FCCee Polarization simulations with SITROS

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Introduction

- *Resonant de-polarization* has been proposed for accurate beam energy calibration at 45 and 80 GeV beam energy.
  It relies on the relationship $\boldsymbol{\nu_{spin}} = \boldsymbol{a\gamma}$ [a].
- Beam polarization is obtained "for free" through Sokolov-Ternov effect.
  The effect is in practice restricted to a limited range of values of machine size and beam energy because
  - of the build-up rate
  - it is jeopardized by machine imperfections (spin/orbital motion resonances) which affects the reachable level of polarization in particular at high energy.
- 5%-10% beam polarization is estimated to be enough for the purpose of energy calibration.

[a] $\boldsymbol{a}$ = gyromagnetic anomaly

Sokolov-Ternov polarization

Beam get vertically polarized in the ring guiding field

$$P_\infty^{\mathrm{ST}} = 92.3\% \qquad \tau_p^{-1} = \frac{5\sqrt{3}}{8}\frac{r_e\gamma^5\hbar}{m_0 C}\oint\frac{ds}{|\rho|^3}$$

For FCC-e^+e^- with $\rho \simeq$ 10424 m,it is

E (GeV)	τ_{pol} (h)	$\tau_{10\%}$ (*) h
45	256	29
80	14	1.6

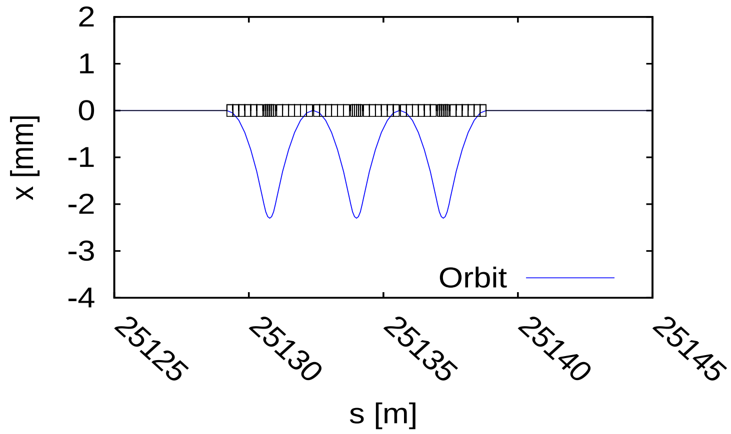
(*) Time needed to reach P=10% for energy calibration

$$\tau_{10\%} = -\tau_p \times \ln(1 - 0.1/P_\infty)$$

Polarization wigglers

τ_p is reduced by introducing *wigglers*, a chain of horizontal bending magnets with alternating field sign.

(orbit for B_w^+=1.7 T)



Using more than one period:

- Smaller impact on ϵ_x,
- Energy spread as with one single period for the same τ_p.

Polarization in real storage rings

A perfectly planar machine (w/o solenoids) is always *spin transparent*.

In practice:

Sokolov-Ternov effect in the guiding dipole field	Perturbations (v-bends, vertical orbit in quads etc.)
↓	↓
Polarisation	Depolarisation
↘	↙

Equilibrium polarisation: $\hat{n}_0(s) P_\infty \quad (P_\infty < P_\infty^{\mathrm{ST}})$

Spin diffusion is larger at high energy and may be particularly large when spin and orbital motions are in resonance

$$\nu_{spin} \pm m Q_x \pm n Q_y \pm p Q_s = \text{integer}$$

- ϵ_y must be small and $\hat{n}_0(s) \approx \hat{y}$
  - closed orbit, spurious D_y and betatron coupling must be well corrected!

Computational tools

Accurate simulations are necessary for evaluating the polarization level to be expected in presence of misalignments.

- MAD-X used for simulating quadrupole misalignments and orbit correction
- SITROS (by J. Kewisch) used for computing the resulting polarization.
  - Tracking code with 2th order orbit description and non-linear spin motion.
  - Used for HERA-e in the version upgraded by M. Böge and M. Berglund.
  - It contains SITF (fully 6D) for analytical polarization computation with *linearized* spin motion.
    - Useful tool for preliminary checks before embarking in time consuming tracking.
    - Computation of polarization related to the 3 degree of freedom separately: useful for disentangling problems!

P_∞ computation in SITROS

Polarization vs. time

$$P(t) = P_\infty(1 - \mathrm{e}^{-t/\tau_p}) + P(0)\mathrm{e}^{-t/\tau_p}$$

In presence of depolarizing effects

asymptotic polarization (unknown)

$$\frac{1}{\tau_p} \simeq \frac{1}{\tau_{\mathrm{BKS}}} + \frac{1}{\tau_{\mathrm{d}}} \quad \text{and} \quad P_\infty \simeq \frac{\tau_p}{\tau_{\mathrm{BKS}}} P_{\mathrm{BKS}}$$

diffusion time (unknown)

- P_{BKS} and τ_{BKS} (Baier-Katkov-Strakhovenko generalization of Sokolov-Ternov quantities when $\hat{n}_0$ is not everywhere perpendicular to the velocity) are known for the *nominal* lattice.
- τ_d and thus P_∞ depend on the *actual* machine.

SITROS tracks the spins of an initially fully polarized beam in the *absence* of polarization build-up. For small t it is

$$P(t) \simeq P(0)\mathrm{e}^{-t/\tau_d} = P_{\mathrm{BKS}}\mathrm{e}^{-t/\tau_d}$$

The TBT average spin projection onto $\hat{n}_0$ is fitted to find τ_d. Several fitting algorithms are implemented in the code.

Last method added fits also the starting polarization.
For $t_0 \geq 0$ it is

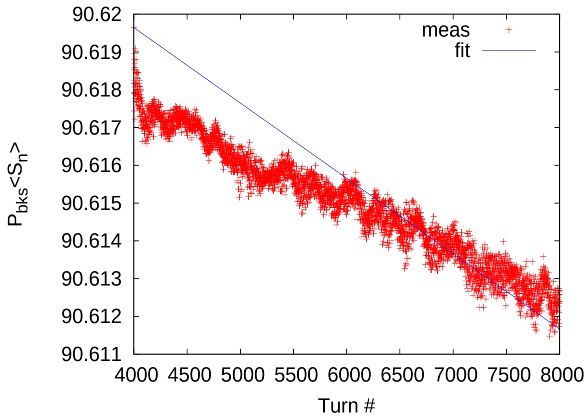
$$P(t') \simeq P_{\mathrm{BKS}}\mathrm{e}^{-(t'+t_0)/\tau_d} = P(t_0)\mathrm{e}^{-t'/\tau_d}$$

with $t' \equiv t - t_0$.

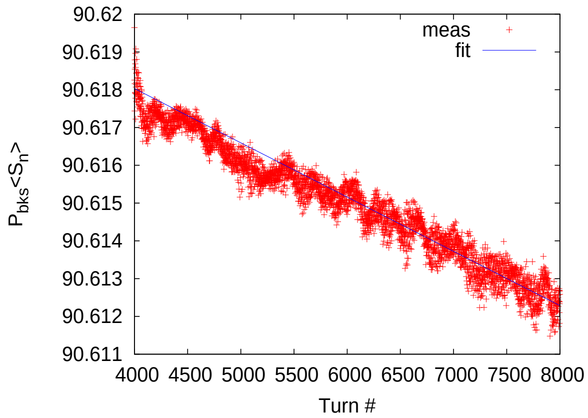
If the fit is done only for τ_d, the value $P(t_0)$ may be critical.
We shall fit the data for τ_d *and* $P(t_0)$.

Results for SITROS data by skipping the first 2000 turns tracking

method	P_∞ (%)
default	21.00
old	21.08
new	26.76



default



new

The value found for $P(t_0)$ is 90.6152% to be compared to P_{2000}=90.6157% from SITROS tracking.

Simulations for FCC-ee 2018 optics

FCC-$e^{\pm}$ design relies on ultra-flat beams.

(January 2018)

	Z	WW
Beam energy [GeV]	45	80
FODO	$60^0/60^0$	$60^0/60^0$
ϵ_x [nm]	0.27	0.84
ϵ_y [pm]	1	1.7
β_x^* [m]	0.15	0.2
β_y^* [mm]	0.8	1
σ_x^* [μm]	6.4	13
σ_y^* [nm]	28	41

For squeezing β_y^* strong quadrupoles are needed in the IR where β_y is large.

$\rightsquigarrow$ Large impact on chromaticity and response to misalignments in the vertical plane.

Additional related problems

- Beam offsets in the strong IRs sextupoles may produce betatron coupling.
- Small offsets of the IRs quads may lead to an anti-damped machine.

Simulations in presence of misalignments

MADX files provided by T. Charles in 2019. Misalignments:

	IR Quads	other Quads	Sexts
δx (μm)	50	100	100
δy (μm)	50	100	100
$\delta\theta$ (μrad)	50	100	100

- BPMs are supposed perfectly aligned to the near-by quadrupole and perfectly calibrated.
- Tune shift and coupling are corrected by 1204 normal + 1204 skew *thin lenses* quadrupoles.

SITROS can't treat thin lenses $\longrightarrow$ replaced in MADX files by 5 mm long quadrupoles, in lack of more space. SITROS edited for dropping

- magnets shorter than 10 mm in emittance and damped transport matrix calculation;
- quadrupole component of misaligned sextupoles in the closed orbit calculation (for compatibility with MADX).

For some seeds the thin lenses substitution in MADX went well:

Seed 13, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	23	22	276.4	0.04
MADX (thick)	35	22	278.4	0.04

Seed 13, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	23	22	239.7	0.114
MADX (thick)	35	22	241.5	0.114

Seed 1, with radiation, B_w=0

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	278.2	0.366
MADX (thick)	35	21	280.2	0.375

Seed 1, with radiation , B_w for $\tau_{10\%}$=1.7 h

	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (pm)	ϵ_y (pm)
MADX (thin)	35	21	242.8	0.281
MADX (thick)	35	21	244.6	0.288

Seed 1, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1457	0.2181	34.9	21.5	0.245	0.288
SITF	0.1459	0.2175	34.4	20.7	0.231	10.3 (*)

(*) Due to CV798 ! Dropping it is ϵ_y=0.34 pm.

Seed 13, with radiation and 8 wigglers

	Q_x	Q_y	x_{rms} (μm)	y_{rms} (μm)	ϵ_x (nm)	ϵ_y (pm)
MADX (thick)	0.1447	0.2097	35.2	22.1	0.241	0.112
SITF	0.1447	0.2099	35.2	21.3	0.231	0.394

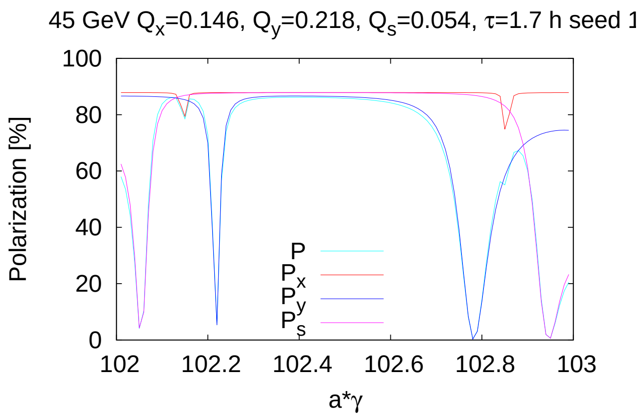
For some seeds the substitution with 5 mm lenses did not work.

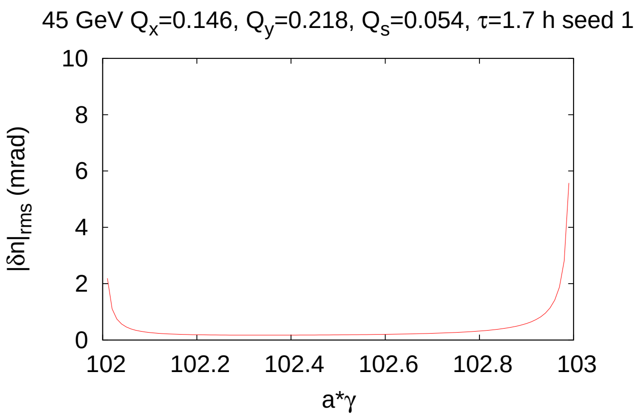
Seed 17, 45 GeV

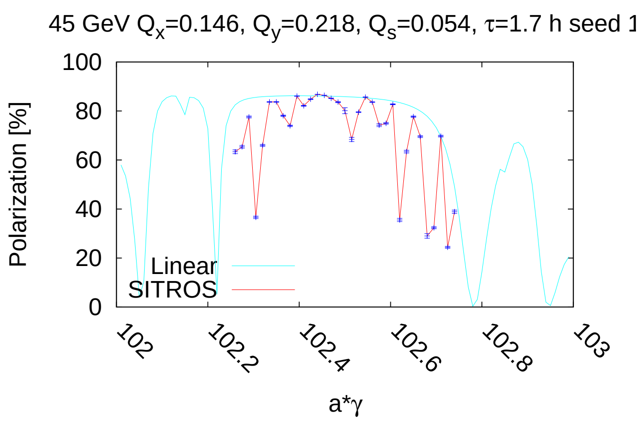
	x_{rms} (μm)	y_{rms} (μm)	J_x	J_y	J_s	ϵ_x (nm)	ϵ_y (pm)
MADX (thin)	34.3	21.7	1.001	1.000	1.998	0.240	0.14
MADX (thick)	35.4	23.3	1.200	1.395	1.402	0.234	84.5

Those seeds have been skipped.

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 1.



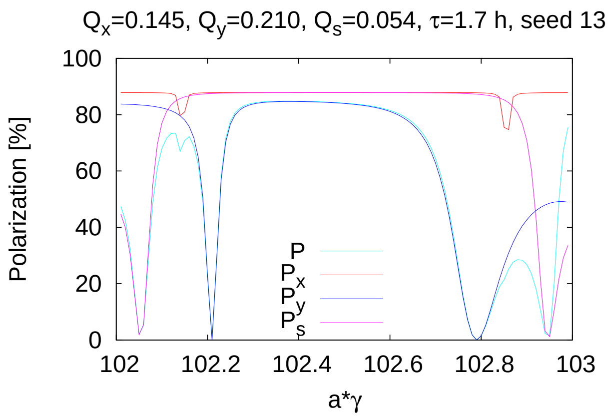


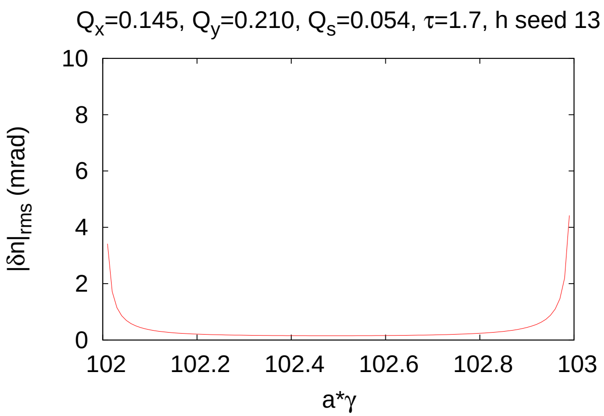


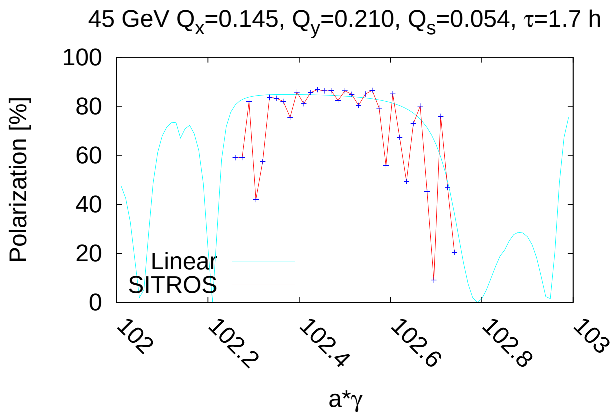
Beam size at IP1

	σ_x (μm)	σ_y (nm)	σ_ℓ (mm)
analytical	5.994	97.6	5.857
SITROS Tracking	7.274	14.9	5.917

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 13.



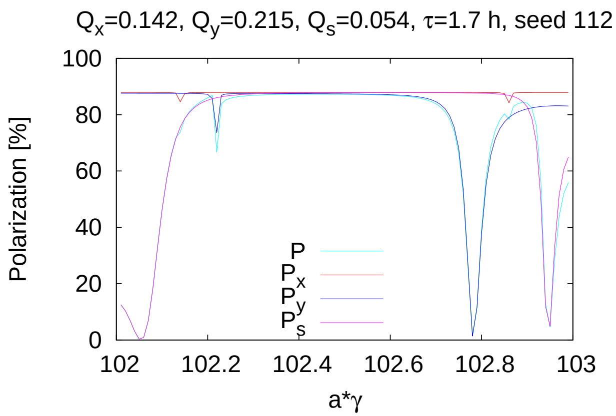


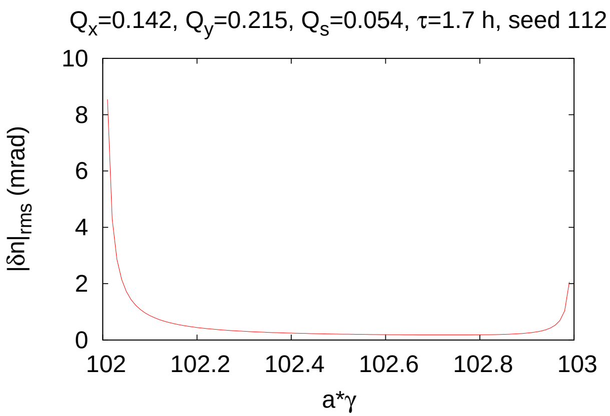


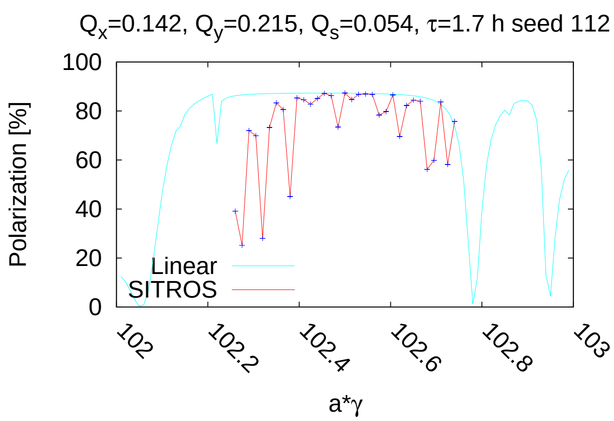
Beam size at IP1

	σ_x	σ_y	σ_ℓ
	(μm)	(nm)	(mm)
analytical	5.966	19.7	5.721
SITROS Tracking	7.114	21.2	5.681

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 112.



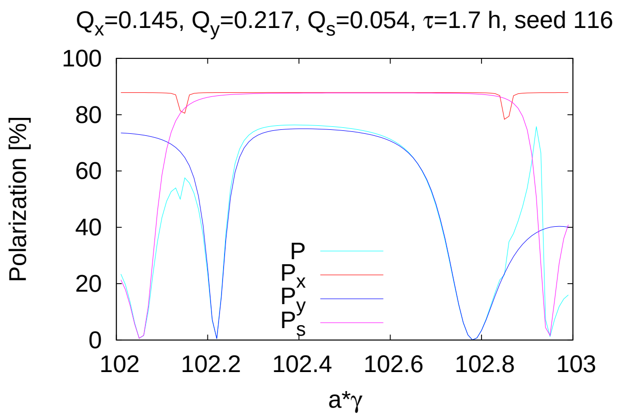


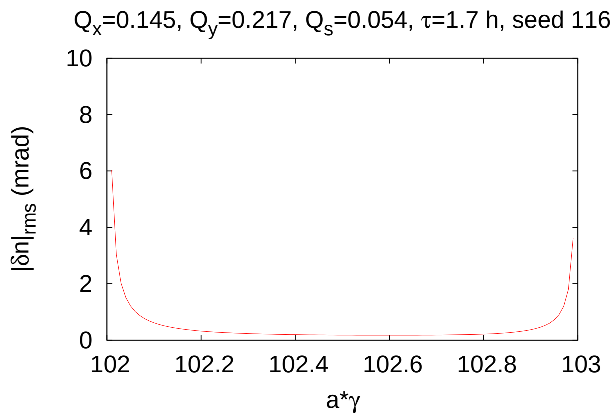


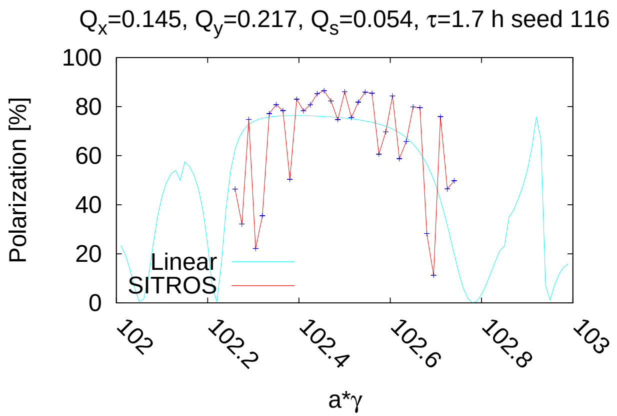
Beam size at FRF.1

	σ_x (μm)	σ_y (μm)	σ_ℓ (mm)
analytical	188.7	1.021	5.738
SITROS Tracking	264.1	1.947	5.717

Tessa 45 GeV optics with 8 wigglers for $\tau_{10\%}$=1.7 h, seed 116.







Beam size at FRF.1

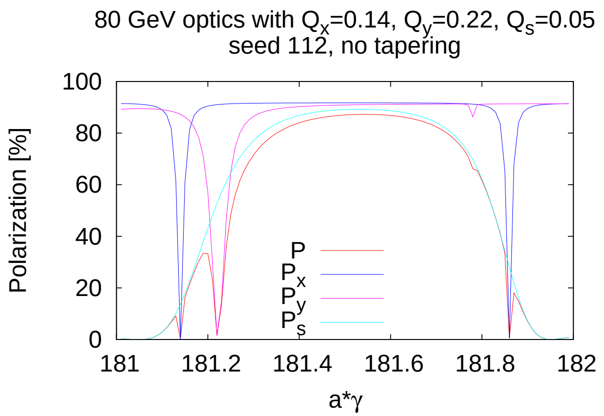
	σ_x (μm)	σ_y (μm)	σ_ℓ (mm)
analytical	183.3	9.863	5.855
SITROS Tracking	259.7	3.313	5.915

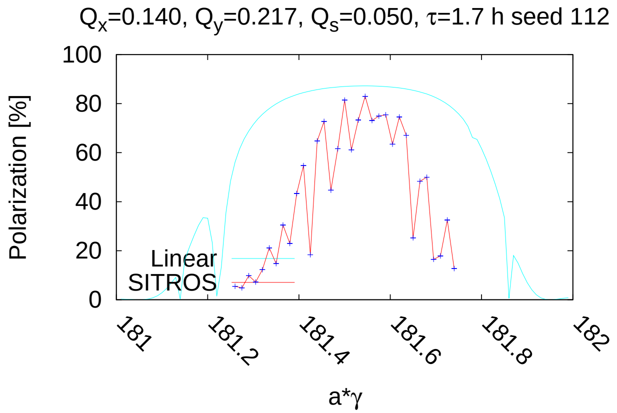
80 GeV

The same 45 GeV optics have been scaled to 80 GeV

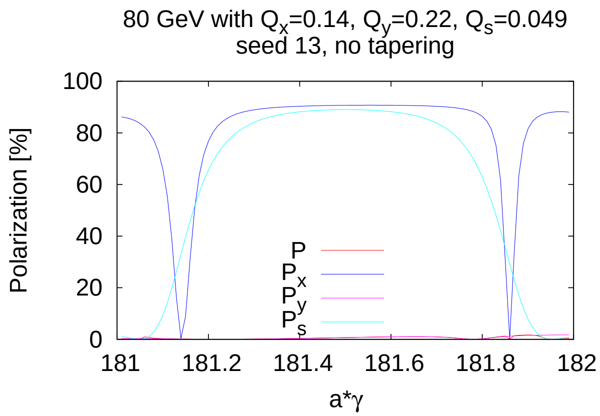
- no wigglers
- no tapering (from previous simulations it seemed not crucial):
  - main quads adjusted for compensating the sextupoles feed-down effect.

Seed 112



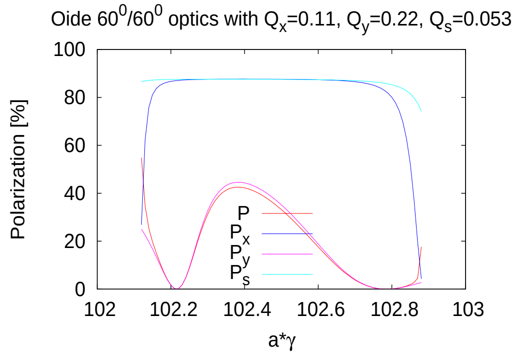


Seed 13



Why is $P_y \simeq 0$?

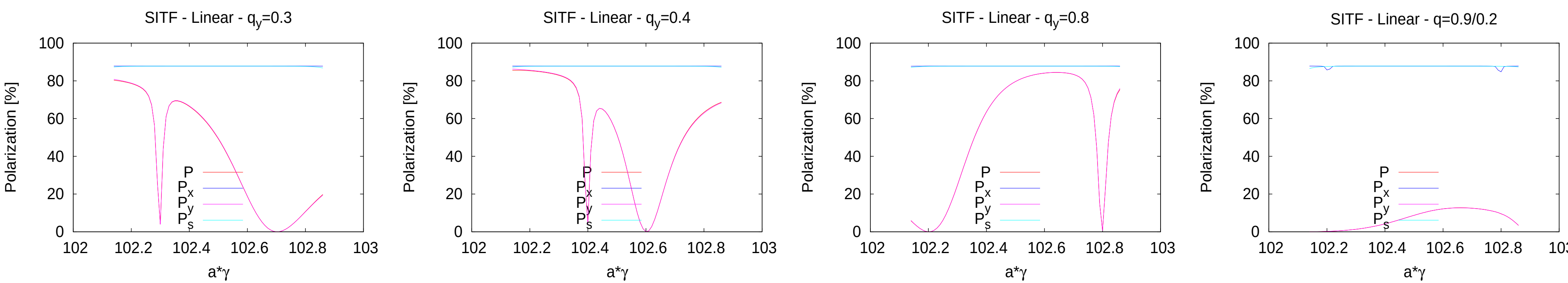
An example (October 2017 60/60 deg optics) at 45 GeV:



Despite a well corrected optics, $\boldsymbol{P_y}$ is relatively small.

Vertical tune scan:

⇓



Why is 0.1/0.8 better then 0.1/0.2 ?

Linear approximation for spin diffusion (betatron-dispersion formalism):

$$\frac{\partial \hat{n}}{\partial \delta}(\vec{u};s) = \vec{d}(s) = \frac{1}{2}\Im\Big\{(\hat{m}_0 + i\hat{l}_0)^* \sum_{k=\pm x,\pm y,\pm s} \boldsymbol{\Delta}_k\Big\}$$

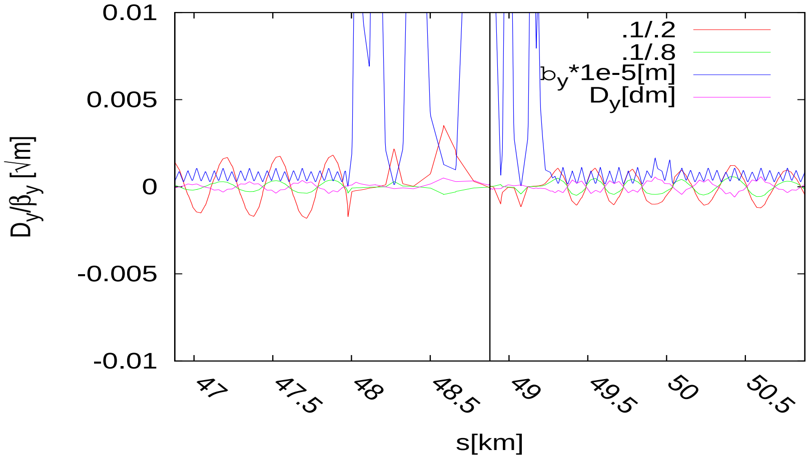
$$\boldsymbol{\Delta}_{\pm x,\pm y} = (1+a\gamma)\frac{e^{\mp i\mu_{x,y}}}{e^{2i\pi(\nu\pm Q_{x,y})}-1}\,\frac{[-D \pm i(\alpha D + \beta D')]_{x,y}}{\sqrt{\beta_{x,y}}}\;J_{x,y}$$

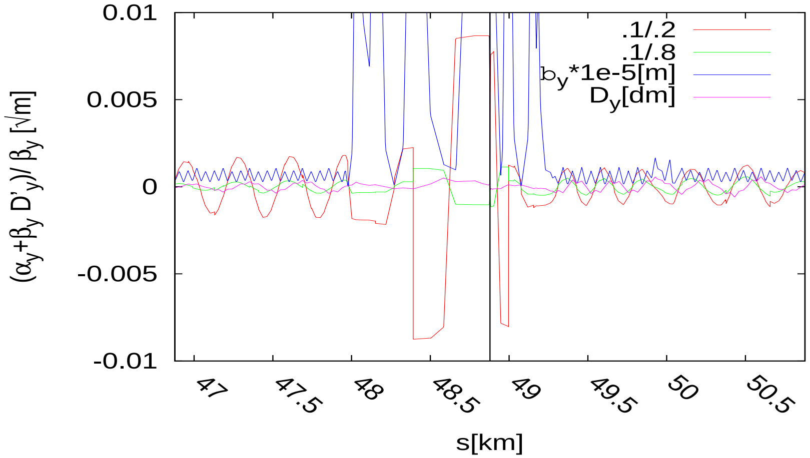
$$\boldsymbol{\Delta}_{\pm s} = (1+a\gamma)\frac{e^{\pm i\mu_s}}{e^{2i\pi(\nu\pm Q_s)}-1}\;J_s$$

$$J_{\pm x,\pm y} = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot\left\{\begin{array}{c}\hat{y}\sqrt{\beta_x}\\ \hat{x}\sqrt{\beta_y}\end{array}\right\} K e^{\pm i\mu_{x,y}}$$

$$J_s = \int_s^{s+L} ds'(\hat{m}_0 + i\hat{l}_0)\cdot(\hat{y}D_x + \hat{x}D_y)\,K$$

Plotting the factor $\frac{[-D_y \pm i(\alpha_y D_y + \beta D'_y)]}{\sqrt{\beta_y}}$ which multiplies $J_{\pm y}$:



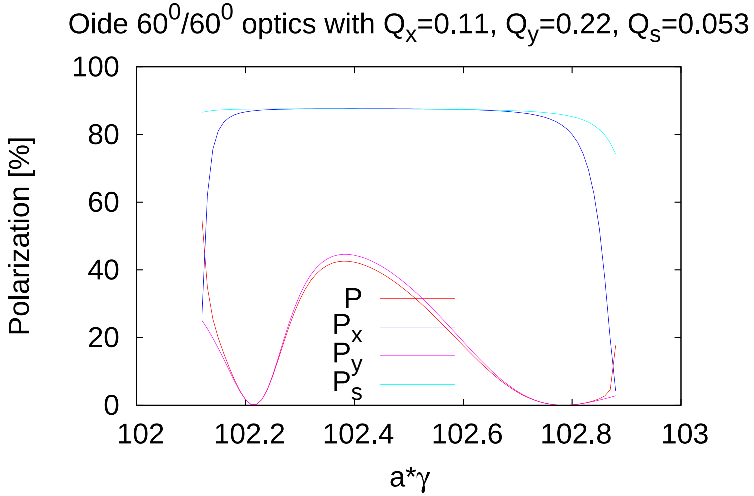


The factor is much smaller when the tunes are moved to .1/.8

$\rightsquigarrow$ Instead of correcting linear coupling and spurious vertical dispersion, we should try minimizing $[-D_y \pm i(\alpha_y D_y + \beta D'_y)]/\beta_y$.

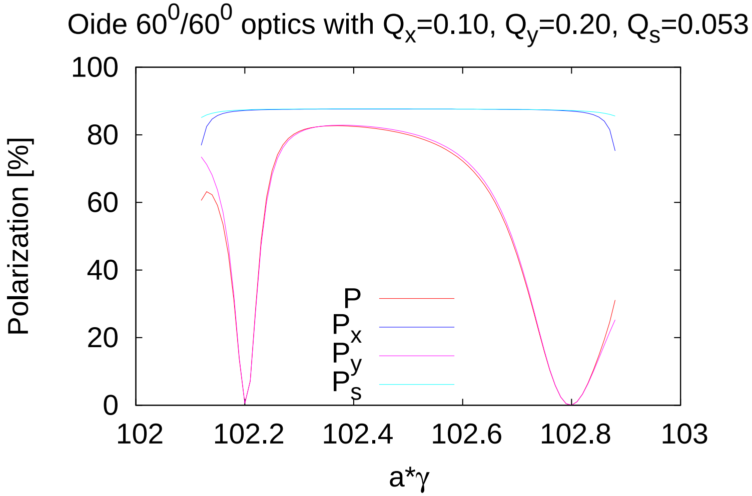
Found skew quadrupole settings improving $\boldsymbol{\Delta}_{\pm y}$ with .1/.2 tunes, but at expenses of betatron coupling.

Before



→

After improving $\boldsymbol{\Delta}_{\pm y}$



Consistency check for Tessa seed 13 case.

$$\mathbf{45\ GeV:} \qquad \boldsymbol{\tau_d^{(45)} = 0.12 \times 10^7\ \mathrm{sec}} \qquad \text{(SITF)}$$

Extrapolating to 80 GeV

$$\boldsymbol{\tau_d^{(80)} \approx \left(\frac{45}{80}\right)^5 \tau_d^{(45)} = 0.07 \times 10^6\ \mathrm{sec}}$$

instead of $0.4{\times}10^{3}$ sec (SITF at 80 GeV) and from

$$\boldsymbol{P_\infty \approx P_{\mathrm{ST}} \frac{\tau_d}{\tau_{\mathrm{ST}} + \tau_d}}$$

with $\boldsymbol{\tau_{\mathrm{ST}} = 5.4 \times 10^4\ \mathrm{sec}}$ at 80 GeV, it should be $\boldsymbol{P_\infty \approx 51\%}$, instead of $\approx$0.

Is the low polarization an artifact?

The importance of damping in the 8x8 matrix

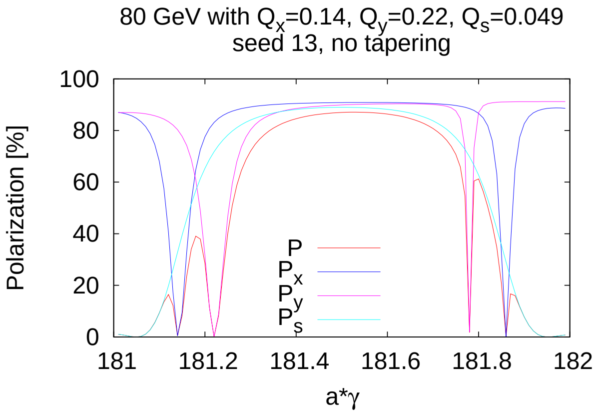
Comparisons with Yi Wu results using Tao revealed that SITF vertical resonances were stronger.
In linear approximation spin diffusion is extracted from the 8×8 transport matrix eigenvectors.

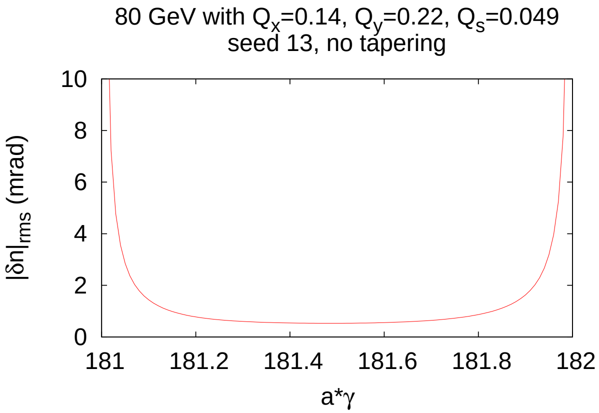
The original Chao paper uses the *undamped* transport matrix for computing the particle distributions and the linear polarization.

In SITROS tracking:

- After photon emission the particle is transported along the ring using the *damped* matrix.
- The emission of photons does not follow the statistical distribution of synchrotron radiation. The average emitted energy in SITROS is zero. Only the stochasticity is taken into account by the tracking.

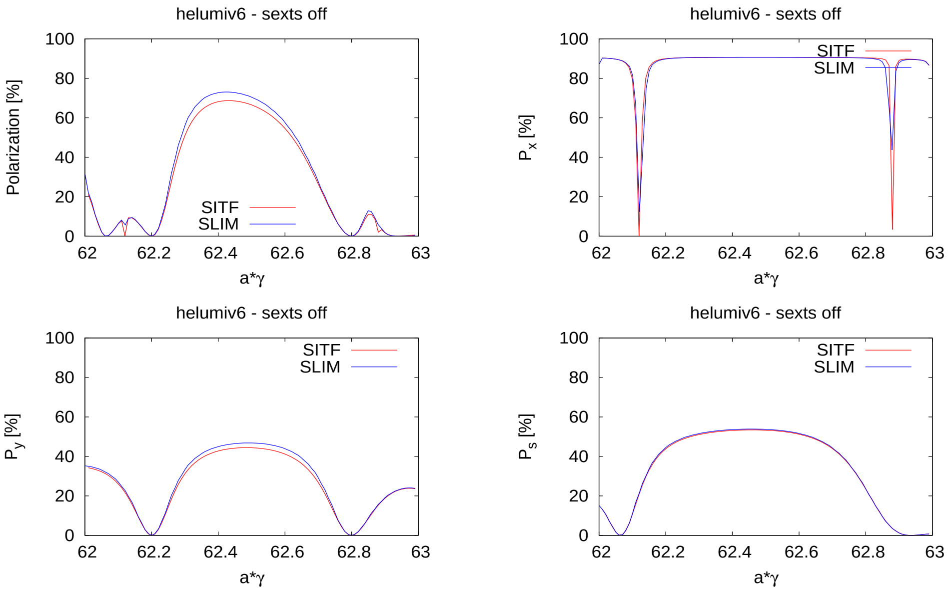
Using the undamped transport matrix for the linear polarization computation we get large polarization also for seed 13 at 80 GeV:





Further observations:

- By using the 8×8 damped transport matrix also in SLIM, the two codes agree:



- The impact of the damped/undamped transport matrix is evident only for a mis-aligned ring.
- It affects only (?) $\boldsymbol{P_y}$.
- The impact is larger at larger energy.

Summary

Due to the demanding IR optics design and the machine size, establishing a closed orbit and keeping a stable machine look challenging.

- Beam polarization is obtained “for free” through Sokolov-Ternov effect.
  - At 45 GeV wigglers are required to get $\tau_{10\%} \approx$ 2-3 h.
    They do not harm polarization.
- P_∞ depends on how well is the machine aligned/corrected, requirements becoming stricter at high energy.
  - Extremely well corrected orbit/optics is required for a large chromatic machine with β^*_y=0.8 - 1 mm as FCC-ee to work and meet required performance.
    - This benefits also polarization.
- The puzzling small P_y, in particular at 80 GeV, has been likely understood.

Thanks!