Luminosity Spectrum Reconstruction with Bhabha Events

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CERN-EP-SFT

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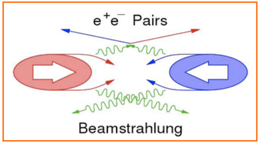
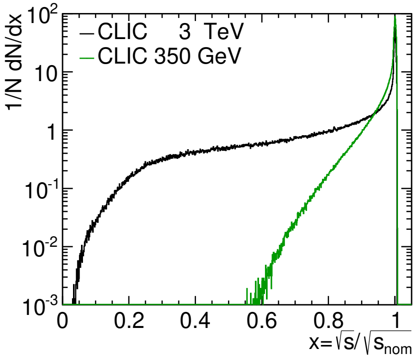
Reminder: Beam–Beam Interactions

- Large luminosities require high bunch charge and small beams $L \propto \frac{N^2}{\sigma_x \sigma_y}$
- Electromagnetic fields during bunch crossing $B \propto \frac{\gamma N}{\sigma_z(\sigma_x+\sigma_y)}$ cause deflection of beam particles
- Deflection of particles by the other bunch leads to synchrotron radiation (Beamstrahlung)



- Energy loss leads to luminosity spectrum
  - For 3 TeV CLIC, still 30% of luminosity above 99% of nominal energy



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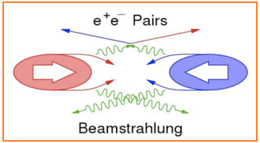

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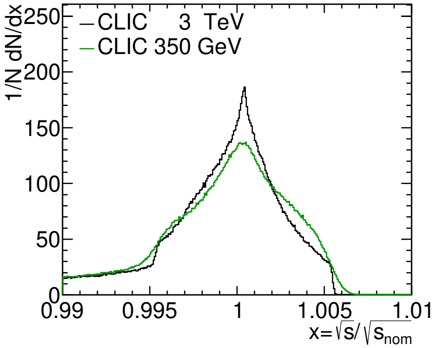

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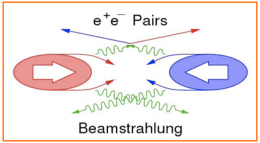

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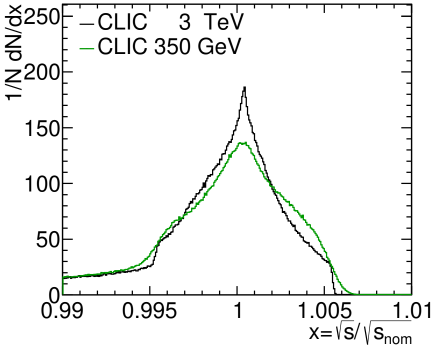
  - For 3 TeV CLIC, still 30% of luminosity above 99% of nominal energy
- How well can the luminosity spectrum be reconstructed?



Measuring the Luminosity Spectrum

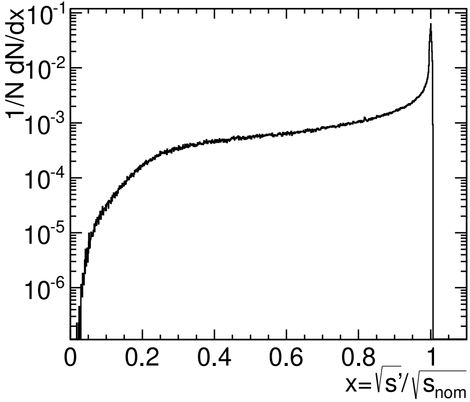
Beam–beam effects (and thus the luminosity spectrum) are highly dependent on bunch geometries

- Cannot measure bunch geometry to sufficient detail
- Bunch geometry changes over time
- If geometry is not known, simulation is not possible
- Downstream measurement of Beamstrahlung photons give no direct access to luminosity spectrum

Therefore: Have to measure luminosity spectrum at the IP with the detector

The Luminosity Spectrum: Definitions

CLIC $\sqrt{s_{\text{nom}}} = 3$ TeV luminosity spectrum as simulated by GUINEAPIG



- Given two particles with the energies E_1 and E_2 colliding head-on, the centre-of-mass energy is $\sqrt{s'} = 2\sqrt{E_1 E_2} = 2E_{\text{Beam}}\sqrt{x_1 x_2}$ $(x_{1,2} = E_{1,2}/E_{\text{Beam}})$
- The luminosity spectrum is the probability distribution of centre-of-mass energies $\mathscr{L}\left(\sqrt{s'}\right)$

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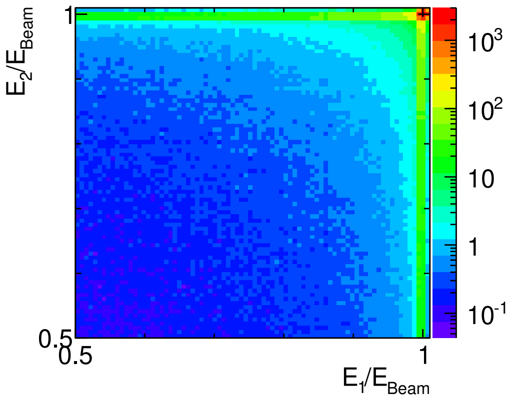

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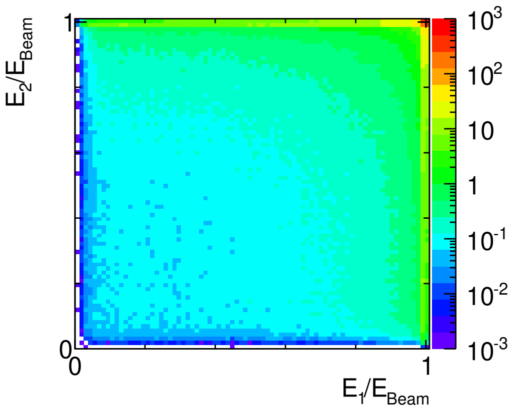
- The luminosity spectrum is the probability distribution of centre-of-mass energies $\mathscr{L}(\sqrt{s})$
- Better: The luminosity spectrum is the probability distribution of the energies of the particle pair $\mathscr{L}(E_1, E_2)$

What is the Goal of this Reconstruction?

- Goal: The distribution of the pairs of particle energies prior to the 'hard interaction'
  - Only reconstructing the centre-of-mass energy ignores the longitudinal boost of the system
  - Strong correlation between the two particle energies
  - Account for (potentially) asymmetric beams

Particle Energy Spectrum from GUINEAPIG



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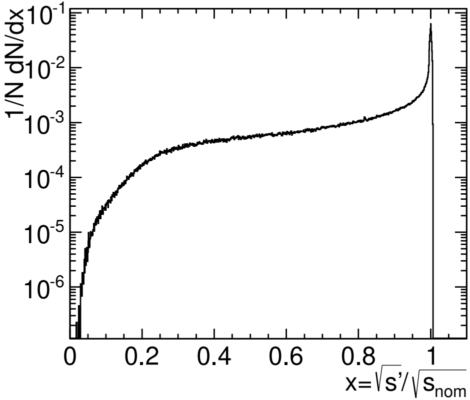
  - Only reconstructing the centre-of-mass energy ignores the longitudinal boost of the system
  - Strong correlation between the two particle energies
  - Account for (potentially) asymmetric beams
- Mostly show the centre-of-mass system (c.m.s.) luminosity spectrum $\mathscr{L}(\sqrt{s'})$ because it is easier to visualise and interpret

Luminosity Spectrum from GUINEAPIG



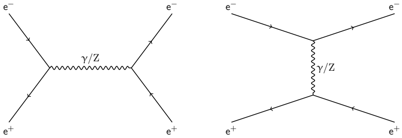
$$\mathscr{L}(\sqrt{s'}) = \int \mathrm{d}x_1 \int \mathrm{d}x_2 \mathscr{L}(x_1, x_2)\,\delta\!\left(\frac{\sqrt{s'}}{\sqrt{s_{\mathrm{nom}}}} - \sqrt{x_1 x_2}\right)$$

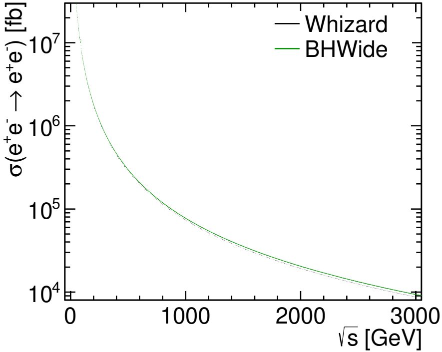
Bhabha Scattering

- Bhabha scattering $e^+e^- \to e^+e^-(\gamma)$ has:
  - Large cross-section
  - Well known cross-section (calculable to high precision)
- Cross-section: 10 000 fb at 3 TeV (with polar angle of electrons above 7°)
  - Proportional to $1/(s \sin^3\theta)$
- Can reconstruct relative centre-of-mass energy from polar angle difference (acollinearity)

$$\frac{\sqrt{s'_{\mathrm{acol}}}}{\sqrt{s_{\mathrm{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$$

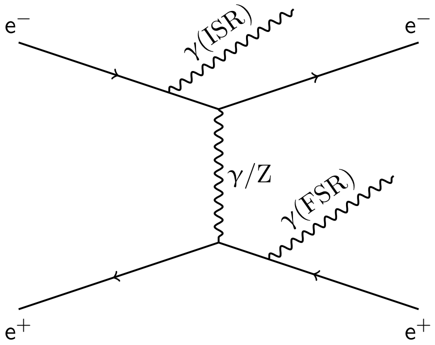
- Also measure the energy of final state electron and positron





What Distribution is Measured in the Detector?

- The detector measure the final state electron and positron affected by the cross-section (initial state radiation (ISR), final state radiation (FSR), $\sqrt{s}$ dependence)
- There is no way, for an individual event, to know if the energy was lost from initial state radiation or Beamstrahlung
- The measured values are also affected by the resolution of the respective sub-detector



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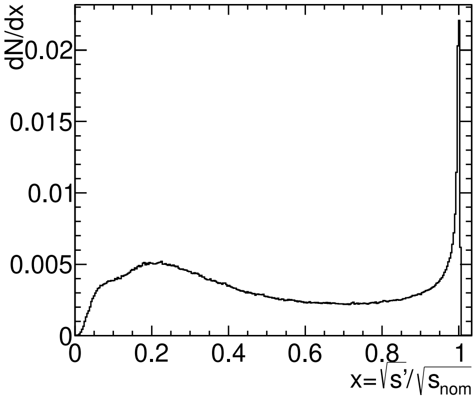
Distributions after Bhabha scattering (+ISR) and cross-section (without detector resolutions)



How to Extract the Luminosity Spectrum ($\mathscr{L}$) from Measurements?

With the distribution $f(E_1, E_2)$ of a vector of observables

$$f(E_1, E_2) \approx \sigma(E_1, E_2) \times \mathscr{L}(E_1, E_2) \otimes \mathrm{ISR}(E_1, E_2) \otimes \mathrm{FSR}(E_1, E_2) \otimes \mathrm{D}(E_1)\mathrm{D}(E_2)$$

connected to the luminosity spectrum and measurable in the detector.
One can then do either:

- De-convolute the measured (2D) spectrum to remove the initial state radiation energy loss, and detector resolutions, un-weight cross-section dependence
- Model the measured spectrum including cross-section, initial state radiation, and luminosity spectrum
  - Create a 2D function for the complete model and fit the measured spectrum to extract the luminosity spectrum
  - Let Bhabha generator take care of cross-section and initial state radiation, do GEANT4 simulation, and only model the luminosity spectrum
    - Do a template fit (normal models have 1 or 2 free parameters (e.g., mass and width), here one would need to have templates in a $\approx$ 25D phase space)
    - Use a reweighting technique for *efficient* fitting
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Reweighting Fit in Words

Reweighting technique uses χ^2-fit of two histogram with a distribution like

$$f(E_1,E_2) \approx \sigma(E_1,E_2) \times \mathscr{L}(E_1,E_2) \otimes \mathrm{ISR}(E_1,E_2) \otimes \mathrm{FSR}(E_1,E_2) \otimes \mathrm{D}(E_1)\mathrm{D}(E_2)$$

- Data histogram: measured in detector (simulated by GUINEAPIG) (also apply Bhabha-scattering and detector simulation)
- MC histogram: Luminosity spectrum according to a parametrisation
  - Apply Bhabha scattering/ISR/Detector resolutions on event-by-event basis via MC Generator and detector simulation
  - Remember initial probability based on luminosity spectrum of each event $\mathscr{L}(x_1^i, x_2^i; [p]_0)$
  - Vary all event probabilities (via model parameters $[p]_N$) until minimum χ^2 is found

$$\text{event weight: } w^i = \frac{\mathscr{L}(x_1^i, x_2^i; [p]_N)}{\mathscr{L}(x_1^i, x_2^i; [p]_0)}$$

- Advantage
  - Only have to do (very time consuming) Bhabha-scattering and detector simulation once

Section 2:

- The Model: Parametrisation of the Luminosity Spectrum
  - Beta-Distributions
  - Beam-Energy Spread
  - Beamstrahlung
  - The Full Model
  - Model Validation

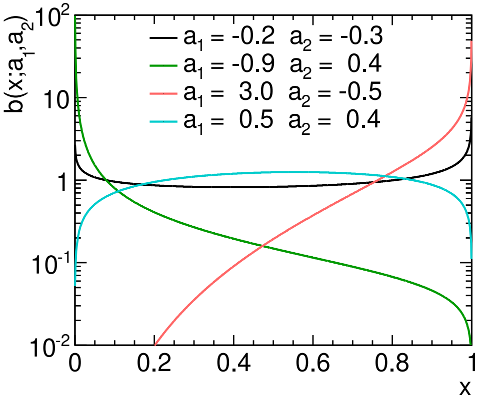
Beta-Distributions

- For the model of the luminosity spectrum mostly using Beta-Distributions

$$b(x) = \frac{1}{N} x^{a_1} (1-x)^{a_2}$$

with different parameter bounds

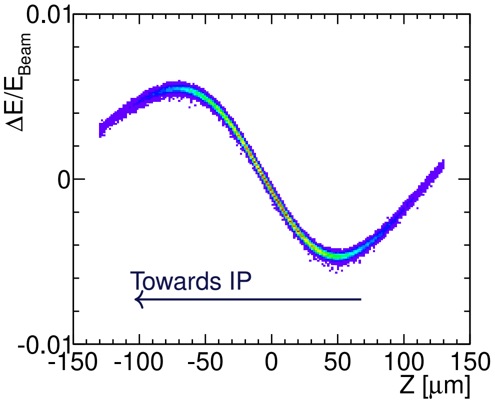
- Range: $0 < x < 1$
- Beta-Distribution can represent wide variety of shapes
- Two free parameters: a_1 and a_2, Normalisation N



Beam-Energy Spread I

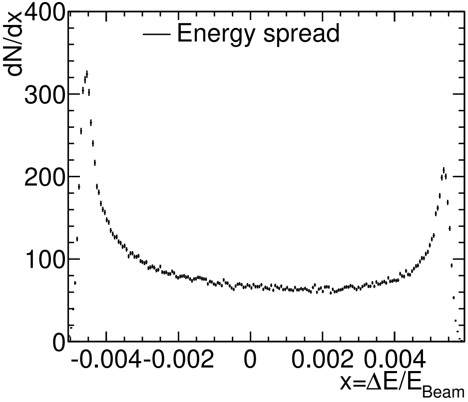
- Energy distribution in the bunch mostly due to intra-bunch wakefields and RF phase offset in main Linac
- Front of bunch gains more energy, because wakefields reduce effective gradient for the tail

Particle energy vs. longitudinal position from the accelerator simulation (3 TeV CLIC)



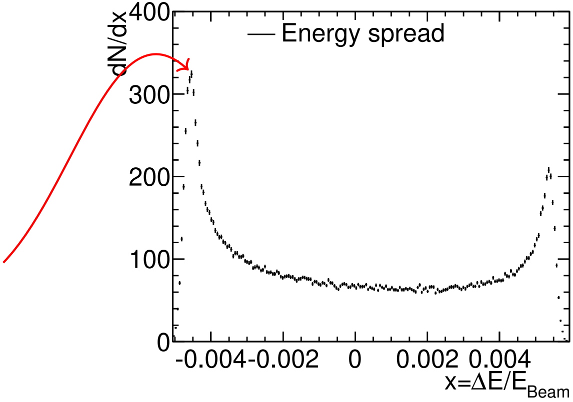
Beam-Energy Spread II

- Beam-Energy spread shows two peaks
- Mean around the nominal beam-energy



Beam-Energy Spread II

- Beam-Energy spread shows two peaks
- Mean around the nominal beam-energy
- Lower energy peak is *back* of the bunch



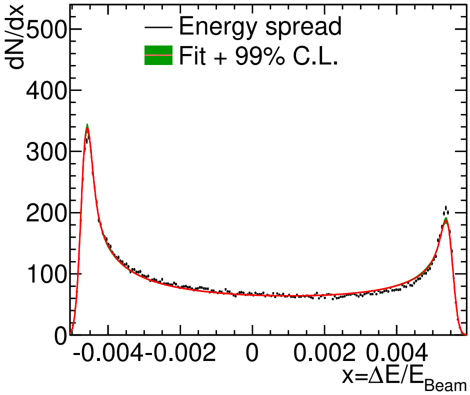
Beam-Energy Spread Function

- Beam-Energy Spread: Beta-distribution convoluted with Gauss

$$\mathrm{BES}(x) = \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} b(\tau)\mathrm{Gauss}(x-\tau)\mathrm{d}\tau$$

- 5 parameters, including min. and max. of beta-distribution range
- χ^2/ndf = 764/195
- Tried many other functions (Cosh, Polynomials), none of them work as well with a limited number of parameters

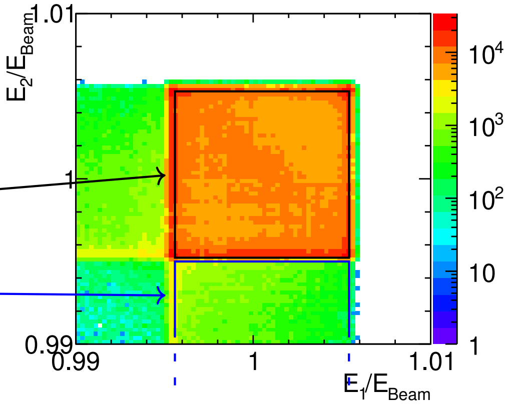
Particle energy distribution from accelerator simulation



Luminosity-weighted Beam-Energy Spread

- Due to the correlation of particle energy and longitudinal position, Beamstrahlung, and beam-beam effects, two vastly different beam-energy spread distributions emerge for the luminosity spectrum
- *Peak* Region: Both particles with $E > 0.995 E_{\mathrm{Beam}}$
- *Arms* Region: Only one of the particles with $E > 0.995 E_{\mathrm{Beam}}$
- Both can be fit with a beta-distribution convoluted with a Gauss (keeping x_{min}, x_{max}, and σ fixed)

Peak of the luminosity spectrum



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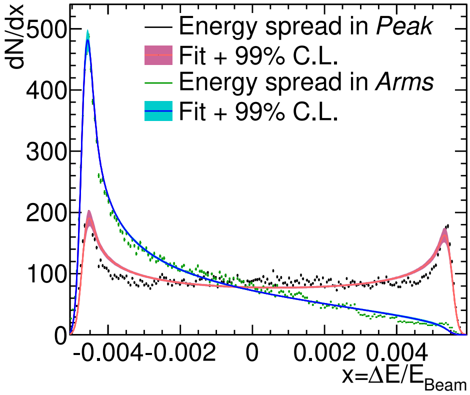
Particle energy distribution from the GUINEAPIG simulation 3 TeV



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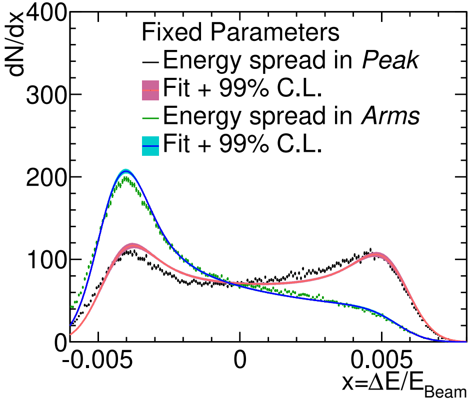
Particle energy distribution from the GUINEAPIG simulation 350 GeV

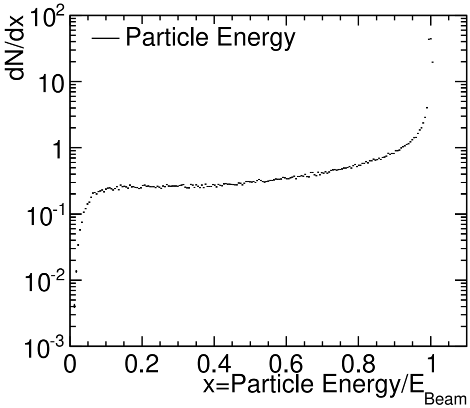


Beamstrahlung

- Second contribution to luminosity spectrum is energy loss due to Beamstrahlung
- Potentially large loss of energy for some particles

Fitting the particle Energy Spectrum

- Upper bound of $0.995E_{\mathrm{Beam}}$, because of impact of beam-energy spread (Particle energy is convolution of Beamstrahlung and beam-energy spread effect)
- Single Beta-Distribution not enough to describe full range of particle energies
- Keep small number of parameters: Limit model to $0.5E_{\mathrm{Beam}}$ and a single beta-distribution, but could extend in the future



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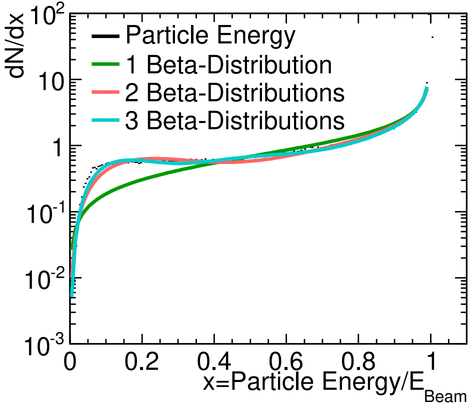

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$$b_{\text{linear}}(x) = \sum_{i=1}^{N_{\text{Beta}}} p_i \, b(x;[p]_i)$$

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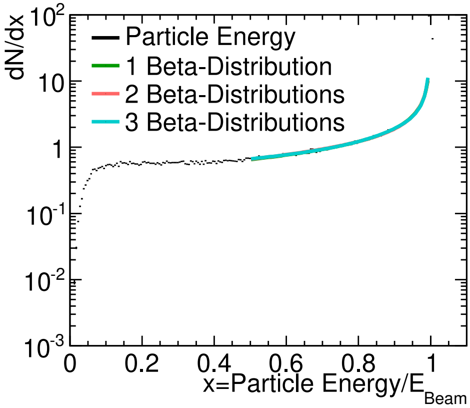

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The Model: Putting the Individual Parts Together

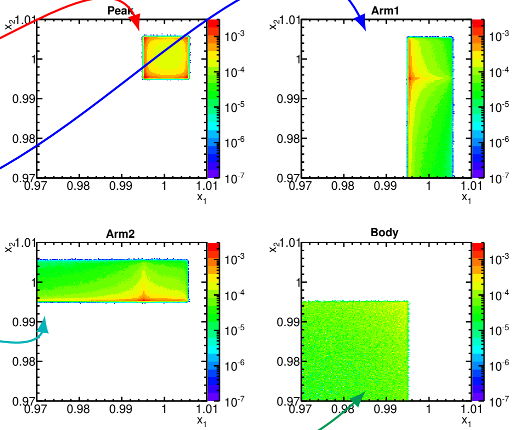
$$\mathcal{L}(x_1, x_2) =$$

$$p_{\text{Peak}}\,\delta(1-x_1) \otimes \text{BES}\left(x_1; [p]_1^{\text{Peak}}\right) \; \delta(1-x_2) \otimes \text{BES}\left(x_2; [p]_2^{\text{Peak}}\right)$$

$$+ p_{\text{Arm1}}\,\delta(1-x_1) \otimes \text{BES}\left(x_1; [p]_1^{\text{Arm}}\right) \; \text{BB}\left(x_2; [p]_2^{\text{Arm}}, \beta^1_{\text{limit}}\right)$$

$$+ p_{\text{Arm2}}\,\text{BB}\left(x_1; [p]_1^{\text{Arm}}, \beta^1_{\text{limit}}\right) \; \delta(1-x_2) \otimes \text{BES}\left(x_2; [p]_2^{\text{Arm}}\right)$$

$$+ p_{\text{Body}}\,\text{BG}\left(x_1; [p]_1^{\text{Body}}, \beta^2_{\text{limit}}\right) \; \text{BG}\left(x_2; [p]_2^{\text{Body}}, \beta^2_{\text{limit}}\right)$$



Model: 19 free parameters, here drawn with arbitrary parameter values

The Model: Putting the Individual Parts Together

$$
\begin{aligned}
\mathscr{L}(x_1, x_2) = \\
& p_{\mathrm{Peak}}\,\delta(1-x_1)\otimes \mathrm{BES}\left(x_1; [p]_1^{\mathrm{Peak}}\right) \\
& \delta(1-x_2)\otimes \mathrm{BES}\left(x_2; [p]_2^{\mathrm{Peak}}\right) \\
& + p_{\mathrm{Arm1}}\,\delta(1-x_1)\otimes \mathrm{BES}\left(x_1; [p]_1^{\mathrm{Arm}}\right) \\
& \mathrm{BB}\left(x_2; [p]_2^{\mathrm{Arm}}, \beta^1_{\mathrm{limit}}\right) \\
& + p_{\mathrm{Arm2}}\,\mathrm{BB}\left(x_1; [p]_1^{\mathrm{Arm}}, \beta^1_{\mathrm{limit}}\right) \\
& \delta(1-x_2)\otimes \mathrm{BES}\left(x_2; [p]_2^{\mathrm{Arm}}\right) \\
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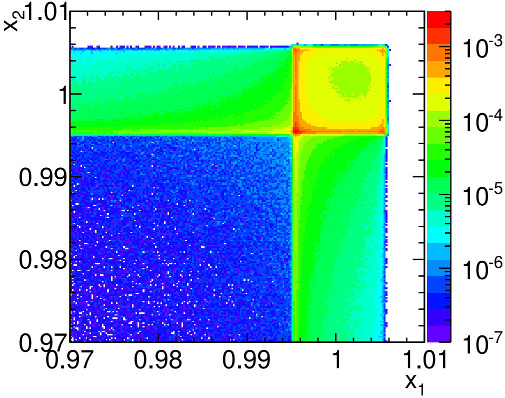
\end{aligned}
$$



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The Model, continued

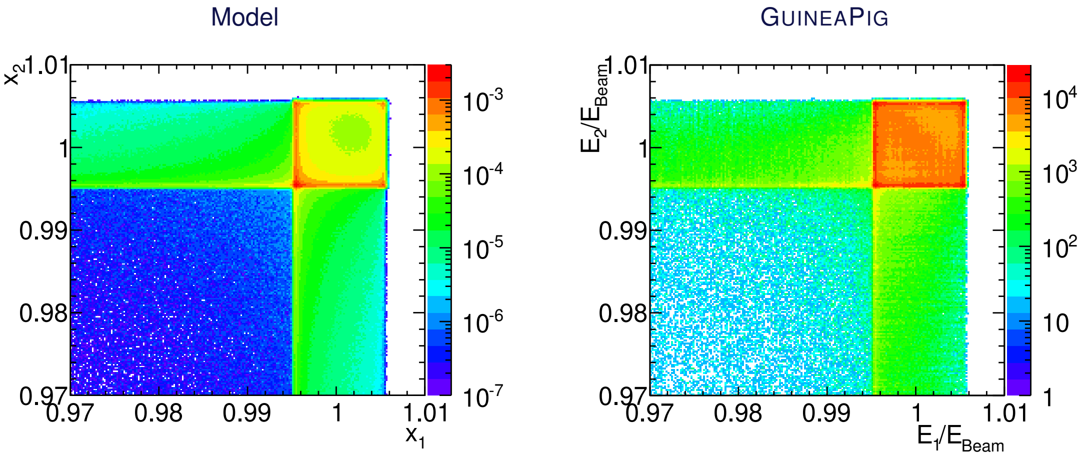
With

$$\mathrm{BES}(x) = \int_{x_{\mathrm{min}}}^{x_{\mathrm{max}}} b(\tau)\mathrm{Gauss}(x-\tau)\mathrm{d}\tau$$

$$\mathrm{BB}(x) = (b \otimes \mathrm{BES})(x)$$

$$\mathrm{BG}(x) = (b \otimes g)(x)$$

Model vs. GUINEAPIG



Arbitrary parameter values for the Model

Reweighting Fit Details

- Do not have to calculate any (numerical) convolutions:
  The distribution of a random variate (x_h), which is based on the convolution of two probability density functions (PDFs) is equal to the distribution of the sum of the individual random variates (x_f and x_g).

$$h(x) \equiv (f \otimes g)(x) \rightarrow x_h = x_f + x_g.$$

Then the new weights can be calculated from the products of the individual PDFs

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Model Validation

- Fit the 2D distribution of *Initial Particle Energies*
- 3 million GP events and 10 million according to the model
- No cross-section, initial state radiation, or detector effects
- Spectrum described within 5% down to $0.6\sqrt{s_{\mathrm{nom}}}$
- Difference in the width of the peak, but averages out
- Some problem with the width of the peak
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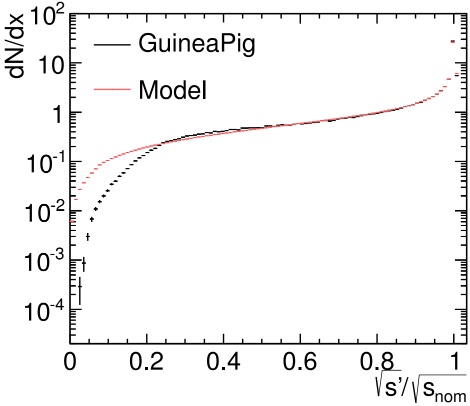

Results for 150×150 (E_1, E_2) bins and cut $\sqrt{s'} > 1.5$ TeV

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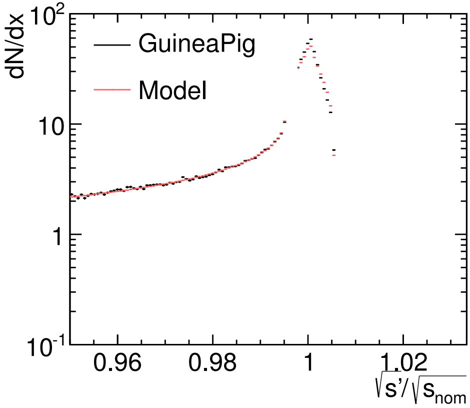

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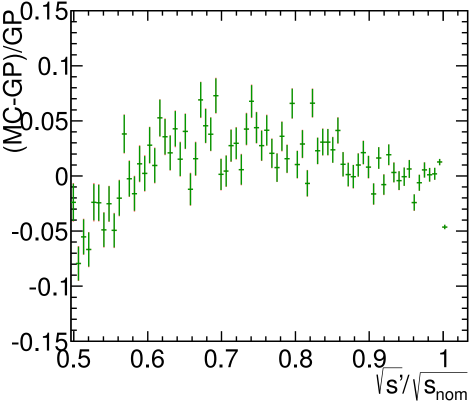

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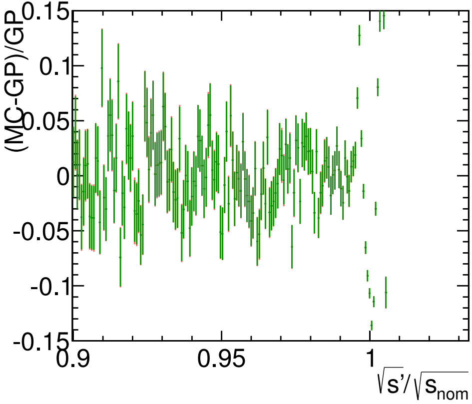

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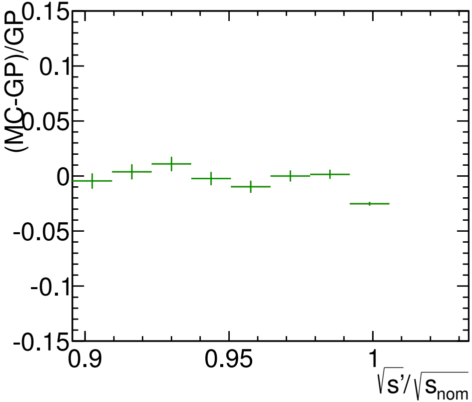

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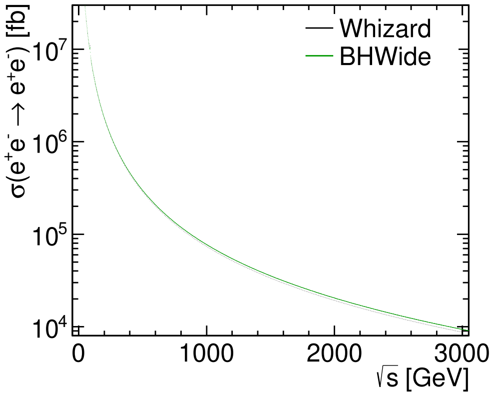
Section 3:

- Adding Bhabha Cross-Section, ISR, Detector Effects
  - Cross-Section
  - Detector Effects
  - Results

Luminosity Spectrum with Cross-Section

- Bhabha cross-section proportional to $1/s$
- Cross-section calculated by WHIZARD and BHWIDE $7^{\circ} < \theta_{e^{\pm}} < 173^{\circ}$, without luminosity spectrum
- Need Luminosity Spectrum scaled according to cross-section
- Feed these energy pairs to BHWIDE for ISR/FSR and Bhabha-scattering



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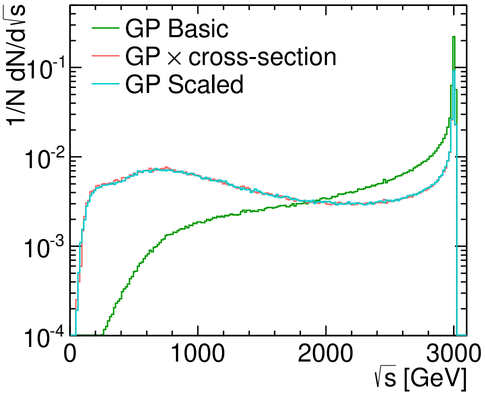
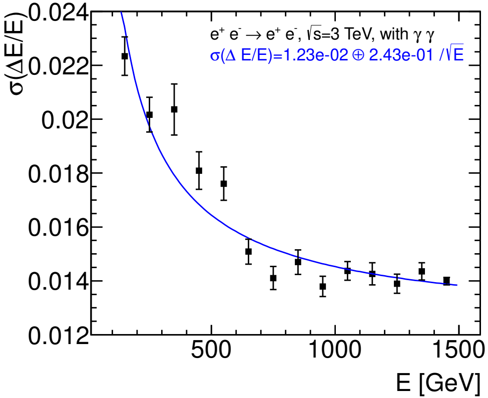
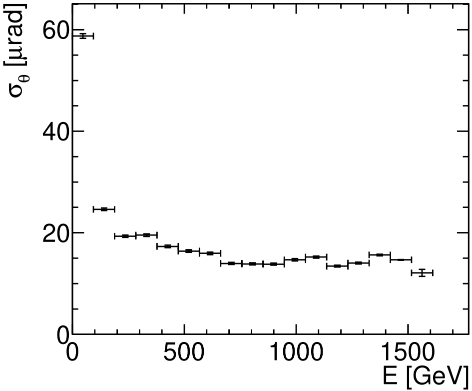


Detector Effects

Particle Energy



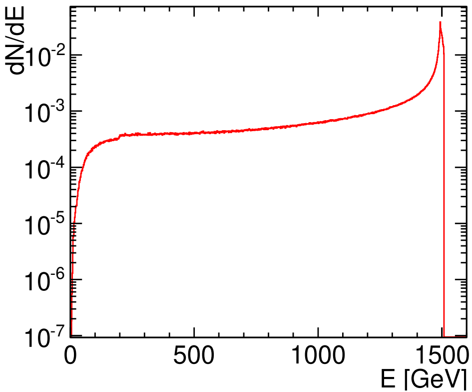
Angular Resolution ($e^{\pm}, \theta \geq 7^{\circ}$)



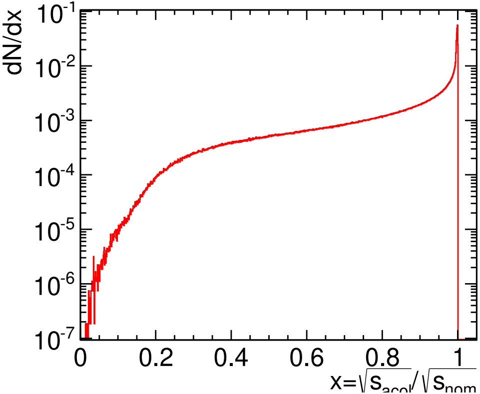
- Full simulation of millions of Bhabha events not feasible, use 4-vector smearing
- Detector resolutions obtained with full simulation/reconstruction with $\gamma\gamma \rightarrow$ hadron background overlay thanks to J.J. Blaising

Observables

Unsmeared
Energy of the electron/positron:



Relative centre-of-mass energy (c.m.e.):

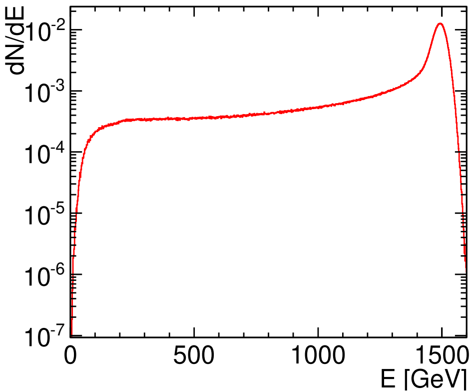


Very large effect on energy, small on relative c.m.e. because of better angular resolution

$$\frac{\sqrt{s'_{acol}}}{\sqrt{s_{nom}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}},$$

Observables

Smeared
Energy of the electron/positron:



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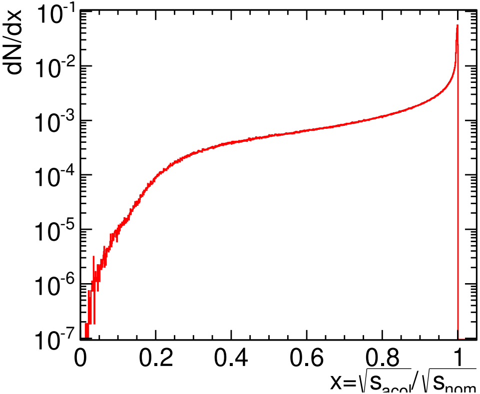

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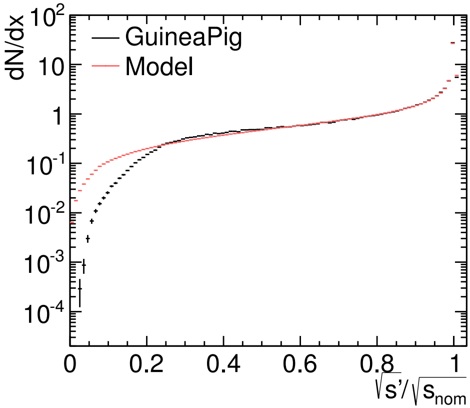
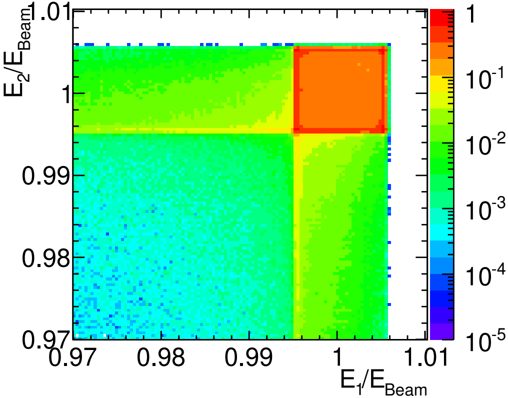
Final Fit: All Effects

- Includes cross-section scaling, ISR, FSR, detector resolutions
- Binning $60 \times 30 \times 30$ (Rel. c.m.s., E_1, E_2)
- 2 million GP (current number of available events, approx. 400fb^{-1}) , 10 million model
- Cut on: $\sqrt{s'} > 1.5$ TeV, $E_1 > 150$ GeV, $E_2 > 150$ GeV

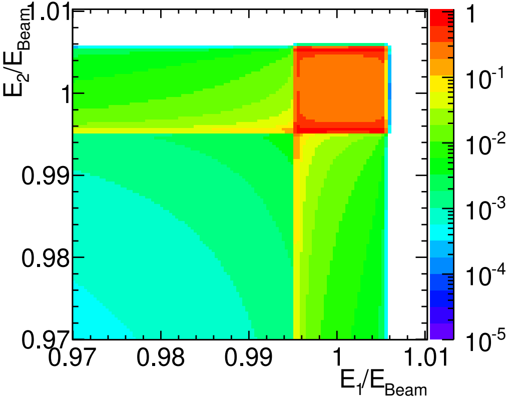


Reconstructed 2D Spectrum

GUINEAPIG



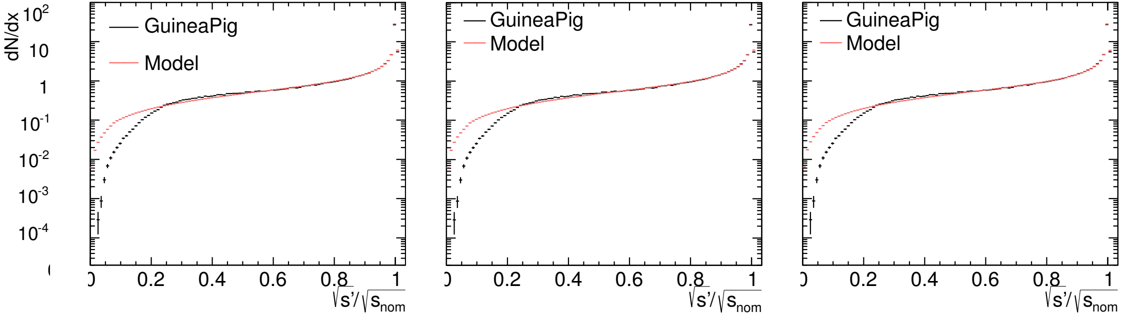
model after fit



Fit with all effects $60 \times 30 \times 30$ bins

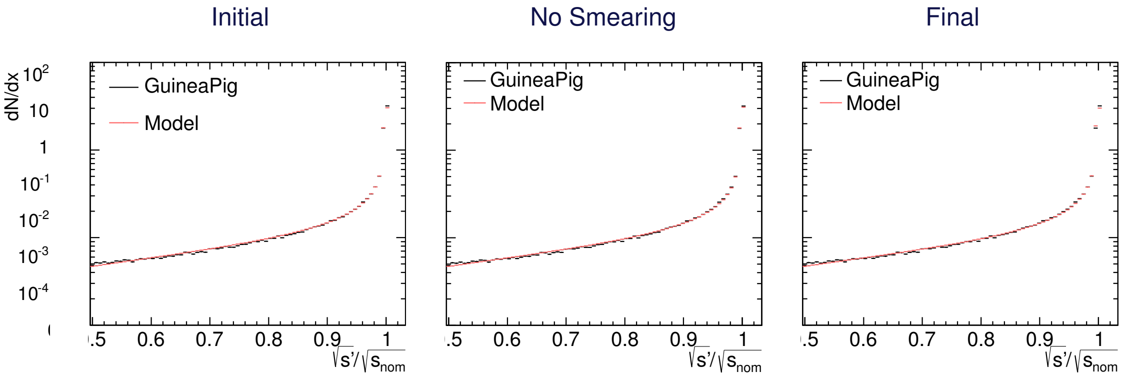
Comparison: Initial vs. Final

Initial | No Smearing | Final



- Initial: Initial Particle Energies Fit (for model validation)
- No Smearing: Bhabha observables and cross-section, no detector resolutions
- Final: Bhabha observables and cross-section, including detector resolutions
- N.B.: The GuineaPig sample for all these plots is the same.
- The differences between the GuineaPig and the model spectra are very similar for all stages of the reconstruction

Comparison: Initial vs. Final



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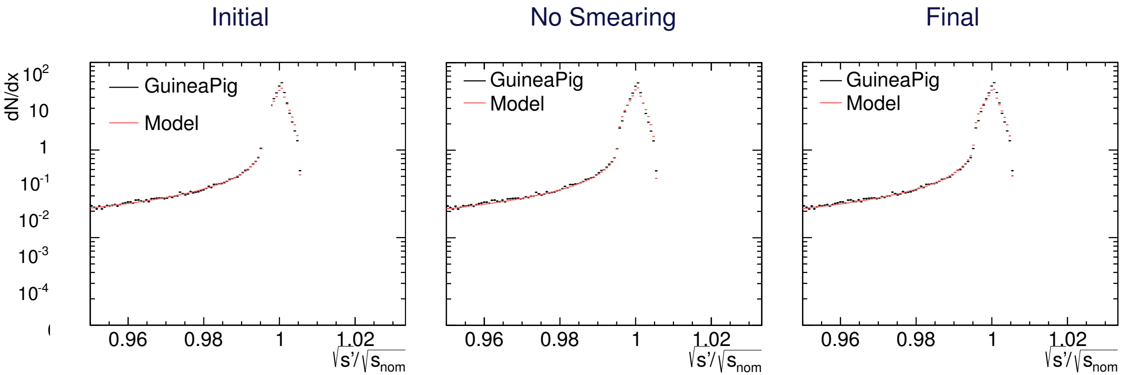

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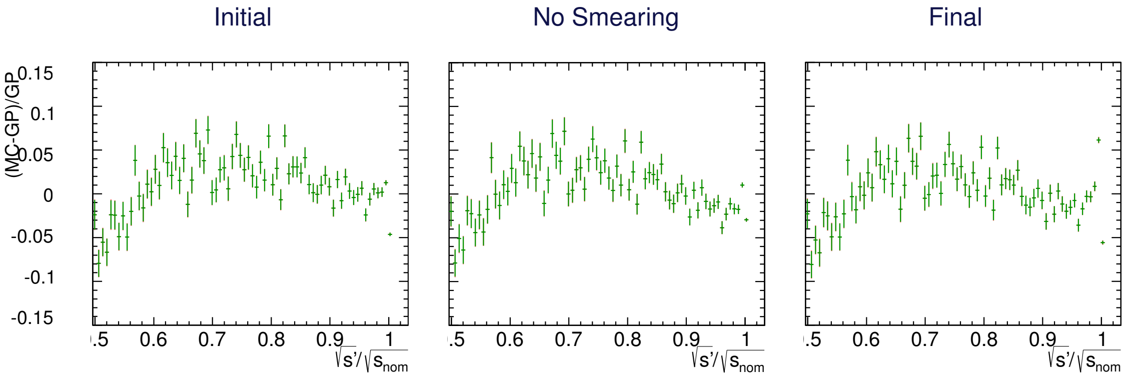

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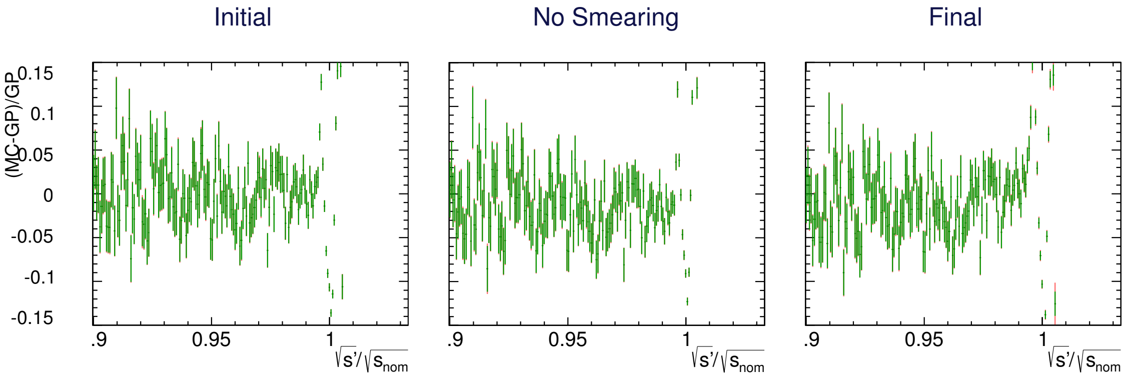

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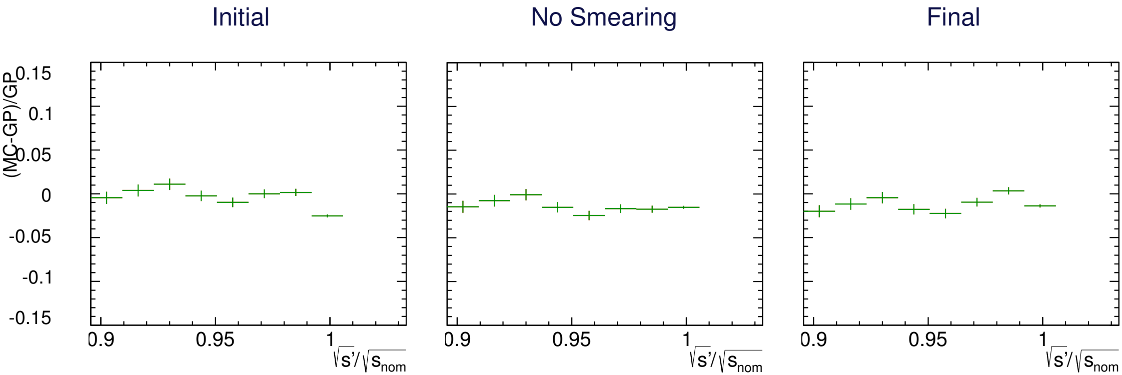

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Section 4:

- Conclusions

Conclusions

- Luminosity spectrum reconstruction is possible with Bhabha events
- Studies for 3 TeV CLIC are documented in comprehensive paper [1]
- Some studies for 350 GeV / 380 GeV CLIC were done as well [2]
  - Reconstruction at the lower energy
  - Systematic effects from Detector resolution mis-modelling
- One would need some luminosity spectra for FCC to start understand if this approach can be used
- Source Code: https://gitlab.cern.ch/CLICdp/CLICDetSVN/DiffLumi

References

[1] S. Poss and A. Sailer. "Luminosity Spectrum Reconstruction at Linear Colliders". In: *Eur. Phys. J. C* 74 (2014), p. 2833.

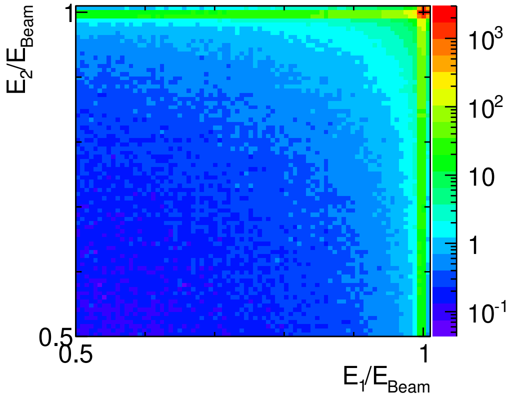
[2] E. Fullana and P. Zehetner. *Top quark mass measurement in the continuum + luminosity spectrum*. 2018. URL: https://indico.cern.ch/event/703821/contributions/3102578/.

Thank you for your attention

Backup Slides

Binning

- Luminosity spectrum has strong peak and long tail
- χ^2-fit requires binned events and sufficient number of events in each bin
- Too coarse binning smears the peak, too fine binning leaves not enough events per bin in the tail
- Use *equiprobability* binning: Varying bin size, but the same number of entries in each bin



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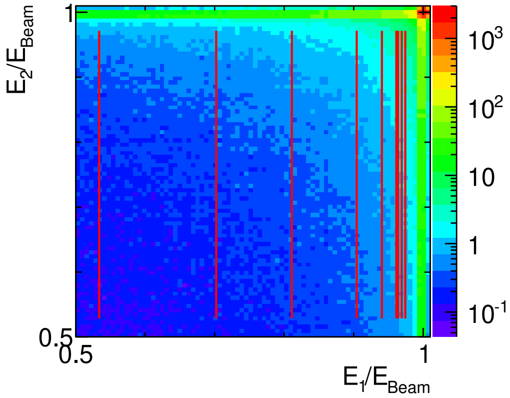

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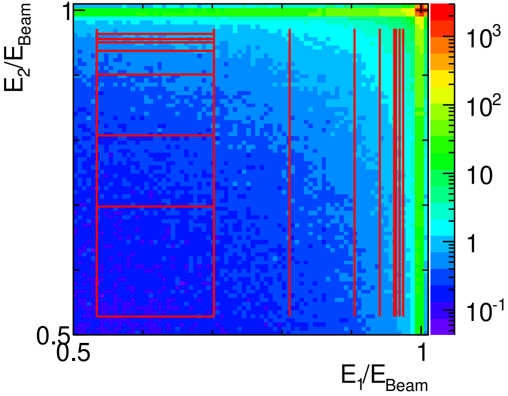

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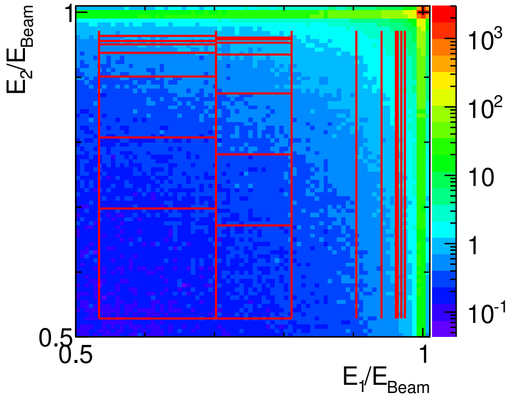

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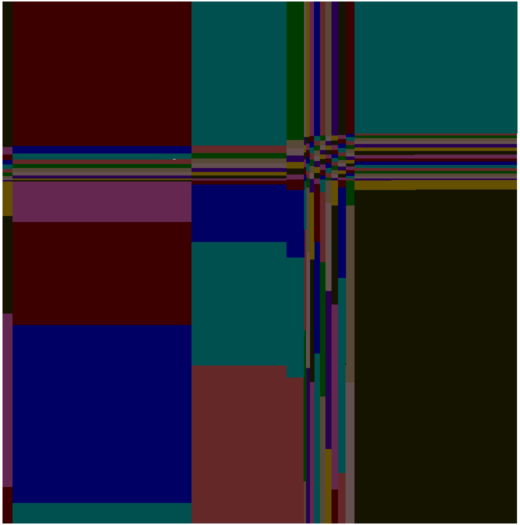
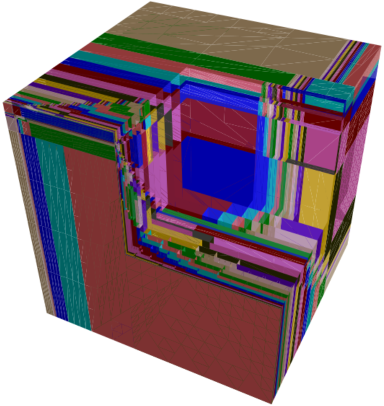
- Slice events first along dimension 1 into equal parts
- Slice parts of dimension 1 into equal parts along dimension 2
- Wrote program to create, store, and fill equiprobability in 2D and 3D



Binning 3D

- The relative centre-of-mass energy calculated from the angles $\frac{\sqrt{s'_{\text{acol}}}}{\sqrt{s_{\text{nom}}}} = \sqrt{\frac{\sin(\theta_1)+\sin(\theta_2)+\sin(\theta_1+\theta_2)}{\sin(\theta_1)+\sin(\theta_2)-\sin(\theta_1+\theta_2)}}$ gives not enough information to reconstruct 2D spectrum
- Additionally use the electron and positron energy measured with calorimeter to see which of the particles lost energy
- These three observables are filled into 3D equiprobability histogram



Fitting with Chebyshev Polynomials

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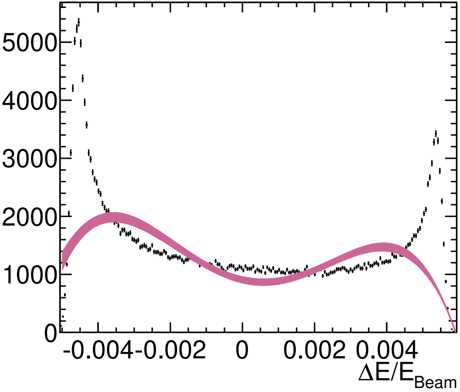

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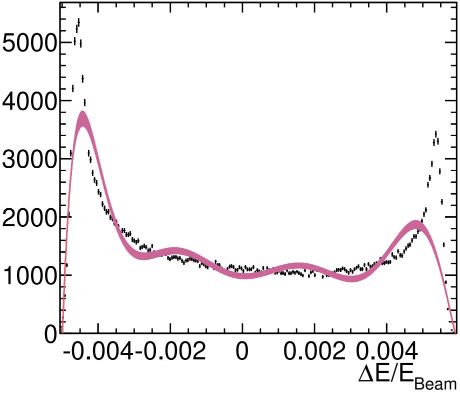
  - 10 Parameters



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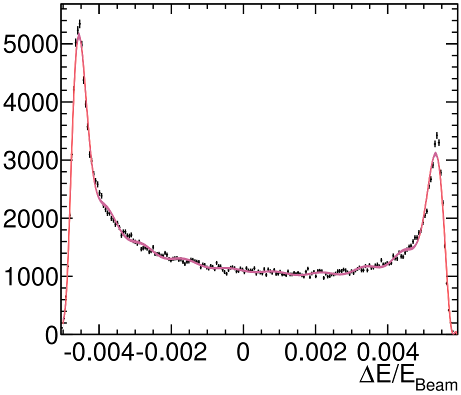
  - 26 Parameters: $\chi^2/\mathrm{ndf} = 668/173$



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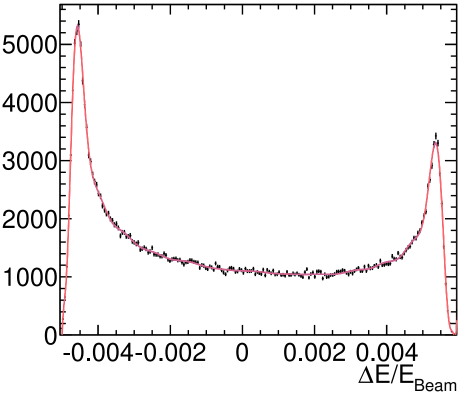

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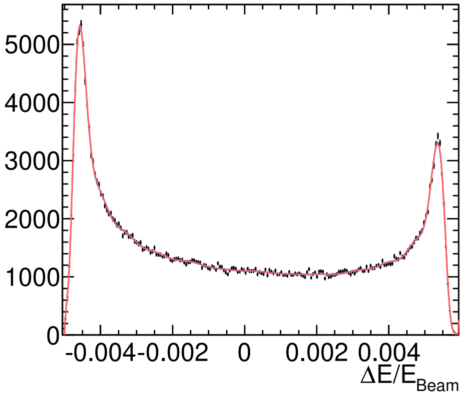
  - 26 Parameters: $\chi^2/\mathrm{ndf} = 668/173$
  - 35 Parameters: $\chi^2/\mathrm{ndf} = 226/164$
- Saves trouble of convolution, but at the cost of many parameters
- Could also fit centre only and do convolution with Gauss, but still need larger number of parameters



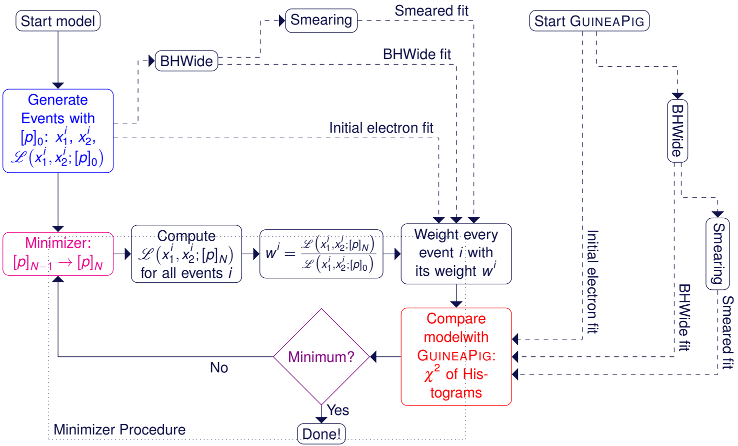
Reweighting Fit Details

- Do not have to calculate any (numerical) convolutions:
  The distribution of a random variate (x_h), which is based on the convolution of two probability density functions (PDFs) is equal to the distribution of the sum of the individual random variates (x_f and x_g).

$$h(x) \equiv (f \otimes g)(x) \rightarrow x_h = x_f + x_g.$$

Then the new weights can be calculated from the products of the individual PDFs

$$w^i = \frac{p^N_{\text{region}}\, b\left(x^{i,1}_{\text{Strahlung}}, [p]_N\right) b\left(x^{i,1}_{\text{Spread}}, [p]_N\right) g\left(x^{i,1}_{\text{G}}, [p]_N\right) b\left(x^{i,2}_{\text{Strahlung}}, [p]_N\right) b\left(x^{i,2}_{\text{Spread}}, [p]_N\right) g\left(x^{i,2}_{\text{G}}, [p]_N\right)}{p^0_{\text{region}}\, b\left(x^{i,1}_{\text{Strahlung}}, [p]_0\right) b\left(x^{i,1}_{\text{Spread}}, [p]_0\right) g\left(x^{i,1}_{\text{G}}, [p]_0\right) b\left(x^{i,2}_{\text{Strahlung}}, [p]_0\right) b\left(x^{i,2}_{\text{Spread}}, [p]_0\right) g\left(x^{i,2}_{\text{G}}, [p]_0\right)}$$



$$N^j_{\mathrm{GP}} = \sum_{\text{GP Events } i \text{ in Bin } j} 1$$

$$N^j_{\mathrm{Model}} = \sum_{\text{Model Events } i \text{ in Bin } j} w^i$$

$$f_S = \frac{\sum_{\text{GP Events}} 1}{\sum_{\text{Model Events } i} w^i}$$

$$\chi^2 = \sum_{\text{Bins } j} \frac{\left(N^j_{\mathrm{GP}} - f_S \cdot N^j_{\mathrm{Model}}\right)^2}{(\sigma^j_{\mathrm{GP}})^2 + (f_S \cdot \sigma^j_{\mathrm{Model}})^2}$$

Effect on the Smuon Mass Measurement (I) (LCD-Note-2011-018)

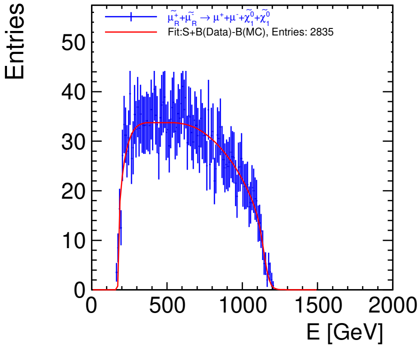
- $e^+e^- \to \tilde{\mu}^+\tilde{\mu}^- \to \mu^+\mu^-\tilde{\chi}\tilde{\chi}$
- Fit background subtracted muon energy distribution to extract smuon and neutralino mass with $f(E_\mu; m_{\tilde{\mu}}, m_{\tilde{\chi}}) = \text{Box} \times \sigma(\sqrt{s'}) \otimes \mathscr{L}(\vec{p}) \otimes \text{ISR} \otimes \text{DetRes}$
- Fit with all parameters of luminosity spectrum varied by $\pm\sigma_p^i/2$ individually

$$\sigma^2_{m_{\tilde{\mu}}} = \sum_{i,j} \delta_i C_{ij} \delta_j \qquad \delta_i = m_{+_i} - m_{-_i}$$

$$m_{+_i} = f\left(\vec{p} + \vec{e}_i \frac{\sigma_{p_i}}{2}\right) \qquad m_{-_i} = f\left(\vec{p} - \vec{e}_i \frac{\sigma_{p_i}}{2}\right)$$

with the correlation matrix

$$C = \begin{pmatrix} 1 & -0.6 & \dots & -0.02 \\ -0.6 & 1 & \dots & 0.04 \\ \dots & \dots & \dots & \dots \\ -0.02 & 0.04 & \dots & 1 \end{pmatrix}$$



Effect on the Smuon Mass Measurement (II)(LCD-Note-2011-018)

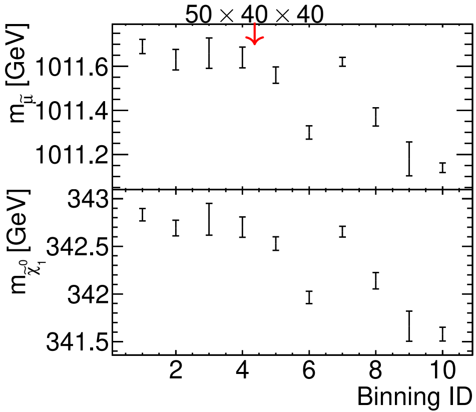
Results:

- Using our reconstructed spectrum (e.g.,$50 \times 40 \times 40$ bins):

$$m_{\tilde{\mu}} = \left(1011.6 \pm 3.0(\text{stat}) \pm 0.04(\text{par})\right)\ \text{GeV},$$
$$m_{\tilde{\chi}} = \left(\ 342.5 \pm 6.8(\text{stat}) \pm 0.07(\text{par})\right)\ \text{GeV}$$

Conclusion:

- Small dependence on number of bins used during reconstruction, but changes smaller than statistical error
- The luminosity spectrum reconstruction has no significant effect on $\tilde{\mu}/\chi$ mass measurements



Systematic error from parameter reconstruction only