EIC ESR Tracking Studies

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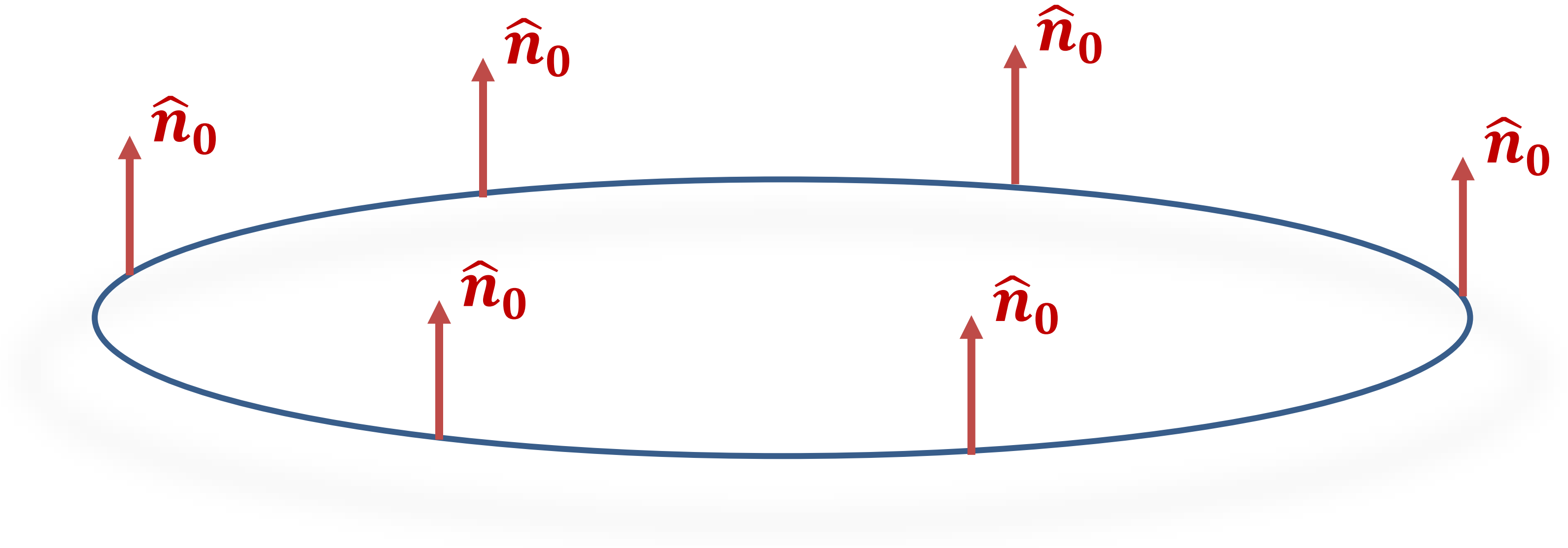
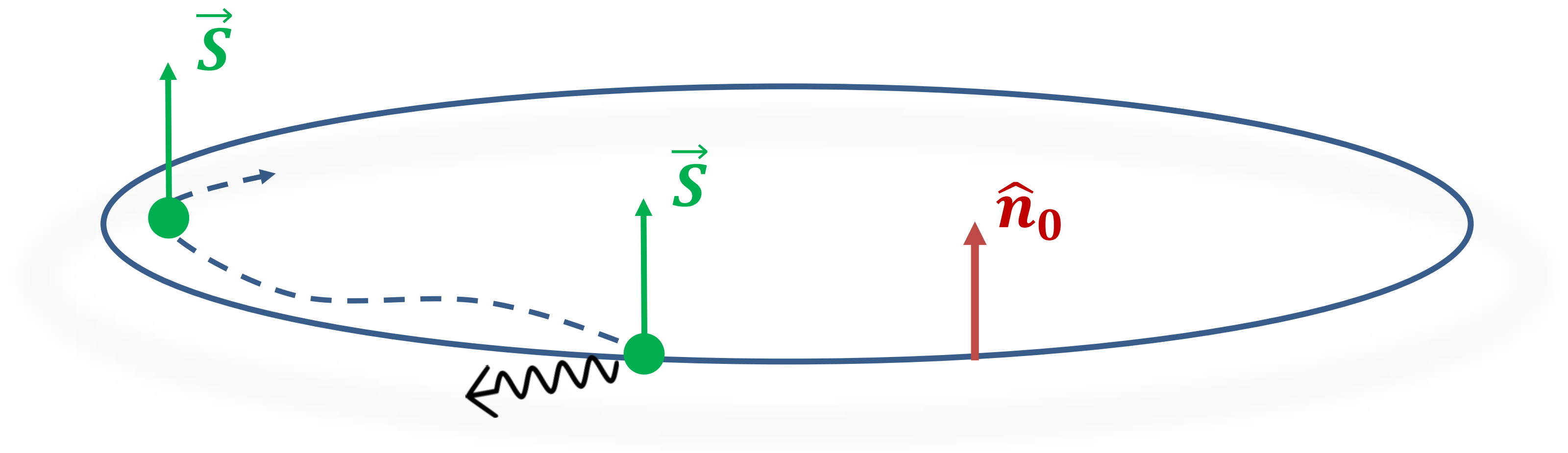
Electron Polarization in a Storage Ring



- Periodic spin direction $\hat{n}_0$

From [1-8]

Electron Polarization in a Storage Ring

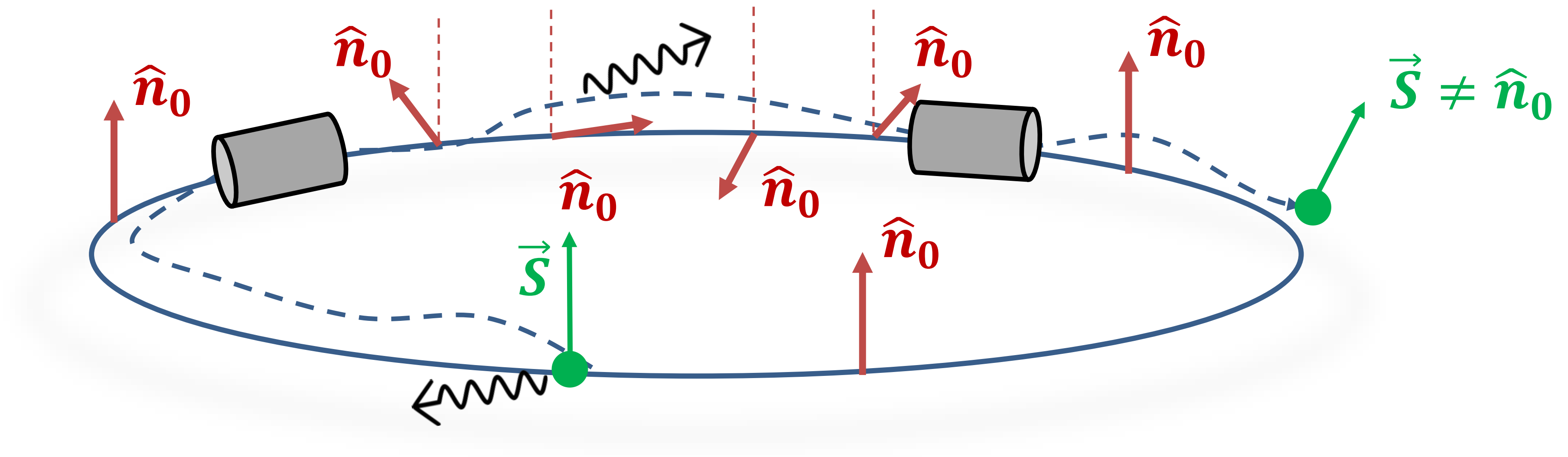


- No depolarizing effects of radiation in perfectly planar ring

From [1-8]

Background

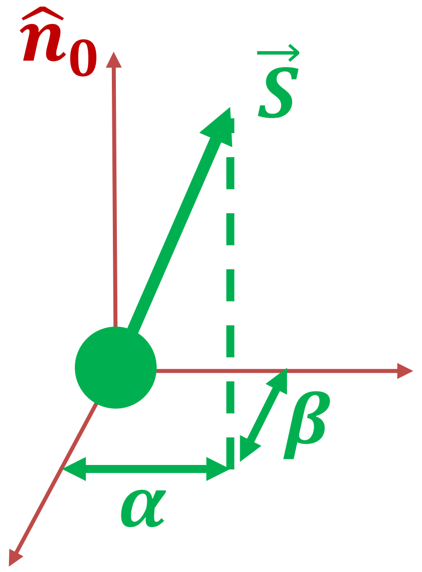
Electron Polarization in a Storage Ring



- “Spin diffusion”

From [1-8]

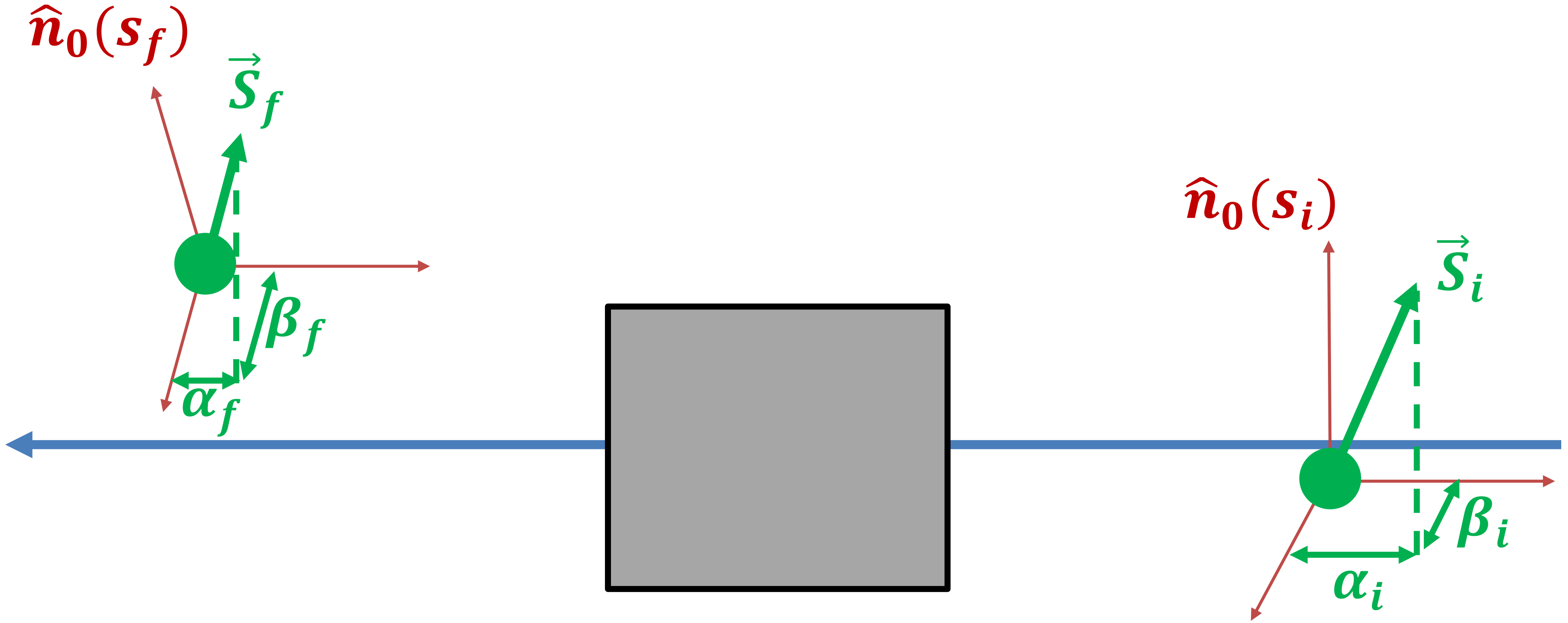
Electron Polarization in a Storage Ring



From [1-8]

Background

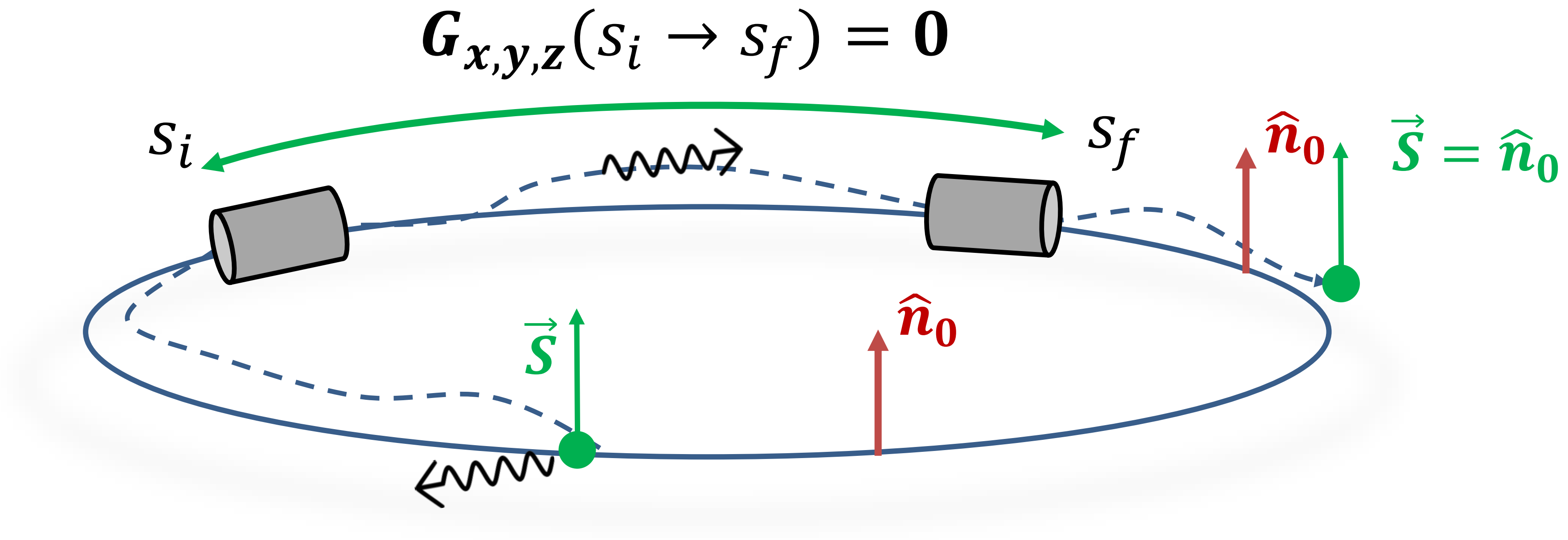
Electron Polarization in a Storage Ring



$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_f = \boldsymbol{G_x}\begin{pmatrix}x\\ p_x\end{pmatrix}_i + \boldsymbol{G_y}\begin{pmatrix}y\\ p_y\end{pmatrix}_i + \boldsymbol{G_z}\begin{pmatrix}z\\ p_z\end{pmatrix}_i + \boldsymbol{D}\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_i$$

From [1-8]

Electron Polarization in a Storage Ring



- “Spin matching”

See [9] for more details.

From [1-8]

Background

Electron Polarization in a Storage Ring

$$P(t) = P_\infty\left(1 - e^{-t/\tau_{eq}}\right) + P_0 e^{-t/\tau_{eq}}$$

$$\tau_{eq}^{-1} = \tau_{pol}^{-1} + \tau_{dep}^{-1}$$

✓ Can be accurately approximated from the closed orbit with analytical formulas

× Hard to estimate analytically. May be affected significantly by nonlinearities

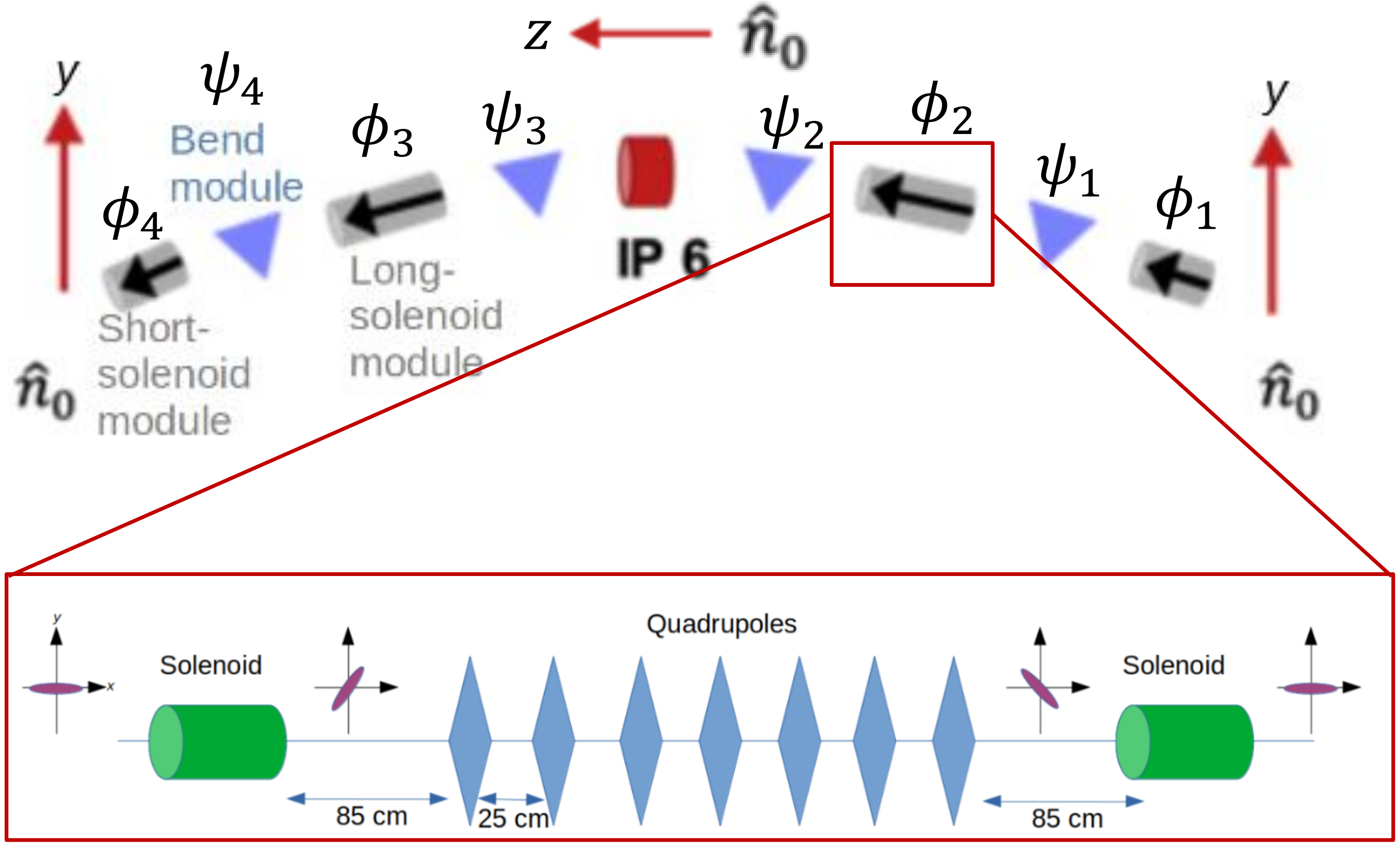
To estimate τ_{dep}^{-1}, do Monte Carlo tracking with *only* spin diffusion effects

$$P_{tr}(t) = P_0 e^{-t/\tau_{dep}} \approx P_0 - t/\tau_{dep}$$

From [1-8]

Background

EIC-ESR Spin Rotator System

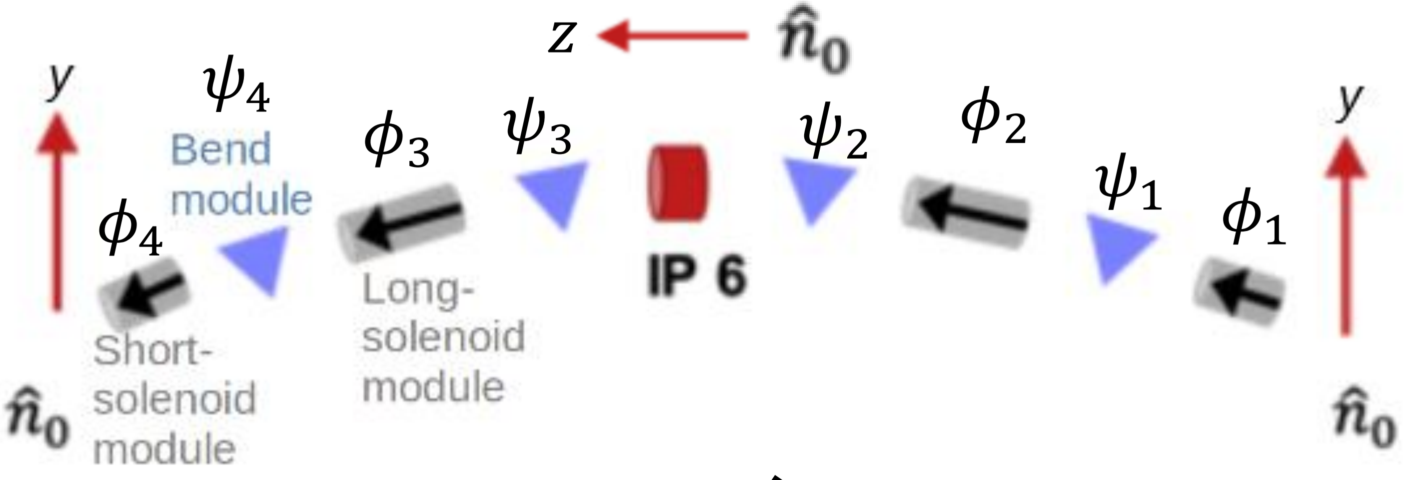


- Rotates $\hat{n}_0$ to longitudinal at the IP for a wide range of e-beam energies (5-18 GeV)

Images from [10]

Background

EIC-ESR Spin Matching Conditions



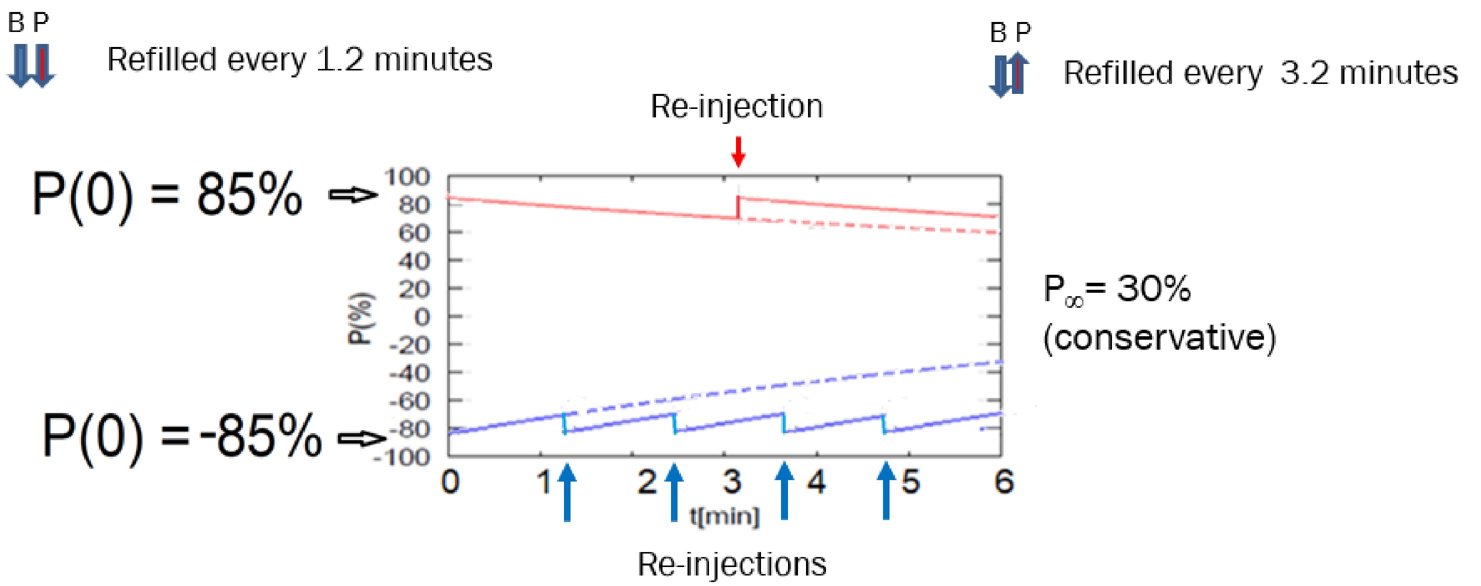
$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_f = \boldsymbol{G_x}\begin{pmatrix}x\\ p_x\end{pmatrix}_i + \boldsymbol{G_y}\underbrace{\begin{pmatrix}y\\ p_y\end{pmatrix}_i}_{0} + \boldsymbol{G_z}\begin{pmatrix}z\\ p_z\end{pmatrix}_i + \boldsymbol{D}\underbrace{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}_i}_{0}$$

$\boldsymbol{G_x} = \boldsymbol{0}$ by conditions on quads between solenoids

$\boldsymbol{G_z} = \boldsymbol{0}$ by conditions on periodic dispersion η, η'

Images from [10], ESR spin matching conditions in [11] or [9].

EIC-ESR Polarization Requirements



- Maintain average polarization of at least 70%
- Bunches should be replaced on average every 2.2 minutes
- For $P_\infty = 30\%$, need $\tau_{eq} > 4.6$ min

Images from [10]

Motivation for Tracking Studies

- Most accurate statistics including all nonlinearities
- Verify effects/significance of first-order spin matching
- Must cross-check between various modern codes
- Verify polarization robustness (i.e. with misalignments, ϵ_y-creator)

Methods

Monte-Carlo Spin Tracking Methods with Radiation

- **Map Tracking –** damped maps generated between each bend center (radiation points*) by PTC w/ user-specified order
- **Bmad Tracking –** element-by-element damped nonlinear maps w/ radiation points after each element
- **PTC Tracking –** element-by-element symplectic integration w/ radiation points at each step within the element
- Bmad toolkit conveniently implements all the above tracking methods and can be run in parallel on a GPU cluster

**bend-splitting for radiation – "SLICKTRACK" – is necessary for accurate spin tracking*

Methods

ESR 18 GeV Lattice Tracking Studies

- **v5.3: $G_x = 0,\ G_z = 0, \eta, \eta' \neq 0$ in solenoid modules**
  - 1IP

- **v5.6: $G_x = 0,\ G_z \neq 0, \eta, \eta' = 0$ in solenoid modules**
  - 1IP
  - ϵ_y-creator
  - 2IP

All trackings started with 5,000–10,000 particle distribution generated from analytical equilibrium $\epsilon_a, \epsilon_b, \epsilon_c$

v5.3 Results

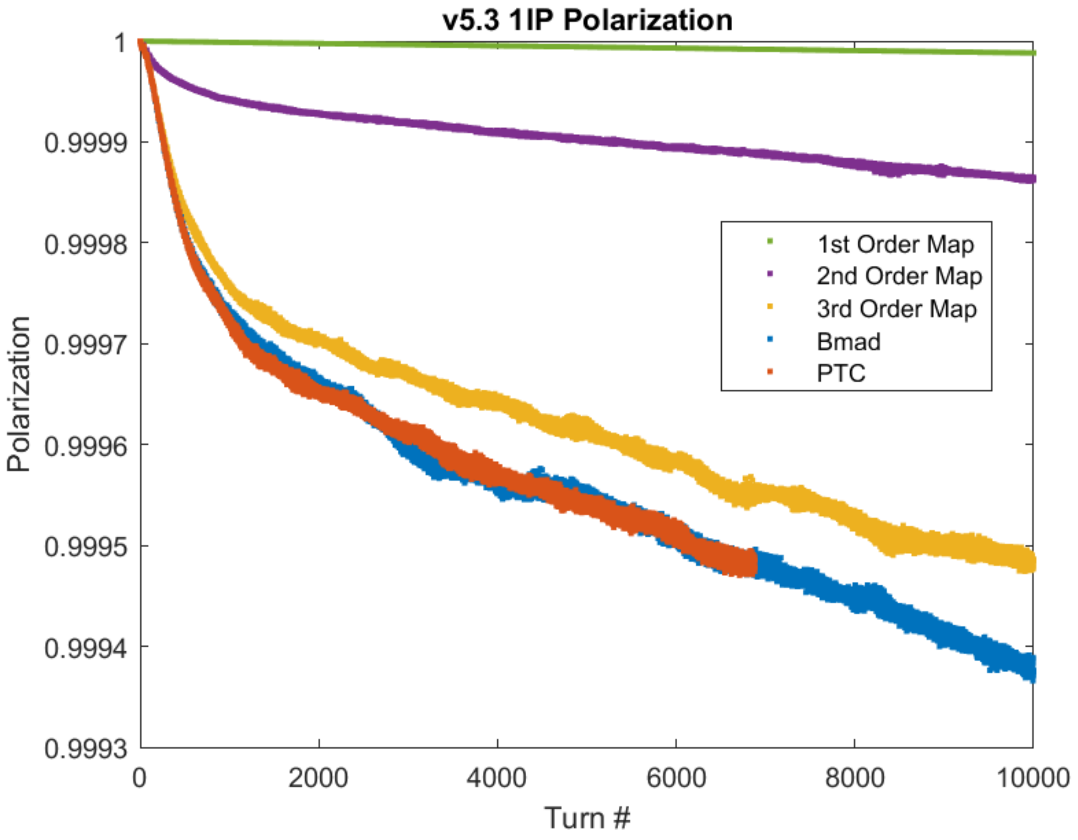
$G_x = 0, \;\; G_z = 0$

$\eta, \eta' \neq 0$ in solenoid modules

Results – v5.3

Polarization

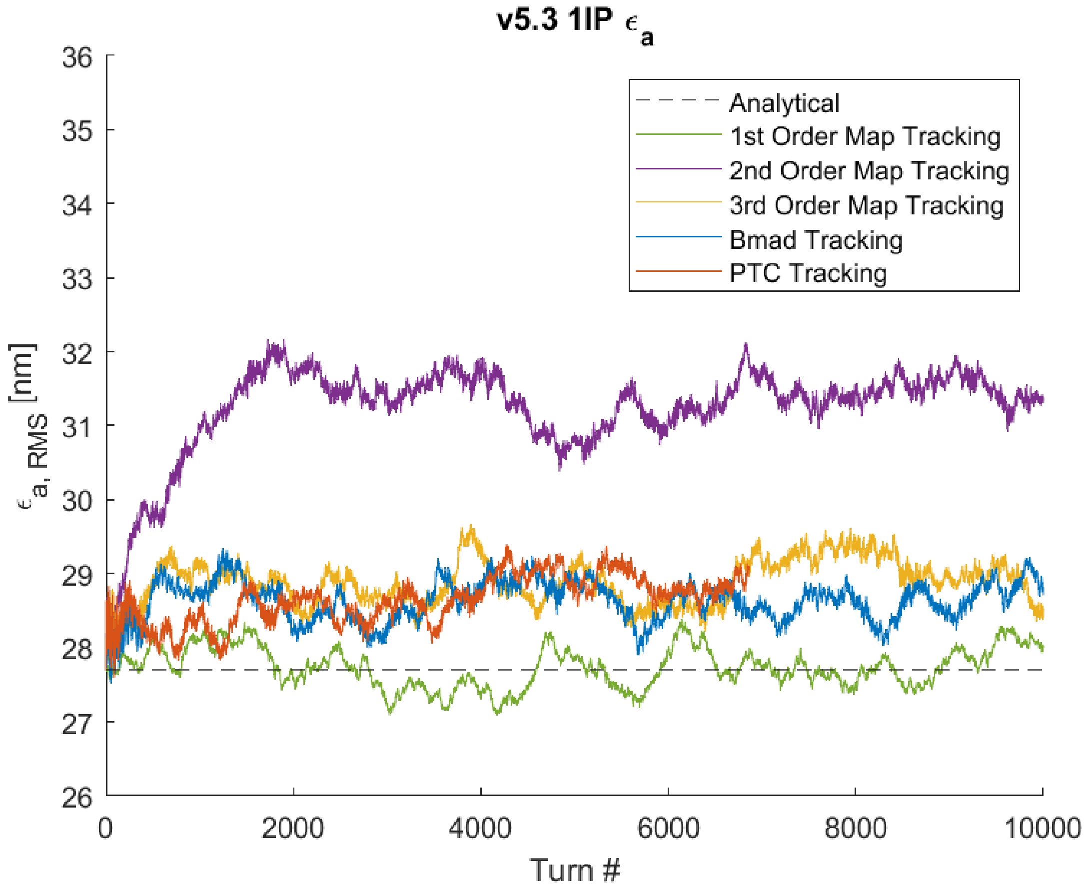
	v5.3 1IP	
	τ_{eq} [min]	P_∞
Analytical	31.1	61.3%
1st Order Map Tracking	30.7	66.4%
2nd Order Map Tracking	15.7	33.8%
3rd Order Map Tracking	6.5	14.0%
Bmad Tracking	5.6	12.1%
PTC Tracking	5.7	12.3%

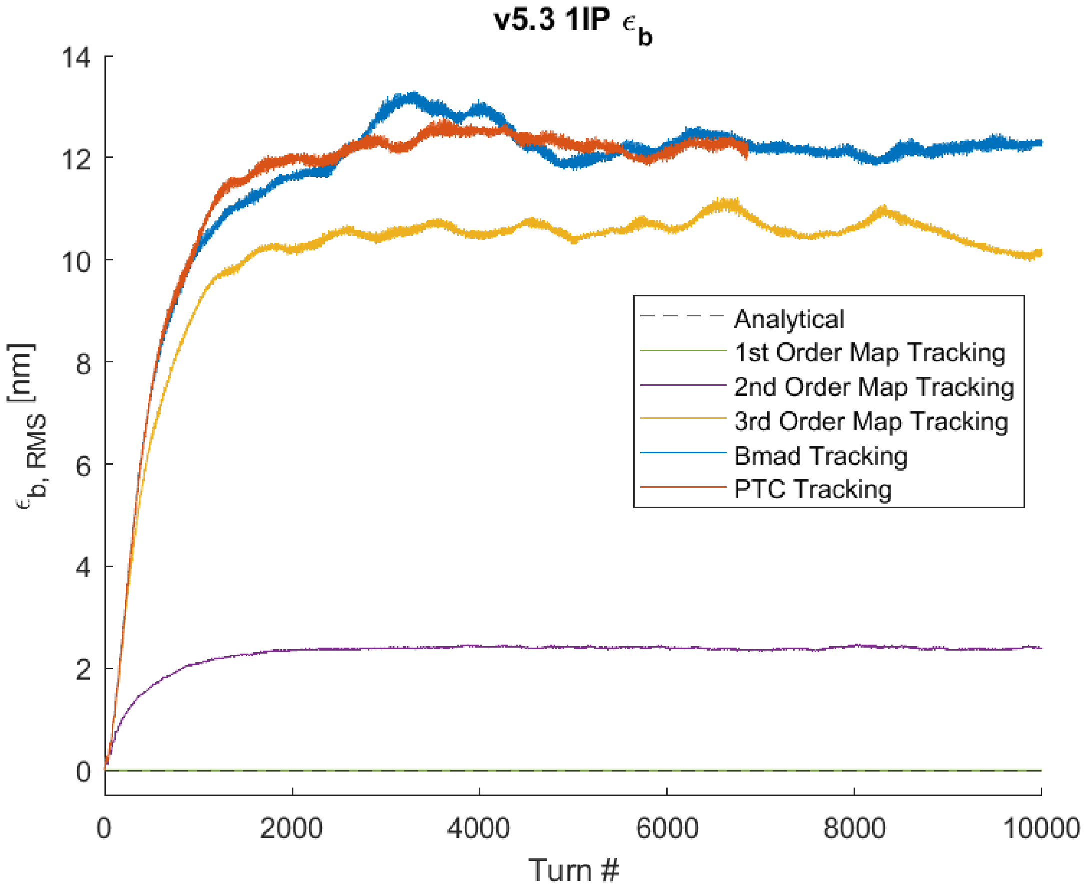


- Polarization significantly worse in nonlinear case
- Such significant damping should not occur if starting w/ equilibrium distribution. Is this a clue on what's happening?

Results – v5.3

Turn-by-turn RMS emittances





Nonlinearities might be driving tails of y-distribution to high amplitudes
→ **Core emittance** likely a better measure…

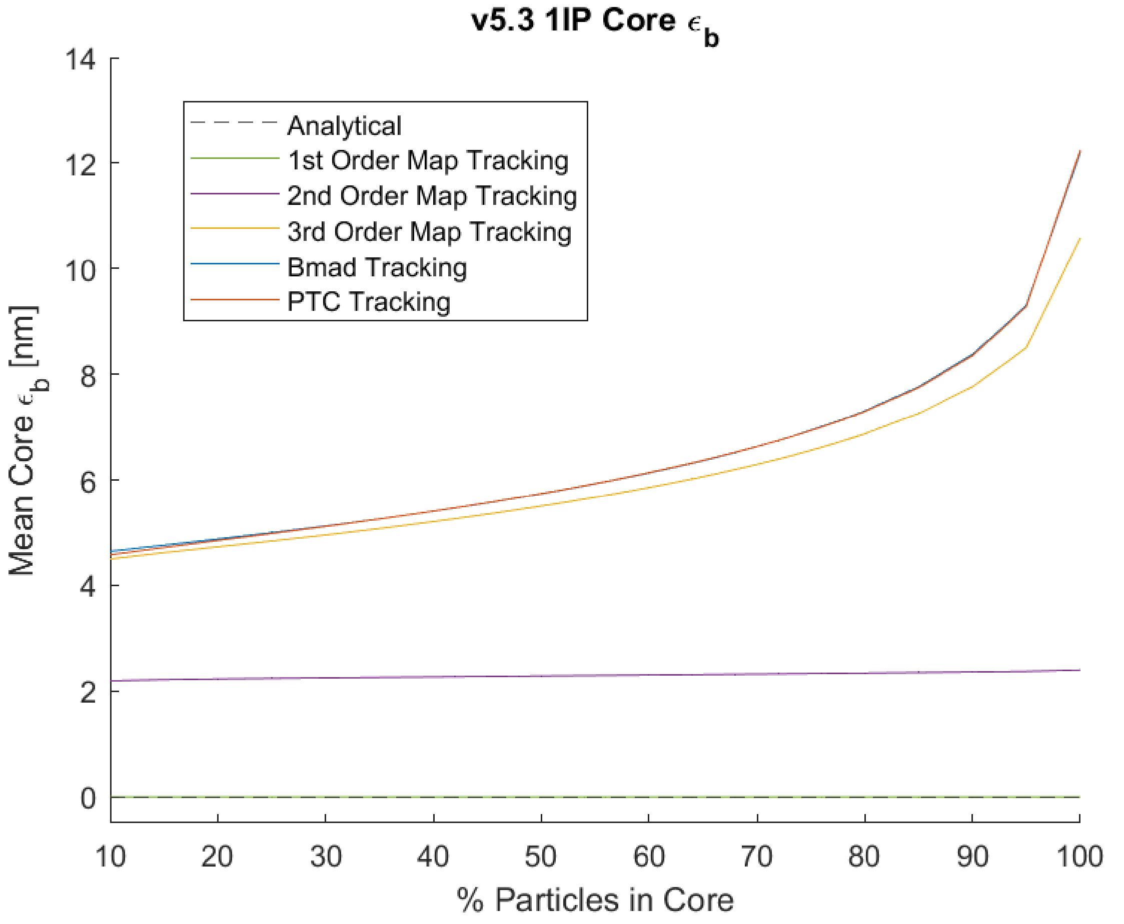
Results – v5.3

Core emittance

- Emittance obtained by fitting a Gaussian to particles within some cutoff amplitude
- If perfectly Gaussian distribution, $\epsilon_{core} = \epsilon_{RMS}$ for all cutoff amplitudes

- Core emittances calculated as means of core emittance over turns 4,000 to end

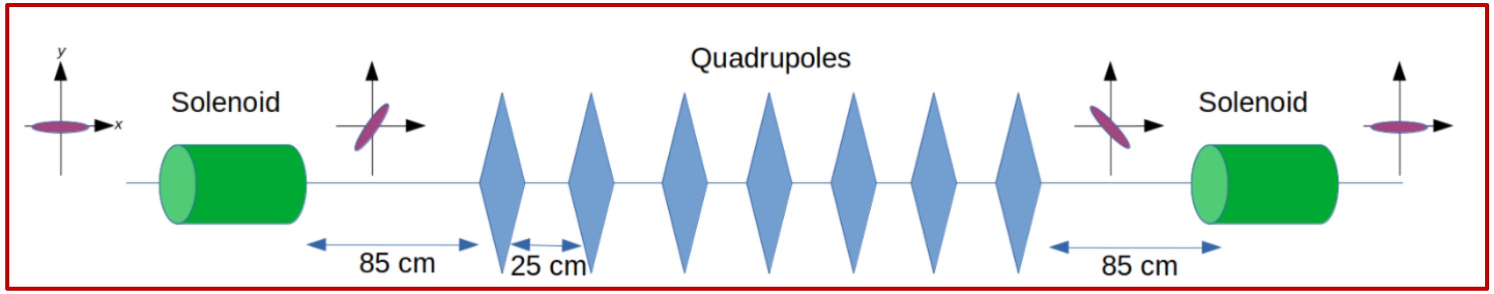
- **In nonlinear case, obtaining ~5 nm of vertical emittance even in the core**



Results – v5.3

- **There is some nonlinear effect present that:**
  → Creates 5nm ϵ_b even in the core
  → Reduces DK polarization from 60% to 12%

- Only regions in ring where ϵ_b might be created is where there is coupling

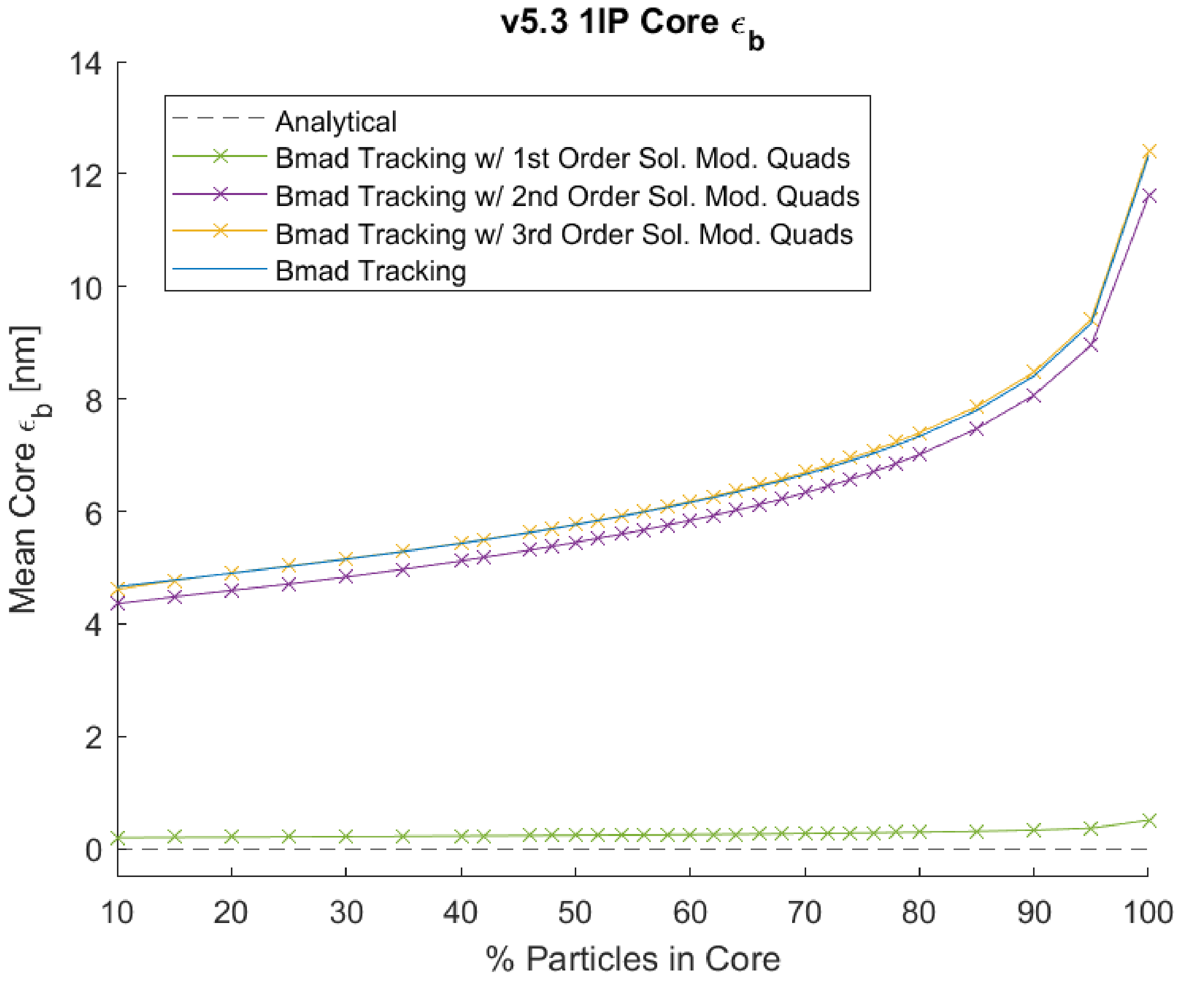


- Try fully nonlinear trackings (including nonlinear solenoids) but with 1st, 2nd and 3rd order quadrupoles in between solenoids (settable in Bmad)

Results – v5.3

Core Emittance

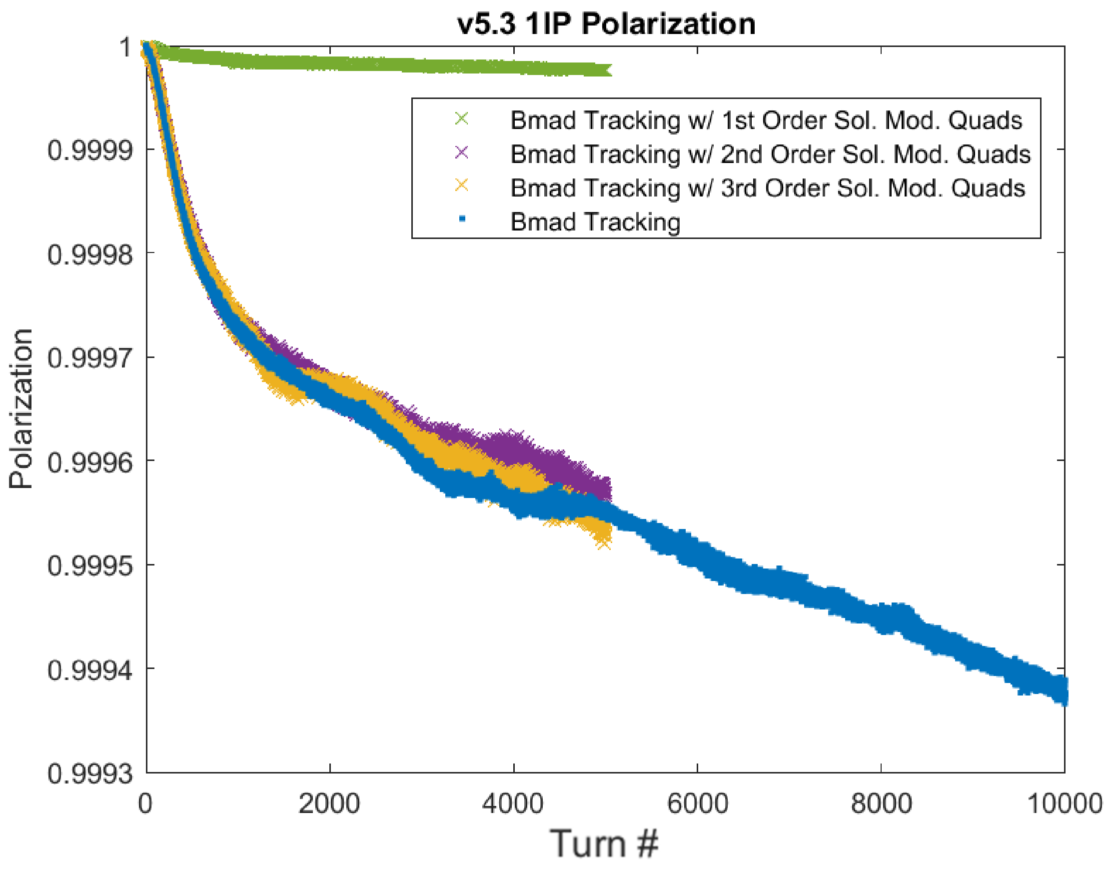
- Almost entire effect presents with 2nd order quadrupoles in coupled regions
- **Coupling in solenoid modules appears to not cancel for off-energy particles, creating ~5 nm of vertical emittance**
- **Polarization?**



Results – v5.3

Polarization

	v5.3 1IP	
	τ_{eq} [min]	P_∞
Analytical	31.1	61.3%
Bmad w/ 1st Order S.M. Quads	28.3	61.1%
Bmad w/ 2nd Order S.M. Quads	7.0	15.1%
Bmad w/ 3rd Order S.M. Quads	5.0	10.8%
Bmad Tracking	5.6	12.1%



- Chromatic effects in solenoid module quadrupoles the primary culprit

Conclusions – v5.3

- **Coupling in solenoids appears to not fully cancel for off-energy particles**
  - ⟶ Creates ~5 nm of core vertical emittance
  - ⟶ Reduces P_∞ to 12%

- **DA studies suggest having $\boldsymbol{\eta, \eta' = 0}$ in solenoids highly beneficial**
  - ⟶ Must turn off the short solenoid & lose the longitudinal spin match

- **Raises two questions:**
  1. Does having $\eta, \eta' = 0$ in the solenoids fix off-energy coupling correction?
  2. Can we live without a longitudinal spin match at 18 GeV?

v5.6 Results

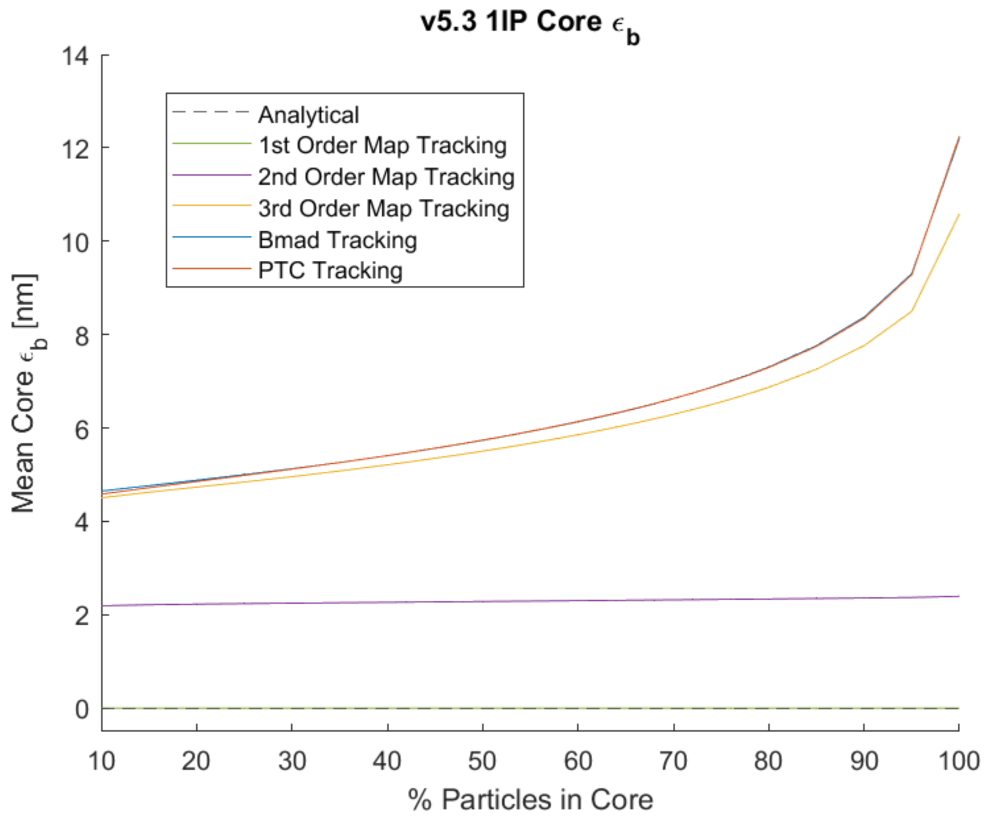
$$G_x = 0, \;\; G_z \neq 0$$

$\eta, \eta' = 0$ in solenoid modules

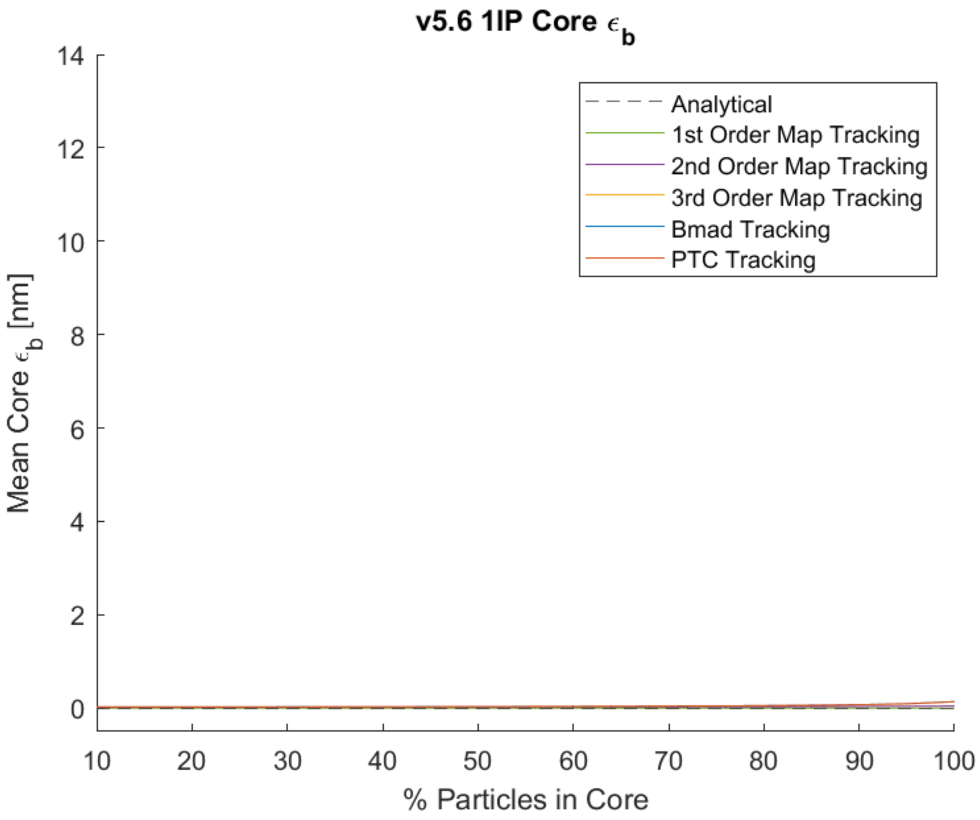
Results – v5.6 1IP

Does having $\eta, \eta' = 0$ in the solenoids fix off-energy coupling correction? ✓

Vertical core emittances:



$\eta, \eta' \neq 0$



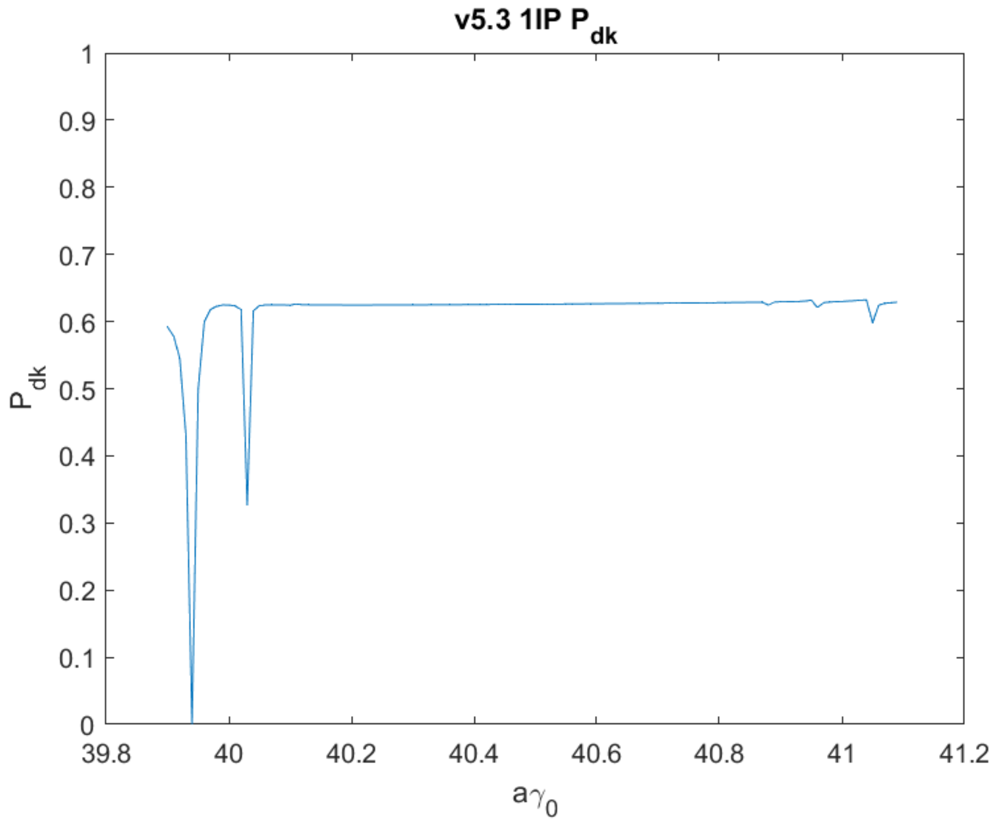
$\eta, \eta' = 0$

Results – v5.6 1IP

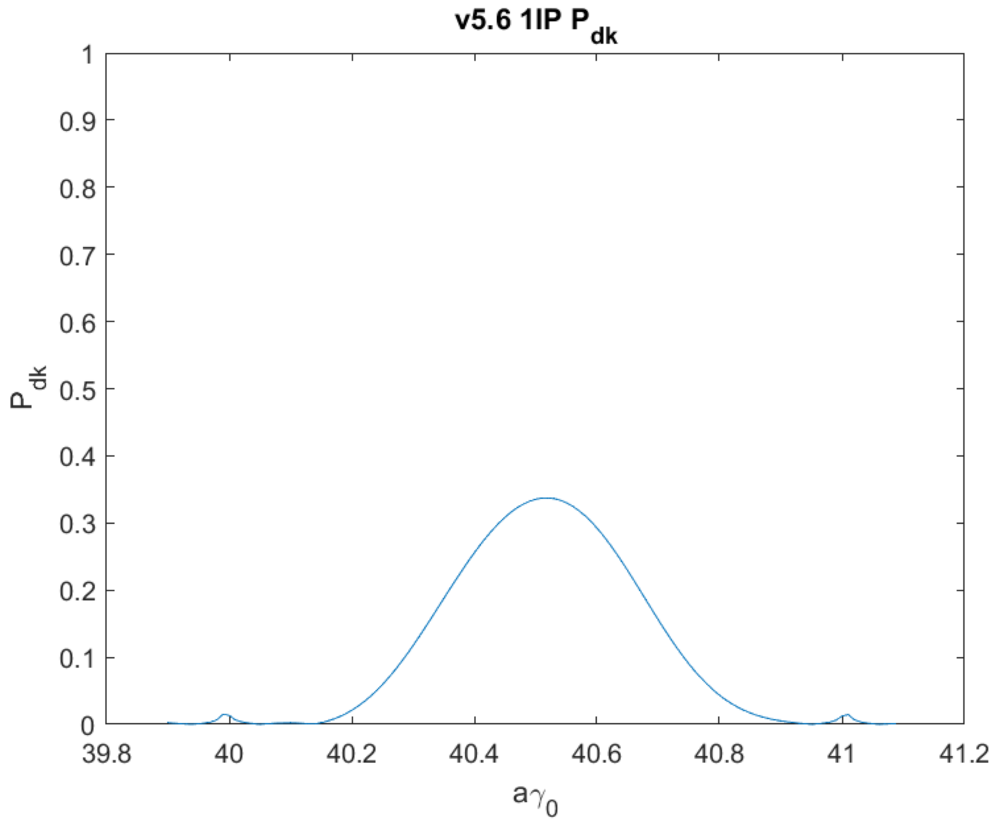
Can we live without a longitudinal spin match at 18 GeV?

Maybe – need to check nonlinear tracking

Linear P_∞:



$$G_z = 0$$



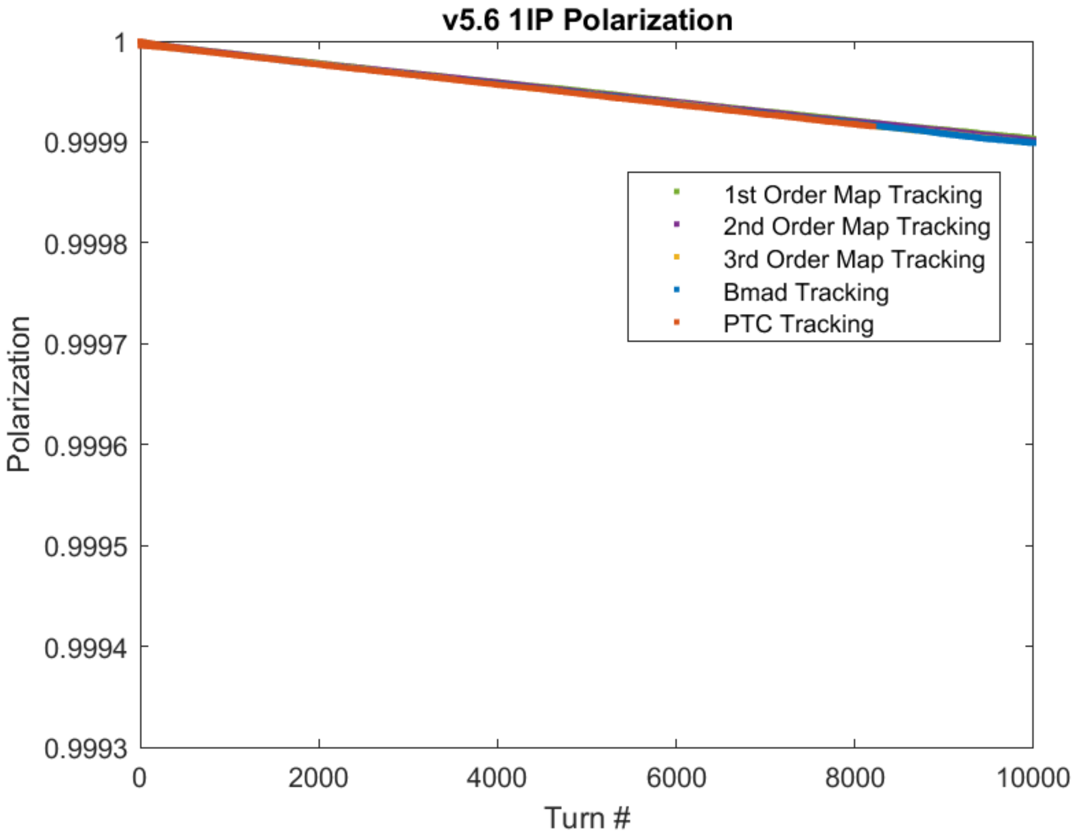
$$G_z \neq 0$$

*nonlinearities give much lower actual P_∞

Results – v5.6 1IP

Polarization

	v5.6 1IP	
	τ_{eq} [min]	P_∞
Analytical	15.0	*33.4%**
1st Order Map Tracking	14.0	32.9%
2nd Order Map Tracking	13.9	32.7%
3rd Order Map Tracking	13.7	32.1%
Bmad Tracking	13.7	32.1%
PTC Tracking	13.6	31.9%



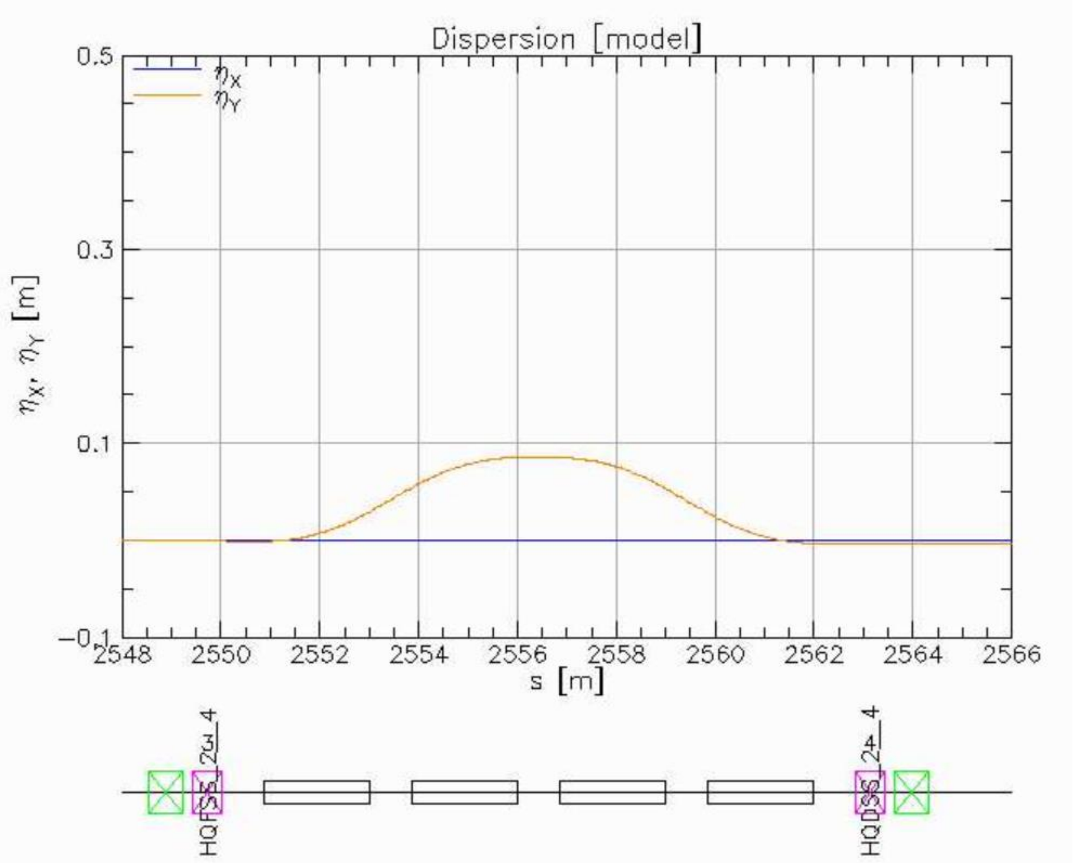
- Polarization holds up well for 1IP v5.6 in fully nonlinear case
- Will the same robustness be observed when including a ϵ_y-creator?

Results – v5.6 ϵ_y-creator

Several methods to create $\boldsymbol{\epsilon_y}$

1. Localized closed η_y bump ⟵
2. Delocalized η_y
3. Localized coupling near the IR

- Inserted closed η_y bump in IP2 drift space that creates 2.5 nm ϵ_y
- Optimized so $\boldsymbol{G}_y = 0$ for 1-turn from center of chicane
- Spin match was tricky: ϵ_y grew to ~ 5 nm



Results – v5.6 ϵ_y-creator

Polarization

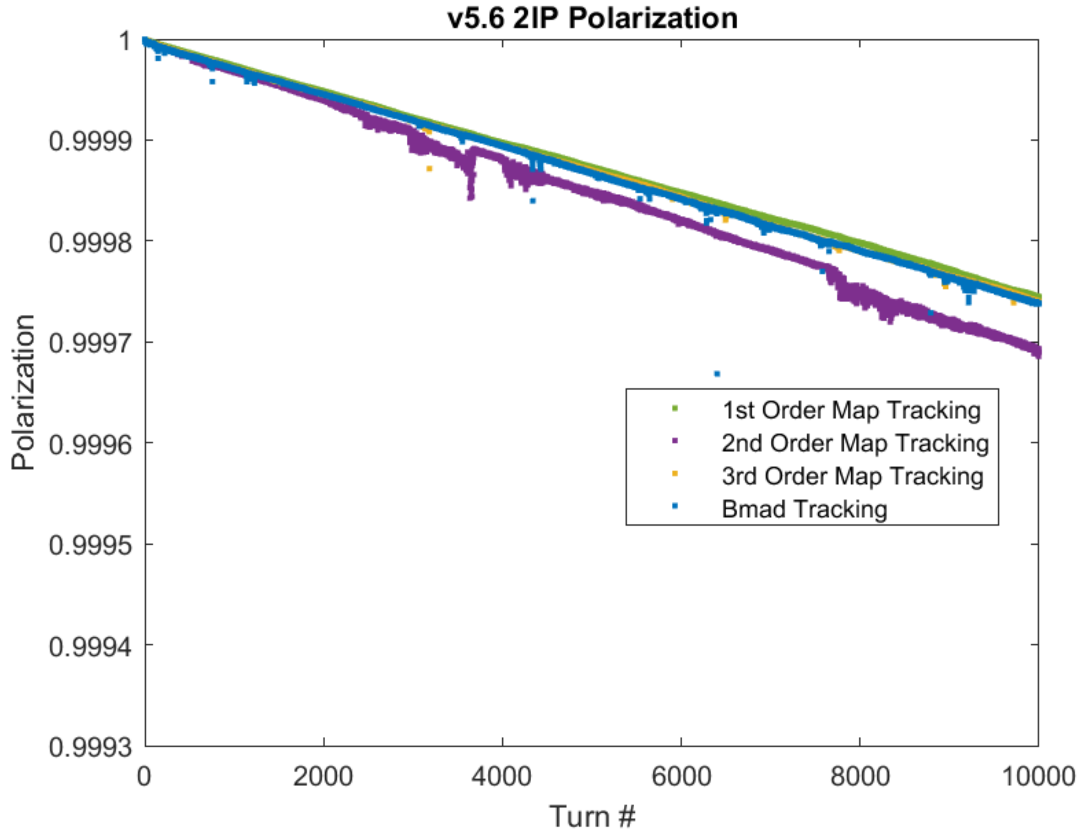
	v5.6 1IP			v5.6 1IP + η_y bump			v5.6 1IP + η_y bump + $G_y = 0$	
	τ_{eq} [min]	P_∞		τ_{eq} [min]	P_∞		τ_{eq} [min]	P_∞
Analytical	15.0	33.4%		6.8	29.3%		12.2	31.9%
1st Order Map Tracking	14.0	32.9%		6.4	14.5%		8.9	24.5%
2nd Order Map Tracking	13.9	32.7%		5.8	13.4%		6.2	17.1%
3rd Order Map Tracking	13.7	32.1%		5.6	13.0%		6.6	18.0%
PTC Tracking	13.6	31.9%		5.4	12.5%		6.4	17.5%

- Careful implementation and vertical spin matching will be necessary if closed η_y-bump used as vertical emittance creator

Results – v5.6 2IP

Polarization

	v5.6 2IP	
	τ_{eq} [min]	P_∞
Analytical	7.6	16.9%
1st Order Map Tracking	6.8	15.6%
2nd Order Map Tracking	5.6	13.0%
3rd Order Map Tracking	6.7	15.4%
Bmad Tracking	6.7	15.4%



Conclusions

- **Zero dispersion in the solenoid modules is necessary**
  - Else, coupling is not fully corrected for off-energy particles
  - However, the longitudinal spin match is unachievable with $\eta, \eta' = 0$
- **v5.6 1IP ($G_z \neq 0$) maintains sufficient polarization in fully nonlinear case**
- **More work to be done on ϵ_y-creation: determine most feasible method with least significant effect on polarization**
  - Closed η_y-bump would require spin matching, which proved difficult
- **v5.6 2IP polarizations lower than 1IP**

References

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Thank you!

Questions?