Beam energy (spread) measurement with dimuons

WG4 – Status and Goals

- **WG4 = (EPOL) Measurements in particle-physics experiments**
  - **Status of**
    - Centre-of-mass energy absolute determination
    - Centre-of-mass energy relative determination (a.k.a. point-to-point)
    - Crossing angle and centre-of-mass energy spread
    - Longitudinal boost
    - Absolute angle determination
    - QED predictions
  - **Main goal of the workshop : Get new and young physicists interested in these studies**
    - Restart, reproduce, improve, and complete existing studies
    - Develop new studies to improve the precision
  - **Main goals of these studies**
    - Precision EW / Higgs / top measurements

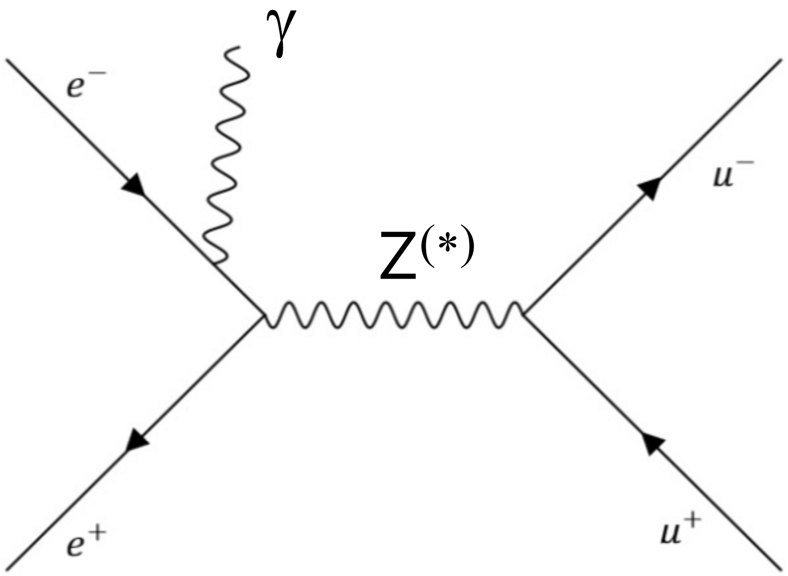
See my introductory slides on Monday afternoon

Status mostly unchanged since arXiv:1909.12245

In this talk

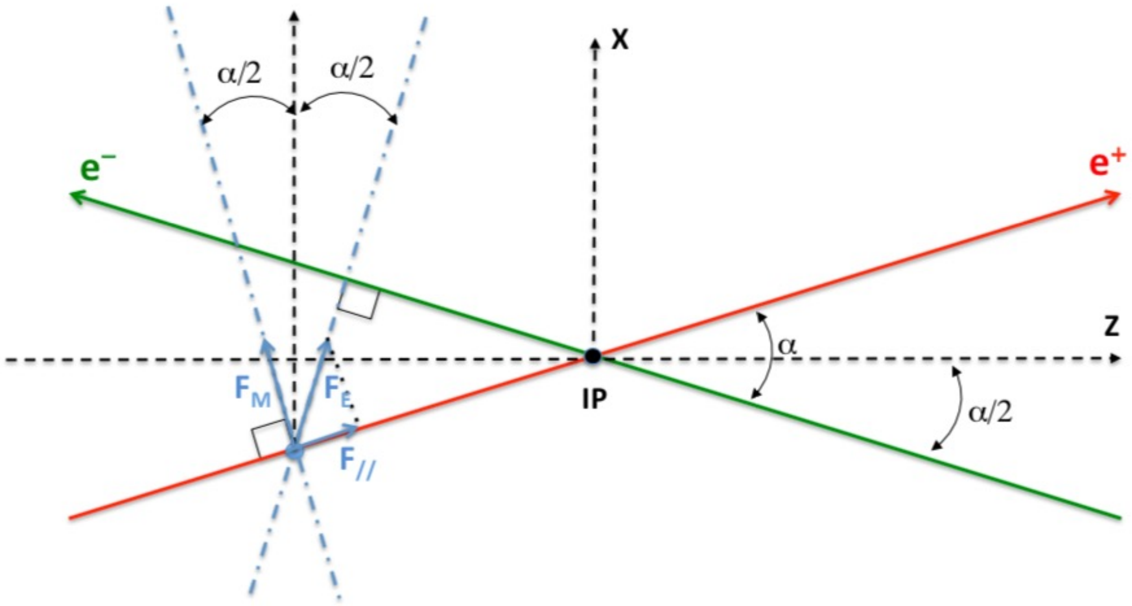
- **Step-by-step "tutorial" to determine**
  - The absolute centre-of-mass energy above the Z pole
  - The crossing angle
  - The centre-of-mass energy spread
  - The longitudinal boost (difference of energy between incoming electrons and positrons)
    - With $e^+e^- \to \mu^+\mu^- (\gamma)$ events
      - ➔ The ISR photon is emitted mostly along the beam direction
        - Assumption taken in all calculations
        - But in real life, it has an angular distribution
        - And there might be photons emitted from both beams
        - There might also be photons emitted by the muons (FSR)
      - ➔ The Z might be on-shell or off-shell
        - On-shell : a.k.a. radiative returns to the Z pole
        - On-shell: Can use the Z mass constraint to determine $\sqrt{s}$ with precision
      - ➔ The calculations that follow apply to on-shell and off-shell cases

ISR must be predicted with precision to account for these "deviations"



Centre-of-mass energy √s and crossing angle α

- **Beams cross at an angle α in the horizontal plane**
  - **The horizontal plane is defined as the plane subtended the two beams**



$$s = (p_e^+ + p_e^-)^2$$

$$e^+\left(E_e^+ \sin\frac{\alpha}{2}, 0, \quad E_e^+ \cos\frac{\alpha}{2}, E_e^+\right)$$

$$e^-\left(E_e^- \sin\frac{\alpha}{2}, 0, -E_e^- \cos\frac{\alpha}{2}, E_e^-\right)$$

⇩

$$\sqrt{s} = 2\sqrt{E_e^+ E_e^- \cos\frac{\alpha}{2}}$$

- **The z axis is the bisector of the two beam directions**
- **The y axis is perpendicular to the (x,z) plane**
  - → Polar angle θ defined wrt the z axis
  - → Azimuthal angle φ defined in the (x,y) plane

with $E_e^\pm = E(1 \pm \epsilon)$, $\sqrt{s} = 2E\sqrt{1-\epsilon^2}\cos\frac{\alpha}{2}$

Absolute $\sqrt{s}$ determination with $e^+e^- \to Z\gamma$

- **Four energy-momentum equation (p_z, p_y, p_x, E) conservation**

$$
\begin{aligned}
E^+\cos\theta^+ \quad + E^-\cos\theta^- \quad + p_z^\gamma &= (E_e^+ - E_e^-)\cos\alpha/2 &&= \varepsilon\sqrt{s/1-\varepsilon^2} \\
E^+\sin\theta^+\sin\varphi^+ + E^-\sin\theta^-\sin\varphi^- &= 0, &&= 0 \\
E^+\sin\theta^+\cos\varphi^+ + E^-\sin\theta^-\cos\varphi^- + |p_z^\gamma|\tan\alpha/2 &= (E_e^+ + E_e^-)\sin\alpha/2, &&= \sqrt{s/1-\varepsilon^2}\tan\alpha/2 \\
E^+ \quad + E^- \quad + |p_z^\gamma|/\cos\alpha/2 &= E_e^+ + E_e^-, &&= \sqrt{s/1-\varepsilon^2}/\cos\alpha/2
\end{aligned}
$$

- **One mass constraint : the Z mass**
  - Well measured at the Z pole (~100 keV)
- **Four very-well measured $\mu^\pm$ angles**
  - $\theta^+, \theta^-, \varphi^+, \varphi^-$
- **Six unknowns (or less well measured momenta)**
  - p_z^γ, ε, E^+, E^-, α, $\sqrt{s}$
    - One unknown too many for the four equations and the Z mass constraints
      - ➔ But p_z^γ and ε cannot be determined independently from each other – only $p_z^\gamma - \varepsilon\sqrt{s}/\sqrt{1-\epsilon^2}$ can

Let's start with $\varepsilon = 0$ in the next slide, and see where it brings

Solving the conservation equations with $\varepsilon = 0$

$$\begin{aligned}
E^+ \cos\theta^+ \quad + E^- \cos\theta^- \quad + p_z^\gamma &= 0\\
E^+ \sin\theta^+ \sin\varphi^+ + E^- \sin\theta^- \sin\varphi^- &= 0,\\
E^+ \sin\theta^+ \cos\varphi^+ + E^- \sin\theta^- \cos\varphi^- + |p_z^\gamma| \tan\alpha/2 &= \sqrt{s}\tan\alpha/2,\\
E^+ \quad + E^- \quad + |p_z^\gamma|/\cos\alpha/2 &= \sqrt{s}/\cos\alpha/2,
\end{aligned}$$

- Eliminate p_z^γ and $\sqrt{s}$ from the last two equations (p_x and E)

$$E^+(\sin\theta^+\cos\varphi^+ - \sin\alpha/2) = -E^-(\sin\theta^-\cos\varphi^- - \sin\alpha/2)$$

- Rewrite the p_y conservation equation

$$E^+\sin\theta^+\sin\varphi^+ = -E^-\sin\theta^-\sin\varphi^-$$

- Eliminate E^+ and E^- in the ratio, and get the crossing angle from the muon angles

$$\boxed{\alpha = 2\arcsin\left[\frac{\sin(\varphi^- - \varphi^+)\sin\theta^+\sin\theta^-}{\sin\varphi^-\sin\theta^- - \sin\varphi^+\sin\theta^+}\right]}$$

  - This result holds even for $\varepsilon \neq 0$ (substituting $\sqrt{s}\,/\,\sqrt{1-\epsilon^2}$ for $\sqrt{s}$ in the last two equations)

Solving the conservation equations with $\varepsilon = 0$

- **Define reduced muon energies $x_\pm$**

$$x_\pm = E^\pm \cos(\alpha/2)/(\sqrt{s} - |p_Z^\gamma|)$$

- **Rewrite p_y and E conservation equations accordingly**

$$\begin{aligned} x_+ \sin\theta^+ \sin\varphi^+ + x_- \sin\theta^- \sin\varphi^- &= 0 \\ x_+ \qquad\qquad\qquad + x_- &= 1 \end{aligned}$$

- **Solve for $x_\pm$**

$$\boxed{x_\pm = \frac{\mp \sin\theta^\mp \sin\varphi^\mp}{\sin\theta^+ \sin\varphi^+ - \sin\theta^- \sin\varphi^-}}$$

  - Again, this result holds for $\varepsilon \neq 0$ (substituting $\sqrt{s}$ / $\sqrt{1-\epsilon^2}$ for $\sqrt{s}$ in the last equation)

Solving the conservation equations with $\varepsilon = 0$

- We are left with the p_z conservation equation

- Define the reduced ISR photon energy x_γ as

$$x_\gamma = p_z^\gamma / \sqrt{s}$$

- Rewrite the p_z conservation equation as a function of the reduced variables

$$x_+ \cos\theta^+ + x_- \cos\theta^- + \frac{x_\gamma \cos\alpha/2}{1 - |x_\gamma|} = 0$$

- And solve it for x_γ as a function of the sole muon angles

$$\boxed{x_\gamma = -\frac{x_+ \cos\theta^+ + x_- \cos\theta^-}{\cos(\alpha/2) + |x_+ \cos\theta^+ + x_- \cos\theta^-|}}$$

with

$$x_\pm = \frac{\mp \sin\theta^\mp \sin\varphi^\mp}{\sin\theta^+ \sin\varphi^+ - \sin\theta^- \sin\varphi^-}$$

$$\alpha = 2 \arcsin\left[\frac{\sin(\varphi^- - \varphi^+) \sin\theta^+ \sin\theta^-}{\sin\varphi^- \sin\theta^- - \sin\varphi^+ \sin\theta^+}\right]$$

- This result does NOT hold for $\varepsilon \neq 0$

And for $\varepsilon \neq 0$?

- **Substituting $\sqrt{s}$ / $\sqrt{1-\epsilon^2}$ for $\sqrt{s}$ in the definition of the reduced variables**
  - The p_z conservation equation becomes a little more involved

$$x_+ \cos\theta^+ + x_- \cos\theta^- + \frac{(x_\gamma - \varepsilon)\cos\alpha/2}{1-|x_\gamma|} = 0$$

  - Cannot solve for x_γ and ε independently …
    - The best that I could come up with so far (but I started this morning)

$$\frac{x_\gamma - \varepsilon}{1 \pm \varepsilon} = -\frac{x_+ \cos\theta^+ + x_- \cos\theta^-}{\cos(\alpha/2) \pm (x_+ \cos\theta^+ + x_- \cos\theta^-)} \quad \text{where} \quad \pm = \mathrm{sgn}(x_\gamma)$$

➔ More work needed to make it user friendly and actually useable: Take it up!

  - In 1st approximation, the previous-slde formula gives a symmetrized sum of ISR and boost
    - With small correction(s) to be worked out

(mistakes not 100% excluded from this slide!)

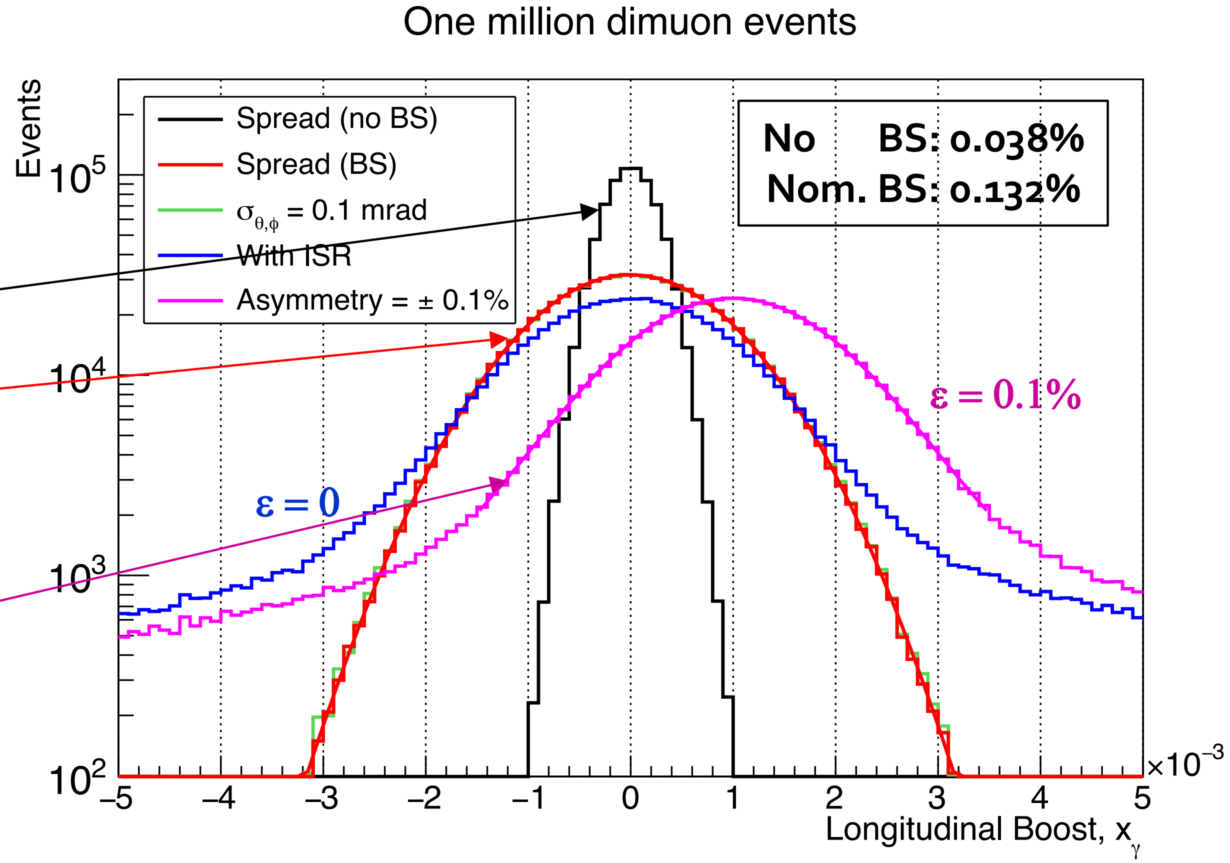
Example: √s spread and average boost

- Resulting distributions of x_γ, for 10^6 dimuon events (every 5 minutes at the Z pole)
  - Solved with $\varepsilon = 0$

$$\sigma_\delta = (0.03804 \pm 0.00004)\%$$

$$\sigma_\delta = (0.13185 \pm 0.00011)\%$$

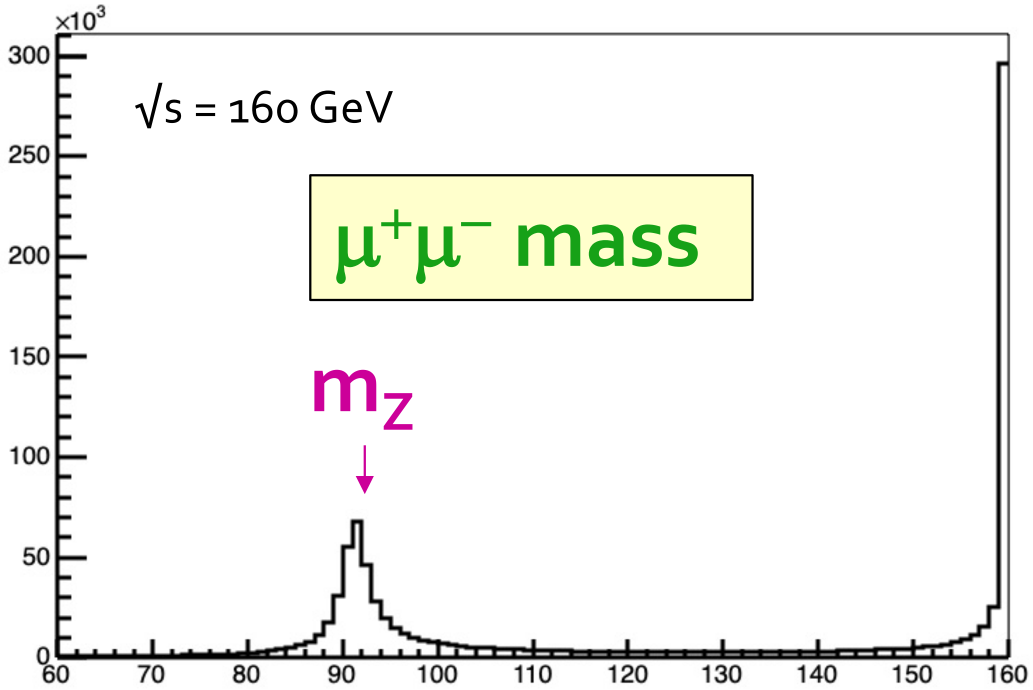
$$\langle x_\gamma \rangle = (0.9991 \pm 0.0015) \times 10^{-3}.$$



- The distribution of x_γ contains ISR + √s spread + muon angular resolution
  - Interesting project: solve the equations with $\varepsilon \neq 0$ (even numerically) to make them useful

Back to: Absolute $\sqrt{s}$ determination with $e^+e^- \rightarrow Z\gamma$

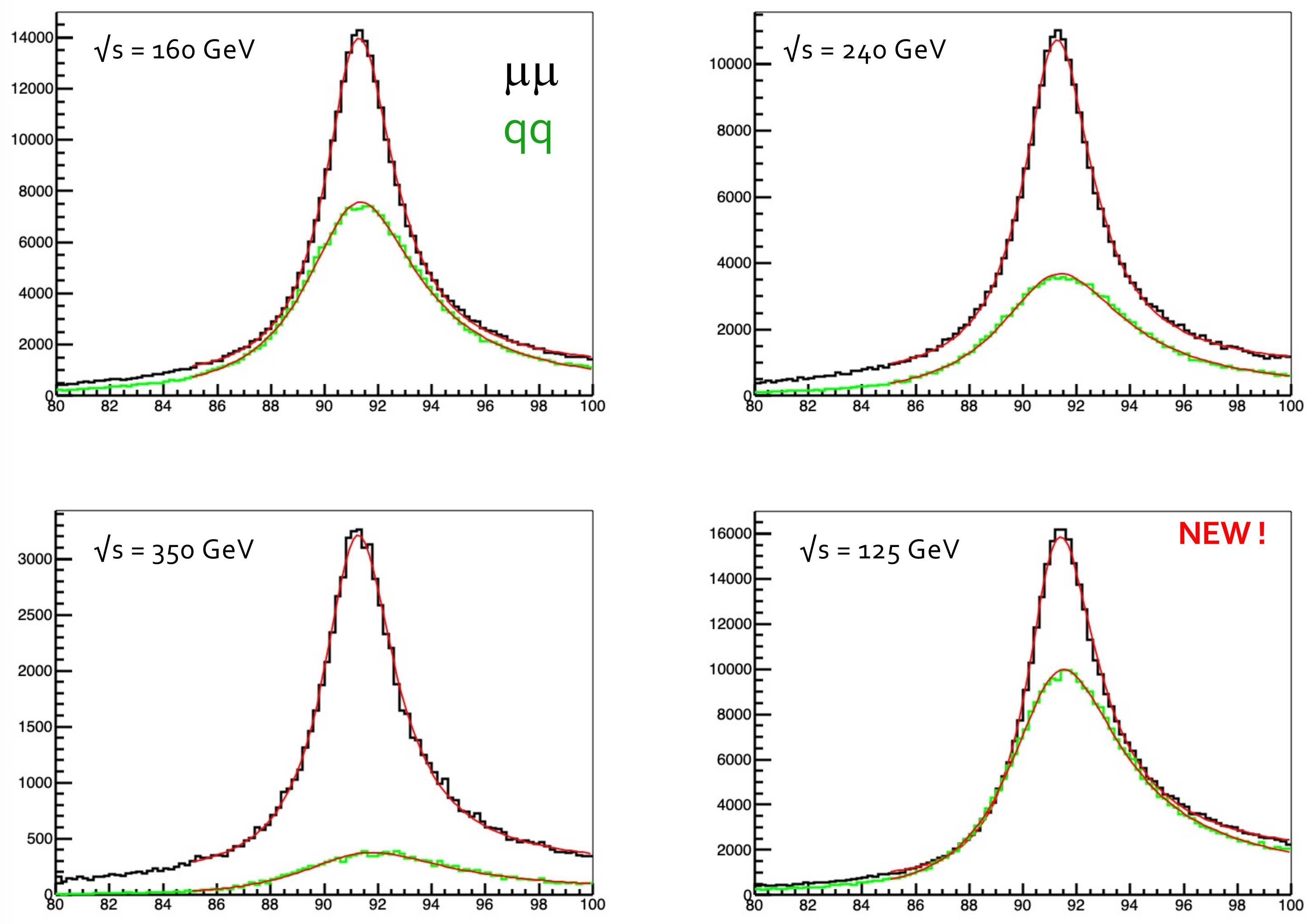
- **Above the Z pole : $\sqrt{s} = m_H, 2m_W$, 240 GeV, $2m_{top}$, 365 GeV**
  - The radiated photon energy is large, from 29 to 171 GeV: $E_\gamma \sim \sqrt{s}/2 \times (1 - m_Z^2/s)$
    - And x_γ varies from 0.23 to 0.47 : can safely ignore the boost due to $\sqrt{s}$ spread
  - Distribution of the true dimuon mass (from the generator, no FSR) at $\sqrt{s}$ = 160 GeV



- $M_{\mu\mu} / \sqrt{s} = (1 - 2\,x_\gamma)^{1/2}$ with $\varepsilon = 0$: A fit of the dimuon mass distribution gives the $\sqrt{s}$ rescaling factor

Absolute $\sqrt{s}$ determination with $e^+e^- \to Z\gamma$

- **Examples of distributions (10^6 events)**
  - Dileptons
    - p > 10 GeV
    - 10 < θ < 170 degrees
  - Dijets
    - E > 20 GeV
    - 25 < θ < 165 degrees
  - Fit to a Breit-Wigner
    - × 2nd order polynomial



Absolute $\sqrt{s}$ determination with $e^+e^- \to Z\gamma$

- First estimates for precision on the average $\sqrt{s}$

	$\sqrt{s}$	E_γ (GeV)	$N_{\mu\mu}$ ($\times 10^6$)	N_{qq} ($\times 10^6$)	$\sigma_{\sqrt{s}}$ ($\mu\mu$)	$\sigma_{\sqrt{s}}$ (qq)	$\sigma_{\sqrt{s}}$ (comb.)	$\sigma_{\sqrt{s}}$ (EPOL)	
6 ab^{-1}	m_H	29	107	173	660 keV	280 keV	225 keV	200 keV ?	NEW !
12 ab^{-1}	$2m_W$	54	47	667	900 keV	340 keV	285 keV	300 keV	
5 ab^{-1}	240 GeV	102	5.6	53	4.2 MeV	2.4 MeV	1.7 MeV	–	
0.2 ab^{-1}	$2m_{top}$	163	0.1	0.3	51 MeV	60 MeV	26 MeV	–	

- Bonus: RDP available at the H resonance / WW threshold, with similar precision
  - RDP can be used to calibrate / validate the radiative return method

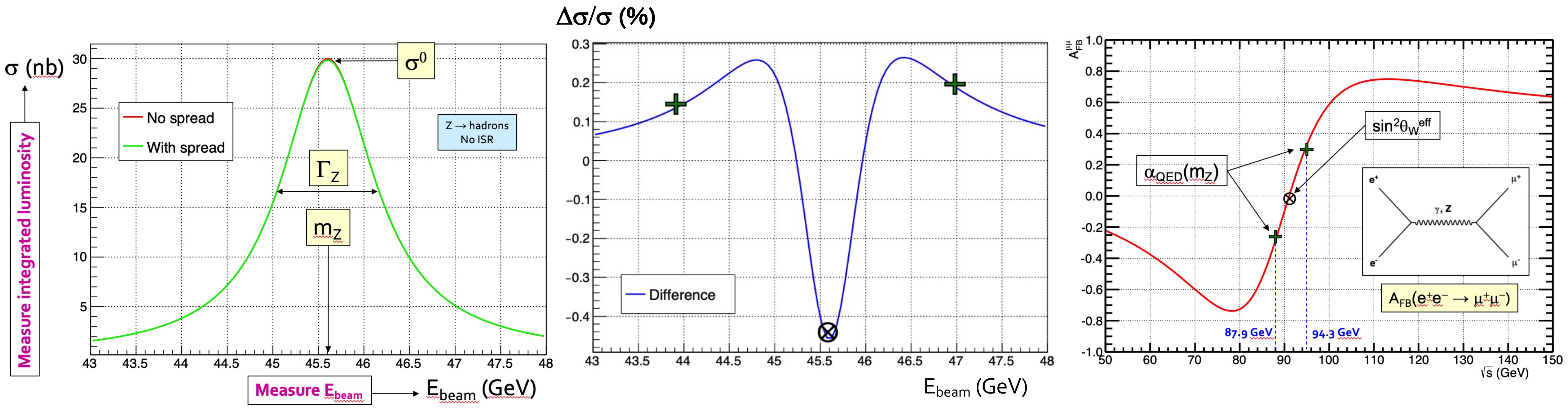
- All reported numbers obtained with
  - A home-made event generator, including home-made ISR generation (no FSR)
  - Gaussian smearing of muon / quarks momenta/energies and angles
  - Standalone analysis code / only statistical uncertainties / no systematic studies (ISR!)
  - Everything should be done professionally now

Improving the √s precision at 350 GeV

- At the top-pair threshold, can use the 2 million WW events (+E,p conservation)
  - With known √s, can be used to measure the W mass with a statistical precision of 2.2 MeV
    - And even 1.1 MeV with the fully hadronic final state
      - ➔ See Marina Béguin's thesis : https://tel.archives-ouvertes.fr/tel-02490574
  - Alternatively, with a known m_W (from the threshold measurement)
    - Can be used to measure √s with a precision of 10 MeV (5 MeV)
      - ➔ Which translates to a top mass systematic uncertainty of 5 MeV (2.5 MeV)
  - According to Marina's thesis, the W mass is best measured at the WW threshold
    - With the cross-section lineshape (all final states used)
    - With direct reconstruction (from the lepton momentum with the semi-leptonic final state)
      - ➔ Study performed with DELPHES, and its CLD parameterization
  - The colour reconnection effects in the fully hadronic final state should be controllable (?)
    - To better than 1 MeV with 100 million WW events collected at √s = 240 GeV
      - ➔ Maybe also use ZZ events in the fully hadronic final state + knowledge of the Z mass?
  - Project: Repeat, cross check and improve Marina's analysis
    - For √s at the top-pair threshold ; for mW at 160 and 240 GeV;

Back to: √s spread

- √s spread strongly affects √s-dependent observables (e.g., Z, W top widths, α_{QED})
  - Must therefore be measured with adequate precision
    - So that related uncertainty be smaller than the expected statistical precision



- If not attended, the centre-of-mass energy spread:
  - Increases Γ_Z, reduces σ^0, increases A_{FB}(87.9 GeV), decreases A_{FB}(94.3 GeV)

Back to: √s spread

- Extract from the conclusions of arXiv:1909.12245

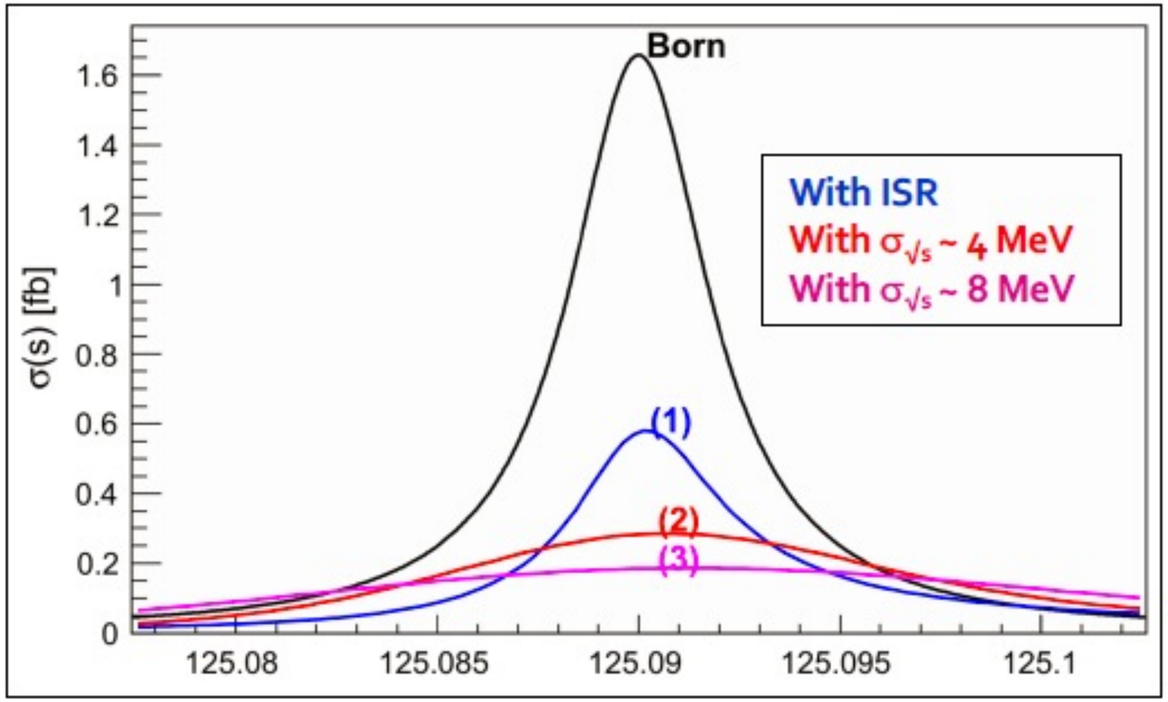
To be revised

Pseudo Observable	Γ_Z			$\alpha_{QED}(m_Z^2)$		Γ_W	Γ_{top}
Acceptable error	35 keV			10^{-5}		0.5 MeV	18 MeV
$\sqrt{s}$ (GeV)	87.9	91.2	93.8	87.9	93.8	161	350
$\sigma(\delta E)/\delta E$	0.8%	0.2%	0.8%	0.7%		11%	35%
$N_{e^+e^-\to\mu^+\mu^-}$	$5\,10^4$	$8\,10^5$	$5\,10^4$	$6.5\,10^4$		260	25
L (10^{34} cm^{-2}s^{-1})	230					28	1.8
$\sigma_{\mu\mu}$ (pb)	185	1450	460	185	460	4.0	0.8
Dimuon rate (Hz)	425	3325	1050	425	1050	1.1	0.015
Time needed	2 min	4 min	< 1 min	3 min	1 min	4 min	30 min

- All these numbers ought to be checked with professional analysis (gen, sim, reco, ana)
- A column for the Higgs direct production at the H pole ($\sqrt{s} = m_H$) must be added

Back to: √s spread

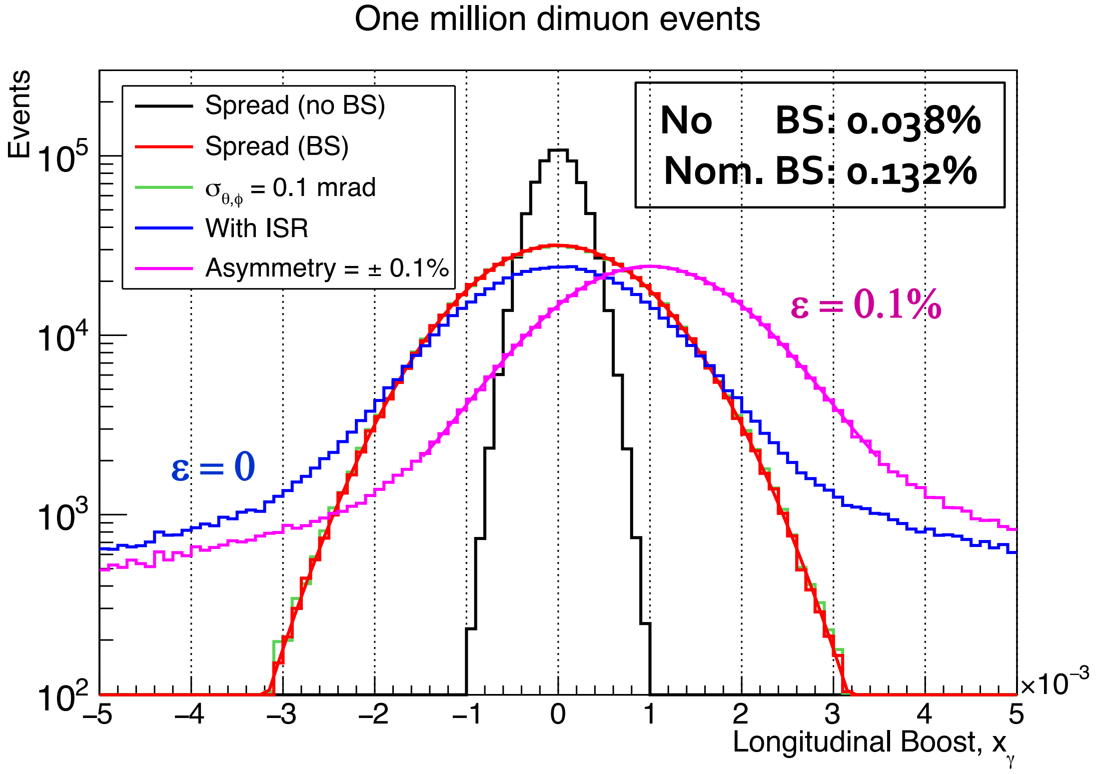
- √s spread strongly affects √s-dependent observables (e.g., $e^+e^- \to H$ peak XS)



- Understand with what precision √s can be measured in situ
- Understand with what precision the energy spread must be measured
  - And check if there are enough dimuon (dilepton) events to provide this precision regularly

Back to: √s spread

- In real life, we'll deal with a distribution that gathers many effects
  - ISR, angular resolution, √s spread, longitudinal boost
  - Need to disentangle all these effects
    - Master the boost impact
      - ➔ Analytically (best) or numerically
    - Predict ISR with adequate precision
      - ➔ Improvements needed ?
    - Improve statistical uncertainty
      - ➔ All lepton species ?
    - Map angular resolution from data
      - ➔ What precision is needed ?
      - ➔ What mapping is needed ?
      - ➔ What resolution is needed ?
  - Need high-quality generation / simulation



A lot of work ahead !

- But also a lot of fun (speaking from experience)
  - And a possibility for many single-author publications

- IMPORTANT ! A tutorial is foreseen on Thursday afternoon (Marcin Chrząszcz)
  - Learn how to generate, simulate, analyse dimuon events and more in FCCSW
    - Come with your computer !
  - And apply what you have learnt to determine $\sqrt{s}$, spread, boost, angles, axes, etc.

FCC-ee precision measurements

- Strong √s dependence at all centre-of-mass energies!

Table from arXiv:2106.13885

Or is it 0.15 MeV?

	Observable	present value ± error	FCC-ee Stat.	FCC-ee Syst.	Comment and leading exp. error	
√s	m_Z (keV)	91186700 ± 2200	**4**	100	From Z line shape scan Beam energy calibration	$\sqrt{s} \sim m_Z$
Spread	Γ_Z (keV)	2495200 ± 2300	**4**	25	From Z line shape scan Beam energy calibration	
√s	$\sin^2\theta_W^{\rm eff} (\times 10^6)$	231480 ± 160	**2**	2.4	from $A_{FB}^{\mu\mu}$ at Z peak Beam energy calibration	
Spread	$1/\alpha_{QED}(m_Z^2)(\times 10^3)$	128952 ± 14	**3**	small	from $A_{FB}^{\mu\mu}$ off peak QED&EW errors dominate	
√s	m_W (MeV)	80350 ± 15	**0.25**	0.3	From WW threshold scan Beam energy calibration	$\sqrt{s} \sim 2m_W$
Spread	Γ_W (MeV)	2085 ± 42	**1.2**	0.3	From WW threshold scan Beam energy calibration	
√s	m_H (MeV)	125250 ± 170	**2.5**	0.8	From ZH direct reconstruction $\sqrt{s}$ calibration	$\sqrt{s} \sim 240$ GeV
√s	m_{top} (MeV)	172740 ± 500	**17**	small	From $t\bar{t}$ threshold scan QCD errors dominate	$\sqrt{s} \sim 2\, m_{top}$