FCC-ee 3D Polarimetry

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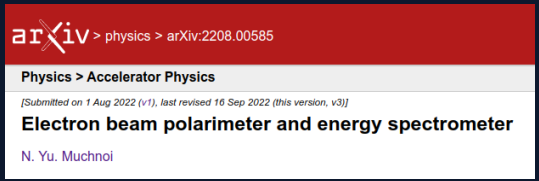
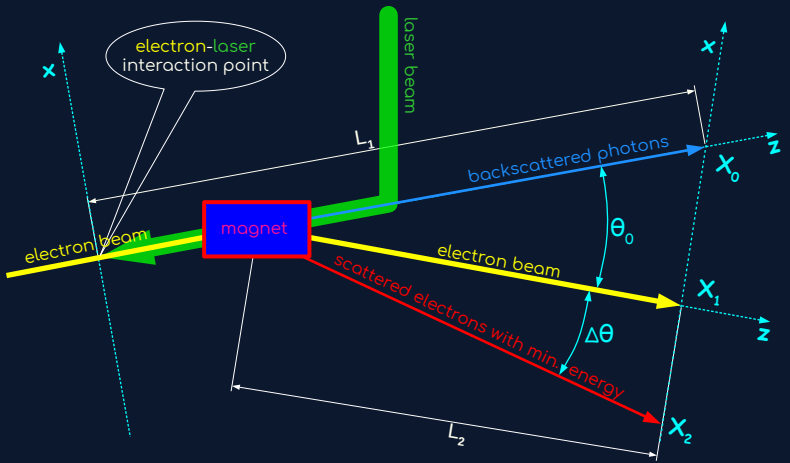
2nd FCC Polarization Workshop

The detailed article (arXiv:2208.00585)
has been submitted to JINST and is under review now



x-z layout



ε_0 - beam e- energy
$\hbar\omega_0$ - laser γ energy

$$\theta_0 = \frac{mc^2}{\varepsilon_0}\left[\frac{\int B_\perp dl}{mc/e}\right]$$

$$\frac{mc}{e} \simeq 1.7 \cdot 10^{-3}[\mathrm{T\ m}]$$

$\alpha \simeq \pi$ collision :

$$\Delta\theta = \frac{4\hbar\omega_0}{mc^2}\left[\frac{\int B_\perp dl}{mc/e}\right]$$

$$\varepsilon_0 = \frac{(mc^2)^2}{4\hbar\omega_0}\frac{\Delta\theta}{\theta_0}$$

Inverse Compton Scattering

$$\gamma = \frac{\varepsilon_0}{mc^2}$$

$$\kappa = 4\gamma \frac{\hbar\omega_0}{mc^2} \sin^2 \frac{\alpha}{2}$$

$$u \equiv \frac{\hbar\omega}{\varepsilon} \in [0, \kappa]$$

If $\kappa \ll \gamma$:

$$\varepsilon_0, \varepsilon, \hbar\omega \gg \hbar\omega_0$$

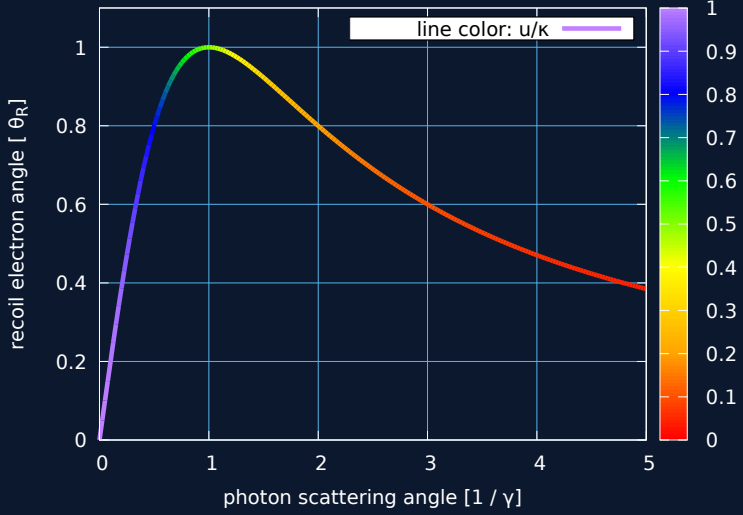
$$u = \frac{\theta_e}{\theta_\gamma} = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}$$

	scattered photon:	recoil electron:
Energy:	$\hbar\omega = \frac{\varepsilon_0 u}{1+u},$	$\varepsilon = \frac{\varepsilon_0}{1+u}.$
Edge energy:	$\hbar\omega_{max} = \frac{\varepsilon_0 \kappa}{1+\kappa},$	$\varepsilon_{min} = \frac{\varepsilon_0}{1+\kappa}.$
Scattering angle:	$\theta_\gamma = \frac{1}{\gamma}\sqrt{\kappa/u - 1},$	$\theta_e = \frac{u}{\gamma}\sqrt{\kappa/u - 1}.$
Angle from bend:	$\theta_0,$	$u\theta_0.$
Maximum bend:	$\theta_0,$	$\Delta\theta = \kappa\theta_0.$
Azimuthal angle:	$\varphi,$	$\varphi + \pi.$
Horizontal angle:	$\eta_x = \theta_\gamma \cos\varphi - \theta_0,$	$\theta_x = u\theta_0 - \theta_e \cos\varphi.$
Vertical angle:	$\eta_y = \theta_\gamma \sin\varphi \quad ,$	$\theta_y = \quad - \theta_e \sin\varphi.$

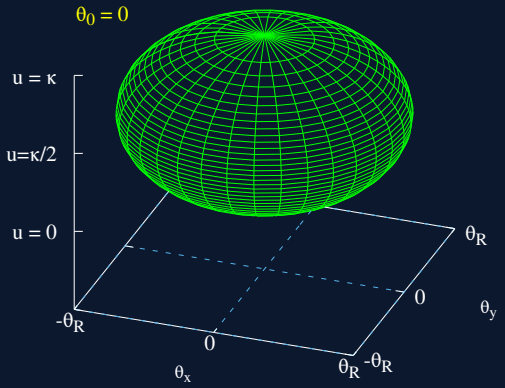
θ_e vs θ_γ vs u/κ (3D)

$$\theta_R = \max(\theta_e) = \frac{2\hbar\omega_0}{mc^2}$$

for green light θ_R
is about 10 urad

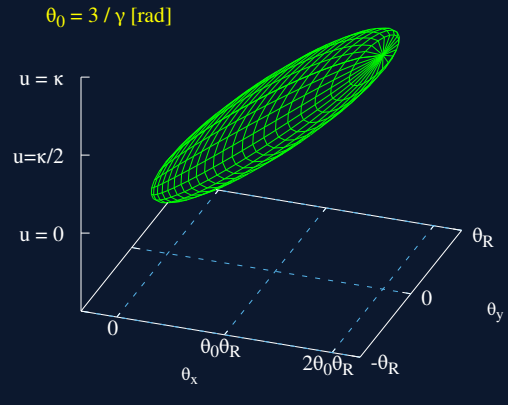


Bending of recoil electrons (3D)



$$\min(\theta_x) = -\theta_R$$

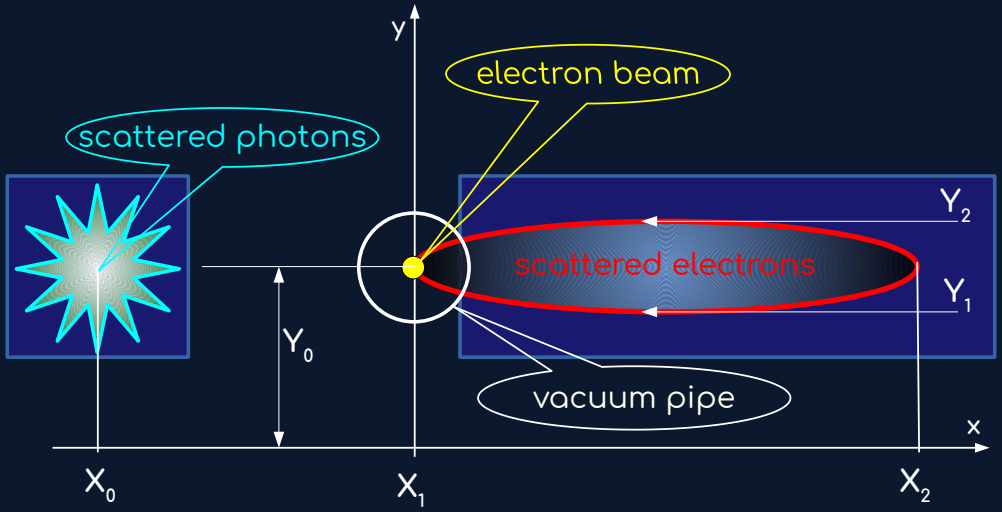
$$\max(\theta_x) = +\theta_R$$



$$\min(\theta_x) = \theta_R \left(\gamma\theta_0 - \sqrt{1 + (\gamma\theta_0)^2}\right)$$

$$\max(\theta_x) = \theta_R \left(\gamma\theta_0 + \sqrt{1 + (\gamma\theta_0)^2}\right)$$

x-y layout & detectors



Compton x-section

$$\frac{1}{r_e^2}\frac{d\sigma_0}{du\,d\varphi} = \frac{1}{\kappa(1+u)^3}\left[1+(1+u)^2-4\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)(1+u)\right]$$

$$\frac{1}{r_e^2}\frac{d\sigma_{\xi_{1,2}}}{du\,d\varphi} = \frac{4}{\kappa(1+u)^2}\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)\left[\xi_1\cos(2\varphi)+\xi_2\sin(2\varphi)\right]$$

$$\frac{1}{r_e^2}\frac{d\sigma_{x,y}}{du\,d\varphi} = \frac{-2\,\xi_3}{(1+u)^3}\;\frac{u}{\kappa}\sqrt{\frac{u}{\kappa}\left(1-\frac{u}{\kappa}\right)}\left[\zeta_x\cos(\varphi)+\zeta_y\sin(\varphi)\right]$$

$$\frac{1}{r_e^2}\frac{d\sigma_z}{du\,d\varphi} = \frac{\xi_3\,\zeta_z}{(1+u)^3}\;\frac{u}{\kappa}(u+2)\left(1-2\frac{u}{\kappa}\right)$$

$$\sigma_0 = \frac{2\pi r_e^2}{\kappa}\left[\left(1-\frac{4}{\kappa}-\frac{8}{\kappa^2}\right)\log(1+\kappa)\;+\frac{1}{2}\left(1-\frac{1}{(1+\kappa)^2}\right)+\frac{8}{\kappa}\right]$$

$$\sigma_z = \xi_3\,\zeta_z\frac{2\pi r_e^2}{\kappa}\left[\left(1+\frac{2}{\kappa}\right)\log(1+\kappa)-\frac{1}{2}\left(4+\left(\frac{\kappa}{1+\kappa}\right)^2\right)\right]$$

X-section: recoil e^- inside the ellipse

$$\left.\frac{1}{r_e^2}\frac{d\sigma_0}{dxdy}\right|_{\pm} = \frac{1+(1+u_\pm)^2-(1+u_\pm)(1-\Delta_\pm^2)}{2(1+u_\pm)^3\sqrt{1-x^2-y^2}},$$

$$\left.\frac{1}{r_e^2}\frac{d\sigma_{\xi_1}}{dxdy}\right|_{\pm} = \xi_1\frac{\delta_\pm^2-y^2}{2(1+u_\pm)^2\sqrt{1-x^2-y^2}},$$

$$\left.\frac{1}{r_e^2}\frac{d\sigma_{\xi_2}}{dxdy}\right|_{\pm} = \xi_2\frac{-\delta_\pm y}{(1+u_\pm)^2\sqrt{1-x^2-y^2}},$$

$$\left.\frac{1}{r_e^2}\frac{d\sigma_x}{dxdy}\right|_{\pm} = \xi_3\zeta_x\frac{-u_\pm\delta_\pm}{2(1+u_\pm)^3\sqrt{1-x^2-y^2}},$$

$$\left.\frac{1}{r_e^2}\frac{d\sigma_y}{dxdy}\right|_{\pm} = \xi_3\zeta_y\frac{u_\pm y}{2(1+u_\pm)^3\sqrt{1-x^2-y^2}},$$

$$\left.\frac{1}{r_e^2}\frac{d\sigma_z}{dxdy}\right|_{\pm} = \xi_3\zeta_z\frac{-u_\pm(u_\pm+2)\Delta_\pm}{2(1+u_\pm)^3\sqrt{1-x^2-y^2}}.$$

Function:

$$f(x,y) = 1/\rho;\ \rho = \sqrt{1-x^2-y^2}$$

Antiderivative:

$$F(x,y) = x\,\mathrm{atan2}(y,\rho) +$$

$$+y\,\mathrm{atan2}(x,\rho) - \mathrm{atan2}(xy,\rho)$$

Integral over pixel:

$$I(x_0,y_0,x_1,y_1) = F(x_0,y_0) +$$

$$+F(x_1,y_1) - F(x_0,y_1) - F(x_1,y_0)$$

MC simulations parameters

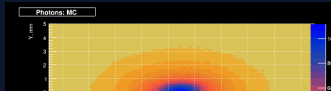
Table: Simulation parameters.

$\varepsilon_0 = 45.6$ GeV	$\gamma = 89237$	$\epsilon_x = 270$	pm rad	$\beta_x = 100$ m	$L_1 = 117$ m
$\lambda_0 = 532$ nm	$\kappa = 1.6279$	$\epsilon_y = 1$	pm rad	$\beta_y = 20$ m	$L_2 = 100$ m
$\omega_0 = 2.331$ eV	$\vartheta_0 = 190.44$	$\theta_0 = 2.1341$	mrad	$D_x = 25$ mm	$\sigma_\gamma/\gamma = 0.001$

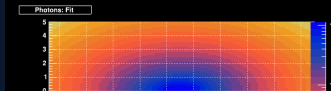
Table: Detectors: geometry, number of pixels, size of pixels.

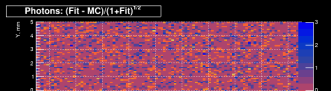
Detector	Size (X $\times$ Y)	Npix (X $\times$ Y)	Pixel size (X $\times$ Y)
Photons	10×10 mm	100×100	100×100 um
Electrons	400×4 mm	1600×80	250×50 um

Fitting the distributions.

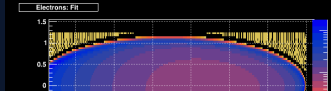


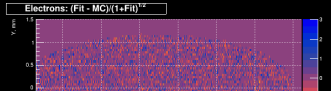
Monte-Carlo Parameters:
Laser λ_0 = 0.532 um
Electron E_0 = 45.600 GeV
Electron γ = 89.237×10^3
Compton κ = 1.628
Bend: $\gamma\theta_0$ = 190.441
(ξ_1, ξ_2, ξ_3) = (0.100, 0.100, 0.990)
(ζ_x, ζ_y, ζ_z) = (0.100, 0.250, 0.100)





Intel(R) Core(TM) i3-6100U CPU @ 2.30GHz
Photons fit: t = 49 s (CPU 49 s)
χ^2/NDF = 9935.8/9990 | Prob = 0.6477
X_0 = -213.539 ± 0.002 mm
ξ_1 = 0.102 ± 0.002 ξ_2 = 0.100 ± 0.001
$\xi_3\zeta_x$ = 0.095 ± 0.007 $\xi_3\zeta_y$ = 0.247 ± 0.006
$\xi_3\zeta_z$ = 0.105 ± 0.002
σ_x = 245.7 ± 3.6 μm σ_y = 12.57 ± 70.37 μm





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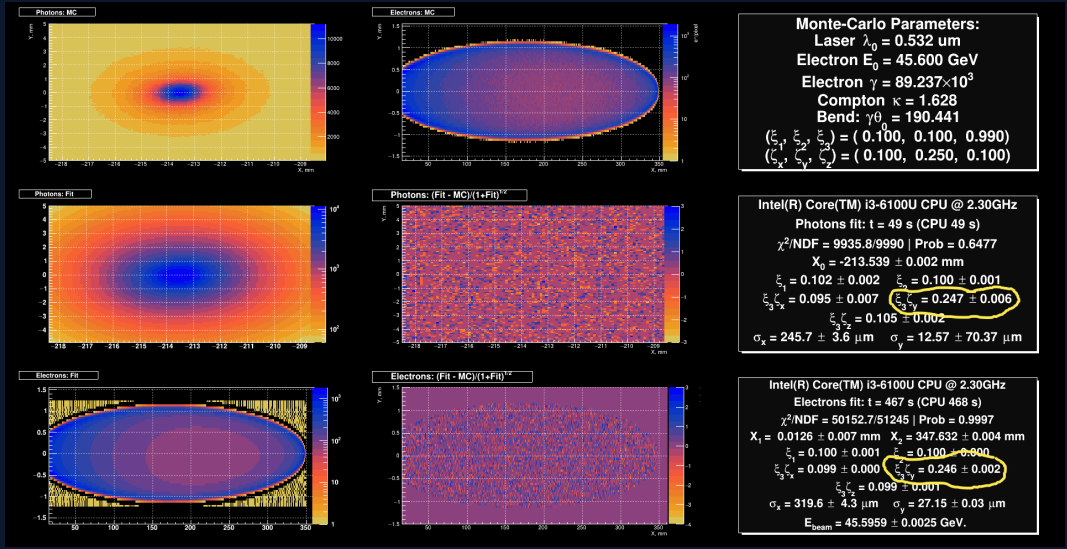
Electrons fit: t = 467 s (CPU 468 s)
χ^2/NDF = 50152.7/51245 | Prob = 0.9997
X_1 = 0.0126 ± 0.007 mm X_2 = 347.632 ± 0.004 mm
ξ_1 = 0.100 ± 0.001 ξ_2 = 0.100 ± 0.000
$\xi_3\zeta_x$ = 0.099 ± 0.000 $\xi_3\zeta_y$ = 0.246 ± 0.002
$\xi_3\zeta_z$ = 0.099 ± 0.001
σ_x = 319.6 ± 4.3 μm σ_y = 27.15 ± 0.03 μm
E_{beam} = 45.5959 ± 0.0025 GeV.

Polarimeter



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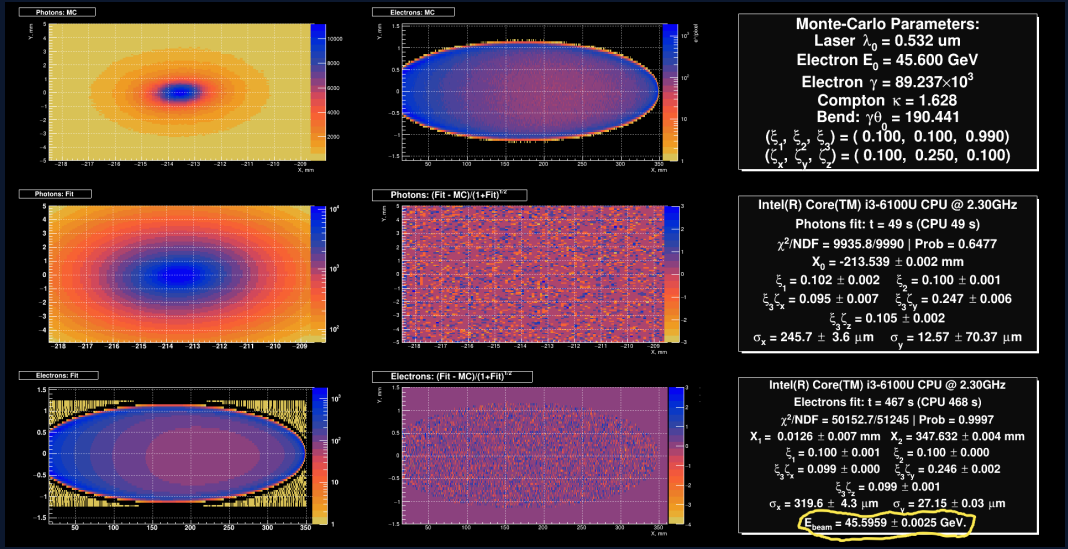
Energy Spectrometer



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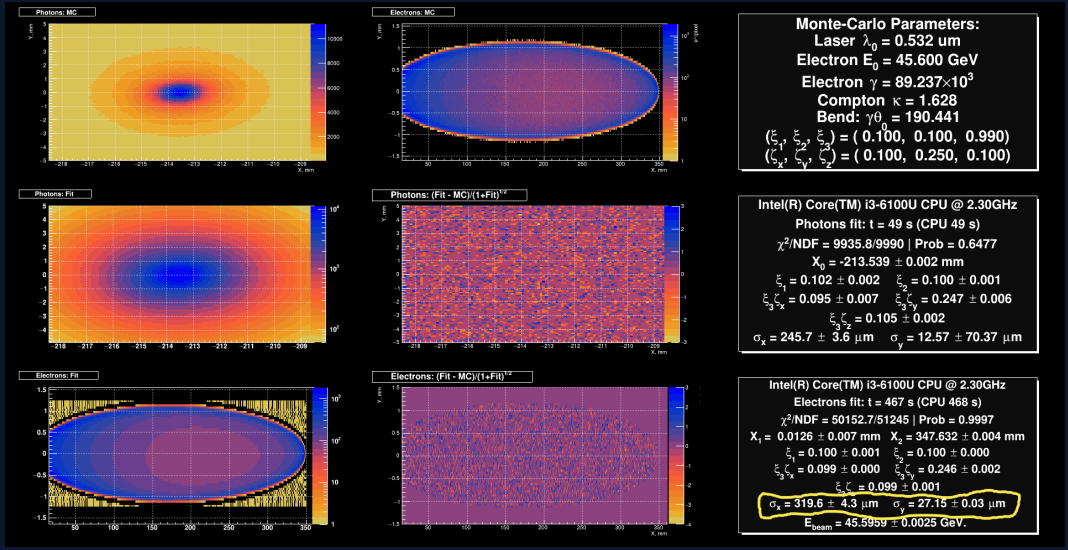
Beam size monitor



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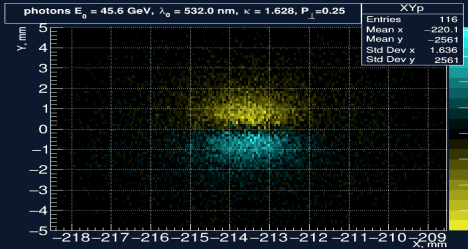
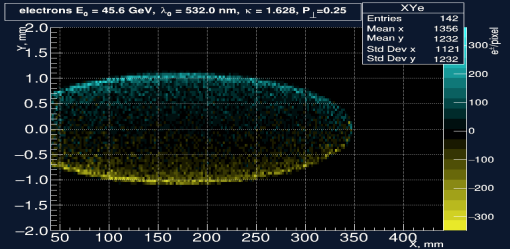
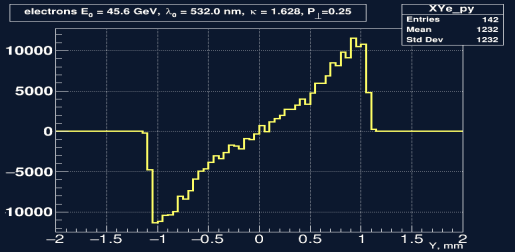
Toggling $\xi_3 = \pm 1$ example.







Summary

Thank you for your attention!
Questions are welcome...