Spin Polarization Simulations for the Future Circular Collider e+e- using BMAD

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EPFL CERN CHART Swiss Accelerator Research and Technology

Outline

1. Introduction
2. Linear Spin Polarization Simulations in BMAD
3. Benchmark between Tao (BMAD) and SITF (SITROS)
4. Nonlinear Spin Tracking in BMAD
5. Optical Bumps
6. Outlook

Motivation

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- FCC-ee, the first step of the FCC project, will offer high precision explorations of physics at four center-of-mass energies.
- The high precision center-of-mass energy calibration is feasible at Z and W energies by means of resonant depolarization.
- Spin simulations for the validation of the energy calibration method
- Effects of lattice perturbations on spin polarization should be investigated.
- Sufficient polarization levels under various possible lattice conditions
- BMAD, a simulation tool that allows full lattice control and the spin simulations, being actively developed and sustains an active group of users and developers.

BMAD Home Page, https://www.classe.cornell.edu/bmad/

Beam Energy Measurement in FCC-ee

- Resonant depolarisation as a beam energy measurement method relies on the relationship between beam energy and spin tune.

$$\nu \approx a \frac{E}{mc^2}$$

- Possible bias in beam energy due to machine imperfections.
- Latest precision target is 4 keV at Z and 100 keV at W
- Proposed running mode: around 200 non-colliding pilot bunches per beam will be injected first and polarized by wigglers, then inject bunches for luminosity running ⇒ frequent measurement of beam energy during luminosity data taking

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FCC collaboration. "FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2." European Physical Journal: Special Topics 228.2 (2019): 261-623.

Polarization Build-Up with Radiative Depolarization

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- ST effect + radiative depolarization → equilibrium polarization
- Derbenev–Kondratenko–Mane (DKM) formula when radiative depolarization is considered

$$P_{DK} = -\frac{8}{5\sqrt{3}} \times \frac{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \hat{b} \cdot \left(\hat{n} - \frac{\partial \hat{n}}{\partial \delta}\right) \right\rangle_s}{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \left(1 - \frac{2}{9}\left(\hat{n}\cdot\hat{s}\right)^2 + \frac{11}{18}\left(\frac{\partial \hat{n}}{\partial \delta}\right)^2\right) \right\rangle_s}$$

$$\tau_{DK}^{-1} = \tau_{BKS}^{-1} + \tau_{dep}^{-1}$$

$$\tau_{dep}^{-1} = \frac{5\sqrt{3}}{8} \frac{r_e \gamma^5 \hbar}{m_e} \frac{1}{C} \oint \mathrm{d}s \left\langle \frac{\frac{11}{18}\left(\frac{\partial \hat{n}}{\partial \delta}\right)^2}{|\rho(s)|^3} \right\rangle_s$$

- $\partial \hat{n}/\partial \delta$: the spin-orbit coupling function

Desmond Barber, Radiative Polarization in Electron Storage Rings

Linear Polarization Calculation

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SLIM formalism for linearized orbital and spin motions

- 6×6 orbital transfer matrix $\rightarrow$ 8×8 spin-orbit transfer matrix

$$\mathbf{T}_{8\times8} = \begin{pmatrix} \mathbf{M}_{6\times6} & \mathbf{0}_{6\times2} \\ \mathbf{G}_{2\times6} & \mathbf{D}_{2\times2} \end{pmatrix}$$

- spin-orbit vector $(x, x', y, y', z, \delta, \alpha, \beta)$ with respect to the closed orbit
- $\vec{S} \approx \hat{n}_0 + \alpha\hat{m} + \beta\hat{l}$, unit along $\hat{n}_0$, small deviation from $\hat{n}_0$

$$P_{DK} = -\frac{8}{5\sqrt{3}} \times \frac{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \hat{b} \cdot \left(\hat{n} - \frac{\partial\hat{n}}{\partial\delta}\right) \right\rangle_s}{\oint \mathrm{d}s \left\langle \frac{1}{|\rho(s)|^3} \left(1 - \frac{2}{9}\left(\hat{n}\cdot\hat{s}\right)^2 + \frac{11}{18}\left(\frac{\partial\hat{n}}{\partial\delta}\right)^2\right) \right\rangle_s}$$

- $\langle\hat{n}\rangle_s \rightarrow \hat{n}_0(s)$
- neglect $\hat{b} \cdot \partial\hat{n}/\partial\delta$
- $\partial\hat{n}/\partial\delta$, ignores its dependence on the phase space position

Tao (BMAD)

SLIM formalism from A.W. Chao, Evaluation of radiative spin polarization in an electron storage ring

Nonlinear Spin Tracking Simulations

- Avoid the introduction of $\hat{n}$
- Independent of spin diffusion theory
- Obtain τ_{dep} via Monte-Carlo spin tracking simulations
- P_{BKS} and τ_{BKS} are computed at closed orbit

$$P(t) = P_{DK}\left[1 - e^{-t/\tau_{DK}}\right] + P_0 e^{-t/\tau_{DK}} \simeq P_0 e^{-t/\tau_{dep}}$$

$$P_{eq} \simeq P_{BKS}\frac{\tau_{dep}}{\tau_{BKS} + \tau_{dep}}$$

Long-Term Tracking

Main Lattice Parameters

Sequence 217 at Z energy is used in the simulations

Circumference (km)	97.756
Beam energy (GeV)	45.6
β_x^* (m)	0.15
β_y^* (mm)	0.8
ϵ_x (nm)	0.27
ϵ_y (pm)	1
Synchrotron tune Q_z	0.025
Horizontal tune Q_x	269.139
Vertical tune Q_y	269.219

Table: Main parameters at Z energy

Old version lattice with 60° FODO cells

FCC collaboration. (2019). FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2. European Physical Journal: Special Topics, 228(2), 261-623.

Effective Model

- Use an effective model to simulate realistic orbits after lattice correction
- The errors are randomly distributed obeying the truncated Gaussian distributions (truncated at 2.5 σ)

Residual errors after lattice correction

Type	$\sigma_{\Delta X}$ (μm)	$\sigma_{\Delta Y}$ (μm)	$\sigma_{\Delta S}$ (μm)	$\sigma_{\Delta PSI}$ (μrad)	$\sigma_{\Delta THETA}$ (μrad)	$\sigma_{\Delta PHI}$ (μrad)
Arc quadrupole	0.1	0.1	0.1	2	2	2
Arc sextupole	0.1	0.1	0.1	2	2	2
Dipoles	0.1	0.1	0.1	2	0	0
IR quadrupole	0.1	0.1	0.1	2	2	2
IR sextupole	0.1	0.1	0.1	2	2	2

Table: An effective model for the small error generation used in the spin-orbit simulations

Preliminary Global Parameters Matching in BMAD

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- Match the global parameters with the designed values
- Simplified matching: using the elements in RF section
- Optimized matching: adding BPMs, kickers and correctors

	Step order	"Data"	"Variables"
No err	1	x and z at IPs, Q_z	phi0, voltage
	2	β^*, Q_x, Q_y	correctors, RF Quad
	3	(recheck Data in step 1)	(phi0, voltage)
	4	save orbits at BPMs	
Add err	5	orbits at BPMs and IPs (higher weight)	kickers
	6	β^*, Q_x, Q_y	correctors, RF quad
	7	x and z at IPs, Q_z	phi0, voltage

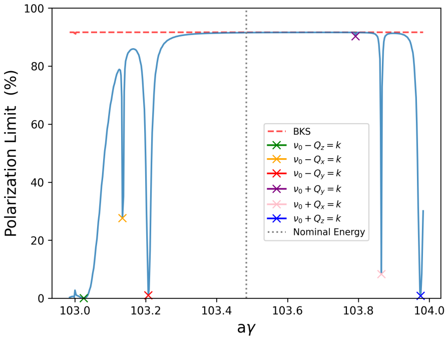
Table: The optimized procedures for the parameter matching

Future orbit corrections and parameter matching will be done in MADX

Energy Scan in Tao (BMAD)

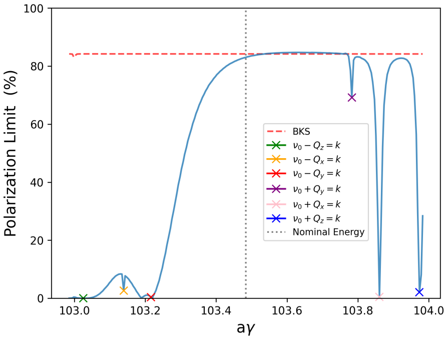
- Tao computes the polarization in linear regime using DKM formula
- Energy scans using two error seeds generated from the effective model
- Six first order spin-orbit resonances between two integer spin tunes

$(\Delta y)_{\rm rms} = 43.7\,\mu$m



91.6% near nominal energy

$(\Delta y)_{\rm rms} = 148\,\mu$m



84.6% near nominal energy

Energy Scan in Tao (BMAD)

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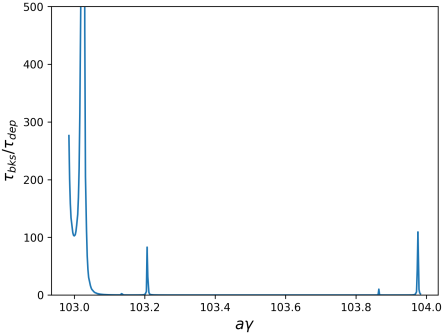
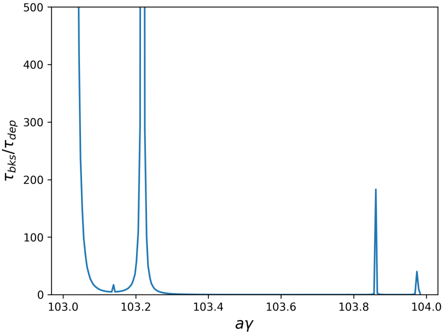
$$P_{eq} \simeq P_{bks} \frac{1}{1 + \tau_{bks}/\tau_{dep}}$$

$$\tau_{tot} = \tau_{bks} \frac{1}{1 + \tau_{bks}/\tau_{dep}}$$

$(\Delta y)_{\mathrm{rms}} = 43.7\,\mu\mathrm{m}$



$(\Delta y)_{\mathrm{rms}} = 148\,\mu\mathrm{m}$



Spin Tune Bias

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measured spin tune $\neq a\gamma$

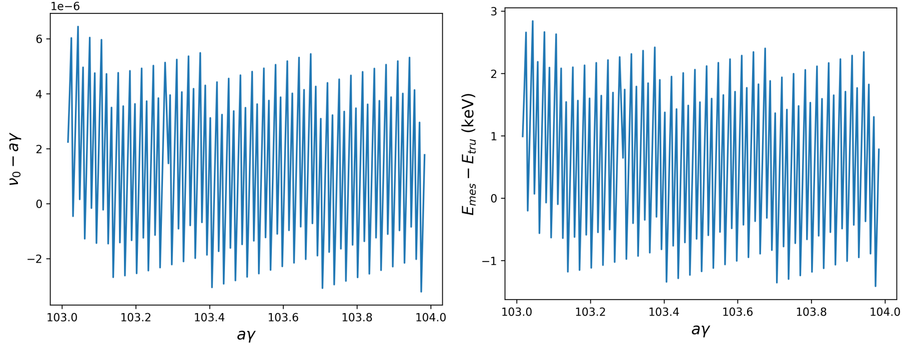

Figure: Spin tune shift from $a\gamma$ (left) and measured energy deviation (right) using the error seed that creates an orbit distortion of $(\Delta y)_{\text{rms}} = 148\,\mu\text{m}$

Requirement for center-of-mass energy determination is ± 4 keV at Z energy*

* Alain Blondel, PED Overview: Centre-of-mass energy calibration, FCC Week 2022

Benchmark between Tao (BMAD) and SITF

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- SITF, the linear spin simulation module in SITROS
- Underlying differences between two codes exist → check step by step

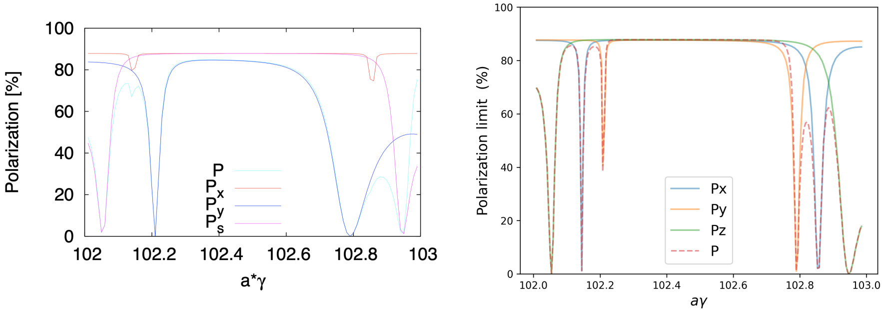

Figure: Energy scan using sequence version 213 seed 13 in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Global Parameter Comparisons

- FCC-ee clean lattice No.217 without misalignments at 45.6 GeV

	Q_x	Q_y	Q_z	x_{rms} [mm]	y_{rms} [mm]	β_x at IP.1 [m]	β_y at IP.1 [mm]
MADX	269.1354	269.2105	0.0247	0.027	0	0.1495	0.8
Tao	269.1354	269.2105	0.0247	0.027	0	0.1495	0.8
SITF	269.1354	269.2108	0.0247	0.027	0	0.1495	0.8

- Simple lattice with 10 nm x and y misalignments in one IR quadrupole (QC1L1.1)

	Q_x	Q_y	Q_z	x_{rms} [mm]	y_{rms} [mm]	β_x at IP.1 [m]	β_y at IP.1 [mm]
MADX	269.1354	269.2105	0.0247	0.027	0.004	0.1495	0.8
Tao	269.1354	269.2105	0.0247	0.027	0.004	0.1495	0.8
SITF	269.1354	269.2106	0.0247	0.027	0.004	0.1495	0.8

Closed Orbit Comparisons

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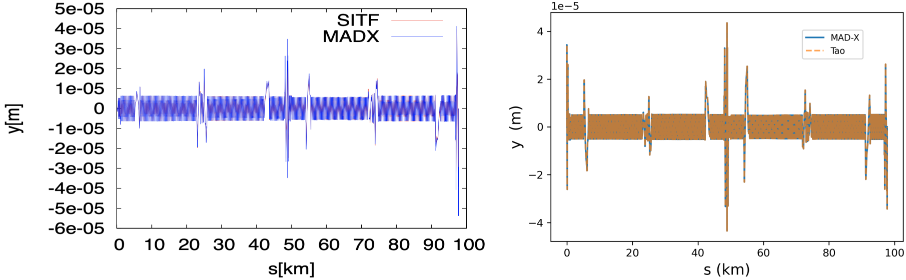

Figure: Vertical closed orbits comparison between MAD-X and SITF (left), and MAD-X and Tao (right)

Tao and SITF create nearly the same closed orbit

SITF plot is from Eliana Gianfelice-Wendt

$\hat{n}_0$ Deviation Comparison

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- $\hat{n}_0$, the central quantity for the spin polarization description
- Away from integer spin tune $\Rightarrow$ $\hat{n}_0$ almost aligned with the vertical
- Near integer spin tune $\Rightarrow$ $\hat{n}_0$ deviates from the vertical

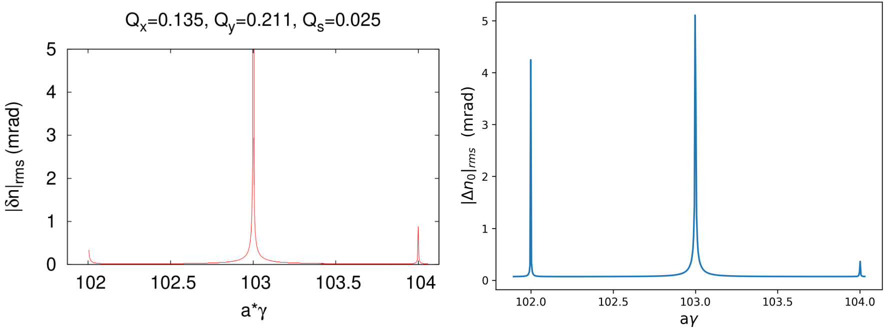

Figure: Variation of the rms $\hat{n}_0$ deviation from the vertical in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Benchmark between Tao, SITF and SLIM

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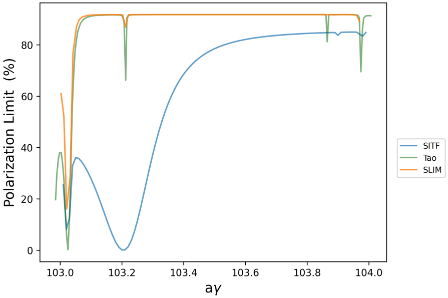


Figure: Energy scan of the equilibrium polarization (left) and the vertical mode polarization (right) by three codes

Tao and SITF share the same BKS level.

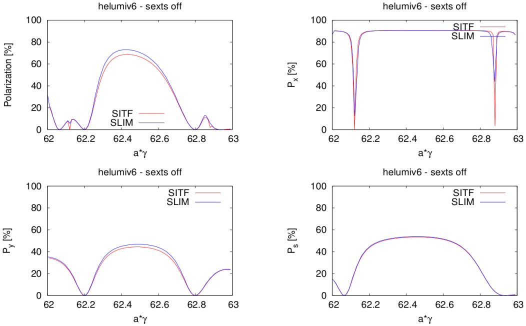
The difference may lie in the computation for the spin-orbit coupling function $\partial\hat{n}/\partial\delta$.

SITF and SLIM data are from Eliana Gianfelice-Wendt

Discussions Regarding the Damping in Transport Matrix

Thanks to Eliana Gianfelice-Wendt and Desmond Barber!

- In SLIM/Tao linear calculation undamped 8×8 transport matrix is used for polarization.
- In SITF/SITROS tracking the damped transport matrix is used between emission points.
- Two codes agree when damped matrix is used



Details will be presented by Eliana Gianfelice-Wendt

Nonlinear Spin Tracking

- The higher order resonances may become prominent at high energies and affect the achievable polarization level
- Reveal all effects of lattice imperfections on spin polarization
- Long-Term Tracking module in BMAD
- Track the polarization level turn by turn and extract τ_{dep}

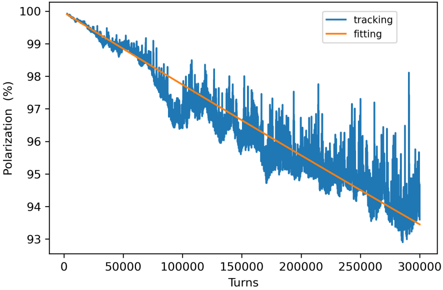
$$P_{eq} \simeq P_{BKS} \frac{\tau_{dep}}{\tau_{BKS} + \tau_{dep}}$$

Long-Term Tracking in BMAD

PTC, $\nu_0 = m + Q_y - Q_s$

10 electrons

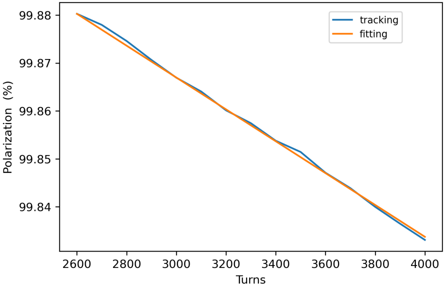
$P_{eq} = 0.15\%$



RMSE=0.59
$R^2 = 0.89$

500 electrons

$P_{eq} = 0.099\%$



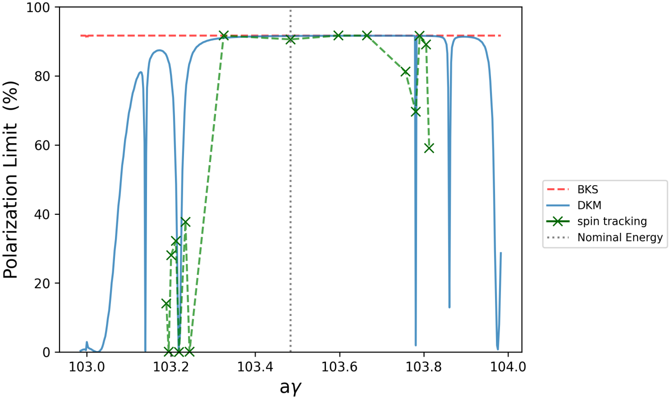
RMSE=4.4×10^{-4}
$R^2 = 0.999$

Using more particles improves the fitting precision but needs more time.

Root-mean-square error, RMSE = $\sqrt{\sum_{i=1}^{N}(P - P^*)^2/N}$

Preliminary Results of Nonlinear Spin Tracking

nonlinear: 1000 particles, 7000 turns, PTC BMAD



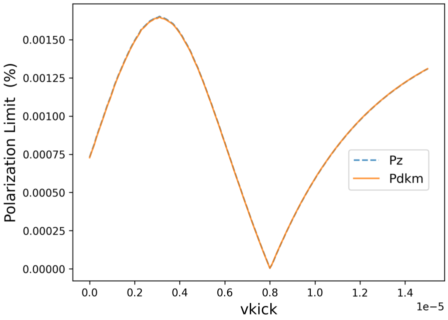
More data points are needed near nominal energy

2π Bump

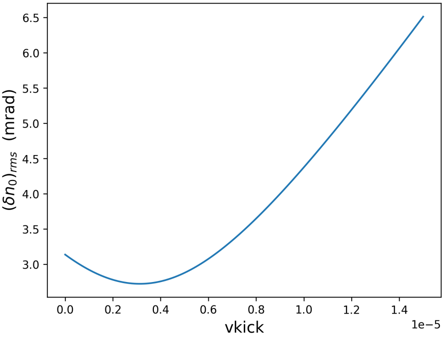
- Three correctors in arc, with vertical kicks to be κ, 2κ, κ
- Error seed that creates around $43\,\mu$m vertical orbit distortion
- 2π phase advance

$$\nu_0 = m + Q_s$$

P_{dkm} and P_z



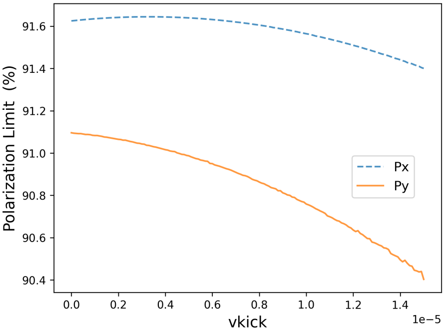
n_0 deviation



Optimized at $vkick = 3.1 \times 10^{-6}$

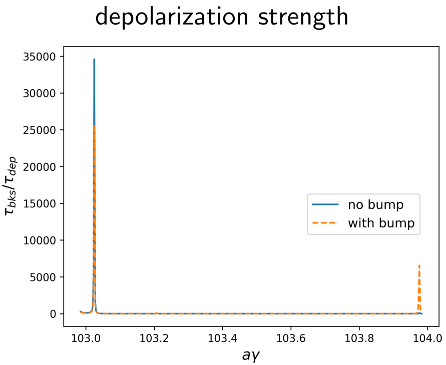
2π Bump

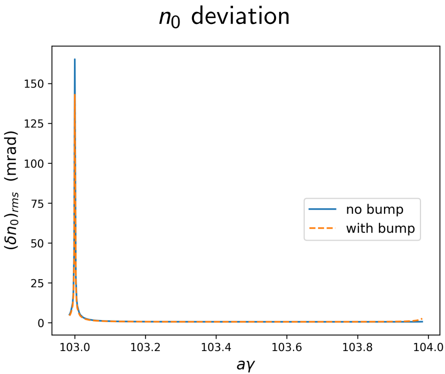
P_x and P_y



- P_{dkm} and P_z and $(\delta n_0)_{rms}$ could be optimized
- P_x and P_y could be lowered

2π Bump





Questions:

- Is 2π bump still necessary for correcting δn_0?
- How much optimization shall we expect using 2π bumps?

Main Problems Now

- Achievable polarization level is based on orbits $\Rightarrow$ robust lattice
- How can a high transverse polarization level be guaranteed?
- How precise can the beam energy be measured using resonant depolarization?

Outlook

- Spin dynamics theory (with Desmond Barber)
- Benchmark with SITROS in nonlinear spin trackings (Eliana Gianfelice-Wendt)
- Resonant depolarization simulation (with David Sagan)
- Lattice corrections of FCC-ee in MADX (with optics group)
- Orbit bumps for transverse polarization optimization
- Optics corrections and spin simulations in LEP (with Werner Herr)

Thank you!

Match the main parameters with the designed value

- Simplified matching: using the elements in RF section
- Optimized matching: adding BPMs, kickers and correctors

Attributes	Designed value	With RF Section	With Kickers, Correctors	Deviation (%)
β_x^* at IP.1/4 (m)	0.15	0.15	0.15	0
β_y^* at IP.1/4 (mm)	0.8	0.7977	0.79941	0.074
β_x^* at IP.2/3 (m)	0.15	0.15	0.15	0
β_y^* at IP.2/3 (mm)	0.8	0.79	0.79947	0.066
x at IP.1/4 (nm)	0	-180	10	N.A.
z at IP.1/4 (nm)	0	20	1.5	N.A.
x at IP.2/3 (nm)	0	-270	390	N.A.
z at IP.2/3 (nm)	0	-20	1.5	N.A.
Synchrotron tune Q_s	0.025	0.0247	0.025	0
Horizontal tune Q_x	269.139	269.139	269.139	0
Vertical tune Q_y	269.219	269.219003	269.219	0

Spin-Orbit Coupling Function Comparison

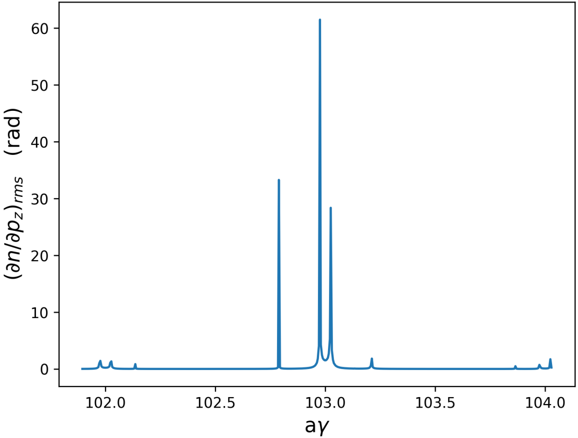


Figure: Variation of the rms spin-orbit coupling function $\partial\hat{n}/\partial\delta$ computed by Tao

Energy Scan Comparison with Simple Lattice

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- Main difference comes from the vertical mode polarization

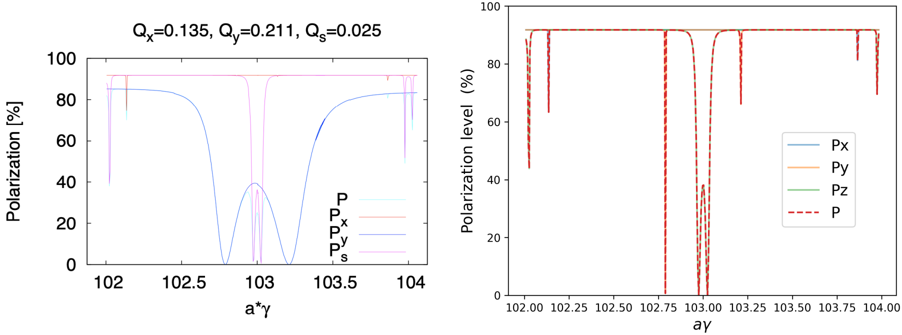

Figure: Energy scans using the simple lattice with one misalignment in SITF (left) and Tao (right)

SITF plot is from Eliana Gianfelice-Wendt

Spin Resonances

Integer resonance $\nu_0 = m$

- the small perturbations have an overwhelming impact
- $\hat{n}_0(s)$ deviates from vertical direction
- loss of polarization accumulation

Spin-orbit resonances $\nu_0 = m + m_x Q_x + m_y Q_y + m_z Q_z$

- $|m_x| + |m_y| + |m_z| = 1$ first order spin-orbit resonances
- Away from resonance $\Rightarrow \hat{n}(\vec{u}; s)$ almost aligned with $\hat{n}_0(s)$
- Near resonances $\Rightarrow \hat{n}(\vec{u}; s)$ deviates from $\hat{n}_0(s) \Rightarrow$ large $\partial \hat{n}/\partial \delta \Rightarrow$ lower polarization

Desmond Barber, Radiative Polarization in Electron Storage Rings