Bayesian SUSY fits to face LHC data

Leszek Roszkowski

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Outline

- Bayesian approach
- CMSSM, CNMSSM, NUHM, prior dependence
- case study: ATLAS SU3 benchmark point
- impact of additional info
- posterior vs profile likelihood
- issues to resolve confronting LHC data
- summary

A conjecture

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...or abandoned

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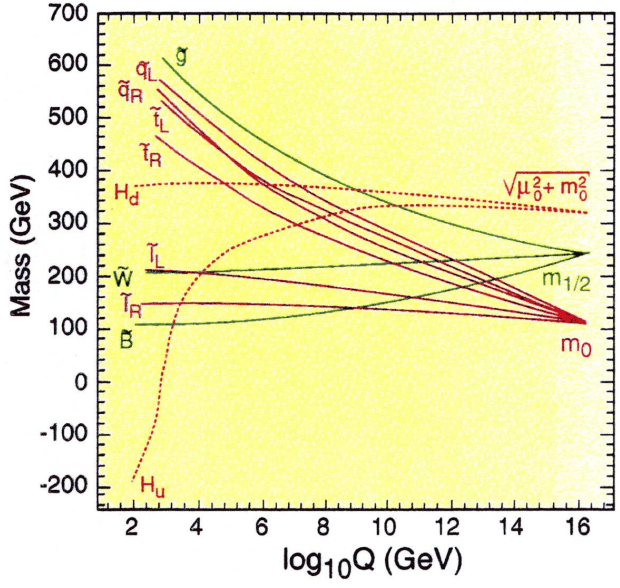

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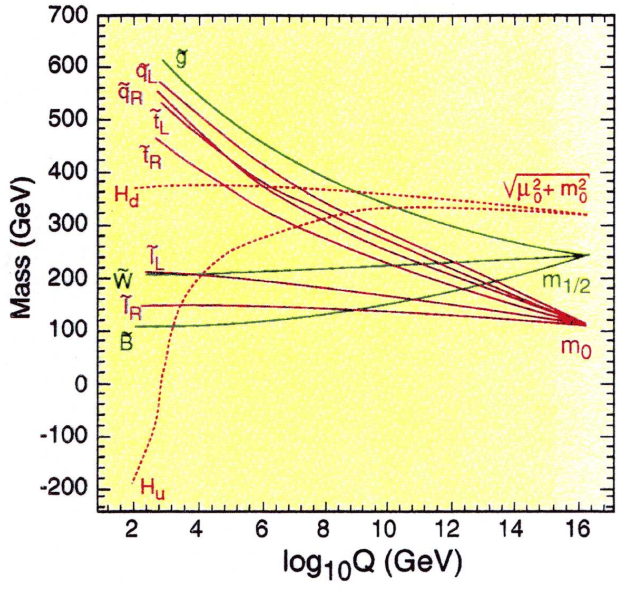

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- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
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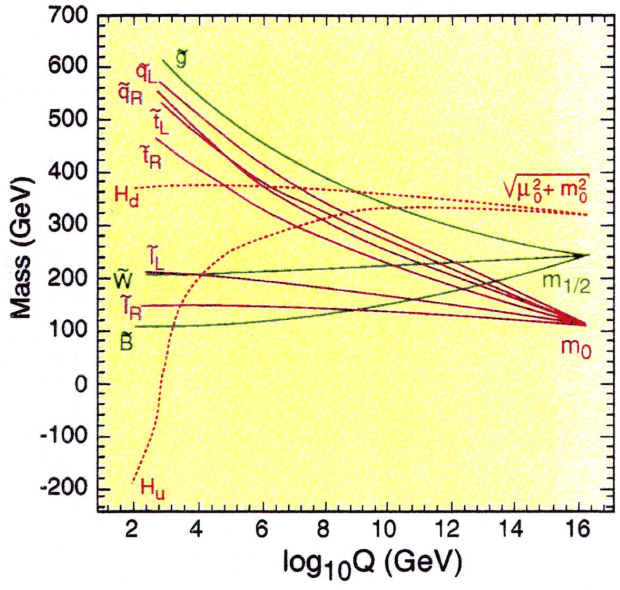

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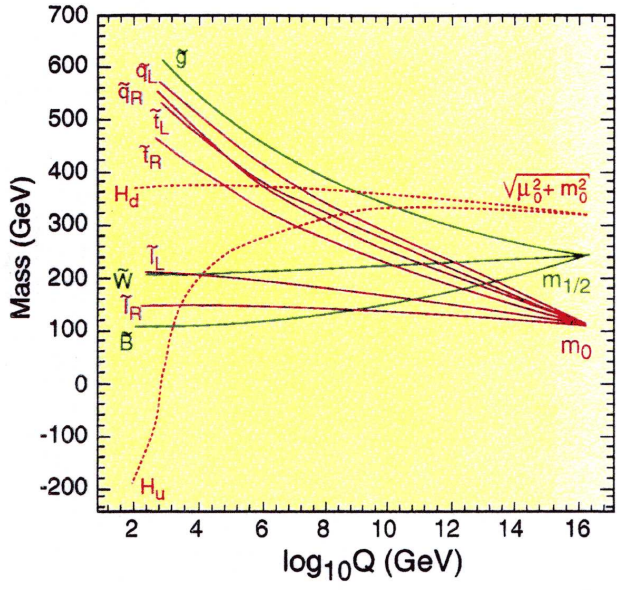

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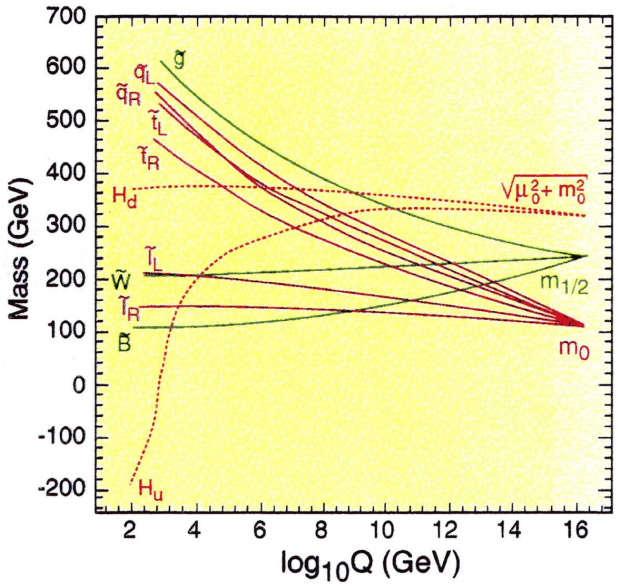

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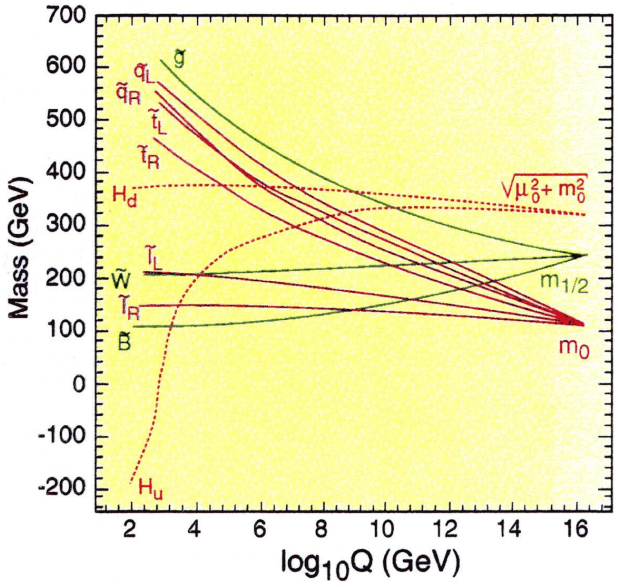

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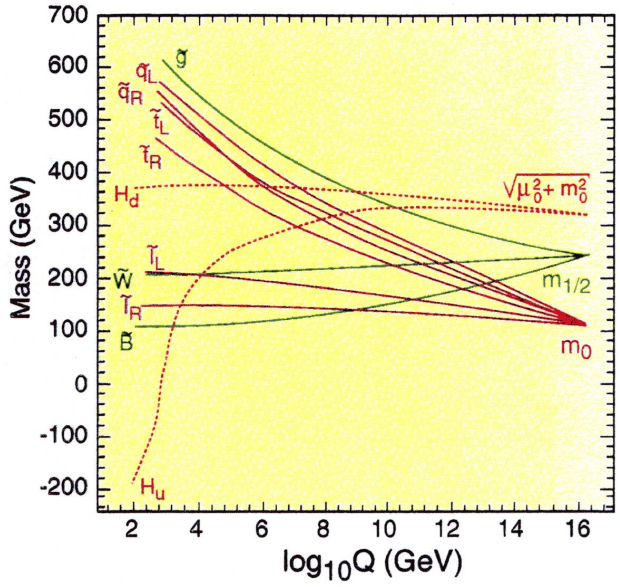

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some useful mass relations:

- bino: $m_\chi \simeq 0.4 m_{1/2}$
- gluino $\tilde{g}$: $m_{\tilde{g}} \simeq 2.7 m_{1/2}$
- supersymmetric tau (stau) $\tilde{\tau}_1$: $m_{\tilde{\tau}_1} \simeq \sqrt{0.15 m^2_{1/2} + m^2_0}$

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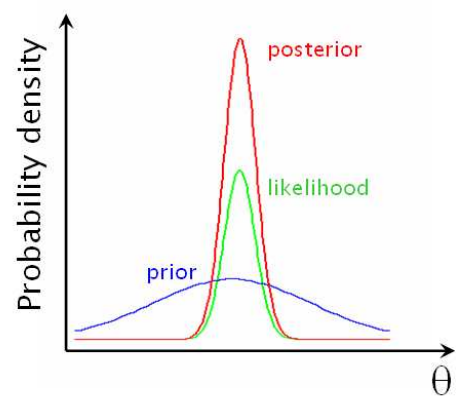

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- $p(d|\xi) = \mathcal{L}$: likelihood
- $\pi(\theta, \psi)$: prior pdf
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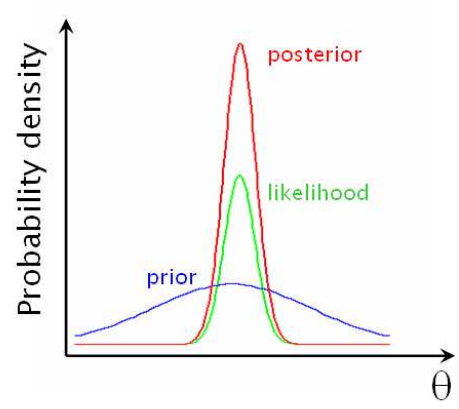

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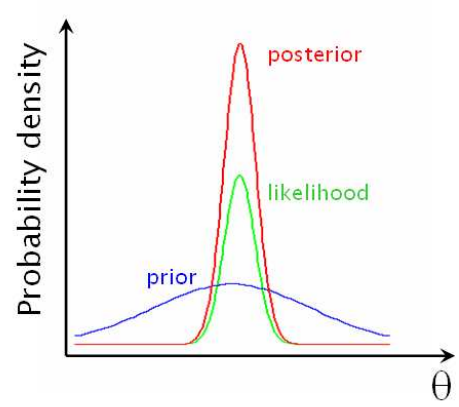

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- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

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SM (nuisance) parameter	Mean	Error
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M_t	172.6 GeV	1.4 GeV
$m_b(m_b)^{\overline{MS}}$	4.20 GeV	0.07 GeV
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Derived observable	Mean μ	Errors σ (expt)	τ (th)
M_W	$\mathbf{80.398\,GeV}$	$\mathbf{25\,MeV}$	$\mathbf{15\,MeV}$
$\sin^2\theta_{\rm eff}$	$\mathbf{0.23153}$	$\mathbf{16 \times 10^{-5}}$	$\mathbf{15 \times 10^{-5}}$
$\delta a_\mu^{\rm SUSY} \times 10^{10}$	$\mathbf{29.5}$	$\mathbf{8.8}$	$\mathbf{1}$
$\mathbf{BR}(\bar{\mathrm{B}} \to \mathrm{X_s}\gamma) \times 10^4$	$\mathbf{3.55}$	$\mathbf{0.26}$	$\mathbf{0.21}$
ΔM_{B_s}	17.33	0.12	4.8
$\Omega_\chi h^2$	0.1099	0.0062	$\mathbf{0.1\,\Omega_\chi h^2}$

take w/o error: $M_Z = \mathbf{91.1876(21)\,GeV}$, $G_F = \mathbf{1.16637(1) \times 10^{-5}\,GeV^{-2}}$

Experimental Limits

Derived observable	upper/lower limit	Constraints $\xi_{\rm lim}$	τ (theor.)
$\mathbf{BR(B_s \to \mu^+\mu^-)}$	UL	$\mathbf{1.5 \times 10^{-7}}$	14%
$\boldsymbol{m_h}$	LL	$\mathbf{114.4\,GeV\ (91.0\,GeV)}$	$\mathbf{3\,GeV}$
$\boldsymbol{\zeta_h^2 \equiv g^2_{ZZh}/g^2_{ZZH_{\rm SM}}}$	UL	$\boldsymbol{f(m_h)}$	3%
$\boldsymbol{m_\chi}$	LL	$\mathbf{50\,GeV}$	5%
$\boldsymbol{m_{\chi_1^\pm}}$	LL	$\mathbf{103.5\,GeV\ (92.4\,GeV)}$	5%
$\boldsymbol{m_{\tilde{e}_R}}$	LL	$\mathbf{100\,GeV\ (73\,GeV)}$	5%
$\boldsymbol{m_{\tilde{\mu}_R}}$	LL	$\mathbf{95\,GeV\ (73\,GeV)}$	5%
$\boldsymbol{m_{\tilde{\tau}_1}}$	LL	$\mathbf{87\,GeV\ (73\,GeV)}$	5%
$\boldsymbol{m_{\tilde{\nu}}}$	LL	$\mathbf{94\,GeV\ (43\,GeV)}$	5%
$\boldsymbol{m_{\tilde{t}_1}}$	LL	$\mathbf{95\,GeV\ (65\,GeV)}$	5%
$\boldsymbol{m_{\tilde{b}_1}}$	LL	$\mathbf{95\,GeV\ (59\,GeV)}$	5%
$\boldsymbol{m_{\tilde{q}}}$	LL	$\mathbf{318\,GeV}$	5%
$\boldsymbol{m_{\tilde{g}}}$	LL	$\mathbf{233\,GeV}$	5%
$(\boldsymbol{\sigma_p^{\rm SI}}$	UL	WIMP mass dependent	$\sim \mathbf{100\%})$

Note: DM direct detection $\boldsymbol{\sigma_p^{\rm SI}}$ not applied due to astroph'l uncertainties (eg, local DM density)

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Bayesian posterior pdf

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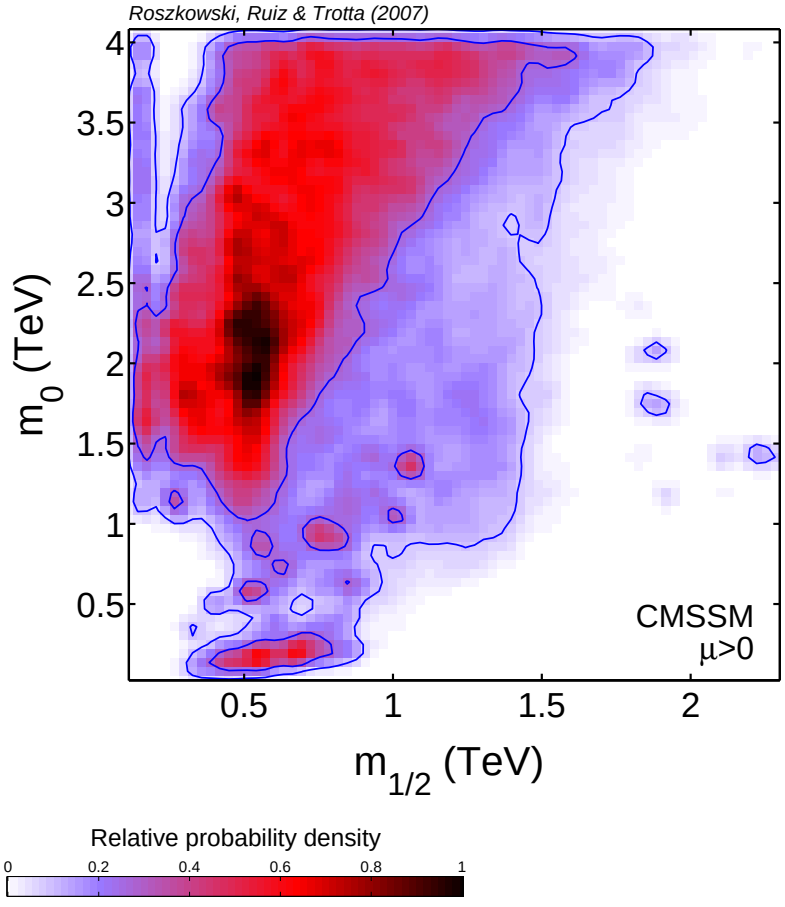
arXiv:0705.2012, flat prior



Bayesian posterior pdf

- MCMC scan (4 CMSSM + 4 SM param's)
- Bayesian analysis
- relative probability density fn (pdf)
- flat priors
- 68% total prob. – inner contours
- 95% total prob. – outer contours
- 2-dim pdf $p(m_0, m_{1/2}|d)$
- favored: $m_0 \gg m_{1/2}$ (FP region)

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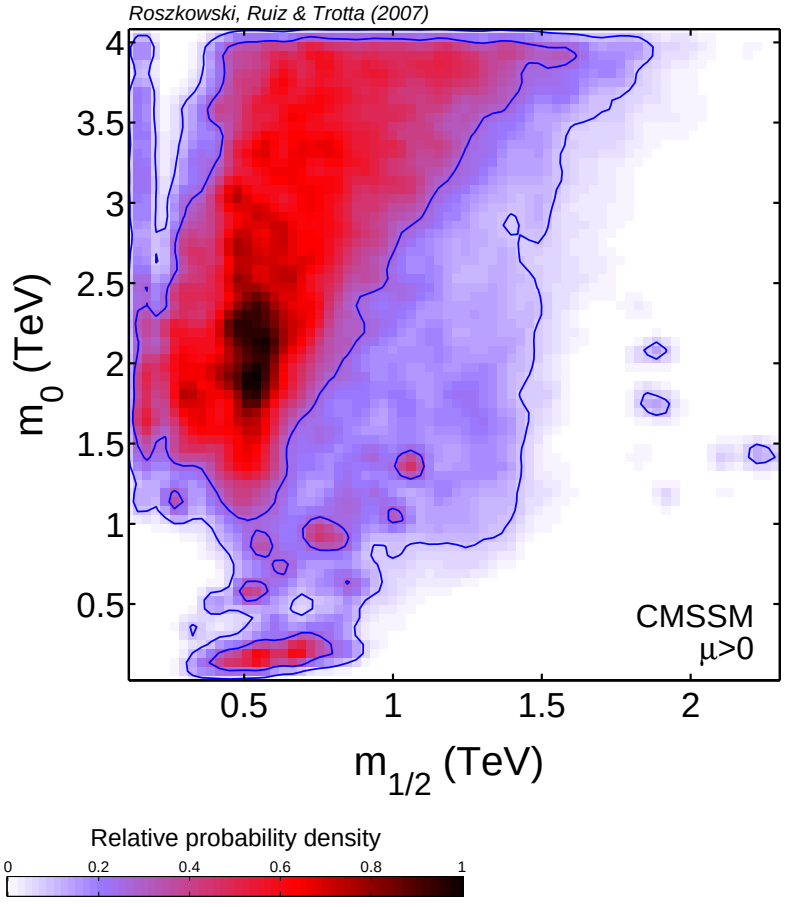

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similar study by Allanach+Lester(+Weber)

see also, Ellis et al (EHOW, χ^2 approach, no MCMC, fixed SM parameters)

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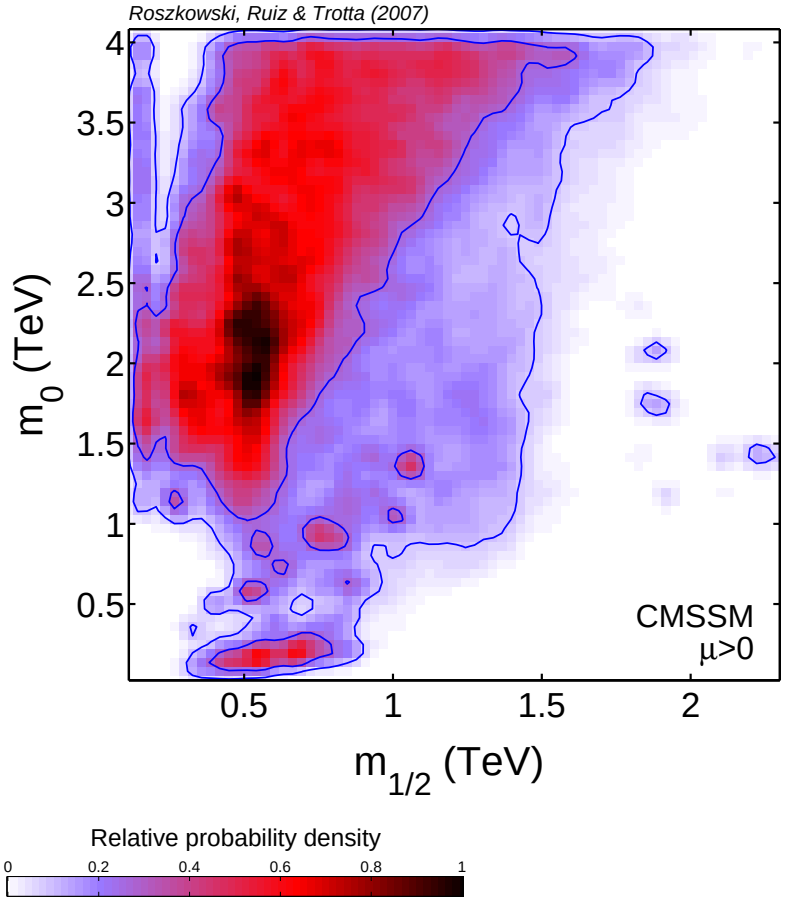

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unlike others (except for A+L), we vary also SM parameters

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Bayesian pdfs depend on prior choice

Trotta et al, arXiv:0809.3792

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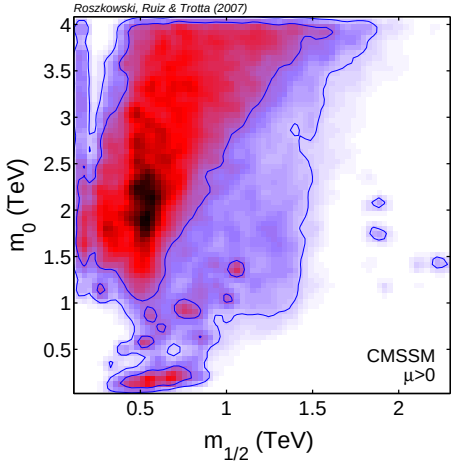
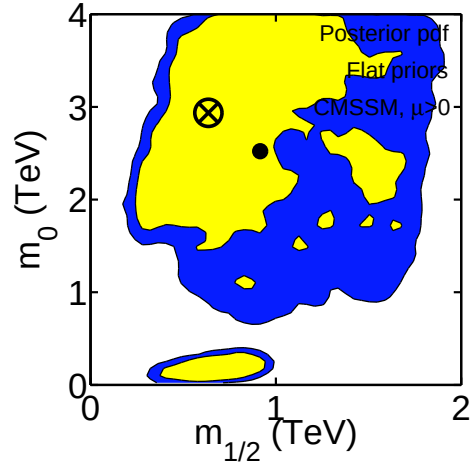


MultiNest, flat prior



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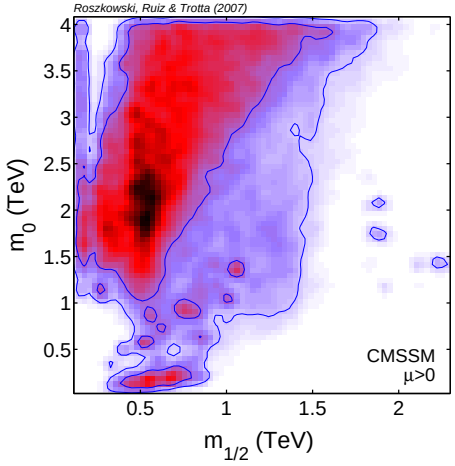
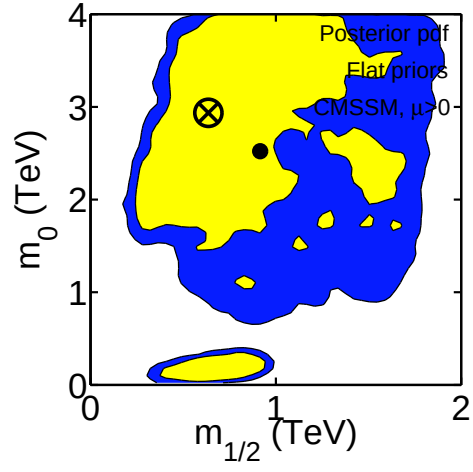
MCMC, flat prior



MultiNest, flat prior



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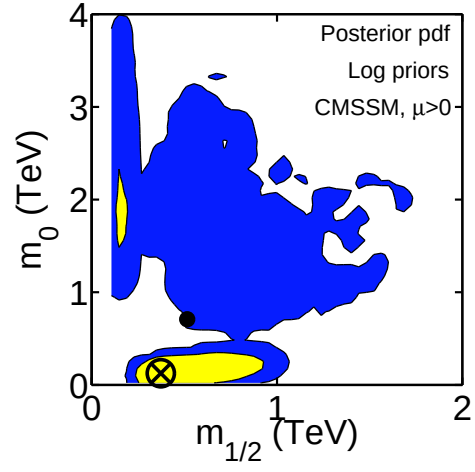

Impact of Priors

Bayesian pdfs depend on prior choice

Trotta et al, arXiv:0809.3792

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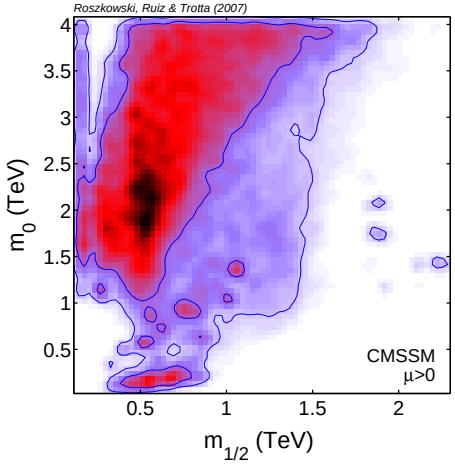

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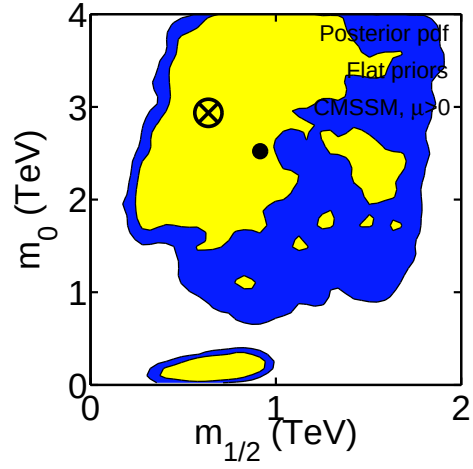

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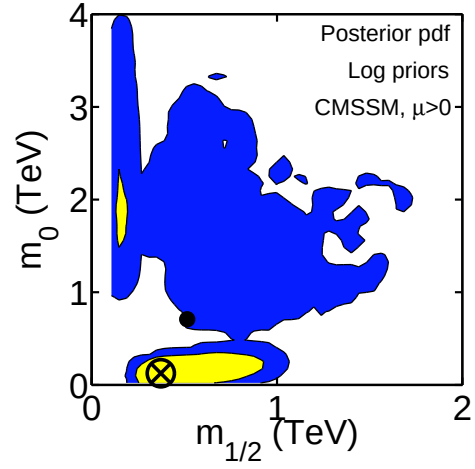

yellow (blue): 68 (95)% total prob. region

cross: best-fit point: thick dot: posterior mean $\langle m\rangle = \int p(m|d)m\mathrm{d}m$

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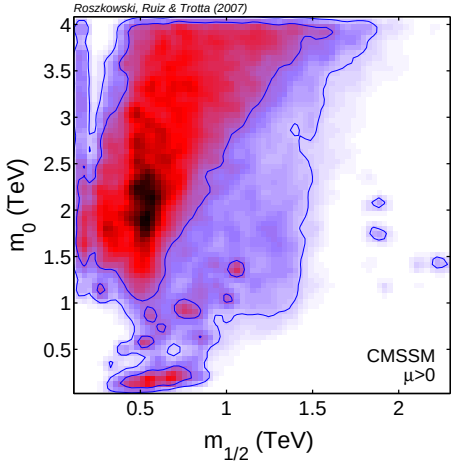

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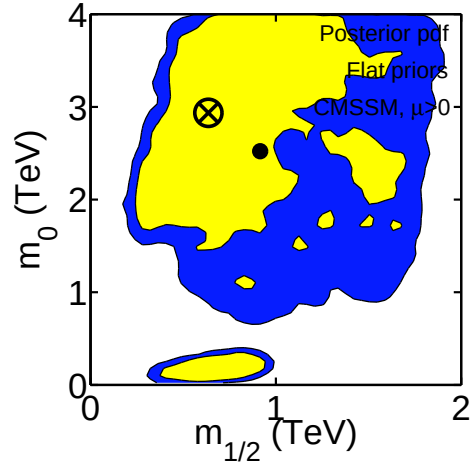

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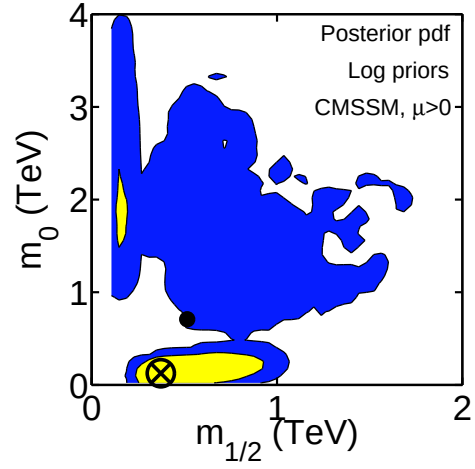

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$\Rightarrow$ rather strong dependence on priors

reason: data not constraining enough (yet)

Impact of Priors on Observables

Trotta et al, arXiv:0809.3792

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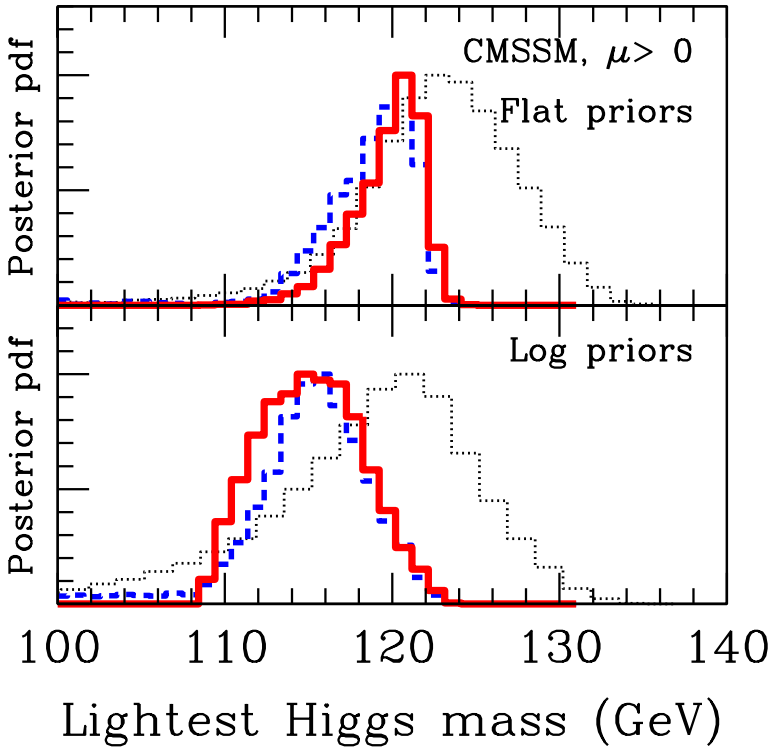
light Higgs mass m_h



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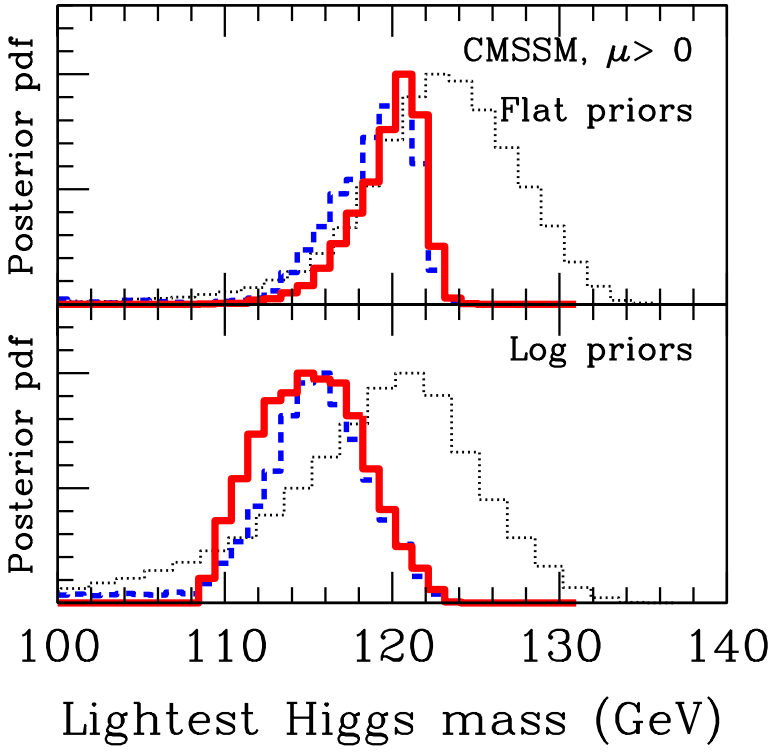
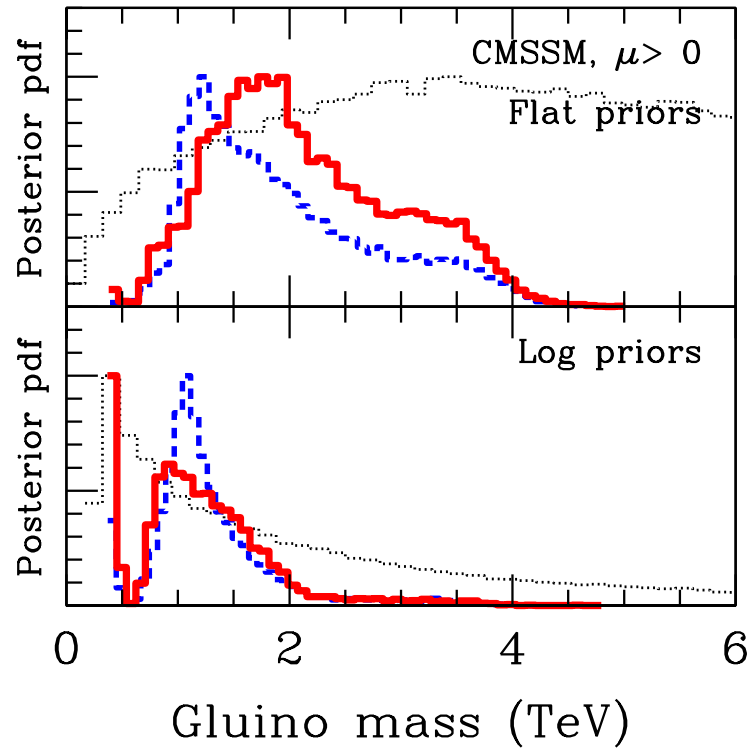


gluino mass $m_{\tilde{g}}$



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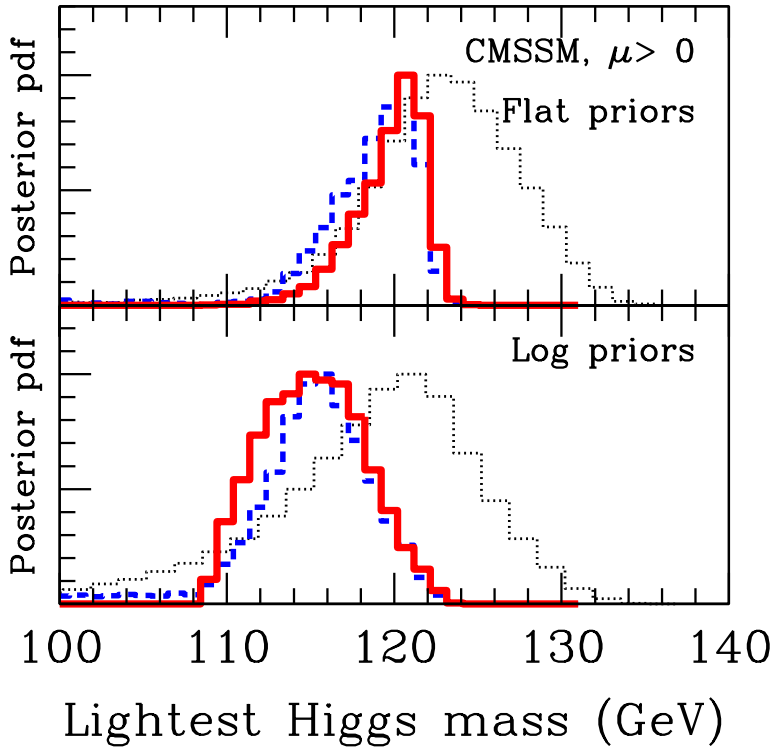

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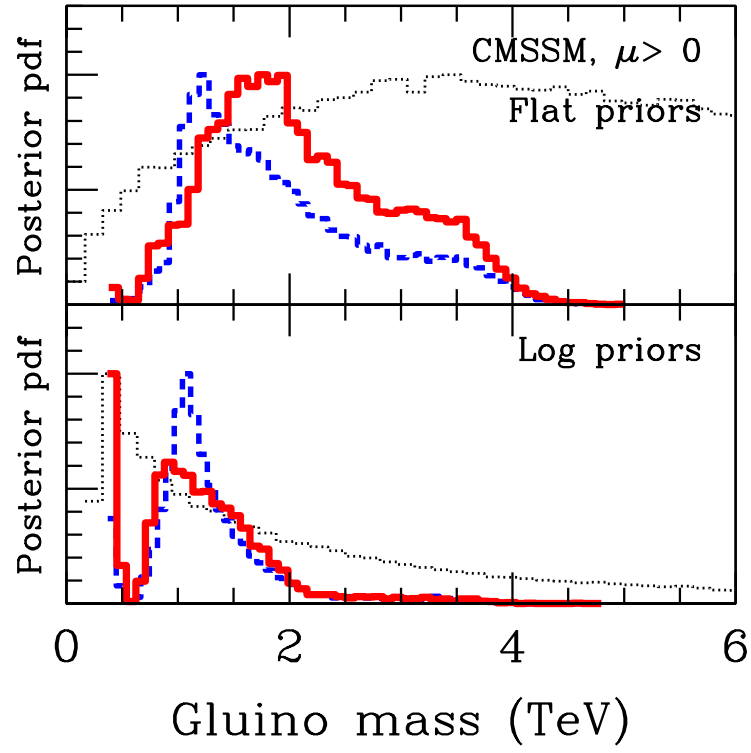

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$\Rightarrow$ impact of priors less strong on observables than on CMSSM parameters

Beyond the CMSSM...

Constrained Next-to-MSSM (CNMSSM)

Higgs: H_u, H_d and singlet S; λS^3

0906.4911

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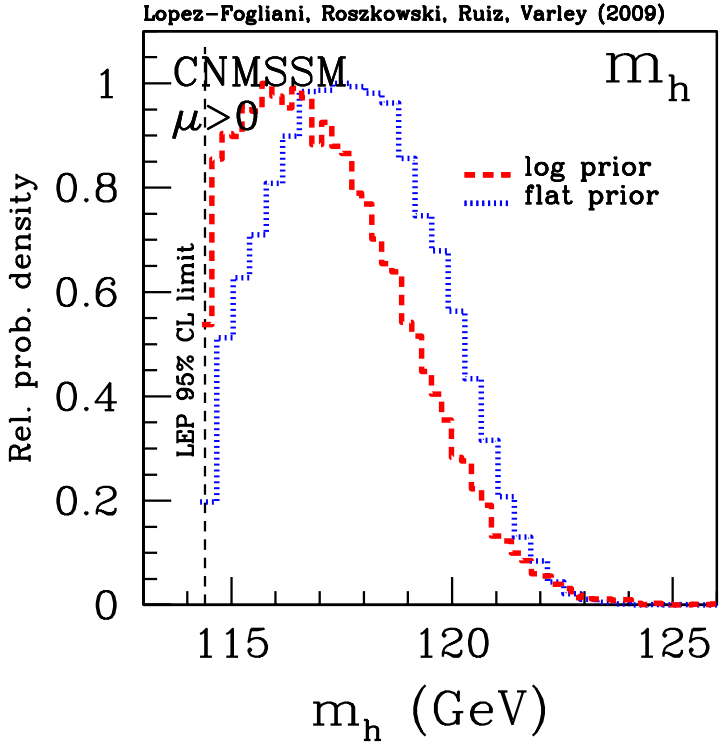

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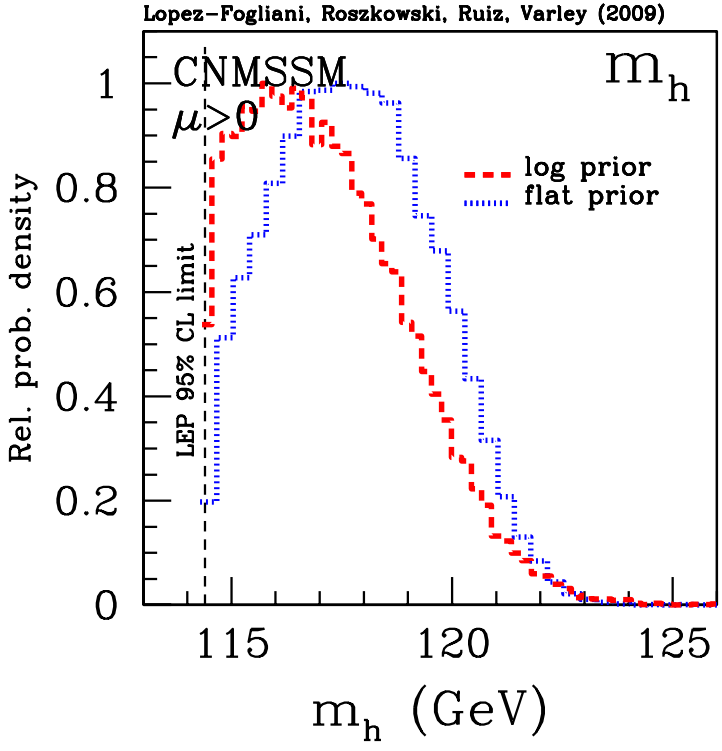

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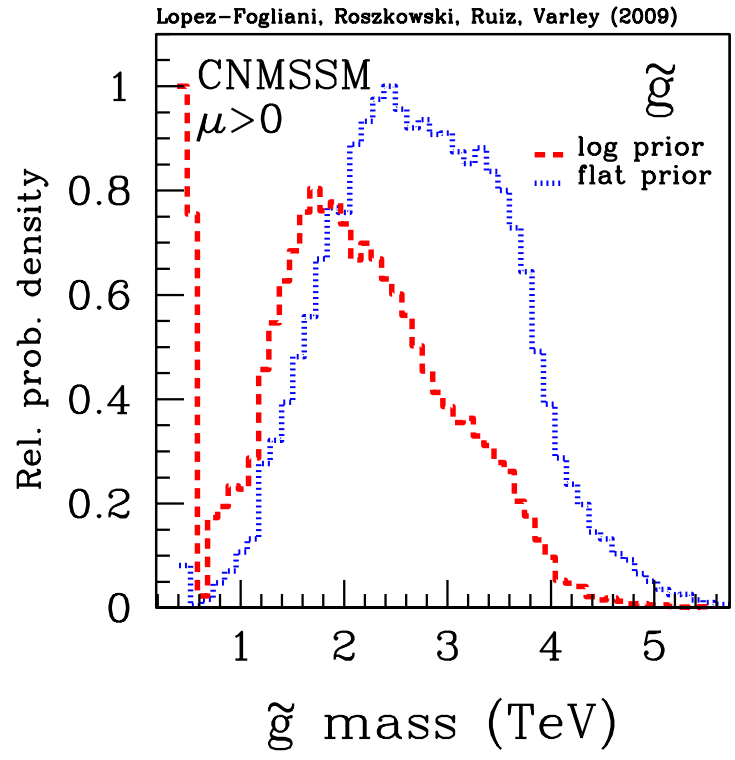

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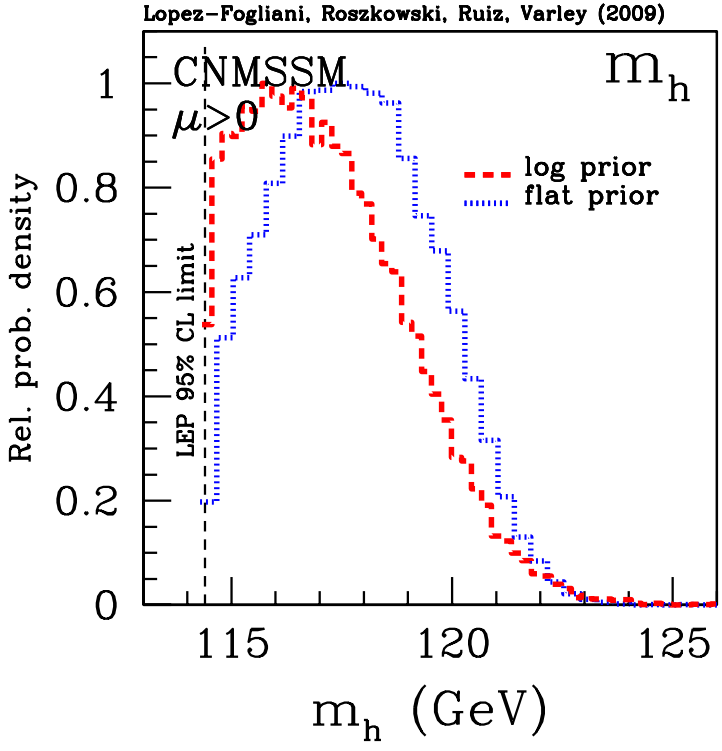

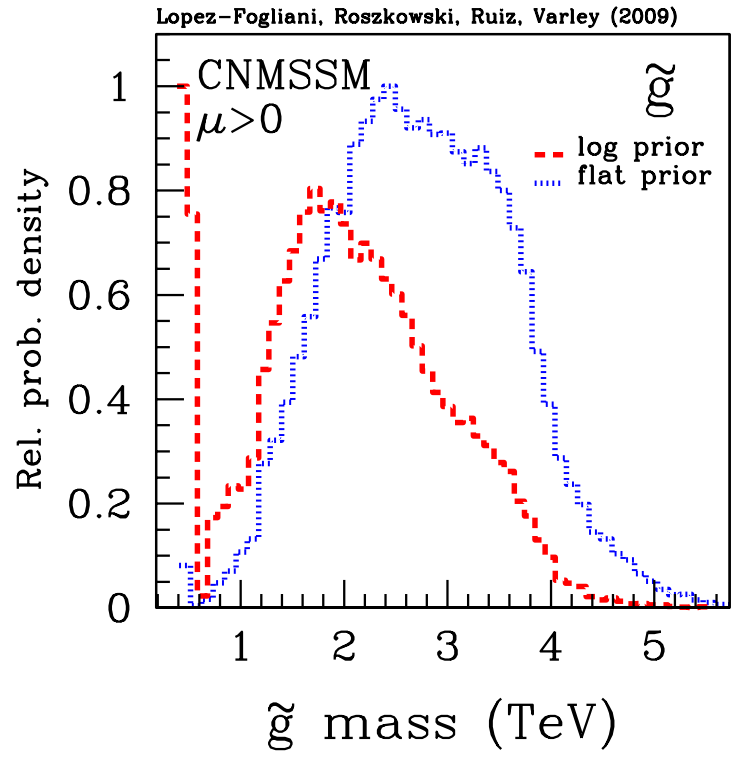

$\Rightarrow$ impact of priors less strong on observables

but for example in NUHM it remains strong

SUSY models and DM direct detection

Bayesian analysis, flat priors

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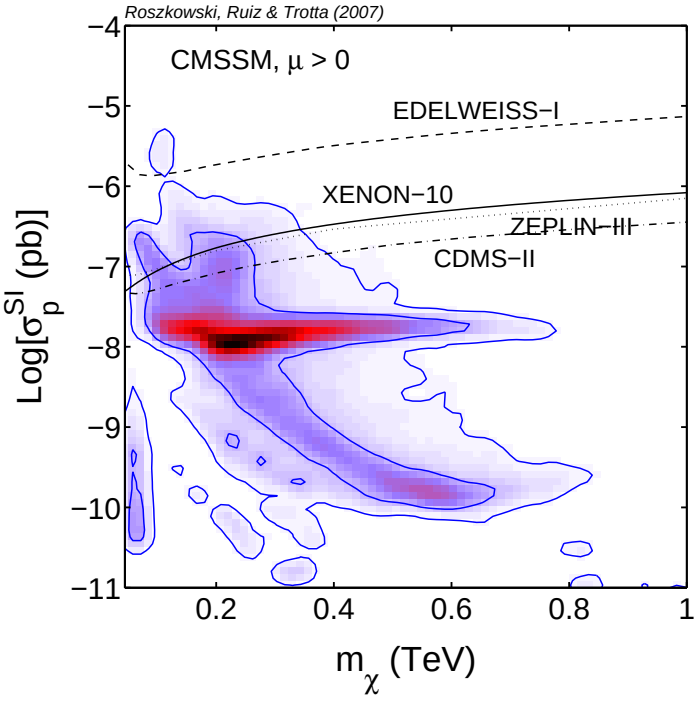
Constrained MSSM (mSUGRA)



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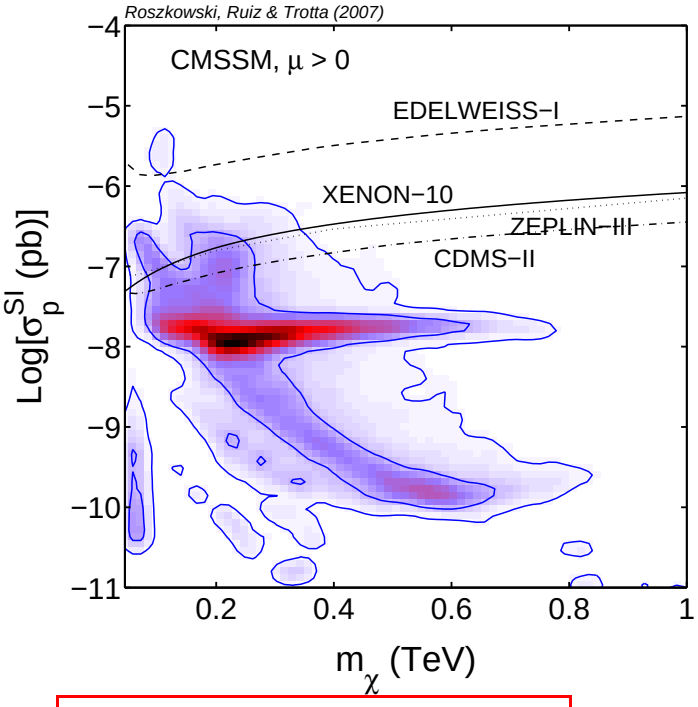

⇒ fairly similar pattern

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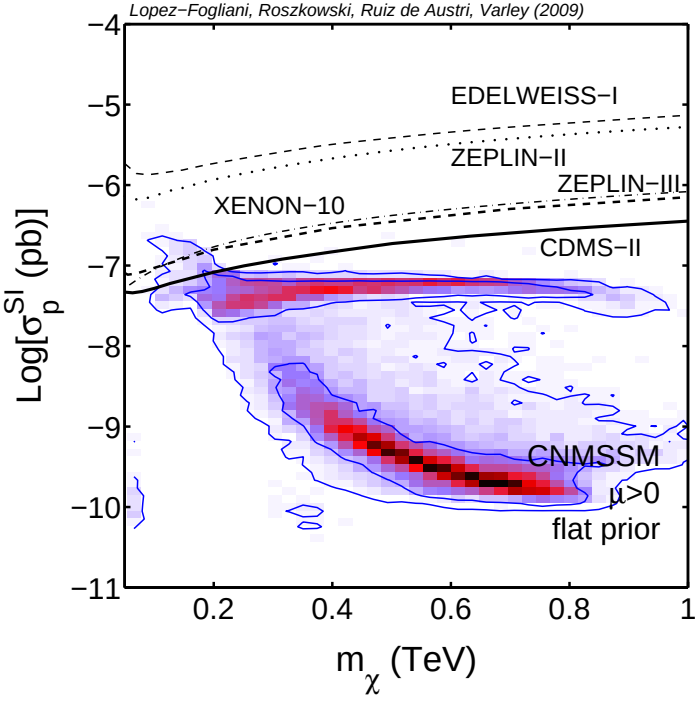

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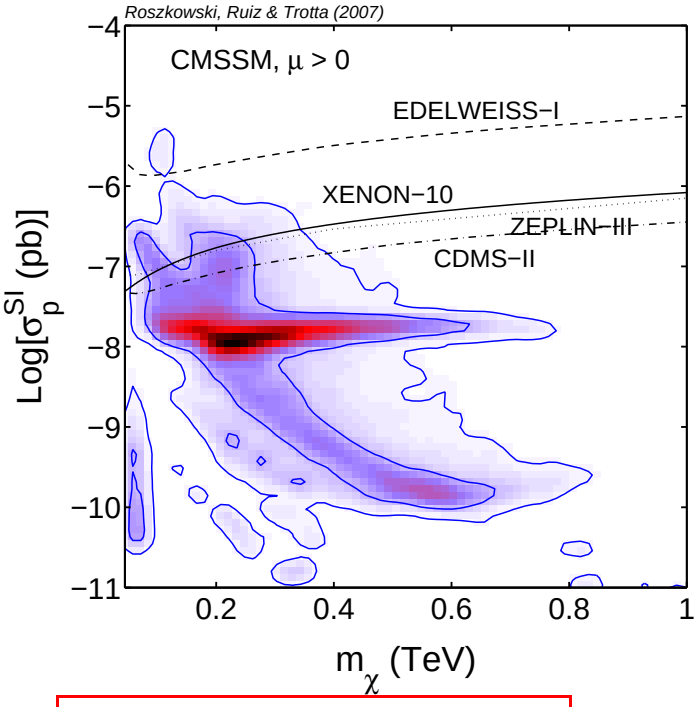

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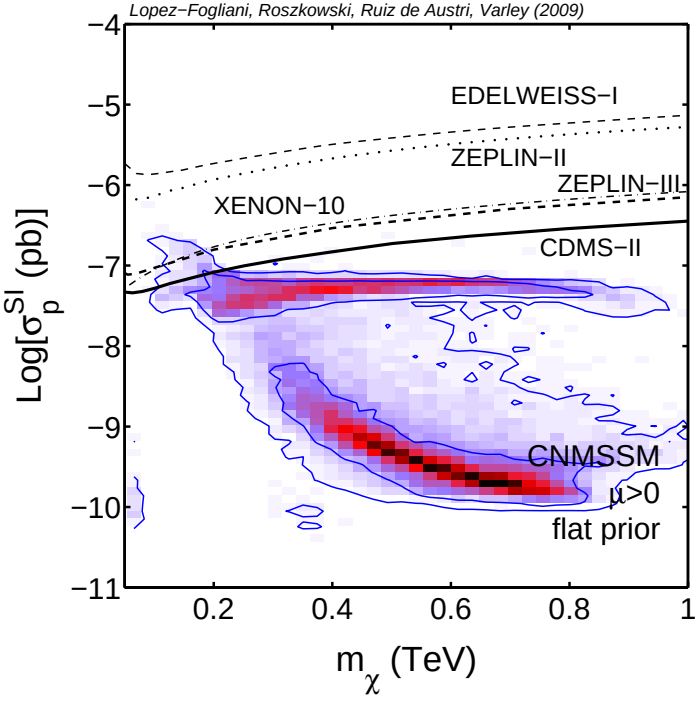

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many collider signatures also (likely to be) similar

⇒ LHC, DM expt: it may be hard to discriminate among models

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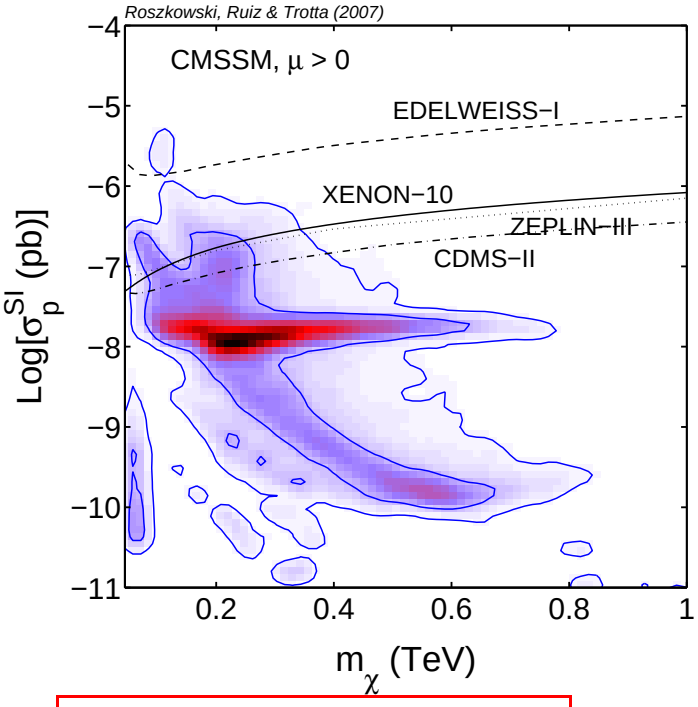

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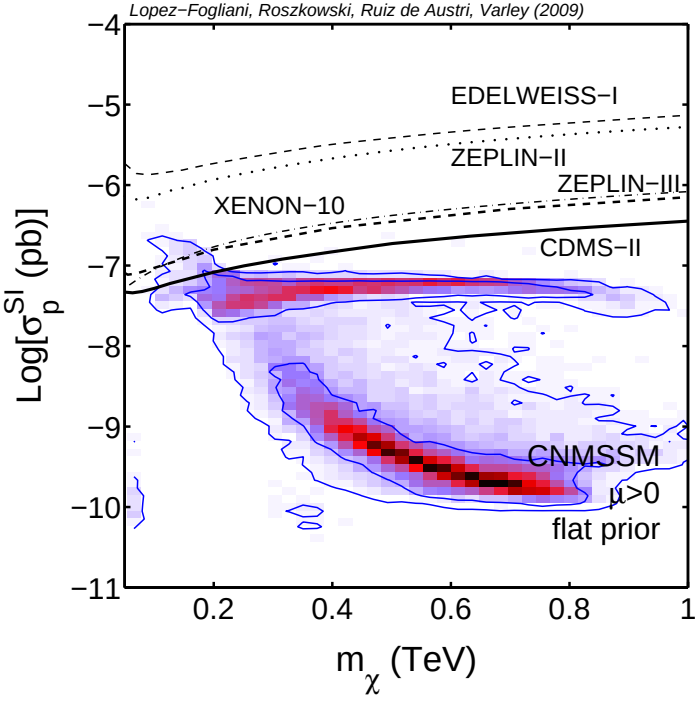

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Parameter reconstruction

Parameter reconstruction

...once positive measurements are made...

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task: reconstruct underlying SUSY parameters

model dependent program

Case study: ATLAS SU3 Point

ATLAS SU3 benchmark point, arXiv:0901.0512

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Parameter	SU3 benchmark value
m_0	100 GeV
$m_{1/2}$	300 GeV
$\tan\beta$	6.0
A_0	-300 GeV
$\Omega_\chi h^2$	0.23319 $\Leftarrow$
SUSY mass spectrum	
$\chi = \chi_1^0$	117.9 GeV
χ_2^0	223.4 GeV
$\widetilde{m}_{\tilde{l}}$	152.2 GeV
$m_{\tilde{q}}$	652.4 GeV

- $\widetilde{m}_{\tilde{l}}$ - lightest slepton mass
- $m_{\tilde{q}}$ - average light squark mass

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- study endpoint measurements
  - dileptons + lepton+jets analysis of the decay chain $\tilde{q}_L \rightarrow \chi_2^0(\rightarrow \tilde{l}^{\pm} l^{\mp}) q \rightarrow \chi_1^0 l^+ l^- q$ and
  - the high-p_T and large missing energy analysis of the decay chain $\tilde{q}_R \rightarrow \chi_1^0 q$
- χ^2 minimization
- int. lum. 1 fb^{-1}

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ATLAS SU3 measurements

Observable	SU3 m_{meas} [GeV]	SU3 m_{MC} [GeV]
$m_{\tilde{\chi}_1^0}$	$88 \pm 60 \mp 2$	118
$m_{\tilde{\chi}_2^0}$	$189 \pm 60 \mp 2$	219
$m_{\tilde{q}}$	$614 \pm 91 \pm 11$	634
$m_{\tilde{\ell}}$	$122 \pm 61 \mp 2$	155
Observable	SU3 Δm_{meas} [GeV]	SU3 Δm_{MC} [GeV]
$m_{\tilde{\chi}_2^0} - m_{\tilde{\chi}_1^0}$	$100.6 \pm 1.9 \mp 0.0$	100.7
$m_{\tilde{q}} - m_{\tilde{\chi}_1^0}$	$526 \pm 34 \pm 13$	516.0
$m_{\tilde{\ell}} - m_{\tilde{\chi}_1^0}$	$34.2 \pm 3.8 \mp 0.1$	37.6

Aad, *et al.*, arXiv:0901.0512

- 1st errors: parabolic
- 2nd errors: jet energy scale

The covariance matrix (ATLAS):

	$m_{\chi_1^0}$	$m_{\chi_2^0} - m_{\chi_1^0}$	$\widetilde{m}_{\tilde{l}} - m_{\chi_1^0}$	$m_{\tilde{q}} - m_{\chi_1^0}$
$m_{\chi_1^0}$	3.72×10^3	53.40	1.92×10^3	10.75×10^2
$m_{\chi_2^0} - m_{\chi_1^0}$		3.6	29.0	-1.3
$\widetilde{m}_{\tilde{l}} - m_{\chi_1^0}$			1.12×10^3	4.65
$m_{\tilde{q}} - m_{\chi_1^0}$				14.1

ATLAS: SU3 parameter reconstruction

Aad, *et al.*, arXiv:0901.0512

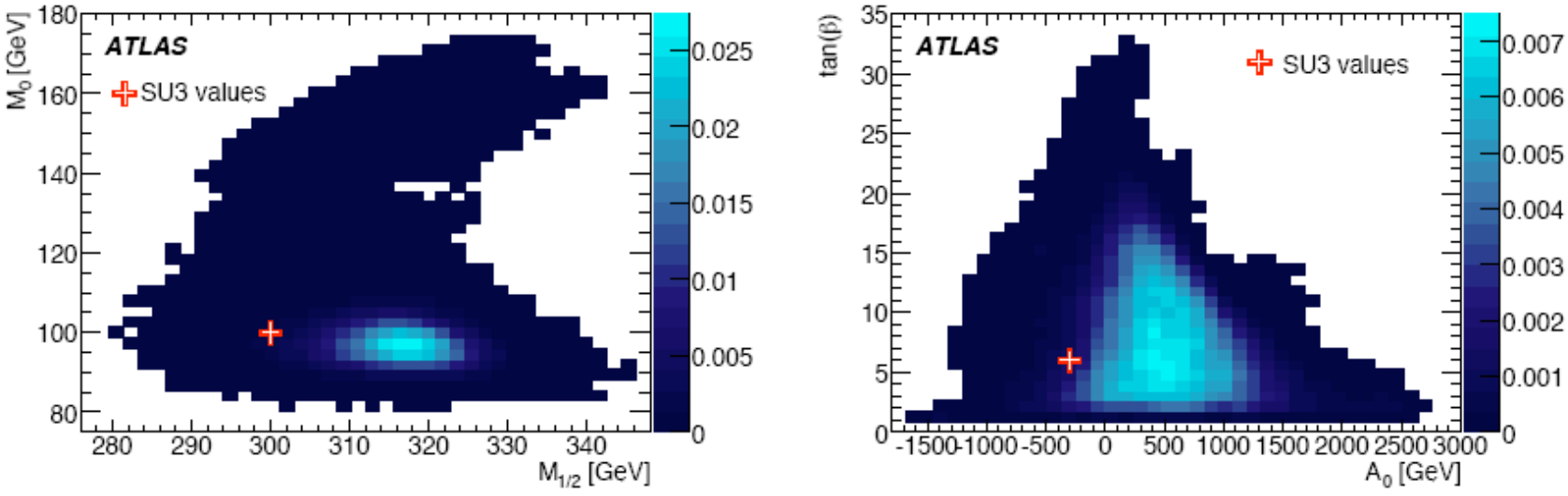

Figure 12: Two-dimensional Markov chain likelihood maps for mSUGRA parameters M_0 and $M_{1/2}$ (left) as well as $\tan\beta$ and A_0 (right) for sign $\mu = +1$, for benchmark point SU3, with integrated luminosity of 1 fb^{-1}. The crosses indicate the actual values of the parameters for that benchmark point.

- 2D likelihood maps (int. lum. $1\ \text{fb}^{-1}$)
- theory errors neglected
- neglect effect of SM parameters
- ranges around the true value found

Reconstruction of $m_{1/2}, m_0$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

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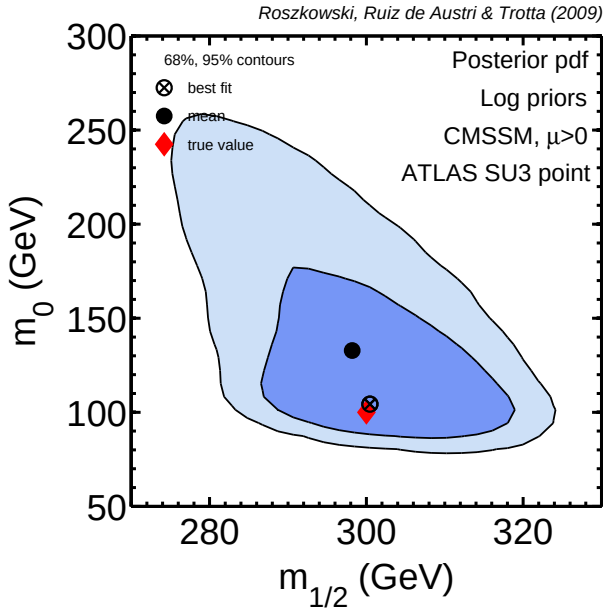
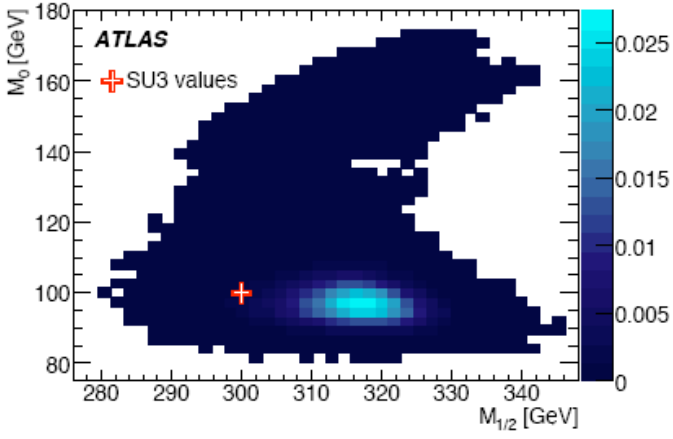


ATLAS analysis



$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \widetilde{m}_{\tilde{l}} - m_{\chi_1^0}, m_{\tilde{q}} - m_{\chi_1^0}\}$$

$$-2 \ln \mathcal{L}_{\mathrm{ATLAS}} = \chi^2_{\mathrm{ATLAS}} = (\theta - \theta_{\mathrm{ML}})^t C^{-1} (\theta - \theta_{\mathrm{ML}})$$

- red diamond: SU3 point
- green cross in circle: best-fit value
- big dot: posterior mean
- dark blue: 68% total prob. region
- light blue: 95% total prob. region

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ATLAS SU3 benchmark point

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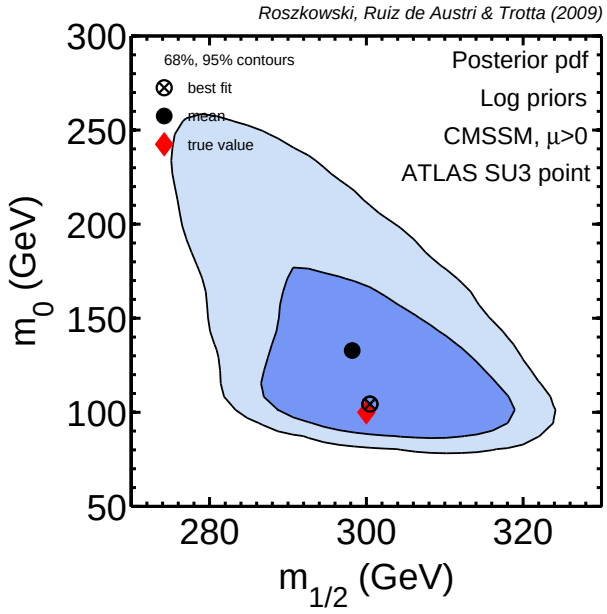

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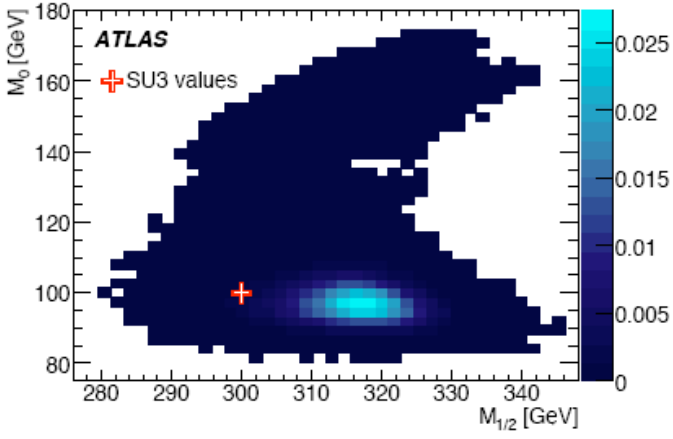

- Nested Sampling (NS) scan
- $\mathbf{50\,GeV \leq m_{1/2}, m_0 \leq 500\,GeV}$, $\boldsymbol{\mu > 0}$
- $\mathbf{-2\,TeV \leq A_0 \leq 2\,TeV}$, $\mathbf{2 \leq \tan\beta \leq 62}$
- follow ATLAS input
- NO exptal constraints applied ($\boldsymbol{b \to s\gamma}$, $\boldsymbol{\Omega_\chi h^2}$, etc)
- similar for flat prior and profile likelihood (akin to $\boldsymbol{\chi^2}$)

Reconstruction of A_0, $\tan\beta$ with SB

ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

Reconstruction of A_0, $\tan\beta$ with SB

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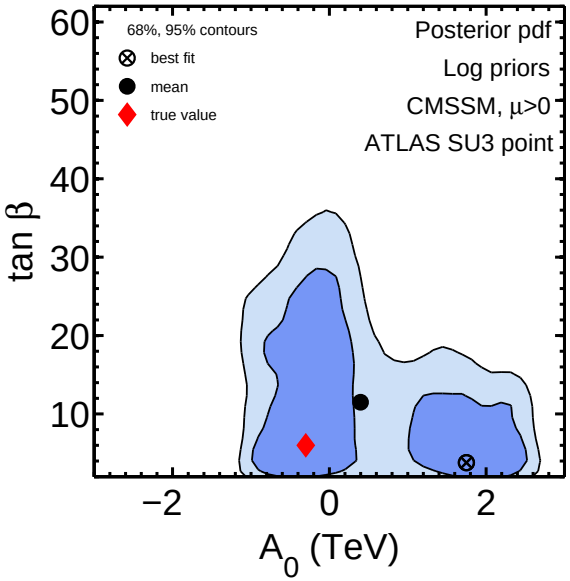
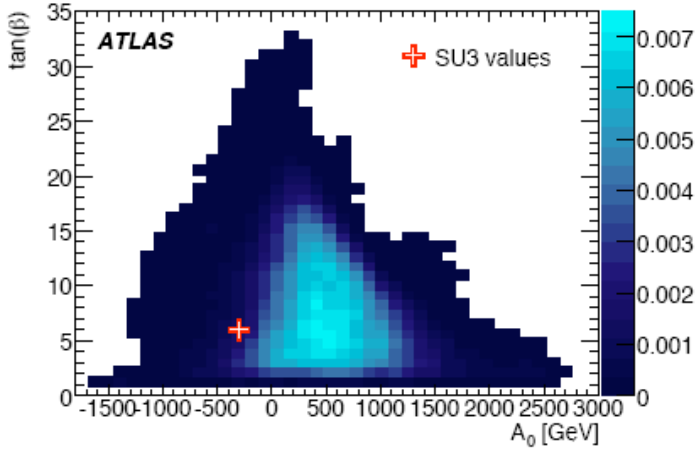
Roszkowski, Ruiz de Austri & Trotta (2009)



ATLAS analysis



$$\theta = \{m_{\chi_1^0}, m_{\chi_2^0} - m_{\chi_1^0}, \widetilde{m}_{\tilde{l}} - m_{\chi_1^0}, m_{\tilde{q}} - m_{\chi_1^0}\}$$

$$-2\ln\mathcal{L}_{\mathrm{ATLAS}} = \chi^2_{\mathrm{ATLAS}} = (\theta - \theta_{\mathrm{ML}})^t C^{-1} (\theta - \theta_{\mathrm{ML}})$$

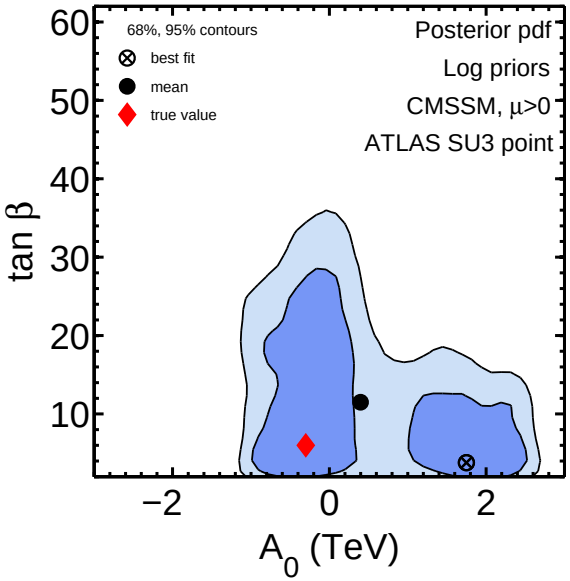
- red diamond: SU3 point
- green cross in circle: best-fit value
- big dot: posterior mean
- dark blue: 68% total prob. region
- light blue: 95% total prob. region

Reconstruction of A_0, $\tan\beta$ with SB

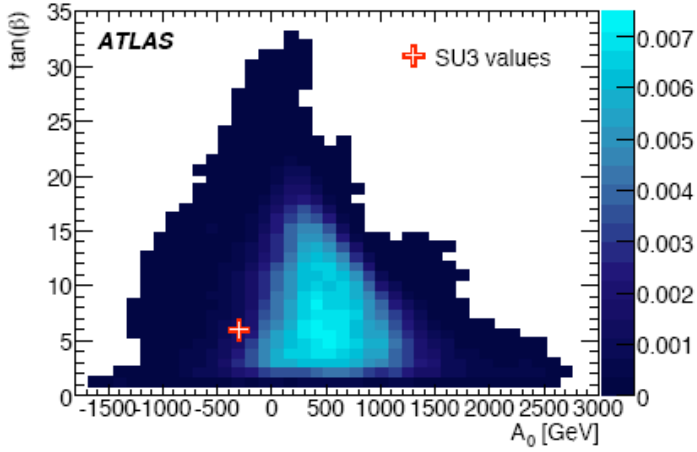
ATLAS SU3 benchmark point

Bayesian analysis, use Gaussian approx. with publicly available info

Roszkowski, Ruiz de Austri & Trotta (2009)



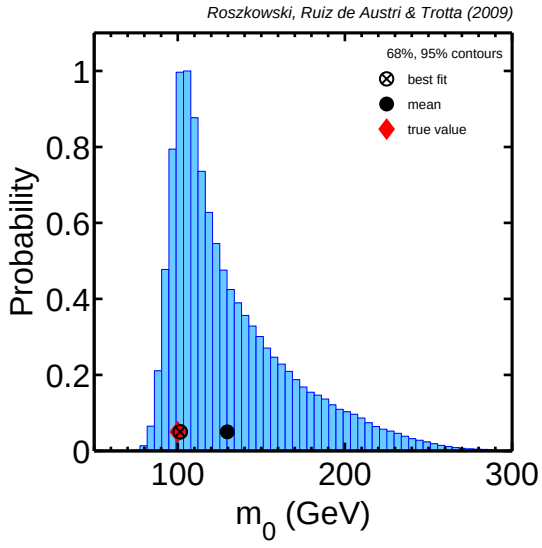
ATLAS analysis

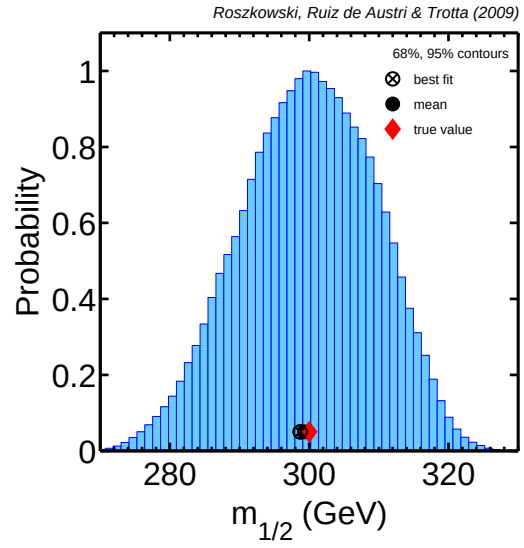


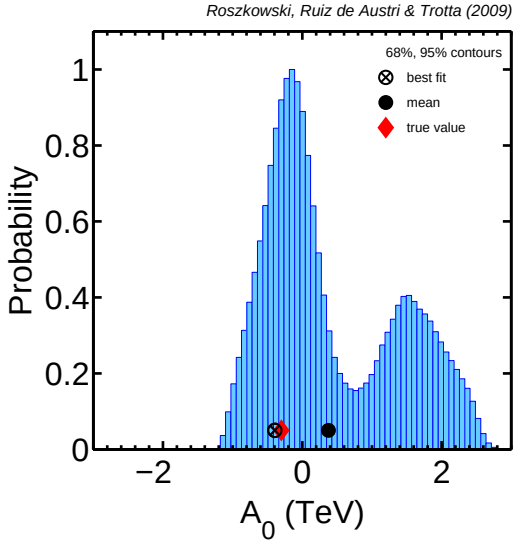
- NS scan
- $50\,\mathrm{GeV} \leq m_{1/2}, m_0 \leq 500\,\mathrm{GeV}$, $\mu > 0$
- $-2\,\mathrm{TeV} \leq A_0 \leq 2\,\mathrm{TeV}$, $2 \leq \tan\beta \leq 62$
- follow ATLAS input
- fix SM (nuisance) parameters
- NO exptal constraints applied ($b \to s\gamma$, $\Omega_\chi h^2$, etc)
- similar result for flat prior and profile likelihood (akin to χ^2)
- cannot resolve sign of A_0

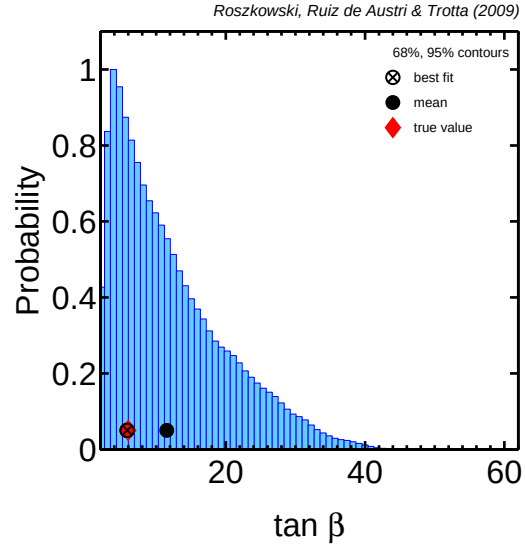
1dim posterior pdfs

ATLAS SU3 benchmark point





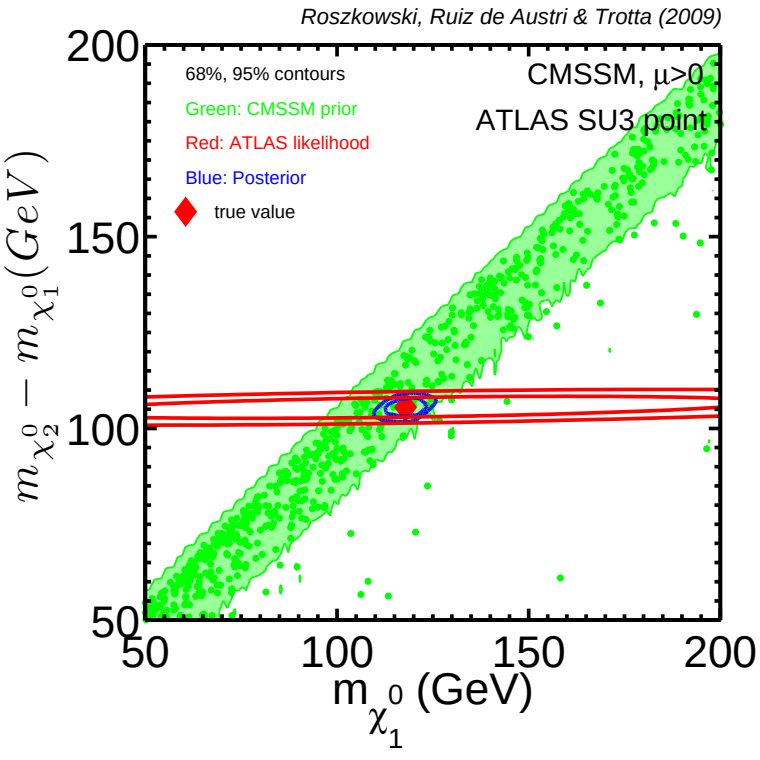




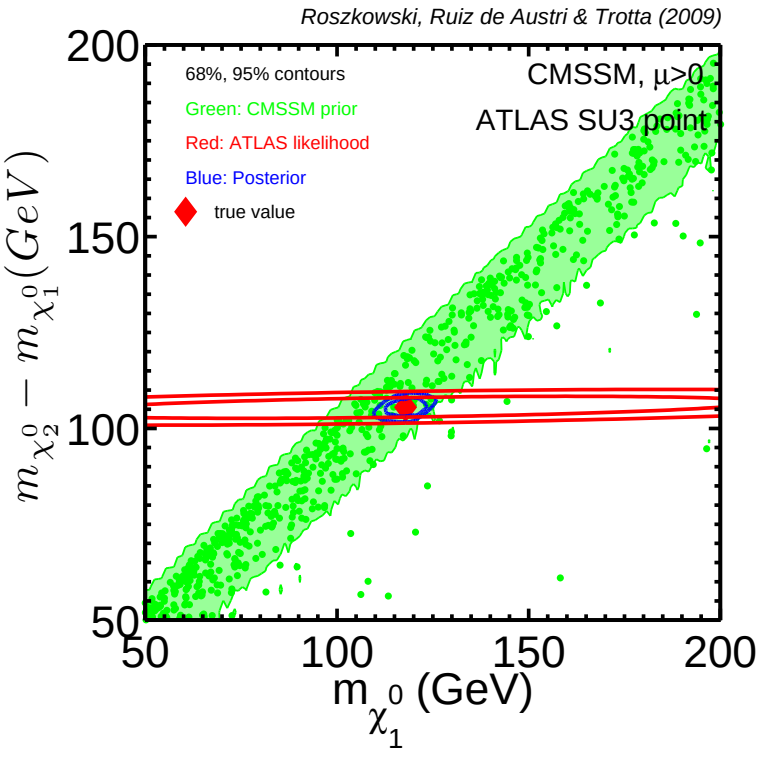
SU3: CMSSM vs MSSM

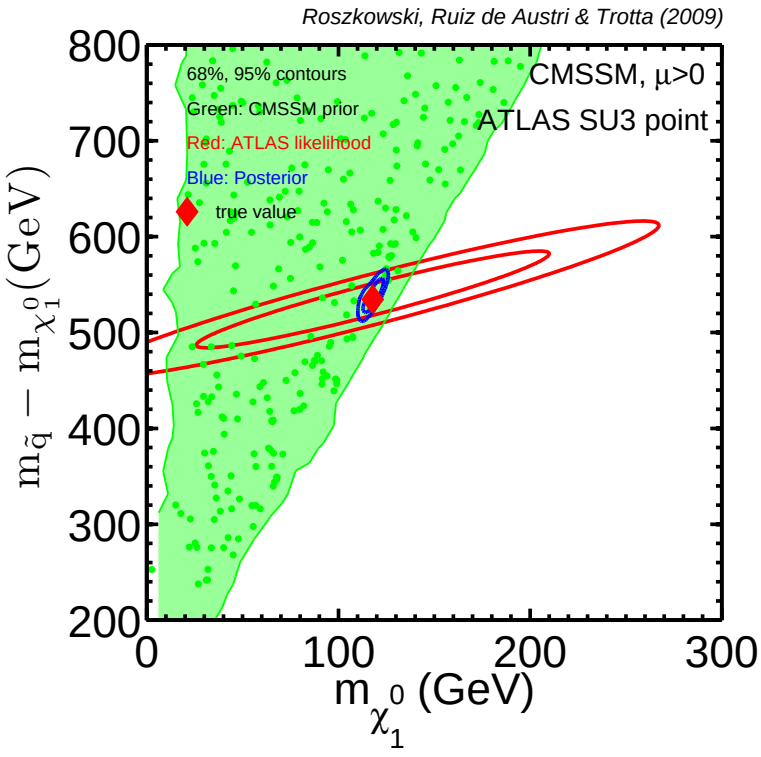
SU3: CMSSM vs MSSM

Roszkowski, Ruiz de Austri & Trotta (2009)



SU3: CMSSM vs MSSM





- green points: allowed by CMSSM
- red ellipses: ATLAS likelihood
- blue ellipses: posterior constraints

theory advantage:

$\Rightarrow$ using posterior allows much better determination of $m_{\chi_1^0}$

Add info about $\Omega_\chi h^2$

take WMAP error on $\Omega_\chi h^2$: 0.0062

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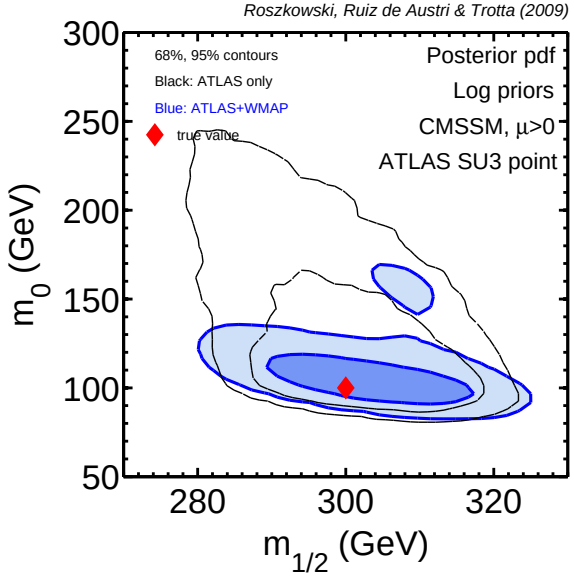
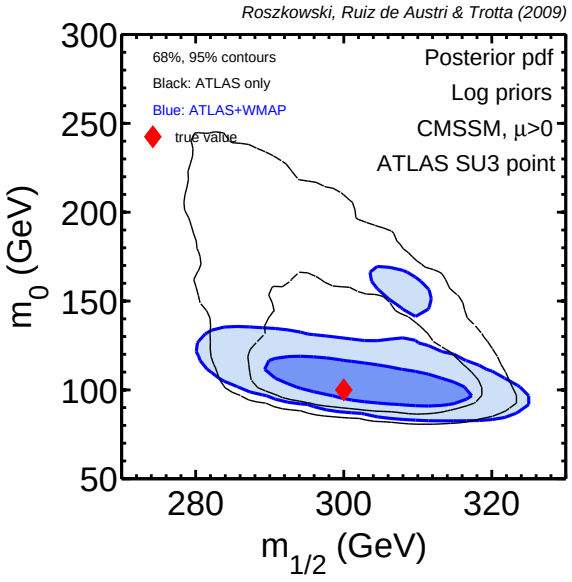
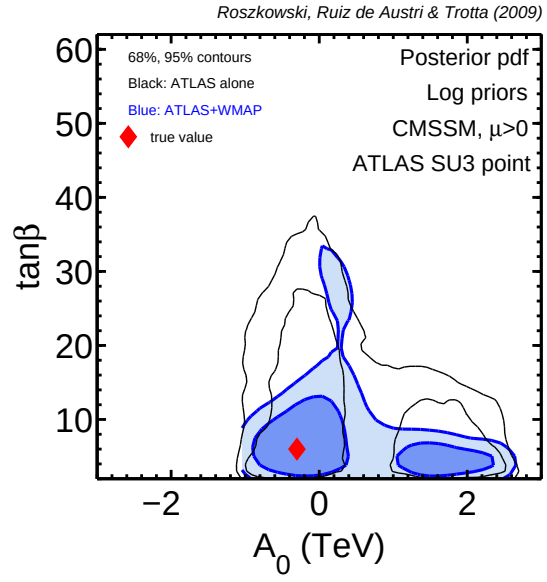
take WMAP error on $\Omega_\chi h^2$: 0.0062



Add info about $\Omega_\chi h^2$

take WMAP error on $\Omega_\chi h^2$: 0.0062





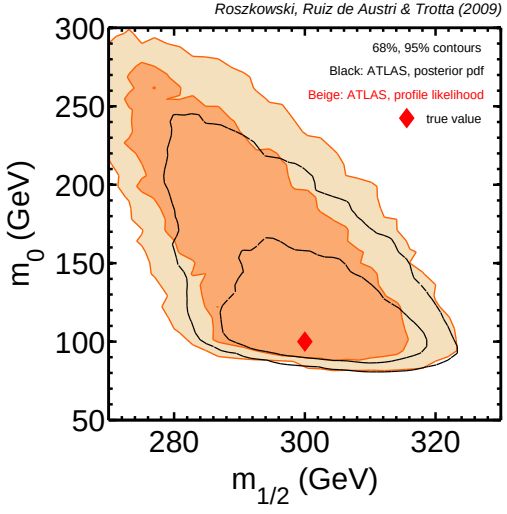
similar result for flat prior and profile likelihood

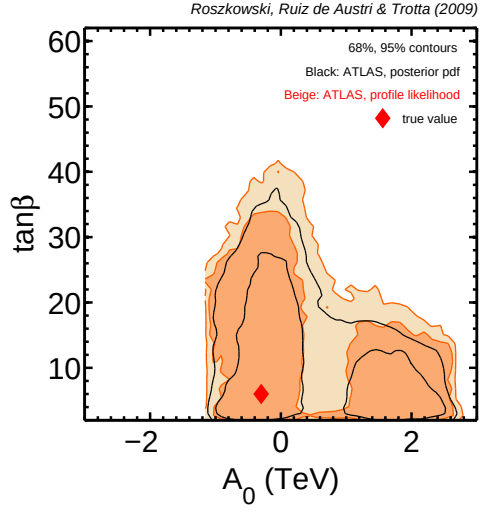
- determination of $m_{1/2}, m_0$ spot on!
- $\tan\beta$ resolved reasonably well
- determination of A_0 remains poor
- still cannot resolve sign of A_0

Posterior pdf vs. profile likelihood

Posterior pdf vs. profile likelihood

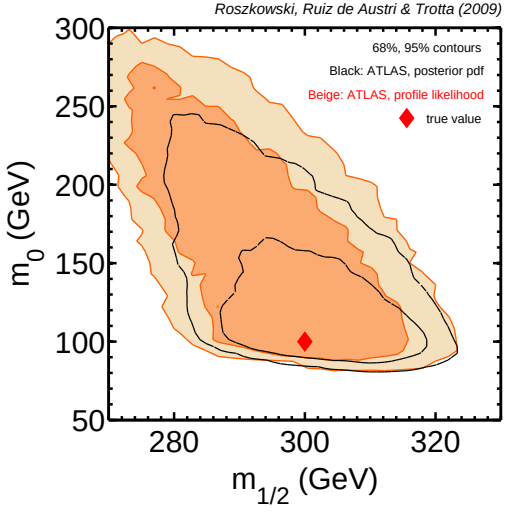
ATLAS data only

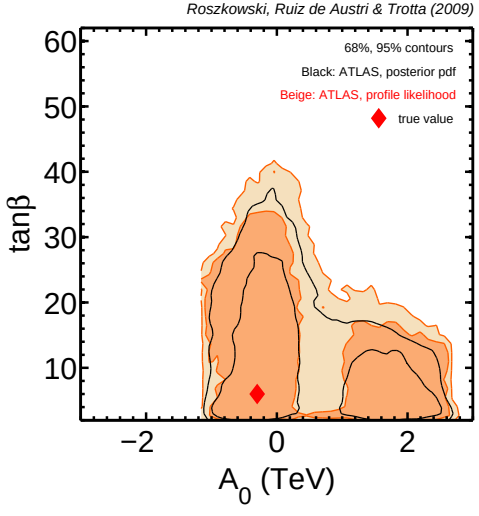




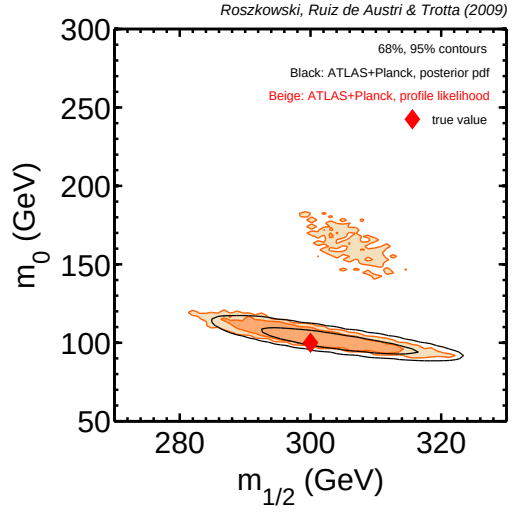
Posterior pdf vs. profile likelihood

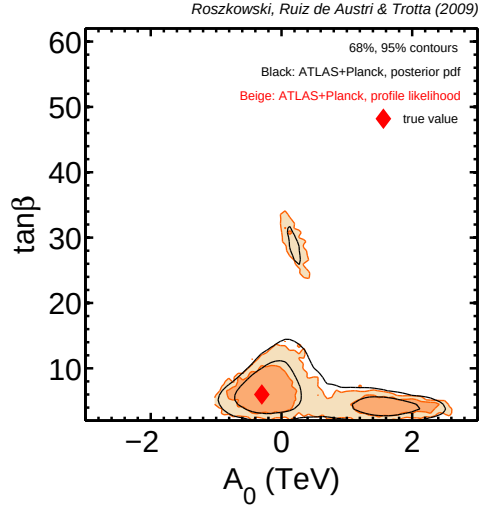
ATLAS data only





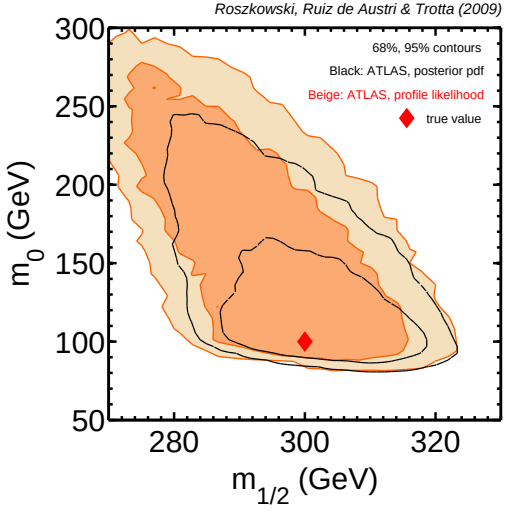
add $\Omega_\chi h^2$+ Planck-like error

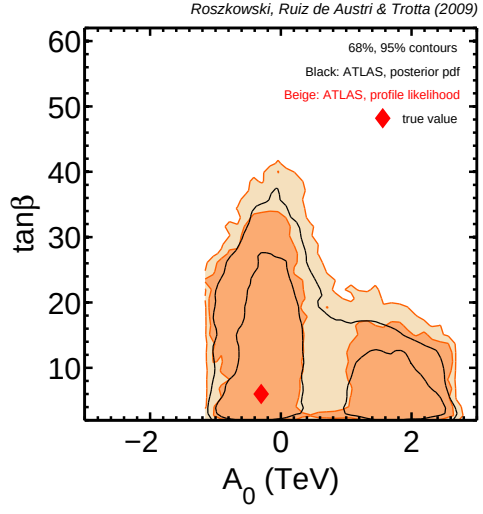




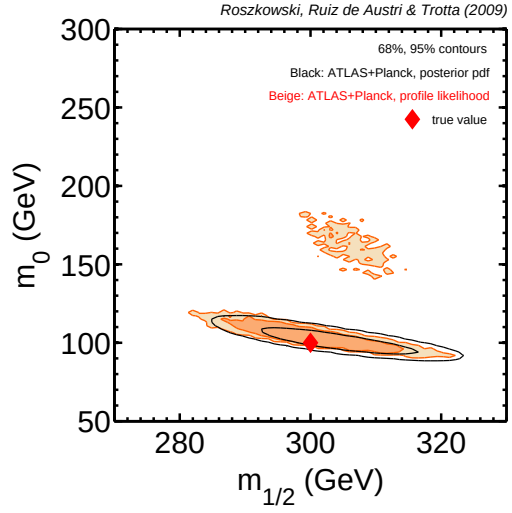
Posterior pdf vs. profile likelihood

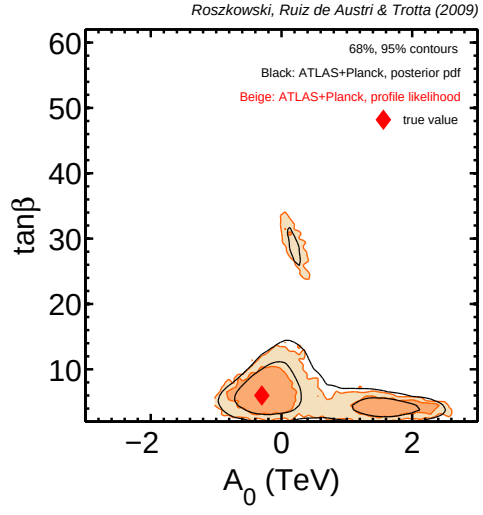
ATLAS data only





add $\Omega_\chi h^2$+ Planck-like error





good agreement but residual prior dependence in PL

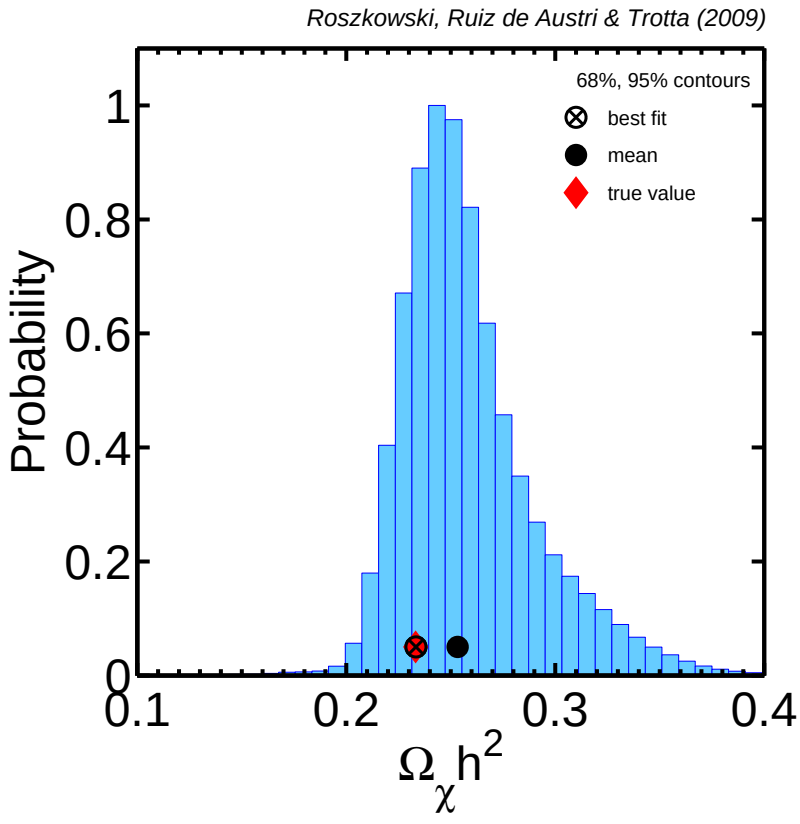
some improvement via MultiNest proposed in arXiv:1101.3296

Determination of $\Omega_\chi h^2$

ATLAS SU3 point

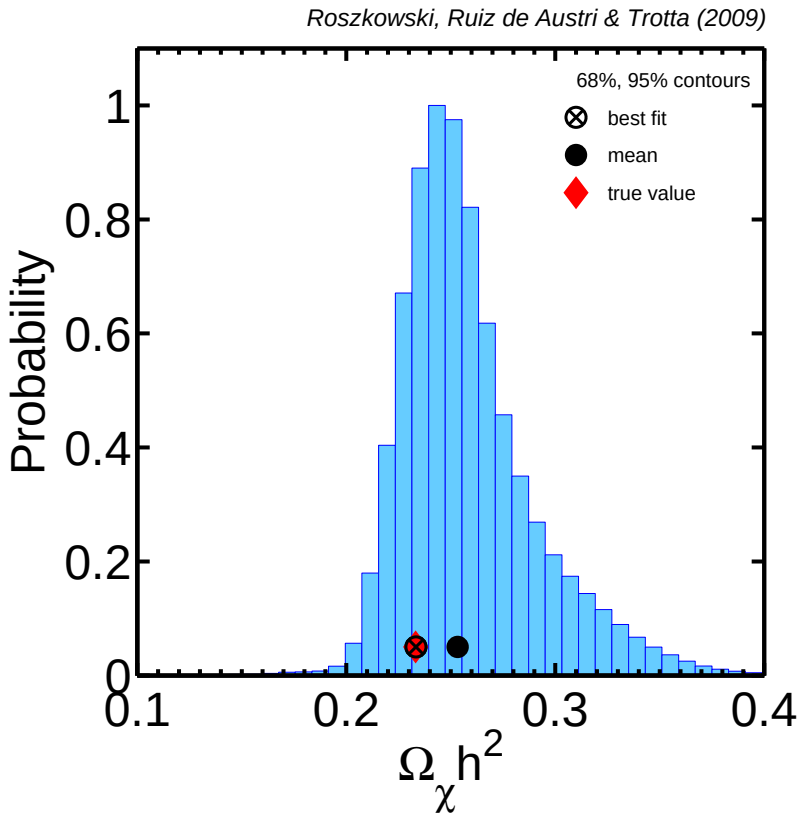
Determination of $\Omega_\chi h^2$

ATLAS SU3 point



Determination of $\Omega_\chi h^2$

ATLAS SU3 point

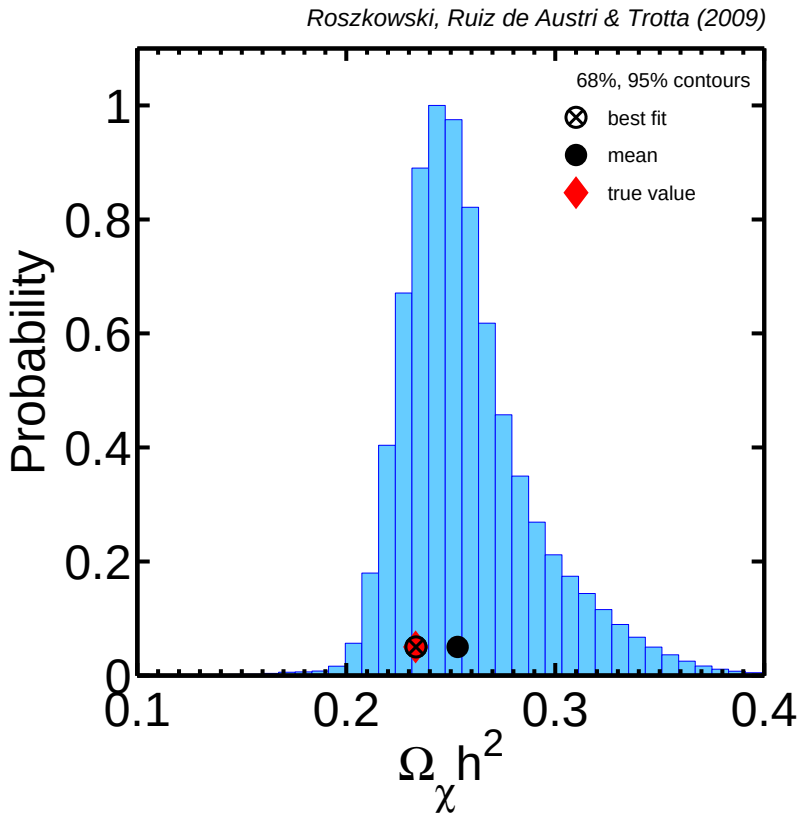


- use only ATLAS data
- similar result for log prior and profile likelihood

- red diamond: SU3 point
- green cross in circle: best-fit value
- dark blue dot: posterior mean

Determination of $\Omega_\chi h^2$

ATLAS SU3 point



- use only ATLAS data
- similar result for log prior and profile likelihood

- red diamond: SU3 point
- green cross in circle: best-fit value
- dark blue dot: posterior mean

$$\Rightarrow \quad \Omega_\chi h^2 = 0.253 \pm 0.034$$

relative accuracy of $\sim 10\%$

Determination of $\sigma_p^{\rm SI}$

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)

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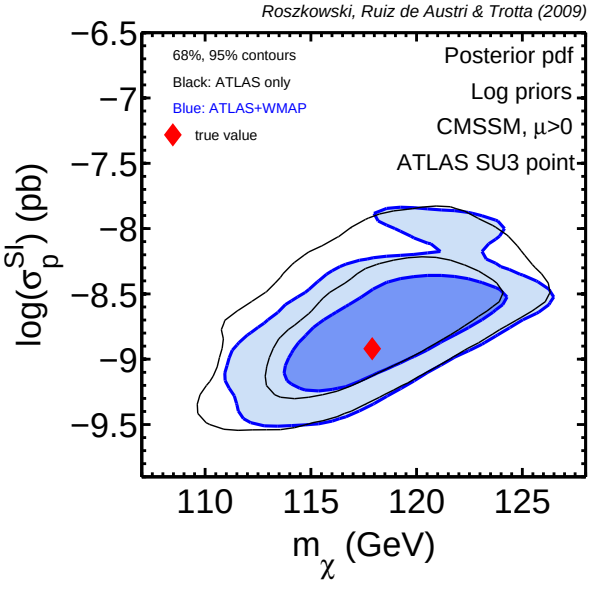
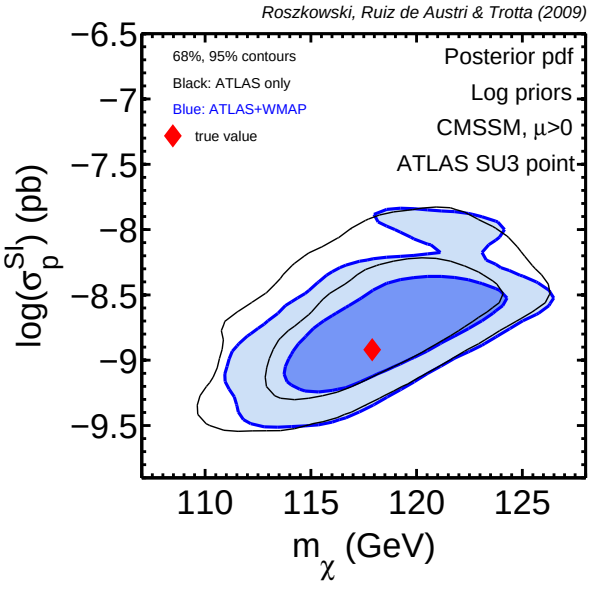
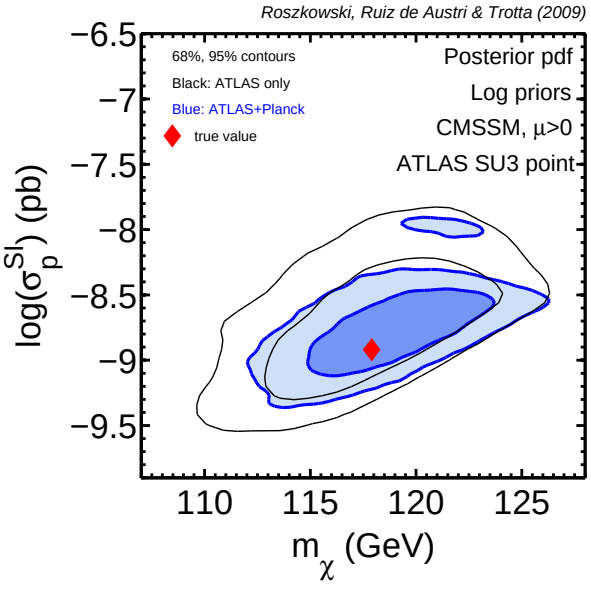
assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)



Determination of σ_p^{SI}

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)



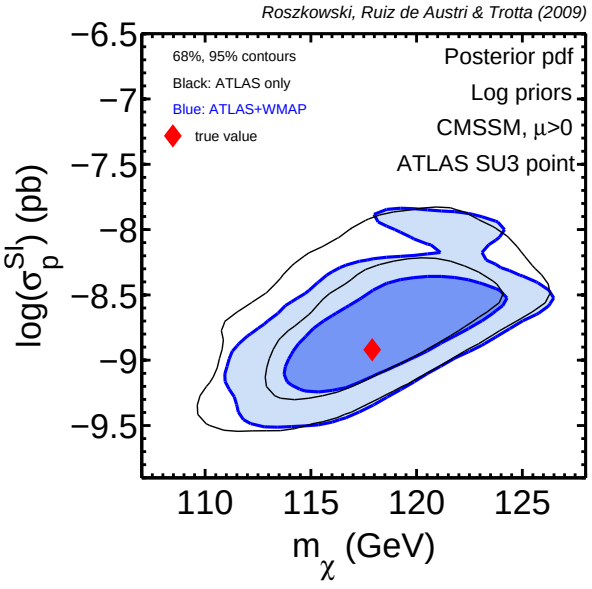


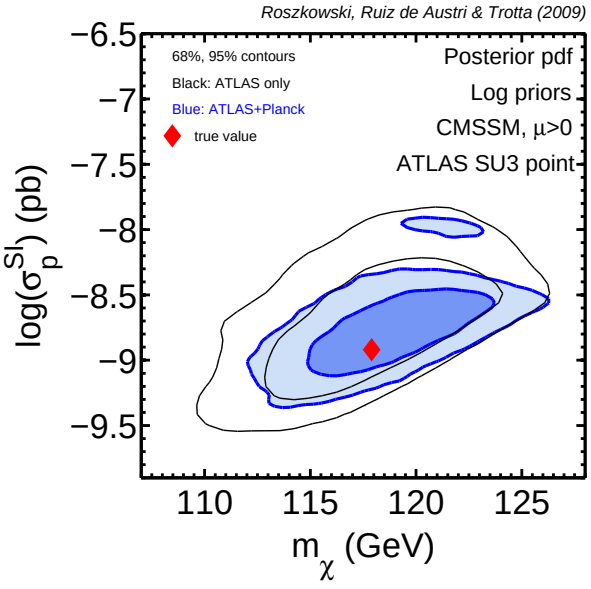
similar result for flat prior and profile likelihood

- determination of σ_p^{SI} much improved by adding WMAP error on $\Omega_\chi h^2$
- Planck: limited impact

Determination of σ_p^{SI}

assume Planck-like error: reduce WMAP error on $\Omega_\chi h^2$ by ~ 5 ($\lesssim 0.0016$)





similar result for flat prior and profile likelihood

- determination of σ_p^{SI} much improved by adding WMAP error on $\Omega_\chi h^2$
- Planck: limited impact

The LHC is on...

CMS, arXiv:1101.1628

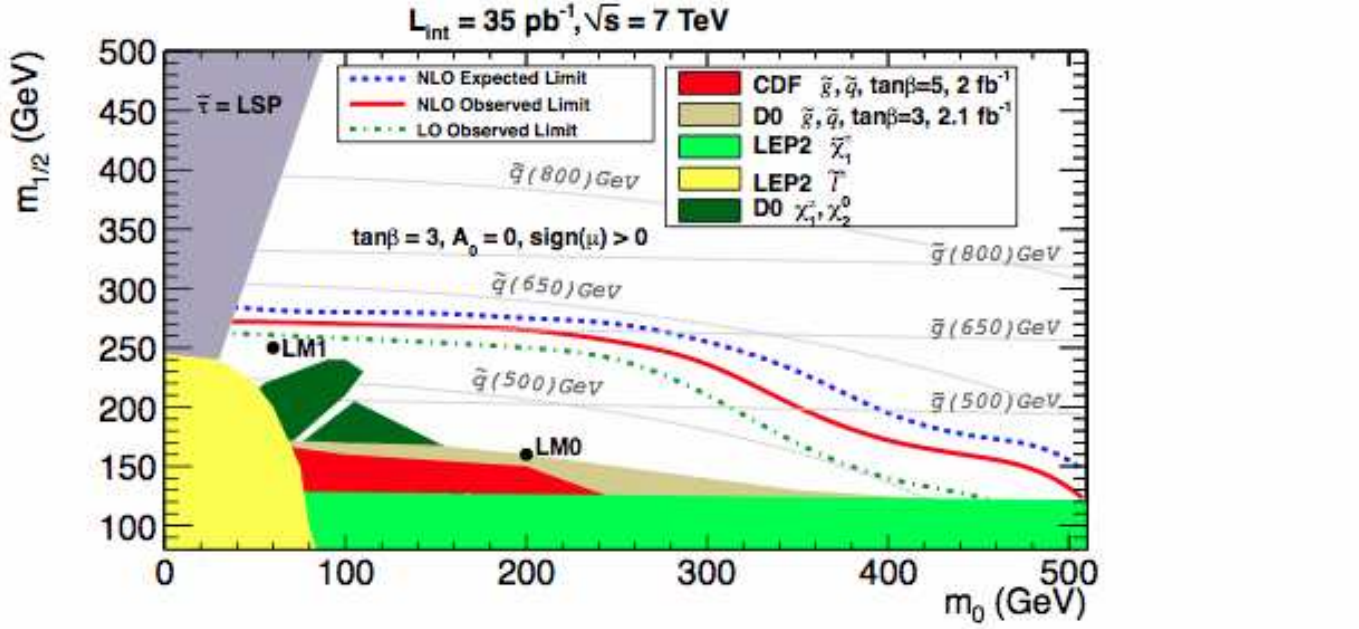


Figure 5: Measured (red line) and expected (dashed blue line) 95% CL exclusion contour at NLO in the CMSSM $(m_0, m_{1/2})$ plane for $\tan\beta = 3$, $A_0 = 0$ and $\text{sign}(\mu) > 0$. The measured LO exclusion contour is shown as well (dot-dashed green line). The area below the curves is excluded by this measurement. Exclusion limits obtained from previous experiments are presented as filled areas in the plot. Grey lines correspond to constant squark and gluino masses. The plot also shows the two benchmark points LM0 and LM1 for comparison.

Prepare!

Include LHC results on SUSY searches in the likelihood f'n

- analysis SUSY model dependent

  CMSSM, CNMSSM, NUHM, general MSSM, ...

- need to evaluate the likelihood f'n
- depends on what CMS and ATLAS will publish

  need (model-dependent) datasets and pdfs

- CMSSM of general interest: obvious start, but need to go beyond
- adopt Les Houches accord?
- can expts publish data with and and w/o detector unfolding?
- employ workspace of RooFit/RooStats?
- ???

Summary

- MCMC + Bayesian statistics: powerful tool for LHC/Planck era to properly analyze multi-dim. "new physics" models like SUSY
- tool: SuperBayes package, available from www.superbayes.org
- CMSSM, CNSSM, NUHM, ... – currently underconstrained
- ATLAS CMSSM SU3 benchmark point with $\mathbf{1}$ fb^{-1}
  - Bayesian reconstruction of CMSSM parameters with simple Gaussian approximation and public info (except covariance matrix) comparable to that claimed in ATLAS analysis
  - almost no prior dependence left
  - theory extras: adding WMAP or Planck info on $\Omega_\chi h^2$, etc: strong impact on further reducing CMSSM parameter errors
- need to discuss/agree on the form/amount of info to be released by ATLAS and CMS