Energy & Polarisation workshop, Sep 21, 2022, CERN

# Crossing-angle and beam-beam effects at FCC-ee

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Based on work done 3 years ago with P. Janot, D. Shatilov, Y.Voutsinas

For details, see :
"Polarization and center-of-mass energy calibration at FCC-ee",
A. Blondel, P. Janot, J. Wenninger et al, arXiv:1909.12245

See also the talk from D. Shatilov on Tuesday in WP2

When not specified otherwise, numbers refer to what happens at the Z peak.

# Center of mass energy

Uncertainty on √s is the driving systematic uncertainty on many key EW precision measurements at FCC-ee.

- Beam energies can be measured with an exquisite precision at the Z peak and the WW threshold, thanks to the Resonant Depolarisation method
  - unique to circular colliders
  - uses non-colliding bunches
  - leads to $\delta ( \sqrt{s} )$ = 100 keV at the Z peak (i.e. $10^{-6}$ rel.), < 300 keV at WW

- Crossing angle $\alpha$ = 30 mrad:

$$\sqrt{s} = 2\sqrt{E_+ E_-} \cos\frac{\alpha}{2}$$

$$\frac{d\sqrt{s}}{\sqrt{s}} = \frac{1}{4}\,\alpha\, d\alpha$$

To contribute e.g. 10 keV to the uncertainty on √s at the Z peak, the crossing angle must be known to about 13 μrad i.e. to 0.4 ‰.

# Measurement of the crossing angle

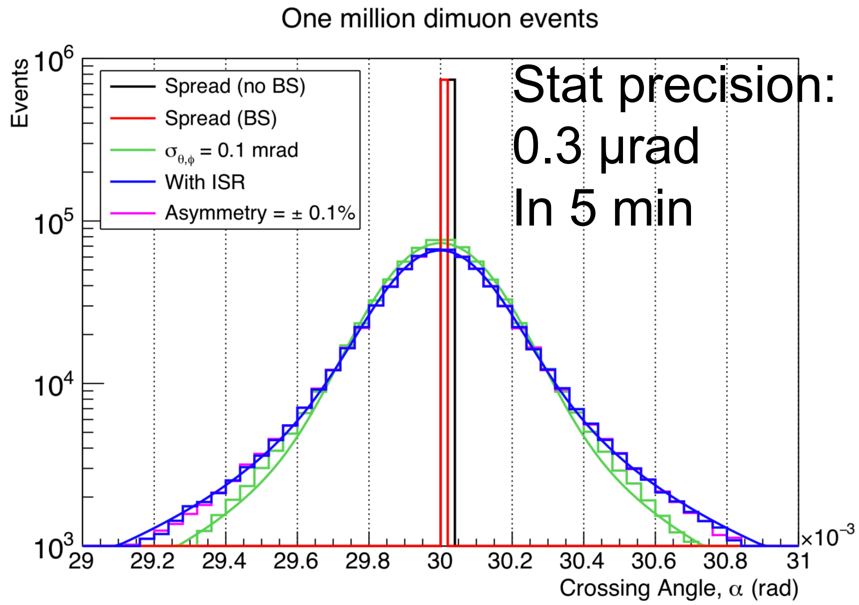
- Beam Position Monitors placed on the quads close to the IP measure $\alpha$
  - But expected precision not better than O ( 0.1 ) mrad
  - At the Z peak, corresponds to O ( 100 keV ) on √s

- $\alpha$ can be measured much better by the experiment using the constrained kinematics of dimuon events ee → μμ ($\gamma$)

$$\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]$$

Syst. uncertainty of O ( 0.1 μrad )
Negligible contribution to δ( √s )

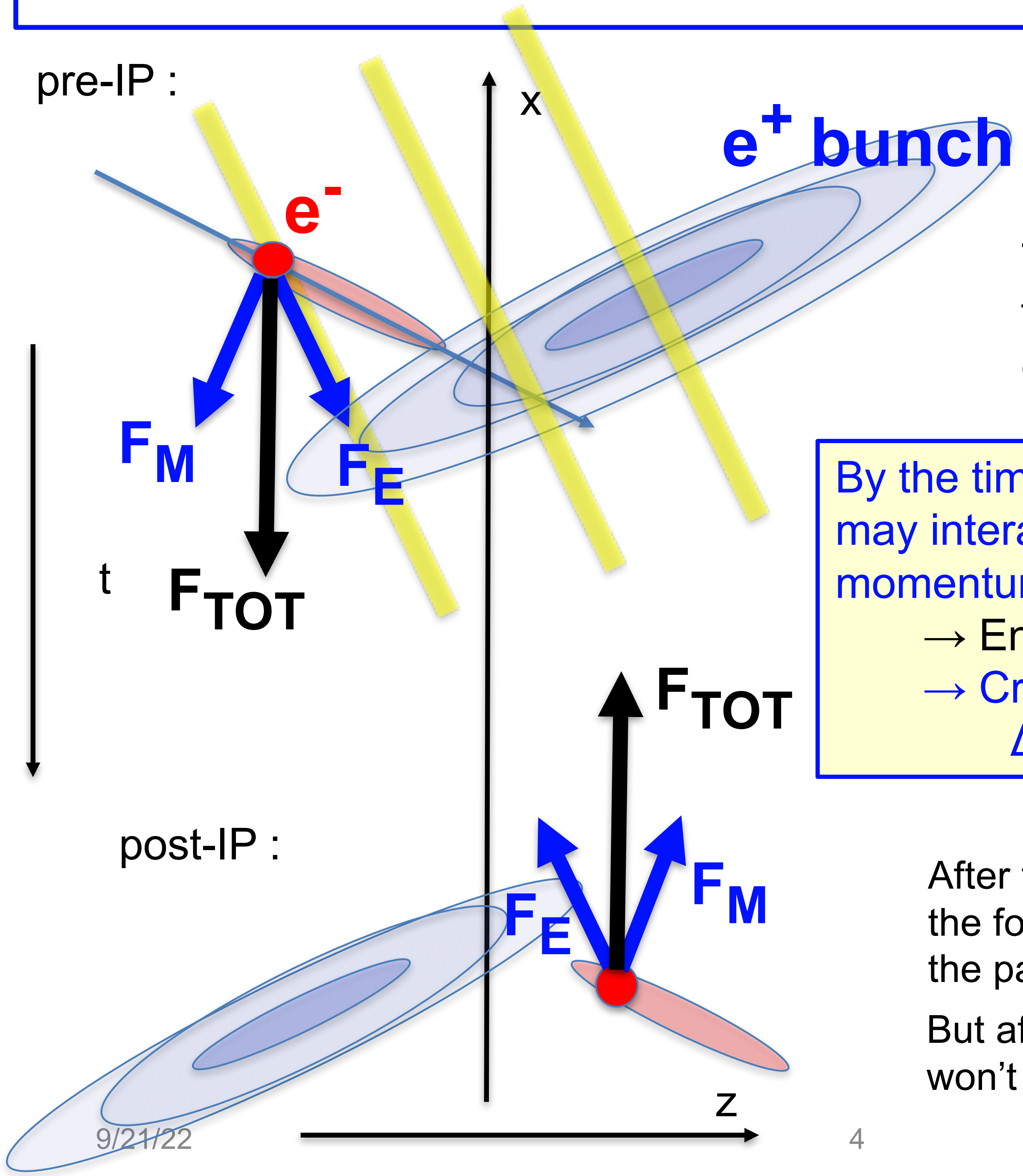
(See later talk by PJ: absolute detector alignment)




Stat precision:
0.3 μrad
In 5 min

NB: the same events also allow the energy spread to be determined in-situ

# Beam-beam effects complicate the picture…




Before it reaches the IP :
The Lorentz force felt by the electron
is along the x axis, pointing downwards.

The particle is accelerated by this force along -x, and it gains energy.

By the time the particles reach the IP and may interact, they have acquired a net momentum ("kick") along ( - ) x.

→ Energy increases $\Delta E$ = Kick x $\sin\alpha/2$

→ Crossing angle increases:

$\Delta(\alpha/2)$ = Kick / $E_e$

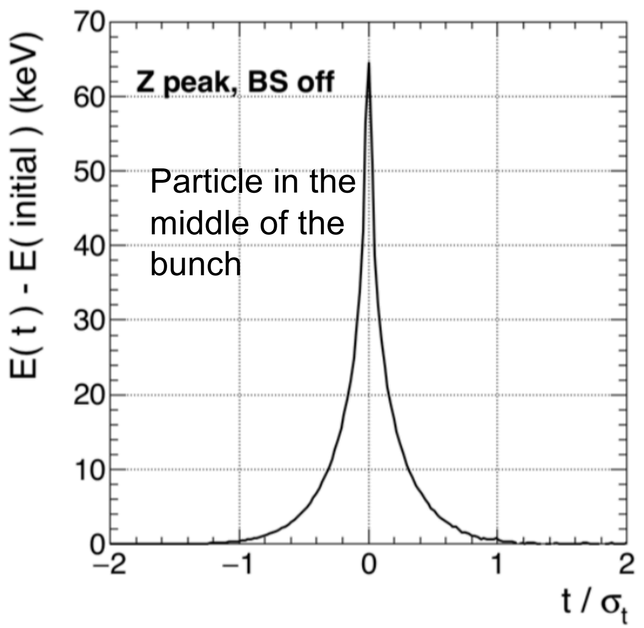
After the IP :
the force is in the other direction,
the particle is decelerated and looses energy.

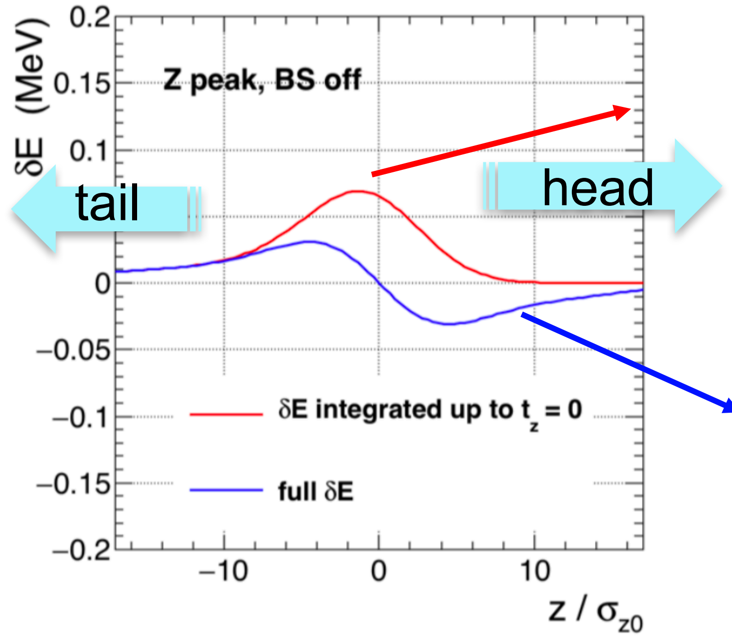
But after it has crossed the IP, the particle won't be able to collide anymore in this BX.

# Numerical determinations

Numerical tools to determine / study the beam-beam effects:

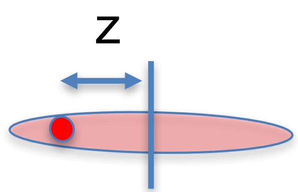
- LifeTrack (D. Shatilov) : the reference. Accelerator physics code, multi-turn.
- Guinea-Pig (D. Schulte): single-turn. Need to pass equilibrium beam parameters.
- Analytical model (E.P.) : analytical calculations using the well-known expressions of the field created by a gaussian charge distribution.




Kick integrated by the particles up to the time when they won't be able to collide anymore.

Integration up to t = ∞. The averaging over all particles in the bunch is zero.




- Average increase of Ee = 60.5 keV
- Average increase of $\alpha$: $\Delta\alpha$ = 0.17 mrad, i.e. $\Delta\alpha / \alpha \sim 0.6\%$

# Importance of beam-beam effects for √s determination

$$\sqrt{s} = 2\sqrt{E_+ E_-}\cos\frac{\alpha}{2} = 2\sqrt{|\,p_{z,+}\,p_{z,-}\,|}$$

$$\delta\alpha = \frac{1}{\tan \alpha/2}\left(\frac{\delta E_+}{E_+} + \frac{\delta E_-}{E_-}\right),$$

BB effects do not affect the $p_z$. I.e. exact compensation of :

- The increase of Ee
- The increase of $\alpha$ ( decrease of cos $\alpha/2$ )

However these effects can not be ignored, because :

$$\sqrt{s} = 2\sqrt{E_+^0 E_-^0}\cos \alpha_0/2 = 2\sqrt{E_+ E_-}\cos \alpha/2,$$

E in absence of BB effects, measured with RDP

??

$\alpha$ with BB effects, measured with dimuons

To go to √s: one needs to know $\alpha_0$, i.e. in addition to $\alpha$, the xing angle increase induced by the BB effects, $\Delta\alpha = \alpha - \alpha_0$.

Want to know $\Delta\alpha$ to ~ 13 μrad, hence to a relative precision of O( 10 % ).

# Measurement of the crossing-angle increase

Beam-beam effects scale linearly with the bunch intensities when everything else is equal. E.g. energy increase of electrons prop. to intensity of positron bunch.

→ measure $\Delta\alpha$ by measuring the crossing angle in bunches of different intensities

Filling period of the machine, at the beginning of each fill : naturally offers collisions with bunches with N < nominal.

N/bunch is gradually increased, starting from 50% of Nnominal, e.g. adding 10% of the nominal N per step, every O(50 sec) in e- or e+. The beams do collide during this filling, with nominal optics.

Measure $\alpha$ in each filling step, extrapolate to N = 0.

Table: LifeTrack simulation, D. Shatilov.

intensities | Bunch length (i.e. E spread) | ΔE in keV

| $N^+_{\text{part}}$ | $N^-_{\text{part}}$ | $\mathcal{L}$ | $\sigma^+_\delta$ | $\sigma^-_\delta$ | $\sigma_{\sqrt{s}}$ | $\delta E_+$ | $\delta E_-$ | $\alpha$ | $N_{\mu^+\mu^-}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.50 | 0.50 | 0.37 | 0.68 | 0.68 | 0.680 | 39.2 | 39.2 | 30.1147 | 49210 |
| 0.50 | 0.55 | 0.38 | 0.79 | 0.61 | 0.705 | 47.9 | 33.7 | 30.1193 | 50540 |
| 0.60 | 0.55 | 0.44 | 0.64 | 0.84 | 0.747 | 35.5 | 51.5 | 30.1273 | 58250 |
| 0.60 | 0.65 | 0.50 | 0.87 | 0.68 | 0.781 | 52.9 | 39.2 | 30.1347 | 66500 |
| 0.70 | 0.65 | 0.56 | 0.69 | 0.93 | 0.819 | 40.1 | 56.5 | 30.1413 | 74480 |
| 0.70 | 0.75 | 0.62 | 0.94 | 0.74 | 0.846 | 57.5 | 43.8 | 30.1480 | 82460 |
| 0.80 | 0.75 | 0.68 | 0.76 | 0.99 | 0.883 | 44.7 | 61.6 | 30.1553 | 90440 |
| 0.80 | 0.85 | 0.74 | 1.02 | 0.80 | 0.917 | 63.4 | 45.6 | 30.1593 | 98420 |
| 0.90 | 0.85 | 0.81 | 0.82 | 1.04 | 0.936 | 49.2 | 65.2 | 30.1673 | 107730 |
| 0.90 | 0.95 | 0.87 | 1.09 | 0.84 | 0.973 | 67.5 | 49.2 | 30.1707 | 115710 |
| 1.00 | 0.95 | 0.91 | 0.86 | 1.12 | 0.998 | 49.2 | 67.5 | 30.1707 | 121030 |
| 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.000 | 60.2 | 60.2 | 30.1760 | 133000 |

Normalised to nominal | ( keV ) | ( mrad )

# Measurement of $\Delta\alpha$: extrapolation to N = 0

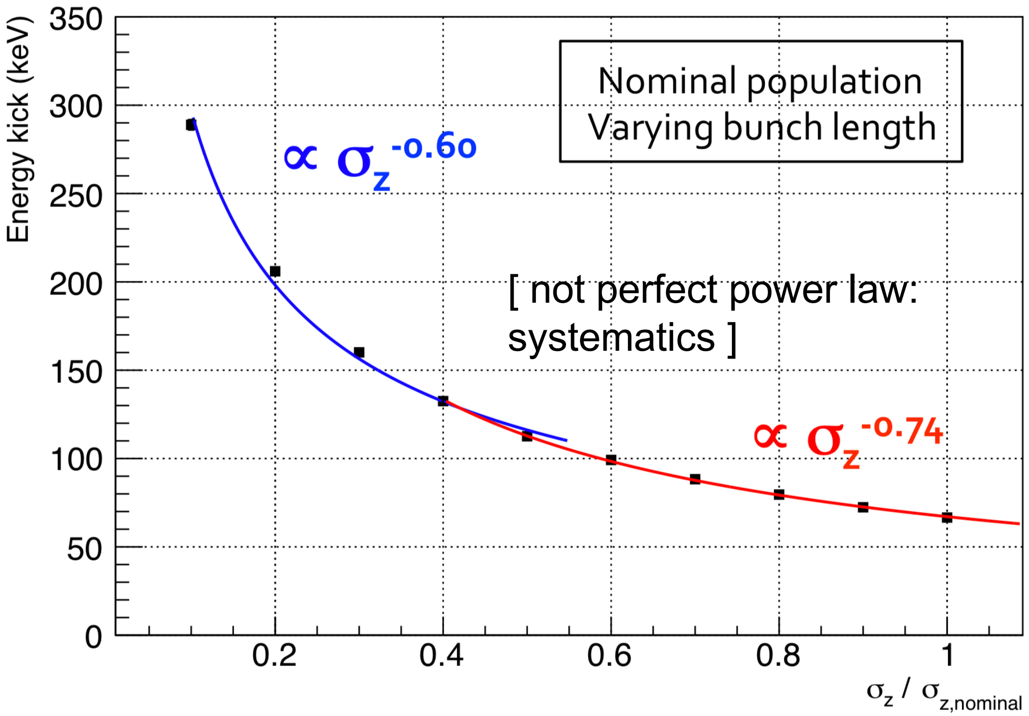
At Z and WW : bunch length σ at equilibrium is dominated by beamstrahlung.

- hence it varies during the filling steps
- and when σ increases: energy kick on the particles of the opposite bunch decreases (smaller charge density)

Variation of energy kicks with the length σ of the opposite bunch studied (everything else being constant) in analytical calculations.

For FCC bunches at the Z, in the range of interest:

$$\delta E^{\pm} \propto \frac{N_{\text{part}}^{\mp}}{\sigma_{\delta}^{\mp\,2/3}}.$$



# Scaling of the energy kicks

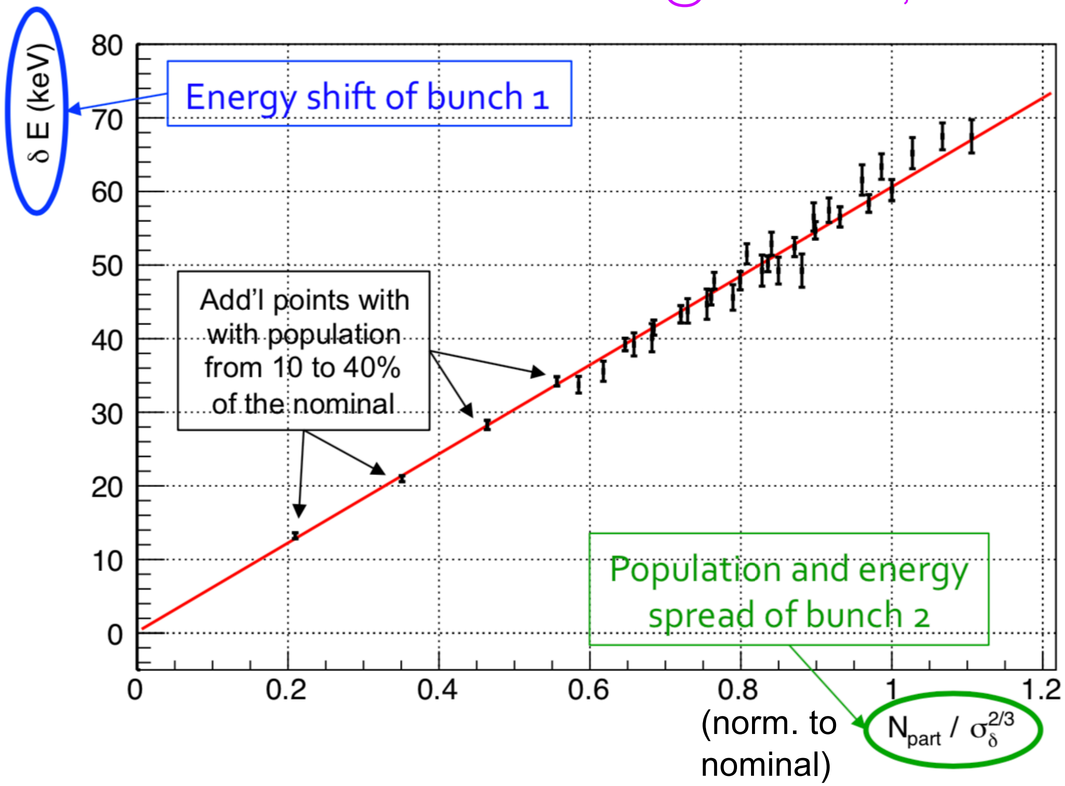
Cross-check of the scaling given on the last slide, with the numbers coming from the LifeTrack simulation :

Slide PJ @ FCC week, Brussels

- **Linear fit and extrapolation**

$\delta E_o$ (keV)= 0.2 ± 0.2 (stat.) ± 1.0 (syst.)
$\delta E_{nom}$ (keV)= 60.5 ± 0.5 (stat.) ± 0.6 (syst.)

- **Statistical uncertainty**
  - From Lifetrac MC statistics
- **Systematic uncertainty**
  - From $\sigma_\delta$ exponent uncertainty




When intensities of e+ and e- bunches are equal:

$$\delta\alpha \quad \propto \quad \delta E \propto \quad \frac{N}{\sigma^{2/3}}$$

N measured very precisely (e.g. beam pick-up), but also in-situ:

$$\mathcal{L} \quad \propto \quad \frac{N^2}{\sigma} \quad \propto \quad N_{\mu\mu}$$

$$\delta\alpha \quad \propto \quad \frac{N_{\mu\mu}^{1/2}}{\sigma^{1/6}}$$

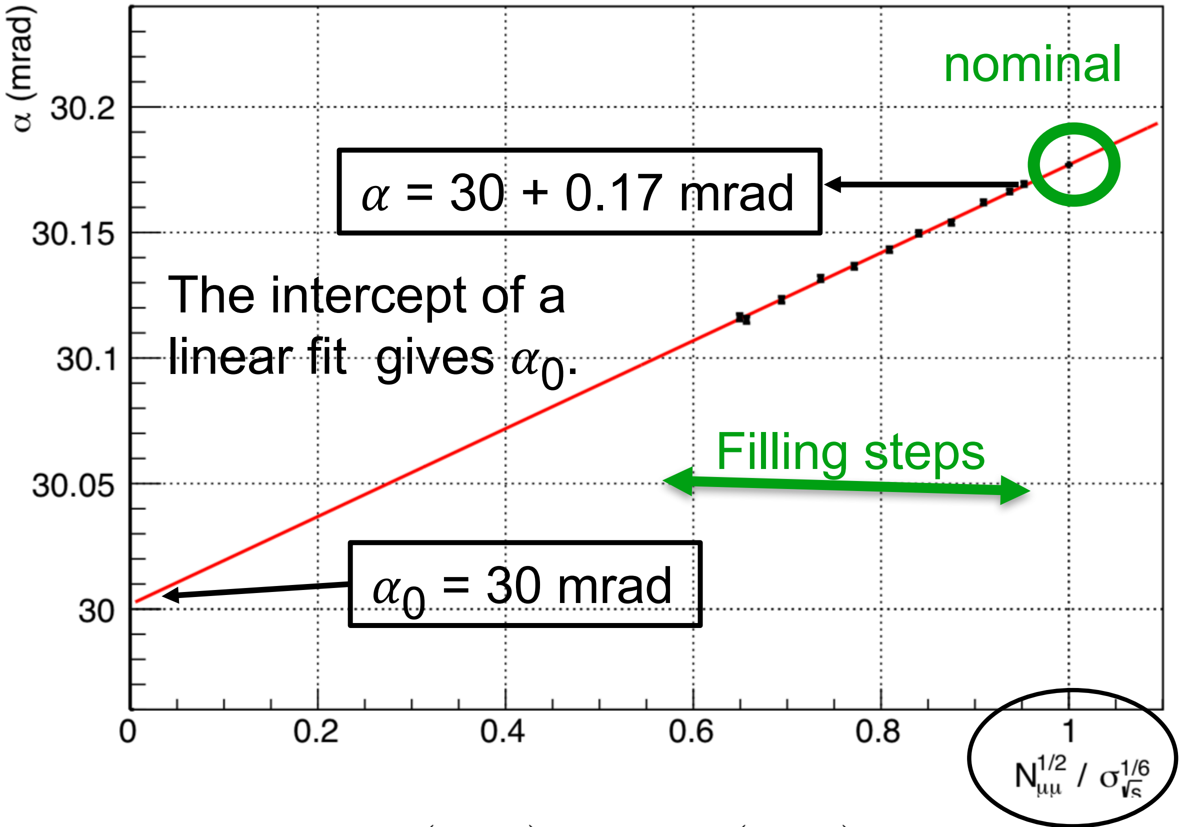
# Correction of beam-beam effects: determination of $\alpha_0$

LifeTrack simulation: shows that the scaling of

$\alpha$ versus $\sqrt{N_{\mu\mu}} \;/\; \sigma^{1/6}$

with $\sigma = \sigma_+ \oplus \sigma_-$

does remain.




(norm. to nominal)

σ is prop. to the beam energy spread.
Can be measured in situ very precisely, see next talk by PJ.

With an intensity ramp of O(10) steps, each of 40 sec measurements:

$$\alpha_0 = 30.0008 \pm 0.0016(\mathrm{stat.}) \pm 0.0031(\mathrm{syst.})\ \mathrm{mrad},$$
$$\delta\alpha = 0.1761 \pm 0.0016(\mathrm{stat.}) \pm 0.0032(\mathrm{syst.})\ \mathrm{mrad},$$

Can determine $\alpha_0$ with a precision of about 3 μrad ( and $\Delta\alpha$ within 2%)

i.e. δ ( $\alpha$ ) negligible ( a few keV) to δ ( √s )

# Alternatives

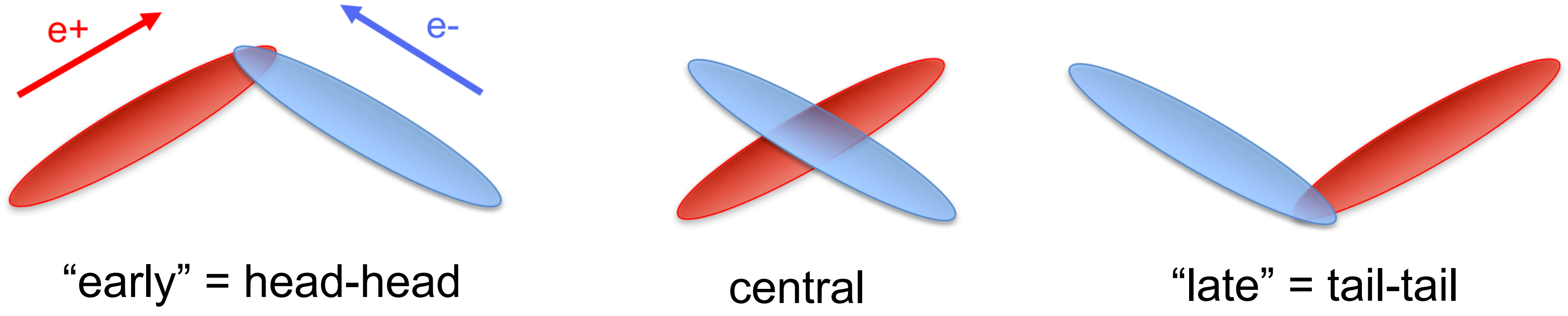
In case the filling period could not be used (e.g. beam instabilities): can still exploit the dependence of beam-beam effects w.r.t. bunch intensities with e.g. :

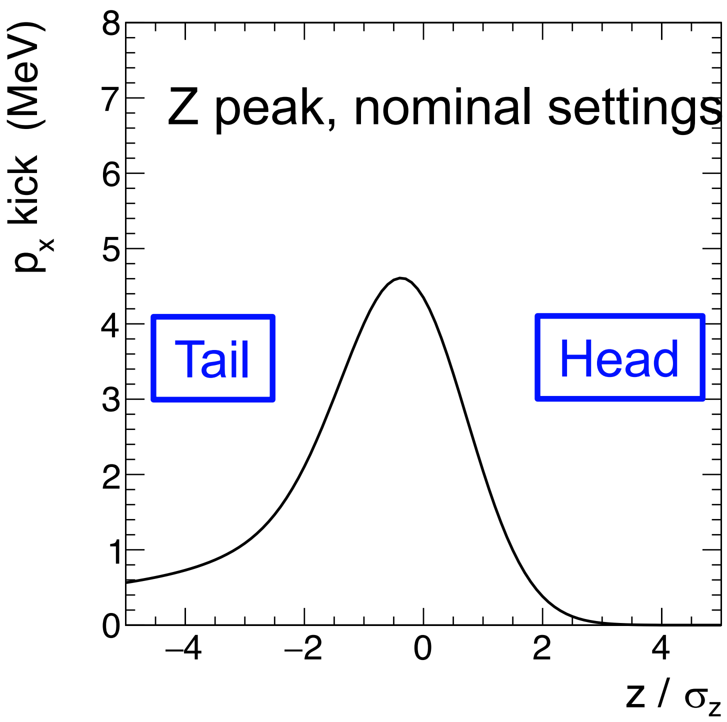
- **Use natural bunch population spread, or have half of the bunches with 99% nominal current**
  - **Inducing a minute loss of luminosity of 0.75%**
- **Or better, use the fact that each bunch population varies between 101% and 99% of the nominal over every period of 104 seconds, with alternate $e^{\pm}$ injection every 52 seconds.**
  - **Measure $\alpha$, $\sigma_{\sqrt{s}}$ and $N_{\mu\mu}$ every 26 seconds (just before and just after any top-up)**
    - ➡ **Precision on $\alpha$ of 0.016 mrad /√hours at the Z pole**

      **Corresponding to a precision on √s on 10 keV/√hours at the Z pole**

# Other alternative ( ? ) using timing…

Due to the xing angle: The longitudinal position (within its bunch) of an interacting e+/- is determined by the time of the interaction ( with $\sigma_z$ = 12 mm, $\sigma_t$ = 30 ps ) :

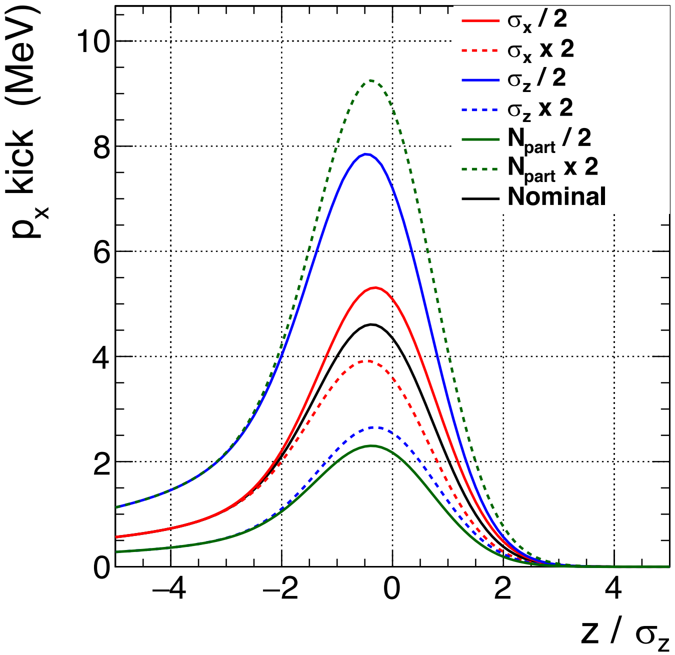



With some measurement of the interaction time (hence of z) : we can exploit the shape of this curve.

The time of the collision can be inferred by measuring the time of the charged particles in final state, e.g. in dedicated timing layer.

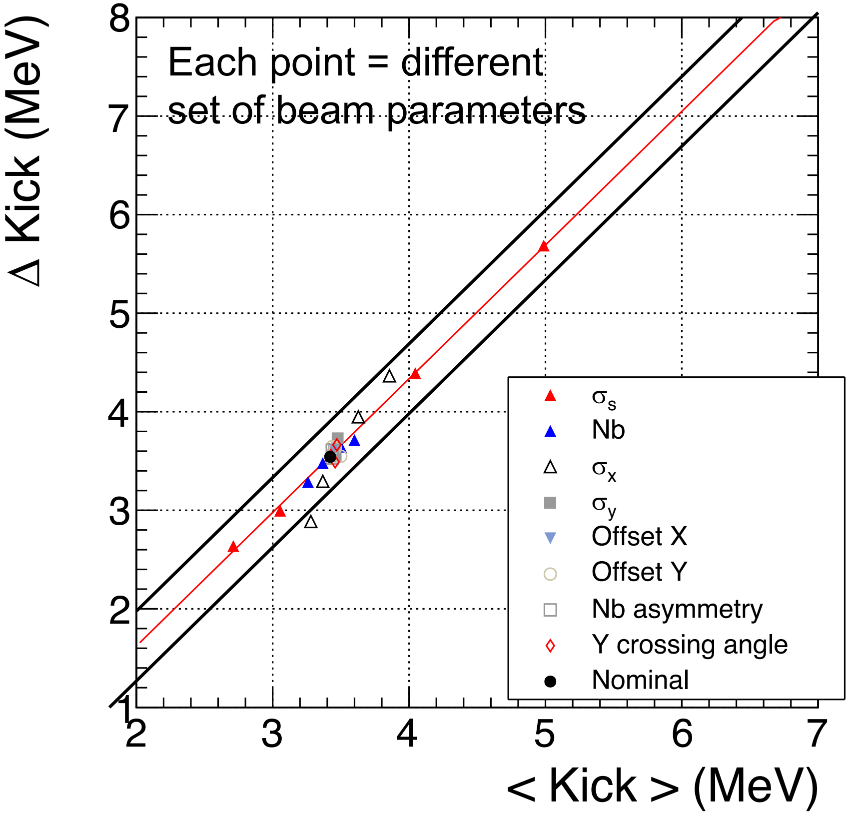
# "Central" vs "head" collisions




Vary some bunch parameters:

The difference between the maximum ( reached at z ~ 0 ) and the value at z = 1 $\sigma_z$ is well correlated with the average.

- linear behaviour




MC events (Guinea-Pig):
For each event, one has the time of the interaction.

Make three bins in the timing distribution:

- Head : time > $\sigma_t$
- Central : - $\sigma_t$ < t < $\sigma_t$
- Tail : time < - $\sigma_t$

ΔKick = Kick (central bin) – Kick (head bin).
Δkick ~ linear with the average kick.

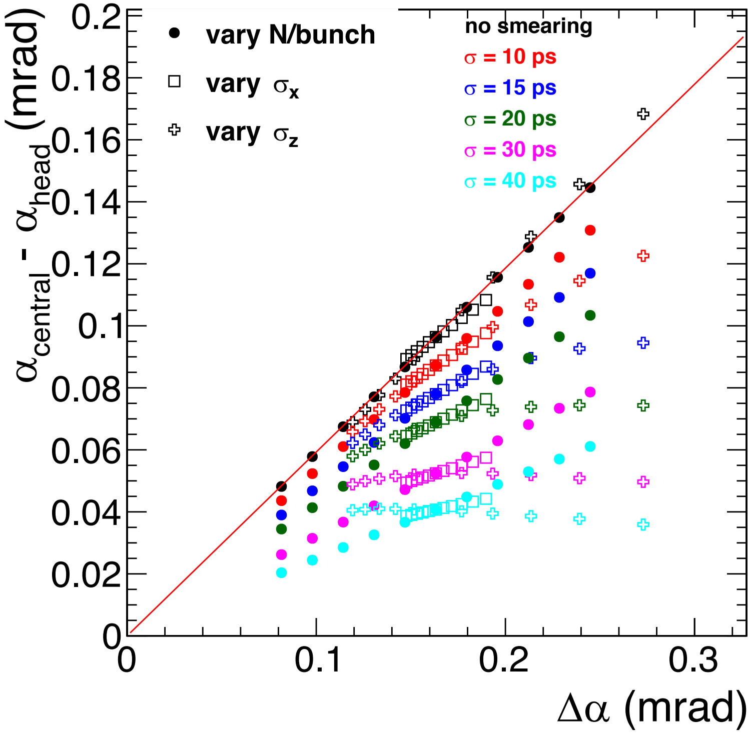
# Experimental sensitivity ?

Assume that we can bin the dimuon events according to the timing :

- Measure the effective crossing angle separately for the "head" and the "central" collisions
- $\alpha_{central} - \alpha_{head} = (\alpha_{central} - \alpha_0) - (\alpha_{head} - \alpha_0)$ = ΔKick of the previous slide
- From the correlation shown on the previous slide:
  - the measurement of $\alpha_{central} - \alpha_{head}$ (i.e. of Δkick) gives the average kick (i.e. the average $\Delta\alpha$, i.e. $\alpha_0$ )
- Can be a complementary check of the method described in the paper (does not require bunches with different intensities).

Plugging in some timing resolution:

- With a worse and worse resolution: $\alpha_{central} - \alpha_{head}$ decreases as expected (resolution washes out the difference)
- Effect of resolution is bad when $\sigma_z$ is decreased (expected)
- Apart for large $\sigma_z$ variations, linear correlation is maintained and $\alpha_{central} - \alpha_{head}$ remains large enough to be measured, even with a resolution of tens of ps.




May be worth pursuing a bit… ?

# Conclusions

- The crossing angle can be measured precisely from dimuon events.

- Beam-beam effects lead to an increase of the effective crossing angle.
    - This increase must be known, in order to exploit the very precise resonant depolarisation measurements of the energy of non-colliding bunches.

- The crossing angle increase can be measured in-situ using bunches of variable intensity
    - During the intensity ramp of each fill (filling period)
    - Or during stable collisions (thanks to top-up)

- Resulting uncertainties on $\Delta\alpha$, at the Z peak, lead to a few keV on $\delta( \sqrt{s} )$
    - A bit higher for off-peak points (88 and 94 GeV), but uncertainty O(30 keV) in the worst case

- Can not be used at higher energies (low dimu rate & faster filling)
    - But precision from BPM is enough (larger uncertainty from RPD)
    - Could anyway be calculated (numerical tools would have been calibrated at the Z)