

# FCC-ee 3D Polarimetry

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2nd FCC Polarization Workshop

The detailed article (arXiv:2208.00585)  
has been submitted to JINST and is under review now

arXiv > physics > arXiv:2208.00585

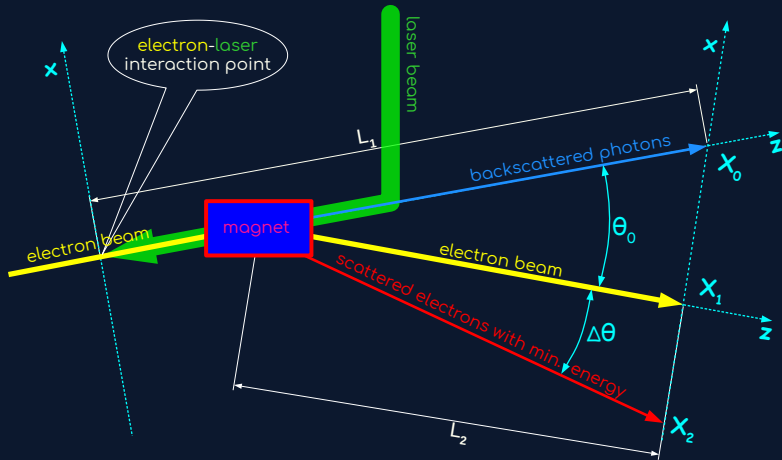
Physics > Accelerator Physics

*[Submitted on 1 Aug 2022 (v1), last revised 16 Sep 2022 (this version, v3)]*

**Electron beam polarimeter and energy spectrometer**

N. Yu. Muchnoi

# x-z layout



$\varepsilon_0$  - beam e- energy  
 $\hbar\omega_0$  - laser  $\gamma$  energy

$$\theta_0 = \frac{mc^2}{\varepsilon_0} \left[ \frac{\int B_{\perp} dl}{mc/e} \right]$$

$$\frac{mc}{e} \simeq 1.7 \cdot 10^{-3} [\text{T m}]$$

$\alpha \simeq \pi$  collision :

$$\Delta\theta = \frac{4\hbar\omega_0}{mc^2} \left[ \frac{\int B_{\perp} dl}{mc/e} \right]$$

$$\varepsilon_0 = \frac{(mc^2)^2}{4\hbar\omega_0} \frac{\Delta\theta}{\theta_0}$$

# Inverse Compton Scattering

$$\gamma = \frac{\varepsilon_0}{mc^2}$$

$$\kappa = 4\gamma \frac{\hbar\omega_0}{mc^2} \sin^2 \frac{\alpha}{2}$$

$$u \equiv \frac{\hbar\omega}{\varepsilon} \in [0, \kappa]$$

If  $\kappa \ll \gamma$ :

$$\varepsilon_0, \varepsilon, \hbar\omega \gg \hbar\omega_0$$

$$u = \frac{\theta_e}{\theta_\gamma} = \frac{\varepsilon_0 - \varepsilon}{\varepsilon}$$

scattered photon:

recoil electron:

Energy:

$$\hbar\omega = \frac{\varepsilon_0 u}{1 + u},$$

$$\varepsilon = \frac{\varepsilon_0}{1 + u}.$$

Edge energy:

$$\hbar\omega_{max} = \frac{\varepsilon_0 \kappa}{1 + \kappa},$$

$$\varepsilon_{min} = \frac{\varepsilon_0}{1 + \kappa}.$$

Scattering angle:

$$\theta_\gamma = \frac{1}{\gamma} \sqrt{\kappa/u - 1},$$

$$\theta_e = \frac{u}{\gamma} \sqrt{\kappa/u - 1}.$$

Angle from bend:

$$\theta_0,$$

$$u\theta_0.$$

Maximum bend:

$$\theta_0,$$

$$\Delta\theta = \kappa\theta_0.$$

Azimuthal angle:

$$\varphi,$$

$$\varphi + \pi.$$

Horizontal angle:

$$\eta_x = \theta_\gamma \cos \varphi - \theta_0,$$

$$\theta_x = u\theta_0 - \theta_e \cos \varphi.$$

Vertical angle:

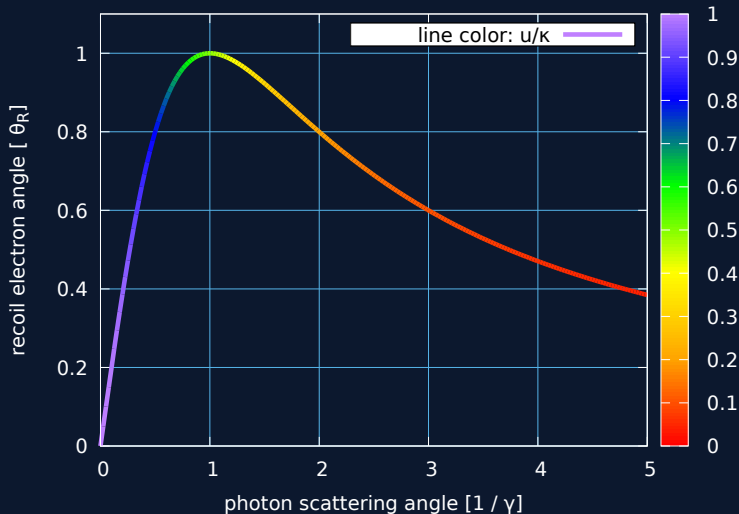
$$\eta_y = \theta_\gamma \sin \varphi,$$

$$\theta_y = -\theta_e \sin \varphi.$$

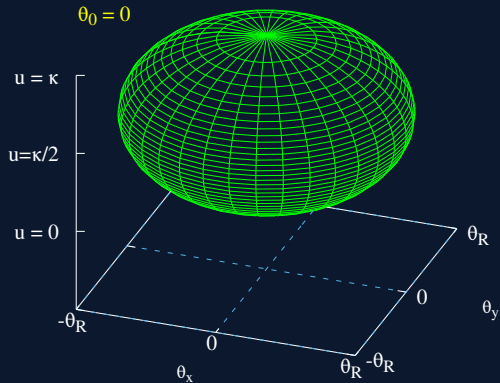
# $\theta_e$ vs $\theta_\gamma$ vs $u/\kappa$ (3D)

$$\theta_R = \max(\theta_e) = \frac{2\hbar\omega_0}{mc^2}$$

for green light  $\theta_R$   
is about 10 urad

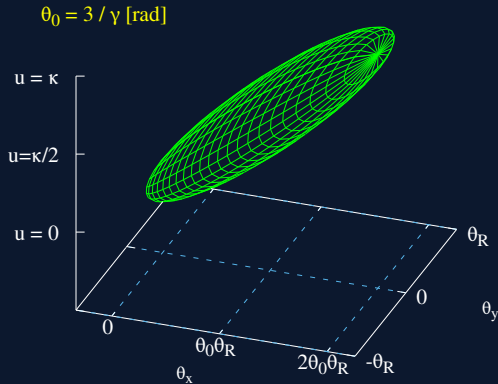


# Bending of recoil electrons (3D)



$$\min(\theta_x) = -\theta_R$$

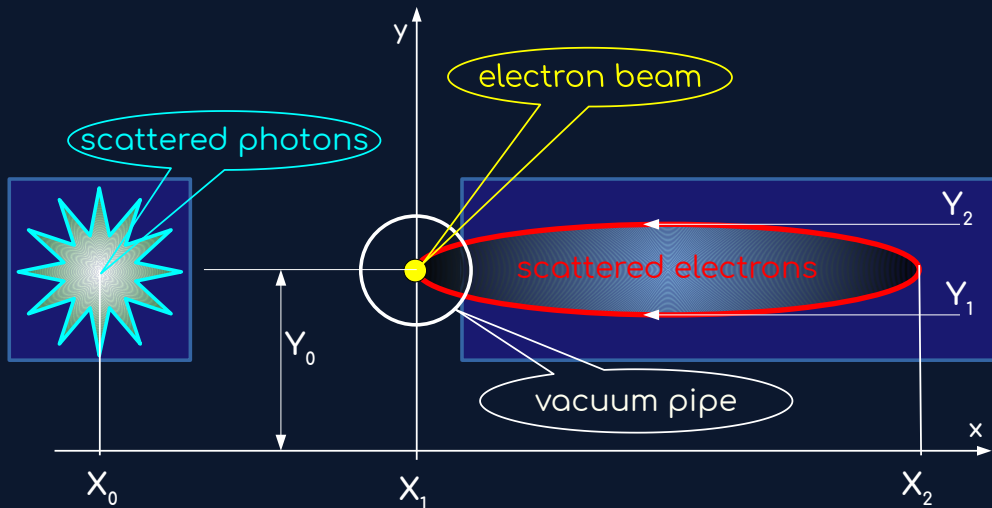
$$\max(\theta_x) = +\theta_R$$



$$\min(\theta_x) = \theta_R \left( \gamma\theta_0 - \sqrt{1 + (\gamma\theta_0)^2} \right)$$

$$\max(\theta_x) = \theta_R \left( \gamma\theta_0 + \sqrt{1 + (\gamma\theta_0)^2} \right)$$

# x-y layout & detectors



# Compton x-section

$$\frac{1}{r_e^2} \frac{d\sigma_0}{du d\varphi} = \frac{1}{\kappa(1+u)^3} \left[ 1 + (1+u)^2 - 4\frac{u}{\kappa} \left(1 - \frac{u}{\kappa}\right) (1+u) \right]$$

$$\frac{1}{r_e^2} \frac{d\sigma_{\xi_{1,2}}}{du d\varphi} = \frac{4}{\kappa(1+u)^2} \frac{u}{\kappa} \left(1 - \frac{u}{\kappa}\right) [\xi_1 \cos(2\varphi) + \xi_2 \sin(2\varphi)]$$

$$\frac{1}{r_e^2} \frac{d\sigma_{x,y}}{du d\varphi} = \frac{-2\xi_3}{(1+u)^3} \frac{u}{\kappa} \sqrt{\frac{u}{\kappa} \left(1 - \frac{u}{\kappa}\right)} [\zeta_x \cos(\varphi) + \zeta_y \sin(\varphi)]$$

$$\frac{1}{r_e^2} \frac{d\sigma_z}{du d\varphi} = \frac{\xi_3 \zeta_z}{(1+u)^3} \frac{u}{\kappa} (u+2) \left(1 - 2\frac{u}{\kappa}\right)$$

$$\sigma_0 = \frac{2\pi r_e^2}{\kappa} \left[ \left(1 - \frac{4}{\kappa} - \frac{8}{\kappa^2}\right) \log(1+\kappa) + \frac{1}{2} \left(1 - \frac{1}{(1+\kappa)^2}\right) + \frac{8}{\kappa} \right]$$

$$\sigma_z = \xi_3 \zeta_z \frac{2\pi r_e^2}{\kappa} \left[ \left(1 + \frac{2}{\kappa}\right) \log(1+\kappa) - \frac{1}{2} \left(4 + \left(\frac{\kappa}{1+\kappa}\right)^2\right) \right]$$



# X-section: recoil $e^-$ inside the ellipse

$$\frac{1}{r_e^2} \frac{d\sigma_0}{dxdy} \Big|_{\pm} = \frac{1 + (1 + u_{\pm})^2 - (1 + u_{\pm})(1 - \Delta_{\pm}^2)}{2(1 + u_{\pm})^3 \sqrt{1 - x^2 - y^2}},$$

$$\frac{1}{r_e^2} \frac{d\sigma_{\xi_1}}{dxdy} \Big|_{\pm} = \xi_1 \frac{\delta_{\pm}^2 - y^2}{2(1 + u_{\pm})^2 \sqrt{1 - x^2 - y^2}},$$

$$\frac{1}{r_e^2} \frac{d\sigma_{\xi_2}}{dxdy} \Big|_{\pm} = \xi_2 \frac{-\delta_{\pm} y}{(1 + u_{\pm})^2 \sqrt{1 - x^2 - y^2}},$$

$$\frac{1}{r_e^2} \frac{d\sigma_x}{dxdy} \Big|_{\pm} = \xi_3 \zeta_x \frac{-u_{\pm} \delta_{\pm}}{2(1 + u_{\pm})^3 \sqrt{1 - x^2 - y^2}},$$

$$\frac{1}{r_e^2} \frac{d\sigma_y}{dxdy} \Big|_{\pm} = \xi_3 \zeta_y \frac{u_{\pm} y}{2(1 + u_{\pm})^3 \sqrt{1 - x^2 - y^2}},$$

$$\frac{1}{r_e^2} \frac{d\sigma_z}{dxdy} \Big|_{\pm} = \xi_3 \zeta_z \frac{-u_{\pm}(u_{\pm} + 2)\Delta_{\pm}}{2(1 + u_{\pm})^3 \sqrt{1 - x^2 - y^2}}.$$

Function:

$$f(x, y) = 1/\rho; \quad \rho = \sqrt{1 - x^2 - y^2}$$

Antiderivative:

$$F(x, y) = x \operatorname{atan2}(y, \rho) + \\ + y \operatorname{atan2}(x, \rho) - \operatorname{atan2}(xy, \rho)$$

Integral over pixel:

$$I(x_0, y_0, x_1, y_1) = F(x_0, y_0) + \\ + F(x_1, y_1) - F(x_0, y_1) - F(x_1, y_0)$$

# MC simulations parameters

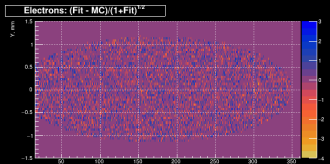
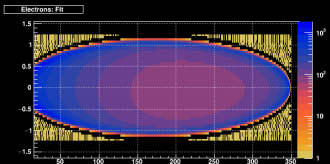
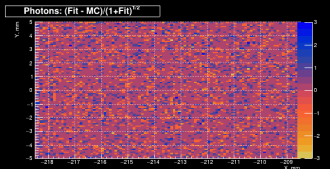
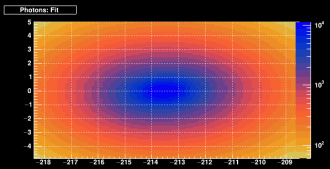
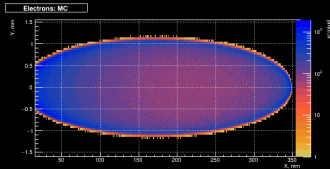
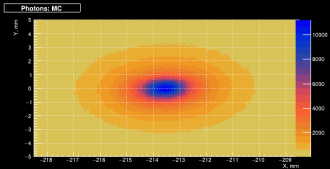
Table: Simulation parameters.

$\varepsilon_0 = 45.6 \text{ GeV}$	$\gamma = 89237$	$\epsilon_x = 270$	pm rad	$\beta_x = 100 \text{ m}$	$L_1 = 117 \text{ m}$
$\lambda_0 = 532 \text{ nm}$	$\kappa = 1.6279$	$\epsilon_y = 1$	pm rad	$\beta_y = 20 \text{ m}$	$L_2 = 100 \text{ m}$
$\omega_0 = 2.331 \text{ eV}$	$\vartheta_0 = 190.44$	$\theta_0 = 2.1341$	mrad	$D_x = 25 \text{ mm}$	$\sigma_\gamma/\gamma = 0.001$

Table: Detectors: geometry, number of pixels, size of pixels.

Detector	Size (X × Y)	Npix (X × Y)	Pixel size (X × Y)
Photons	10 × 10 mm	100 × 100	100 × 100 $\mu\text{m}$
Electrons	400 × 4 mm	1600 × 80	250 × 50 $\mu\text{m}$

# Fitting the distributions.

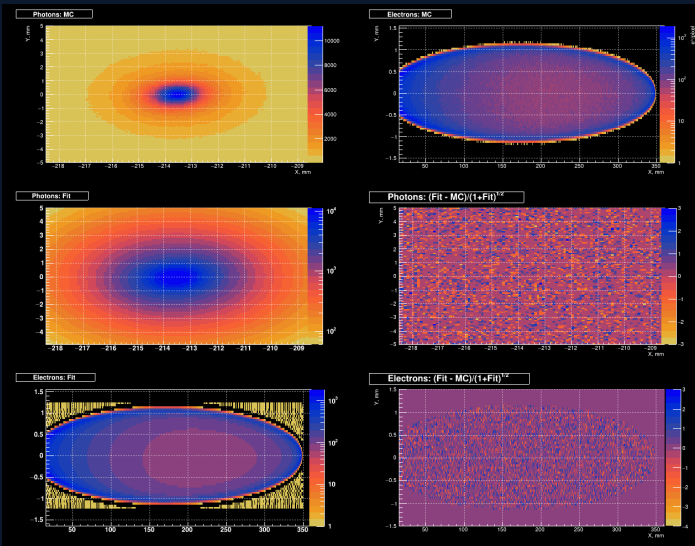


**Monte-Carlo Parameters:**  
 Laser  $\lambda_0 = 0.532 \mu\text{m}$   
 Electron  $E_0 = 45.600 \text{ GeV}$   
 Electron  $\gamma = 89.237 \times 10^3$   
 Compton  $\kappa = 1.628$   
 Bend:  $\gamma\theta = 190.441$   
 $(\zeta_x^{\text{MC}}, \zeta_y^{\text{MC}}, \zeta_z^{\text{MC}}) = (0.100, 0.100, 0.990)$   
 $(\zeta_x^{\text{Fit}}, \zeta_y^{\text{Fit}}, \zeta_z^{\text{Fit}}) = (0.100, 0.250, 0.100)$

**Intel(R) Core(TM) i3-6100U CPU @ 2.30GHz**  
 Photons fit: t = 49 s (CPU 49 s)  
 $\chi^2/\text{NDF} = 9935.8/9990 \mid \text{Prob} = 0.6477$   
 $X_0 = -213.539 \pm 0.002 \text{ mm}$   
 $\zeta_x^{\text{Fit}} = 0.102 \pm 0.002 \quad \zeta_y^{\text{Fit}} = 0.100 \pm 0.001$   
 $\zeta_x^{\text{MC}} = 0.095 \pm 0.007 \quad \zeta_y^{\text{MC}} = 0.247 \pm 0.006$   
 $\zeta_x^{\text{Fit}} \zeta_y^{\text{Fit}} = 0.105 \pm 0.002$   
 $\sigma_x = 245.7 \pm 3.6 \mu\text{m} \quad \sigma_y = 12.57 \pm 70.37 \mu\text{m}$

**Intel(R) Core(TM) i3-6100U CPU @ 2.30GHz**  
 Electrons fit: t = 467 s (CPU 468 s)  
 $\chi^2/\text{NDF} = 50152.7/51245 \mid \text{Prob} = 0.9997$   
 $X_1 = 0.0126 \pm 0.007 \text{ mm} \quad X_2 = 347.632 \pm 0.004 \text{ mm}$   
 $\zeta_x^{\text{Fit}} = 0.100 \pm 0.001 \quad \zeta_y^{\text{Fit}} = 0.100 \pm 0.000$   
 $\zeta_x^{\text{MC}} = 0.099 \pm 0.000 \quad \zeta_y^{\text{MC}} = 0.246 \pm 0.002$   
 $\zeta_x^{\text{Fit}} \zeta_y^{\text{Fit}} = 0.099 \pm 0.001$   
 $\sigma_x = 319.6 \pm 4.3 \mu\text{m} \quad \sigma_y = 27.15 \pm 0.03 \mu\text{m}$   
 $E_{\text{beam}} = 45.5959 \pm 0.0025 \text{ GeV}$

# Polarimeter



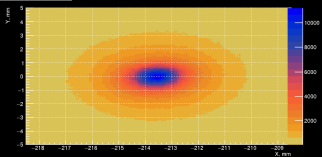
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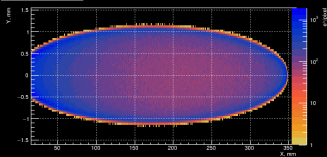
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# Energy Spectrometer

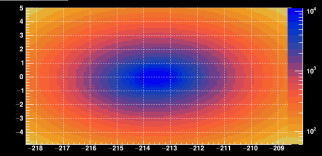
Photons: MC



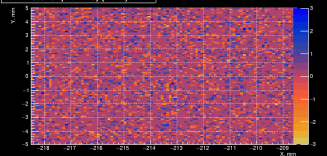
Electrons: MC



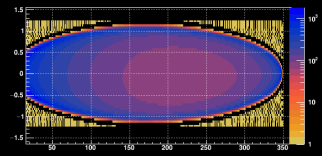
Photons: Fit



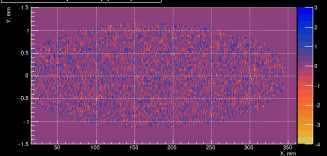
Photons: (Fit - MC)/(1+Fit)^k



Electrons: Fit



Electrons: (Fit - MC)/(1+Fit)^k



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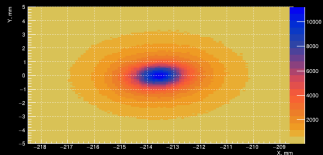
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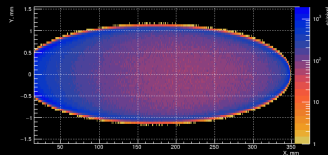
$E_{\text{beam}} = 45.5959 \pm 0.0025 \text{ GeV}$

# Beam size monitor

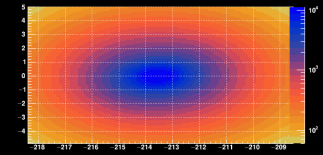
Photons: MC



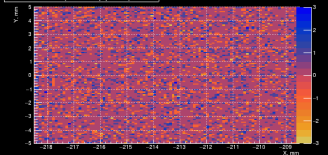
Electrons: MC



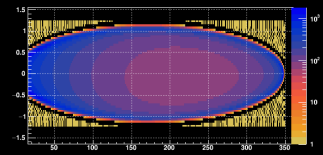
Photons: Fit



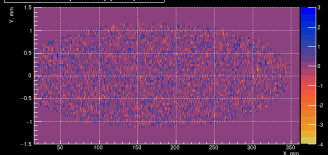
Photons: (Fit - MC)/(1+Fit)^x



Electrons: Fit



Electrons: (Fit - MC)/(1+Fit)^x



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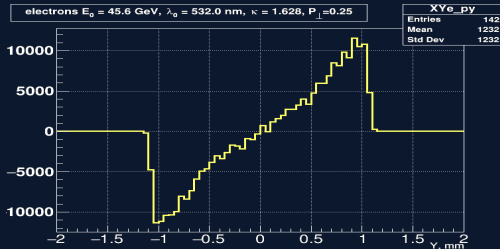
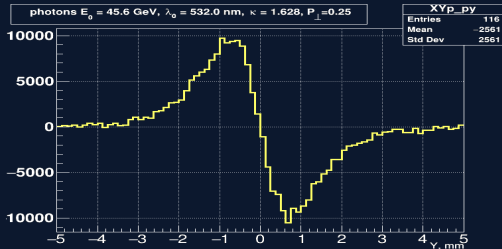
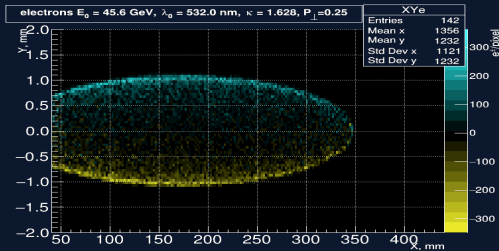
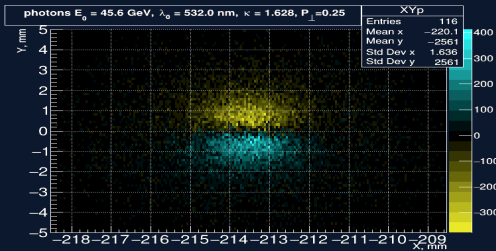
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$\xi_3^x \xi_3^y = 0.099 \pm 0.001$

$\sigma_x = 319.6 \pm 4.3 \text{ \mu m} \quad \sigma_y = 27.15 \pm 0.03 \text{ \mu m}$

$E_{\text{beam}} = 45.5959 \pm 0.0025 \text{ GeV}$

# Toggling $\xi_3 = \pm 1$ example.



# Summary

Thank you for your attention!  
Questions are welcome...