Enabling Precision EW Measurements at High Energy e^+e^- Colliders with in situ Center-of-Mass Energy Measurements

Lessons from LEP applied to ILC, and to FCC-ee?

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- Center-of-mass energy determination using $e^+e^- \to \mu^+\mu^-(\gamma)$ events at future e^+e^- colliders (2209.03281). Comments welcome.
- Similar methodology* to that presented by Patrick. But emphasize using muon **momenta** to measure the absolute $\sqrt{s}$ scale without assuming collinear ISR.
- Needs exquisite control of tracker momentum scale. Can work at all $\sqrt{s}$.
- Focus is **ILC**, but relevant to any e^+e^- collider. Eg. C^3, HELEN, ReLiC, FCC-ee (so both linear and circular topologies).

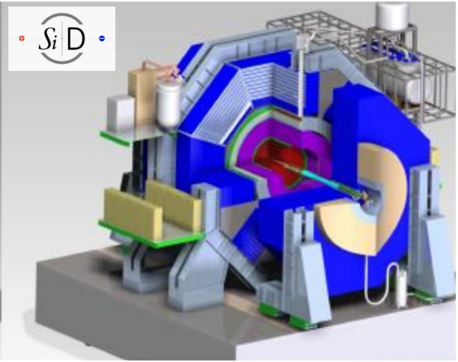
ILC Detectors

Modern detectors designed for ILC

ILD = International Large Detector

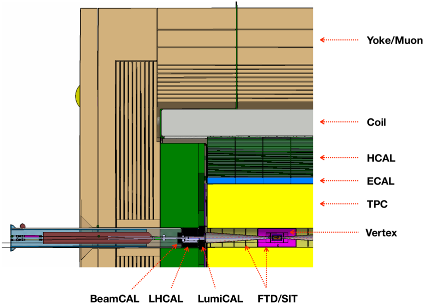
(also ILD Interim Design Report (IDR))

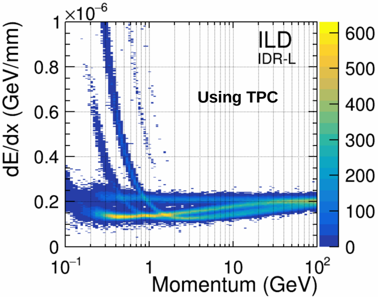
SiD = Silicon Detector



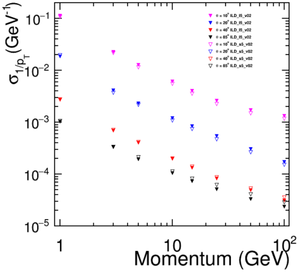
- B=3.5–5T. Particle-flow for hadronic jets. **Very hermetic**.
- Low material. Precision vertexing.
- ILD tracking centered around a Time Projection Chamber (TPC).

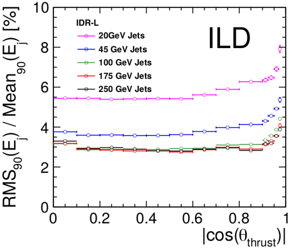
ILD Detector (See IDR: 2003.01116)

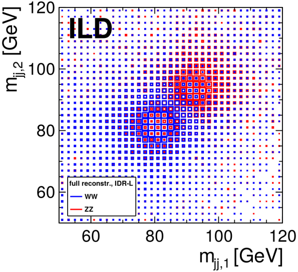




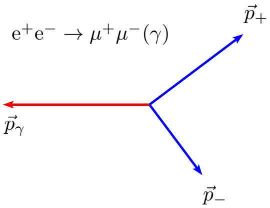
Momentum Resolution







$\sqrt{s}_p$ Method in a Nutshell



Measure $\sqrt{s}_p$ using,
$(|\vec{p}_+|, |\vec{p}_-|, |\vec{p}_+ + \vec{p}_-|)$

Assuming,

- **Equal** beam energies, $E_{\rm b}$
- The lab **is** the CM frame, ($\sqrt{s} = 2\,E_{\rm b}$, $\sum \vec{p}_i = 0$)
- The system recoiling against the dimuon is **massless**

$$\sqrt{s} = \sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_+ + \vec{p}_-|$$

$$\sqrt{s}_p = \sqrt{p_+^2 + m_\mu^2} + \sqrt{p_-^2 + m_\mu^2} + |\vec{p}_+ + \vec{p}_-|$$

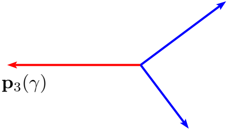
An estimate of $\sqrt{s}$ using only the (precisely measurable) muon momenta

[Now, $\sqrt{s}$ estimators previously extended to allow a crossing angle and beam energy difference are extended to the general case with a massive recoil. Work in progress on applying constrained fits]

With ILD detector at ILC - expect 0.17% momentum resolution for typical 71 GeV muon in $Z\gamma$ events at $\sqrt{s} = 250$ GeV. Detector-level studies are with full simulation and reconstruction.

Essentials Explained

General case has 3 nuisance parameters: the crossing angle, α, the collision energy asymmetry, $(E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+})/(E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) = \overline{\Delta E_{\mathrm{b}}}/E_{\mathrm{ave}}$, and the recoil mass, M_3.



1. $\sqrt{s} = E_1^* + E_2^* + E_3^* = E_{12}^* + E_3^*$
2. $\sqrt{s} = E_{12}^* + \sqrt{(p_{12}^*)^2 + M_3^2}$ (general M_3)
3. $\sqrt{s} = E_{12}^* + |\mathbf{p}_{12}^*|$ (assuming $M_3 = 0$)

We have the measured dimuon 4-vector in the detector frame $(E_{12}, \mathbf{p}_{12})$. Need to apply the appropriate boost from lab back to the CM frame to obtain $(E_{12}^*, \mathbf{p}_{12}^*)$. The boost velocity (in the horizontal plane) is

$$\boldsymbol{\beta} = (\beta_x, \beta_y, \beta_z) = (\sin(\alpha/2), 0, \frac{\overline{\Delta E_{\mathrm{b}}}}{E_{\mathrm{ave}}} \cos(\alpha/2))$$

$\beta_x = 0.007/0.015$ (ILC/FCC-ee). β_z depends on the collision energy asymmetry.

Generator-level Examples

Event	1	2	3	4	5	6
E_{b}^{-}	125.34	114.55	125.32	124.87	124.75	122.77
E_{b}^{+}	124.82	124.64	121.08	124.49	116.24	110.12
ΔE_{b}	$+0.26$	-5.04	$+2.12$	$+0.19$	$+4.26$	$+6.33$
M_{12}	92.55	238.97	94.62	249.30	82.34	92.26
p_{12}	108.41	10.22	104.74	1.73	101.66	105.43
p_{12}^{x}	$+18.82$	$+1.67$	$+1.25$	$+1.70$	$+0.92$	$+1.03$
p_{12}^{y}	-14.54	0.00	$+0.21$	-0.01	0.00	-0.25
p_{12}^{z}	$+105.77$	-10.08	$+104.73$	$+0.35$	-101.65	$+105.43$
p_3	107.62	0.00	100.49	0.06	110.17	92.78
M_3	0.00	0.00	31.27	0.00	0.55	0.00
$\sqrt{s}$	250.15	238.97	246.35	249.35	240.84	232.53
E_{12}^{*} (β_x)	142.41	239.18	141.15	249.30	130.82	140.10
p_{12}^{*} (β_x)	108.24	10.08	104.73	0.35	101.65	105.43
$\sqrt{s_p}$	250.65	249.26	245.88	249.65	232.47	245.53
E_{12}^{*} $(\boldsymbol{\beta})$	142.20	238.97	139.36	249.30	134.49	134.57
p_{12}^{*} $(\boldsymbol{\beta})$	107.96	0.00	102.32	0.06	106.34	97.96
$\sqrt{s_p}$ (true ΔE_{b})	250.15	238.97	241.60	249.35	240.84	232.53
$\sqrt{s_p}$ (true M_3)	250.65	249.26	250.45	249.65	232.47	245.53

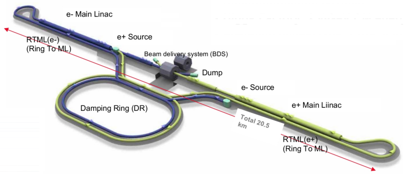
Makes use of radiative-return $(Z\gamma)$ events too.

ILC

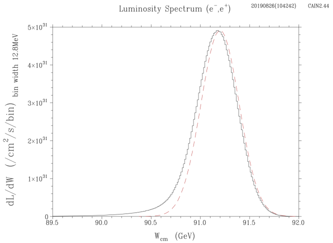
The ILC linear e^+e^- collider has been designed with an emphasis on an **initial-stage Higgs factory** that starts at $\sqrt{s} = 250$ GeV and is **expandable in energy** to run at higher energies for pair production of top quarks and Higgs bosons, and potentially to 1 TeV and more.

Particular strengths: **Longitudinally polarized electron and positron beams** and **higher energies**. Many new measurement possibilities. Very complementary to those feasible with unpolarized & lower energy reach e^+e^- circular colliders.

The ILC is designed primarily to explore the 200 – 1000 GeV energy frontier regime. This has been the focus in making the case for the project.
It is also capable of running at the **Z** and **WW** threshold.



See B. List's talk for ILC details (p22)



Z running – see Yokoya, Kubo, Okugi

Snowmass LoI Studies

Studies were undertaken:

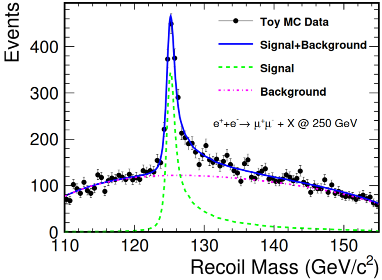
1. to understand ILC capabilities for a precision measurement of the Z lineshape observables with a **scan** using longitudinally polarized beams,
2. to further explore an experimental strategy for $\sqrt{s}$ determination using di-leptons, and
3. to further explore M_{W} capabilities synergistic with a concurrent Higgs program.

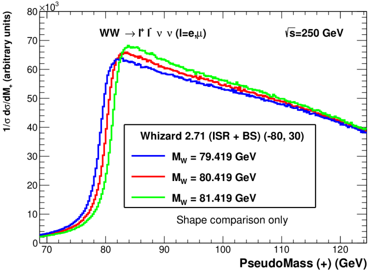
Focus of this talk: reporting progress on experimental issues associated with **center-of-mass energy** (item 2) which is a pre-requisite for fully exploiting a polarized Z scan (item 1) and underpin M_{W} prospects (item 3).

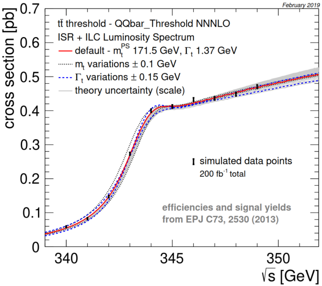
Key Issue: Systematic control for the absolute scale of (**in collision...**) **center-of-mass energy** at **all** C-o-M energies

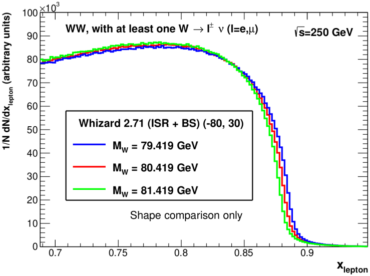
Note: 10^{10} hadronic Z's - 0.001% uncertainties - already a big challenge for absolute observables. Less so for asymmetries and relative cross-sections vs $\sqrt{s}$.

Example Physics Importance of $\sqrt{s}$ Knowledge









ILC Physics Targets — Energy ($\sqrt{s}$) Requirements

Core Program

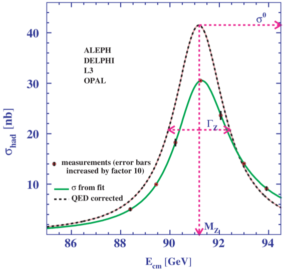
Observable	M_{H}	m_{t}	M_{W}	M_{X}
Method	Recoil mass	Scan	Reconstruction	Scan?
Best $\sqrt{s}$ [GeV]	250	350	250	Highest?
Current precision [MeV]	170	300	15*	–
Target precision [MeV]	10	20	2	?
$\sqrt{s}$ contribution [MeV]	3	6	0.6	?
$\sqrt{s}$ uncertainty goal [ppm]	100	200	10	100?

Ultimate Impact/Reach

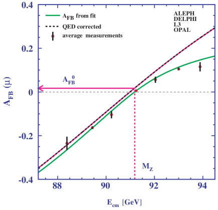
Observable	M_{W}	M_{Z}	Γ_{Z}	A_{LR}
Method	Scan	Scan	Scan	Count/Scan
Best $\sqrt{s}$ [GeV]	161	91	91	91
Current precision	15**	2.1	2.3	1.9×10^{-3}
Target precision	2 MeV	0.2 MeV	0.11 MeV	4.5×10^{-5}
$\sqrt{s}$ contribution	0.8 MeV	0.2 MeV	small	0.9×10^{-5}
$\sqrt{s}$ uncertainty goal [ppm]	10	**2**	5**	5

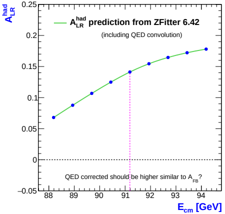
*(post CDF ...), **(point-to-point most relevant)

Essentially, perform LEP/SLC-style measurements in all channels but also with $\sqrt{s}$ dependence of the polarized asymmetries, A_{LR} and $A^{f}_{FB,LR}$, in addition to A_{FB}.
(Also polarized $\nu\bar{\nu}\gamma$ scan.) Not constrained to LEP-style scan points.



LEP: $\Delta M_{\mathrm{Z}} = 2100$ MeV, $\Delta\Gamma_{\mathrm{Z}} = 2300$ MeV



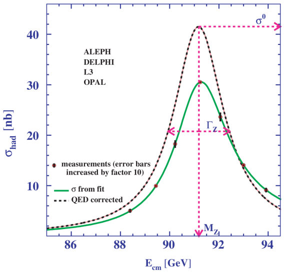


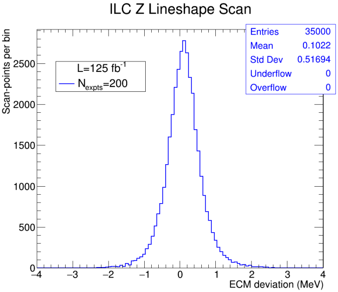
With 0.1 ab^{-1} polarized scan around M_{Z}, find **statistical** uncertainties of 35 keV on M_{Z}, and 80 keV on Γ_{Z}, from LEP-style fit to $(M_{\mathrm{Z}}, \Gamma_{\mathrm{Z}}, \sigma^0_{\mathrm{had}}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER for QED convolution.

Exploiting this fully needs in-depth study of $\sqrt{s}$ **calibration systematics**
ILC $\mathcal{L}$ is sufficient for M_{Z} to be systematics limited
Γ_{Z} systematic uncertainty depends on $\Delta(\sqrt{s}_+ - \sqrt{s}_-)$, so expect $\Delta\Gamma_{\mathrm{Z}} \ll \Delta M_{\mathrm{Z}}$

Polarized Beams Z Scan for Z LineShape Study: WIP I

Initial line-shape study (all 4 channels). Use unpolarized cross-sections for now.





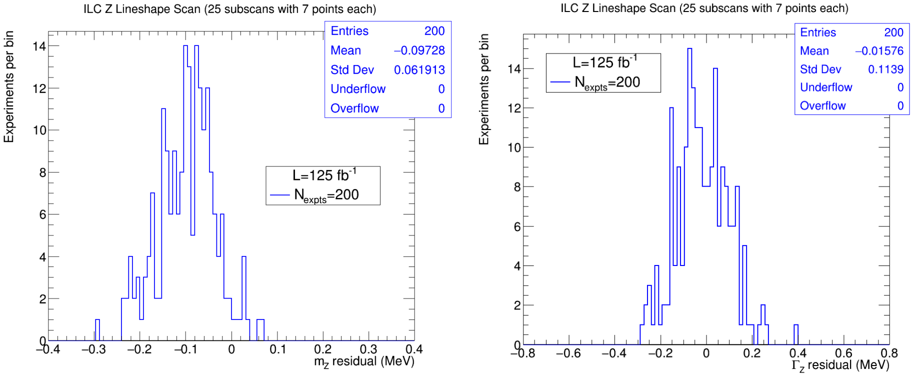
Uses $\sigma_{stat}/\sqrt{s}$ (%) $= 0.25/\sqrt{N_{\mu\mu}} \oplus\ 0.8/\sqrt{N_{\rm h}}$

- Scan has 7 nominal $\sqrt{s}$ points, (peak,$\pm\Delta, \pm 2\Delta \pm 3\Delta$) with $\Delta = 1.05$ GeV
- 25 scans of 5 fb^{-1} per "experiment". $7 \times 25 \times 4 = 700$ $\sigma_{\rm tot}$ measurements.
- Assign luminosity per scan point in (2:1:2:1) ratio. (1 or 0.5 fb^{-1} each).
- Do LEP-style fit to $(M_{\rm Z}, \Gamma_{\rm Z}, \sigma^0_{\rm had}, R^0_e, R^0_\mu, R^0_\tau)$ using ZFITTER
- Model center-of-mass energy systematics and int. lumi syst. of 0.064%.
- Each scan-point (175 per expt.) shifted from $\sqrt{s}_{\rm nominal}$ by a 100% **correlated** overall scale systematic (here +100 keV) and by stat. component driven by stat. uncertainty of $\sqrt{s}$ measurement (typically 0.4 MeV/4.4 ppm).

Polarized Beams Z Scan for Z LineShape Study: WIP II

Ensemble tests with 200 experiments.

Currently, fit the 700 measured cross-sections (actually occuring at shifted $\sqrt{s}$) using assumed nominal $\sqrt{s}$. Ensemble mean χ^2 of 790 for 693 dof.



- As expected M_Z biased down by assumed scale error (here +100 keV) with stat. error of 50–60 keV.
- As expected Γ_Z bias small with stat. dominated error of 100–120 keV.
- Such an experiment has 1.9B hadronic Zs.

ILC A_{LR} Prospects from Z Running

Use 4 cross-section measurements ($\sigma_{\pm\pm}$) to measure simultaneously:

$$A_{\mathrm{LR}},\ |P(e^-)|,\ |P(e^+)|,\ \sigma_u$$

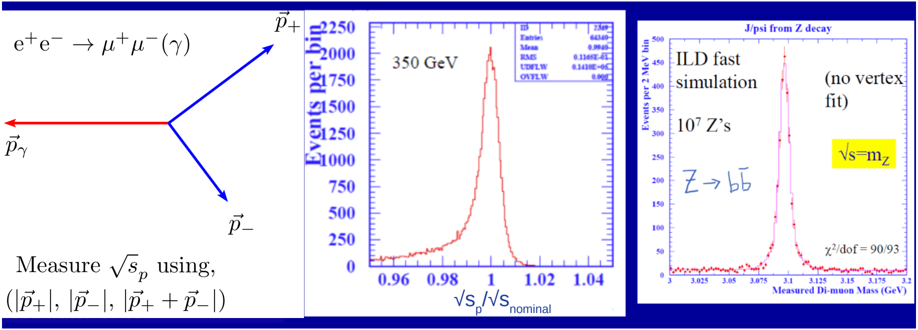
L (fb^{-1})	N_Z^{had} (10^9)	$\|P(e^-)\|$	$\|P(e^+)\|$	ΔA_{LR} (stat.)	ΔA_{LR} (syst).
100	3.3	80%	30%	4.3×10^{-5}	1.3×10^{-5}
100	4.2	80%	60%	2.4×10^{-5}	1.3×10^{-5}
250	8.4	80%	30%	2.7×10^{-5}	1.3×10^{-5}
250	11	80%	60%	1.5×10^{-5}	1.3×10^{-5}

Estimated uncertainties on A_{LR} for 4 different scenarios of Z-pole running with data-taking fractions in each helicity configuration $(-+), (+-), (--), (++)$ chosen to minimize the statistical uncertainty on the asymmetry. The quoted statistical uncertainty includes Bhabha statistics for relative luminosity and Compton statistics for polarization differences. The systematic uncertainty assumes 5 ppm uncertainty on the absolute center-of-mass energy and a 1% understanding of beamstrahlung effects. Estimates assume data taken at a single center-of-mass energy (91.2 GeV).

Total uncertainty on A_{LR} of 4.5×10^{-5} (scenario 1) to 2.0×10^{-5} (scenario 4). Corresponds to uncertainty on $\sin^2\theta^{\ell}_{\mathrm{eff}}$ of 5.6×10^{-6} (1) to 2.5×10^{-6} (4).

$\sqrt{s}_p$ Method for Center-of-Mass Energy

Use dilepton **momenta**, with $\sqrt{s}_p \equiv E_+ + E_- + |\vec{p}_{+-}|$ as $\sqrt{s}$ estimator.



Tie detector p-scale to particle masses (know J/ψ, π^+, p to 1.9, 1.3, 0.006 ppm)

Measure $< \sqrt{s} >$ and luminosity spectrum with same events. Expect statistical uncertainty of 1.0 ppm on p-scale per 1.2M $J/\psi \to \mu^+\mu^-$ (4×10^9 hadronic Z's).

- excellent tracker momentum resolution - can resolve beam energy spread.
- feasible for $\mu^+\mu^-$ and e^+e^- (and ... 4l etc).

Introduction to Center-of-Mass Energy Issues

- Proposed $\sqrt{s}_p$ method uses only the momenta of leptons in dilepton events.
- Critical issue for $\sqrt{s}_p$ method: calibrating the **tracker momentum scale**.
- Can use $\mathrm{K^0_S}$, Λ, $J/\psi \to \mu^+\mu^-$ (mass known to 1.9 ppm).

For more details see studies of $\sqrt{s}_p$ from ECFA LC2013, and of momentum-scale from AWLC 2014. Recent $\mathrm{K^0_S}$, Λ studies at LCWS 2021 – much higher precision feasible ... few **ppm** (not limited by parent mass knowledge or J/ψ statistics). More in depth talks on $\sqrt{s}$: ILC physics seminar and ILC MDI/BDS/Physics talk

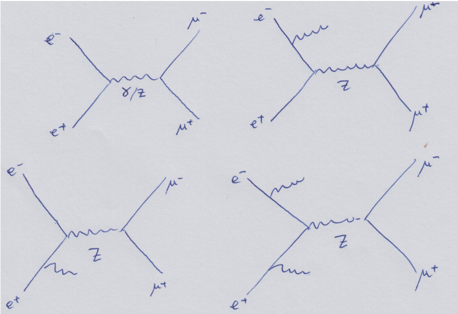
Today,

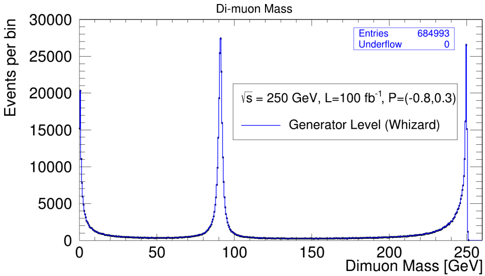
- Overview of the $\sqrt{s}_p$ method prospects with $\mu^+\mu^-$
- Brief overview of the "new" concept in recent tracker momentum scale studies (LCWS2021 talk).
- Bonus. Physics: M_{Z}. Beam knowledge: **luminosity spectrum**, $\mathrm{dL}/\mathrm{d}\sqrt{s}$.

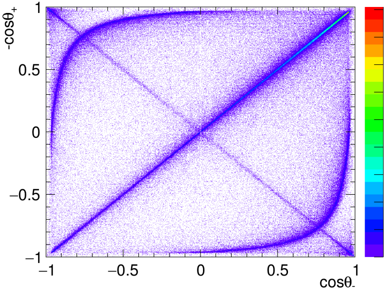
Three main kinematic regimes.

1. **Low** mass, $m_{\mu\mu} < 50$ GeV
2. **Medium** mass, $50 < m_{\mu\mu} < 150$ GeV
3. **High** mass, $m_{\mu\mu} > 150$ GeV

- Back-to-back events in the full energy peak.
- Significant radiative return (ISR) to the Z and to low mass.







New approach to tracker momentum scale

See LCWS2021 talk for details. Use Armenteros-Podolanski kinematic construction for 2-body decays (AP).

1. Explore AP method using mainly $\mathrm{K^0_S} \to \pi^+\pi^-$, $\Lambda \to \mathrm{p}\pi^-$ (inspired by Rodríguez et al.). **Much higher statistics than J/ψ alone.**
2. If proven realistic, **enables precision Z program** (polarized lineshape scan)
3. Bonus: potential for **large improvement in** parent and child particle **masses**

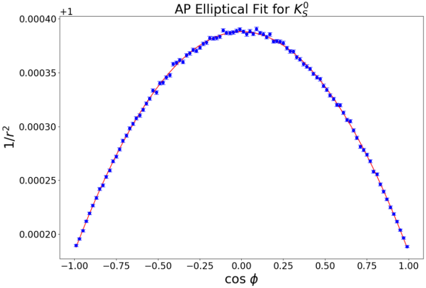
For a "V-decay", $M^0 \to m_1^+ m_2^-$, decompose the child particle lab momenta into components transverse and parallel to the parent momentum. The distribution of (child p_T, $\alpha \equiv \frac{p_L^+ - p_L^-}{p_L^+ + p_L^-}$) is a semi-ellipse with parameters relating the CM decay angle, θ^*, β, and the masses, (M, m_1, m_2), that determine, p^*.

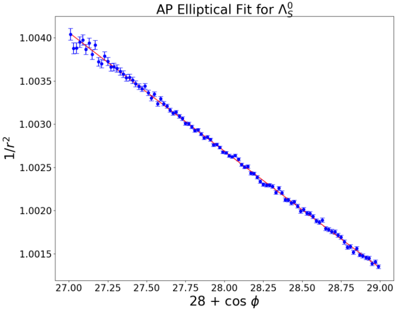
By obtaining sensitivity to both the parent and child masses, and positing improving ourselves the measurements of more ubiquitous parents ($\mathrm{K^0_S}$ and Λ), can obtain high sensitivity to the momentum scale

Proving the feasibility of sub-10 ppm momentum-scale uncertainty needs much work when typical existing experiments are at best at the 100 ppm level

Tracker momentum scale sensitivity estimate

Used sample of 250M hadronic Z's at $\sqrt{s} = 91.2$ GeV. Fit $\mathrm{K^0_S}, \Lambda, \overline{\Lambda}$ in various momentum bins.





1. $m_{\mathrm{K^0_S}}$: 0.48 ppm
2. m_Λ: 0.072 ppm
3. m_π: 0.46 ppm
4. S_p: 0.57 ppm

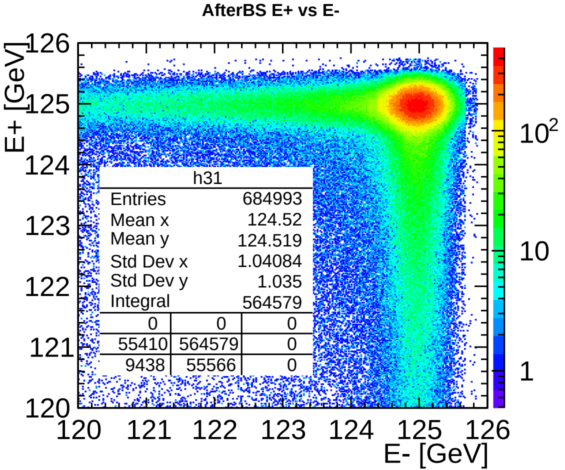
- Fit fixes proton mass
- Factors of (54, 75, 3) improvement over PDG for $(\mathrm{K^0_S}, \Lambda/\overline{\Lambda}, \pi^\pm)$
- Momentum-scale to **2.5 ppm stat.** per 10M hadronic Z, ILC Z run may have 400 such samples.

What do we really want to measure?

Ideally, the 2-d distribution of the **absolute beam energies** after beamstrahlung. From this we would know the distribution of both $\sqrt{s}$ and the initial state momentum vector (especially the z component).

Now let's look at the related 1-d distributions $(E_+, E_-, \sqrt{s}, \ p_z)$ with empirical fits.

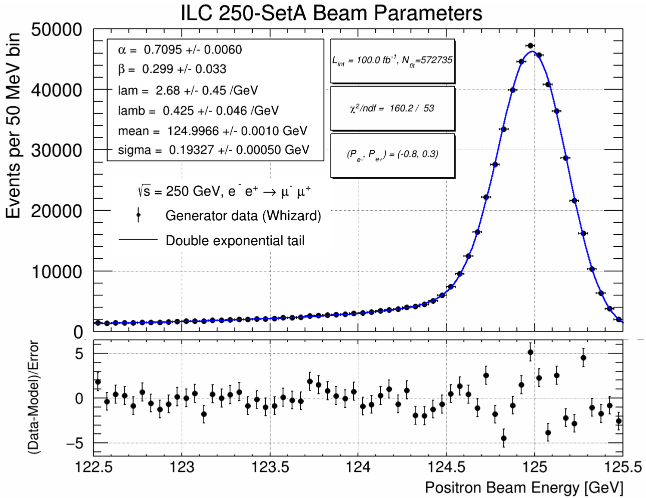
[dL/d$\sqrt{s}$: see work by Frary, Miller, Moenig, Sailer, Poss]



Whizard 250 GeV SetA $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$ events

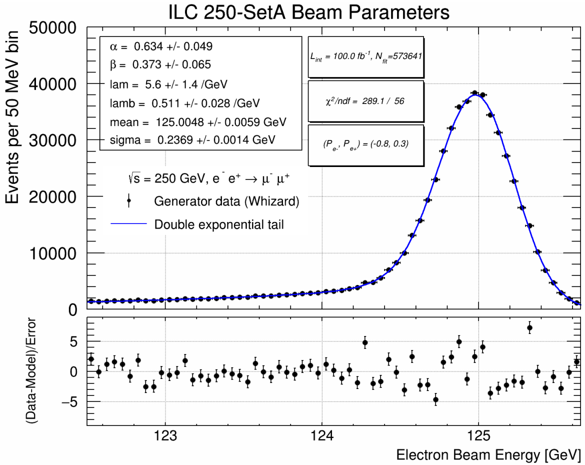
Positron Beam Energy (After Beamstrahlung)

Fits with (double-exponential tail + delta-function) convolved with Gaussian beam energy spread (6 parameters).



$\sigma_{\mathrm{R}}/E = 0.1546 \pm 0.0004\%$ (cf 0.152% in TDR)

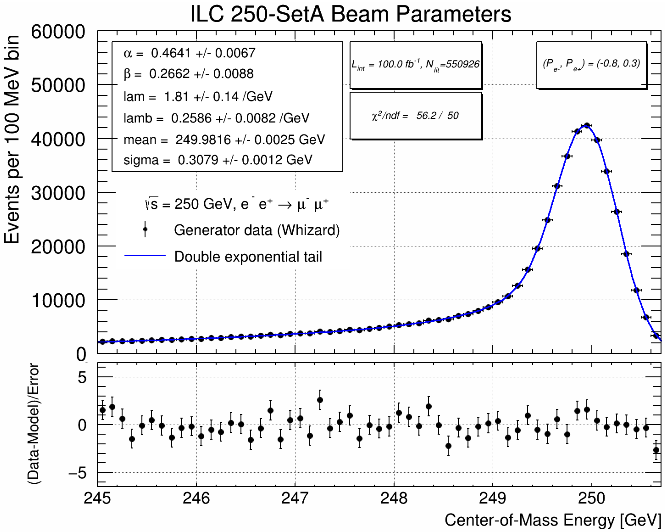
Electron Beam Energy (After Beamstrahlung)



$$\sigma_{\mathrm{R}}/E = 0.1895 \pm 0.0011\% \text{ (cf 0.190\% in TDR)}$$

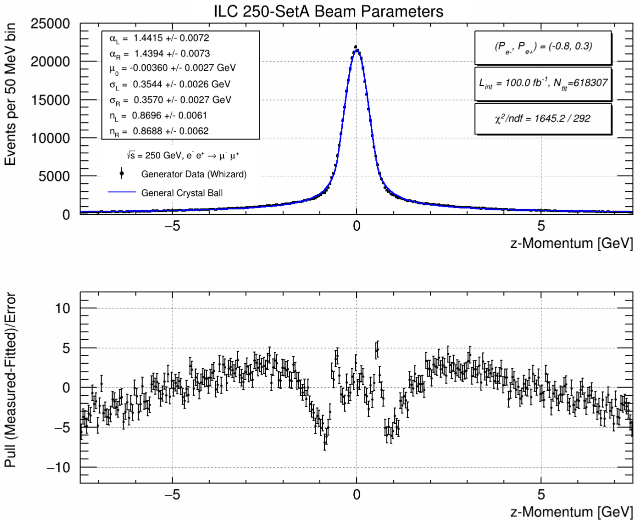
Note an undulator bypass could reduce this spread when one e^- cycle is used purely for e^+ production.

Center-of-Mass Energy (After Beamstrahlung)



$$\sigma/\sqrt{s} = 0.1232 \pm 0.0005\% \text{ (cf } 0.122\% \text{ in TDR (} 0.190\% \oplus 0.152\%)/2)$$

z-Momentum of e^+e^- system (After Beamstrahlung)



$\sigma/\sqrt{s} = 0.1416 \pm 0.0007\%$ (cf 0.122% from beam energy spread alone)

Initial State Kinematics with Crossing Angle

Define the two beam energies (after beamstrahlung) as E_{b}^{-} and E_{b}^{+} for the electron beam and positron beam respectively.
Initial-state energy-momentum 4-vector (neglecting m_{e})

$$E = E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}$$

$$p_x = (E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}) \sin(\alpha/2)$$

$$p_y = 0$$

$$p_z = (E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}) \cos(\alpha/2)$$

The corresponding center-of-mass energy is

$$\sqrt{s} = 2\sqrt{E_{\mathrm{b}}^{-} E_{\mathrm{b}}^{+}} \cos(\alpha/2)$$

Hence if α is known (14 mrad for ILC), evaluation of the collision center-of-mass energy amounts to measuring the two beam energies. Introducing,

$$E_{\mathrm{ave}} \equiv \frac{E_{\mathrm{b}}^{-} + E_{\mathrm{b}}^{+}}{2} \ , \overline{\Delta E_{\mathrm{b}}} \equiv \frac{E_{\mathrm{b}}^{-} - E_{\mathrm{b}}^{+}}{2}$$

then with this notation,

$$\sqrt{s} = 2\sqrt{E_{\mathrm{ave}}^{2} - (\overline{\Delta E_{\mathrm{b}}})^{2}} \cos(\alpha/2)$$

Final State Kinematics and Equating to Initial State

Let's look at the final state of the $\mathrm{e^+e^-} \rightarrow \mu^+\mu^-(\gamma)$ process. Denote the μ^+ as particle 1, the μ^- as particle 2, and the rest-of-the event (RoE) as system 3. We can write this final-state system 4-vector as

$$(E_1 + E_2 + E_3,\ \vec{p_1} + \vec{p_2} + \vec{p_3})$$

Applying $(E, \vec{p})$ conservation we obtain,

$$E_1 + E_2 + \sqrt{p_3^2 + M_3^2} = 2\ E_{\mathrm{ave}} \tag{1}$$

$$\vec{p_1} + \vec{p_2} + \vec{p_3} = (2\ E_{\mathrm{ave}} \sin(\alpha/2), 0, 2\ \overline{\Delta E_{\mathrm{b}}} \cos(\alpha/2)) \equiv \vec{p}_{\mathrm{initial}} \tag{2}$$

The RoE is often not fully detected and needs to be inferred using $(E, \vec{p})$ conservation. We have 4 equations and 6 unknowns:
the 3 components of the RoE momentum $(\vec{p_3})$, E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, and M_3.
Our approach is to solve for E_{ave} for various assumptions on ($\overline{\Delta E_{\mathrm{b}}}$, M_3).
Specifically we then focus on using the simplifying assumptions of the original $\sqrt{s_p}$ method that $M_3 = 0$ and $\overline{\Delta E_{\mathrm{b}}} = 0$. Note: latter is often a poor assumption for the p_z conservation component on an event-to-event basis.

The Averaged Beam Energy Quadratic

This approach results in a quadratic equation in $E_{\rm ave}$, ($AE_{\rm ave}^2 + BE_{\rm ave} + C = 0$), with coefficients of

$$A = \cos^2(\alpha/2)$$

$$B = -E_{12} + p_{12}^x \sin(\alpha/2)$$

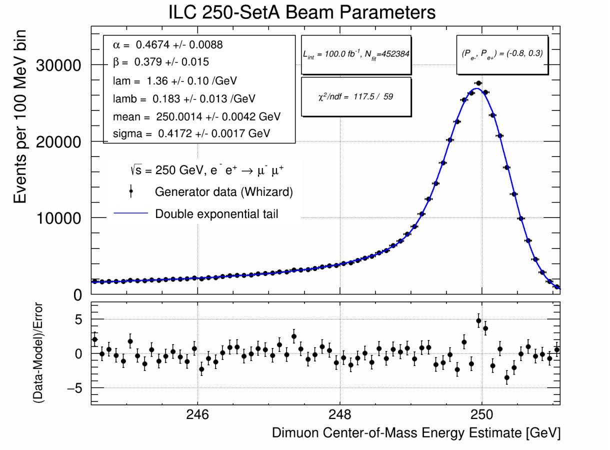
$$C = (M_{12}^2 - M_3^2)/4 + p_{12}^z \overline{\Delta E_{\rm b}} \cos(\alpha/2) - \overline{\Delta E_{\rm b}}^2 \cos^2(\alpha/2)$$

Based on this, there are a number of cases of interest to solve for $E_{\rm ave}$:

1. Zero crossing angle, $\alpha = 0$, $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
2. Crossing angle and $\overline{\Delta E_{\rm b}} = 0$, $M_3 = 0$.
3. Crossing angle and $\overline{\Delta E_{\rm b}}$ non-zero, $M_3 = 0$.
4. Crossing angle and M_3 non-zero, $\overline{\Delta E_{\rm b}} = 0$.
5. Crossing angle and $\overline{\Delta E_{\rm b}}$ and M_3 non-zero.

The original formula, $\sqrt{s} = E_1 + E_2 + |\vec{p}_{12}|$, arises trivially in the first case. In the rest of this talk the $\sqrt{s}$ estimate from the largest positive solution of the second case is what I now mean by $\sqrt{s}_p$. Obviously it is also a purely muon momentum dependent quantity.

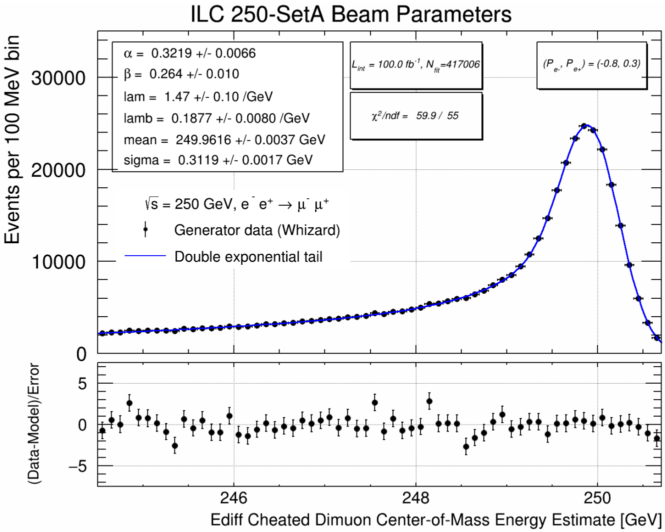
Dimuon Estimate of Center-of-Mass Energy (After BS)



$\sigma/\sqrt{s} = 0.1669 \pm 0.0007\%$ (cf 0.1232% with true $\sqrt{s}$)

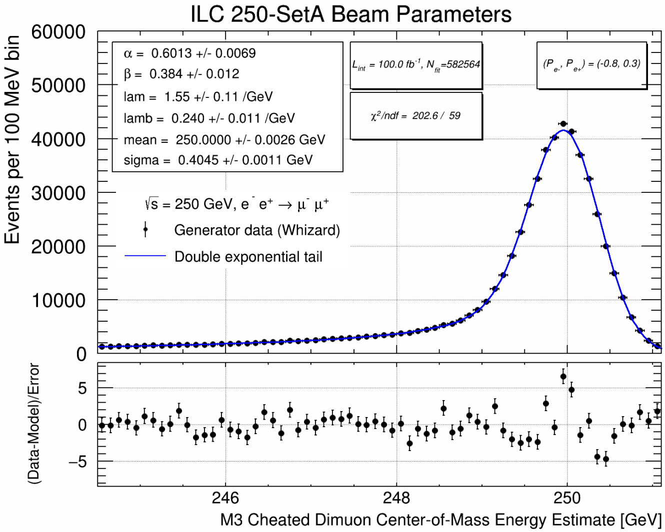
- This is the generator-level $\sqrt{s}_p$ calculated from the 2 muons
- Why so broad? Why fewer events?
- Because some events violate the assumptions that $\overline{\Delta E_{\rm b}} = 0$ and $M_3 = 0$
- The former is no surprise given the p_z distribution
- The latter is associated with events with 2 or more non-collinear ISR/FSR photons

Cheated $\overline{\Delta E_b}$ Center-of-Mass Energy Estimate (After BS)



$$\sigma/\sqrt{s} = 0.1248 \pm 0.0007\% \ (\text{cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Cheated M_3 Center-of-Mass Energy Estimate (After BS)



$$\sigma/\sqrt{s} = 0.1618 \pm 0.0004\% \text{ (cf } 0.1232 \pm 0.0005\% \text{ for } \sqrt{s})$$

Generator-level $\sqrt{s_p}$ Biases

$M_{\mu^+\mu^-}$ range [GeV]	$\mu(\sqrt{s})$ [GeV]	$\mu(\sqrt{s_p})$ [GeV]	$\mu(\sqrt{s_p})$ - $\mu(\sqrt{s})$ [MeV]
$M > 150$	249.9792 ± 0.0011	250.0337 ± 0.0013	$+54.5 \pm 1.7$
$50 < M < 150$	249.9813 ± 0.0010	249.9602 ± 0.0017	-21.1 ± 2.0
$M < 50$	249.9871 ± 0.0015	249.9633 ± 0.0028	-23.8 ± 3.2
All	249.9816 ± 0.0008	250.0014 ± 0.0010	$+19.8 \pm 1.2$

Results of the 1-parameter fits for the μ parameter to the generator-level distributions of $\sqrt{s}$ and $\sqrt{s_p}$ for three different dimuon mass ranges for the 80%/30% LR helicity mixture. The statistical uncertainties of these tests reflect an integrated luminosity of 100 fb^{-1}. The last column gives the difference in MeV of the fit parameters for the two distributions.

Strong evidence that high mass events tend to be over-measured (addition of fictitious photons in genuine 2-body $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ events), and that lower mass events are under-measured (multiple radiation more important).

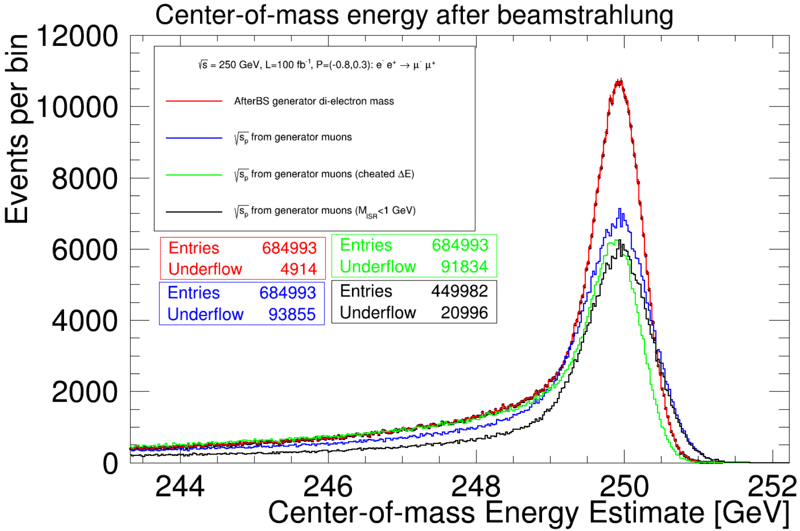
Bias associated with neglecting M_3

Naively with a mean value of M_3 of around 25 GeV, one imagines large biases for $\sqrt{s}_p$, but the median M_3 value is much lower, and examining slide 5, IF the boost is correct, the M_3 related bias goes as:

$$\Delta\sqrt{s} = |\mathbf{p}^*_{12}| - \sqrt{(p^*_{12})^2 + M_3^2}$$

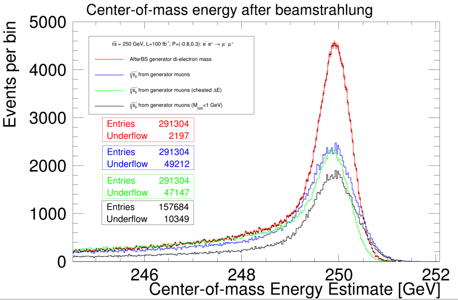
So for $p_{12} = 100$ GeV, the bias for a 10 GeV M_3 is only -0.50 GeV.

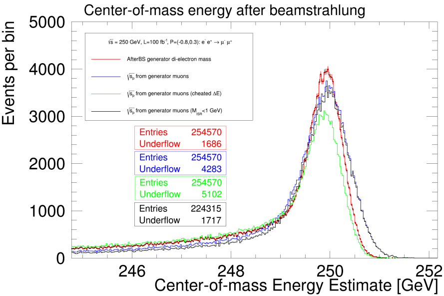
Comparisons (After BS)



$50 < m_{\mu\mu}^{\text{gen}} < 150$ GeV

$m_{\mu\mu}^{\text{gen}} > 150$ GeV





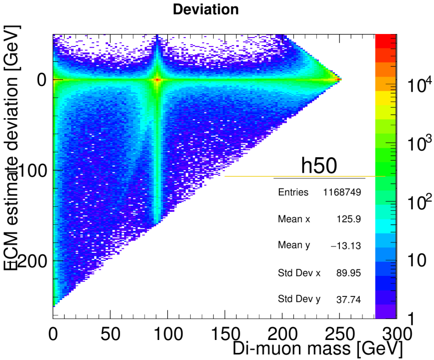
- For lower dimuon mass events, only about half are reconstructed close to $\sqrt{s}$
- Most higher dimuon mass events reconstructed close to the original $\sqrt{s}$

Conclusion

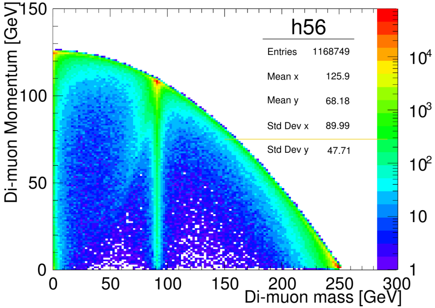
Lower dimuon mass events are more likely to violate the assumptions.

2d Generator Level Plots

Plot of $(\sqrt{s}_p - \sqrt{s})$ vs $M_{\mu\mu}$



Plot of $|p_{\mu\mu}|$ vs $M_{\mu^+\mu^-}$



Most events consistent with $M_3 \approx 0$

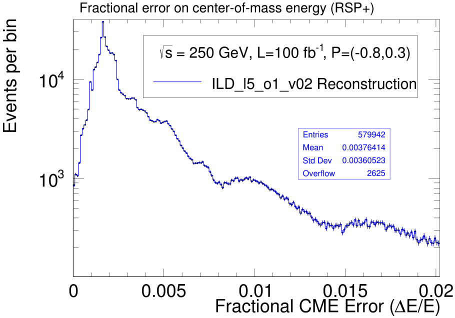
In most events, $\sqrt{s}_p$, is a reasonable estimator. But also can be off by a lot. WIP on identifying problematic events (eg. kinematic fits). It may be feasible to find alternative estimators/methods in those cases, or at least reject them.

Event Selection Requirements

Currently rather simple.

Use latest full ILD simulation/reconstruction at 250 GeV.

- Require exactly two identified muons
- Opposite sign pair
- Require uncertainty on estimated $\sqrt{s}_p$ of the event of less than 0.8% of $\sqrt{s}_{\mathrm{nom}}$ based on propagating track-based error matrices
- Categorize reconstruction quality as **gold** (<0.15%), **silver** ([0.15, 0.30]%), **bronze** ([0.30, 0.80]%)
- Require the two muons pass a vertex fit with p-value > 1 %



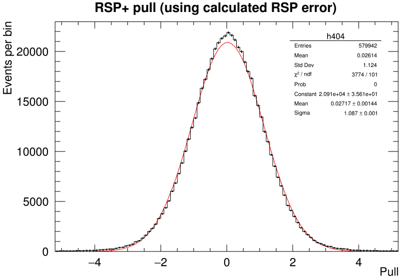
Selection efficiencies for (80%/30%) beam polarizations:

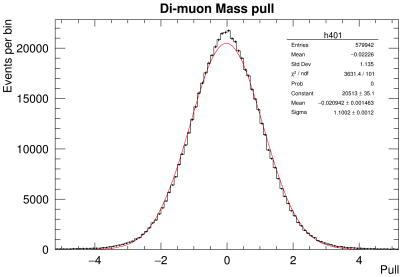
- $\varepsilon_{-+} = 69.77 \pm 0.06$ %
- $\varepsilon_{+-} = 67.35 \pm 0.06$ %
- $\varepsilon_{--} = 69.47 \pm 0.05$ %
- $\varepsilon_{++} = 67.72 \pm 0.06$ %

Backgrounds not yet studied in detail, ($\tau^+\tau^-$ is small:0.15%, of no import for the $\sqrt{s}$ peak region).

Dimuon Pull Distributions

- Pull $\equiv$ (meas - true)/error.
- Track-based estimates of the errors on both the $\sqrt{s}_p$ quantity (left) and the di-muon mass (right) agree well with the modeled uncertainties for reconstructed dimuon events.





- In both cases the fitted rms over this range is about 10% larger than ideal. Central range well described. Suspect tails should be non-Gaussian given the non-Gaussian tails of multiple scattering.
- In practice this is rather encouraging

Vertex Fit: Exploit ILC nanobeams

Given that the track errors are well modeled and the 2 muons should originate from a common vertex consistent with the interaction point, we can perform:

- Vertex Fit: Constrain the two tracks to a common point in 3-d
- Beam-spot Constrained Vertex Fit

The ILC beam-spot size is $(\sigma_x, \sigma_y) = (515, 7.7)$ nm, $\sigma_z = 0.202$ mm

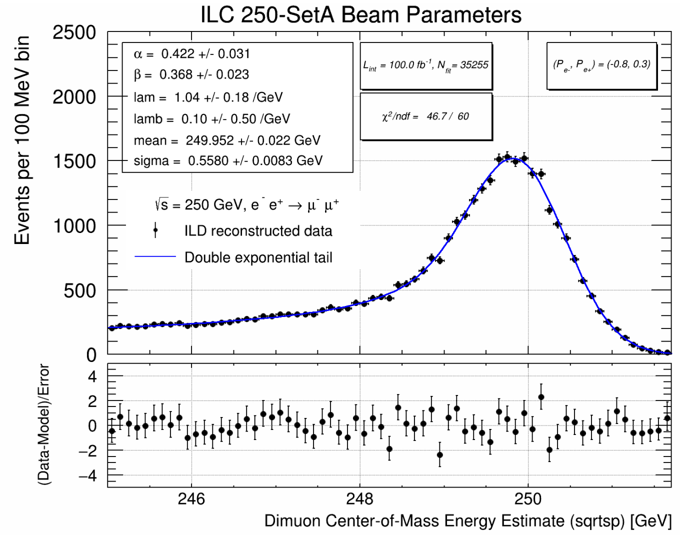
- Vertex fit along same lines as AWLC2014 talk has been re-implemented using the fully simulated data
- Also have explored beam-spot constraints

What good is this?

- Residual background rejection (eg. $\tau^+\tau^-$ reduced by factor of 20)
- Additional handle for rejecting or deweighting mis-measured events
- Some modest improvement in precision of di-muon kinematic quantities
- Also useful for $\mathrm{H} \rightarrow \mu^+\mu^-$ and for ZH recoil
- Interaction point measurement ($\mathcal{O}(1\mu\mathrm{m})$ resolution per event) can be used to correlate with (E_-, E_+) for understanding beamstrahlung

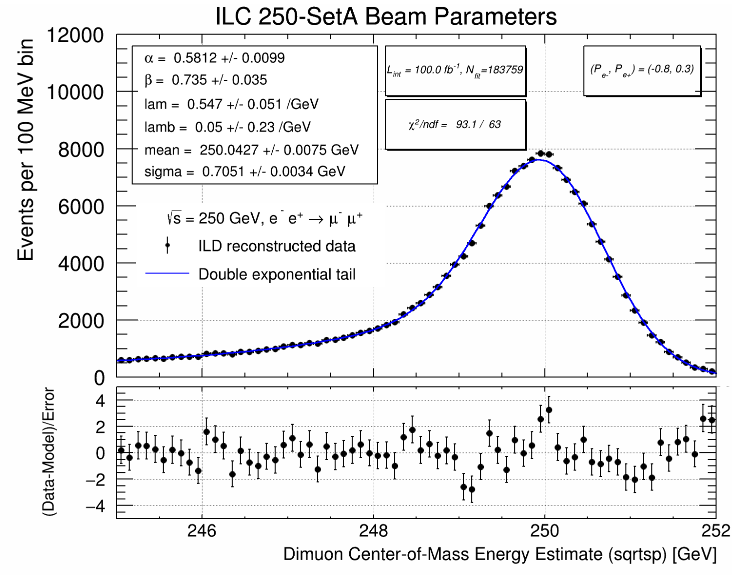
Note: simulated data does not currently simulate the transverse beam-spot ellipse nor the beam energy-z_{vtx} correlations.

Gold Quality Dimuon PFOs (After BS)



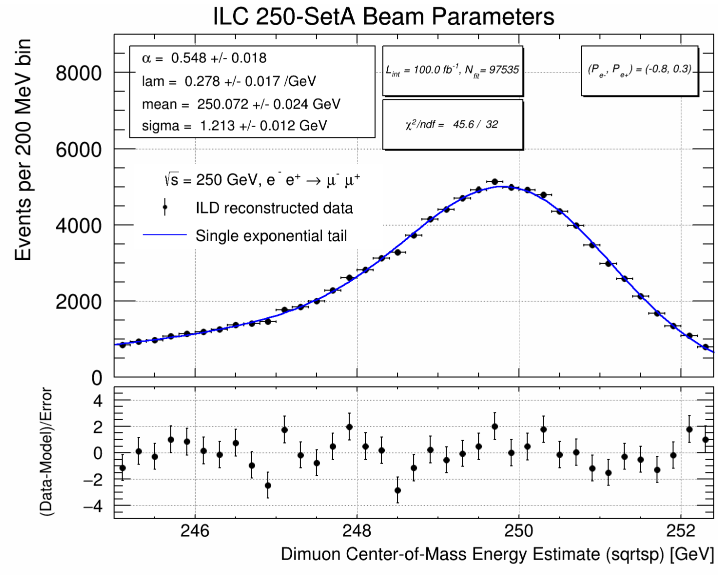
Peak width 1.34 ± 0.02 wider than $\sqrt{s_p}$ (gen).

Silver Quality Dimuon PFOs (After BS)



Peak width 1.69 ± 0.01 wider than $\sqrt{s_p}$ (gen).

Bronze Quality Dimuon PFOs (After BS)



Peak width 2.91 ± 0.03 wider than $\sqrt{s_p}$ (gen).

Strategy for Absolute $\sqrt{s}$ and Estimate of Precision

Prior Estimation Method

- Guesstimate how well the peak position of the Gaussian can be measured using the observed $\sqrt{s}_p$ distributions in bins of fractional error

Current Thinking

- The luminosity spectrum and absolute center-of-mass energy are the same problem or at least very related. How well one can determine the absolute scale depends on knowledge of the shape (input also from Bhabhas).
- Beam energy spread likely to be well constrained by spectrometer data
- Likely need either a convolution fit (CF) or a reweighting fit
- Work is in progress on a CF by parametrizing the underlying (E_-, E_+) distribution, and modeling quantities related to $\sqrt{s}$ and p_z after convolving with detector resolution (and ISR, FSR and cross-section effects)

Current Estimation Method

- Use estimates of the statistical error on the peak position for 6-parameter convolved double exponential tail fits to fully simulated data with the 5 shape parameters fixed to their best fit values.
- Fits are done in the 3 resolution categories.
- Next slide has these estimates

$\sqrt{s}$ Sensitivity Estimates at $\sqrt{s} = 250$ GeV

Statistical uncertainties in ppm on $\sqrt{s}$ for $\mu^+\mu^-$ channel

$L_{\rm int}$ [ab^{-1}]	Poln [%]	ε [%]	Gold	Silver	Bronze	All categories
0.9	$-80, +30$	70.4	6.4	3.1	7.7	2.6
0.9	$+80, -30$	68.0	7.5	3.4	8.7	2.9
0.1	$-80, -30$	70.1	25	12	30	10
0.1	$+80, +30$	68.3	28	13	33	11
2.0	Combined	-	4.7	2.2	5.6	**1.9**

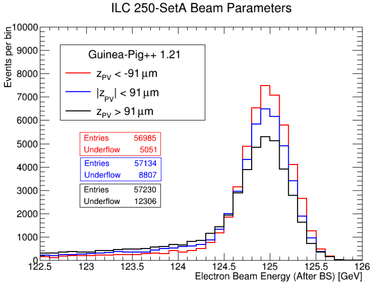
Fractional errors on μ parameter (mode of peak) when fitting with 6-parameter double exponential tail function with all 5 shape parameters fixed to their best-fit values. (4/3 for bronze).

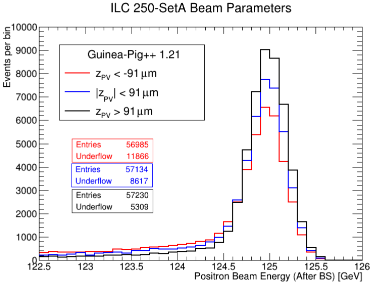
Also the $\mathrm{e}^+\mathrm{e}^-$ channel should be used. The additional benefit of the much larger statistics from more forward Bhabhas will be offset by the poorer track momentum resolution at forward angles.

Beamstrahlung / z-Vertex Effects Explained

Divide interactions in 3 equi-probability parts according to z_{PV}. Preferentially

1. $\mathrm{e}^+\mathrm{e}^-$ collisions occurring more on the initial e^- side ($z < 0$)
2. $\mathrm{e}^+\mathrm{e}^-$ collisions mostly central
3. $\mathrm{e}^+\mathrm{e}^-$ collisions preferentially on the initial e^+ side ($z > 0$)

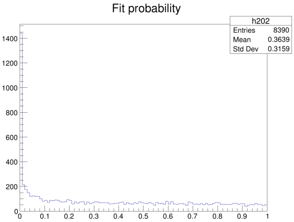


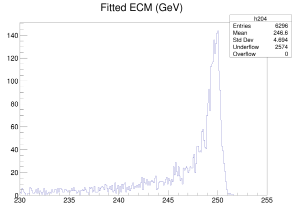


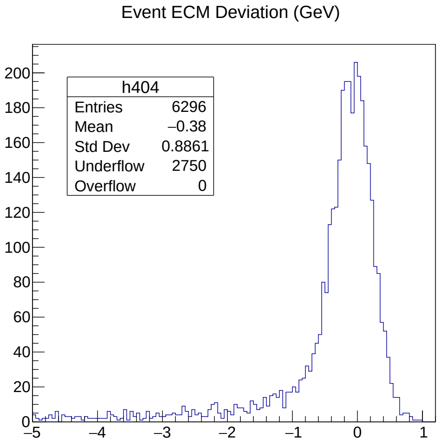
The beamstrahlung tail grows and the peak shrinks for e^- as z increases, and, for e^+ as z decreases. In both cases, the largest beamstrahlung tail occurs when the interacting e^- or e^+ has on average traversed more of the opposing bunch.

Thus both $\sqrt{s}$ and $p_z = E_- - E_+$ distributions depend on z. Likely needs to be taken into account for $\sqrt{s}$, $\mathrm{dL}/\mathrm{d}\sqrt{s}$, Higgs recoil, kinematic fits ...

Test consistency with $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ (no photons) by fitting for E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ as unmeasured parameters (4C/2U/2dof). So measure $\sqrt{s}$ and collision asymmetry.







Plots require $p_{\mathrm{fit}} > 0.05$ (26% of all events). See backup for details. Use 0.15% momentum resolution. Peak width is 0.3 GeV (same as energy spread).

Outlook and Future Work

Lots of opportunities to improve this:

1. Constrained kinematic fits. For example one can test the consistency with the pure 2-body hypothesis of $e^+e^- \to \mu^+\mu^-$ while fitting for the two unmeasured parameters of E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$, and also perform fits with the $e^+e^- \to \mu^+\mu^-\gamma$ hypothesis.

2. Extend the techniques to the $e^+e^- \to e^+e^-$ channel.

3. Exploit fully events with detected photons.

4. Implement complete end-to-end measurement scheme and understand how best to use different kinematic regimes and correct/mitigate observed biases.

5. Characterize better the intrinsic limitations associated with beam energy spread, beamstrahlung, ISR, FSR, backgrounds, and detector acceptance and resolution. This includes studies with more specialized physics event generators such as KKMCee [29].

6. Tracker momentum scale studies using $J/\psi \to \mu^+\mu^-$, $K^0_S \to \pi^+\pi^-$, $\Lambda^0 \to p\pi^-$. We have some preliminary results [30] further applying the technique advocated in [31] based on the Armenteros-Podolanski [32] reconstruction technique. A more novel aspect is that one can aspire to simultaneously improve the measurements of the K^0_S and Λ masses and the momentum scale given that the masses of their decay products are very well known.

7. Understand the relative merit of dimuons for luminosity spectrum determination compared with Bhabhas and integrate both techniques in a global analysis.

8. Characterize further the scope for measuring accelerator parameters such as the crossing angle and beamstrahlung-induced correlations including the observed dependence of the beam energy spectrum on the longitudinal collision vertex. The latter has been shown to be easily measurable with vertex fits in $e^+e^- \to \mu^+\mu^-(\gamma)$ events.

Summary of Progress

Progress

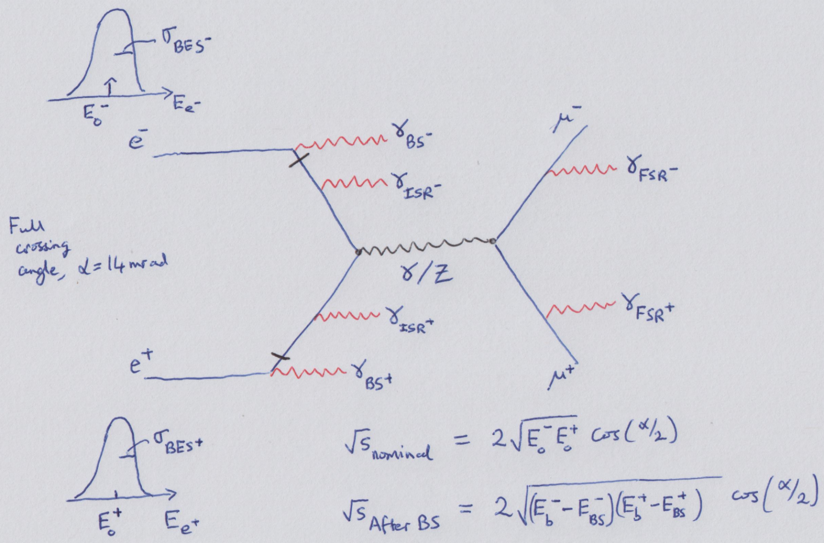
- New high precision method for momentum-scale using especially $\mathrm{K^0_S}$ and Λ. Promises 2.5 ppm uncertainty per 10M hadronic Zs.
- More detailed investigation of dimuons for $\sqrt{s}$ and dL/d$\sqrt{s}$ reconstruction
- Measurement of M_{Z} using dimuon mass for $\sqrt{s} \gg M_{\mathrm{Z}}$ to 1.0 MeV - dominated by $\sqrt{s} = 250$ GeV data

Conclusions

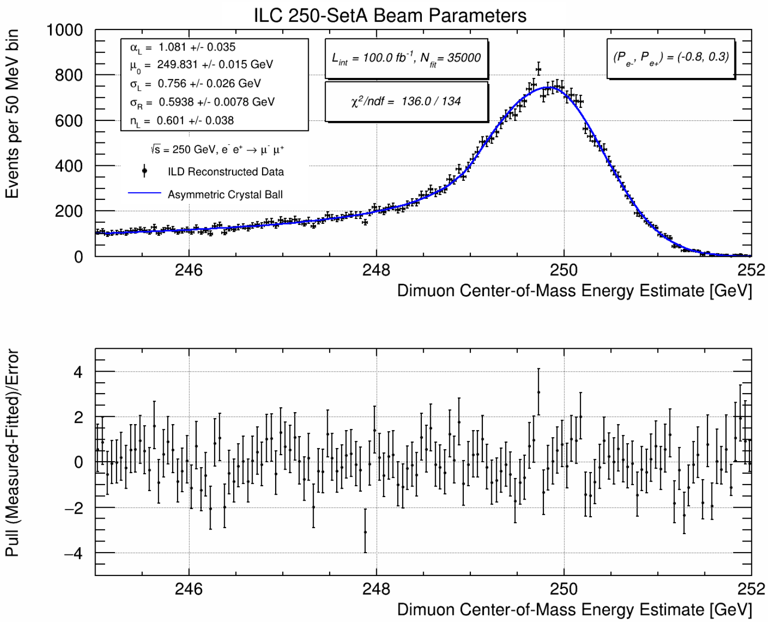
- Tracking detectors designed for ILC have the potential to measure beam energy related quantities with precision similar to the intrinsic energy spread using dimuon events (and also wide-angle Bhabha events)
- At $\sqrt{s} = 250$ GeV, dimuon estimate of 1.9 ppm precision on $\sqrt{s}$. More than sufficient (10 ppm needed) to not limit measurements such as M_{W}.
- Potential to improve M_{Z} by a factor of three using 250 GeV di-lepton data
- Applying the same $\sqrt{s}$ techniques to running at the Z-pole enables a high precision electroweak measurement program for ILC that takes advantage of absolute center-of-mass energy scale knowledge

Backup Slides

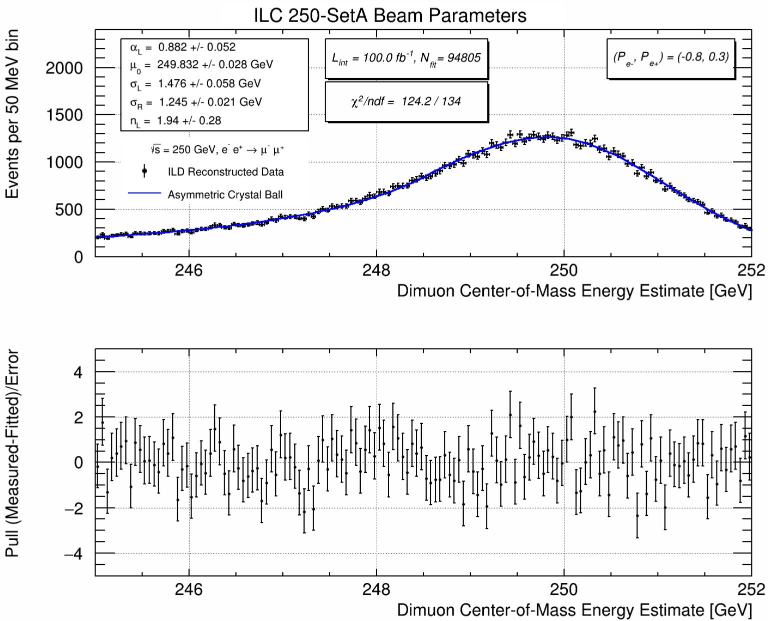
Returning to $\sqrt{s}_p$ and Adding More Realism



Gold Quality Dimuon PFOs (After BS)

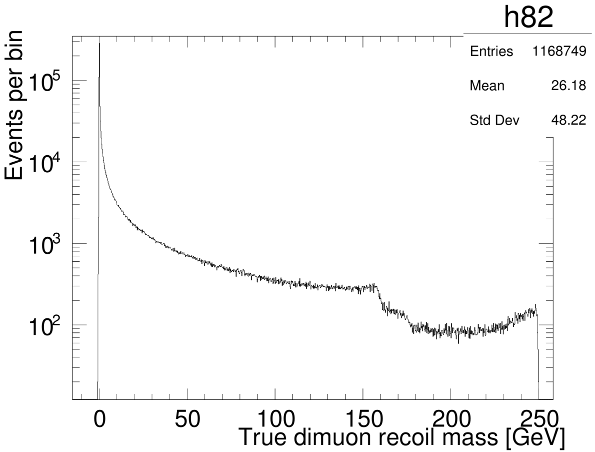


Bronze Quality Dimuon PFOs (After BS)



Recoil Mass (at generator level)

Distribution of M_3.



Events in the tails will be from multiple non-collinear radiation (example ISR from both beams)

Kinematic Fits for $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-(\gamma)$

Inspired by revisiting some of the LEP2 techniques for M_{W} measurement, one can also cast the whole problem as a constrained fit problem. Promises to be very useful in event selection, hypothesis identification, and parameter measurement, but needs excellent object calibration and measurement uncertainties.

Two body fits

Test the hypothesis of $\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-$ with no additional photons.

1. * Specify E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ and fit with the 4 constraints of (E,p) conservation. (4C/4dof fit)
2. * Fit for E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ as unmeasured fit parameters with the 4 constraints. (4C/2U/2dof fit).

Initial test implementation uses easily adaptable constrained fitting code of V. Blobel with toy MC based smearing and uncertainties.

1. Find 10.7% of events satisfy the 2-body hypothesis ($p_{\mathrm{fit}} > 0.01$) IF the correct E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$ are specified (Fit 1). For these events, $M_{\mu\mu}$ is synonymous with $\sqrt{s}$.
2. Find 26% of events satisfy fit 2 ($p_{\mathrm{fit}} > 0.05$).
   Note often the fitted $\sqrt{s}$ is near M_{Z} ... with large $|\overline{\Delta E_{\mathrm{b}}}|$.

Kinematic Fits for $e^+e^- \to \mu^+\mu^-(\gamma)$

Three particle collinear ISR fits

Test the $e^+e^- \to \mu^+\mu^-\gamma$ hypothesis where the γ is an undetected ISR photon collinear with one of the beams with z-hemisphere signed energy, E_{ISR}.

1. Specify E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} and fit with 4 constraints. (4C/4dof fit)
2. * Specify E_{ave} and $\overline{\Delta E_{\mathrm{b}}}$. Fit E_{ISR} as unmeasured parameter and fit with 4 constraints. (4C/1U/3dof fit)
3. Fit for E_{ave}, $\overline{\Delta E_{\mathrm{b}}}$, E_{ISR} as unmeasured fit parameters with the 4 constraints. (4C/3U/1dof fit).