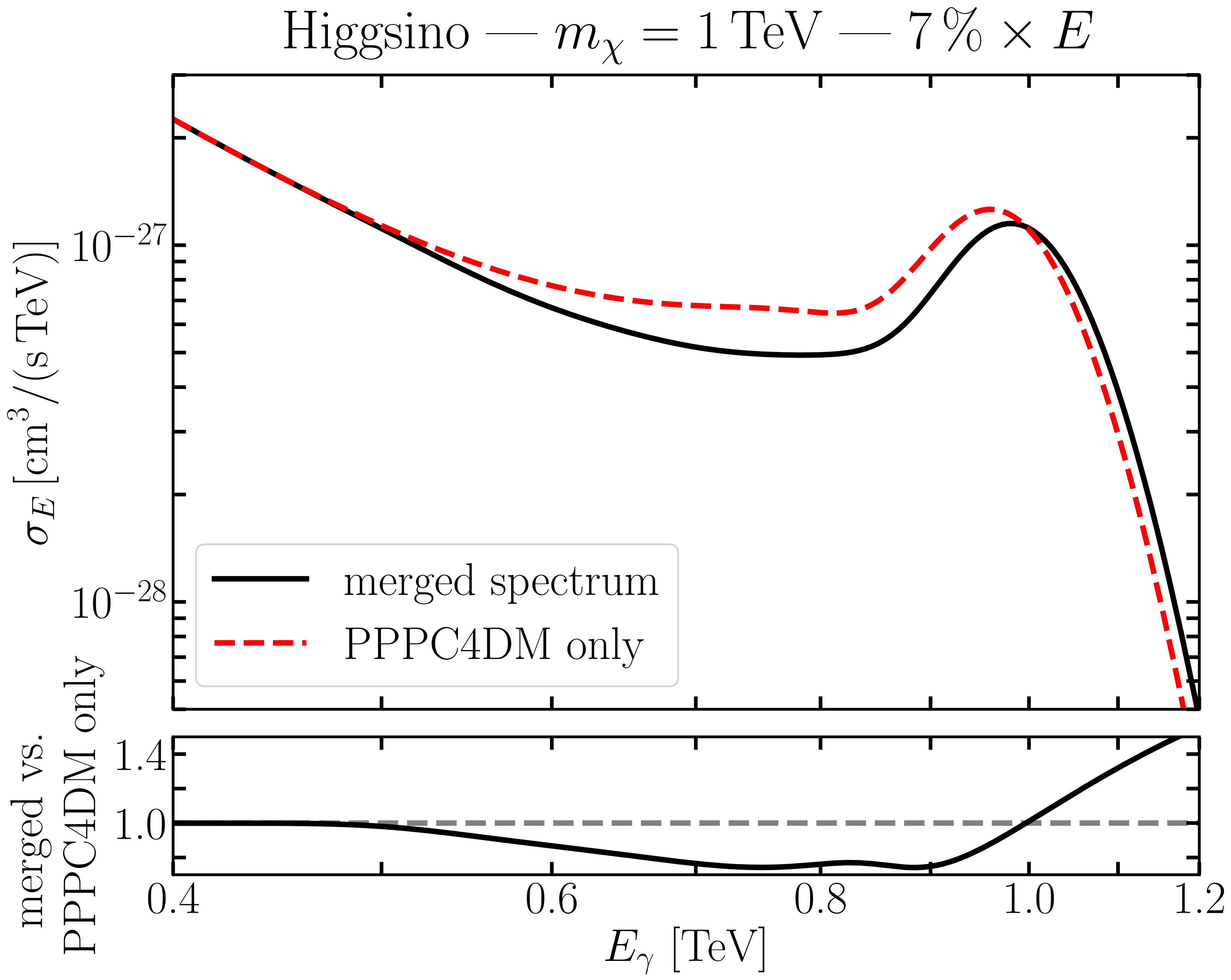
Matching with the continuum (parton showers)



DMγSpec

Instrument response function





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Conclusions

A 50yrs old story of EWimps coming to a slow (50 more years?) end

- Unexplored heavy WIMP parameter-space chunk to be probed by indirect detection observations in the near future
- Electroweak effects are extremely important
  - Besides Sommerfeld enhancements, Sudakov-log resummation at the endpoint plays a very important role
- Provided a complete description of prompt gamma-ray spectra from wimp annihilation for the benchmark wino and higgsino models

DMSpec

  - Demonstrated a perfect matching and consistency between different regimes/calculations apparent in these spectra
- The full MSSM is the next big resummation factory to look at!