A multi-step Fitting Approach to the CMSSM

Outline

Problems in CMSSM Constraints:

Exclusion plots of different groups differ
       DM does NOT constrain SUSY masses, if one allows $\tan\beta$ to be free $\rightarrow$ $\tan\beta$ becomes large in large region of parameter space

Tension with g-2, b$\rightarrow$sγ, Bs$\rightarrow\mu\mu$: STRONG correlation between parameters

Fitting problems:

MCMC likely to miss highly correlated regions, especially in pre-generated scans

χ2 minimization quickly stuck in local minima

Solution:

Multistep Fitting Procedure: fit highly correlated parameters first
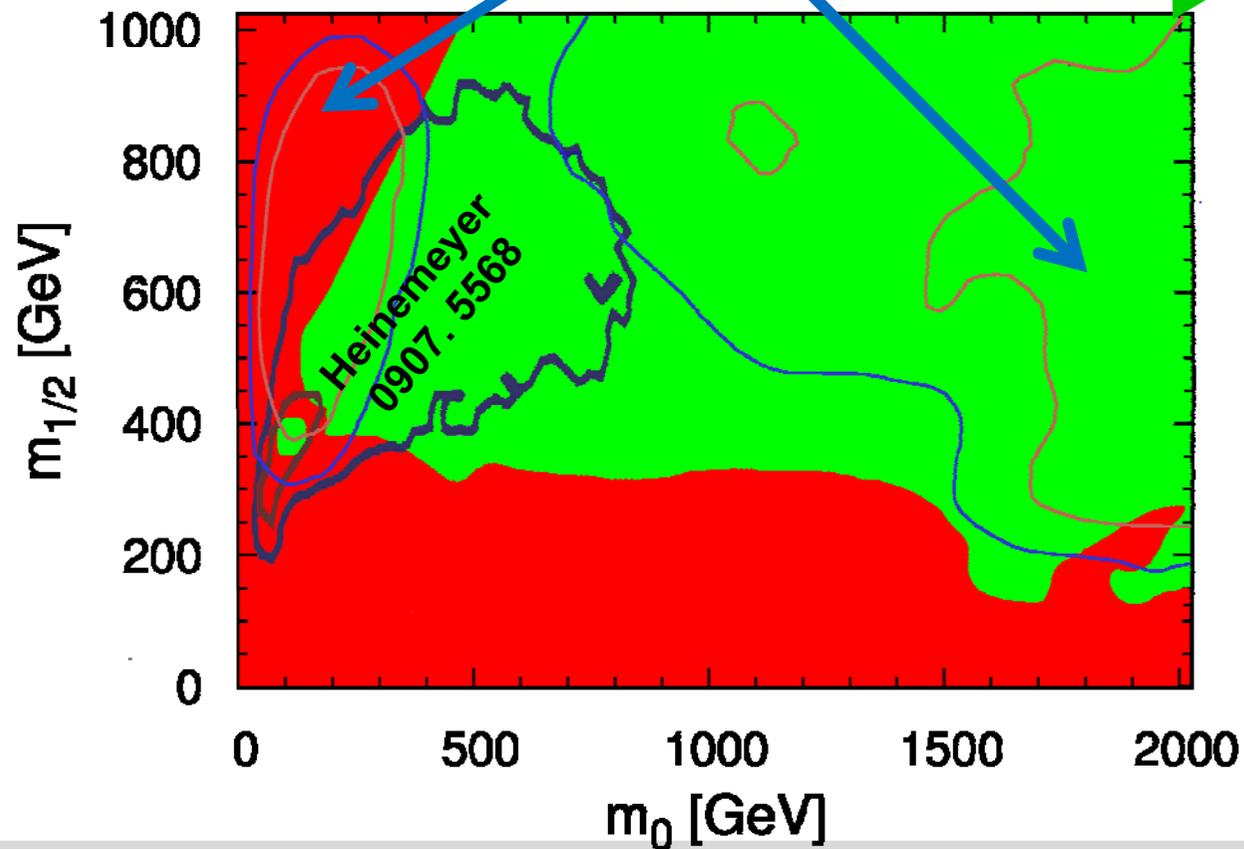
Different approaches → different results?

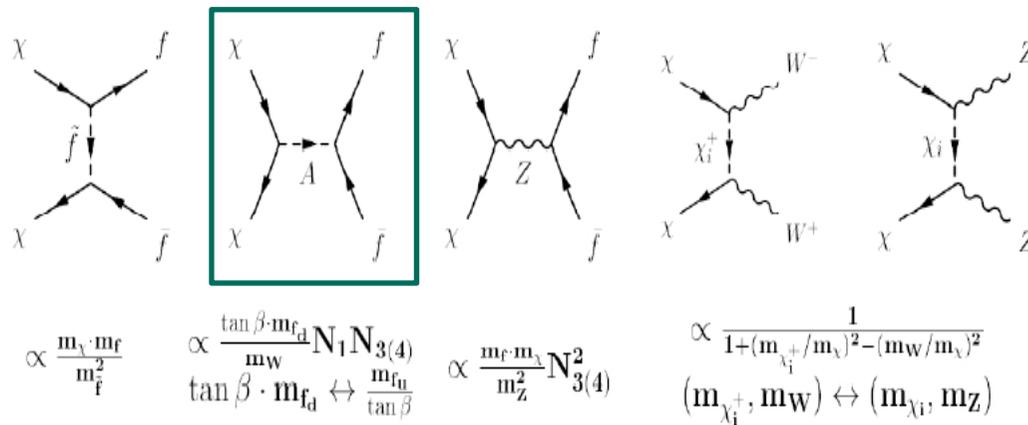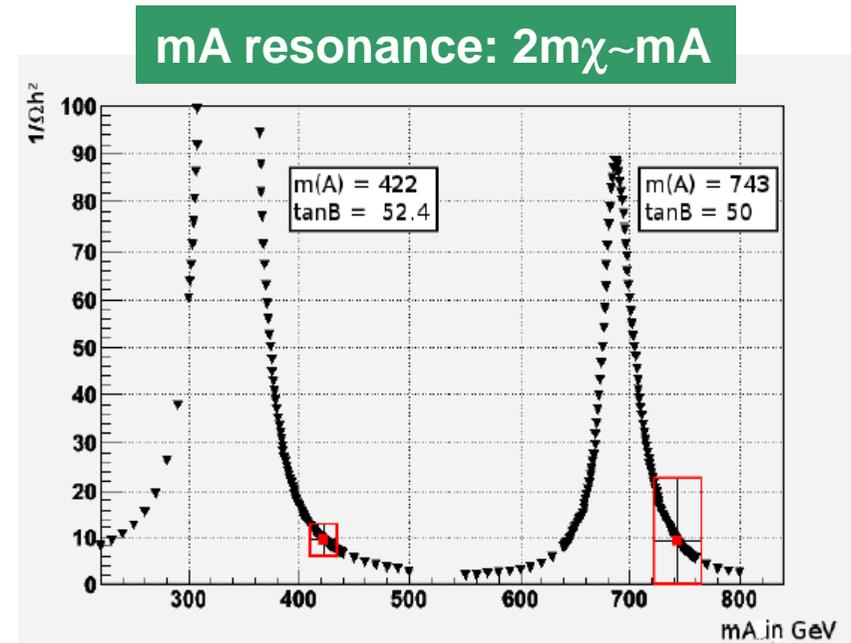Stockholm_0910.3950v3 (genetic alg.)
Roszkowski_0809.3792 (MCMC)
Allanach_0807.4512v2 (Multinest)
Typical allowed regions

Multistep Fits
Green = allowed

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DM constraints

mA resonance: 2mχ~mA

$m(A) = 422$
$tanB = 52.4$

$m(A) = 743$
$tanB = 50$

$$\propto \frac{m_\chi \cdot m_f}{m_{\tilde{f}}^2}$$

$$\propto \frac{\tan\beta \cdot m_{f_d}}{m_W} N_1 N_{3(4)}$$
$$\tan\beta \cdot m_{f_d} \leftrightarrow \frac{m_{f_u}}{\tan\beta}$$

$$\propto \frac{m_f \cdot m_\chi}{m_Z^2} N_{3(4)}^2$$

$$\propto \frac{1}{1+(m_{\chi_i^+}/m_\chi)^2-(m_W/m_\chi)^2}$$
$$(m_{\chi_i^+}, m_W) \leftrightarrow (m_{\chi_i}, m_Z)$$
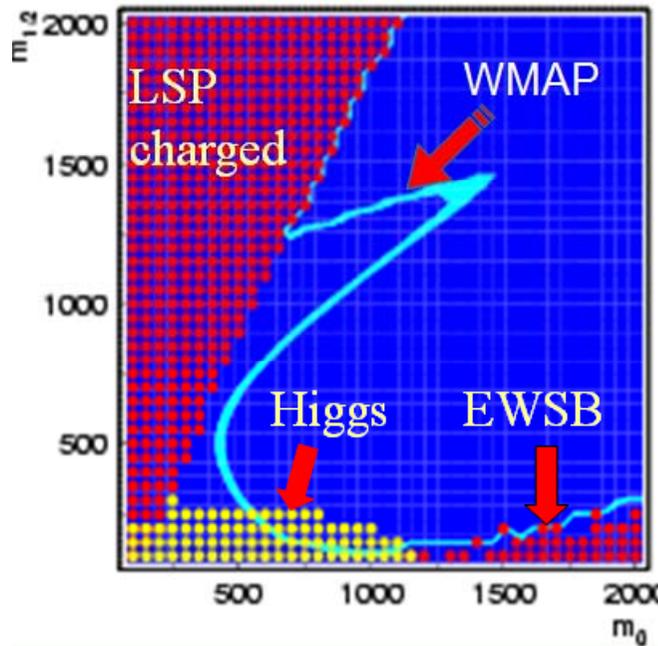
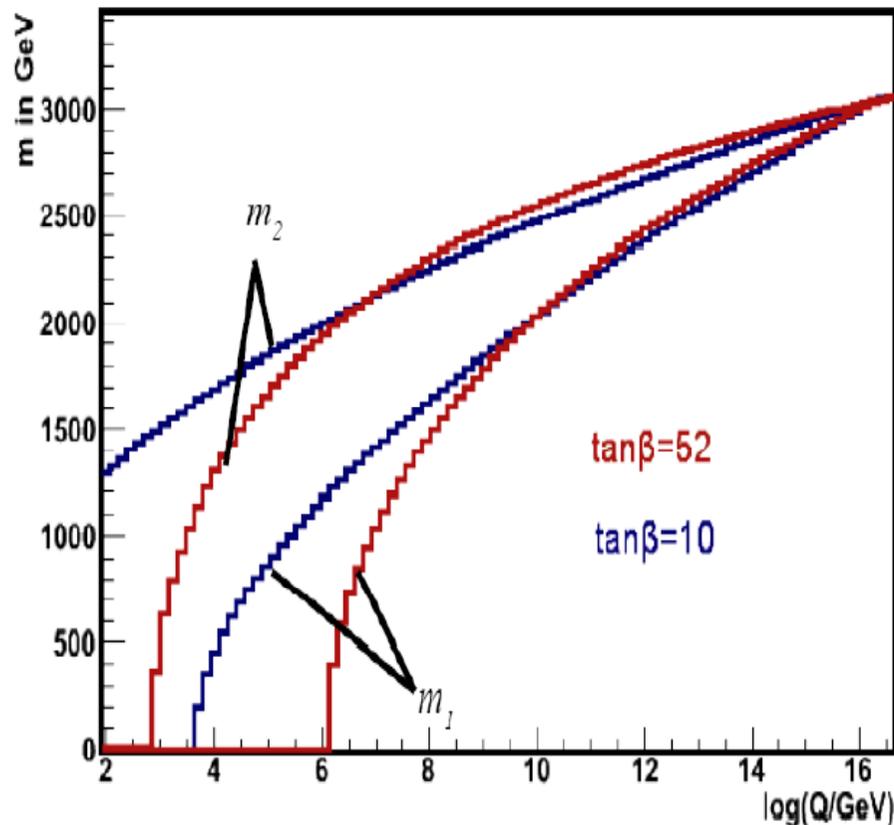**+ co-annihilation**
e.g. $\tilde{\tau}+\chi\rightarrow\tau$

Pseudoscalar Higgs Mass m_A

$$V_{tree}(H_1, H_2) = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2(H_1 H_2 + h.c.)$$

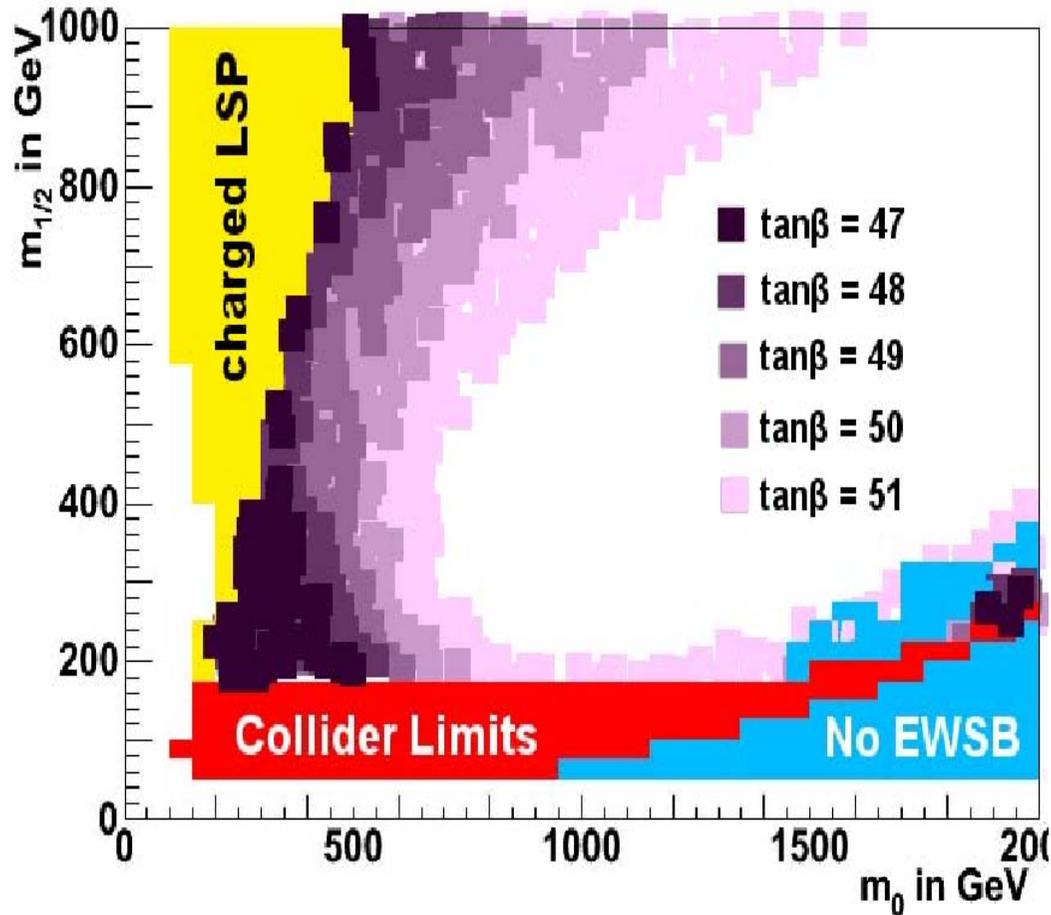$$+ \frac{g^2 + g'^2}{8}(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2$$

CP-odd neutral Higgs A
CP-even charged Higgses H

$$m_A^2 = m_1^2 + m_2^2$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2$$

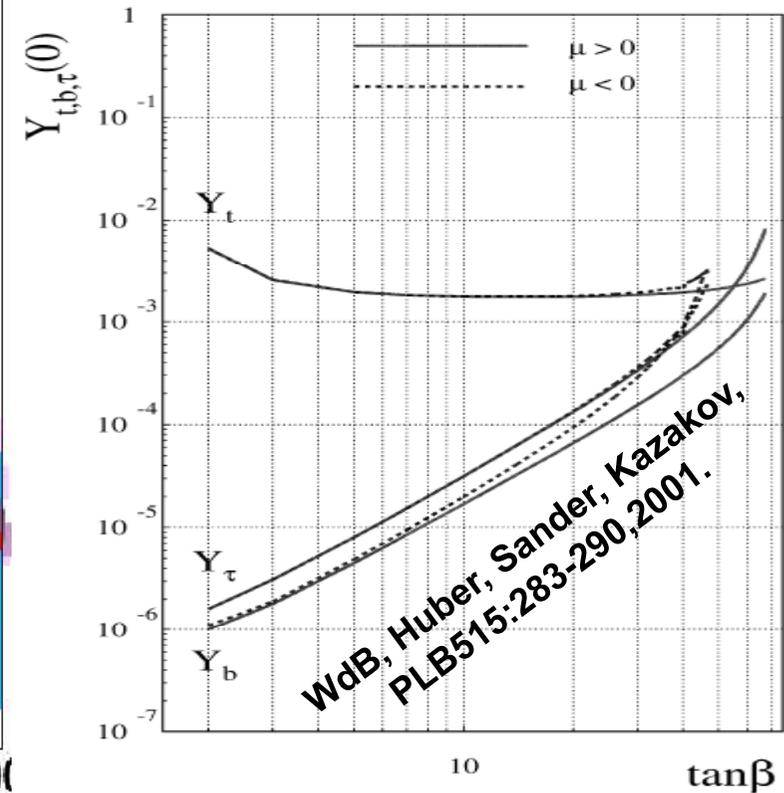m_A becomes ALWAYS small at large tanβ

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DM can be fitted for any SUSY spectrum

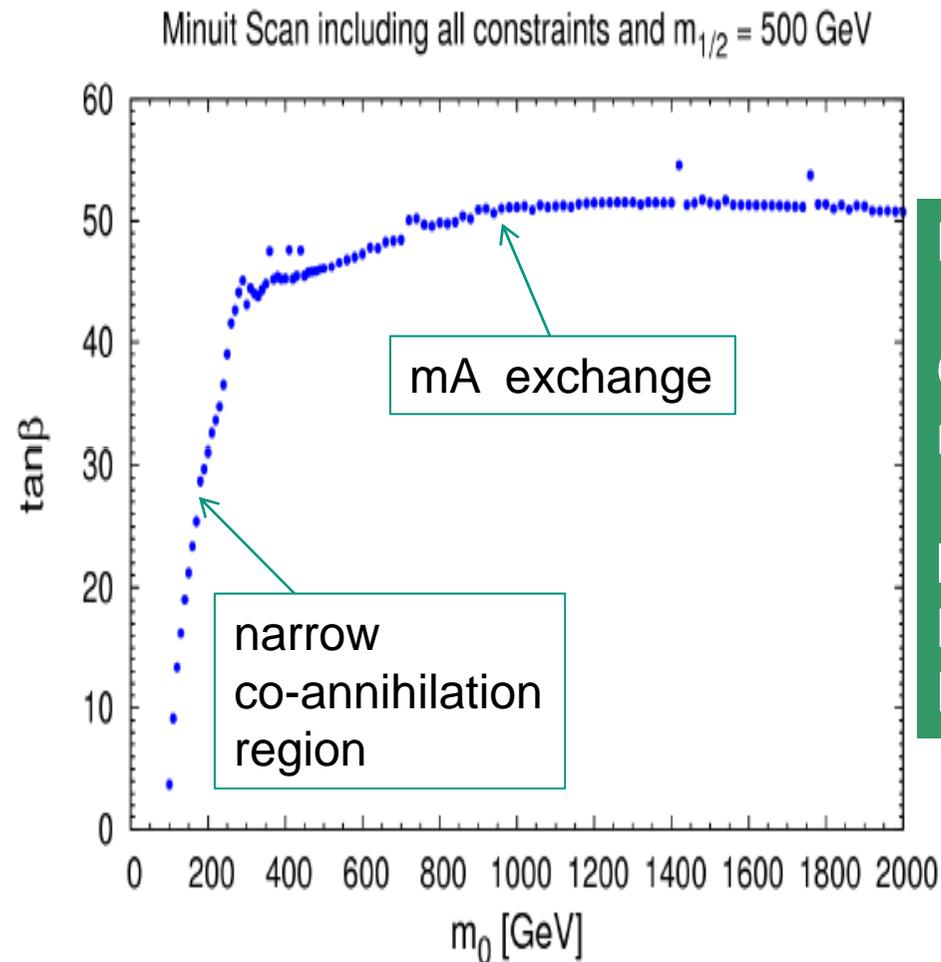**Yukawa couplings**

Tuning tanβ→ change mA→ change funnel region

~Yukawa Unification for large tanβ with correct t,b,τ masses at low energy

Values of m_A and tanb for correct relic density, $\chi^2 = 0$

C.~Beskidt, WdB et al.
arXiv:1008.2150
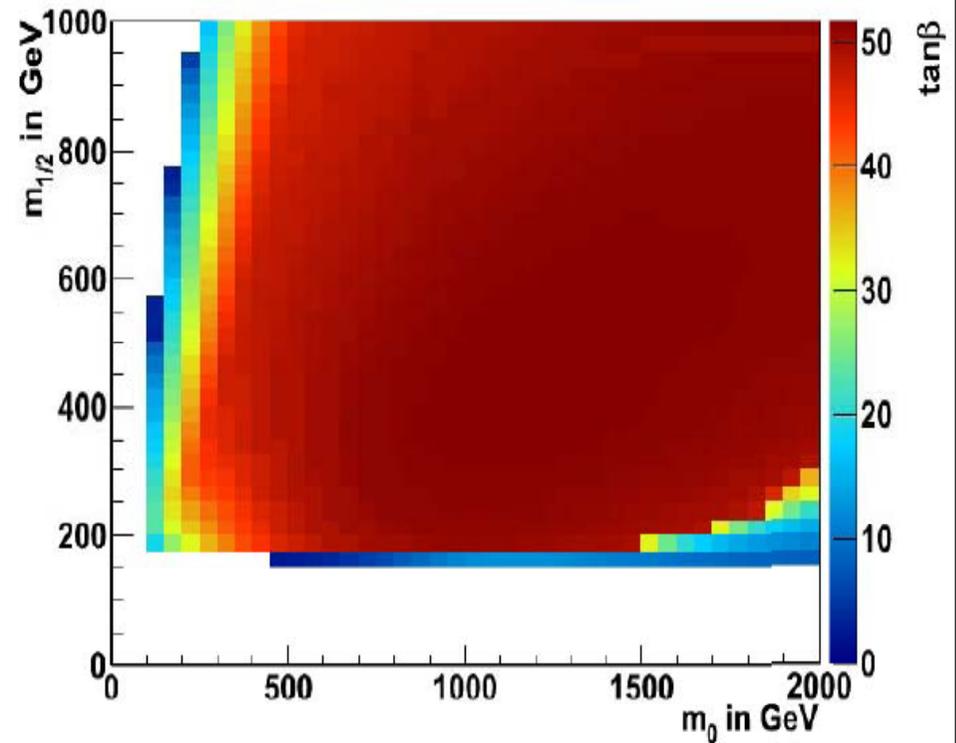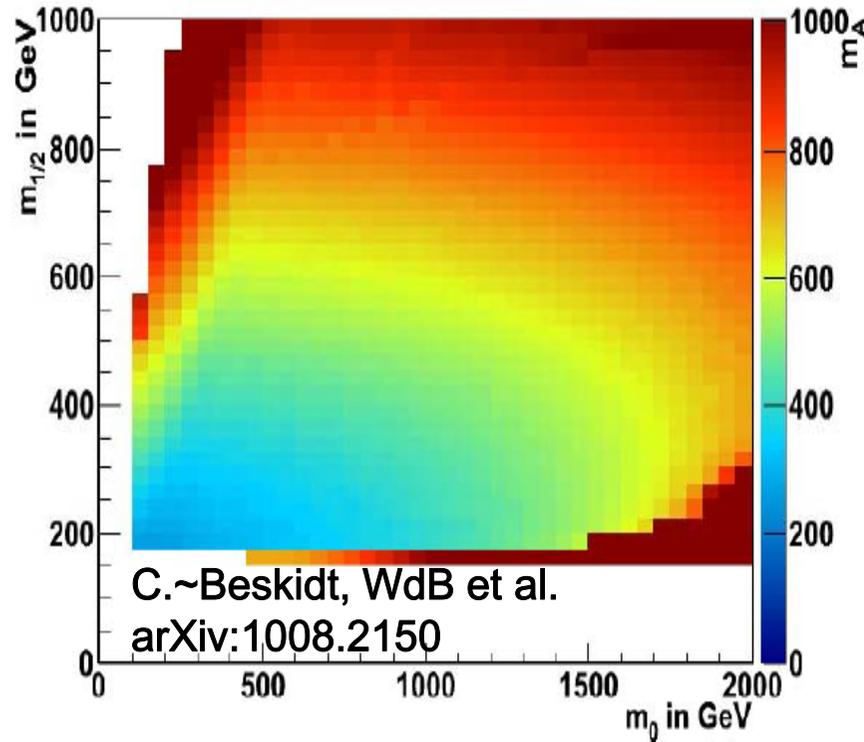
In co-annihilation regions: small tanβ to suppress mA annihilation

Why so strong correlation between m_0 and $\tan\beta$?

Minuit Scan including all constraints and $m_{1/2} = 500$ GeV

mA exchange

narrow co-annihilation region

Main contribution to DMA:

Co-annihilation
mA exchange

If co-annihilation important, have to reduce mA-exchange lowering $\tan\beta$, thus increasing mA
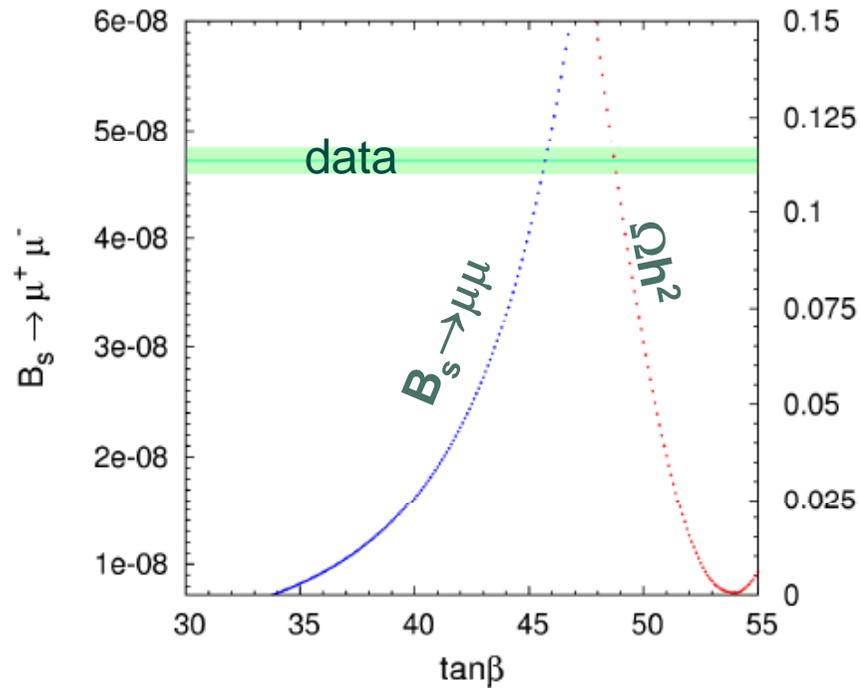
Tension between Ωh^2 and $B_s \to \mu\mu$
(both very sensitive to tanβ)
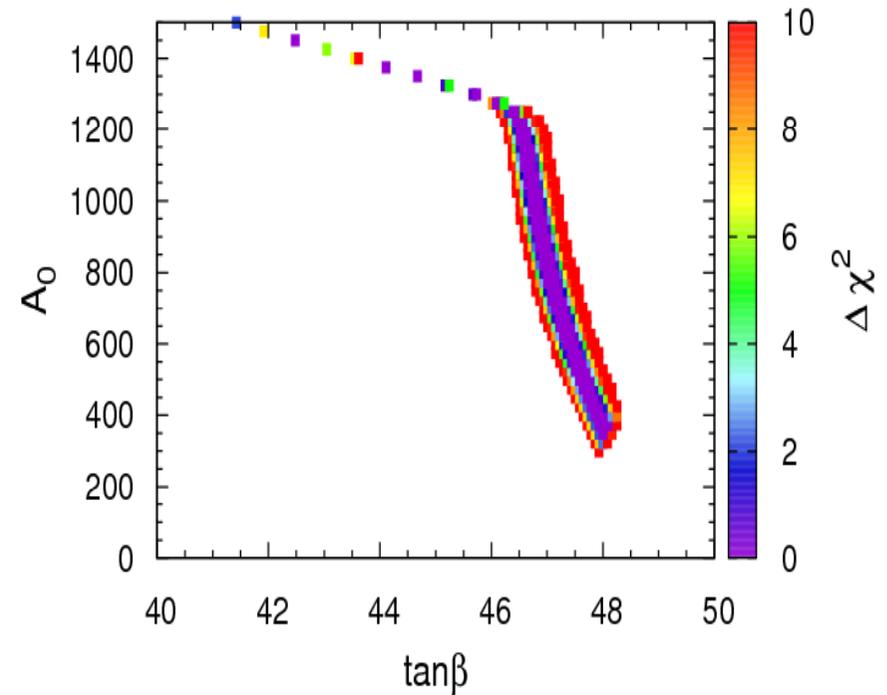
$B_s \to \mu\mu$

Proportional to $\tan^6 \beta$

$BR^{SM}(B_s \to \mu\mu) = 3.5 \cdot 10^{-9}$

$BR^{Ex}(B_s \to \mu\mu) < 4.5 \cdot 10^{-8}$

Minuit Scan including Ωh^2 and $B_s \to \mu^+\mu^-$

VERY steep dependence of Ωh^2 and $B_s \to \mu\mu$ on tan β

VERY strong correlation between A_0 and tan β

Bs→μμ and A₀

Minuit including Ωh^2 and $B_s \to \mu^+ \mu^-$

Main SYSY contribution
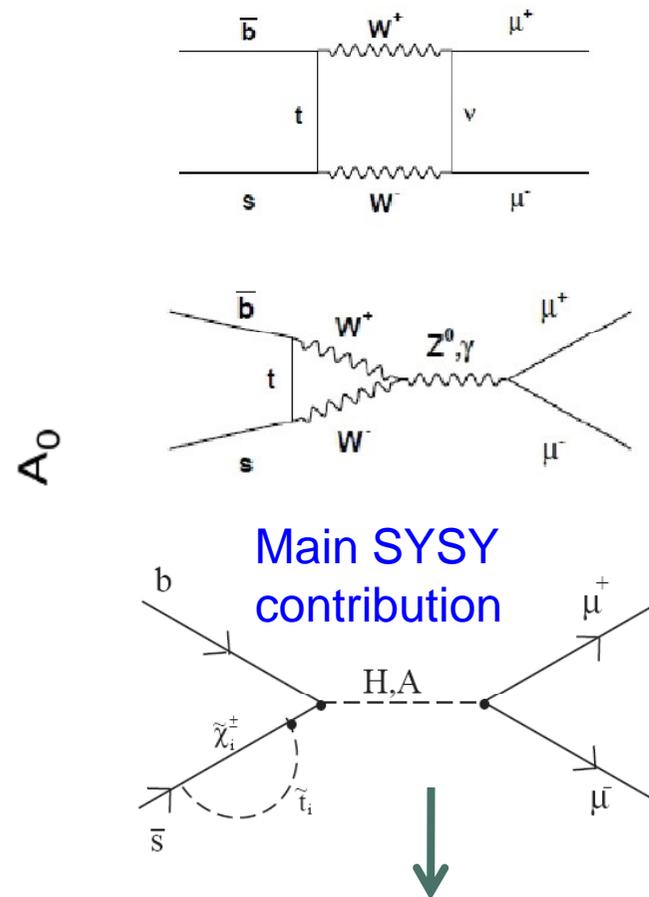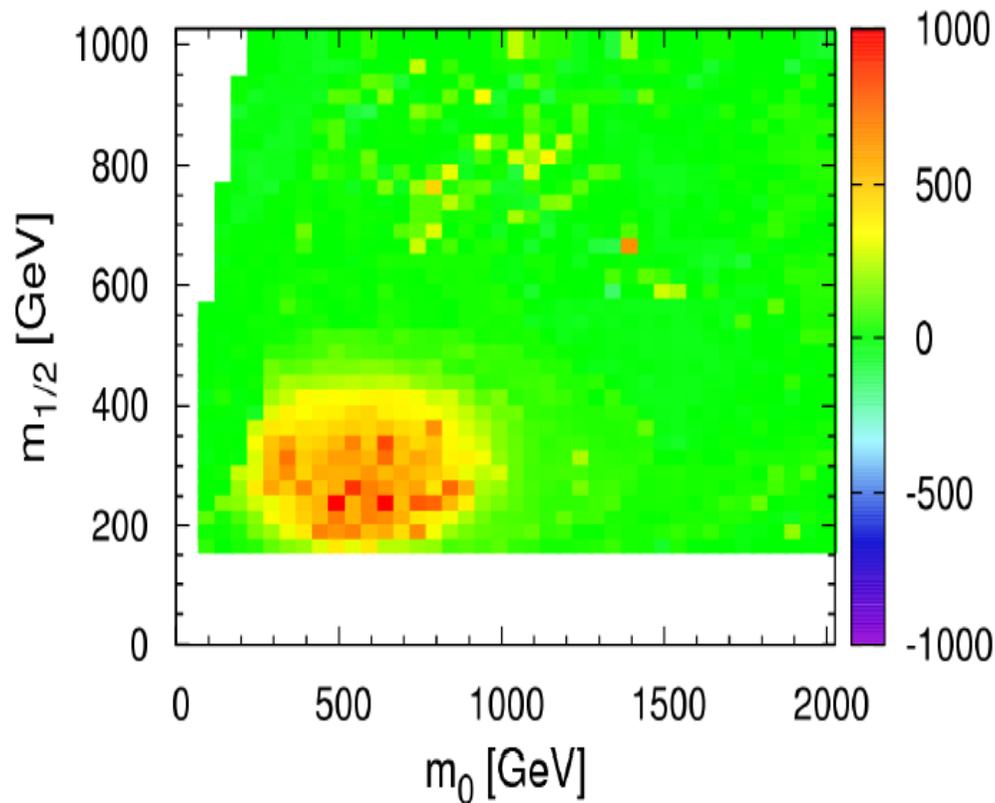
A₀ only needed to suppress Bs→μμ at large tan β, when SUSY particles light (equalizes stop1 and stop2 masses

$$\frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \left(\frac{\tan^3 \beta}{4\sin^2\theta_W} \right) \left(\frac{m_b m_\mu m_t \mu}{M_W^2 M_A^2} \right) \frac{\sin 2\theta_{\tilde{t}}}{2} \left(\frac{m_{\tilde{t}_1}^2 \log\left[\frac{m_{\tilde{t}_1}^2}{\mu^2}\right]}{\mu^2 - m_{\tilde{t}_1}^2} - \frac{m_{\tilde{t}_2}^2 \log\left[\frac{m_{\tilde{t}_2}^2}{\mu^2}\right]}{\mu^2 - m_{\tilde{t}_2}^2} \right)$$

How to handle large correlations?

MCMC:

recommendation in cosmomc:
The proposal density can use a supplied covariance matrix:

use one if possible

It will make correlated steps in correlated variables,so
If m0 steps in one direction, then tanb in a correlated direction
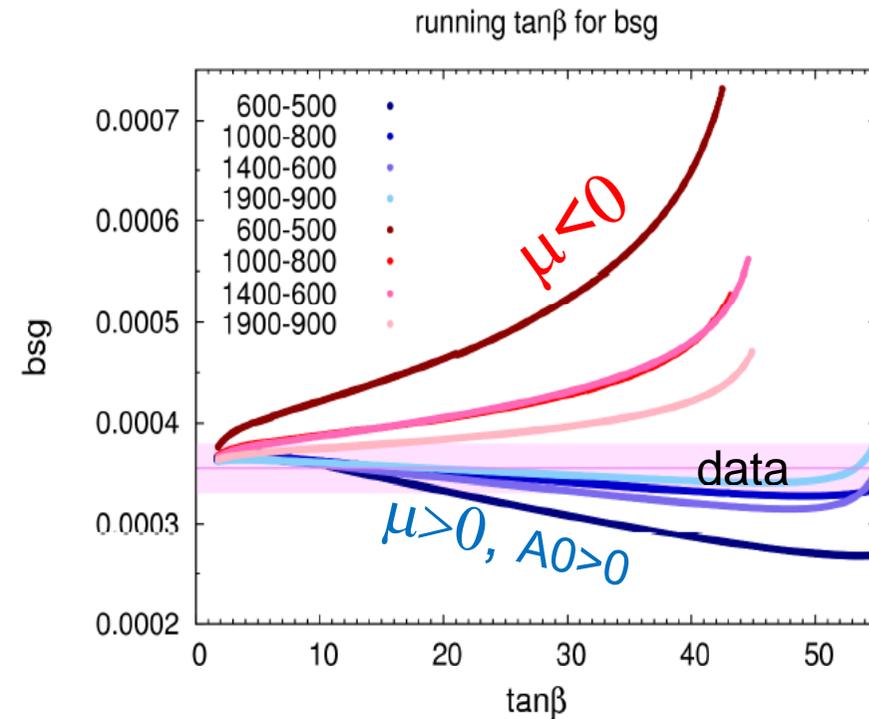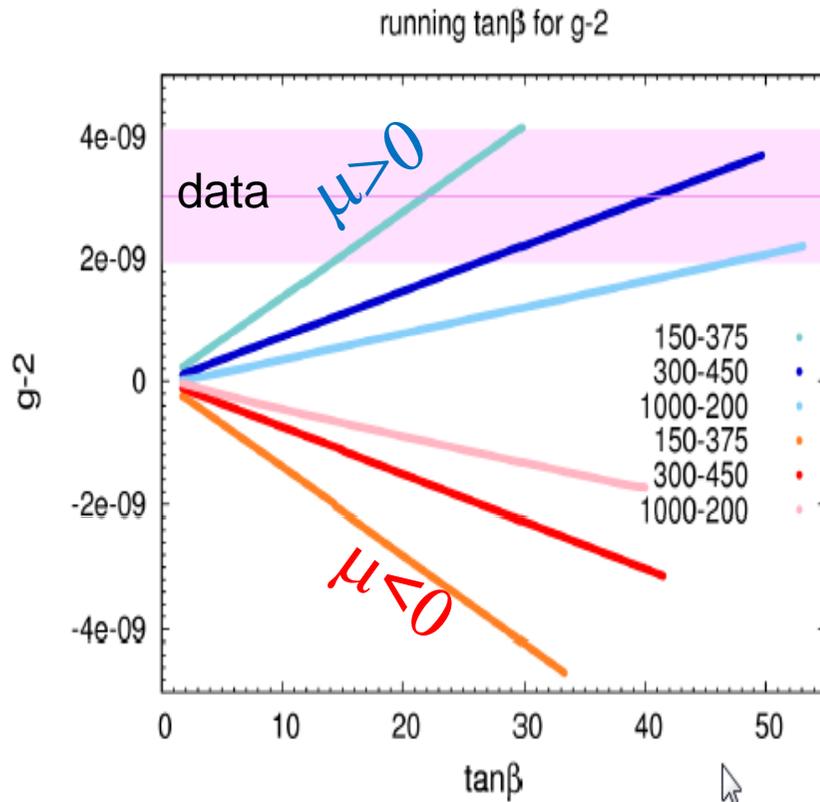followed by A0 in a correlated direction.
Very efficient, much less prior dependence
http://cosmologist.info/cosmomc/readme.html#Kosowsky)**.**

Frequentist $\chi 2$ fitting:

Minimize in steps: first fit parameters with highest correlation first

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Tension between b→sγ and g-2

running tanβ for g-2

data

μ>0

μ<0

150-375
300-450
1000-200
150-375
300-450
1000-200

running tanβ for bsg

600-500
1000-800
1400-600
1900-900
600-500
1000-800
1400-600
1900-900

μ<0

data

μ>0, A0>0

g-2 requires μ>0, b→sγ prefers μ<0 Can only be compensated by large A0>0. Fortunately Bs→μμ needs A_0>0 too! And both need it for light SUSY masses

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20 years ago…

However, it was recently suggested that in the theoretical calculation one should use the running c-quark mass in the ratio m_c/m_b, which reduces this ratio from 0.29 to 0.22 [14]. The SM value for $b \to X_s\gamma$ increases from $(3.35 \pm 0.30) \times 10^{-4}$ to $(3.73 \pm 0.30) \times 10^{-4}$ in this case. This value is 1.7σ above the most recent world average of $(2.96 \pm 0.46) \times 10^{-4}$, which is the average from CLEO $((2.85 \pm 0.35_{stat} \pm 0.22_{sys}) \times 10^{-4})$ [15], ALEPH $((3.11 \pm 0.80_{stat} \pm 0.72_{sys}) \times 10^{-4})$ [16] and BELLE
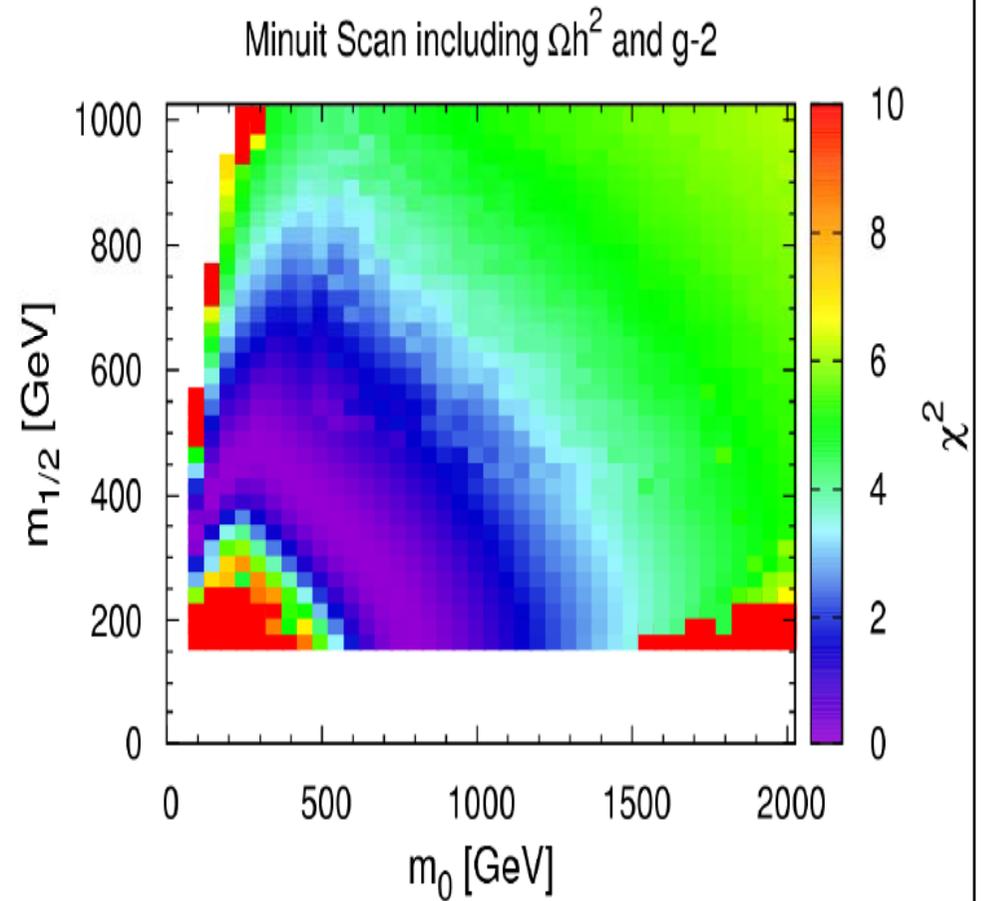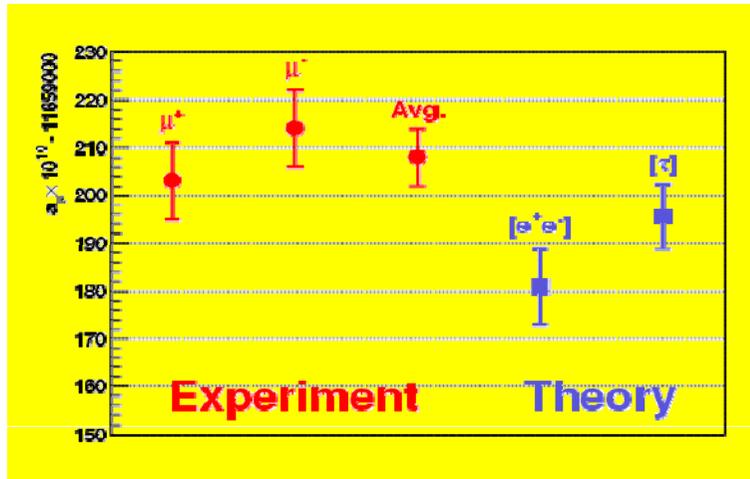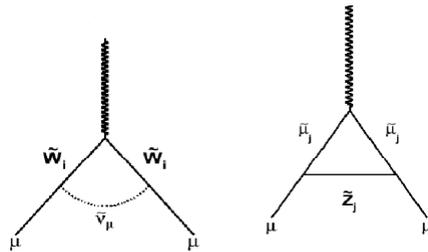
$BR(b \to X_s\gamma) = (2.96 \pm 0.46) \times 10^{-4}$ [15–17] and
$\Delta a_\mu = (43 \pm 16) \times 10^{-10}$ [1], which shows once
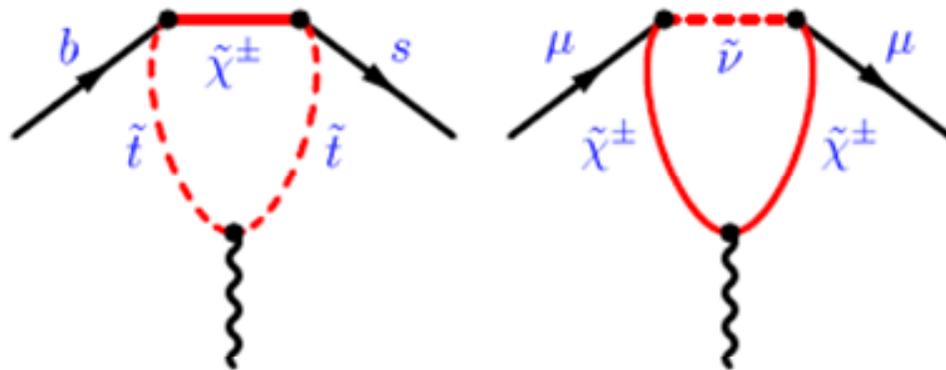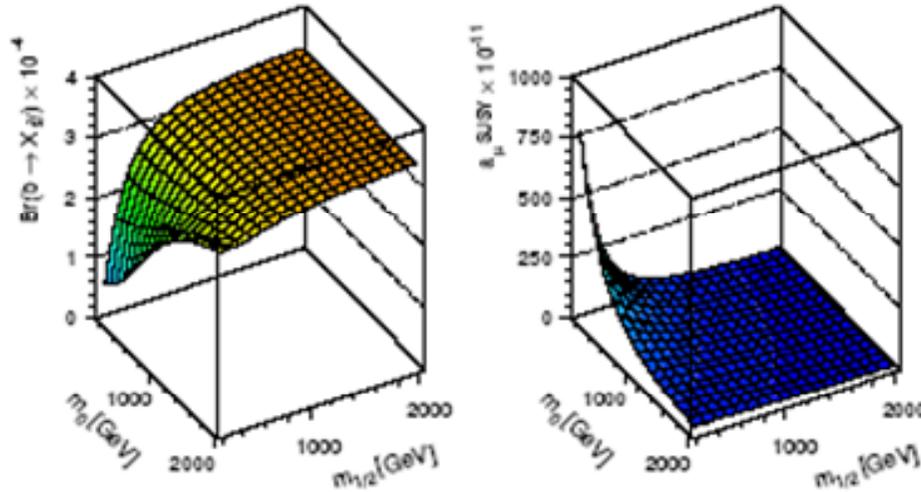
SM: 3.73.10⁻⁴
EXP: 2.96.10⁻⁴

2001: b→sγ <SM, as expected for μ>0 from g-2 and A₀=0

g-2 and tanβ

Minuit Scan including Ωh^2 and g-2

g-2 prefers light masses, but long tail to large SUSY masses allowed

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Beskidt, dB et al.1008.2150, PLB 2011

$mA{\to}\tau\tau$
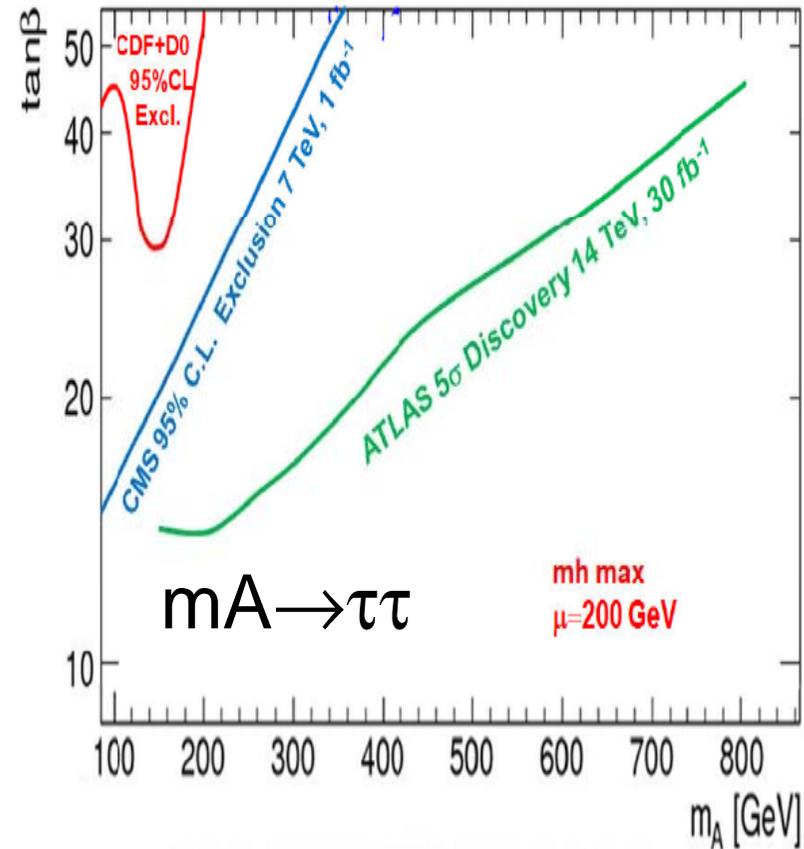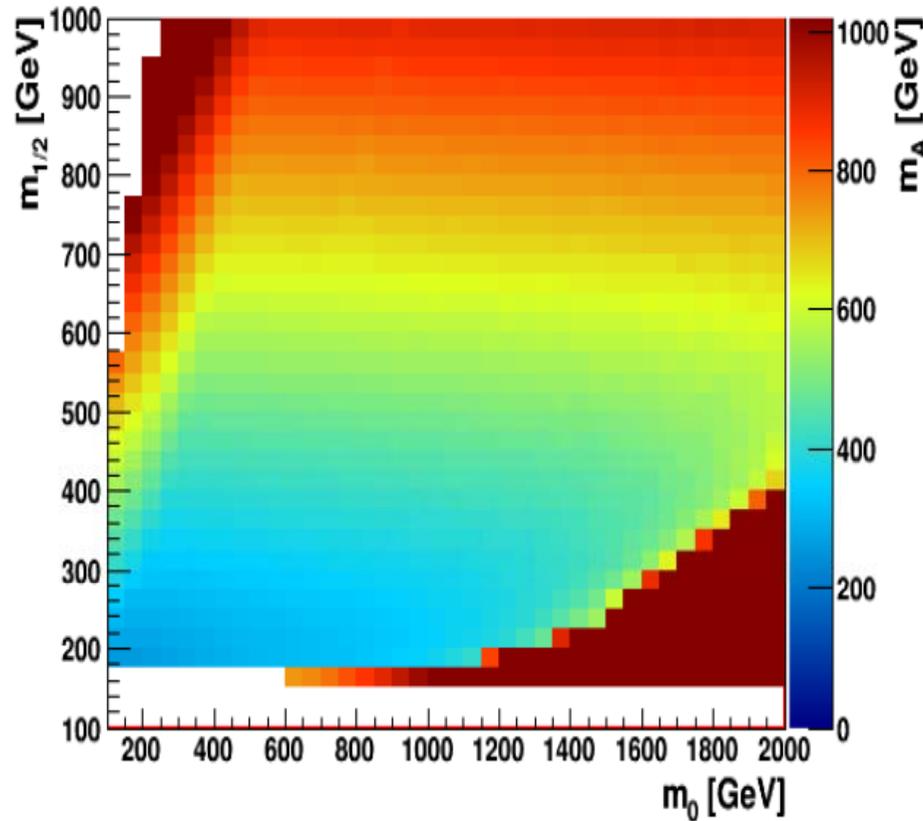
DM constraint yields relatively small mA and large mA cross section

mA cross sections ∝ tanβ²
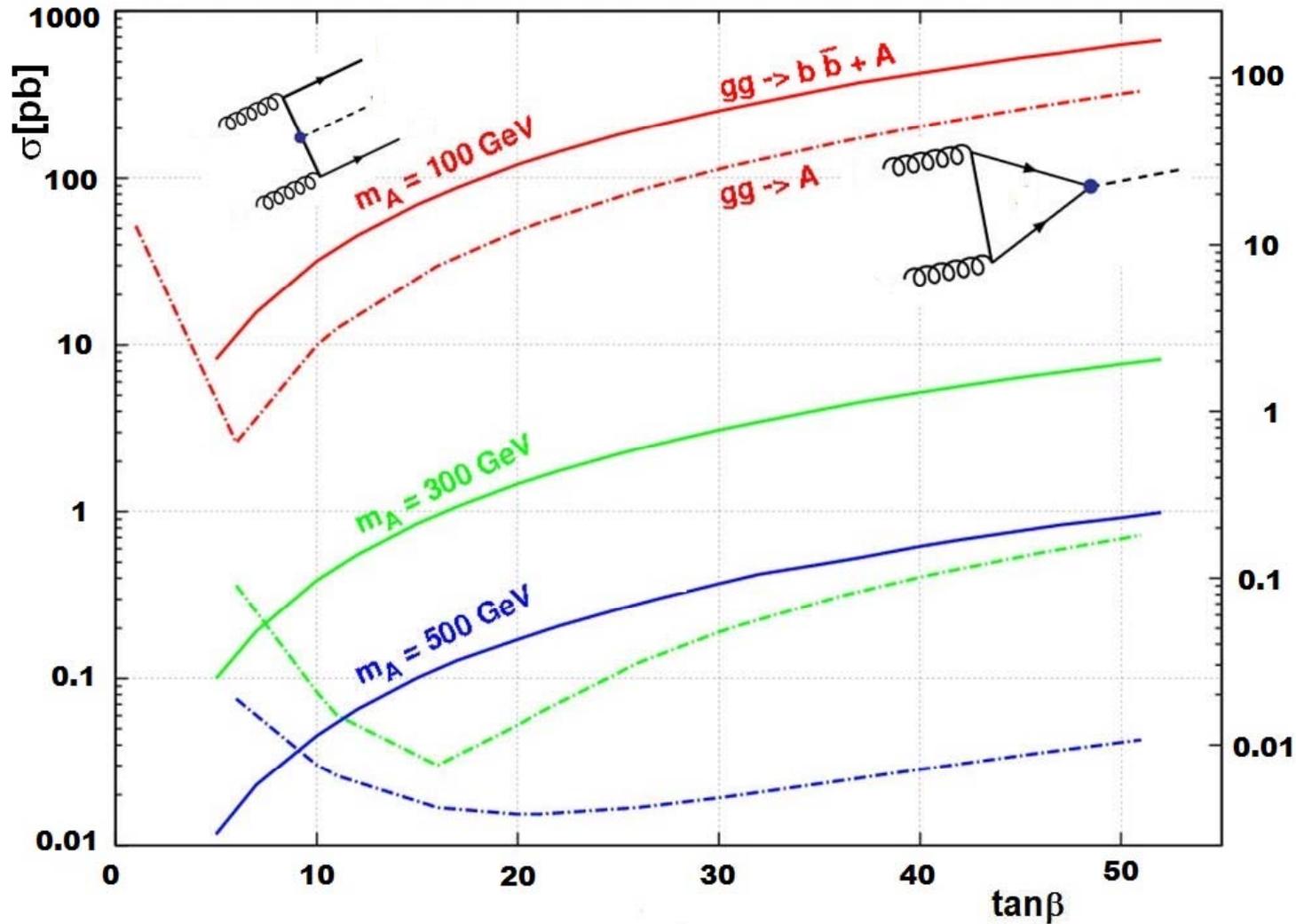
Beskidt, dB et al.1008.2150, PLB 2011
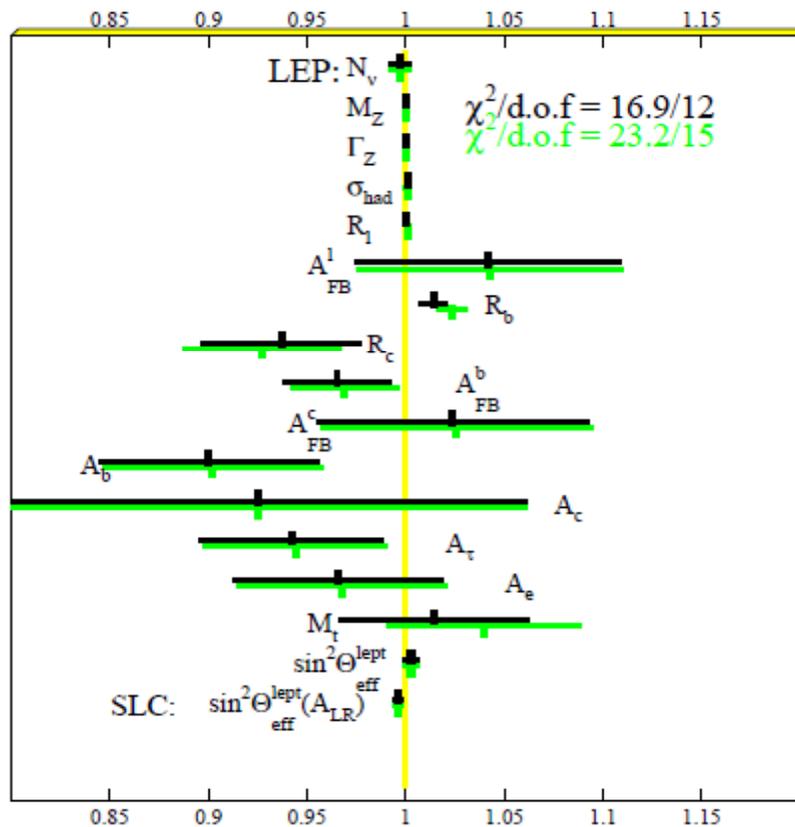
Constraints considered

- Find consistent points by minimizing χ^2 function
- χ^2 function in general defined as $\chi^2 = \left(\dfrac{x_{model} - x_{exp}}{\sigma_{exp}} \right)^2$
- χ^2 function includes
  - Relic Density Ωh^2
  - Higgs mass m_h
  - Muon g-2
  - $b \to s\gamma$
  - $B_s \to \mu\mu$
  - $B \to \tau\nu$

  Variables calculated with MicrOMEGAs 2.4

- Exclusion from the direct dark matter experiment (DDM)

Why leaving out all LEP I data?

Data / MSSM($\tan\beta = 1.6$)
Data / SM

$\chi^2/\text{d.o.f} = 16.9/12$
$\chi^2/\text{d.o.f} = 23.2/15$

Data / MSSM($\tan\beta = 50$)
Data / SM

$\chi^2/\text{d.o.f} = 18.4/12$
$\chi^2/\text{d.o.f} = 23.2/15$
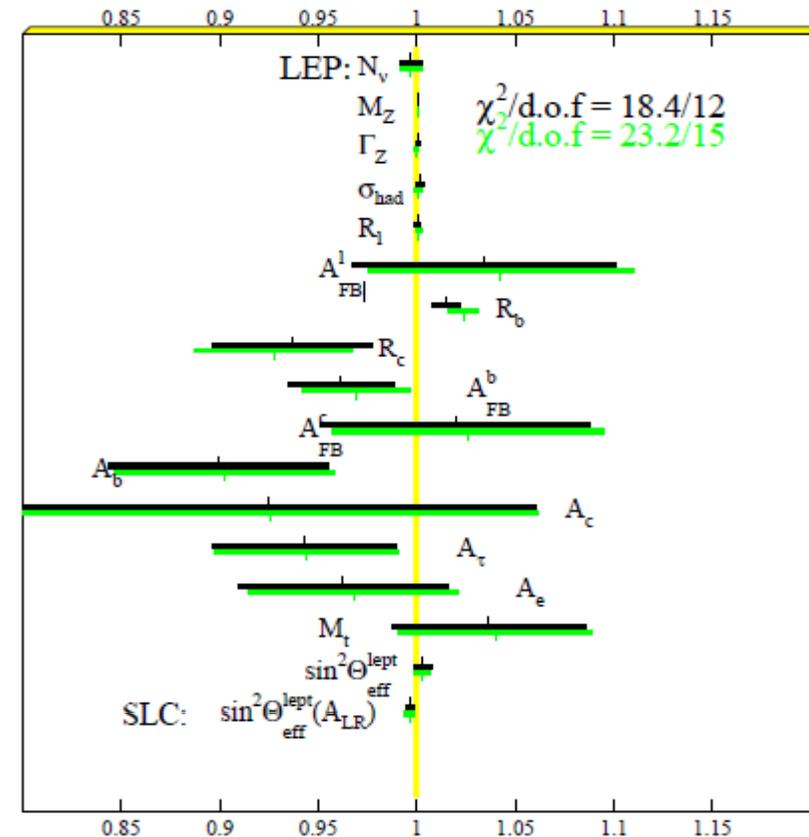
No significant improvement in probability if one compares SM and MSSM

Fitting Approach

Minimization:

Use either MCMC in multi-step approach or use

Minuit in two steps:

gradient search (SIMPLEX-option) to avoid derivatives and local min.
Precise minimum (MIGRAD-option) using derivatives
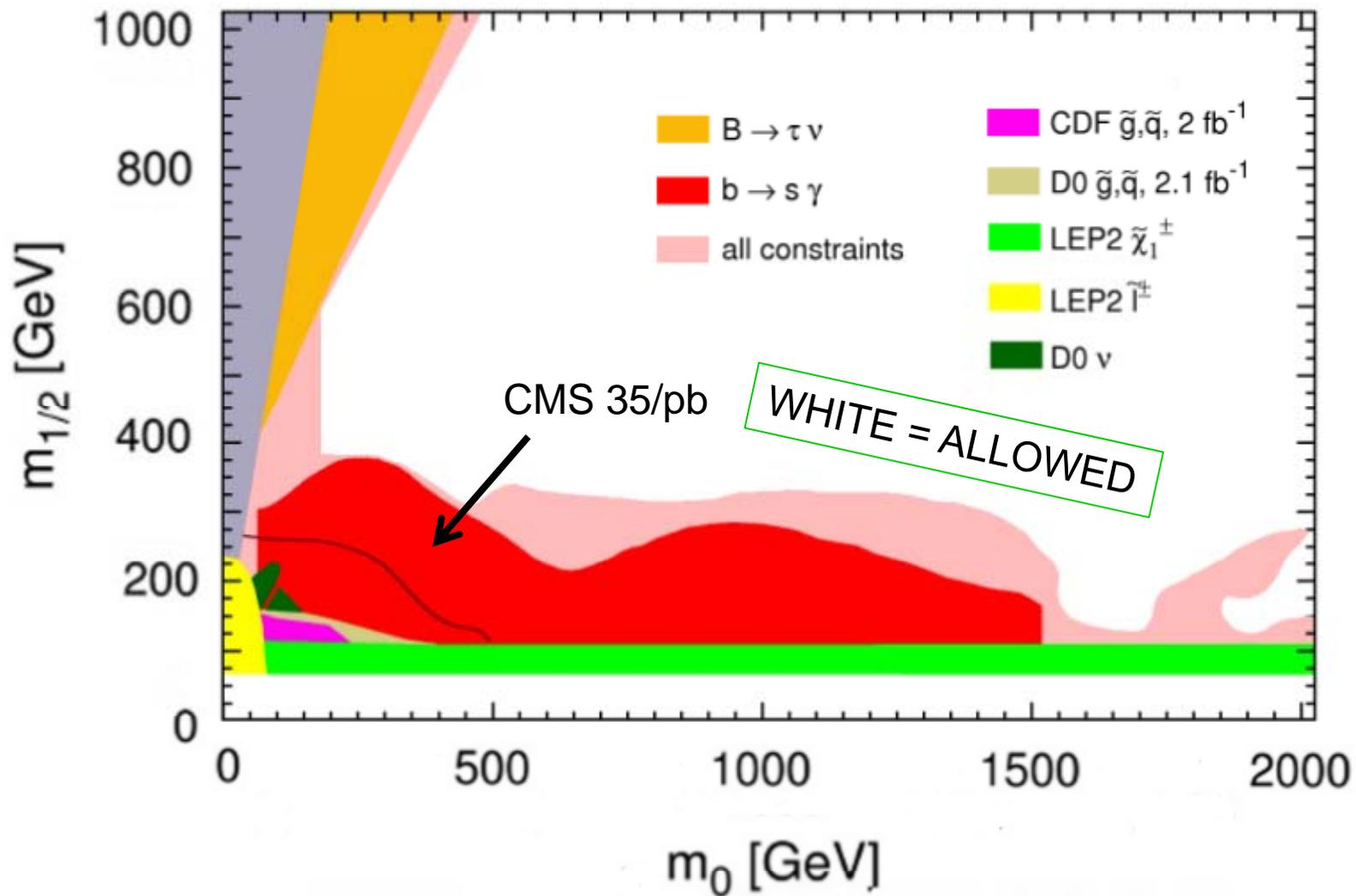
Fit strongly correlated parameters ($\tan\beta$ and A_0) FIRST for given $m_0, m_{1/2}$.
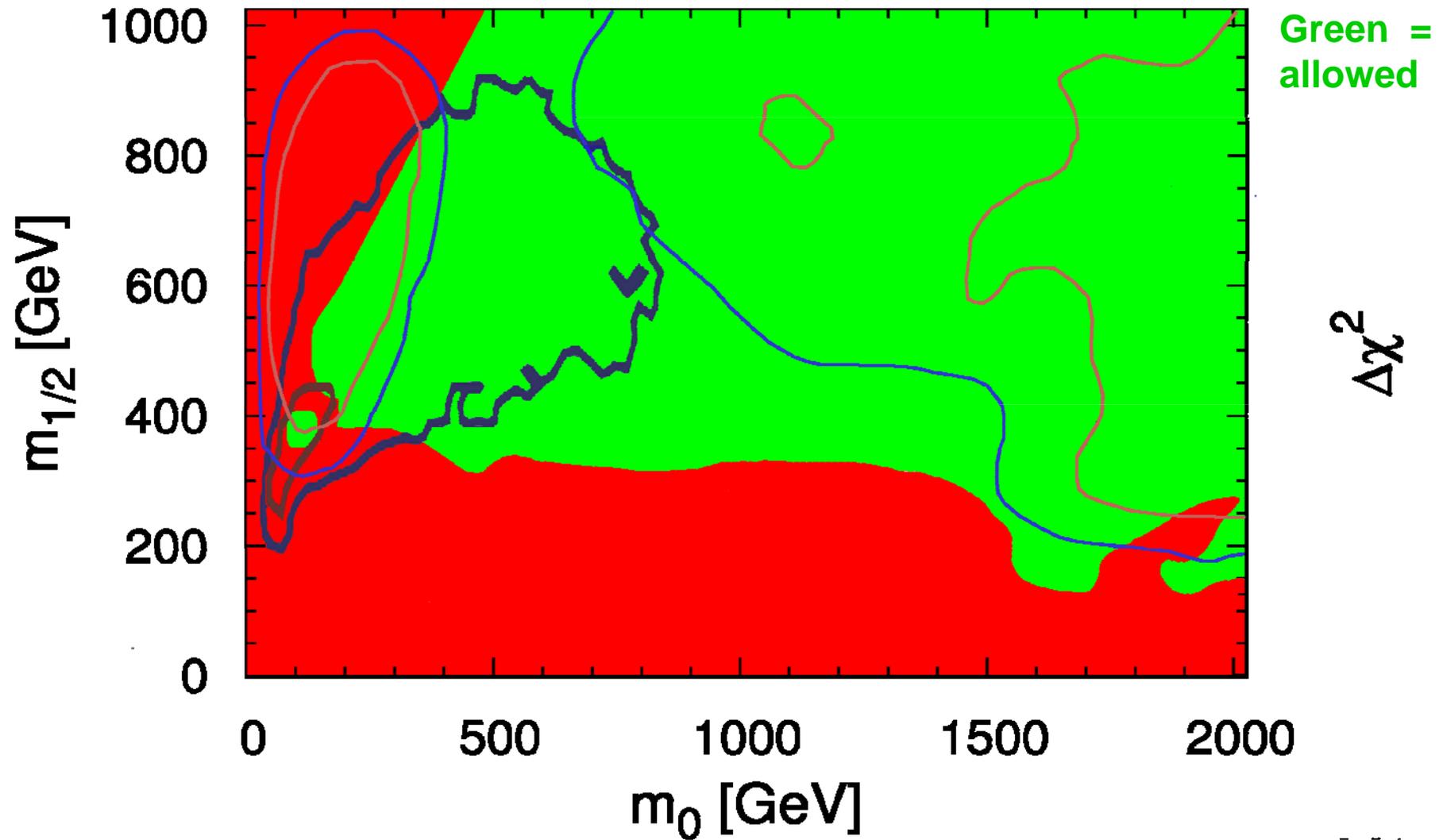
Marginalize or profile nuisance parameters

Repeat for every $m_0, m_{1/2}$.

Very fast. Complete scan in < 1 day

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Result: very flat chi2 for large masses

Green = allowed

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Conclusion

Strong correlations between at least 3 of the 4 CMSSM parameters requires careful fitting strategies.

MCMC or other stepping techniques:
first build a correlation matrix between the parameters

OR: multi-step strategy, i.e. fits strongly correlated parameters first.
Works efficiently for all: Bayesian or frequentist (chi2) and improves scanning efficiency for all scanning techniques (MCMC, Multinest, Genetic Algorithm)