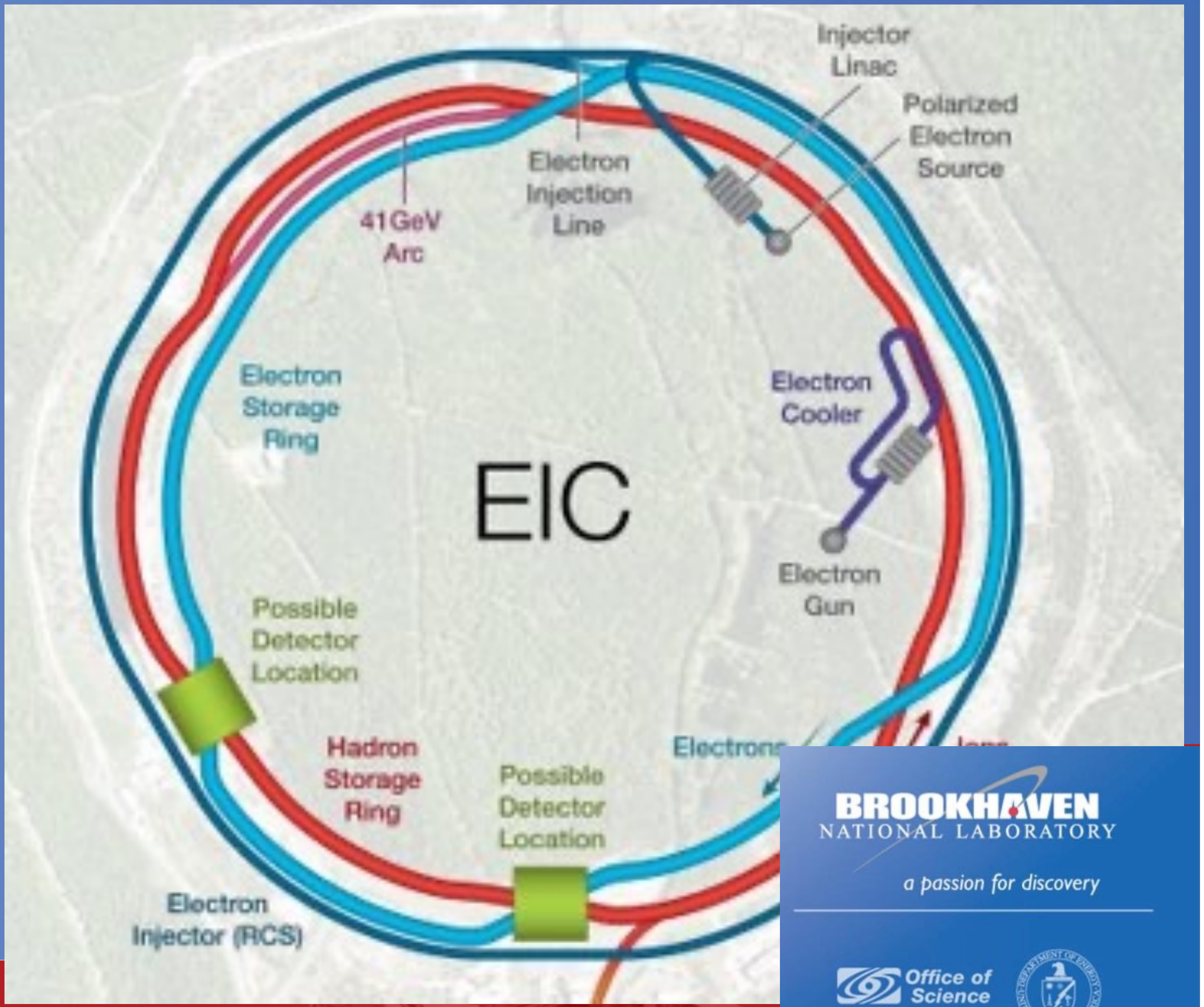
Polarization in e+/e- Storage Rings



Georg Hoffstaetter
ERL & EIC group Cornell / BNL

Cornell Laboratory for
Accelerator-based Sciences and
Education (CLASSE)

BROOKHAVEN
NATIONAL LABORATORY
a passion for discovery
Office of Science
U.S. DEPARTMENT OF ENERGY

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The Cornell ERL/EIC group

Brookhaven National Laboratory

A university group with research on several EIC topics on the undergrad to PhD level, providing workforce development.

Currently 2 research associates, 6 grads, and 5 undergrads, 2 research associates, 1 prof.

- Machine Learning for operations: Lucy Lyn (grad), George Quinn, Vadim Popov (under grads)
- Dynamic aperture for EIC rings: Jonathan Unger (grad)
- Space charge for the EIC cooler ERL: Ningdong Wang (grad)
- Polarized electrons for the EIC: Matt Signorelli (grad), Jacob Asimow (undergrad)
- Polarized protons in RHIC and the EIC: Eiad Hamwi (grad)
- Beam-Based alignment in CESR and the EIC: Jim Crittenden (research associate), Ariel Shaket (grad), James Wang, Ishaan Mishra (undergrads)
- Bmad / Tao simulation code and digital accelerator twin development: David Sagan (research associate)

➔ 4 presentations at this EPOL workshop.

Previous work: Cornell & BNL

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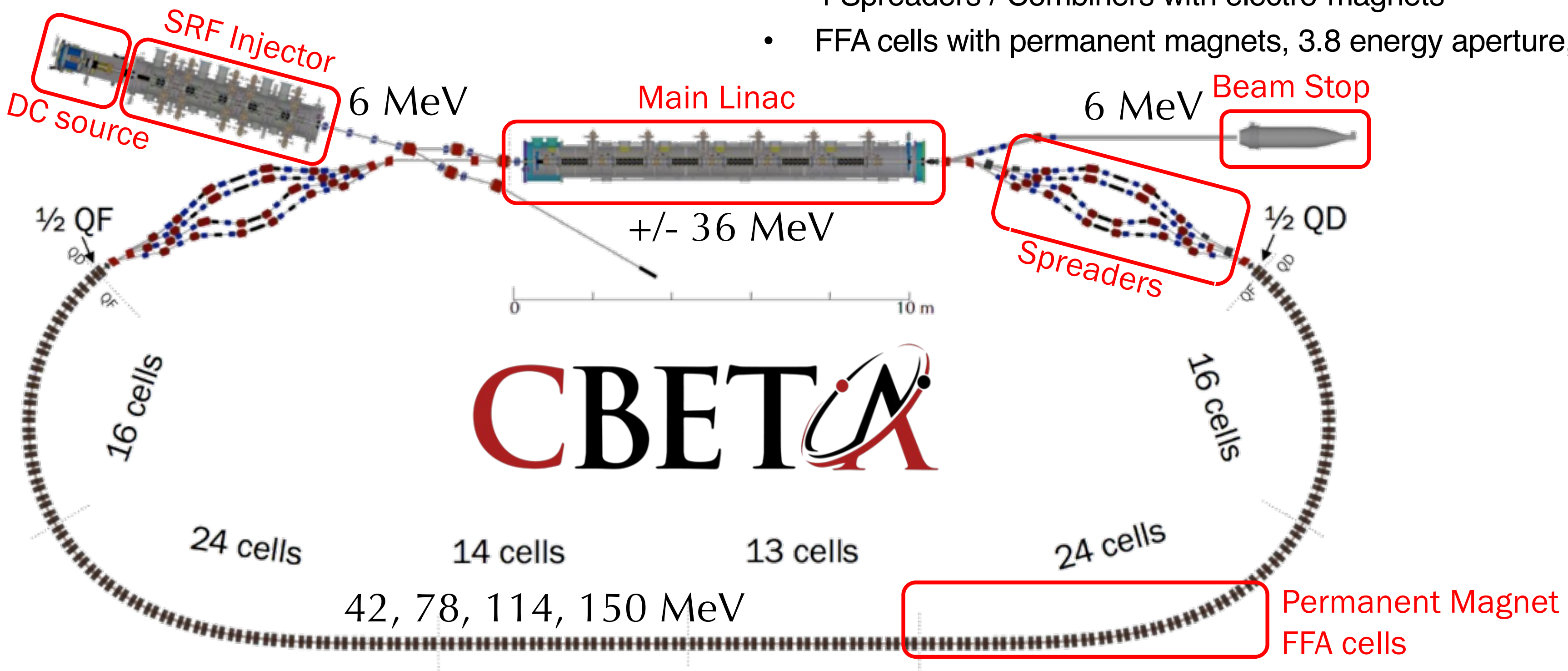
Brookhaven National Laboratory

The Cornell-BNL ERL Test Accelerator

- Cornell DC gun, 2nC peak
- 6MeV SRF injector (ICM), 1.3GHz
- 6-cavity SRF CW Linac (MLC), 1.3GHz
- 4 Spreaders / Combiners with electro-magnets
- FFA cells with permanent magnets, 3.8 energy aperture, 7 beams



First multi-turn ERL operation

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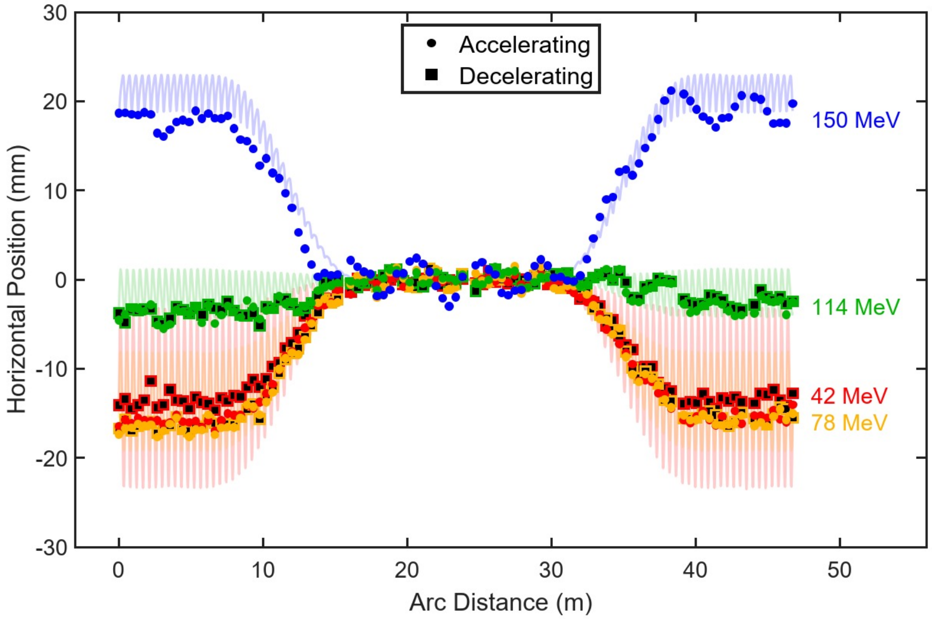
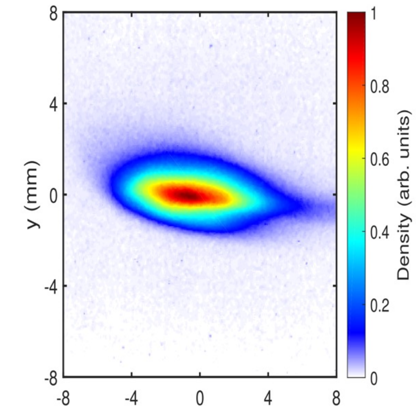
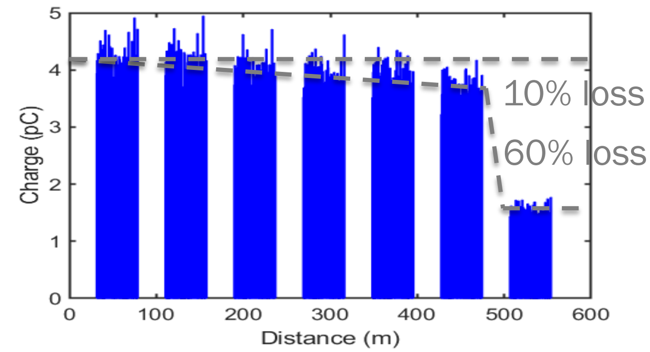


7 beams in the same FFA beamline, accelerated and energy-recovered.

Reports appeared in Nature, Phys. Rev. Letters, Forbes Magazine, EEE Spectrum, reddid.com, and others.

Beam in the beam stop after 8 passes.





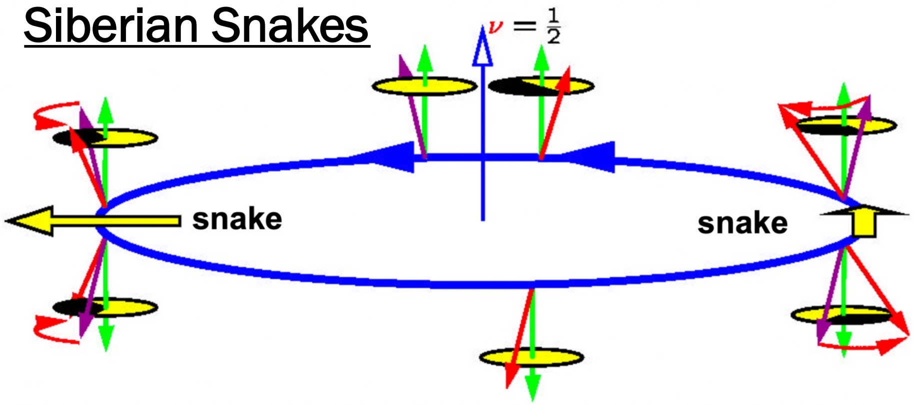
Before the 7th FFA pass, 60% loss

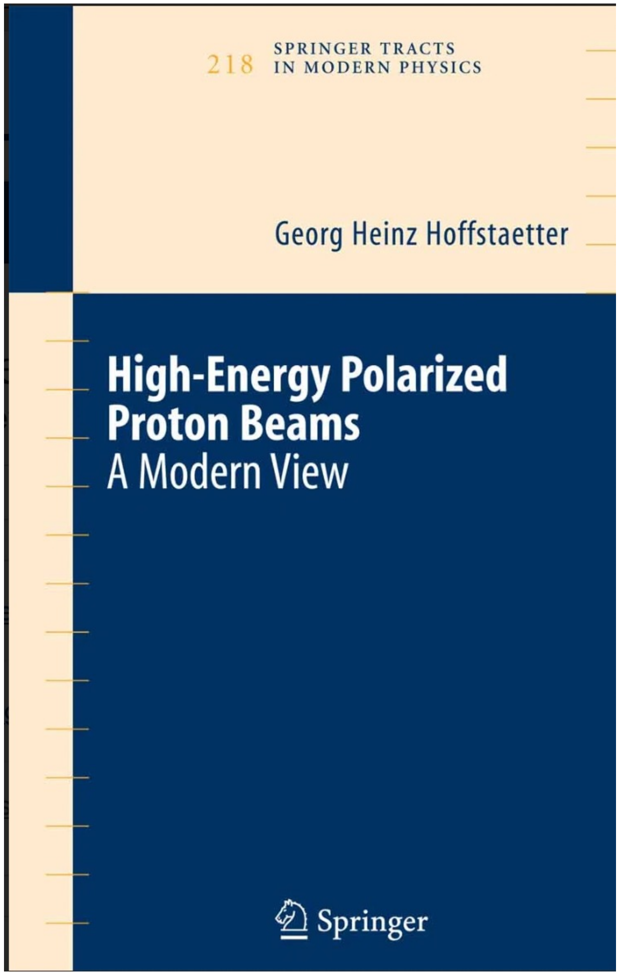
Accelerators with proton polarization

RHIC	250 GeV
AGS	25 GeV
ZGS	12 GeV
COSY	3.65 GeV
IUCS	3.6 GeV
VEPP-4	0.7 GeV
PSI Cyclotron	0.59 GeV

Polarization method:
Injection of polarized beam

Polarization preservation method:
Siberian Snakes





Modes of polarizing electrons rings

Polarized injection

Bates (MIT)
AmPs (Nikhef)
ELSA (Bonn)
SLC
CEBAF
etc.

➔ EIC, SuperKEK-B

Time average decaying polarization

$$< P > \propto P_\infty + (P_0 - P_\infty) < e^{-t/\tau} >_t$$

Sokolov-Ternov Spin flip

HERA
LEP
VEPP
etc.

Kinetic polarization

Bates (MIT)
AmPs (Nikhef)
etc.

Equilibrium polarization buildup by bending fields $\hat{b}$.

$$P_\infty \propto < \hat{b} \cdot \vec{n} >$$

$$P_\infty \propto < \hat{b} \cdot \frac{d\vec{n}}{d\delta} >$$

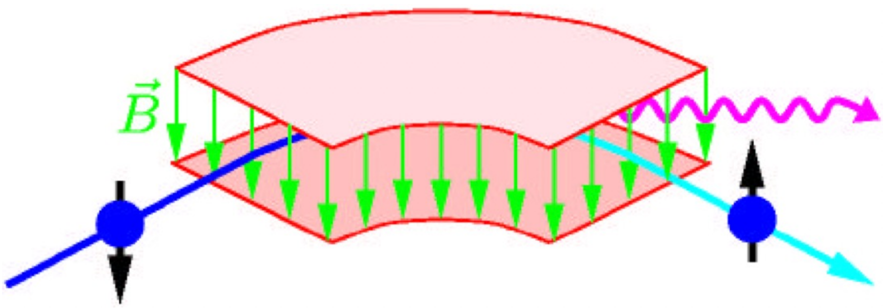
Accelerators with electron polarization

VEPP	1970 vert.	80%	0.65 GeV
ACO	1970 vert.	90%	0.53 GeV
VEPP-2M	1974 vert.	90%	0.65 GeV
SPEAR	1975 vert.	90%	2 GeV
VEPP-3	1976 vert.	80%	3.7 GeV
VEPP-4	1982 vert.	80%	5 GeV
CESR	1983 vert.	30%	5 GeV
PETRA	1982 vert.	70%	16.5 GeV
DORIS	1983 vert.	80%	5 GeV
TRISTAN	1990 vert.	70% (?)	29 GeV
LEP	1993 vert.	57%	47 GeV
HERA	1993 vert.	60%	26.7 GeV
HERA	1994 long.	70%	27.5 GeV
LEP	1999 vert.	7%	60 GeV
VEPP-4M	1990 vert.	(?)	6 GeV
VEPP2000	2010 vert.	(?)	1 GeV

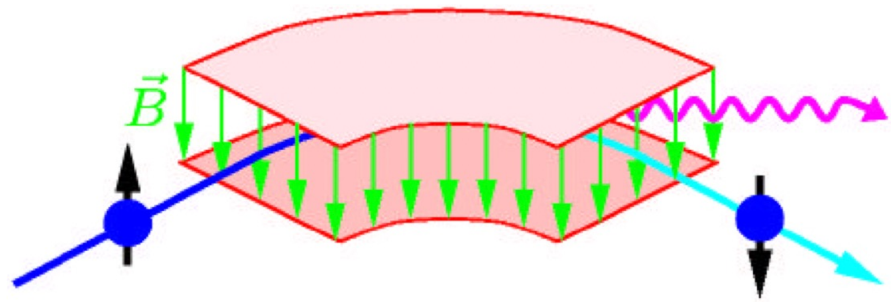
EIC	long.	<70%>	5 to 18GeV
SuperKEK-B	long.		
FCC-ee	vert.		

Self Polarization of Electron Beams

Each 10^{10th} photon flips the spin of the electron, flip/non-flip $\propto \left(\frac{E_{ph}}{E_{el}}\right)^2$



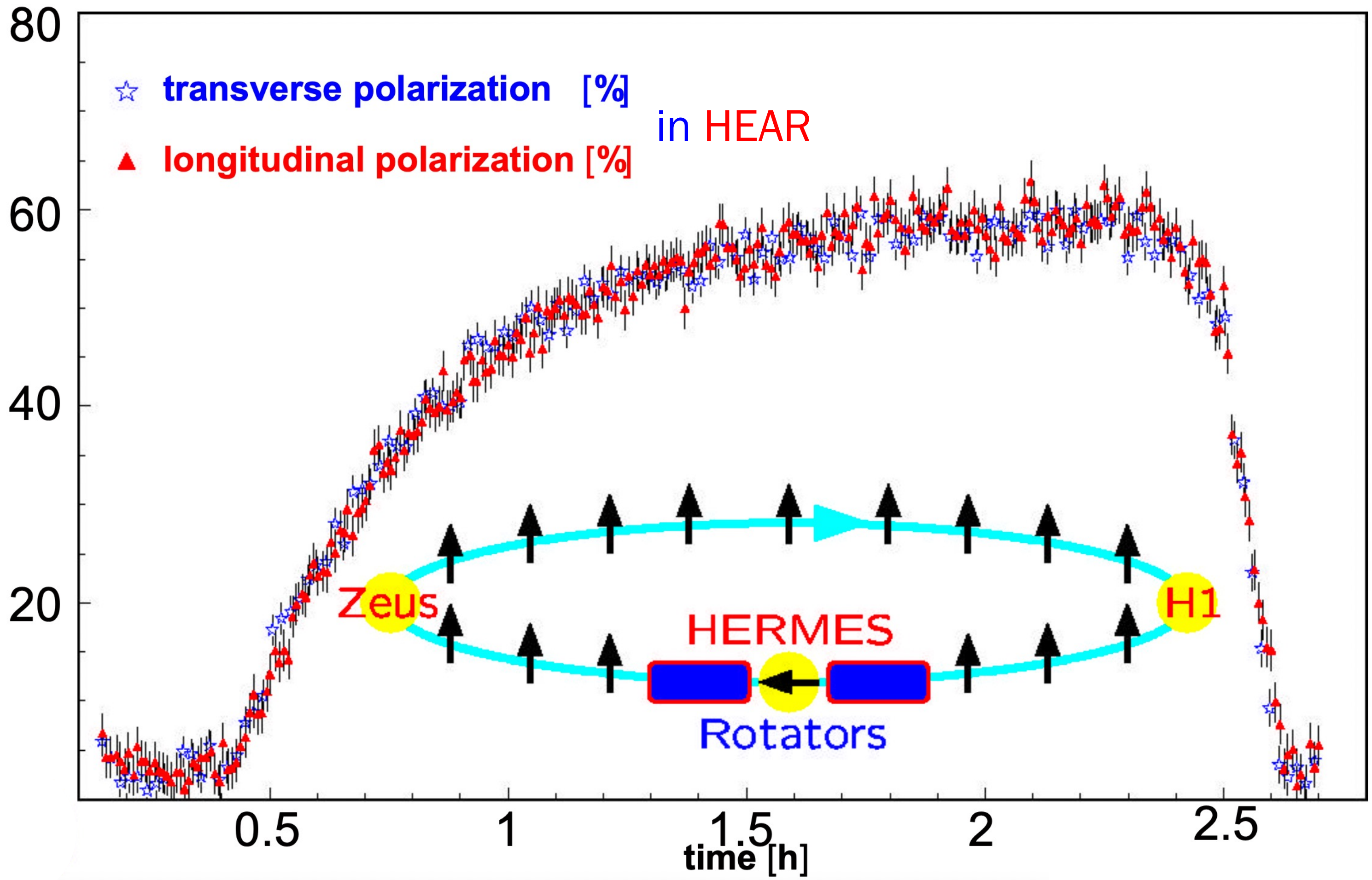
In HEAR every 38.5 minutes
(always 25 times faster)



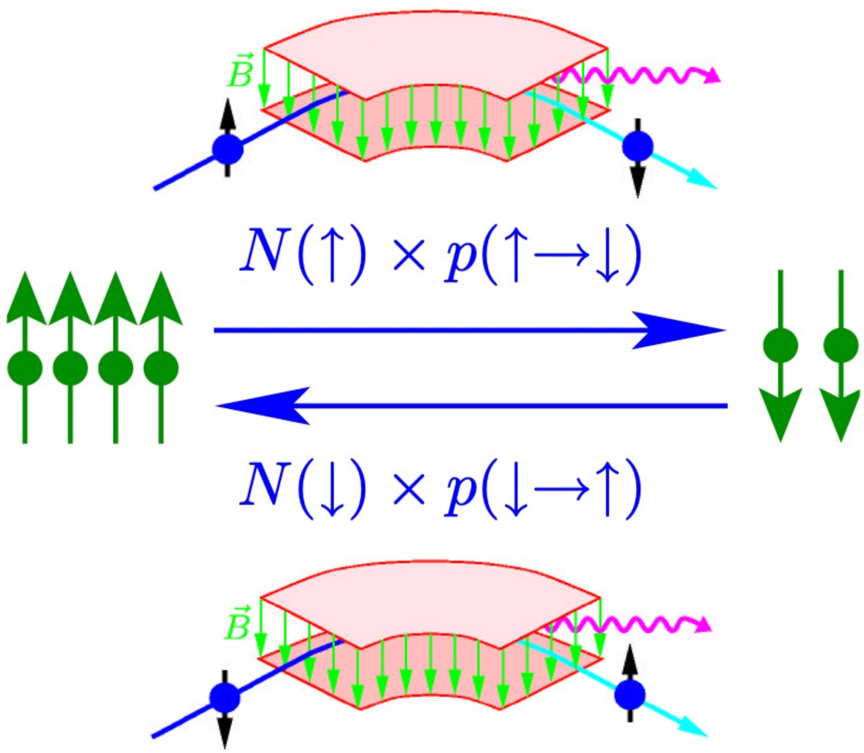
In HEAR every 16.2 hours

Ideal ring: equilibrium polarization of 92.4%
HEAR: routine operation with polarization of 60-65%

Electron Polarization buildup



Up-down flip equilibrium



always 25 times faster ➔ always 92.4%

$$P = \frac{N(\uparrow)-N(\downarrow)}{N(\uparrow)+N(\downarrow)}$$

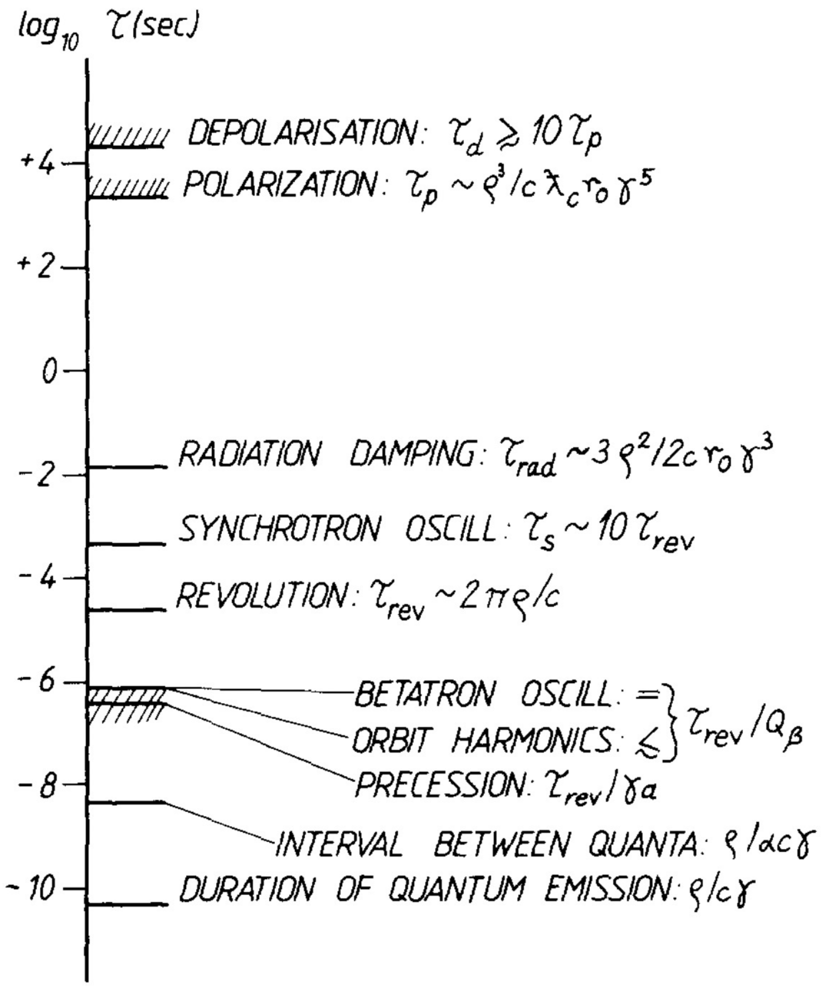
$$\tau_{st}^{-1} = \frac{5\sqrt{3}}{8}\frac{e^2\gamma^5\hbar}{m_e^2 c^2|\rho|^3}$$

$$P = P_{\max} \times (1 - \exp(-\tfrac{t}{\tau}))$$

$$P_{\max} = \frac{8}{5\sqrt{3}} \approx 0.924$$

$$\tau \approx 100\text{s}\,\frac{(R/\text{m})^3}{(E/\text{GeV})^5}$$

Timescales



- Self polarization is one of the slowest processes in an electron accelerator.
- About 6 orders of magnitude slower than radiation damping !
- To observe polarization, unwanted depolarization has to be even slower.

[1] Bryan W. Montague, Polarized Beams in high Energy Storage Rings, CERN

The spin (BMT) equation of motion

Restframe: $\frac{d\vec{s}}{dt'} = \frac{gq}{2m}\vec{s} \times \vec{B}'$ ➔ Boost ➔ $\frac{d}{dt}\vec{s} = \vec{\Omega}_{BMT}(\vec{r},\vec{p}) \times \vec{s}$

$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \vec{B}_\perp \} \times \vec{p}$$

$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma + 1)\vec{B}_\perp + (1+G)\vec{B}_{||}\} \times \vec{S}$$

$G = \frac{g-2}{2} =$

Protons	G = 1.79
Deuterons	G = -0.143
Electrons	G = 0.00116

BMT snippets worth remembering

$$G = \frac{g-2}{2} = \begin{cases} \text{Protons} & G = 1.79 \\ \text{Deuterons} & G = -0.143 \\ \text{Electrons} & G = 0.00116 \end{cases}$$

➔ 10 X harder to manipulate deuteron spin

$$\frac{d\vec{p}}{dt} = \left(\frac{-q}{m\gamma}\right)\{ \vec{B}_\perp \} \times \vec{p}$$

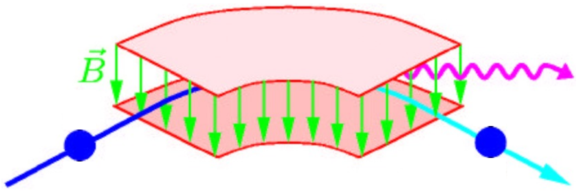
$$\frac{d\vec{S}}{dt} = \left(\frac{-q}{m\gamma}\right)\{(G\gamma + 1)\vec{B}_\perp + (1+G)\vec{B}_{||}\} \times \vec{S}$$

➔ Spin rotates Gγ times faster than the orbit ➔ 1-turn spin rotation = spin tune = Gγ.

In a magnetic field B, the bend angel dp/p decreases with energy.
the spin rotation angle ds/s = q G B dl / mc does not depend on energy.

➔ 4.62 Tm always rotate the electron spin by 180 degrees.

What are the other 10^{10} photons doing?



By far the most photons do not flip spin.
What is their effect on polarization?

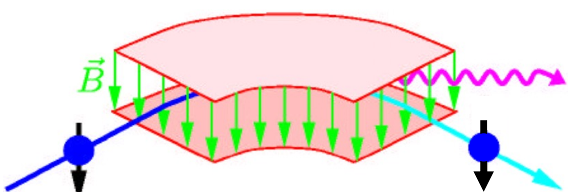
a) Energy loss

➔ quantum noise in phase space

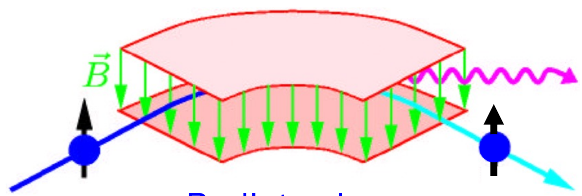
➔ depolarization

b) Spin-dependent Energy loss

➔ Kinetic polarization buildup



Radiates more

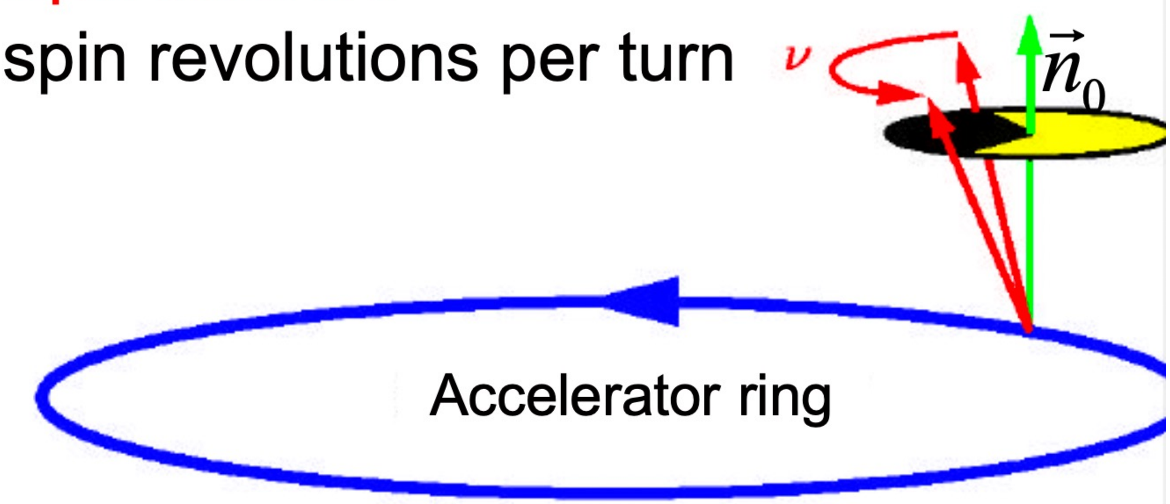


Radiates less

Spin tune and periodic spin direction

A particle traveling on the closed orbit has the closed orbit spin direction $\vec{n}_0$ if its spin is periodic after every turn.

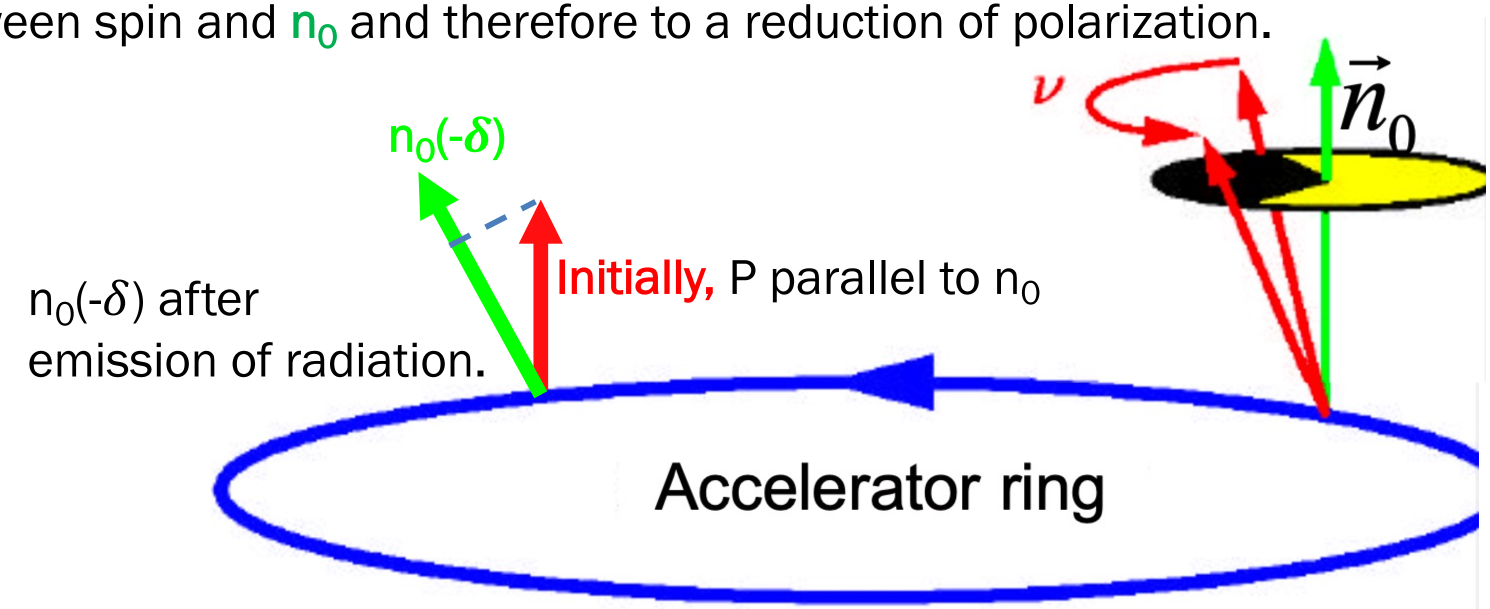
Spin-tune n_0: Number of spin revolutions per turn



Depolarization on the closed orbit

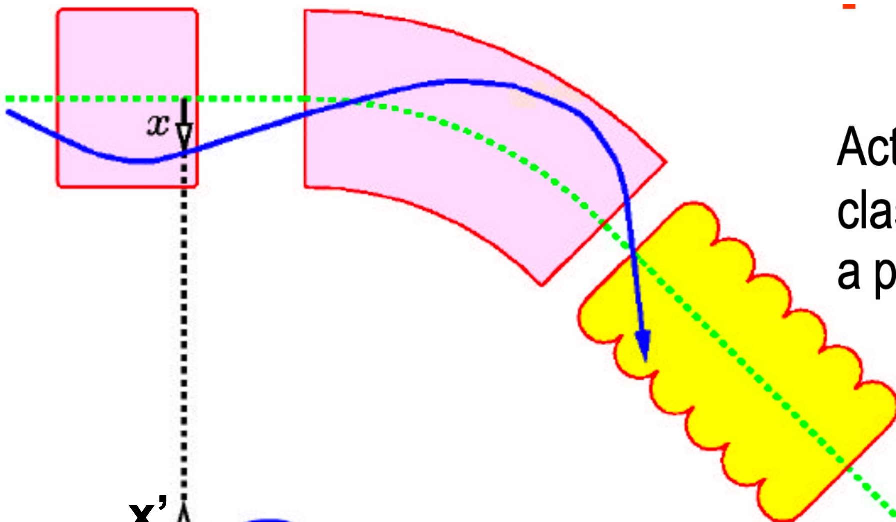
Spin on the closed orbit precesses around n_0, the average polarization is the n_0 projection.

When n_0 depends on energy, the emission of a synchrotron photon leads to an angle between spin and n_0 and therefore to a reduction of polarization.

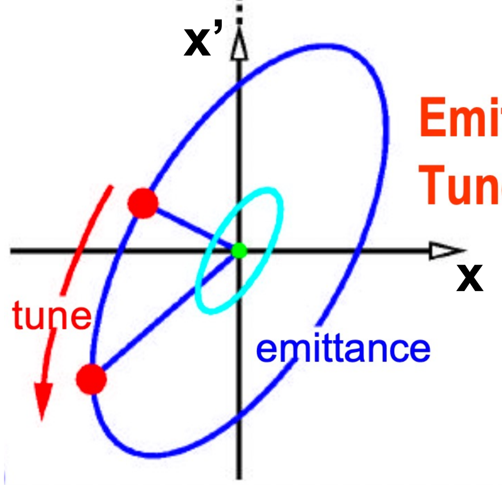


If n_0 is strongly energy dependent, this is the main cause of depolarization.
But often n_0 is vertical in the arcs for all energies. What then causes depolarization?

Invariants of phase space propagation



Action-angle variables of a classical periodic system like a pendulum or planetary motion



Emittances: $\vec{\varepsilon}:\varepsilon_x,\varepsilon_y,\varepsilon_l$

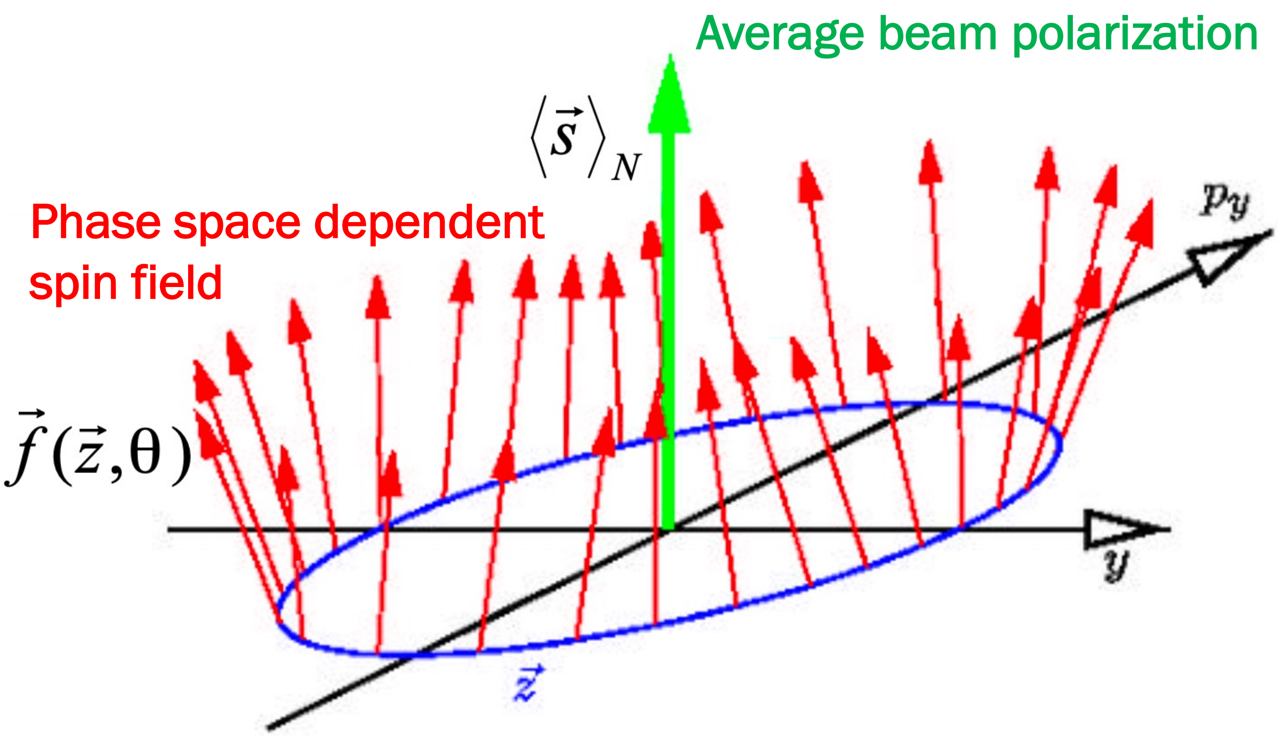
Tunes: $\vec{Q}:Q_x,Q_y,Q_l$

Action variables: $\vec{\Phi}=\vec{Q}\theta$

Angle variables: $\vec{J}=\frac{1}{2}\vec{\varepsilon}$

Propagation of spin fields

Spin field: Spin direction $\vec{f}(\vec{z},\theta)$ for each phase space point $\vec{z}$



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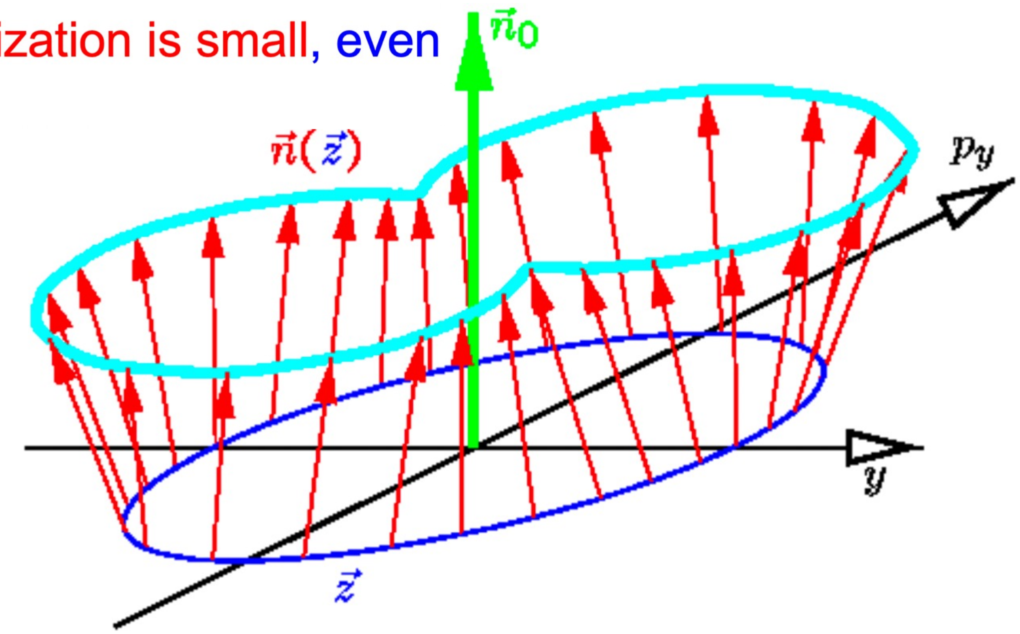
The invariant spin field (ISF)

Brookhaven National Laboratory

A) Maximum polarization: $P_{lim} = < \vec{n}(\vec{z}) >_{\text{Phase space}}$

For a large divergence, the average polarization is small, even if the local polarization is 100%.

B) $\vec{n}(\vec{z}) \cdot \vec{S}$ is an adiabatic invariance !



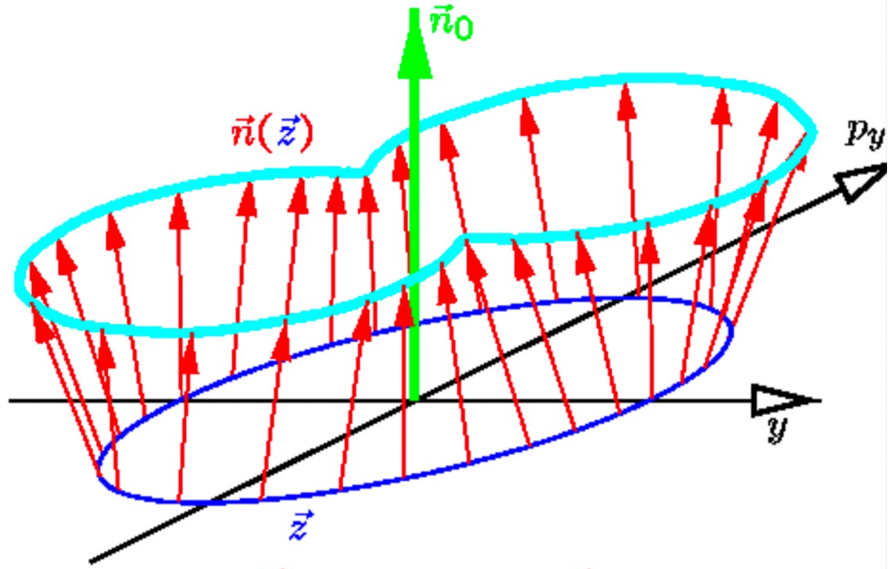
The stable polarization of a beam must be parallel to the ISF at every phase space point.

Linearized $\vec{n}(\vec{z})$ can be analytically computed

Computation of the invariant spin field

Computation of the invariant spin field by analyzing tracking data:

- Fourier analysis
- Stroboscopic averaging
- Anti-damping
- Differential Algebra



$$\vec{S}_{n+1} = \underline{A}(\vec{z}_n)\vec{S}_n$$

defines the $\vec{n}$–axis

$$\vec{n}(\vec{z}_{n+1}) = \underline{A}(\vec{z}_n)\vec{n}(\vec{z}_n)$$

Examples of invariant spin field (ISF)

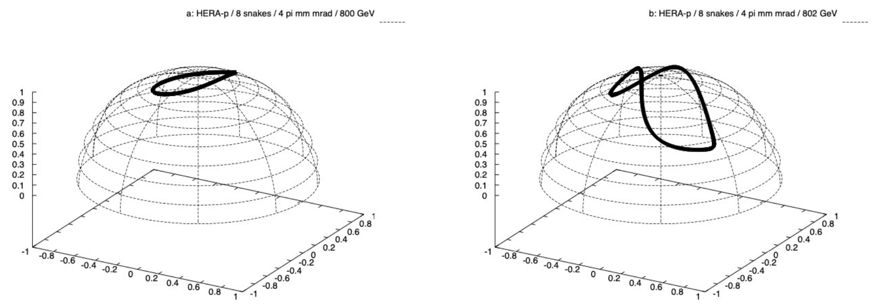


Figure 2: The $\hat{n}$–vector for the 4π mm mrad ellipse at 800 GeV (left) and 802 GeV (right).

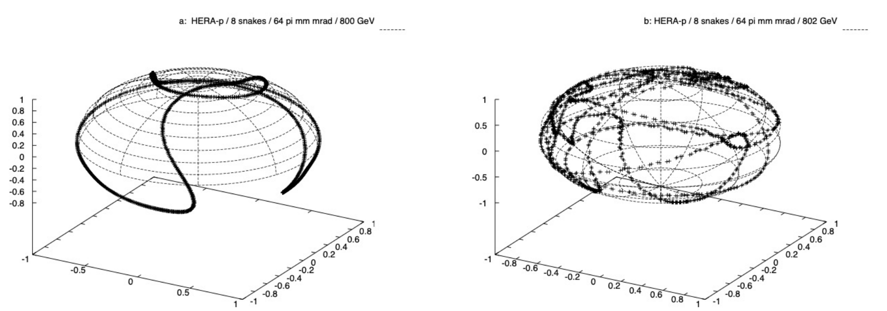
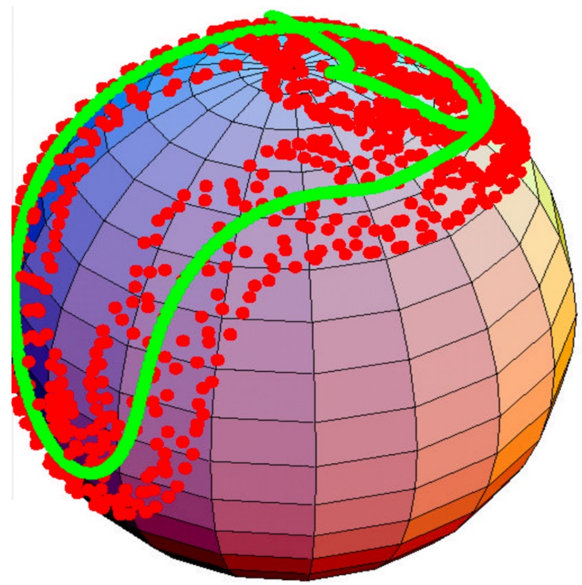


Figure 3: The $\hat{n}$–vector for the 64π mm mrad ellipse at 800 GeV (left) and 802 GeV (right).



Computation by:

- **Fourier analysis,**
- Stroboscopic averaging
- Differential algebra

The Derbenev Kondratenko equilibrium

92.4%

Unit vector of orbit rotation, i.e. B-field

$$P_{\mathrm{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot [\hat{n} - \frac{\partial \hat{n}}{\partial \delta}] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \} \rangle}$$

Invariant spin field (ISF)

Energy dependence of the invariant spin field.

Average over phase space and around the ring

Where do these terms come from, what do they mean?

A flat ring with vertical spin

92.4%

$$P_{\mathrm{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\left\langle \frac{1}{|\rho|^3}\hat{b}\cdot\left[\hat{n} - \frac{\partial\hat{n}}{\partial\delta}\right]\right\rangle}{\left\langle \frac{1}{|\rho|^3}\left\{1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left|\frac{\partial\hat{n}}{\partial\delta}\right|^2\right\}\right\rangle}$$

= 1

Radiation build up rate:

$$\tau_{st}^{-1} = \frac{5\sqrt{3}}{8}\frac{e^2\gamma^5\hbar}{m_e^2c^2|\rho|^3}$$

$$P = P_{\mathrm{max}} \times \left(1 - \exp\left(-\frac{t}{\tau}\right)\right)$$

$$P_{\mathrm{max}} = \frac{8}{5\sqrt{3}} \approx 0.924$$

$$\tau \approx 100\mathrm{s}\frac{(R/\mathrm{m})^3}{(E/\mathrm{GeV})^5}$$

Note: referred to as the "ideal" case.

Energy Independent ISF

$$P_{\text{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot [\hat{n} - \frac{\partial \hat{n}}{\partial \delta}] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \} \rangle}$$

= 0

Necessary for spin rotators

Radiation build up rate: smaller than ideal by the ratio of $\frac{1}{|\rho|^3}$ to $\frac{1}{|\rho|^3}\{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2$

P not too far below 92.4% for good spin rotators and not too much slower (80-90%).

Note: referred to as the "Sokolov-Ternov" case.

Depolarization

$$P_{\text{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot [\hat{n} - \frac{\partial \hat{n}}{\partial \delta}] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \} \rangle}$$

= 0

Energy dependence of the invariant spin field.

Spin in phase space precesses around **n**, the average polarization is the **n** projection.

When **n** depends on energy, the emission of a synchrotron photon leads to an angle between spin and **n** and therefore to a reduction of polarization.

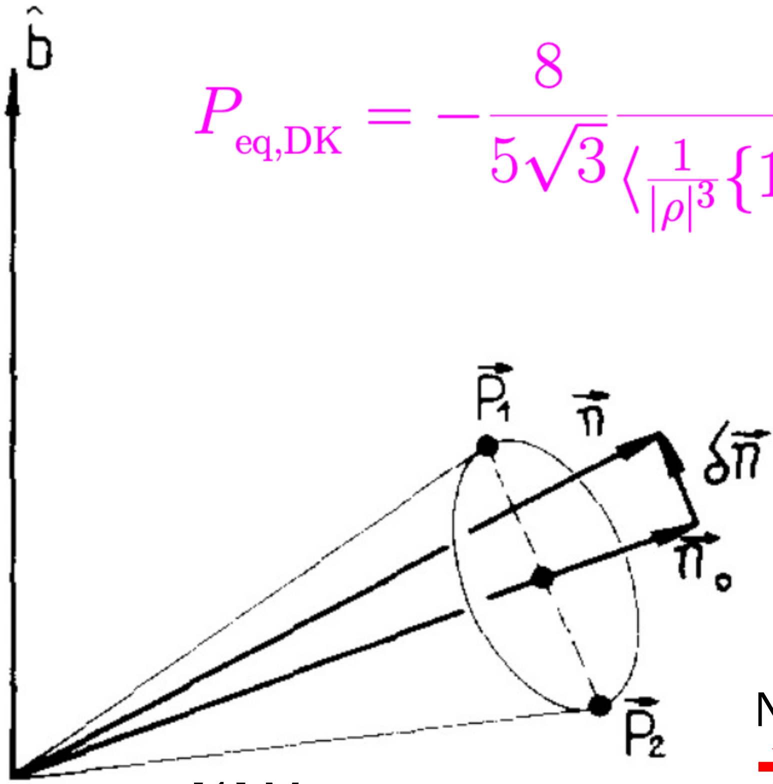
Radiation build up – *or down* rate: often much smaller than ideal by the ratio of $\frac{1}{|\rho|^3}$ to $\frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \}$

Kinetic Polarization

Sokolov-Ternov driving term for polarization buildup anti-parallel to $\hat{b}$.

$$P_{\text{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot [\hat{n} - \frac{\partial \hat{n}}{\partial \delta}] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left| \frac{\partial \hat{n}}{\partial \delta} \right|^2 \} \rangle}$$

New driving term for polarization buildup from the energy dependence of the ISF.



[1] Montague

Because P_1 radiates more than P_2, the average projection on **n** after radiation is increased ➔ polarization buildup !

Note: Usually very small, since mostly $\hat{b} \upuparrows \hat{n}$ and $\hat{b} \perp \hat{n}$.

➔ Not a catch all for disagreements in calcs.

Simplifications

- Drop kinetic polarization.
- Replace n by n0 of the closed orbit to avoid phase space average <...>.
- Retain $\hat{n}$ and <...> average only for depolarization.

$$P_{\mathrm{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot \left[\hat{n} - \frac{\partial \hat{n}}{\partial \delta}\right] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n} \cdot \hat{v})^2 + \frac{11}{18} \left|\frac{\partial \hat{n}}{\partial \delta}\right|^2 \} \rangle}$$

$$\rightarrow \quad P_\infty \approx \frac{8}{5\sqrt{3}} \frac{\oint \frac{\hat{b}\cdot\hat{n}_0}{|\rho|^3} d\theta}{\oint \frac{1}{|\rho|^3}\{1-(\hat{v}\cdot\hat{n}_0)^2\} d\theta + < \frac{1}{|\rho|^3} \frac{11}{18} \left|\frac{\partial \hat{n}}{\partial \delta}\right|^2 >}$$

Note: referred to as the “BKS” case.

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Example from this EPOL workshop

Brookhaven National Laboratory

F. Carlier, E. Gianfelice-Wendt, T. Pieloni, Y. Wu

Polarization and Spin Tune

- Lepton beams polarize naturally transversely over time → Sokolov-Ternov-Effect
- Depolarization naturally from synchrotron radiation, resonances, etc.
- Maximum polarization at about 92.4 % in lepton storage rings

Strong unexpected resonance found for SITROS simulations

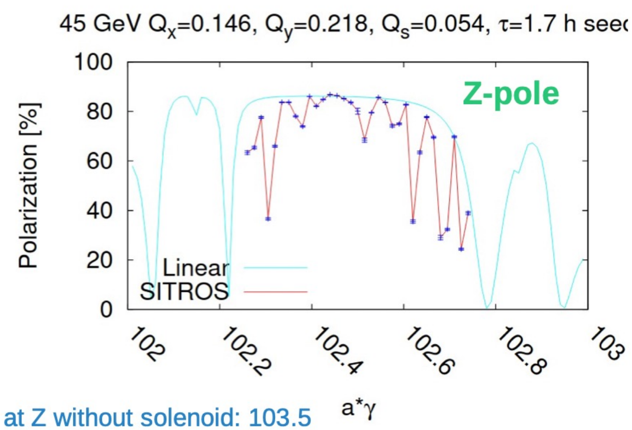
$$\tau^{-1} = \tau_{bks}^{-1} + \tau_{dep}^{-1}$$

Effective polarization rate (τ_{bks}^{-1}); Depolarization rate (τ_{dep}^{-1})

Baier-Katkov-Strakhovenko polarization rate

$$\tau_{bks}^{-1} = \frac{5\sqrt{3}}{8}\frac{\hbar r_e \gamma^5}{m_e C}\oint ds \frac{1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2}{|\rho(s)|^3}$$

Polarization direction in ŷ for planar ring



- Resonances with transverse and longitudinal axis

Q_x … horizontal tune
Q_y … vertical tune
Q_s … synchrotron tune
m_i, k … integer
a … gyromagnetic moment
γ … relativistic gamma

$$a\gamma + m_x Q_x + m_y Q_y + m_s Q_s = k$$

$a\gamma$: Spin tune for ideal machine; $m_x Q_x + m_y Q_y$: Transverse planes; $m_s Q_s$: Longitudinal plane

aγ at Z without solenoid: 103.5

Y. Wu: indico.cern.ch/event/1119730/

E. Gianfelice-Wendt, indico.cern.ch/event/727555/contributions/3468285, 2019.

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Depolarization by resonance crossing

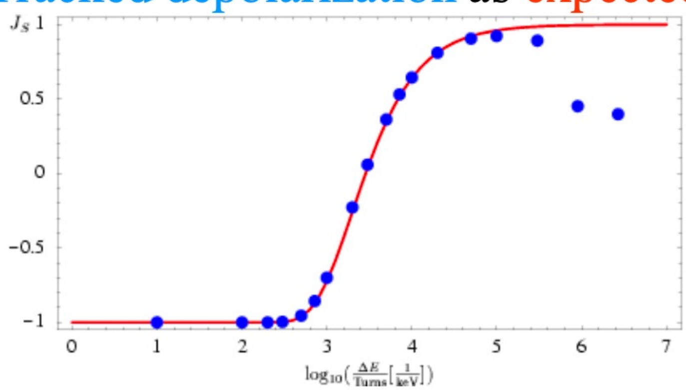
Brookhaven National Laboratory

For proton beams polarization is lost not by radiation but by resonance crossing.

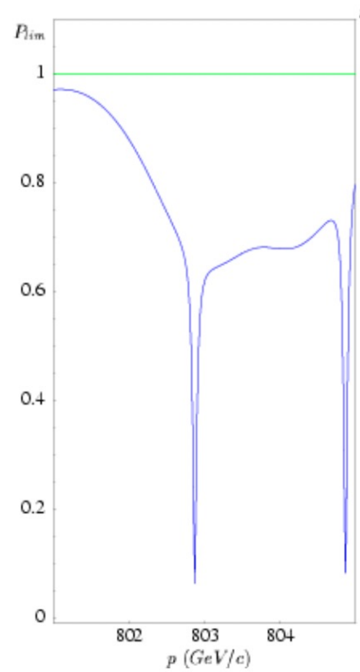
The higher order Froissart-Stora formula

- Resonances up to 19th order can be observed
- Resonance strength can be determined from tune jump.

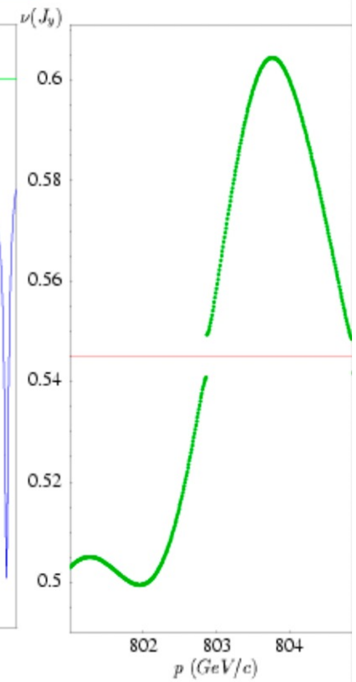
Tracked depolarization as expected



$P_{\lim} = \langle \vec{n}(\vec{z}) \rangle$

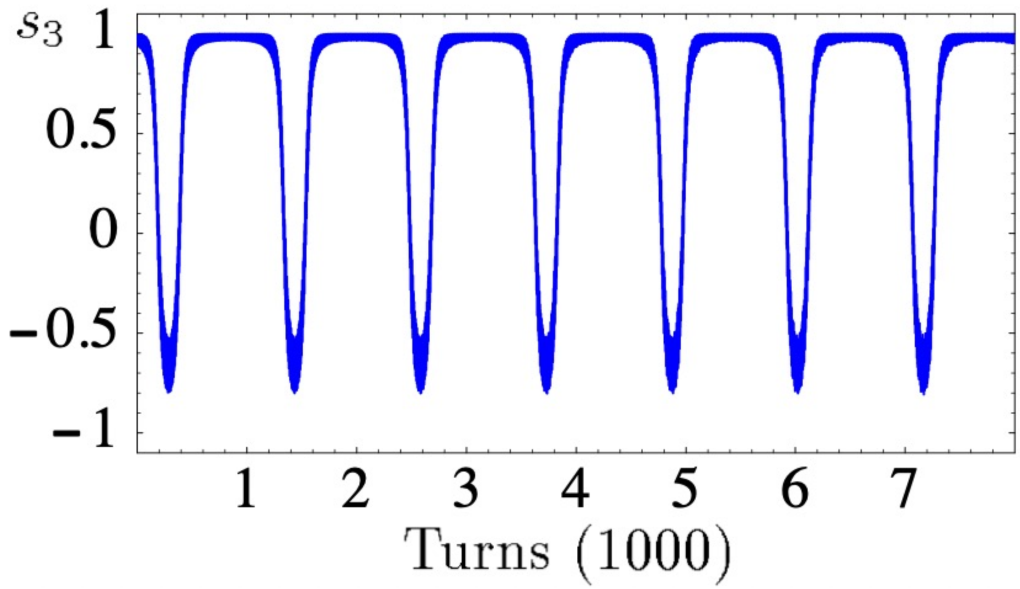


Spin tune



Resonant Depolarization

Can synchrotron oscillations lead to repeated resonance crossing and depolarization?



Examples from a HERA-$\vec{p}$ study without Siberian Snakes.

New research: After the DK-formula leads to good polarization, resonant depolarization has to be checked by tracking.

Computational techniques

$$P_\infty \approx \frac{8}{5\sqrt{3}} \frac{\oint \frac{\hat{b}\cdot\hat{n}_0}{|\rho|^3} d\theta}{\oint \frac{1}{|\rho|^3}\{1-(\hat{v}\cdot\hat{n}_0)^2\} d\theta + < \frac{1}{|\rho|^3} \frac{11}{18} \left|\frac{\partial \hat{n}}{\partial \delta}\right|^2 >}$$

Compute on the closed orbit

Needs the invariant spin field ☹

- Linearize the spin-orbit equations of motion in phase space amplitudes.
  ➔ Codes: SLIM / SLICK / BMAD *(Presentation by Jacob Asimow – next Tuesday)*
- Perturbation theory nonlinear in small phase space amplitudes.
  ➔ SMILE program, did not converge in the past.
- Differential Algebra computation of $\vec{n}$ ➔ did not converge in the past, new research
- Stroboscopic averaging of $\vec{n}$ ➔ new research.
- Fourier analysis of tracking data to get $\vec{n}$ ➔ SODOM program.
- Nonlinear tracking to get depolarization time ➔ BMAD, SITROS, SITF, SLICKtrack

Tracking analysis

Electron Polarization in a Storage Ring

$$P(t) = P_\infty\left(1 - e^{-t/\tau_{eq}}\right) + P_0 e^{-t/\tau_{eq}}$$

$$\tau_{eq}^{-1} = \tau_{pol}^{-1} + \tau_{dep}^{-1}$$

✓ Can be accurately approximated from the closed orbit with analytical formulas

× Hard to estimate analytically. May be affected significantly by nonlinearities

To estimate τ_{dep}^{-1} , do Monte Carlo tracking with *only* spin diffusion effects

$$P_{tr}(t) = P_0 e^{-t/\tau_{dep}} \approx P_0 - t/\tau_{dep}$$

From Matt Signorelli's presentation yesterday.

Tracking Methods

Monte-Carlo Spin Tracking Methods with Radiation

- **Map Tracking** – damped maps generated between each bend center (radiation points*) by PTC w/ user-specified order
- **Bmad Tracking** – element-by-element damped nonlinear maps w/ radiation points after each element
- **PTC Tracking** – element-by-element symplectic integration w/ radiation points at each step within the element
- Bmad toolkit conveniently implements all the above tracking methods and can be run in parallel on a GPU cluster *(Presentation by Dave Sagan next Thursday)*

Note: free Bmad school at Cornell – October 7-9, 2022 after ERL22

From Matt Signorelli's presentation yesterday.

Spin matching

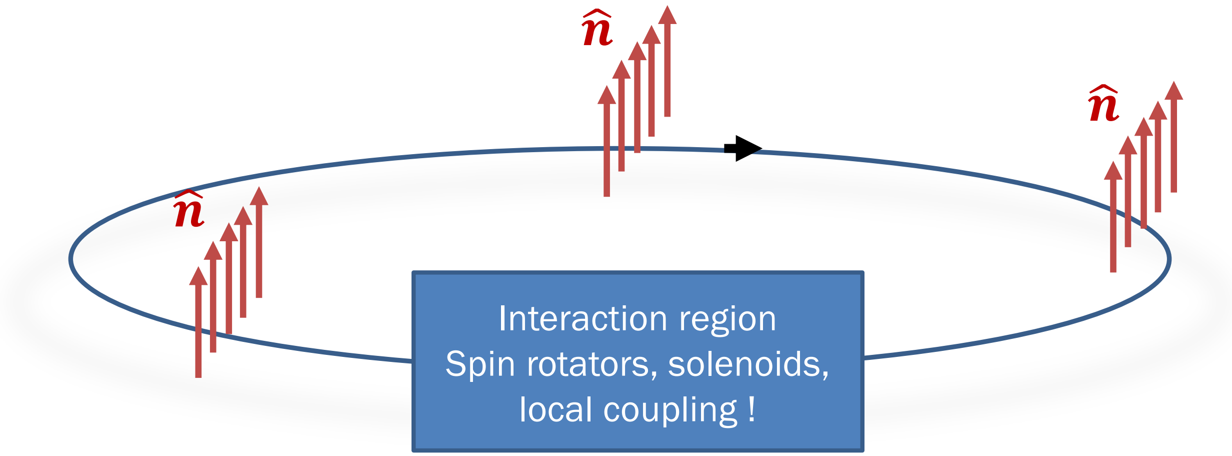
- If the energy dependence of the ISF can be made small throughout phase space, the depolarization rate is small.

$$P_\infty \approx \frac{8}{5\sqrt{3}} \frac{\oint \frac{\hat{b}\cdot\hat{n}_0}{|\rho|^3} d\theta}{\oint \frac{1}{|\rho|^3}\{1-(\hat{v}\cdot\hat{n}_0)^2\} d\theta + < \frac{1}{|\rho|^3} \frac{11}{18} \left|\frac{\partial \hat{n}}{\partial \delta}\right|^2 >}$$

- Often electron breams are flat and <…> does not include the vertical.
- If n is vertical in the midplane at the start of a ring, it stays vertical in the full midplane for all energies, as long as the transport is not x-y coupled.
- Spin matching: Make sure vertical spins in a decoupled arc stay vertical after the IR, for all energies.

Uncoupled arcs

Flat beam electron polarization in an uncoupled arc



Make sure that vertical spins that enter the IR, leave it vertical for all horizontal amplitudes and for all energy deviations.

Usually this is only done in linear phase space approximation.

Questions

Important tasks related to radiation buildup and depolarization.

- Code comparisons (esp. Bmad / experimentally tested old codes.
- Does resonant depolarization occur.
- Include nonlinear beam-beam forces.
- Make n-axis techniques converge, e.g. stroboscopic averaging.
- Obtain strength of kinetic polarization.
- Can spin flip be included for full tracking?