Measurements of Collision Center-of-Mass Energy with Physics Events at ILC

Graham W. Wilson

University of Kansas

September 22, 2022

- Based on "Center-of-mass energy determination using $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ events at future $\mathrm{e}^+\mathrm{e}^-$ colliders" (2209.03281) with Brendon Madison. Comments welcome.
- Similar methodology to that presented in WP4 by Patrick for beam energy spread.
- We emphasize using a muon **momenta** based estimator, $\sqrt{s}_p$, to measure the absolute $\sqrt{s}$ scale without the collinear ISR assumption.
- Need exquisite control of tracker momentum scale and great momentum resolution.
- Can work at all $\sqrt{s}$ and especially for $\sqrt{s} \approx M_{\mathrm{Z}}$.
- Focus is **ILC**, but relevant to any $\mathrm{e}^+\mathrm{e}^-$ collider. Eg. C^3, HELEN, ReLiC, FCC-ee.

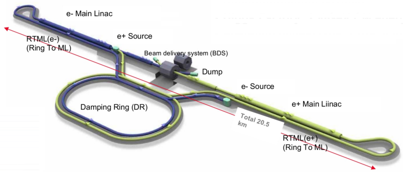
Outline

1. ILC Accelerator + Detectors
2. Physics Targets for $\sqrt{s}$ Knowledge
3. Example: Z observables
4. $E_{\rm beam}$, $\sqrt{s}$, luminosity spectrum
5. Methods Overview
6. Applying $\sqrt{s}_p$ to ILC
7. Results
8. Outlook and Future Work / R&D Directions
9. Summary

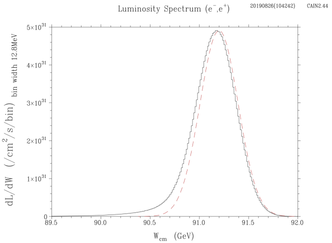
The ILC linear e^+e^- collider has been designed with an emphasis on an **initial-stage Higgs factory** that starts at $\sqrt{s} = 250$ GeV and is **expandable in energy** to run at higher energies for pair production of top quarks and Higgs bosons, and potentially to 1 TeV and more.

Particular strengths: **Longitudinally polarized electron and positron beams** and **higher energies**. Many new measurement possibilities. Very complementary to those feasible with unpolarized & lower energy reach e^+e^- circular colliders.

The ILC is designed primarily to explore the 200 – 1000 GeV energy frontier regime. This has been the focus in making the case for the project.
It is also capable of running at the **Z** and **WW** threshold.



See B. List's talk for ILC details (p22) and ILC Snowmass Report (2203.07622).



Z running – see Yokoya, Kubo, Okugi

ILC Parameters

Quantity	Symbol	Unit	Initial	$\mathcal{L}$ Upgrade	Z pole	Upgrades		
Centre of mass energy	$\sqrt{s}$	GeV	250	250	91.2	500	250	1000
Luminosity	$\mathcal{L}$	$10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	1.35	2.7	0.21/0.41	1.8/3.6	5.4	5.1
Polarization for e^-/e^+	$P_-(P_+)$	%	80(30)	80(30)	80(30)	80(30)	80(30)	80(20)
Repetition frequency	f_{rep}	Hz	5	5	3.7	5	10	4
Bunches per pulse	n_{bunch}	1	1312	2625	1312/2625	1312/2625	2625	2450
Bunch population	N_{e}	10^{10}	2	2	2	2	2	1.74
Linac bunch interval	Δt_{b}	ns	554	366	554/366	554/366	366	366
Beam current in pulse	I_{pulse}	mA	5.8	8.8	5.8/8.8	5.8/8.8	8.8	7.6
Beam pulse duration	t_{pulse}	μs	727	961	727/961	727/961	961	897
Average beam power	P_{ave}	MW	5.3	10.5	1.42/2.84*)	10.5/21	21	27.2
RMS bunch length	σ^*_{z}	mm	0.3	0.3	0.41	0.3	0.3	0.225
Norm. hor. emitt. at IP	$\gamma\epsilon_{\mathrm{x}}$	μm	5	5	6.2	5	5	5
Norm. vert. emitt. at IP	$\gamma\epsilon_{\mathrm{y}}$	nm	35	35	48.5	35	35	30
RMS hor. beam size at IP	σ^*_{x}	nm	516	516	1120	474	516	335
RMS vert. beam size at IP	σ^*_{y}	nm	7.7	7.7	14.6	5.9	7.7	2.7
Luminosity in top 1 %	$\mathcal{L}_{0.01}/\mathcal{L}$		73 %	73 %	99 %	58.3 %	73 %	44.5 %
Beamstrahlung energy loss	δ_{BS}		2.6 %	2.6 %	0.16 %	4.5 %	2.6 %	10.5 %
Site AC power	P_{site}	MW	111	128	94/115	173/215	198	300
Site length	L_{site}	km	20.5	20.5	20.5	31	31	40

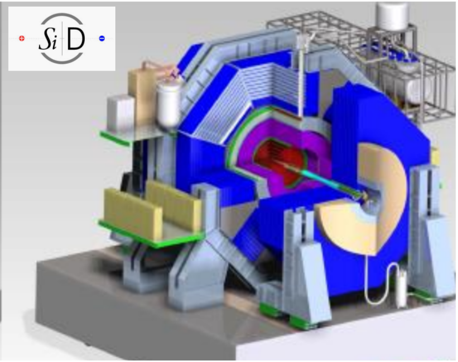
Intrinsic linac beam energy spread is about 200 MeV at all energies.
(140 MeV at 45.6 GeV (0.30%) - longer σ_z).

ILC Detectors

Modern detectors designed for ILC

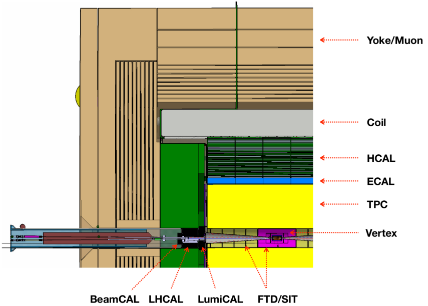
ILD = International Large Detector
(also ILD Interim Design Report (IDR))

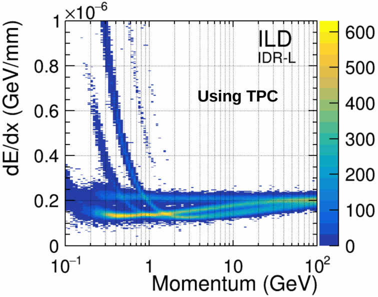
SiD = Silicon Detector



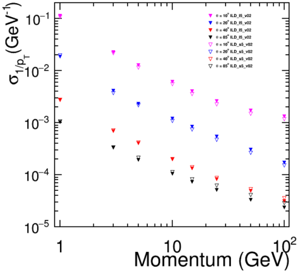
- B=3.5–5T. Particle-flow for hadronic jets. **Very hermetic**.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

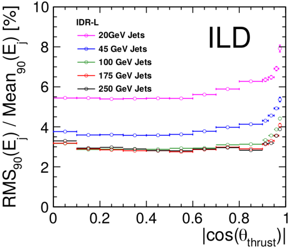
ILD Detector (See IDR: 2003.01116)

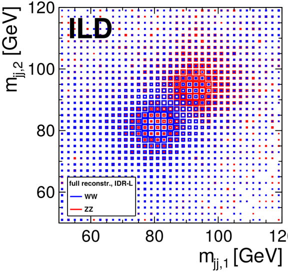




Momentum Resolution







Snowmass LoI Studies

Studies were undertaken:

1. to understand ILC capabilities for a precision measurement of the Z lineshape observables with a **scan** using longitudinally polarized beams,
2. to further explore an experimental strategy for $\sqrt{s}$ determination using di-leptons, and
3. to further explore M_{W} capabilities synergistic with a concurrent Higgs program.

Focus of this talk: reporting progress on experimental issues associated with **center-of-mass energy** (item 2) which is a pre-requisite for fully exploiting a polarized Z scan (item 1) and underpin M_{W} prospects (item 3).

Key Issue: Systematic control for the absolute scale of (**in collision...**) **center-of-mass energy** at **all** C-o-M energies

Note: 10^{10} hadronic Z's - 0.001% uncertainties - already a big challenge for absolute observables. Less so for asymmetries and relative cross-sections vs $\sqrt{s}$.

ILC Physics Targets — Energy ($\sqrt{s}$) Requirements

Core Program

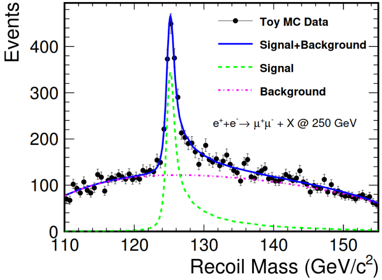
Observable	M_{H}	m_{t}	M_{W}	M_{X}
Method	Recoil mass	Scan	Reconstruction	Scan?
Best $\sqrt{s}$ [GeV]	250	350	250	Highest?
Current precision [MeV]	170	300	12	–
Target precision [MeV]	10	20	2	?
$\sqrt{s}$ contribution [MeV]	3	6	0.6	?
$\sqrt{s}$ uncertainty goal [ppm]	100	200	10	100?

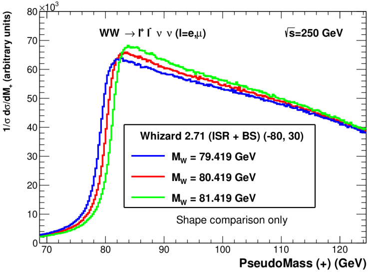
Ultimate Impact/Reach

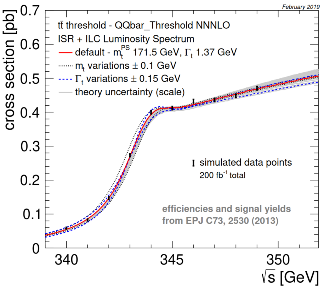
Observable	M_{W}	M_{Z}	Γ_{Z}	A_{LR}
Method	Scan	Scan	Scan	Count/Scan
Best $\sqrt{s}$ [GeV]	161	91	91	91
Current precision	15*	2.1	2.3	1.9×10^{-3}
Target precision	2 MeV	0.2 MeV	0.11 MeV	3.5×10^{-5}
$\sqrt{s}$ contribution	0.8 MeV	0.2 MeV	small	1.8×10^{-5}
$\sqrt{s}$ uncertainty goal [ppm]	10	2	5**	10

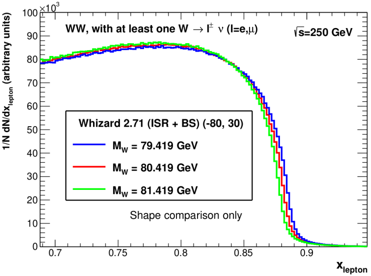
*(post CDF ...), **(point-to-point most relevant)

Example Physics Importance of $\sqrt{s}$ Knowledge



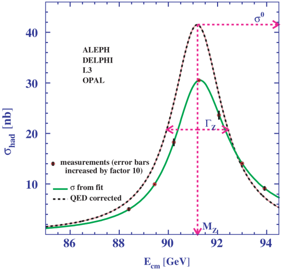




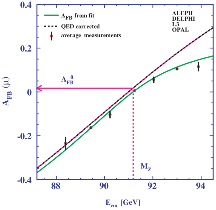


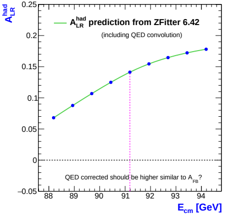
Polarized Beams Z Scan for Z LineShape and Asymmetries

Essentially, perform LEP/SLC-style measurements in all channels but also with $\sqrt{s}$ dependence of the polarized asymmetries, A_{LR} and $A^f_{FB,LR}$, in addition to A_{FB}.
(Also polarized $\nu\overline{\nu}\gamma$ scan.) Not constrained to LEP-style scan points.



LEP: $\Delta M_{\mathrm{Z}} = 2100$ MeV, $\Delta\Gamma_{\mathrm{Z}} = 2300$ MeV





With 0.1 ab^{-1} polarized scan around M_{Z}, find **statistical** uncertainties of 35 keV on M_{Z}, and 80 keV on Γ_{Z}, from LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER for QED convolution.

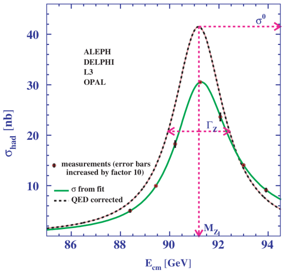
Exploiting this fully needs in-depth study of $\sqrt{s}$ **calibration systematics**

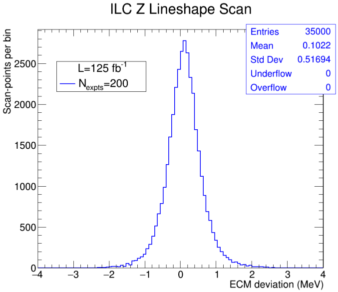
ILC $\mathcal{L}$ is sufficient for M_{Z} to be systematics limited

Γ_{Z} systematic uncertainty depends on $\Delta(\sqrt{s}_+ - \sqrt{s}_-)$, so expect $\Delta\Gamma_{\mathrm{Z}} \ll \Delta M_{\mathrm{Z}}$

Polarized Beams Z Scan for Z LineShape Study: WIP I

Initial line-shape study (all 4 channels). Use unpolarized cross-sections for now.





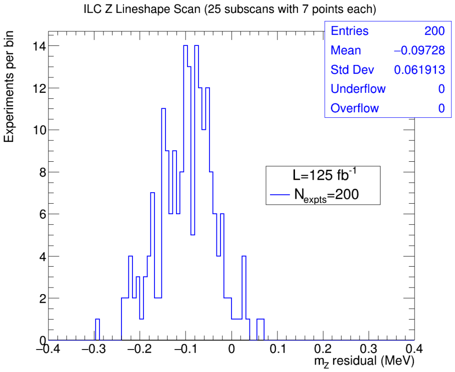
Uses $\sigma_{stat}/\sqrt{s}$ (%) $= 0.25/\sqrt{N_{\mu\mu}} \oplus 0.8/\sqrt{N_{\rm h}}$

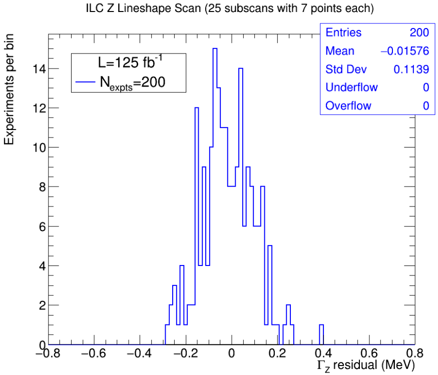
- Scan has 7 nominal $\sqrt{s}$ points, (peak, $\pm\Delta, \pm 2\Delta \pm 3\Delta$) with $\Delta = 1.05$ GeV
- 25 scans of 5 fb^{-1} per "experiment". $7 \times 25 \times 4 = 700$ $\sigma_{\rm tot}$ measurements.
- Assign luminosity per scan point in (2:1:2:1) ratio. (1 or 0.5 fb^{-1} each).
- Do LEP-style fit to ($M_{\rm Z}, \Gamma_{\rm Z}, \sigma^0_{\rm had}, R^0_e, R^0_\mu, R^0_\tau$) using ZFITTER
- Model center-of-mass energy systematics and int. lumi syst. of 0.064%.
- Each scan-point (175 per expt.) shifted from $\sqrt{s}_{\rm nominal}$ by a 100% **correlated** overall scale systematic (here +100 keV) and by stat. component driven by stat. uncertainty of $\sqrt{s}$ measurement (typically 0.4 MeV/4.4 ppm).

Polarized Beams Z Scan for Z LineShape Study: WIP II

Ensemble tests with 200 experiments.

Currently, fit the 700 measured cross-sections (actually occuring at shifted $\sqrt{s}$) using assumed nominal $\sqrt{s}$. Ensemble mean χ^2 of 790 for 693 dof.





- As expected M_Z biased down by assumed scale error (here +100 keV) with stat. error of 50–60 keV.
- As expected Γ_Z bias small with stat. dominated error of 100–120 keV.
- Such an experiment has 1.9B hadronic Zs.

ILC $A_{\rm LR}$ Prospects from Z Running

Use 4 cross-section measurements ($\sigma_{\pm\pm}$) to measure simultaneously:

$$A_{\rm LR},\ |P(e^-)|,\ |P(e^+)|,\ \sigma_u$$

L (fb^{-1})	$N_Z^{\rm had}$ (10^9)	$\lvert P(e^-)\rvert$	$\lvert P(e^+)\rvert$	$\Delta A_{\rm LR}$ (stat.)	$\Delta A_{\rm LR}$ (syst).
100	3.3	80%	30%	4.3×10^{-5}	1.3×10^{-5}
100	4.2	80%	60%	2.4×10^{-5}	1.3×10^{-5}
250	8.4	80%	30%	2.7×10^{-5}	1.3×10^{-5}
250	11	80%	60%	1.5×10^{-5}	1.3×10^{-5}

Estimated uncertainties on $A_{\rm LR}$ for 4 different scenarios of Z-pole running with data-taking fractions in each helicity configuration $(-+), (+-), (--), (++)$ chosen to minimize the statistical uncertainty on the asymmetry. The quoted statistical uncertainty includes Bhabha statistics for relative luminosity and Compton statistics for polarization differences. The systematic uncertainty assumes 5 ppm uncertainty on the absolute center-of-mass energy and a 1% understanding of beamstrahlung effects. Estimates assume data taken at a single center-of-mass energy (91.2 GeV).

Total uncertainty on $A_{\rm LR}$ of 4.5×10^{-5} (scenario 1) to 2.0×10^{-5} (scenario 4). Corresponds to uncertainty on $\sin^2\theta^{\ell}_{\rm eff}$ of 5.6×10^{-6} (1) to 2.5×10^{-6} (4).

Beam Energy and Beam Energy Spread

- Upstream diagnostics. Chicane BPM spectrometer. Energy target: $\mathcal{O}(10^{-4})$.
- Downstream diagnostics. Targets $\mathcal{O}(10^{-4})$. SLC-style synchrotron radiation stripes spectrometer - sees beams after beam-beam effects.
- Beam energy spread?, and distribution?
- Energy-z correlations?
- Also pass-through non-collision mode (to inter-calibrate upstream/downstream)?

While these may not provide the ultimate absolute beam energy uncertainty, they should be extremely useful for tracking **relative** beam energies especially for scans and for short-term variations.

So expect: $< E_-^U >, < E_+^U >, < E_-^D >, < E_+^D >$ on a bunch-by-bunch basis?

Beam/Center-of-Mass Energy, Luminosity Spectrum

Center-of-Mass Energy

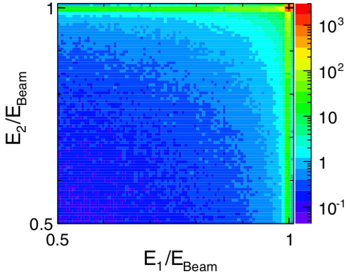
- Naively, $\sqrt{s} = 2E_{\mathrm{b}}$
- Less naively, $\sqrt{s} = 2\sqrt{E_{-}^{C} E_{+}^{C}} \cos(\alpha/2)$ ($\alpha = 14$ mrad crossing-angle)
- E_{-}^{C}, E_{+}^{C} are the actual collision energies (including BES + possible BS)

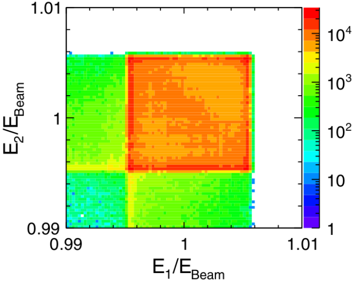
Collision Momentum Imbalance

- Mostly in z, but also in x
- $p_x = (E_{-}^{C} + E_{+}^{C}) \sin(\alpha/2)$
- $p_z = (E_{-}^{C} - E_{+}^{C}) \cos(\alpha/2)$

What is most important is the **distribution** of the collision initial-state 4-vector weighted by luminosity.
This is usually called the **luminosity spectrum**, and is either 1-d ($\sqrt{s}$) or 2-d (E_{-}^{C}, E_{+}^{C}). Potentially even 3-d or more, eg. in (E_{-}^{C}, E_{+}^{C}) for slices in z_{int}.
Needs to be unfolded from collision physics events gathered over long time periods. Necessarily averages over all the variations in conditions.

There are a number of studies of the luminosity spectrum, incl. (Frary, Miller), Moenig, (Boogert, Miller), Sailer, and (Poss, Sailer). Use **Bhabhas** with $\theta > 7^\circ$. State of the published art is Poss and Sailer study for CLIC 3 TeV.





$$
\begin{aligned}
\mathscr{L}(x_1, x_2) = {} & p_{\mathrm{Peak}}\,\delta(1-x_1)\otimes \mathrm{BES}(x_1;[p]^1_{\mathrm{Peak}}) \\
& \delta(1-x_2)\otimes \mathrm{BES}(x_2;[p]^2_{\mathrm{Peak}}) \\
& + p_{\mathrm{Arm1}}\,\delta(1-x_1)\otimes \mathrm{BES}(x_1;[p]^1_{\mathrm{Arm1}}) \\
& \mathrm{BB}(x_2;[p]^2_{\mathrm{Arm1}},\beta^{\mathrm{Arm}}_{\mathrm{Limit}}) \\
& + p_{\mathrm{Arm2}}\mathrm{BB}(x_1;[p]^1_{\mathrm{Arm2}},\beta^{\mathrm{Arm}}_{\mathrm{Limit}}) \\
& \delta(1-x_2)\otimes \mathrm{BES}(x_2;[p]^2_{\mathrm{Arm2}}) \\
& + p_{\mathrm{Body}}\mathrm{BG}(x_1;[p]^1_{\mathrm{Body}},\beta^{\mathrm{Body}}_{\mathrm{Limit}}) \\
& \mathrm{BG}(x_2;[p]^2_{\mathrm{Body}},\beta^{\mathrm{Body}}_{\mathrm{Limit}}),
\end{aligned}
$$

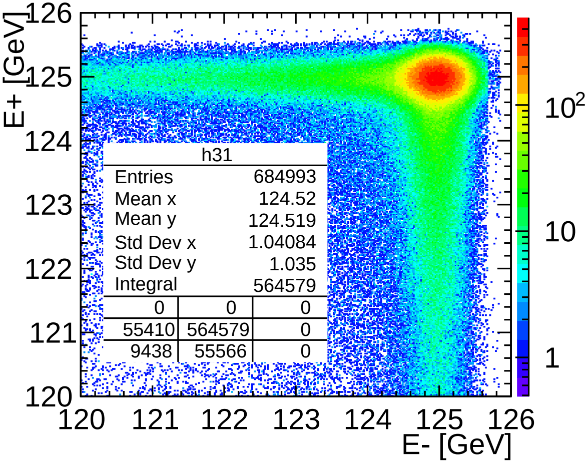
Parametrize the lumi spectrum resulting from beam-beam simulations (Guinea-PIG) and incorporate in measurement using (E_1, E_2, θ_{acol}). [Currently working on related parametrization approach for ILC using reweighting fits.]

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Shortly, we'll look at the related 1-d distributions $(E_+, E_-, \sqrt{s}, p_z)$ with empirical fits.

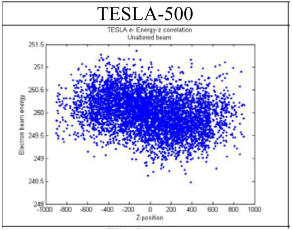
Absolute energies of peak position (E) and shape (LS)

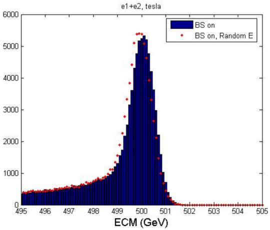
[dL/d$\sqrt{s}$: see work by Boogert, Frary, Miller, Moenig, Sailer, Poss]



Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

- One very important issue is understanding the E-z distribution of the beams presented to the interaction point.
- Wakefield effects can distort the E-z distribution. Also RF phasing/kink instability avoidance? (BNS damping??)
- Plot shows modeled ECM distribution with correlation and without (red) from Woods/Florimonte study of 2005.





Current centralized Whizard simulations assume uncorrelated Gaussian beams as do my initial Guinea-PIG forays.

In situ Methods Related to Center-of-Mass Energy

There are three main techniques currently envisaged using collision physics events. They are inter-related and should be carried out in a global analysis.

Methods

1. $\sqrt{s}_A$: The radiative return to the Z method. (Wilson - Munich96, LEP2, Moenig, Hinze)
2. $\sqrt{s}_p$: The dilepton momenta method. (Barklow - LCWS05, Wilson)
3. $\theta_{\rm acol}$: Bhabha acollinearity angle. (Frary-Miller 91)

Comments

All three use particle direction measurements and a ≤ 3 particle final-state approximation

1: Relies on $M_{\rm Z}$ for energy scale

2: Relies on tracker momentum scale for energy scale

3: More focused on lumi. spectrum to date than energy

1+2: focus of existing studies has been $\mu^+\mu^-$

2: Includes radiative return and full energy events.

$\sqrt{s}_A$ Method for Center-of-Mass Energy

Use radiative return events to the Z with precision angular measurements.

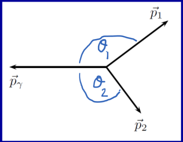
Assume one photon recoiling from $\mu^+\mu^-$

$$x_\gamma \equiv \frac{E_\gamma}{E_b} = 1 - \frac{m_{12}^2}{s}$$

At $\sqrt{s} = 250$ GeV, $x_\gamma = 0.867$, $E_\gamma = 108$ GeV, for $m_{12} = M_\mathrm{Z}$.

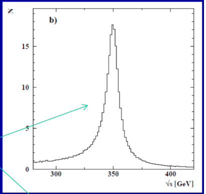
Write $m_{12}^2/s = f(\theta_1, \theta_2)$. Then assume, $m_{12} = M_\mathrm{Z}$.

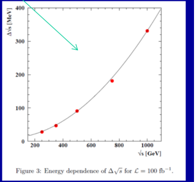
$e^+ e^- \to Z\,(\gamma) \to \mu^+\mu^-(\gamma)$



GWW – MPI 96
LEP Collabs.

Hinze & Moenig





Photon often not detected.
Use muon angles to (photon/beam-axis).
Requires precision polar angle.

$$\sqrt{s} = m_Z \sqrt{\frac{\sin\theta_1 + \sin\theta_2 - \sin(\theta_1+\theta_2)}{\sin\theta_1 + \sin\theta_2 + \sin(\theta_1+\theta_2)}}$$

Statistical error per event of order Γ/M = 2.7%

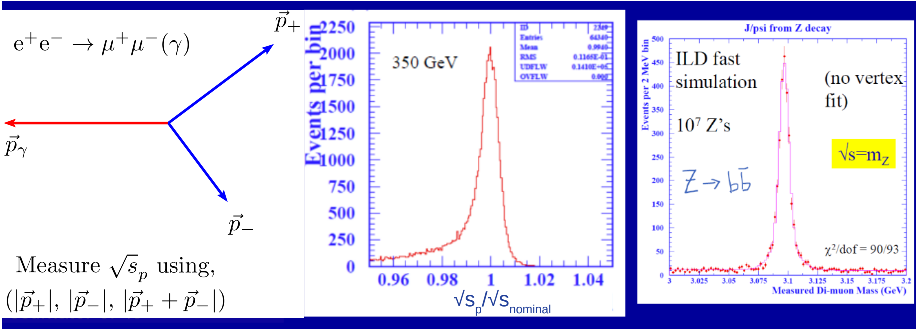
Acceptance degrades quickly at high √s

- uses M_Z and is limited in ultimate precision by its knowledge (23 ppm).
- can also use $\mathrm{e^+e^-}$, and even $\tau^+\tau^-$ decays of the Z (maybe also $\mathrm{Z} \to$)
- per event uncertainty poor given Γ_Z

Most recent study in K. Moenig talk and proceedings from LCWS05.

$\sqrt{s}_p$ Method for Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.



Tie detector p-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc).

Bhabhas and acollinearity

Forward Bhabhas ($\mathrm{e^+e^-} \to \mathrm{e^+e^-}$) with scattering angles above 7° are widely discussed mainly for luminosity spectrum measurements.

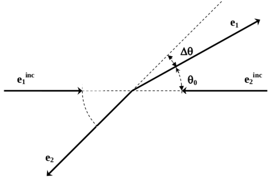
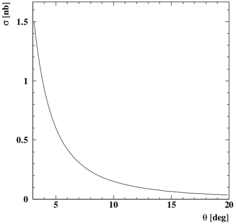


Figure 22 - Bhabha scattering after radiation loss by the incoming particle e_2^{inc} due either to ISR or beamsstrahlung. The acolinearity angle ΔΘ arises from the energy difference in the incoming particles e_1^{inc} and e_2^{inc}.



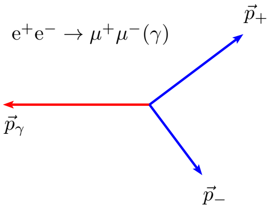
$\sqrt{s} = 500$ GeV

The original literature focused on the acollinearity angle, that measures the **momentum imbalance** of the two beams, (rewritten here using E given $E \approx p$),

$$\Delta p = (E_- - E_+) = \frac{E_b \theta_{\mathrm{acol}}}{\sin\theta_0}$$

One can also use x_γ or s'/s notation as before (with the photon along the direction of lost momentum). No reference energy scale like M_{Z}. Need to rely on spectrometer info or on direct energy measurements. Foreseen endcap E,p resolution not great. Large statistics. Δp uncertainty gets amplified by $1/\sin\theta_0$ term at very forward angle - so not so much to gain with wider acceptance. Can explore $\sqrt{s}_p$ too.

$\sqrt{s}_p$ Method in a Nutshell



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

- **Equal** beam energies, E_b
- The lab **is** the CM frame, $(\sqrt{s} = 2\, E_\mathrm{b},\ \sum \vec{p}_i = 0)$
- The system recoiling against the dimuon is **massless**

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

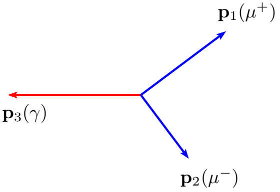
$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

[Now, $\sqrt{s}$ estimators previously extended to allow a crossing angle and beam energy difference are extended to the general case with a massive recoil. Work in progress on applying constrained fits]

With ILD detector at ILC - expect 0.17% momentum resolution for typical 71 GeV muon in $Z\gamma$ events at $\sqrt{s} = 250$ GeV. Detector-level studies are with full simulation and reconstruction.

General case has 3 nuisance parameters: the crossing angle, α, the collision energy asymmetry, $(E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+})/(E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) = \overline{\Delta E_{\mathrm{b}}}/E_{\mathrm{ave}}$, and the recoil mass, M_3.



1. $\sqrt{s} = E_1^* + E_2^* + E_3^* = E_{12}^* + E_3^*$
2. $\sqrt{s} = E_{12}^* + \sqrt{(p_{12}^*)^2 + M_3^2}$ (general M_3)
3. $\sqrt{s} = E_{12}^* + |\mathbf{p}_{12}^*|$ (assuming $M_3 = 0$)

We have the measured dimuon 4-vector in the detector frame $(E_{12}, \mathbf{p}_{12})$. Need to apply the appropriate boost from lab back to the CM frame to obtain $(E_{12}^*, \mathbf{p}_{12}^*)$. The boost velocity (in the horizontal plane) is

$$\boldsymbol{\beta} = (\beta_x, \beta_y, \beta_z) = (\sin(\alpha/2), 0, \frac{\overline{\Delta E_{\mathrm{b}}}}{E_{\mathrm{ave}}} \cos(\alpha/2))$$

$\beta_x = 0.007/0.015$ (ILC/FCC-ee). β_z depends on the collision energy asymmetry.

Generator-level Examples

Event	1	2	3	4	5	6
E_{b}^{-}	125.34	114.55	125.32	124.87	124.75	122.77
E_{b}^{+}	124.82	124.64	121.08	124.49	116.24	110.12
$\overline{\Delta E_{\mathrm{b}}}$	$+0.26$	-5.04	$+2.12$	$+0.19$	$+4.26$	$+6.33$
M_{12}	92.55	238.97	94.62	249.30	82.34	92.26
p_{12}	108.41	10.22	104.74	1.73	101.66	105.43
p_{12}^{x}	$+18.82$	$+1.67$	$+1.25$	$+1.70$	$+0.92$	$+1.03$
p_{12}^{y}	-14.54	0.00	$+0.21$	-0.01	0.00	-0.25
p_{12}^{z}	$+105.77$	-10.08	$+104.73$	$+0.35$	-101.65	$+105.43$
p_3	107.62	0.00	100.49	0.06	110.17	92.78
M_3	0.00	0.00	31.27	0.00	0.55	0.00
$\sqrt{s}$	250.15	238.97	246.35	249.35	240.84	232.53
E_{12}^{*} (β_x)	142.41	239.18	141.15	249.30	130.82	140.10
p_{12}^{*} (β_x)	108.24	10.08	104.73	0.35	101.65	105.43
$\sqrt{s_p}$	250.65	249.26	245.88	249.65	232.47	245.53
E_{12}^{*} (β)	142.20	238.97	139.36	249.30	134.49	134.57
p_{12}^{*} (β)	107.96	0.00	102.32	0.06	106.34	97.96
$\sqrt{s_p}$ (true $\overline{\Delta E_{\mathrm{b}}}$)	250.15	238.97	241.60	249.35	240.84	232.53
$\sqrt{s_p}$ (true M_3)	250.65	249.26	250.45	249.65	232.47	245.53

Makes use of radiative-return ($Z\gamma$) events too.

Introduction to Center-of-Mass Energy Issues

- Proposed $\sqrt{s}_p$ method uses only the momenta of leptons in dilepton events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics). More in depth talks on $\sqrt{s}$: ILC physics seminar and ILC MDI/BDS/Physics talk

Today,

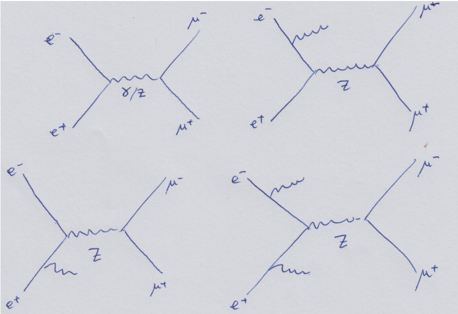
- Overview of the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$
- Brief overview of the "new" concept in recent tracker momentum scale studies (LCWS2021 talk).
- Bonus. Physics: M_{Z}. Beam knowledge: **luminosity spectrum**, $\mathrm{dL/d}\sqrt{s}$.

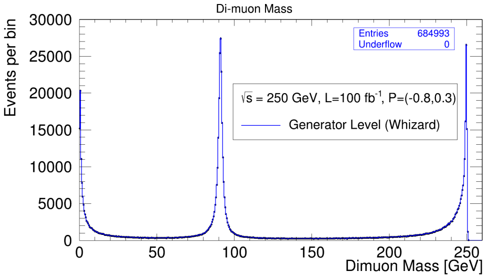
Dimuons

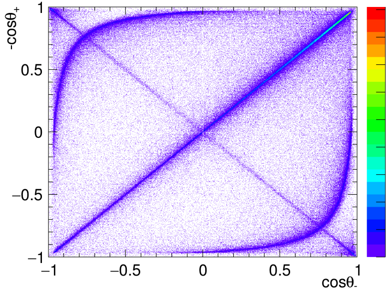
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass,
   $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.







New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

1. Explore AP method using mainly $\mathrm{K_S^0} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

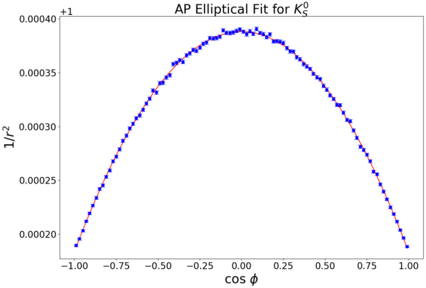
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

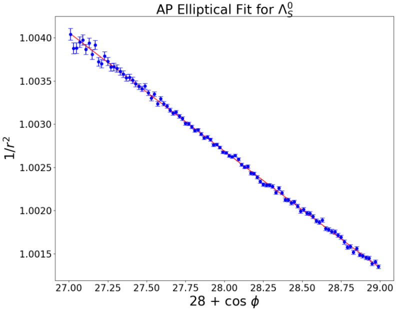
By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents ($\mathrm{K_S^0}$ and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work when typical existing experiments are at best at the 100 ppm level

Tracker momentum scale sensitivity estimate

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K^0_S}, \Lambda, \overline{\Lambda}$ in various momentum bins.





1. $m_{\mathrm{K^0_S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

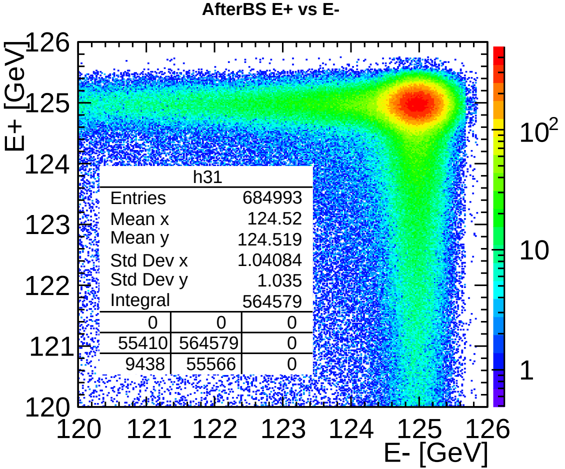
- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K^0_S}, \Lambda/\overline{\Lambda}, \pi^\pm)$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z, ILC Z run may have 400 such samples.

What do we really want to measure?

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Now let's look at the related 1-d distributions $(E_+, E_-, \sqrt{s}, p_z)$ with empirical fits.

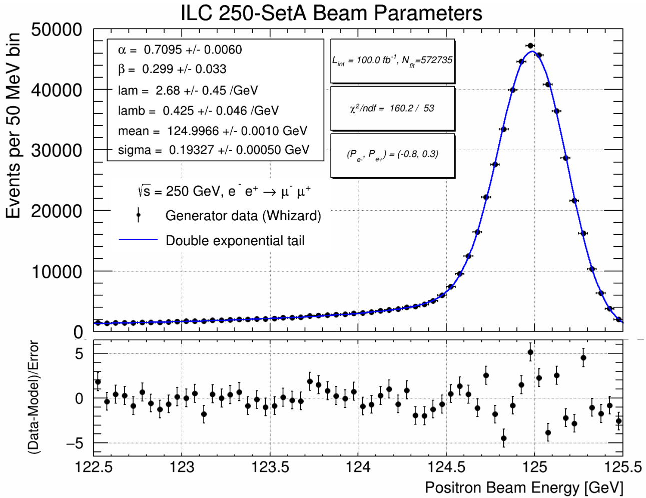
[dL/d$\sqrt{s}$: see work by Frary, Miller, Moenig, Sailer, Poss]



Whizard 250 GeV SetA $e^+e^- \to \mu^+\mu^-(\gamma)$ events

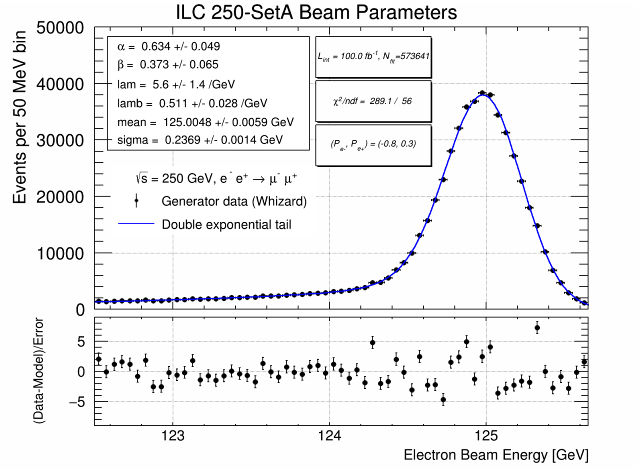
Positron Beam Energy (After Beamstrahlung)

Fits with (double-exponential tail + delta-function) convolved with Gaussian beam energy spread (6 parameters).



$$\sigma/E = 0.1546 \pm 0.0004\% \text{ (cf } 0.152\% \text{ in TDR)}$$

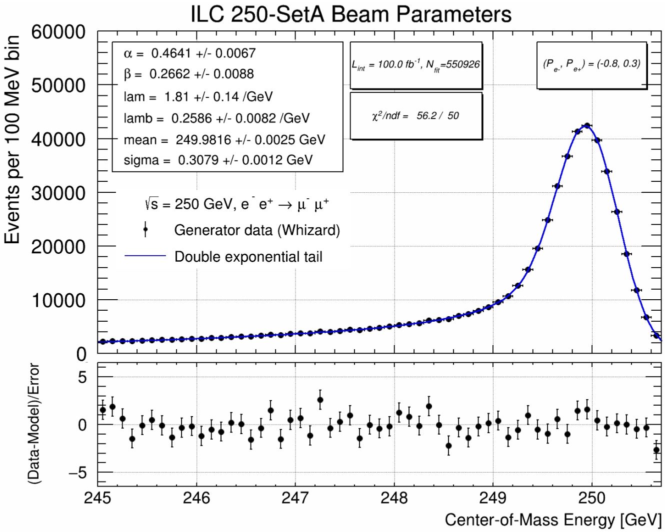
Electron Beam Energy (After Beamstrahlung)



$$\sigma/E = 0.1895 \pm 0.0011\% \text{ (cf } 0.190\% \text{ in TDR)}$$

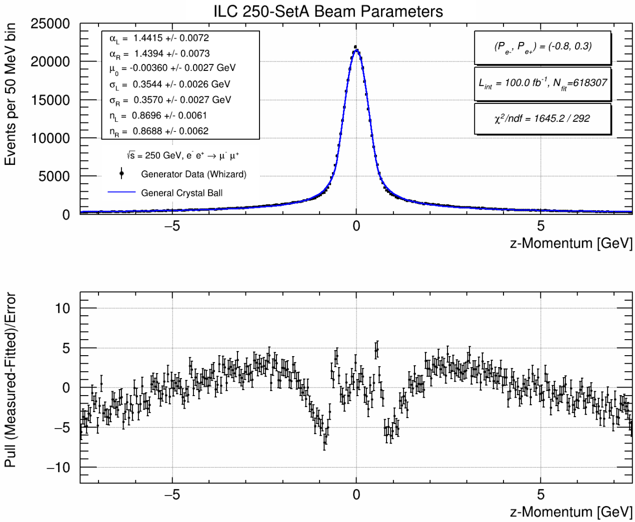
Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.

Center-of-Mass Energy (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1232 \pm 0.0005\%$ (cf 0.122% in TDR ($0.190\% \oplus 0.152\%)/2$)

z-Momentum of e^+e^- system (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the electron beam and positron beam respectively.
Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$
$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$
$$p_y = 0$$
$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α is known (14 mrad for ILC), evaluation of the collision center-of-mass energy amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \ , \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^{2} - (\overline{\Delta E_{\mathrm{b}}})^{2}} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$ process. Denote the μ^+ as particle 1, the μ^- as particle 2, and the rest-of-the event (RoE) as system 3. We can write this final-state system 4-vector as

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Applying $(E, \vec{p})$ conservation we obtain,

$$E_1 + E_2 + \sqrt{p_3^2 + M_3^2} = 2\ E_{\mathrm{ave}} \tag{1}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{2}$$

The RoE is often not fully detected and needs to be inferred using $(E, \vec{p})$ conservation. We have 4 equations and 6 unknowns:
the 3 components of the RoE momentum ($\vec{p_3}$), E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, and M_3.
Our approach is to solve for E_{ave} for various assumptions on ($\overline{\Delta E_{\mathrm{b}}}$, M_3).
Specifically we then focus on using the simplifying assumptions of the original $\sqrt{s}_p$ method that $M_3 = 0$ and $\overline{\Delta E_{\mathrm{b}}} = 0$. Note: latter is often a poor assumption for the p_z conservation component on an event-to-event basis.

The Averaged Beam Energy Quadratic

This approach results in a quadratic equation in $E_{\rm ave}$, ($AE_{\rm ave}^2 + BE_{\rm ave} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

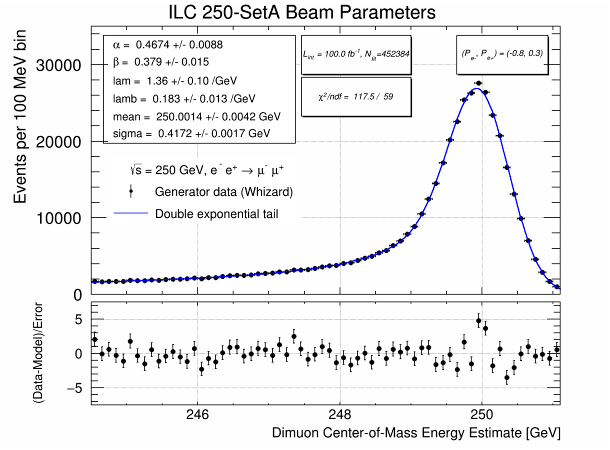
$$C = (M_{12}^2 - M_3^2)/4 + p_{12}^z \overline{\Delta E_{\rm b}} \cos(\alpha/2) - \overline{\Delta E_{\rm b}}^2 \cos^2(\alpha/2)$$

Based on this, there are a number of cases of interest to solve for $E_{\rm ave}$:

1. Zero crossing angle, $\alpha = 0$, $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
2. Crossing angle and $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
3. Crossing angle and $\overline{\Delta E_{\rm b}}$ non-zero, $M_3 = 0$.
4. Crossing angle and M_3 non-zero, $\overline{\Delta E_{\rm b}} = 0$.
5. Crossing angle and $\overline{\Delta E_{\rm b}}$ and M_3 non-zero.

The original formula, $\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$, arises trivially in the first case. In the rest of this talk the $\sqrt{s}$ estimate from the largest positive solution of the second case is what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

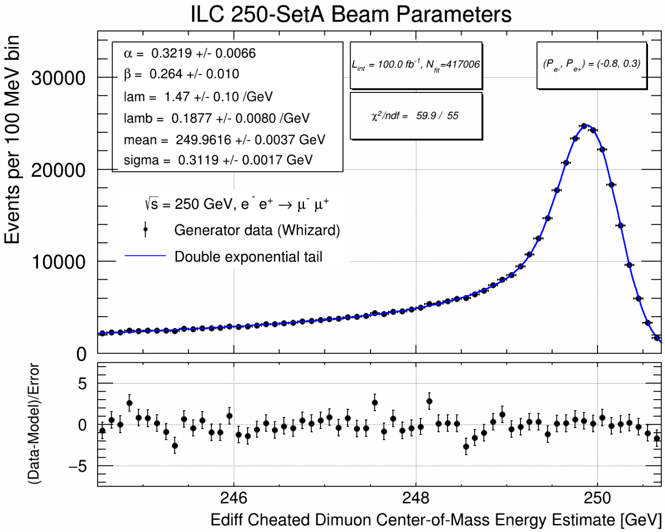
Dimuon Estimate of Center-of-Mass Energy (After BS)



$\sigma/\sqrt{s} = 0.1669 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

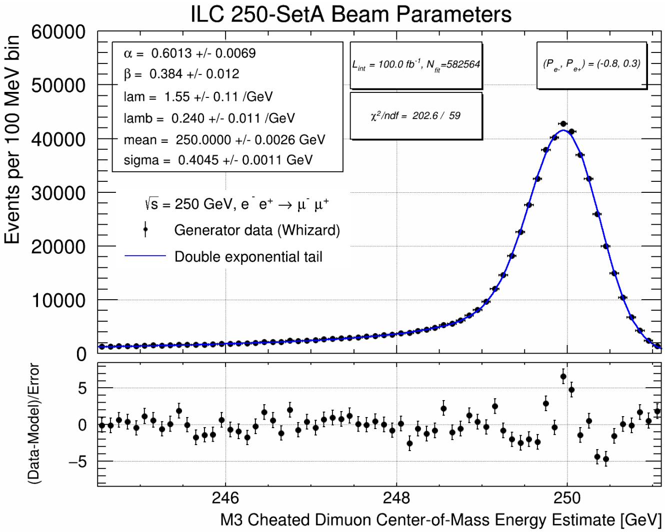
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Because some events violate the assumptions that $\overline{\Delta E_{\rm b}} = 0$ and $M_3 = 0$
- The former is no surprise given the p_z distribution
- The latter is associated with events with 2 or more non-collinear ISR/FSR photons

Cheated $\overline{\Delta E_{\mathrm{b}}}$ Center-of-Mass Energy Estimate (After BS)



$$\sigma/\sqrt{s} = 0.1248 \pm 0.0007\% \text{ (cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Cheated M_3 Center-of-Mass Energy Estimate (After BS)



$$\sigma/\sqrt{s} = 0.1618 \pm 0.0004\% \text{ (cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Generator-level $\sqrt{s_p}$ Biases

$M_{\mu^+\mu^-}$ range [GeV]	$\mu(\sqrt{s})$ [GeV]	$\mu(\sqrt{s_p})$ [GeV]	$\mu(\sqrt{s_p})$ - $\mu(\sqrt{s})$ [MeV]
$M > 150$	249.9792 ± 0.0011	250.0337 ± 0.0013	$+54.5 \pm 1.7$
$50 < M < 150$	249.9813 ± 0.0010	249.9602 ± 0.0017	-21.1 ± 2.0
$M < 50$	249.9871 ± 0.0015	249.9633 ± 0.0028	-23.8 ± 3.2
All	249.9816 ± 0.0008	250.0014 ± 0.0010	$+19.8 \pm 1.2$

Results of the 1-parameter fits for the μ parameter to the generator-level distributions of $\sqrt{s}$ and $\sqrt{s_p}$ for three different dimuon mass ranges for the 80%/30% LR helicity mixture. The statistical uncertainties of these tests reflect an integrated luminosity of 100 fb^{-1}. The last column gives the difference in MeV of the fit parameters for the two distributions.

Strong evidence that high mass events tend to be over-measured (addition of a fictitious photon in genuine 2-body $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ events), and that lower mass events are under-measured (multiple radiation more important).

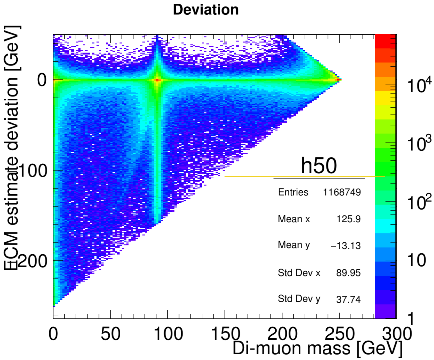
Bias associated with neglecting M_3

Naively with a mean value of M_3 of around 25 GeV, one imagines large biases for $\sqrt{s}_p$, but the median M_3 value is much lower, and examining the relevant equation, IF the boost is correct, the M_3 related bias goes as:

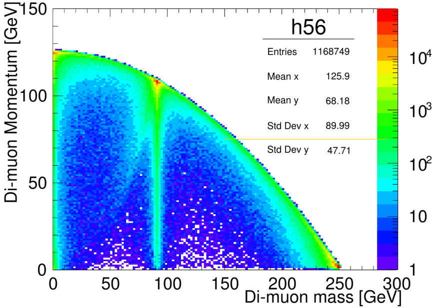
$$\Delta\sqrt{s} = |\mathbf{p}^*_{12}| - \sqrt{(p^*_{12})^2 + M_3^2}$$

So for $p_{12} = 100$ GeV, the bias for a 10 GeV M_3 is only -0.50 GeV.

Plot of $(\sqrt{s}_p - \sqrt{s})$ vs $M_{\mu\mu}$



Plot of $|p_{\mu\mu}|$ vs $M_{\mu^+\mu^-}$



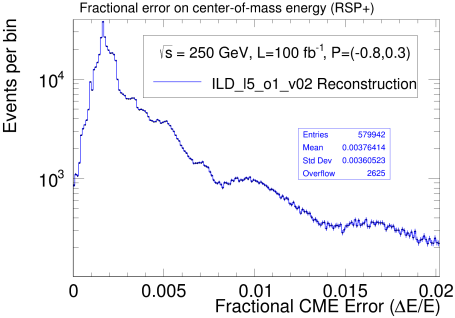
Most events consistent with $M_3 \approx 0$

In most events, $\sqrt{s}_p$, is a reasonable estimator. But also can be off by a lot. WIP on identifying problematic events (eg. kinematic fits). It may be feasible to find alternative estimators/methods in those cases, or at least reject them.

Event Selection Requirements

Currently rather simple.
Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% of $\sqrt{s}_{\mathrm{nom}}$ based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** (<0.15%), **silver** ([0.15, 0.30]%), **bronze** ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %

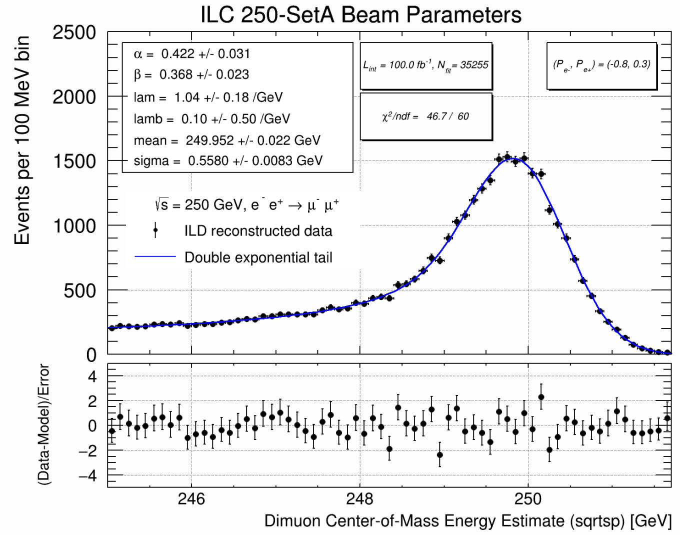


Selection efficiencies for (80%/30%) beam polarizations:

- $\varepsilon_{-+} = 69.77 \pm 0.06$ %
- $\varepsilon_{+-} = 67.35 \pm 0.06$ %
- $\varepsilon_{--} = 69.47 \pm 0.05$ %
- $\varepsilon_{++} = 67.72 \pm 0.06$ %

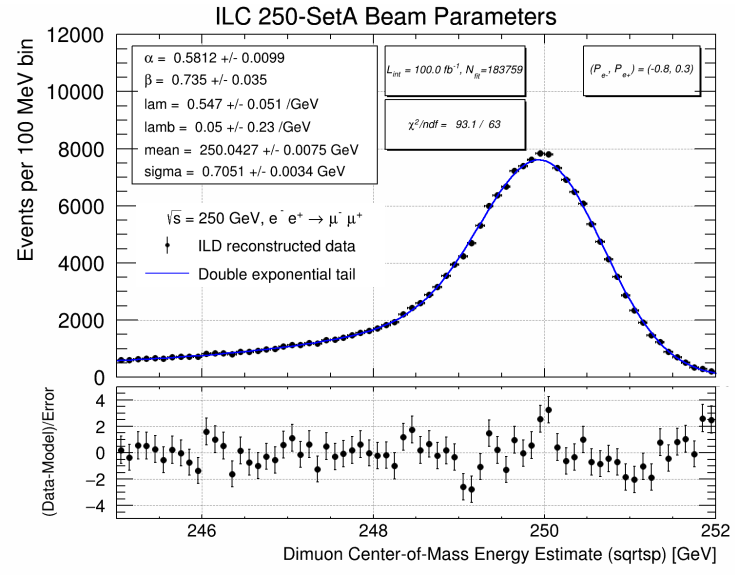
Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Gold Quality Dimuon PFOs (After BS)



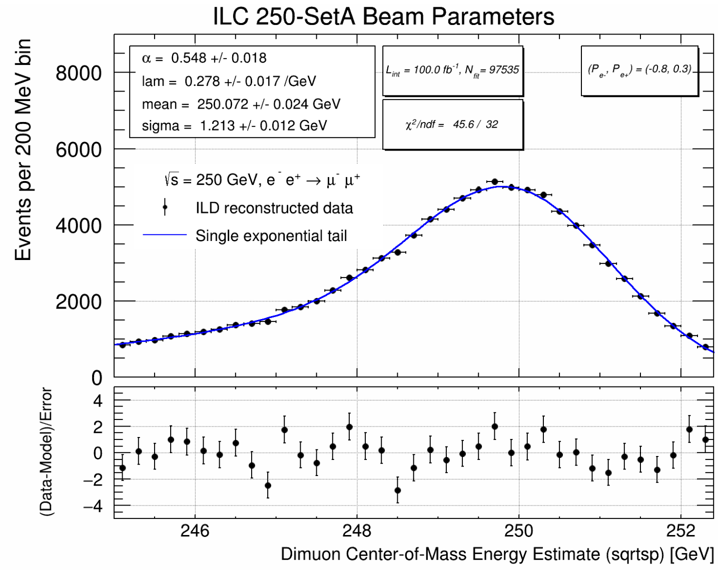
Peak width 1.34 ± 0.02 wider than $\sqrt{s_p}$ (gen).

Silver Quality Dimuon PFOs (After BS)



Peak width 1.69 ± 0.01 wider than $\sqrt{s_p}$ (gen).

Bronze Quality Dimuon PFOs (After BS)



Peak width 2.91 ± 0.03 wider than $\sqrt{s_p}$ (gen).

Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The luminosity spectrum and absolute center-of-mass energy are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- Beam energy spread likely to be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- Work is in progress on a CF by parametrizing the underlying (E_-, E_+) distribution, and modeling quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

Current Estimation Method

- Use estimates of the statistical error on the peak position for 6-parameter convolved double exponential tail fits to fully simulated data with the 5 shape parameters fixed to their best fit values.
- Fits are done in the 3 resolution categories.
- Next slide has these estimates

$\sqrt{s}$ Sensitivity Estimates at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

$L_{\rm int}$ [ab^{-1}]	Poln [%]	ε [%]	Gold	Silver	Bronze	All categories
0.9	$-80, +30$	70.4	6.4	3.1	7.7	2.6
0.9	$+80, -30$	68.0	7.5	3.4	8.7	2.9
0.1	$-80, -30$	70.1	25	12	30	10
0.1	$+80, +30$	68.3	28	13	33	11
2.0	Combined	-	4.7	2.2	5.6	**1.9**

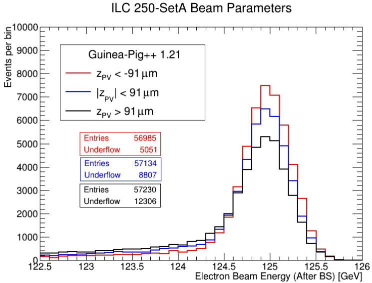
Fractional errors on μ parameter (mode of peak) when fitting with 6-parameter double exponential tail function with all 5 shape parameters fixed to their best-fit values. (4/3 for bronze).

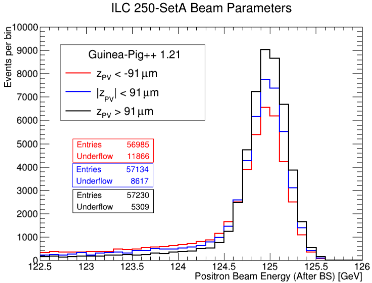
Also the $\mathrm{e}^+\mathrm{e}^-$ channel should be used. The additional benefit of the much larger statistics from more forward Bhabhas will be offset by the poorer track momentum resolution at forward angles.

Beamstrahlung / z-Vertex Effects Explained

Divide interactions in 3 equi-probability parts according to z_{PV}. Preferentially

1. e^+e^- collisions occurring more on the initial e^- side ($z < 0$)
2. e^+e^- collisions mostly central
3. e^+e^- collisions preferentially on the initial e^+ side ($z > 0$)



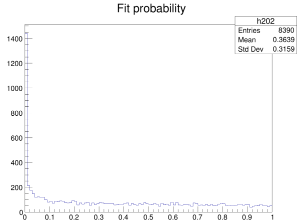


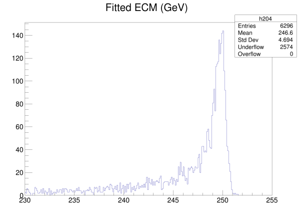
The beamstrahlung tail grows and the peak shrinks for e^- as z increases, and, for e^+ as z decreases. In both cases, the largest beamstrahlung tail occurs when the interacting e^- or e^+ has on average traversed more of the opposing bunch.

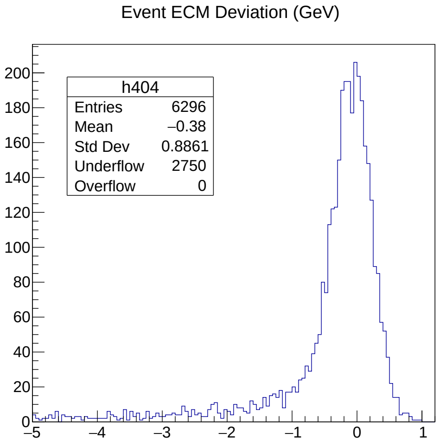
Thus both $\sqrt{s}$ and $p_z = E_- - E_+$ distributions depend on z. Likely needs to be taken into account for $\sqrt{s}$, $dL/d\sqrt{s}$, Higgs recoil, kinematic fits ...

Kinematic Fit Approach: Hot Off The Press

Test consistency with $\mathrm{e}^{+}\mathrm{e}^{-} \to \mu^{+}\mu^{-}$ (no photons) by fitting for E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ as unmeasured parameters (4C/2U/2dof). So measure $\sqrt{s}$ and collision asymmetry.







Plots require $p_{\mathrm{fit}} > 0.05$ (26% of all events). See backup for details. Use 0.15% momentum resolution. Peak width is 0.3 GeV (same as energy spread).

Outlook and Future Work

Lots of opportunities to improve this:

1. Constrained kinematic fits. For example one can test the consistency with the pure 2-body hypothesis of $e^+e^- \to \mu^+\mu^-$ while fitting for the two unmeasured parameters of E_{ave} and $\overline{\Delta E_{\text{b}}}$, and also perform fits with the $e^+e^- \to \mu^+\mu^-\gamma$ hypothesis.
2. Extend the techniques to the $e^+e^- \to e^+e^-$ channel.
3. Exploit fully events with detected photons.
4. Implement complete end-to-end measurement scheme and understand how best to use different kinematic regimes and correct/mitigate observed biases.
5. Characterize better the intrinsic limitations associated with beam energy spread, beamstrahlung, ISR, FSR, backgrounds, and detector acceptance and resolution. This includes studies with more specialized physics event generators such as KKMCee [29].
6. Tracker momentum scale studies using $J/\psi \to \mu^+\mu^-$, $K^0_S \to \pi^+\pi^-$, $\Lambda^0 \to p\pi^-$. We have some preliminary results [30] further applying the technique advocated in [31] based on the Armenteros-Podolanski [32] reconstruction technique. A more novel aspect is that one can aspire to simultaneously improve the measurements of the K^0_S and Λ masses and the momentum scale given that the masses of their decay products are very well known.
7. Understand the relative merit of dimuons for luminosity spectrum determination compared with Bhabhas and integrate both techniques in a global analysis.
8. Characterize further the scope for measuring accelerator parameters such as the crossing angle and beamstrahlung-induced correlations including the observed dependence of the beam energy spectrum on the longitudinal collision vertex. The latter has been shown to be easily measurable with vertex fits in $e^+e^- \to \mu^+\mu^-(\gamma)$ events.

My Take on Appropriate/Relevant R&D Topics/Wish-List

- MDI/BDS: Assess and plan for global energy/luminosity spectrum/beam diagnostics analysis and insights.
- MDI/BDS: Upgrade beam-beam studies/generators to representative complete machine and variations thereof.
- MDI/BDS: Assess and plan for ultimate beam-spot/luminous region diagnostics including vertexing
- MDI/BDS: How do we deal with E-z correlations?
- MDI/BDS: Can we go beyond 100 ppm for energy spectrometers?
- PHYS/DET: Include all channels in physics center-of-mass energy estimates.
- DET: Assess and plan for ultimate tracker momentum-scale capability.
- DET: Assess and plan for ultimate polar angle systematic uncertainty.
- DET: Assess and plan for ultimate detector solenoid field-mapping capability.
- DET: Assess and plan for ultimate tracker alignment.
- DET: Incorporate more appropriate momentum reconstruction for high energy electrons (example: Gaussian Sum Filter a la CMS)

Summary of Progress

Progress

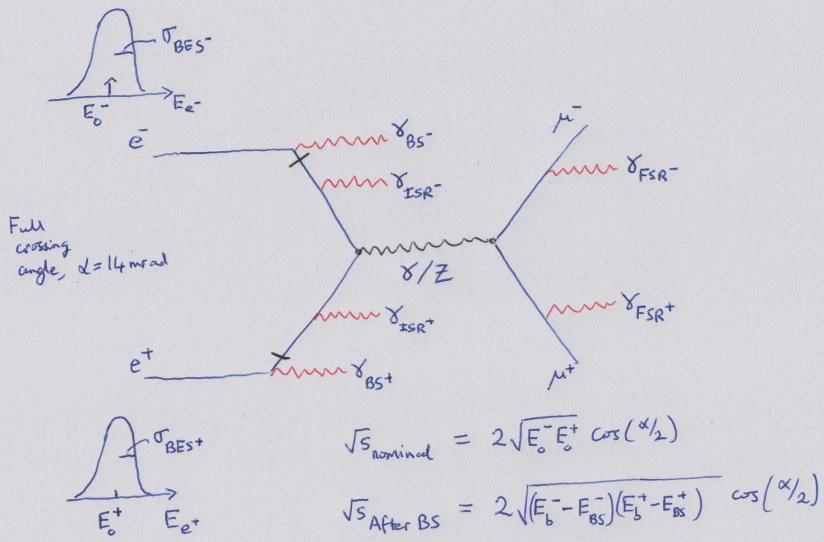
- New high precision method for momentum-scale using especially K^0_S and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and $dL/d\sqrt{s}$ reconstruction
- Measurement of M_Z using dimuon mass for $\sqrt{s} \gg M_Z$ to 1.0 MeV - dominated by $\sqrt{s} = 250$ GeV data

Conclusions

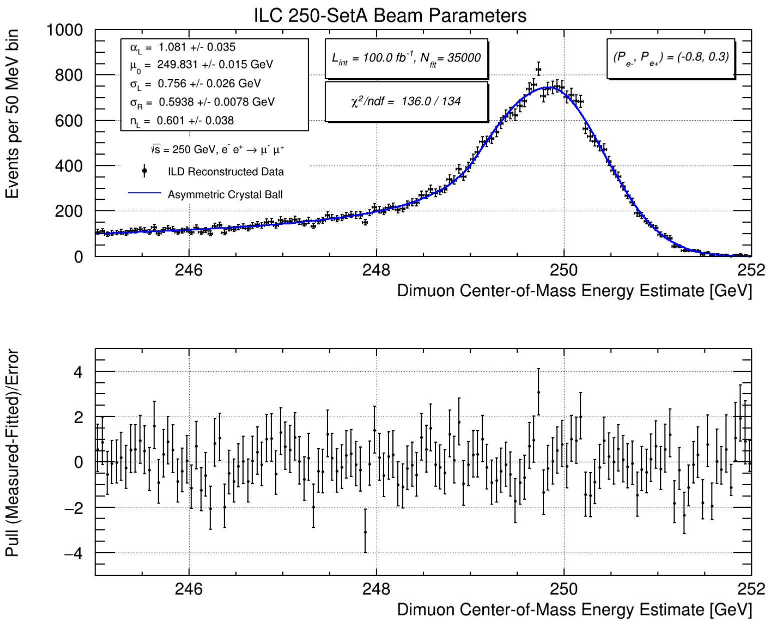
- Tracking detectors designed for ILC have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events)
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 1.9 ppm precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not limit measurements such as M_W.
- Potential to improve M_Z by a factor of three using 250 GeV di-lepton data
- Applying the same $\sqrt{s}$ techniques to running at the Z-pole enables a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge highlighted by potential for Γ_Z to below 100 keV and A_{LR} to 2×10^{-5}.

Backup Slides

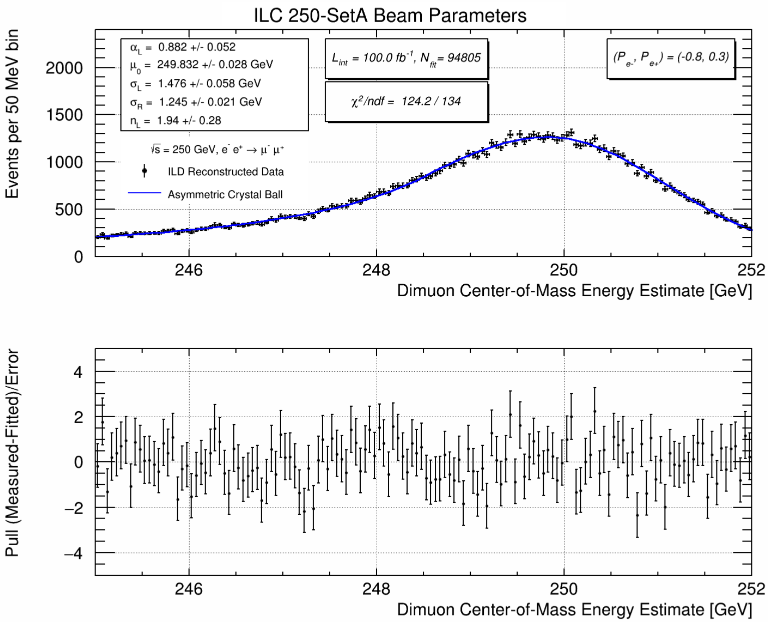
Returning to $\sqrt{s}_p$ and Adding More Realism



Gold Quality Dimuon PFOs (After BS)

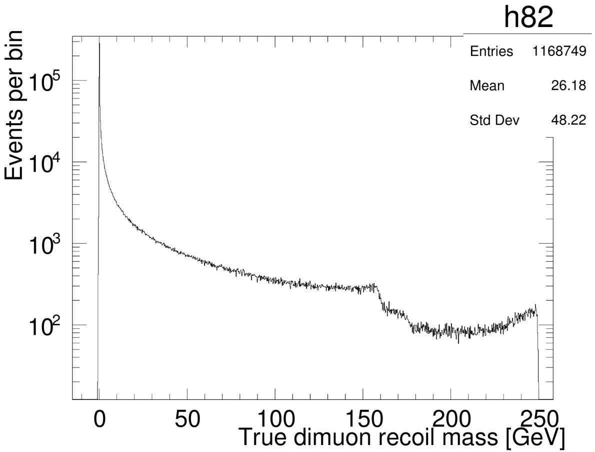


Bronze Quality Dimuon PFOs (After BS)



Recoil Mass (at generator level)

Distribution of M_3.



Events in the tails will be from multiple non-collinear radiation (example ISR from both beams)

Kinematic Fits for $\mathrm{e^+e^-} \to \mu^+\mu^-(\gamma)$

Inspired by revisiting some of the LEP2 techniques for M_W measurement, one can also cast the whole problem as a constrained fit problem. Promises to be very useful in event selection, hypothesis identification, and parameter measurement, but needs excellent object calibration and measurement uncertainties.

Two body fits

Test the hypothesis of $\mathrm{e^+e^-} \to \mu^+\mu^-$ with no additional photons.

1. * Specify E_ave and $\overline{\Delta E_\mathrm{b}}$ and fit with the 4 constraints of (E,p) conservation. (4C/4dof fit)
2. * Fit for E_ave and $\overline{\Delta E_\mathrm{b}}$ as unmeasured fit parameters with the 4 constraints. (4C/2U/2dof fit).

Initial test implementation uses easily adaptable constrained fitting code of V. Blobel with toy MC based smearing and uncertainties.

1. Find 10.7% of events satisfy the 2-body hypothesis ($p_\mathrm{fit} > 0.01$) IF the correct E_ave and $\overline{\Delta E_\mathrm{b}}$ are specified (Fit 1). For these events, $M_{\mu\mu}$ is synonymous with $\sqrt{s}$.
2. Find 26% of events satisfy fit 2 ($p_\mathrm{fit} > 0.05$).
   Note often the fitted $\sqrt{s}$ is near M_Z ... with large $|\overline{\Delta E_\mathrm{b}}|$.

Kinematic Fits for $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-(\gamma)$

Three particle collinear ISR fits

Test the $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-\gamma$ hypothesis where the γ is an undetected ISR photon collinear with one of the beams with z-hemisphere signed energy, E_{ISR}.

1. Specify E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} and fit with 4 constraints. (4C/4dof fit)
2. * Specify E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$. Fit E_{ISR} as unmeasured parameter and fit with 4 constraints. (4C/1U/3dof fit)
3. Fit for E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} as unmeasured fit parameters with the 4 constraints. (4C/3U/1dof fit).