Dark Matter relic abundance with NLO bound state effects: collinear finiteness and gauge invariance

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Based on:
[2002.07145] (JHEP),
[2107.03945] (JHEP).

In collaboration w/:
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Introduction

- Attractive long-range forces lead to quantum mechanical effects: Sommerfeld enhancement, meta-stable bound states

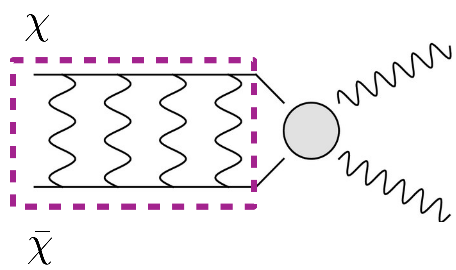
- Relevant for DM relic abundance and indirect detection signal

- Examples: light mediators (e.g., Z’), electroweak or colored coannihilation

- Sommerfeld effect in vacuum, and bound states at LO well understood

- This talk: bound-state formation at higher order (NLO corrections in a thermal field theory approach)

$\alpha m_\chi \gtrsim m_{\mathrm{mediator}}$

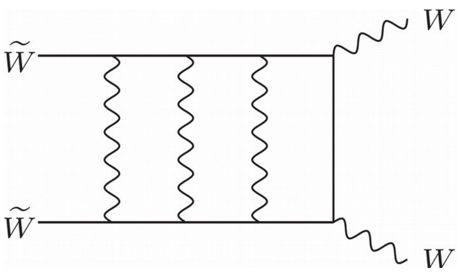


Sommerfeld effect, bound-state decay

Prime example: Wino dark matter

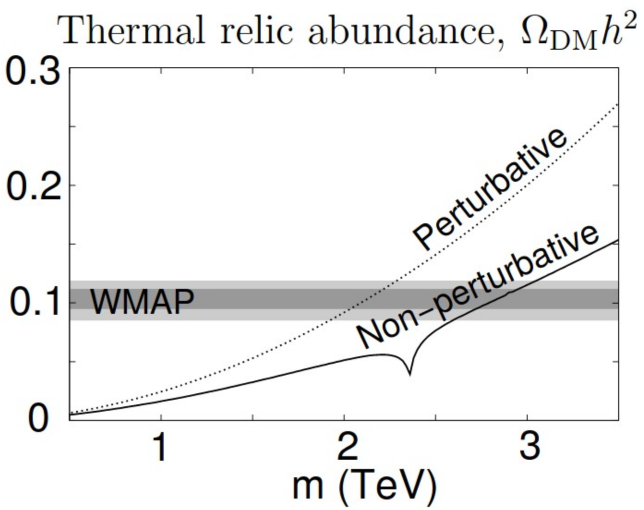
Wino:
- Majorana Fermion
- Triplet under SU(2)
- Hypercharge Y=0
- Most minimal DM!

$$\begin{pmatrix} \widetilde{W}^{+} \\ \widetilde{W}^{0} \\ \widetilde{W}^{-} \end{pmatrix}$$

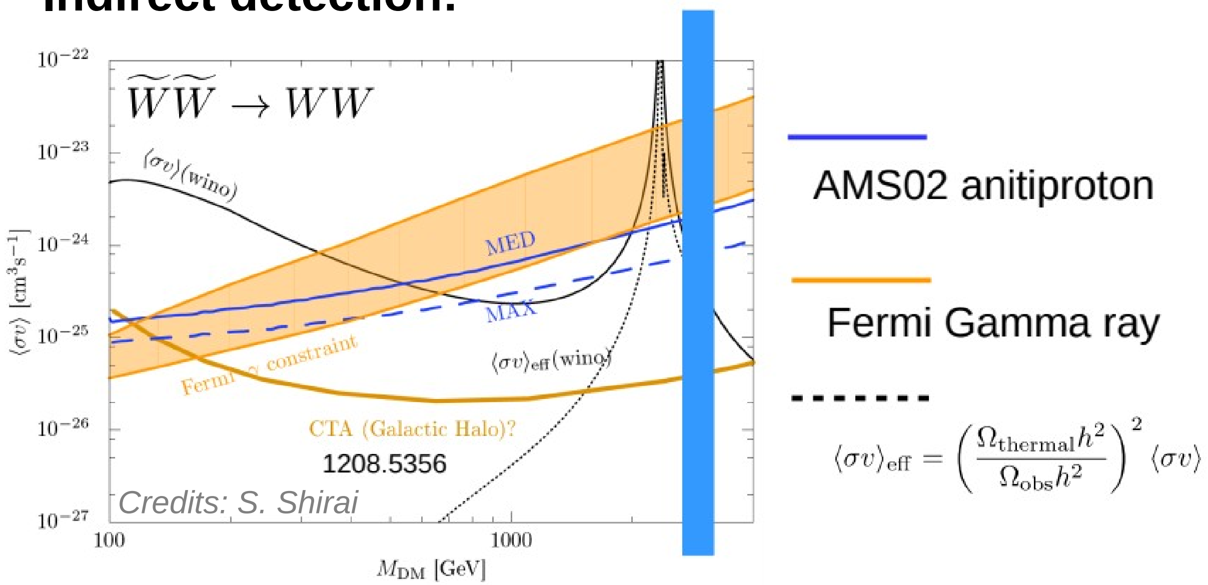


Hisano et al. ‘03, ‘05, ‘06

Predicted mass:



Indirect detection:

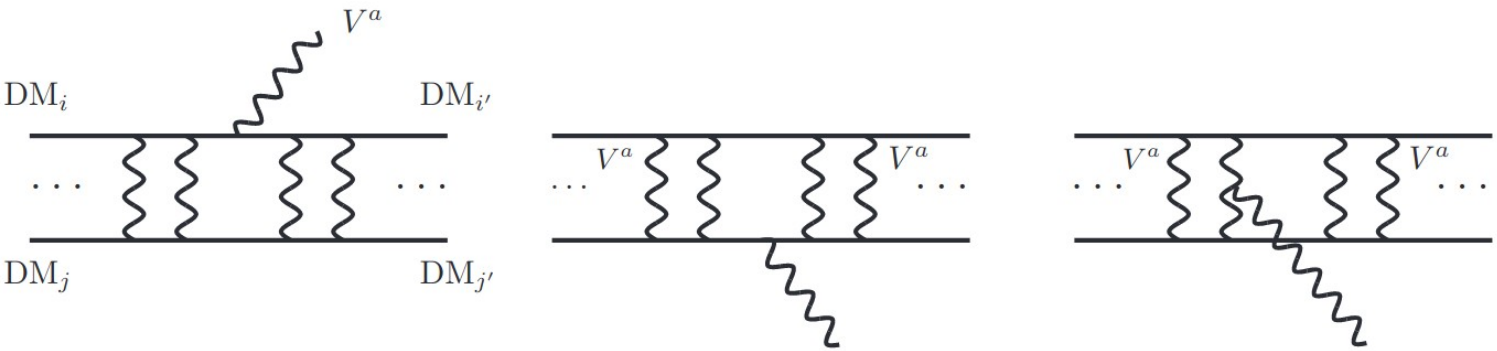


~10 % variation in the DM mass results in ~100 % change of the flux.

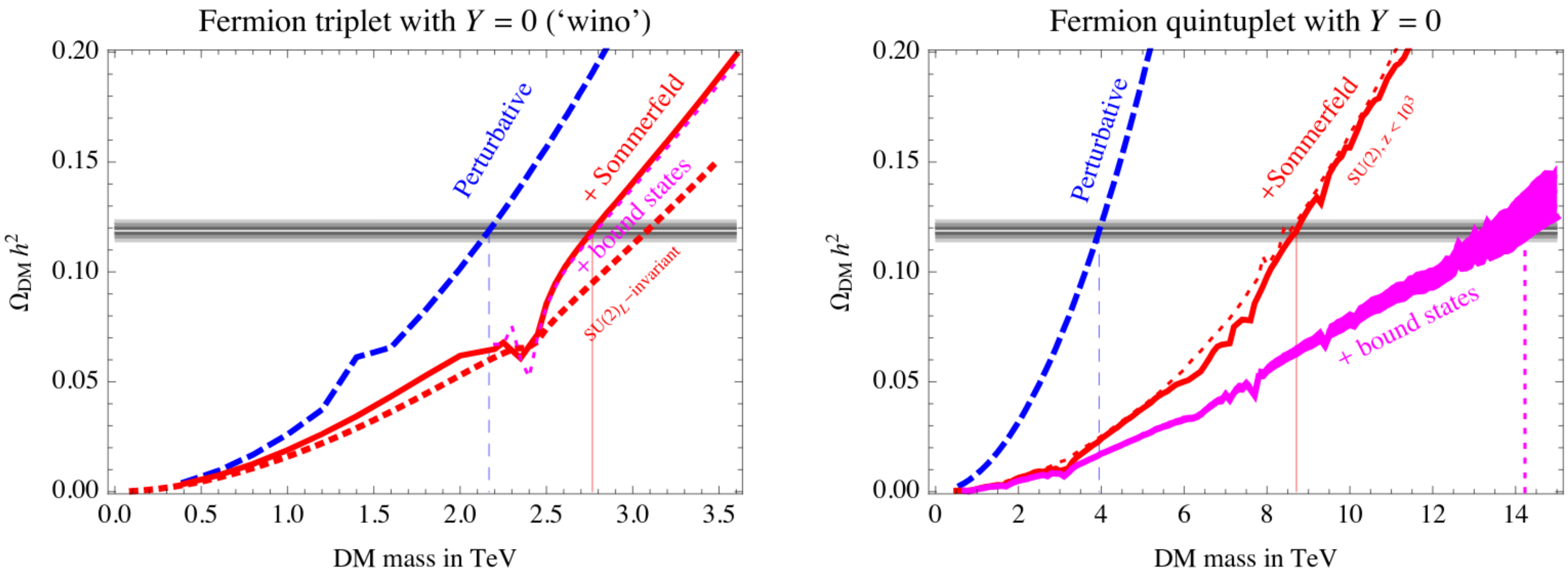
Refinements: mass splitting [Ibe et al. 12], NLO Wino potential & Final State corrections (talk by M. Vollmann), J factor (talk by K. Hayashi), bound states (next slide), ...

Bound-state implications in minimal DM

Radiative bound-state formation at LO:

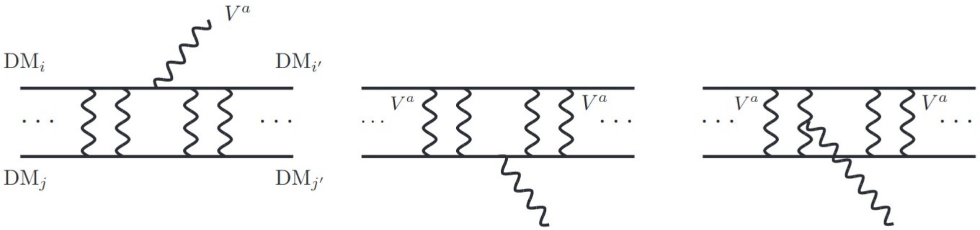


Meta-stable bound states contribute as an additional decay channel.



[Mitridate *et al.* 17]

Bound-state implications colored co-annihilation



- **Co-annihilating partner charged under SU(3)**
  - Squark (scalar triplet)
  - Gluino (fermion octet)

[Ellis *et al.* 15, Liew&Luo 16, Mitridate *et al.* 17]

- **+ Higgs**
  - Additional attractive contribution
  - Color-octet state can have bound state

[Harz & Petraki 18,19]

- **Non-perturbative effects**
  - Relevant for mass splittings below QCD confining scale
  - Enormously large corrections from post-confining effects

[Gross *et al.* 18, Fukuda&Luo&Shirai 18]

Motivation

- At NLO, bound-state formation via bath particle scattering expected to be dominant contribution

- However, amplitudes diverge in forward scattering direction of the bath particles ($\theta \to 0$).
  → Collinear divergence

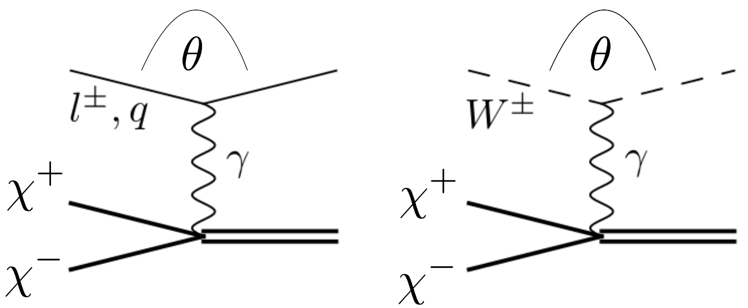
- Insertion of Debye mass as a regulator not allowed for $T \lesssim \Delta E$ (HTL expansion not justified)

- Gauge invariance for non-abelian interactions?

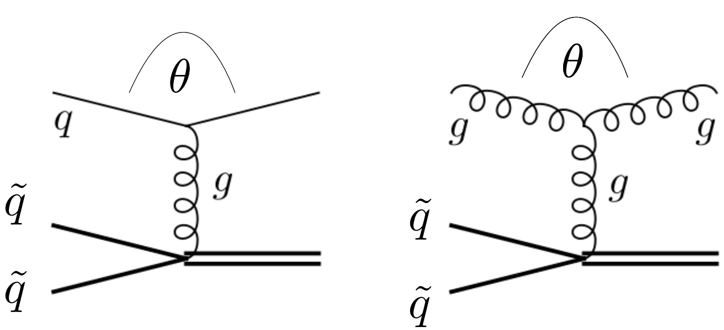
  → Need to derive collision term from thermal field theory to address the issue of collinear divergence and gauge invariance.

NLO examples:

EW charged DM



Colored coannihilation



Electric field correlator

QED toy model:

$$\mathcal{L}_{\text{int}} = g\bar{\chi}\gamma^\mu\chi A_\mu + g\bar{\psi}\gamma^\mu\psi A_\mu$$

Assuming temperature much smaller than Bohr-momentum, we can utilize pNREFT:

$$\mathcal{L}^{\text{pNR}} = \int \mathrm{d}^3 r\ \mathrm{Tr}\{O^\dagger(\mathbf{x},\mathbf{r},t)\,[i\partial_t - h + \ \mathbf{r}\cdot g\mathbf{E}(\mathbf{x},t)]\,O(\mathbf{x},\mathbf{r},t)\} + \mathcal{L}[A,\psi]$$

Ultra-soft transitions among two-body fields O via **electric dipole operator**.

From this effective action, the collision term for the DM number density equation can be derived by using Liouville equation, open-quantum system framework, or CTP formalism. We find for relevant quantities (bound-state formation cross section & de-excitation rate):

$$(\sigma v)_{nl} \sim g^2 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_v\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

$$\Gamma_{nl}^{n'l'} \sim g^2 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{n'l'}\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

Contact with plasma environment is encoded in
the ***Electric Field Correlator***: $D^{-+}(x,y) \equiv \langle E(x)E(y)\rangle$, where $\langle ...\rangle \propto \mathrm{Tr}[e^{-H_{\text{env}}/T}...]$.

Computation of Electric field correlator

KMS relation:

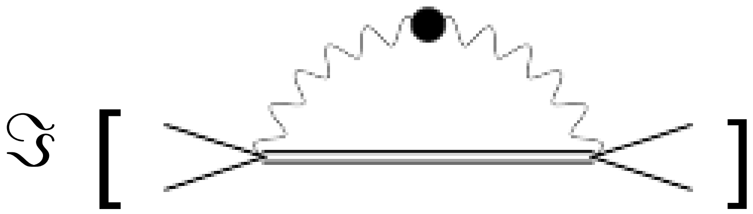
$$D^{-+}(\Delta E, \mathbf{p}) = \left[1 + f_B^{\mathrm{eq}}(\Delta E)\right] D^{\rho}(\Delta E, \mathbf{p})$$

$$D^{\rho} = 2\Im\left[iD^R\right]$$

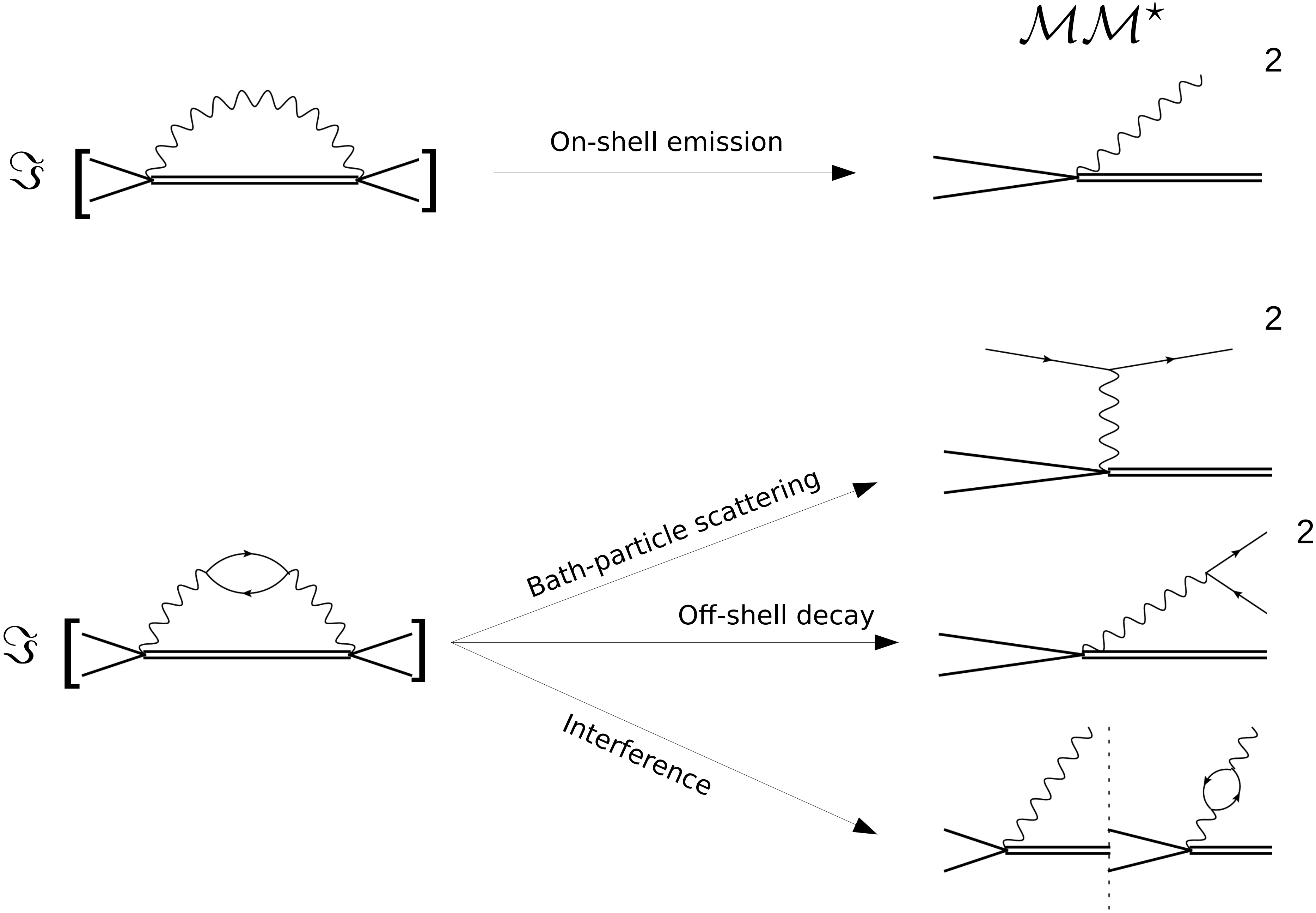
$$D^R = D^{R,0} + D^{R,0}\Pi_R D^{R,0} + ...$$

First contact with the plasma

CTP diagram



Leading and next-to-leading order

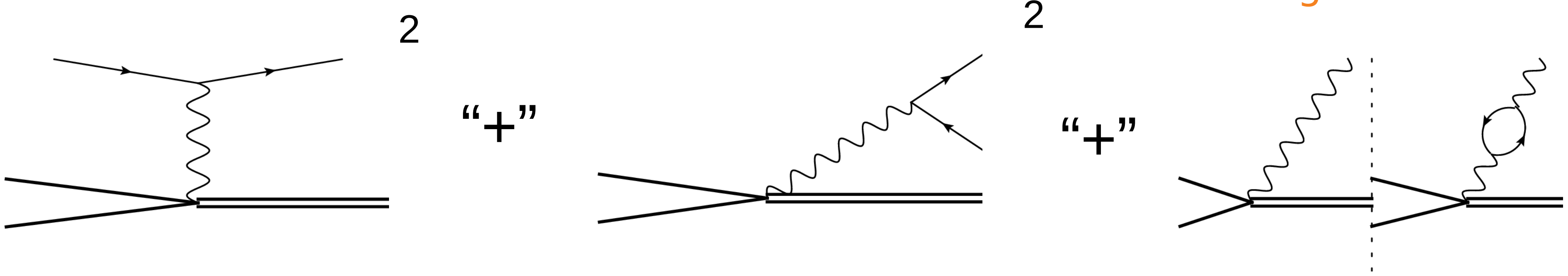


Cancellation of collinear divergences

UV finite, collinear divergent.

UV finite, collinear divergent.

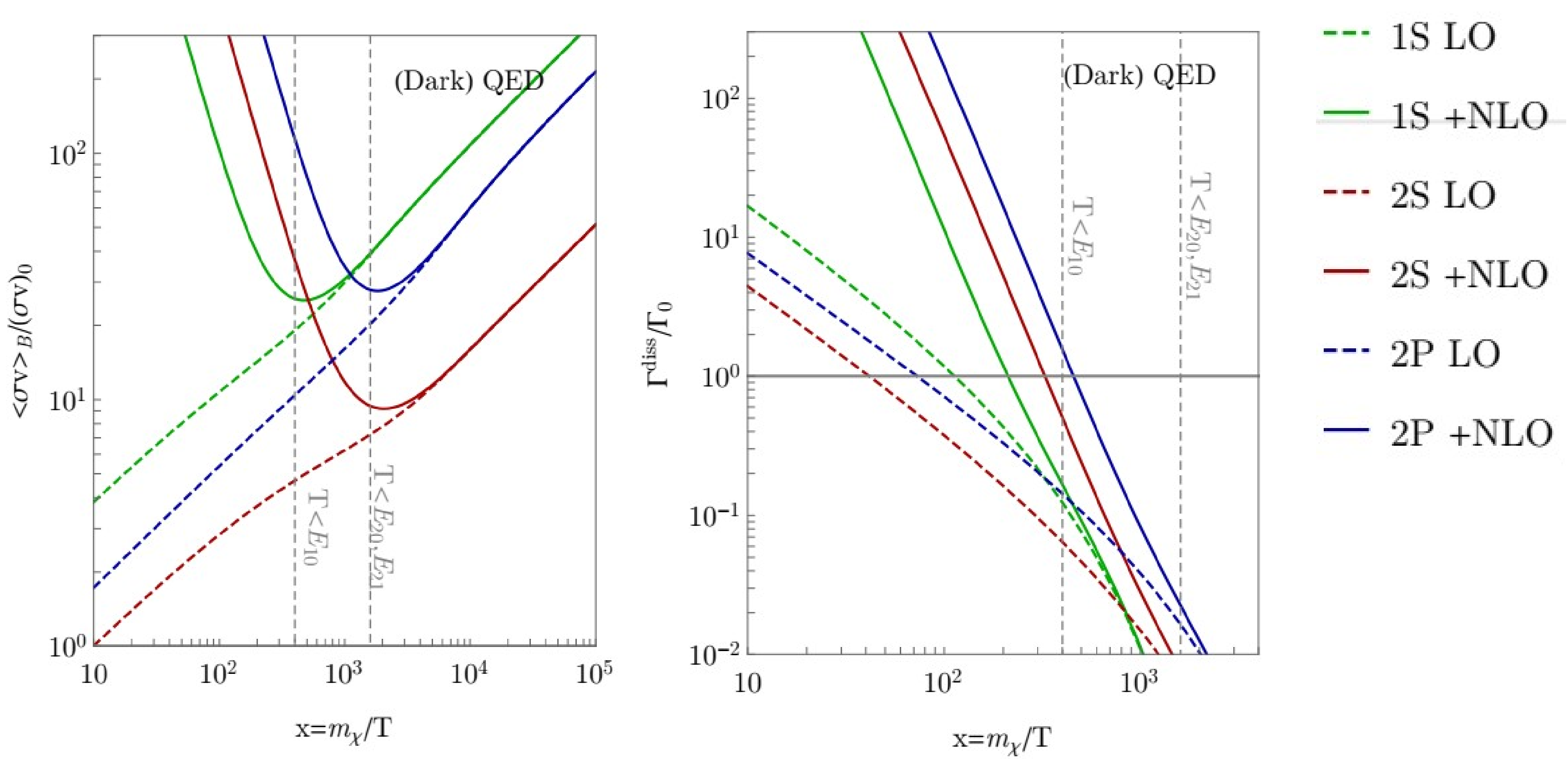
Vacuum part UV divergent, collinear divergent.



= Finite in collinear direction, and UV finite after vacuum renormalization.

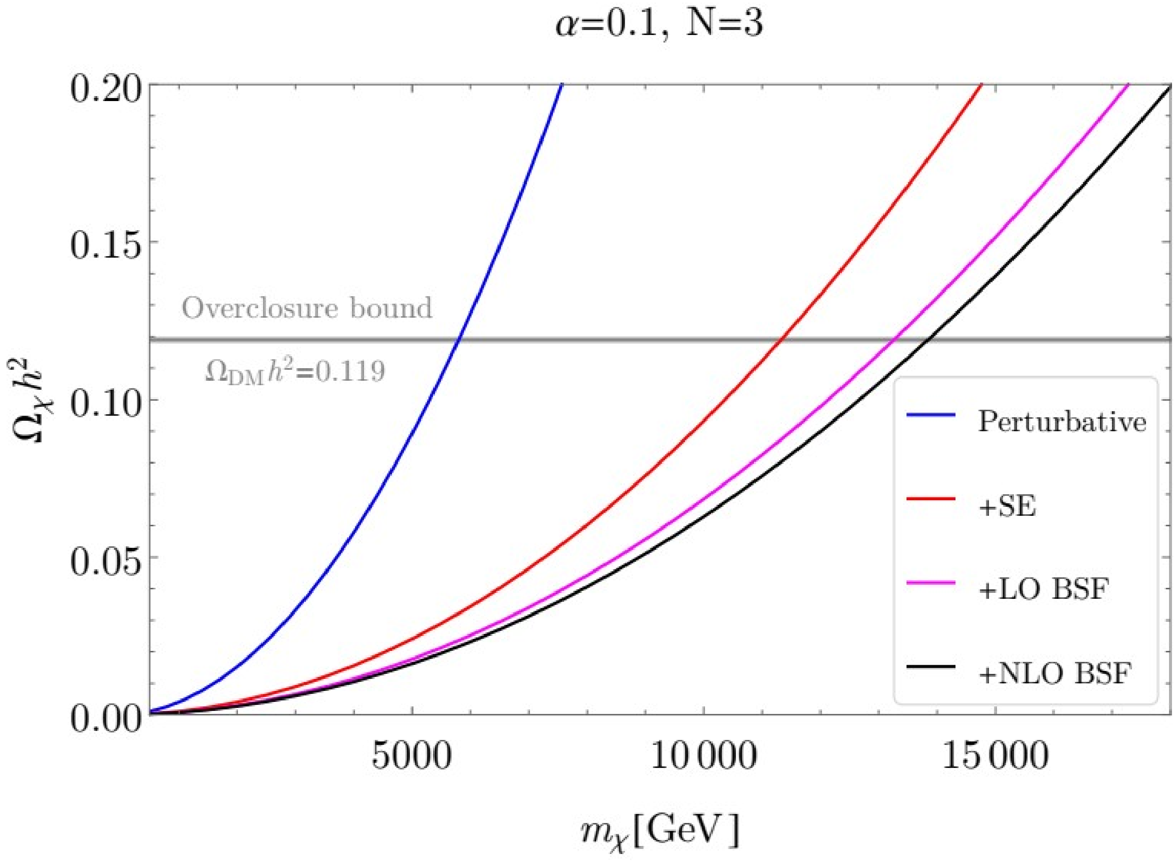
- Provide mathematical proof for cancellation of collinear divergences.
- Holds even for arbitrary phase-space distribution of bath particles, i.e. bath particles do not have to be in thermal equilibrium in order to guarantee finiteness in the forward scattering direction.
- (similar to Bloch-Nordsieck theorem)

Results U(1)



- Strong enhancement for T>>E
- Flipped hierarchies of rates
- Leads to ionization equilibrium

Results U(1)



(only ground state included)

Non-Abelian Electric Field Correlator

Consider SU(N) representation and its conjugate:

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots ,$$

Singlet configuration has tightest bound state.

$$\mathcal{L}_{\rm pNREFT} \supset \int \mathrm{d}^3 r \; \mathrm{Tr} \Big[\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger (iD_0 - H_{\rm adj})\mathrm{Adj} - V_A(\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.}) - \frac{V_B}{2} \mathrm{Adj}^\dagger \{\boldsymbol{r} \cdot g\boldsymbol{E}, \mathrm{Adj}\} + \cdots \Big] .$$

$$\mathcal{S}(\chi\bar{\chi})_{\rm adj} \rightleftharpoons \mathcal{B}(\chi\bar{\chi})_1 , \quad \mathcal{S}(\chi\bar{\chi})_1 \rightleftharpoons \mathcal{B}(\chi\bar{\chi})_{\rm adj} , \quad \mathcal{S}(\chi\bar{\chi})_{\rm adj} \rightleftharpoons \mathcal{B}(\chi\bar{\chi})_{\rm adj} ,$$

Leads to similar BSF cross section but with the replacement:

$$g_{i_1 i_2}^{E++}(t_1, t_2, \mathbf{R}_1, \mathbf{R}_2) = \Big\langle \mathrm{Tr}_{\rm color}\Big(E_{i_1}(\mathbf{R}_1, t_1) \mathscr{W}_{[(\mathbf{R}_1,t_1),(\mathbf{R}_1,+\infty)]} \mathscr{W}_{[(\mathbf{R}_2,+\infty),(\mathbf{R}_2,t_2)]} E_{i_2}(\mathbf{R}_2, t_2)\Big)\Big\rangle_T$$

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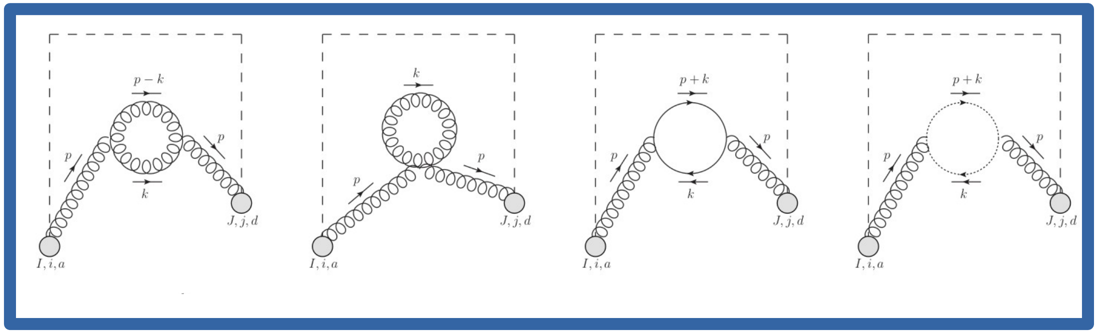
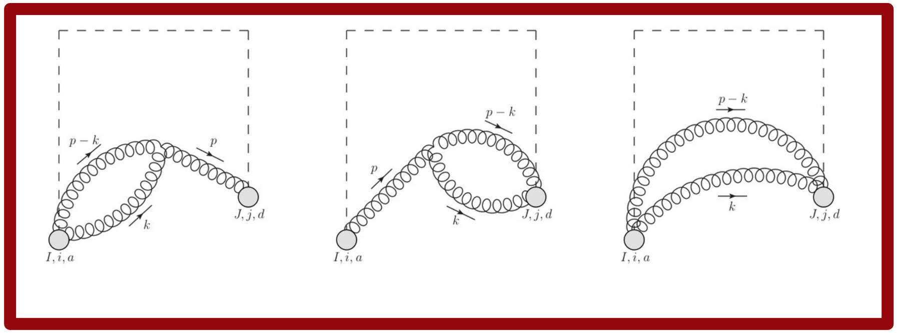
Results SU(N)

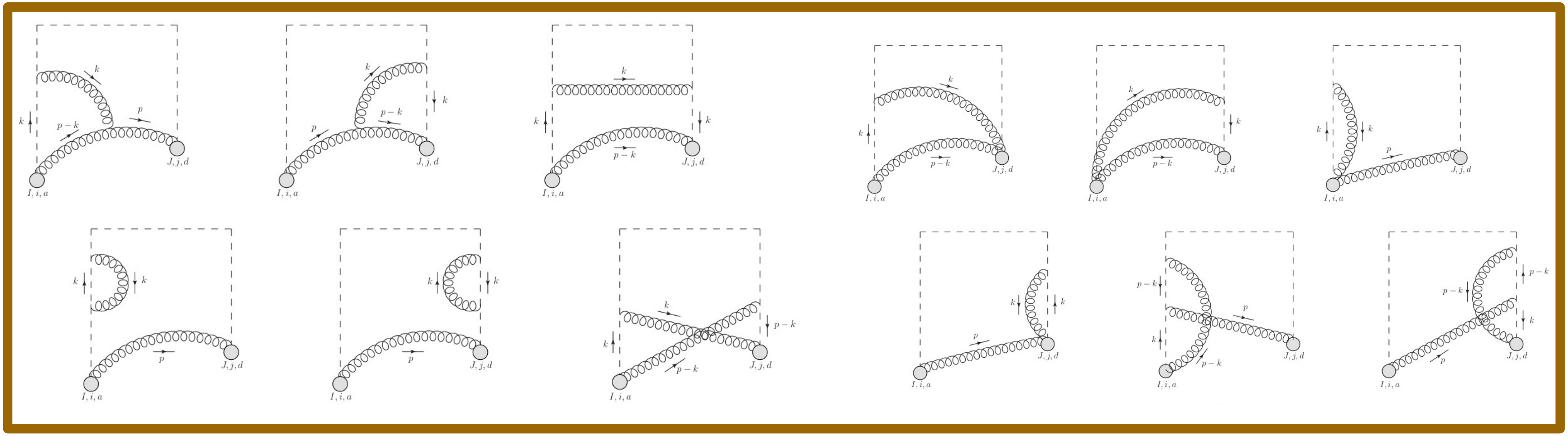
Self-energy



Non-linear



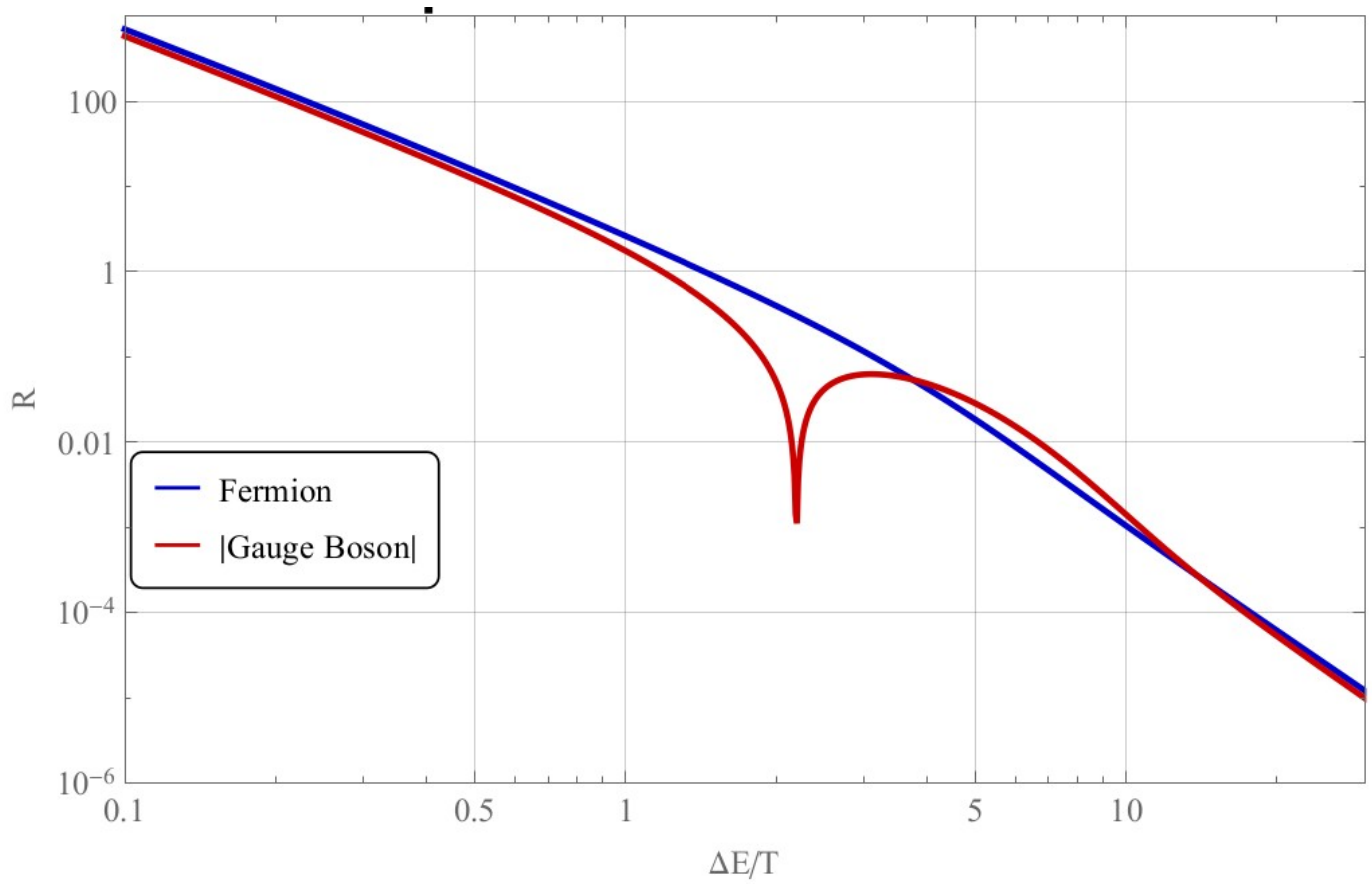
Wilson lines



Gauge invariance, infrared and collinear safety proven.

Results SU(N)

$$(\sigma v_{\rm rel})^{\rm LO+NLO}_{\mathscr{B}} = (\sigma v_{\rm rel})^{\rm LO}_{\mathscr{B}} \times \Big[1 + \alpha N_c R_g^{T=0}(\mu/\Delta E) + \alpha N_c R_g^{T\neq 0}(\Delta E/T) + \alpha N_f R_f^{T=0}(\mu/\Delta E) + \alpha N_f R_f^{T\neq 0}(\Delta E/T)\Big]$$



Summary

- Bound states contribute to a further depletion of relic abundance, allowing for *heavier DM masses*.

- For U(1) and representations of unbroken SU(N), expressed bound-state formation in terms of thermal correlators, showing NLO contributions are **collinear finite and gauge invariant**.

- Provide zero T analytically and finite T as one-integral expressions

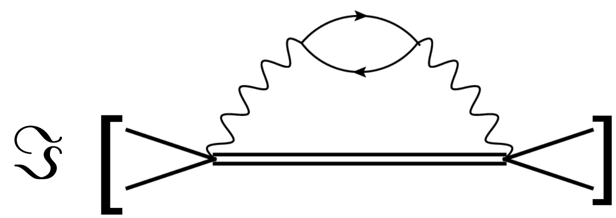
- Finite T part of NLO effects show strong BSF enhancement for $T \gg |E_{nl}|$

- NLO effects can be relevant if:

  - Ionization equilibrium is not fully maintained by LO effects
  (e.g., in QED toy-model, or LO suppressed by kinematical block)

  - Decoupling from ionization equilibrium occurs at a time when NLO is still larger than LO
  (e.g., large coupling, many bath particles)

Sketch of proof



$$\Im[D^R_{\mu\alpha}(P)\Pi^{\alpha\beta}_R(P)D^R_{\beta\nu}(P)]$$

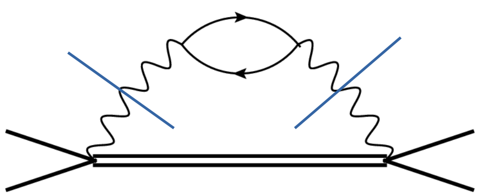
$$G(z) \equiv \frac{H(z)}{(z-z_2)^2(z-z_1)}$$

Collinear limit: $z_1 \to z_2$

double pole cut

single pole cut

$=$



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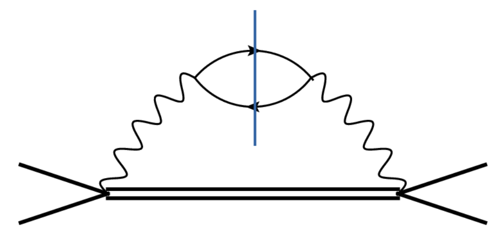

$$\mathrm{Res}(G, z_2) = -\frac{H(z_2)}{(z_1-z_2)^2} - \frac{H'(z_2)}{z_1-z_2}$$

$$\mathrm{Res}(G, z_1) = \frac{H(z_1)}{(z_1-z_2)^2}$$

$$\simeq \frac{H(z_2)}{(z_1-z_2)^2} + \frac{H'(z_2)}{z_1-z_2} + \frac{1}{2}H''(z_2)$$

Collinear divergences cancel in the sum!