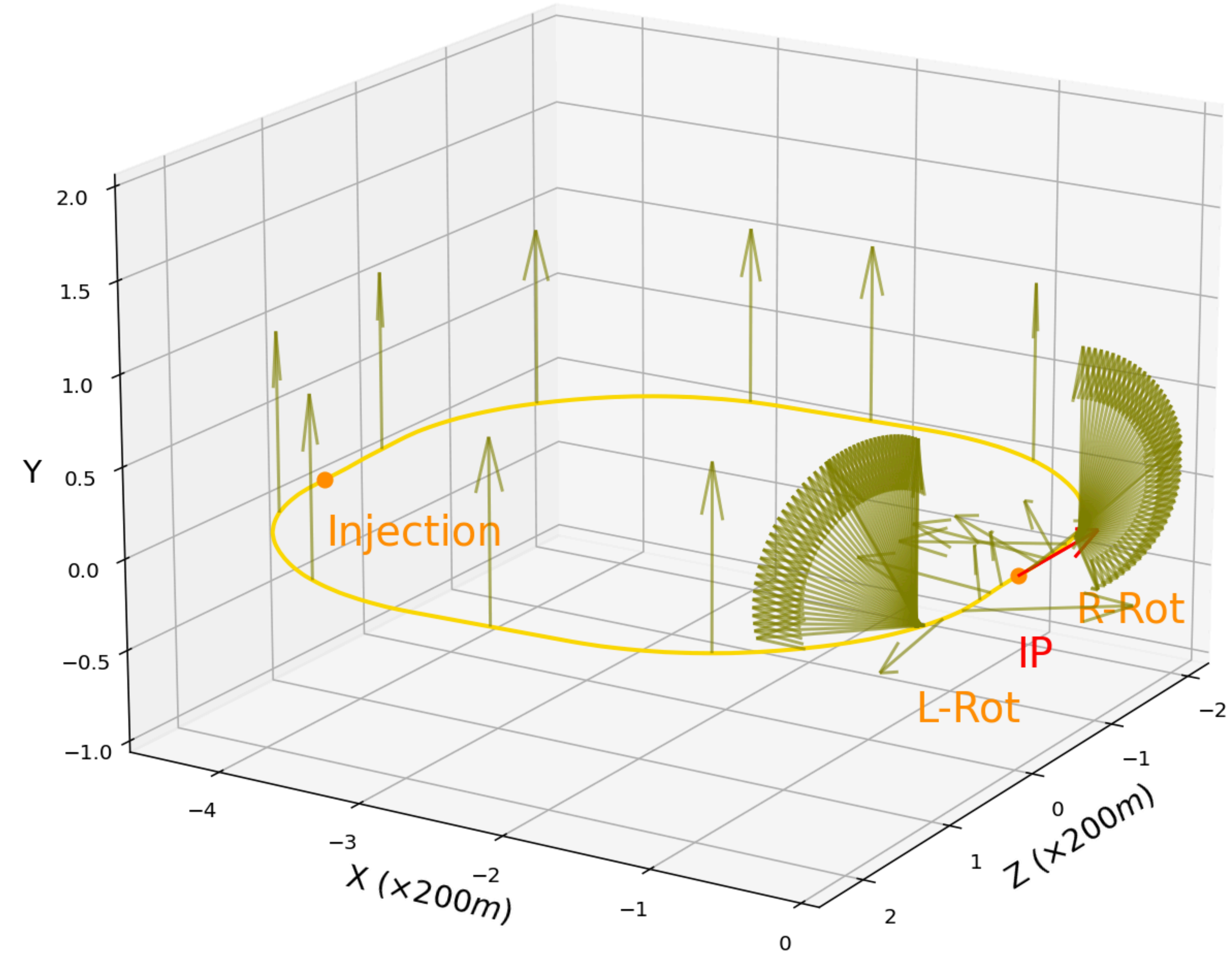
Spin Studies with BMAD for a SuperKEKB Polarization Upgrade



Yuhao Peng

2022.09.22

Outline

- Physics Motivation
- Spin rotator conceptual design overview
- Simulation tool: BMAD
- Simulation procedure and result
- Injection Linac layout
- Polarization studies in the Linac

Physics Motivation

Design a pair of spin rotators for SuperKEKB High Energy Ring (electron ring), to polarize the spin of the electron beam in the longitudinal direction at the interaction point (IP)

- Study of asymmetry between the identical processes with different electron beam handedness, which provides precision electroweak measurements

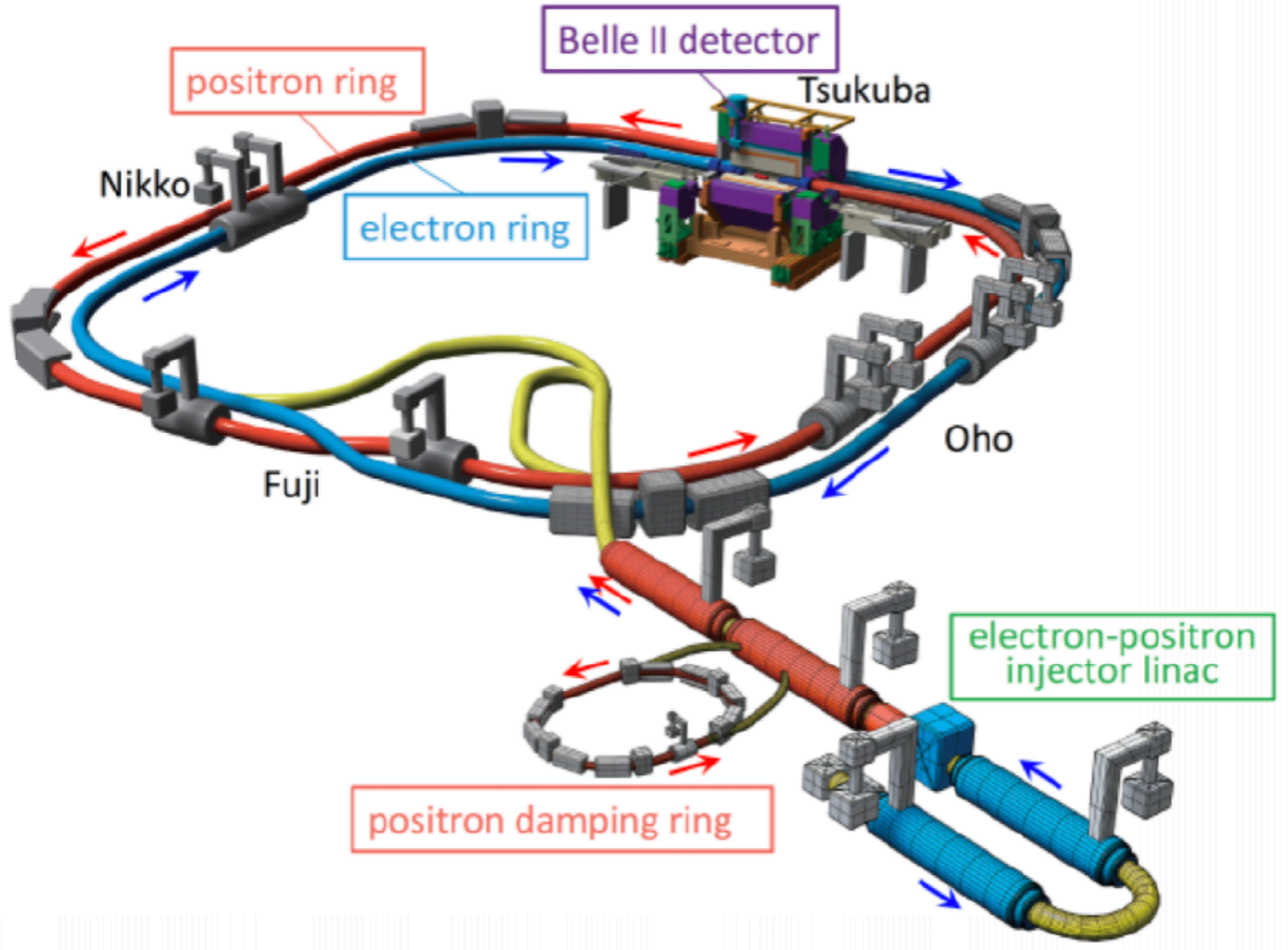
$$A_{LR}^{f} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} = \frac{sG_F}{\sqrt{2}\pi\alpha Q_f} g_A^e g_V^f \langle Pol \rangle \propto T_3^f - 2Q_f \sin\theta_W$$

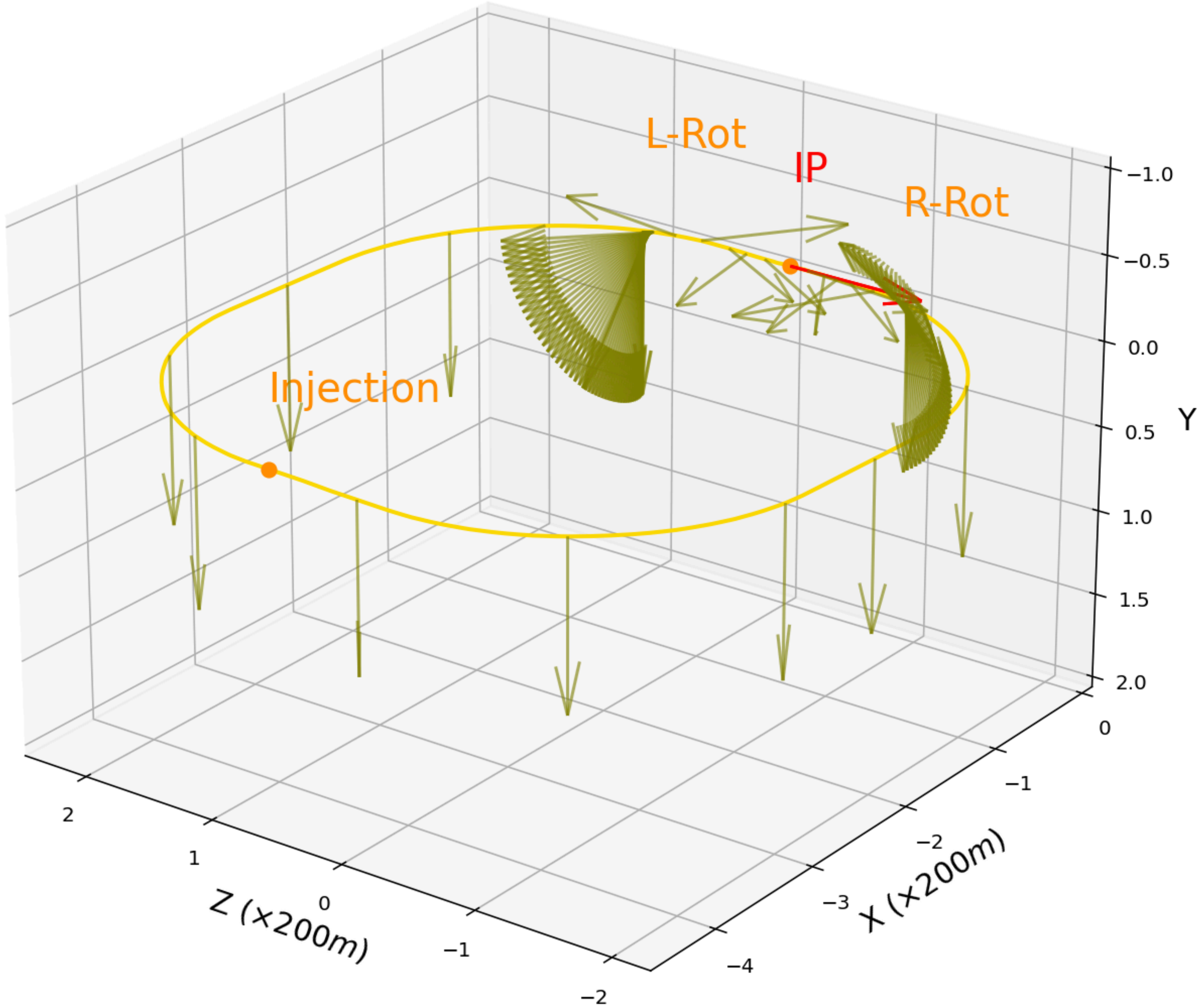
With

$$\langle Pol \rangle = \frac{1}{2}\left[\left(\frac{N_{eR} - N_{eL}}{N_{eR} + N_{eL}}\right)_{\mathrm{R}} - \left(\frac{N_{eR} - N_{eL}}{N_{eR} + N_{eL}}\right)_{\mathrm{L}}\right]$$

See more details in Tuesday’s talk:
“Polarized beams proposal for SuperKEKB” by M. Roney

Overview of the concept for a potential polarization upgrade

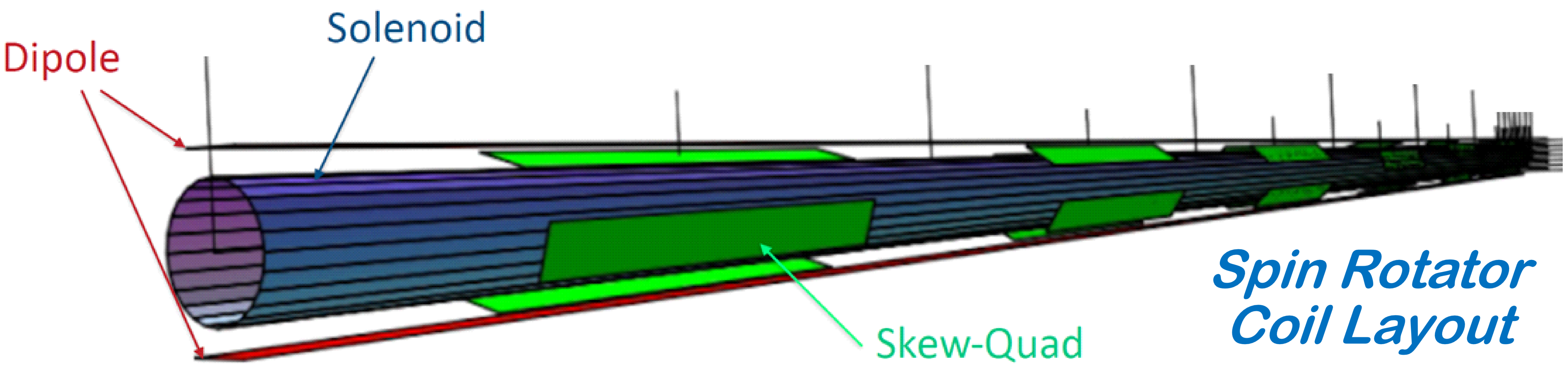




- Inject transverse polarized electron beam into the High Energy Ring (HER)

- A pair of spin rotators installed on both sides of the IP to rotate the spin to the longitudinal direction at the IP and back to transverse after IP

Spin Rotator Magnet Structure



Spin Rotator Coil Layout

Follows Uli Wienands's (Argonne National Laboratory) idea and direction:

- replace some existing ring dipoles on both sides of the IP with the dipole-solenoid combined function magnets and keep the original dipole strength to preserve the machine geometry

- Install 6 skew-quadruple on top of each rotator section to compensate for the x-y plane coupling caused by solenoids

- Original machine can be recovered by turning off sol-quad field

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Constraints of the Design

✤ **Transparency**: Need to maintain the original **beam dynamics,** make the spin rotator transparent to the ring as much as possible (the spin rotator is for the polarization purpose only)

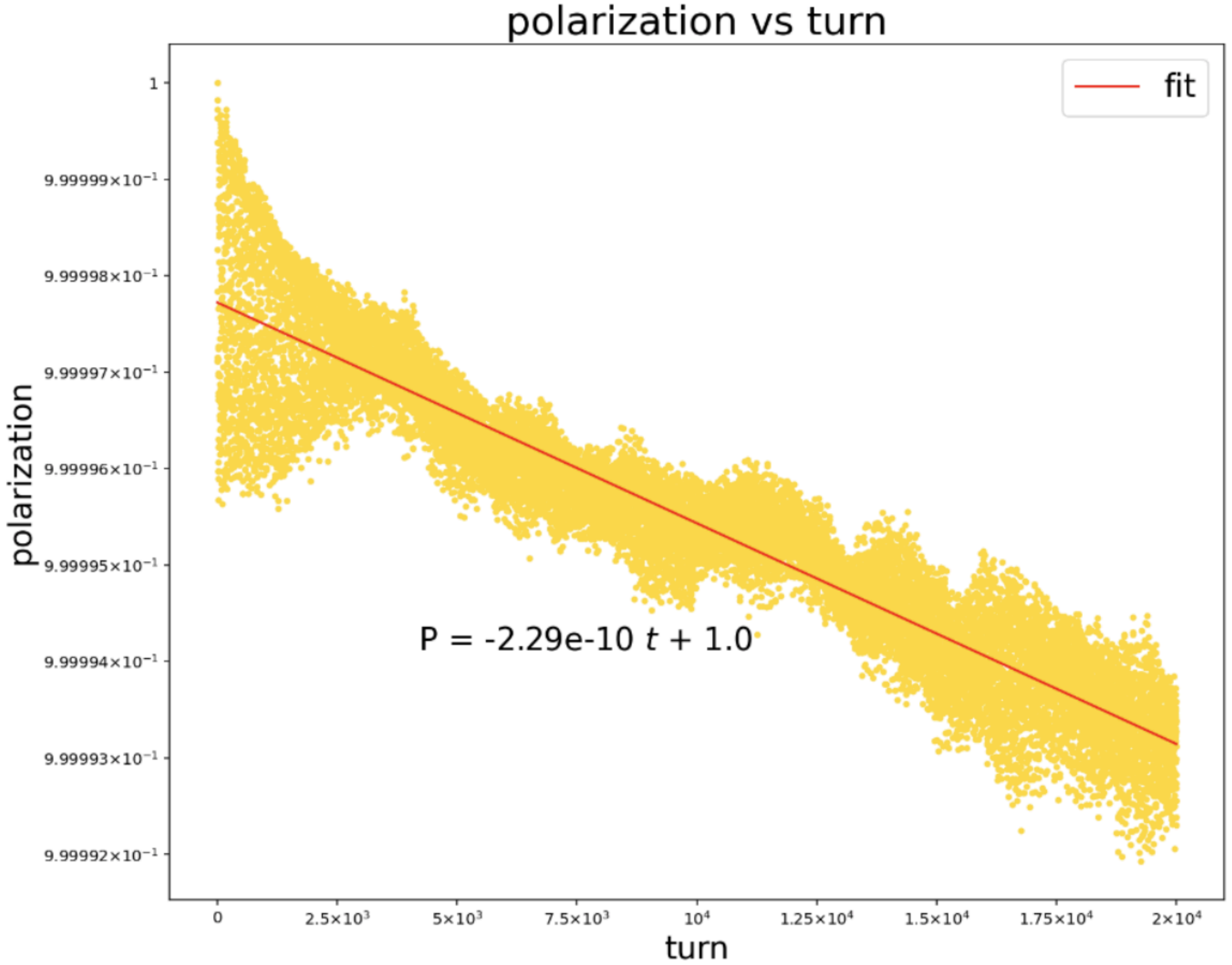
✤ Physical constraints: All new magnets must be manufacturable and installable

- Solenoid strength can not exceed **5** T
- Skew-quad can not exceed **30** T/m (~ 3T at the coil)

Simulation Tool

- **BMAD** is an open-source software library created/maintained by David Sagan at Cornell University for simulating charged particles and X-rays
- Étienne Forest's "Polymorphic Tracking Code" (**PTC**) is incorporated into it
- Use Tao interface to BMAD to perform the optimization
- Optimization Algorithm: **LMDIF** is to minimize the sum of the squares of nonlinear functions by a modification of the Levenberg-Marquardt algorithm
- The BMAD lattice file of the HER with Crab Waist is translated from SAD, which is provided by Demin Zhou (KEK)

Validation of transverse spin lifetime in the HER



- Tracking 100 particles for 20000 turns in the HER with BMAD

- Based on this study, the estimated polarization lifetime > 10 hours

Procedure of the Rot Design

- ✤Identify which dipoles (use 4 “B2E” dipoles) to replace with the spin rotator (dipole-solenoid-quadrupole combined function magnets)

- ✤Build the rotator magnet model with BMAD

- ✤Spin polarization :
  - Fit solenoids to longitudinally polarize the electron beam at the IP, and restore vertical polarization after IP

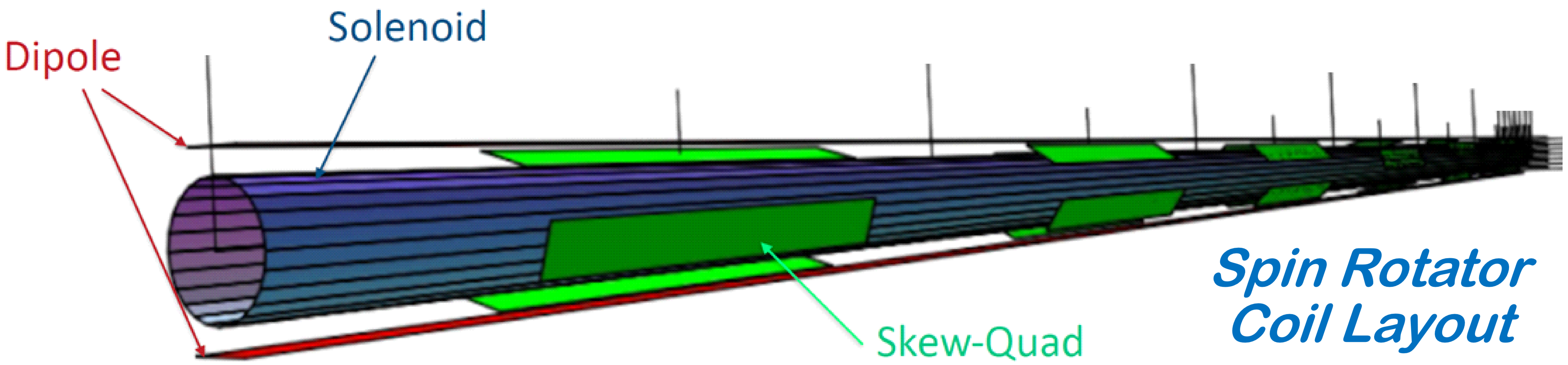
- ✤Transparency:
  - Fit skew-quads to decouple the x-y plane
  - Rematch the optics(Twiss parameters and dispersion functions) by adjusting existing ring quads near the rotator region
  - Maintain Tune value Q
  - Match the first order chromaticity by tuning existing ring sextupoles

Rotator Modelling with BMAD

Rotator modelling requires a combination of dipole (curved element) and solenoid-quadrupole (straight element)

- BMAD has solenoid-quadrupole(Sol_Quad) but does not have dipole-solenoid-quadrupole

- Following David Sagan's suggestion, use hkick(horizontal kick) to simulate the dipole(Sbend) and patches to correct the reference orbit

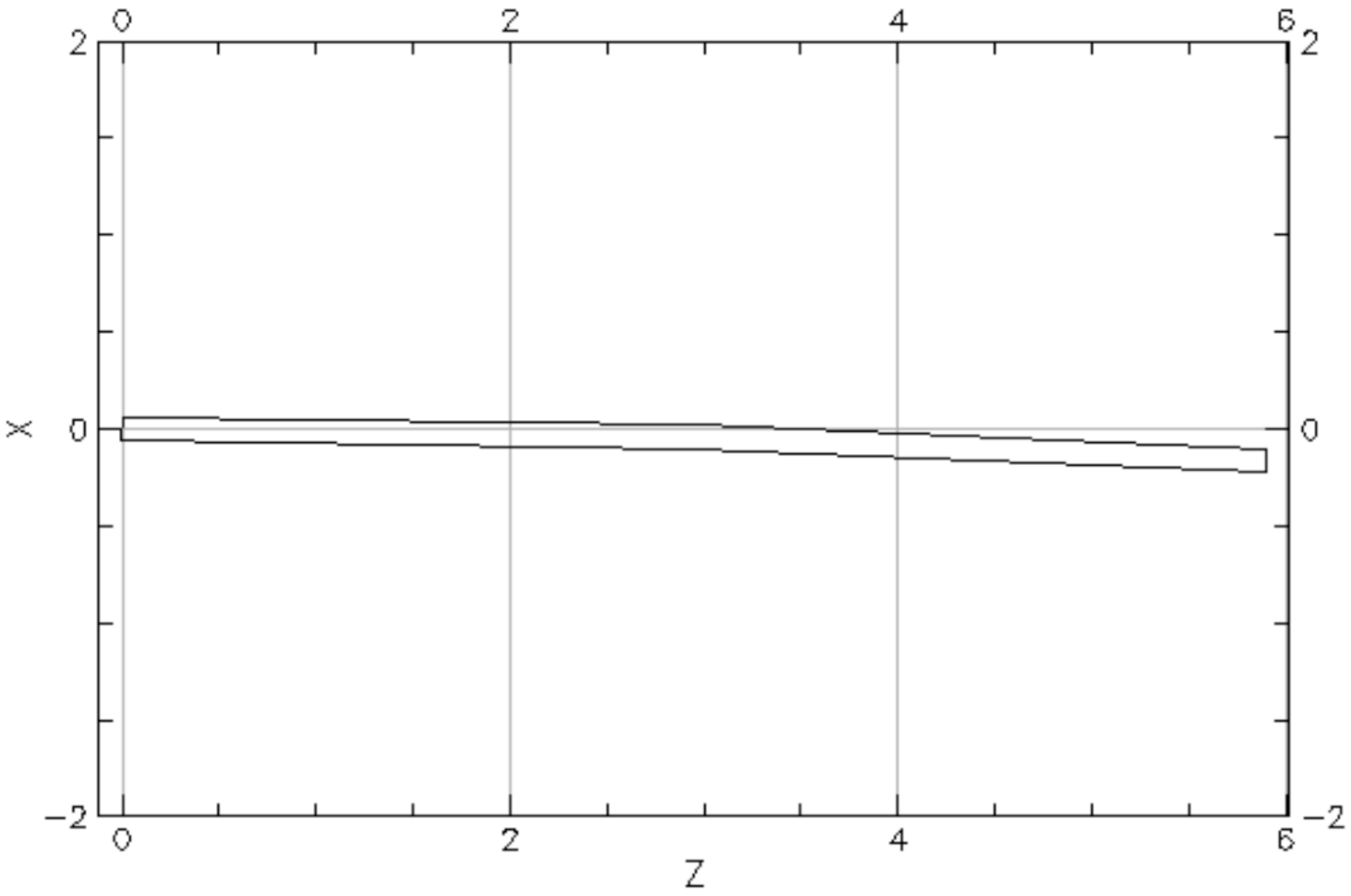


Spin Rotator Coil Layout

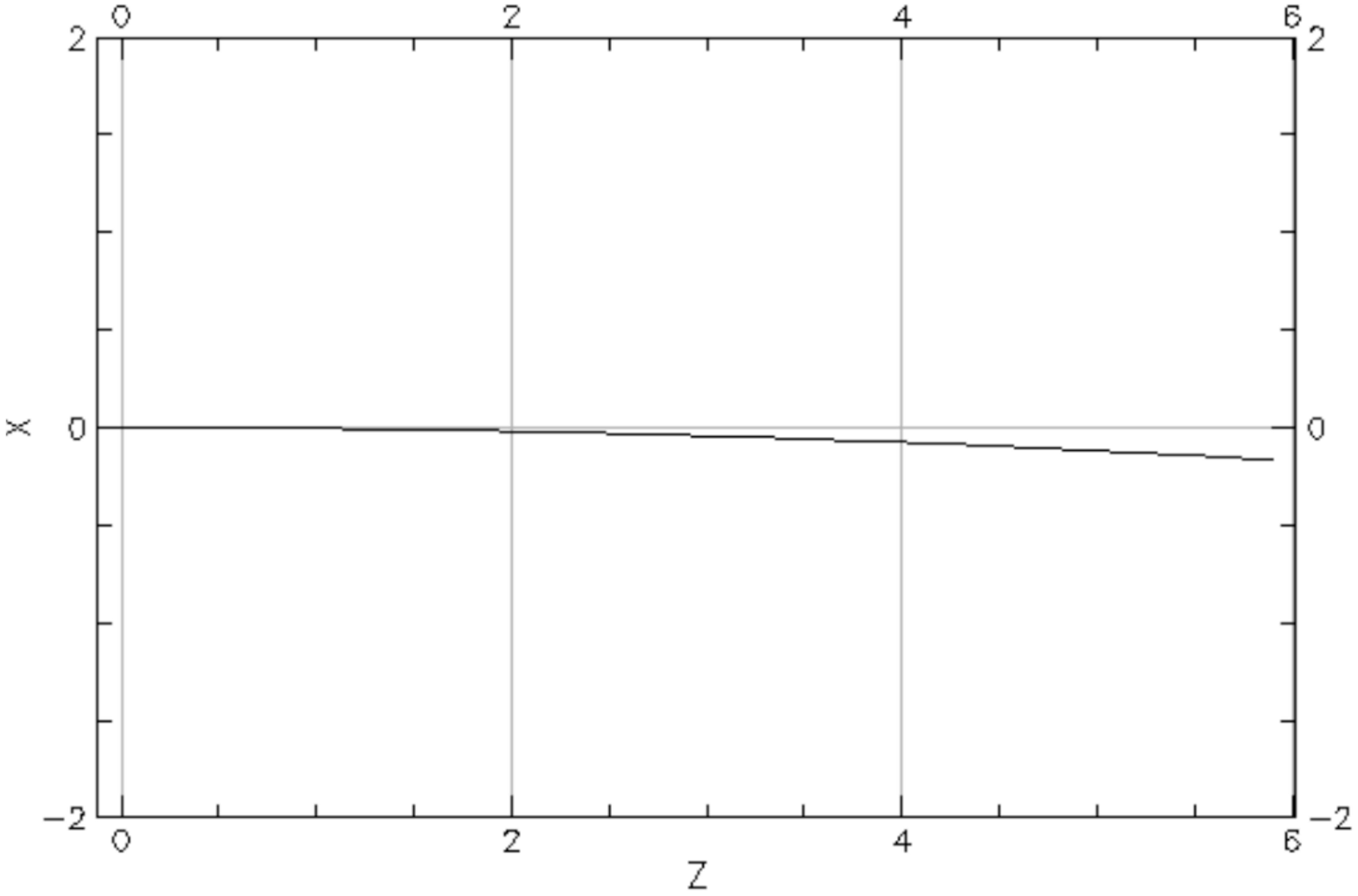
Modelling dipoles with Hkicks in BMAD

- Hkick strength is set to be the same as the dipole
- Initially sliced into 6 pieces to match the number of skew-quads
- Use patches to shift the reference orbit(x,x’,y,y’) at the exit of each piece
- Increase the number of slices to obtain a better model of the dipole

Comparison of geometry between the B2E and the Hkick after fixing the reference orbit with patch

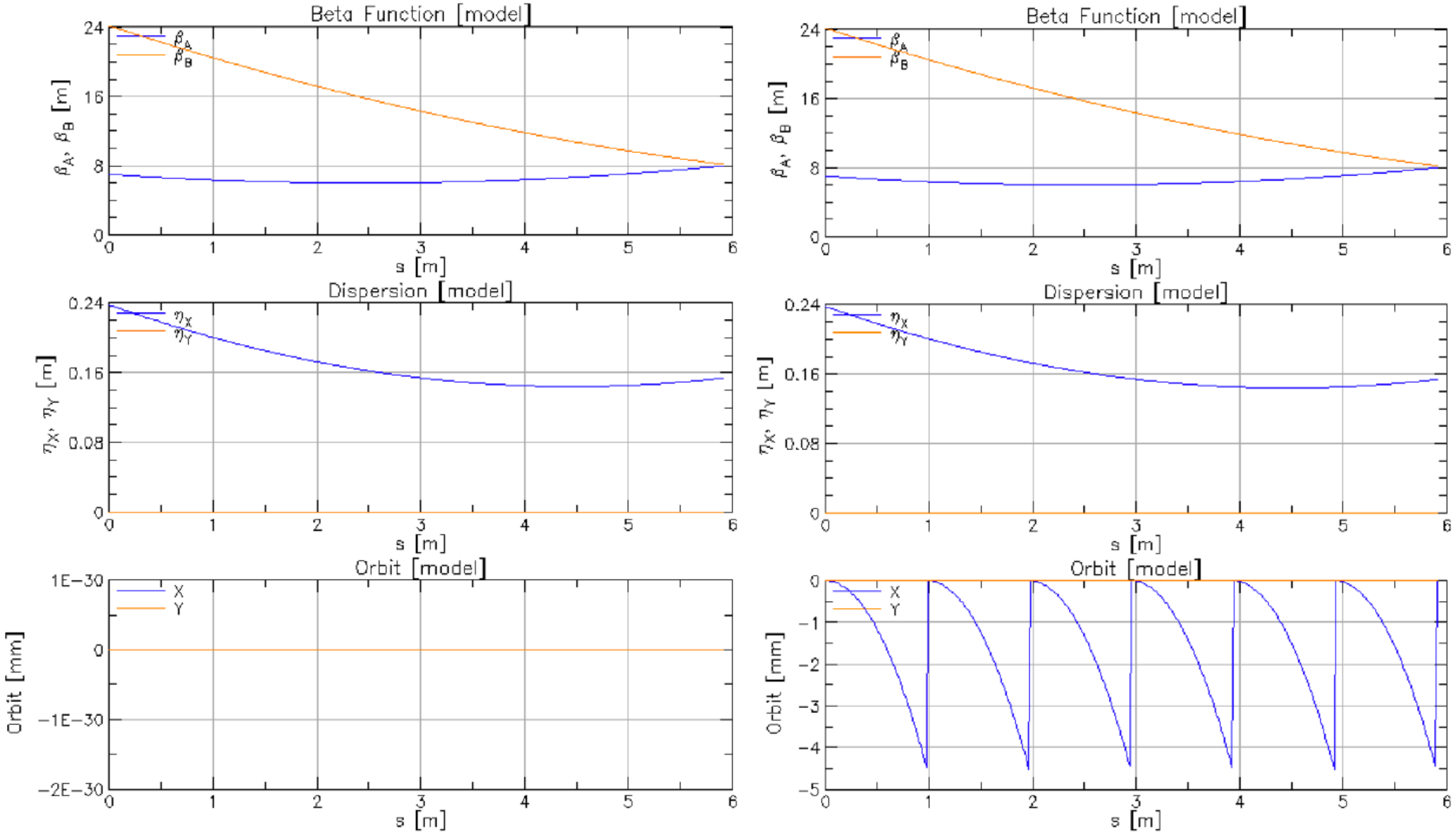


B2E



Hkick(6-piece sliced)

Comparison of orbit and Optical functions



Beta

Dispersion

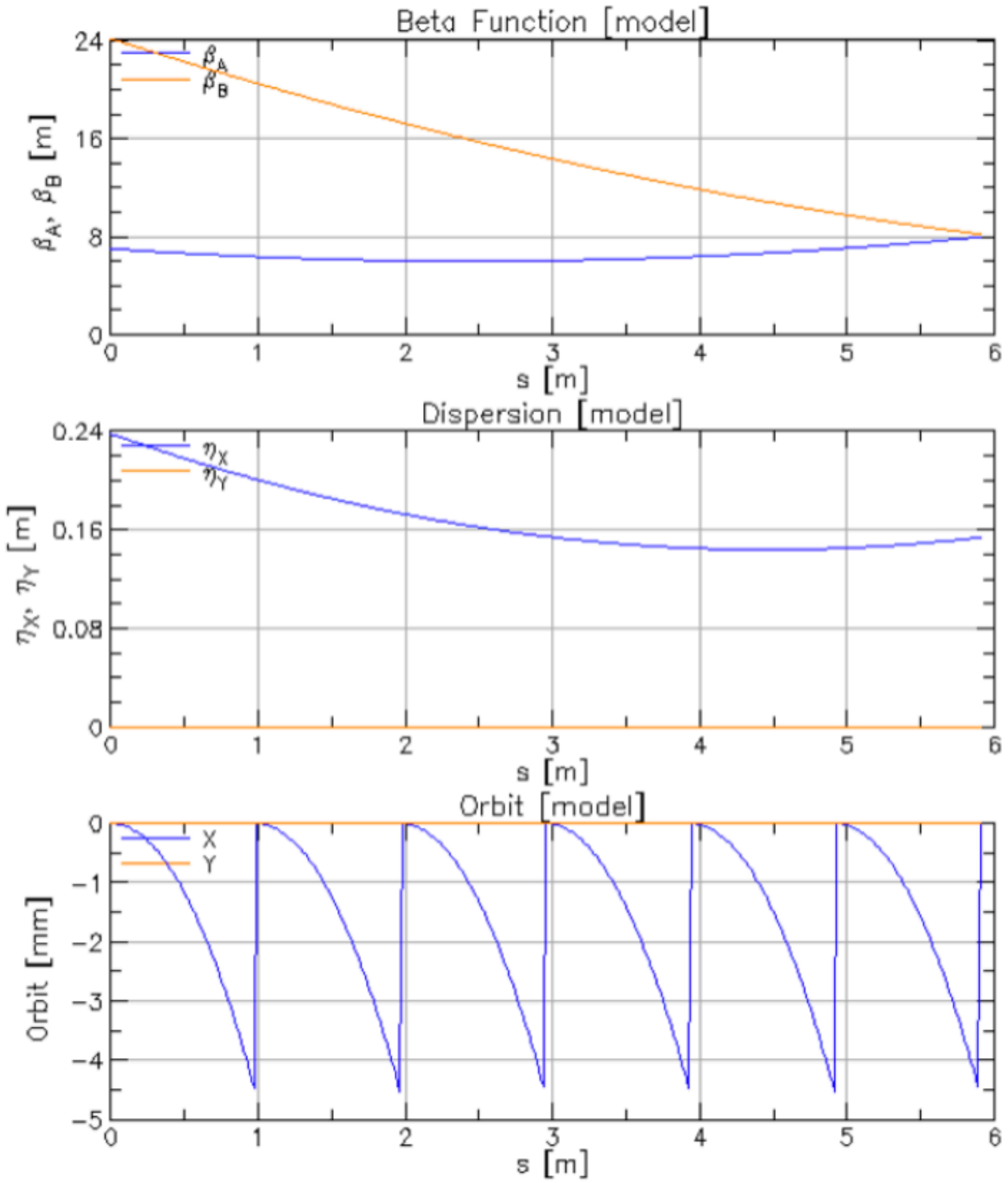
Orbit

Original B2E

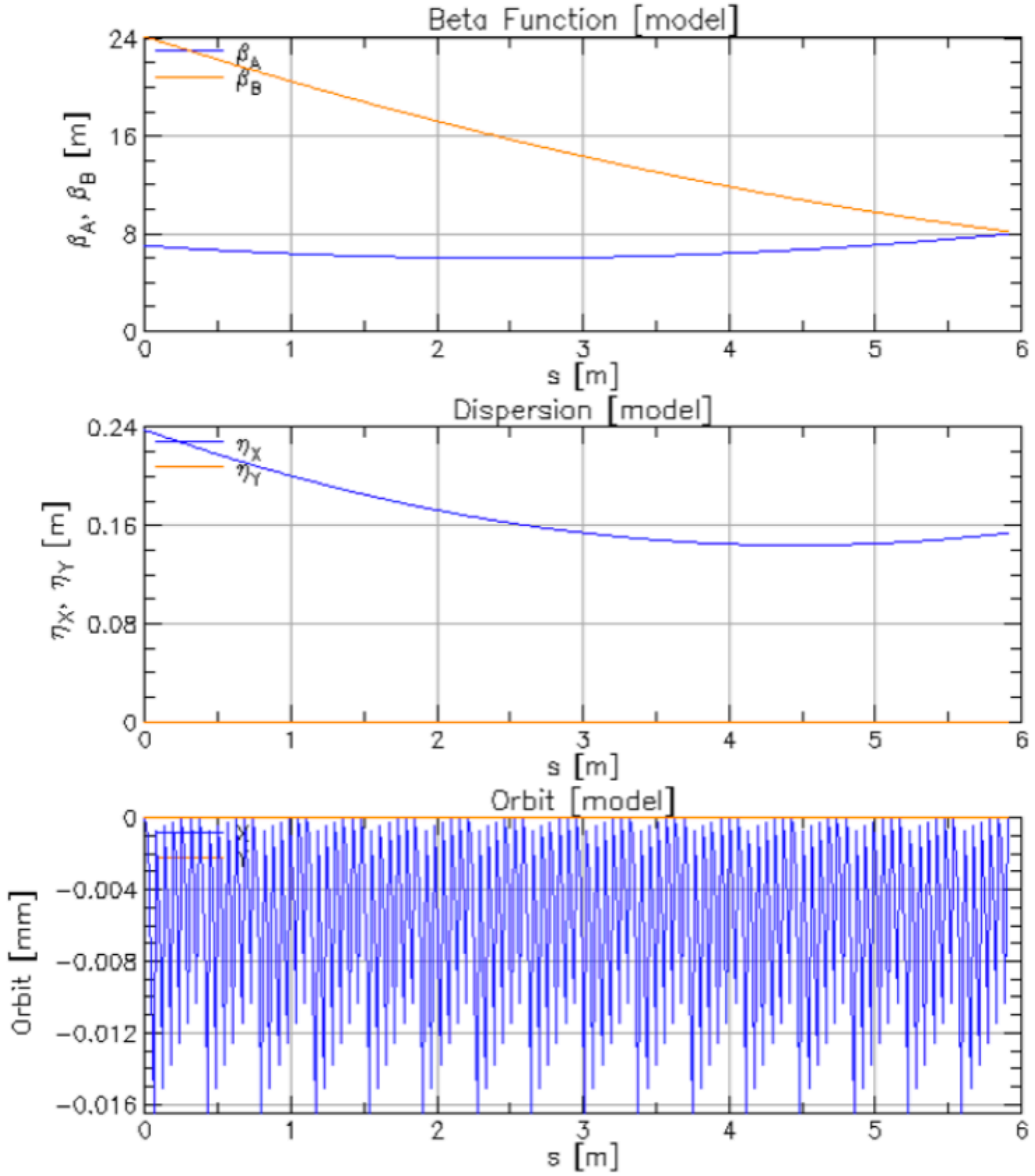
Hkick(6-piece sliced)

The sawtooth shape orbit excursion is not physical, it’s an artificial effect due to using the patch elements

Slice Model



Stand-alone Model(6-pieces)



Slice Model(96-pieces)

In order to reduce the non-physical orbit excursion, each piece of the hkick is further sliced into 16 pieces, 96 in total

Validating Hkick modelling of dipoles

Replace 4 “B2E”(where the rotator magnets will be installed) with hkicks(no solenoid-quadrupole) in the full HER lattice

- Check if the floor coordinate is the same as the original (global geometry)

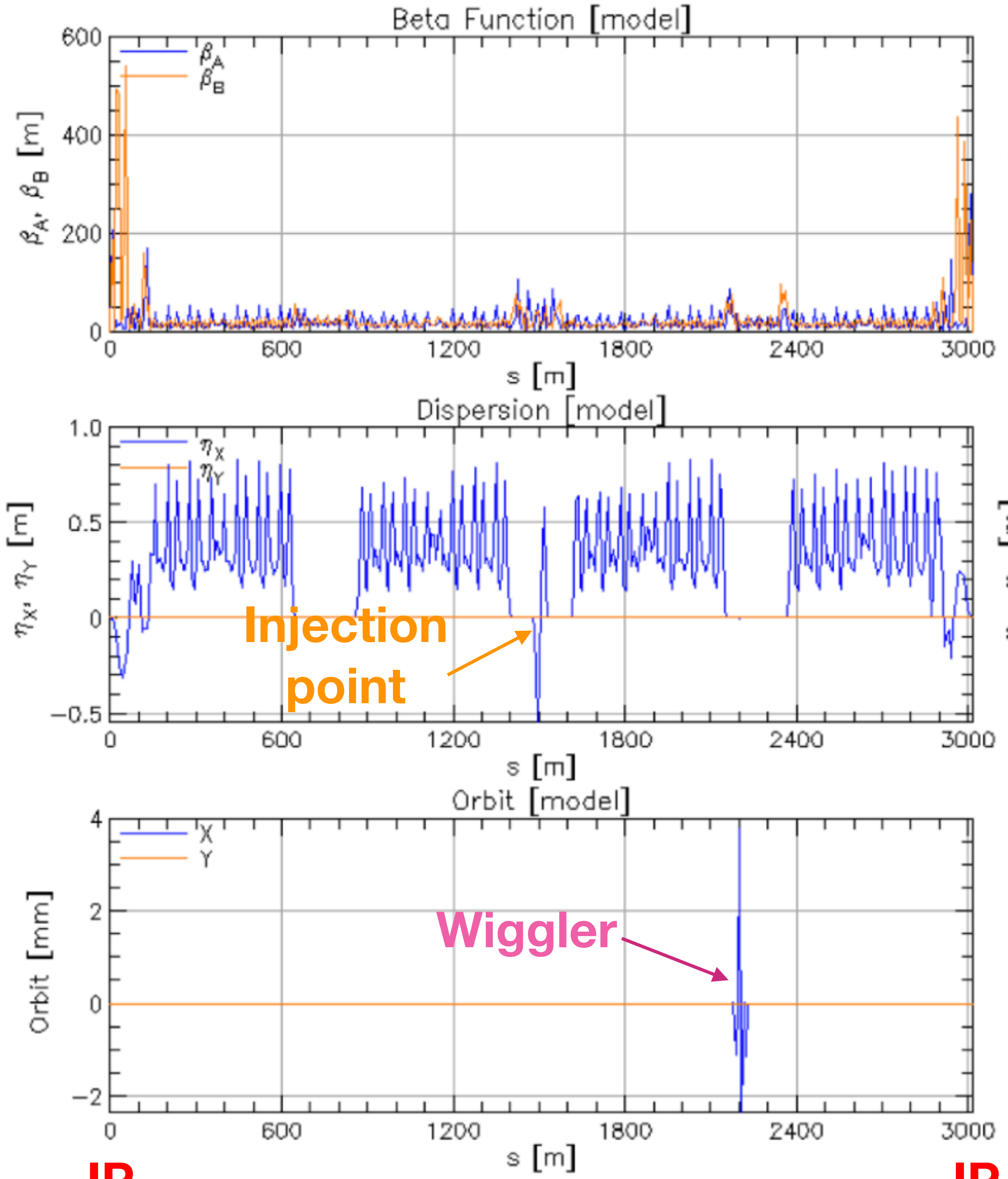
- Check if the orbit, optical functions, and ring parameters… are the same as the original

Comparison of original HER with version having Hkick modelling of the 4 B2E dipoles

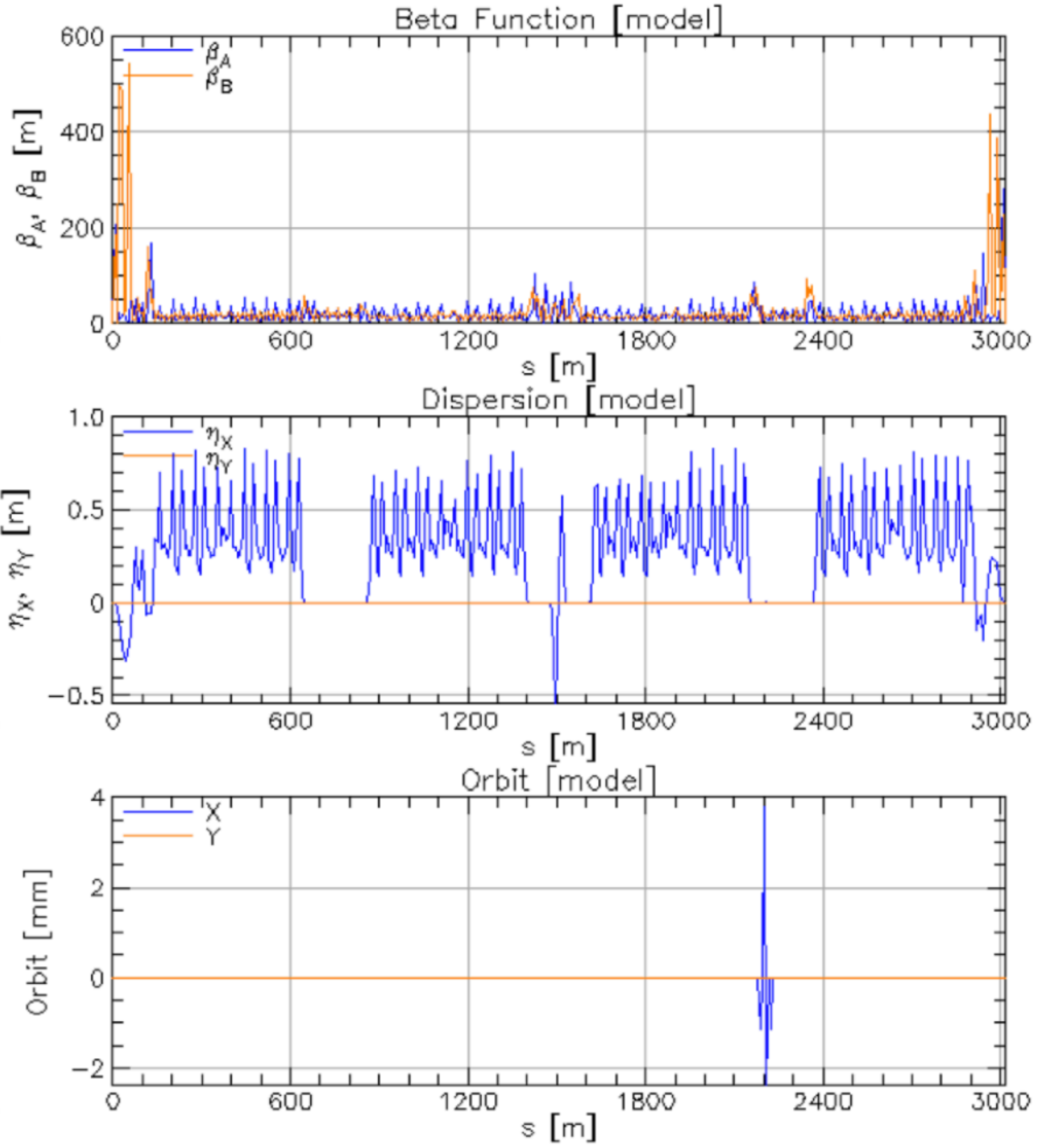
Beta

Dispersion

Orbit

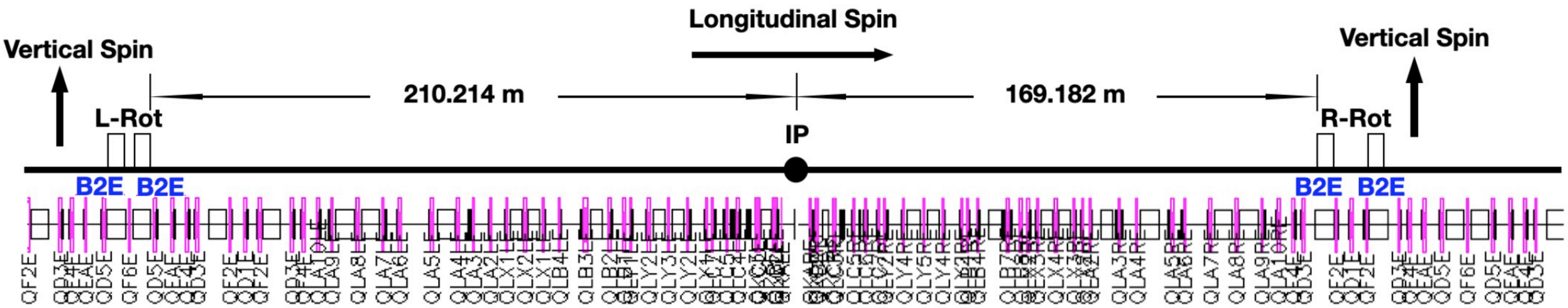


Original



Hkick (Slice Model)

Rotator Structure



- Left Rotator (L-Rot) rotates the spin from the vertical to the horizontal plane
- Right Rotator (R-Rot) rotates the spin back to the vertical direction
- 4 **B2E** dipoles shown above to be replaced with the spin rotator magnets

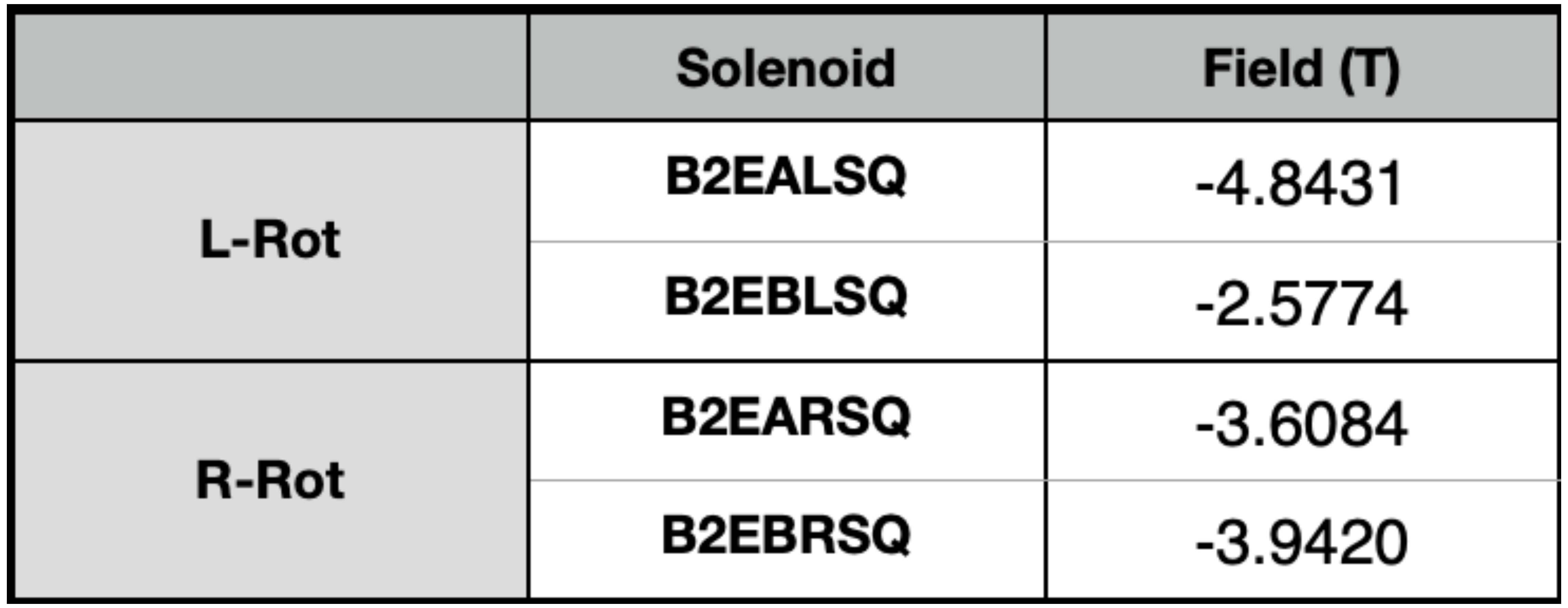
Open-geometry Optimization

Replace B2E with Rotator lattice elements, and perform the optimization in the lattice segment containing the L-Rot and nearby elements, and repeat the same procedure for the R-Rot

- Fit the solenoids to match the spin
- Fit the skew-quads to perform decoupling
- Adjust the ring quads near the rotator region to achieve the optical rematch

Optimization Result

- Solenoid strength, below 5T

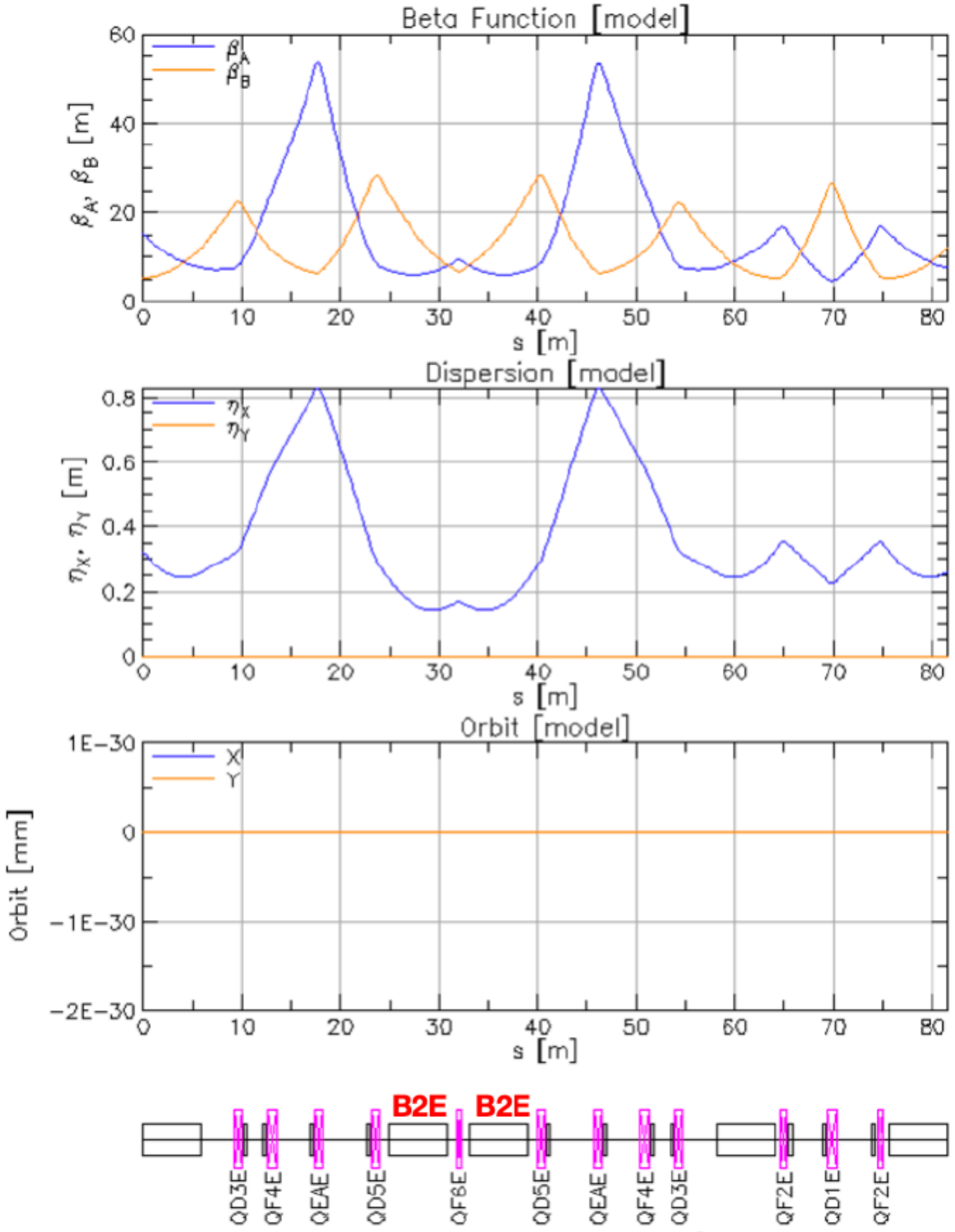


	Solenoid	Field (T)
L-Rot	**B2EALSQ**	-4.8431
	B2EBLSQ	-2.5774
R-Rot	**B2EARSQ**	-3.6084
	B2EBRSQ	-3.9420

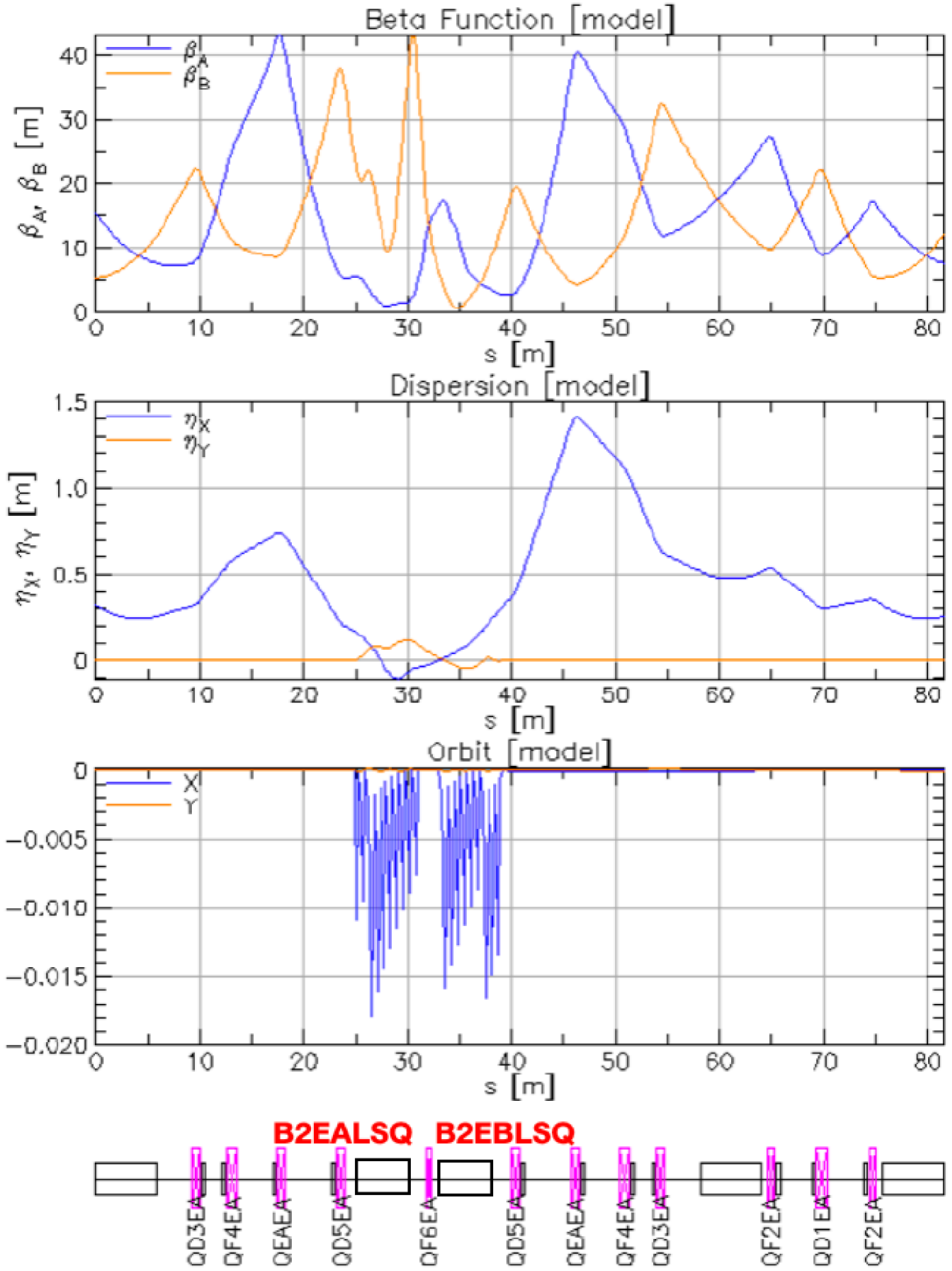
- Maximum skew-quad strength is ~20 T/m, which is below the physical limit 30 T/m, see appendix

- Maximum Ring quad is ~ 14 T/m, which is achievable , see appendix

Comparison at L-Rot tuning area after completing the optical rematch



Original



L-Rot

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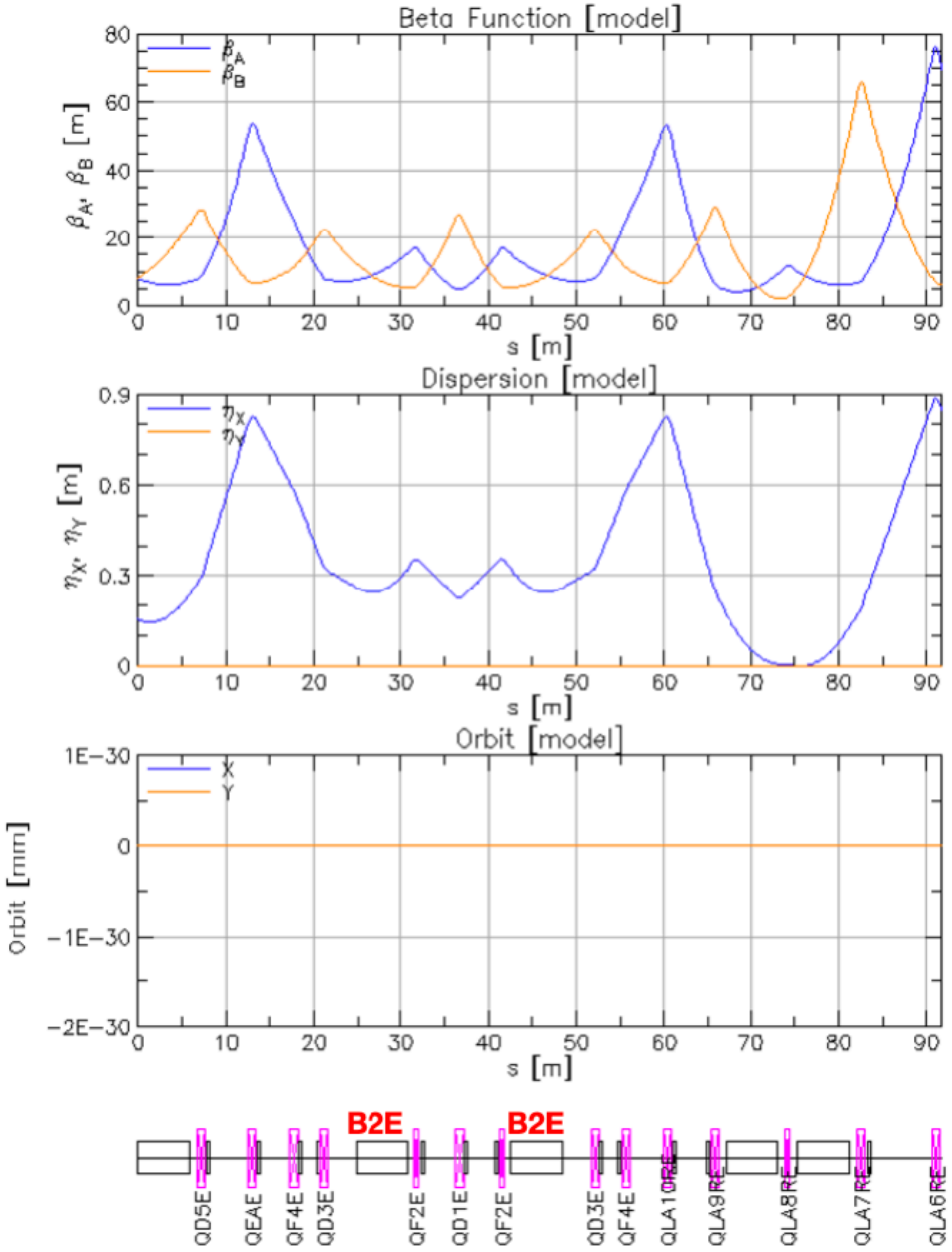
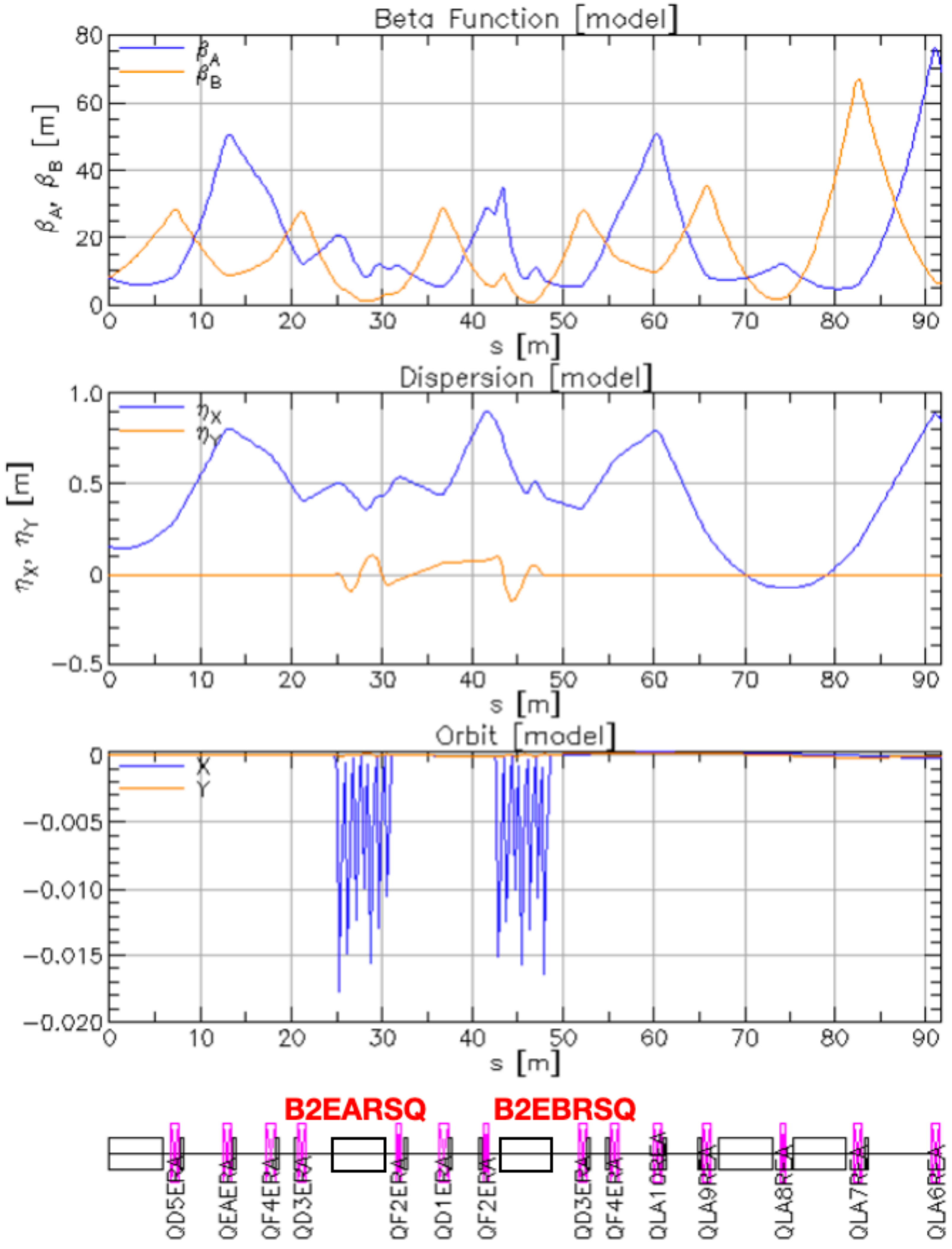
Comparison at R-Rot tuning area after completing the optical rematch



Original



R-Rot

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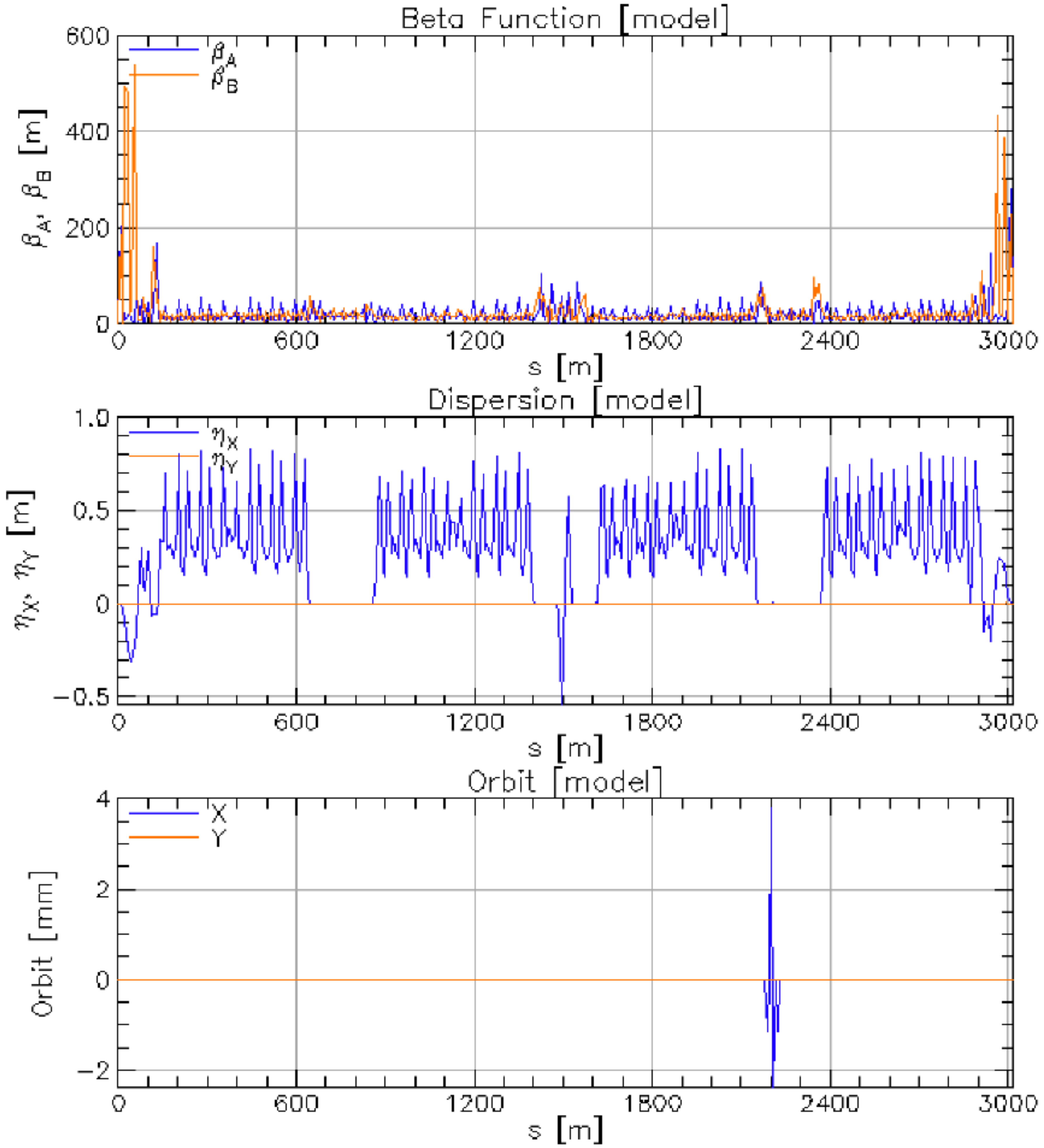
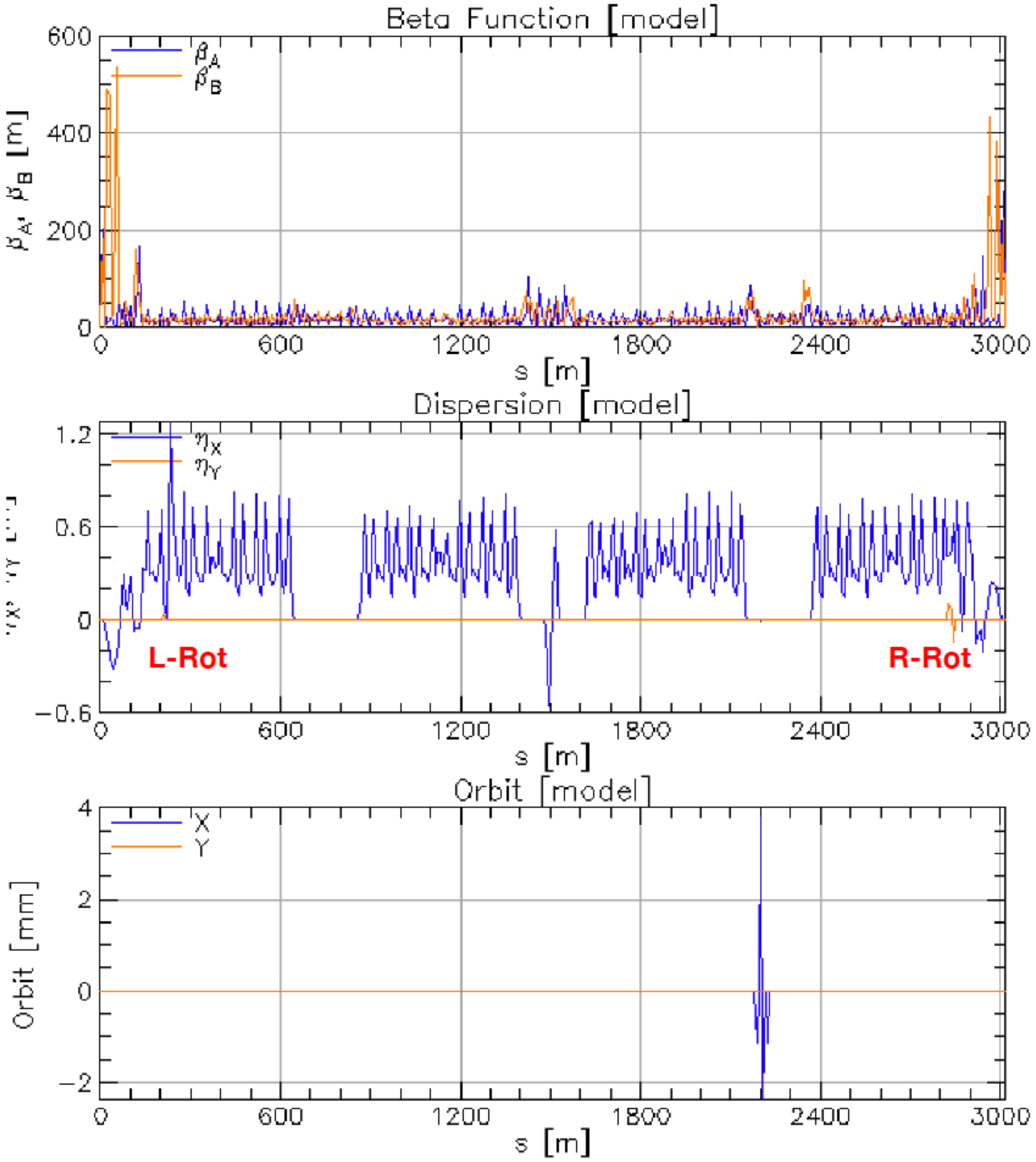
Full lattice Comparison with L/R-Rot installed in the ring

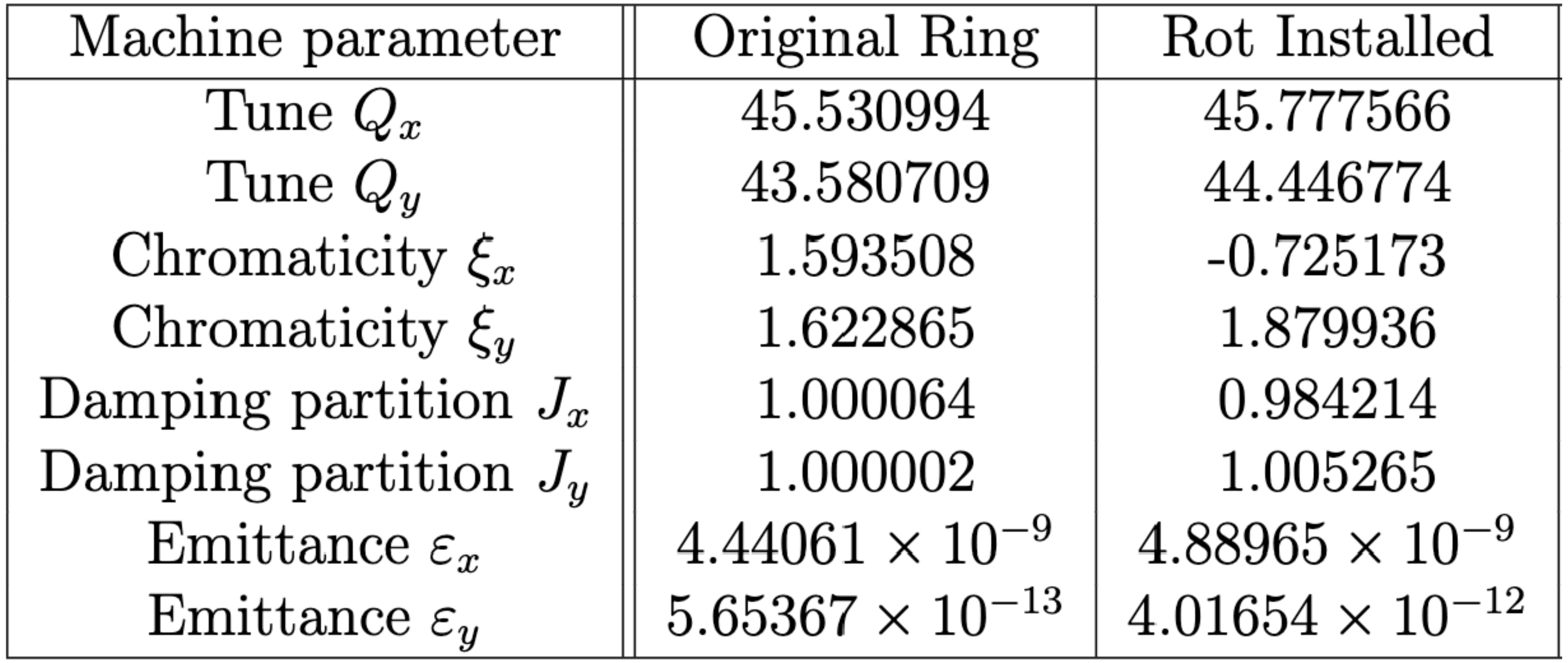


Original Ring



Rotator Ring

Ring Parameters Comparison before performing the closed-geometry optimization



Machine parameter	Original Ring	Rot Installed
Tune Q_x	45.530994	45.777566
Tune Q_y	43.580709	44.446774
Chromaticity ξ_x	1.593508	-0.725173
Chromaticity ξ_y	1.622865	1.879936
Damping partition J_x	1.000064	0.984214
Damping partition J_y	1.000002	1.005265
Emittance ε_x	4.44061×10^{-9}	4.88965×10^{-9}
Emittance ε_y	5.65367×10^{-13}	4.01654×10^{-12}

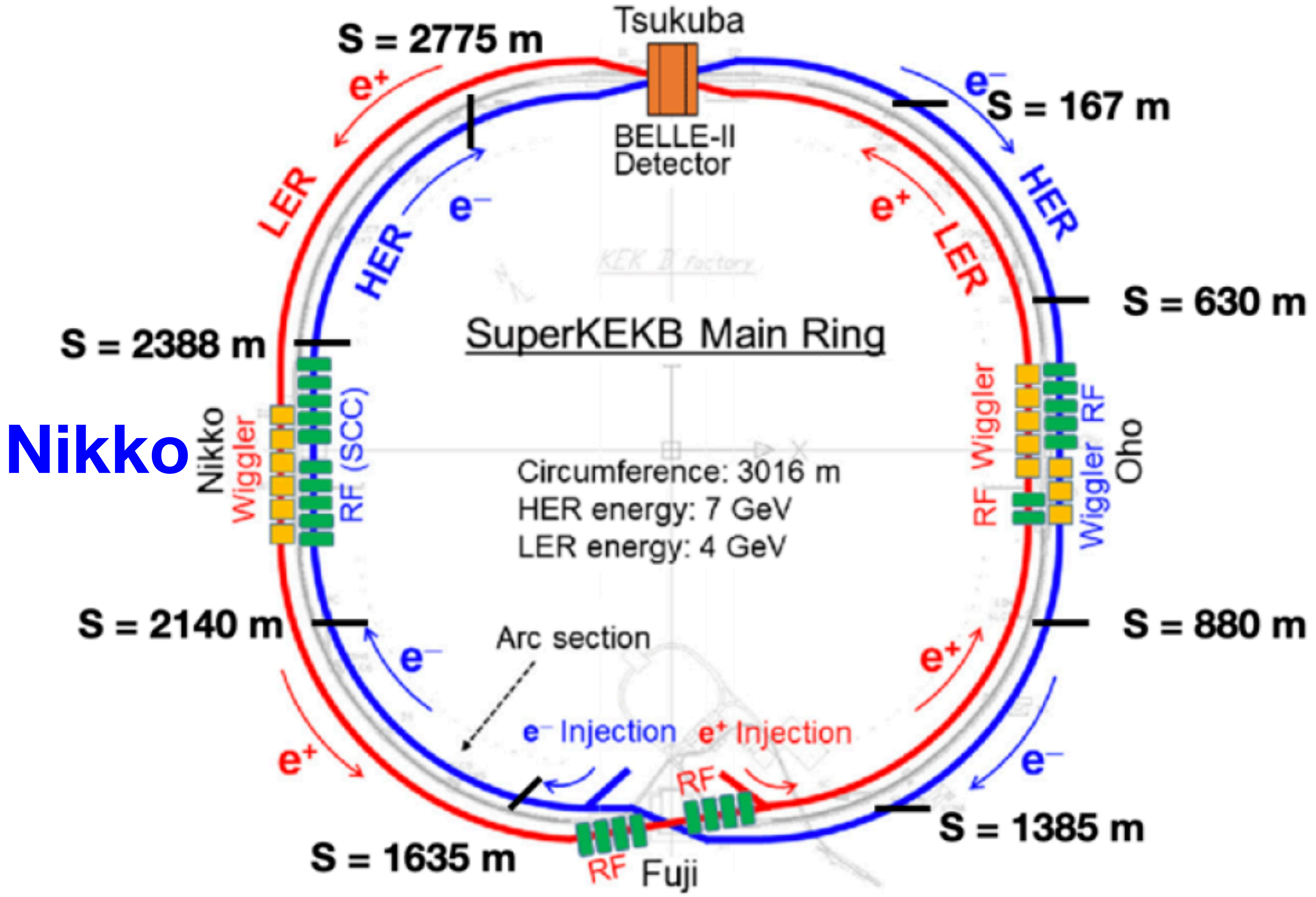
- Tune and Chromaticity needs to be rematched

- Although the vertical emittance is higher than the original, it is 3 times smaller than the current design of 12.9 pm

Closed-geometry Optimization

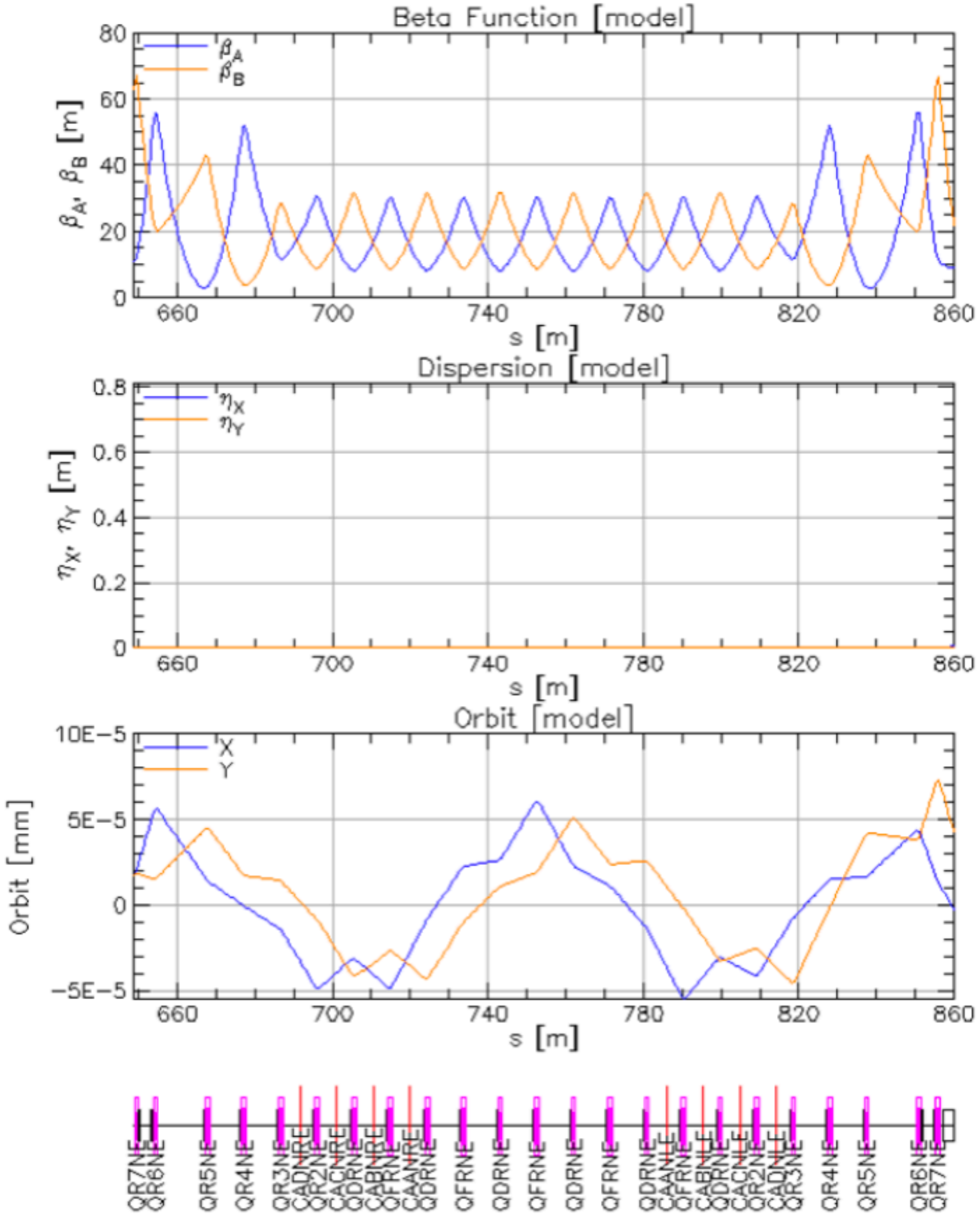
- Tune $Q \equiv \dfrac{\Delta\Psi}{2\pi} = \dfrac{1}{2\pi}\oint \dfrac{ds}{\beta(s)}$

- Chromaticity $\xi \equiv \dfrac{\Delta Q}{\Delta p/p} = -\dfrac{1}{4\pi}\oint k(s)\beta(s)ds$



They are overall ring (circular machine) parameters, can only be calculated in closed-geometry

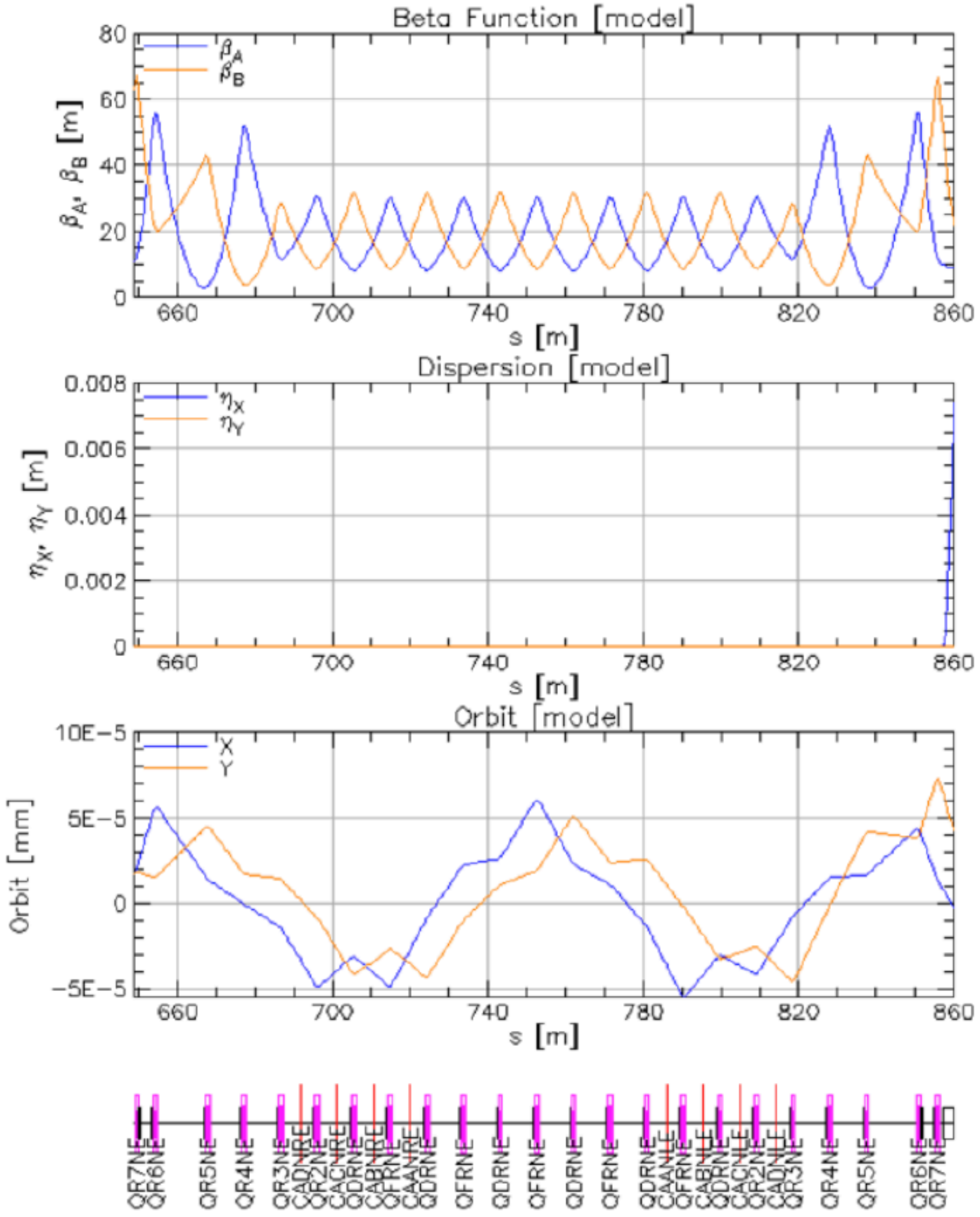
Tune Match



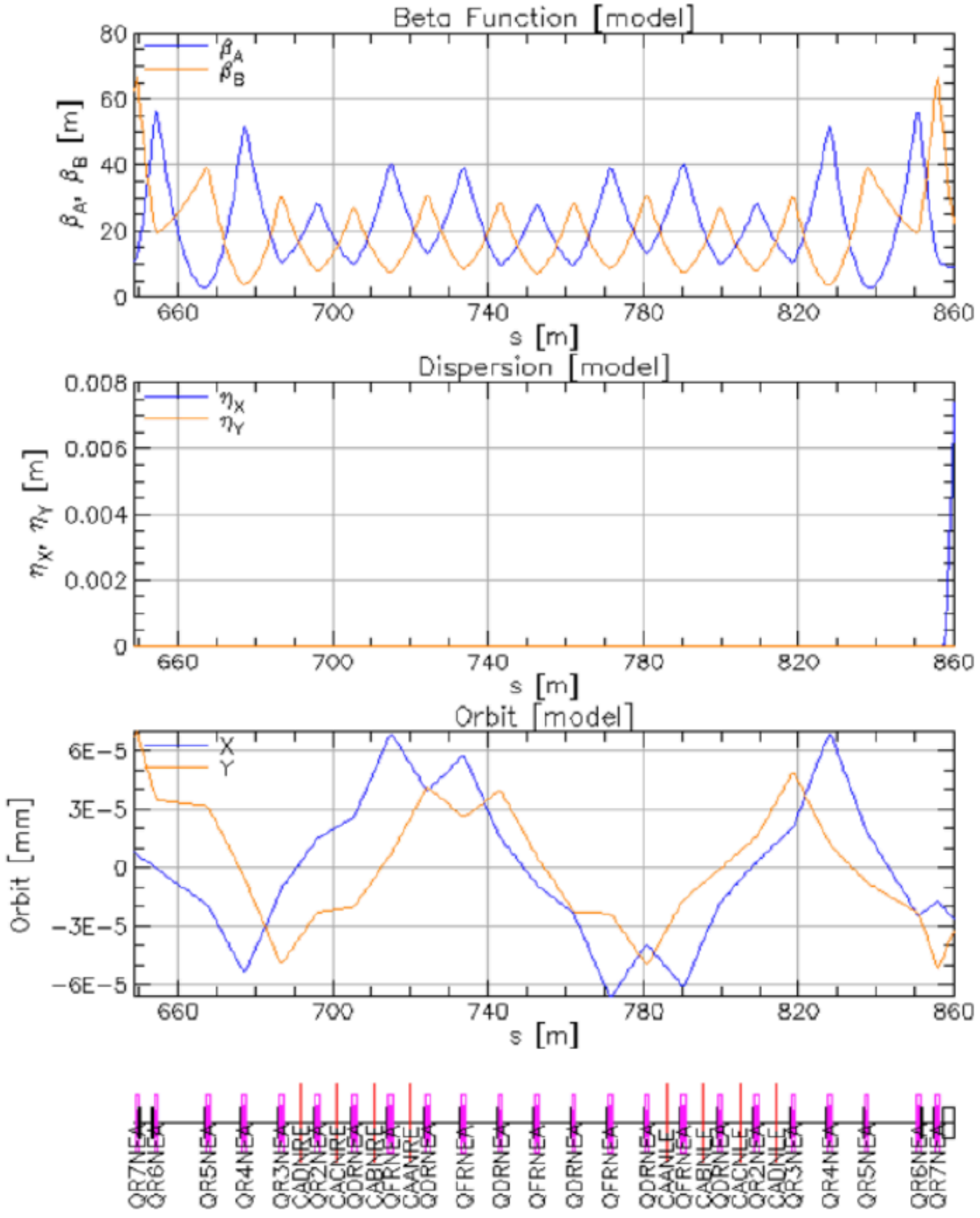
Original

- Adjust quadrupoles at "Nikko" section where the dispersion is zero
- Fitting target: Q_x, Q_y
- Constraints: Matching the Twiss parameters at the exit of the Nikko section ($\beta_{x,y}, \alpha_{x,y}$)
- 8 variables: QR*NE(6 different Quadrupole pairs), QDRNE, QFRNE

Comparison of “Nikko” section after optimization



Original



Rot

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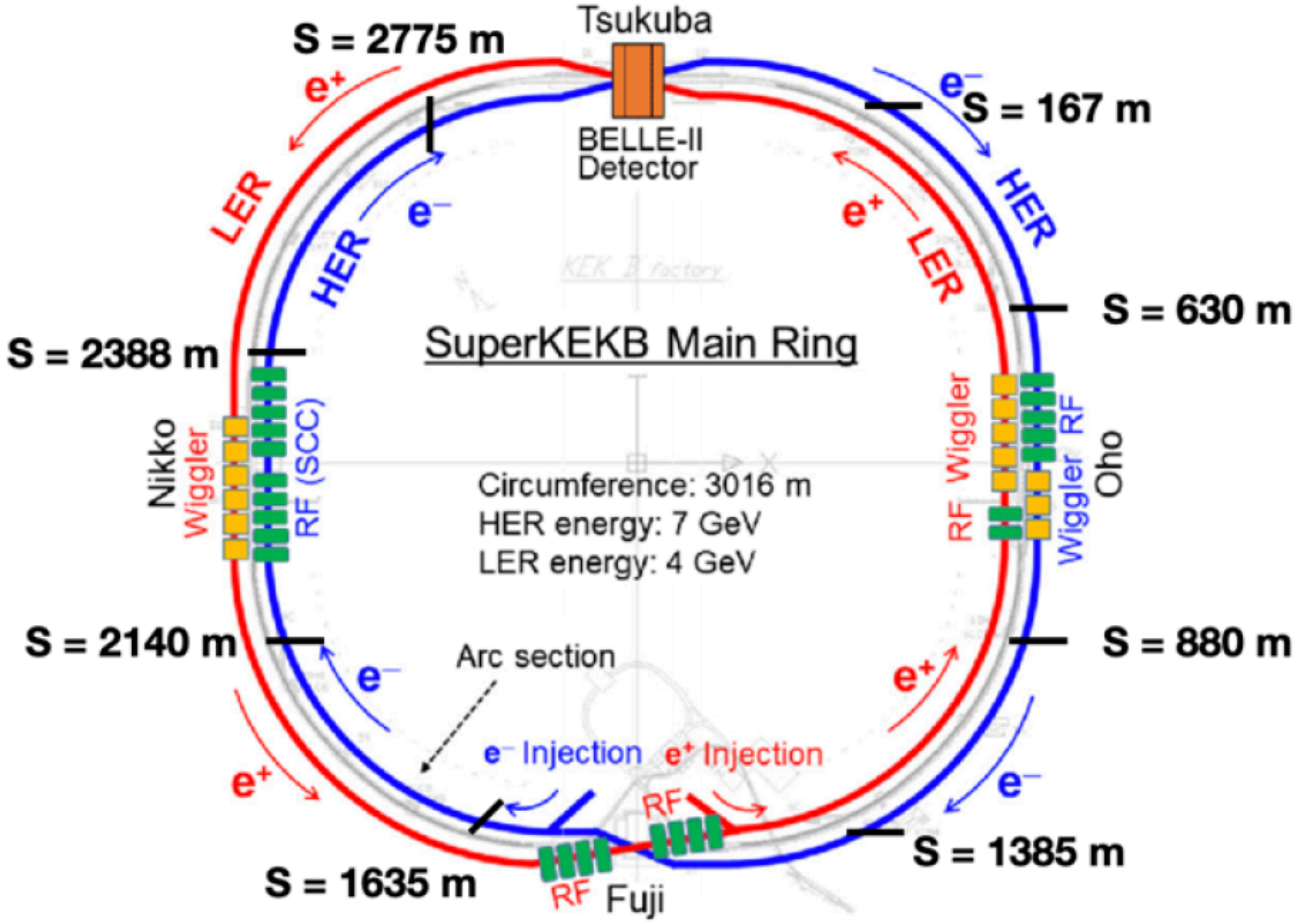
Chromaticity match

- Considers the extra quadrupole effect given by sextupoles, the total Chromaticity is given by:

$$\xi_{tot} = \frac{1}{4\pi} \oint \left[-k(s) + m(s)\eta(s) \right] \beta(s) ds$$

Where k is the quadrupole strength, m is the sextupole strength, and η is the dispersion function

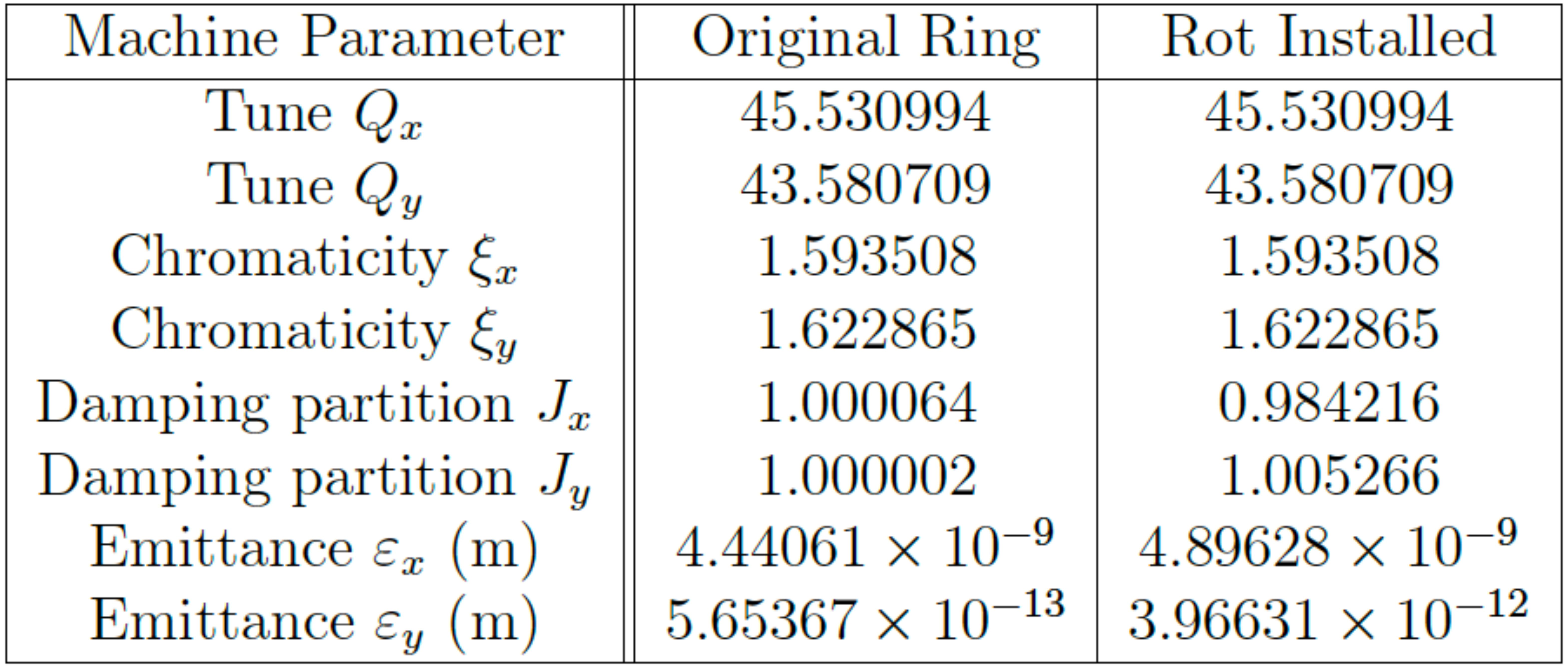
- Changing the sextupole strength does not change the beta function, thus the Chromaticity can be adjusted by tuning the sextupoles



- Sextupole pairs locates at the Rotator tuning area are turned off because the phase difference between these identical pairs is no longer π (the condition to cancel out the non-linear effects)

- Adjust sextuples in 4 arc section (45 pairs) shown in the picture above to match the original Chromaticity

Ring Parameters Comparison after performing the closed-geometry optimization



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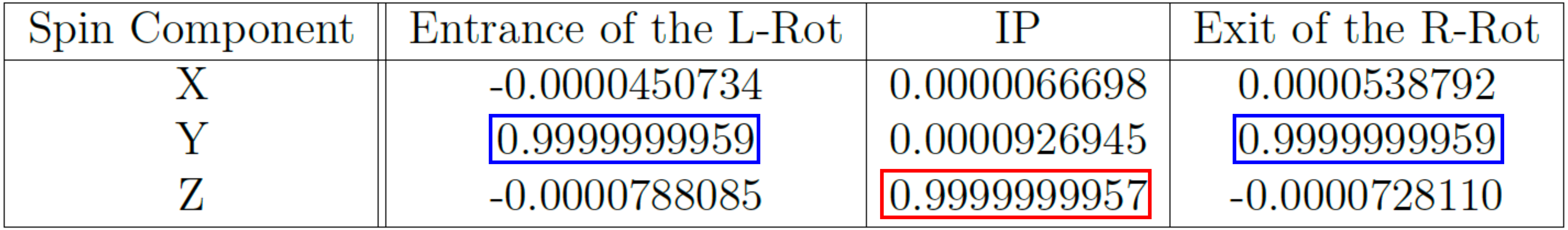
Damping partition J_x	1.000064	0.984216
Damping partition J_y	1.000002	1.005266
Emittance ε_x (m)	4.44061×10^{-9}	4.89628×10^{-9}
Emittance ε_y (m)	5.65367×10^{-13}	3.96631×10^{-12}

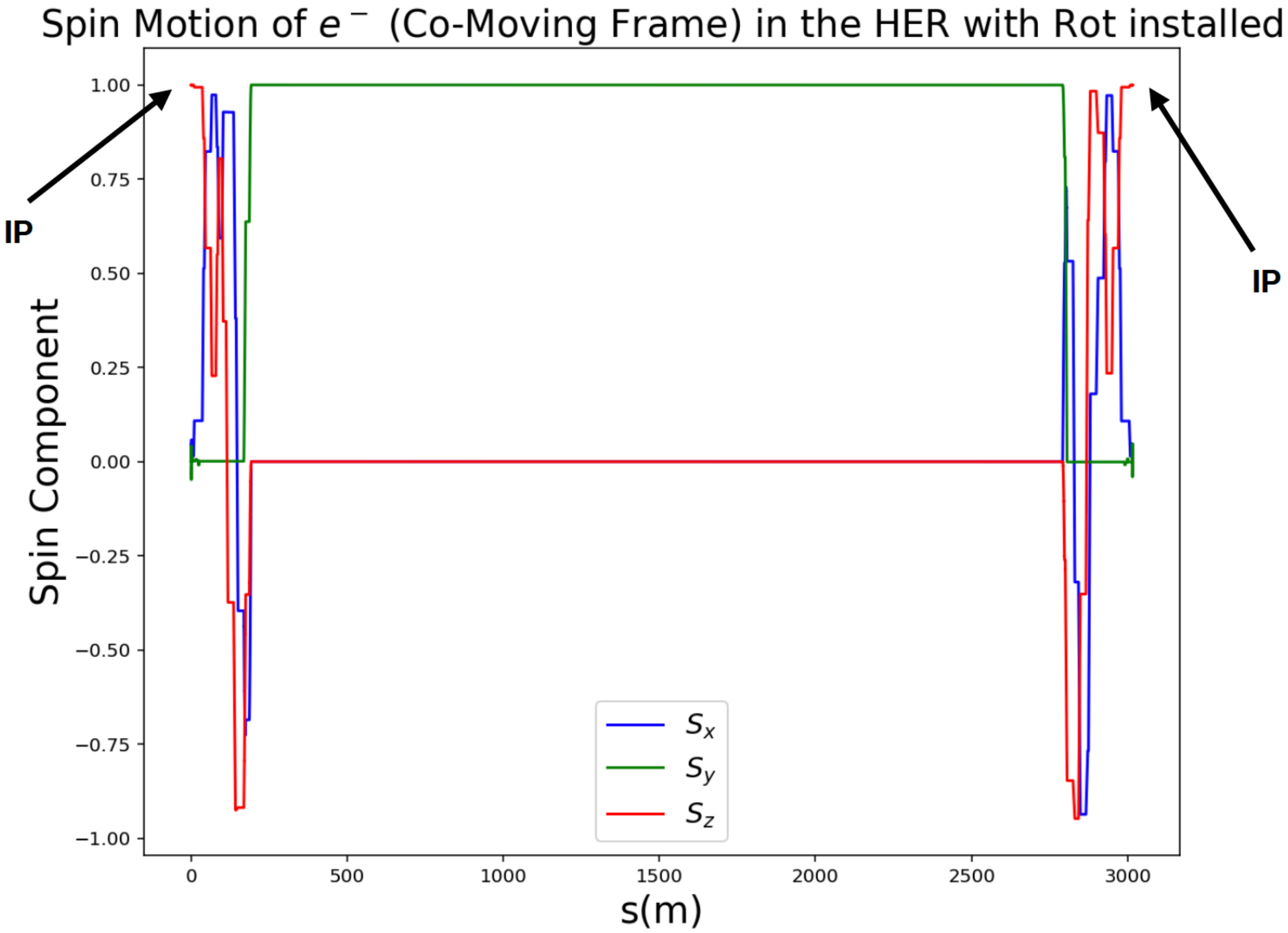
Tune and Chromaticity are matched to the original

Single Particle Spin Tracking Result

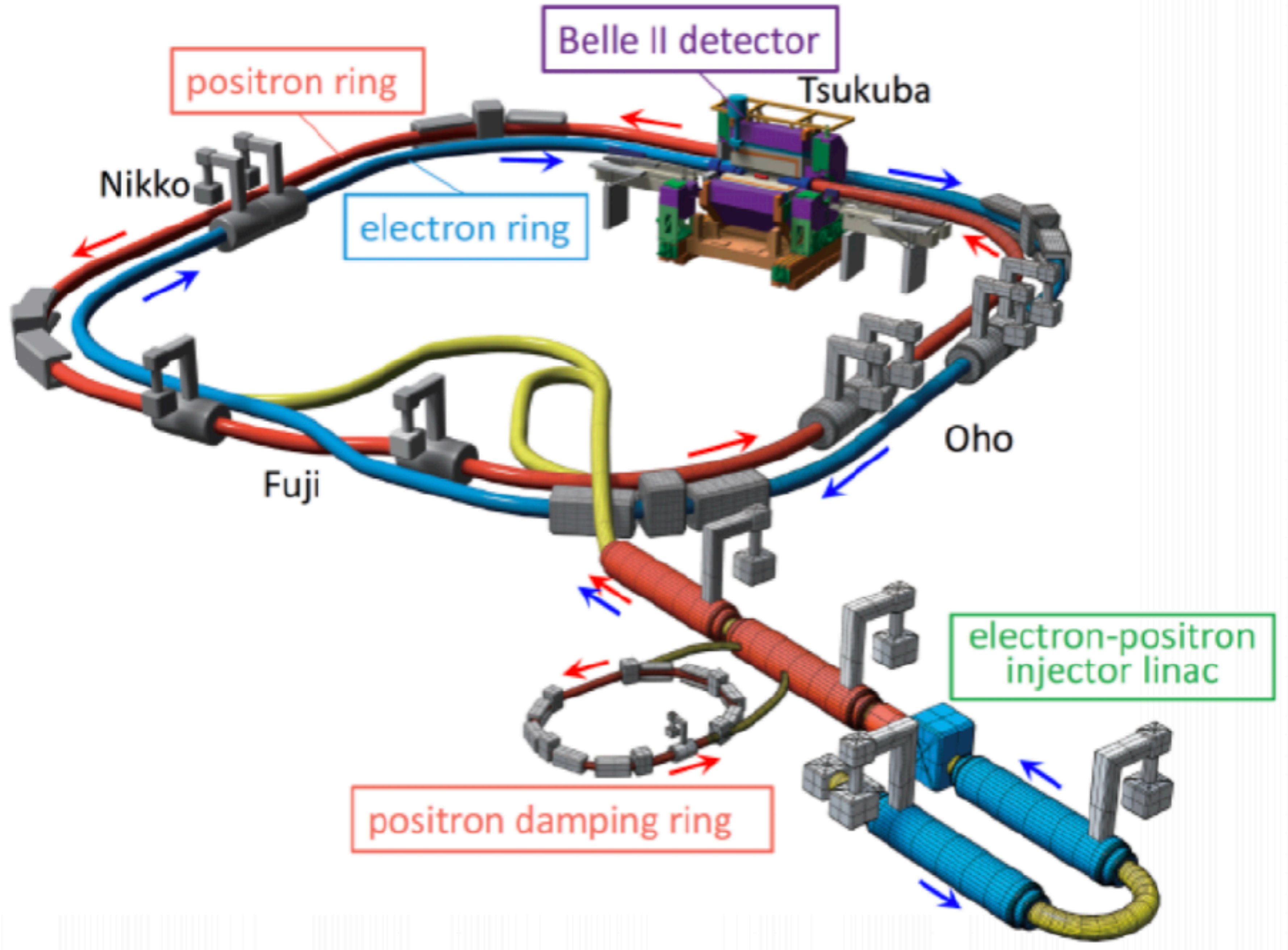


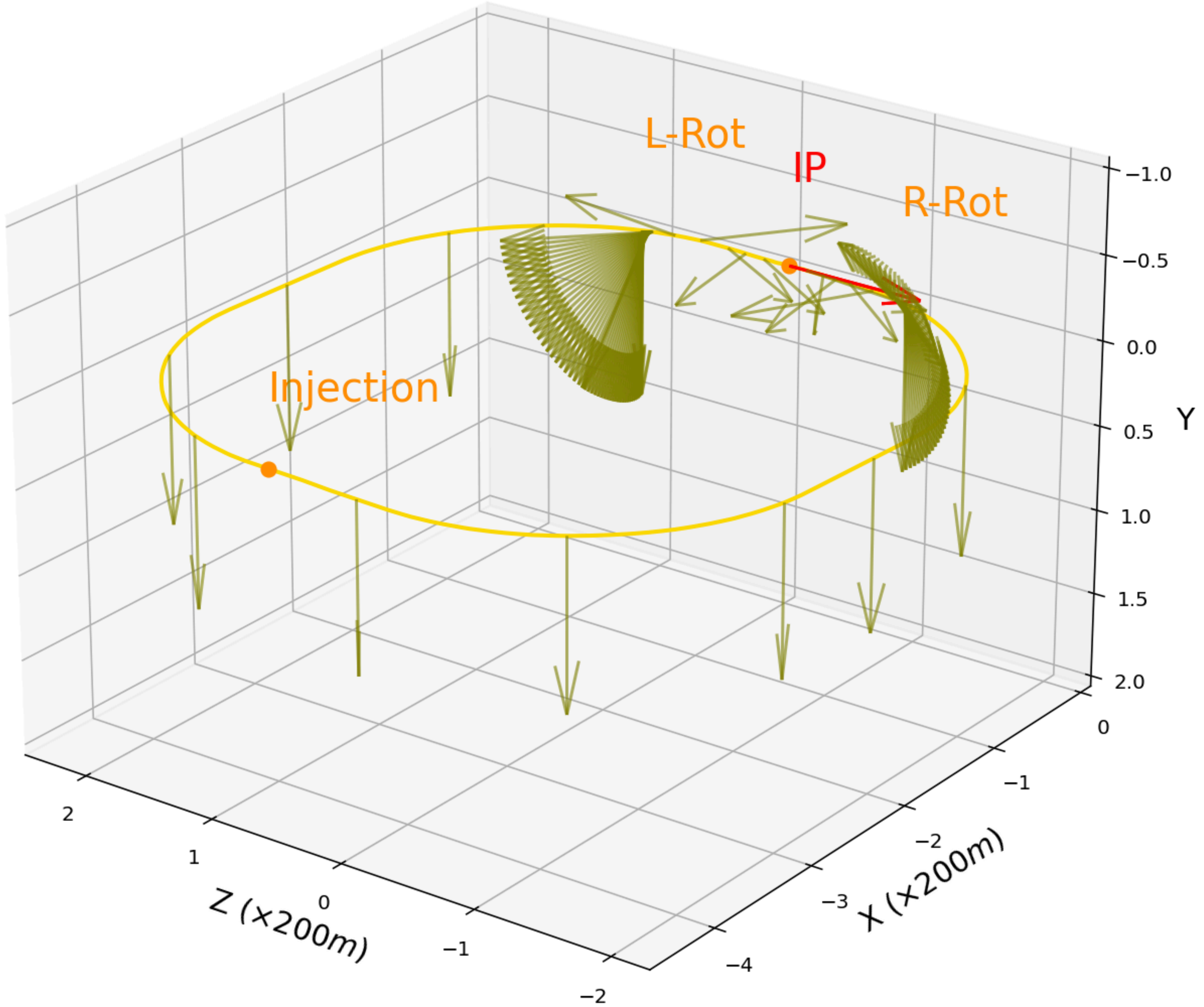
Spin Component	Entrance of the L-Rot	IP	Exit of the R-Rot
X	-0.0000450734	0.0000066698	0.0000538792
Y	0.9999999959	0.0000926945	0.9999999959
Z	-0.0000788085	0.9999999957	-0.0000728110

- The spin track result shows a longitudinal spin alignment >99.99% with the rotator installed in the High Energy Ring


Spin Motion of e^- (Co-Moving Frame) in the HER with Rot installed
Spin Component
s(m)
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0
500
1000
1500
2000
2500
3000
S_x
S_y
S_z
IP
IP

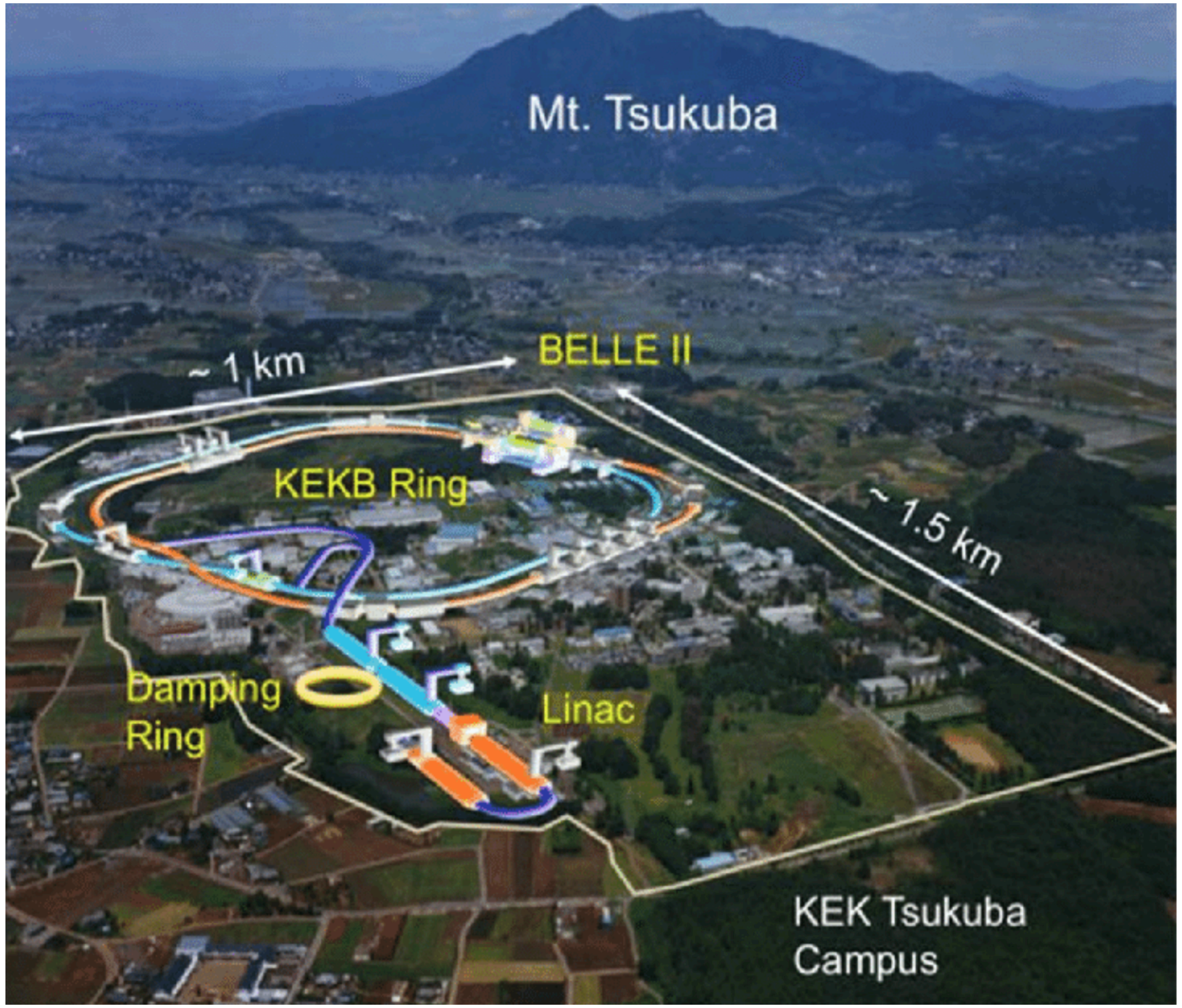
Spin motion of the electron in the Rot Ring (KEK frame)



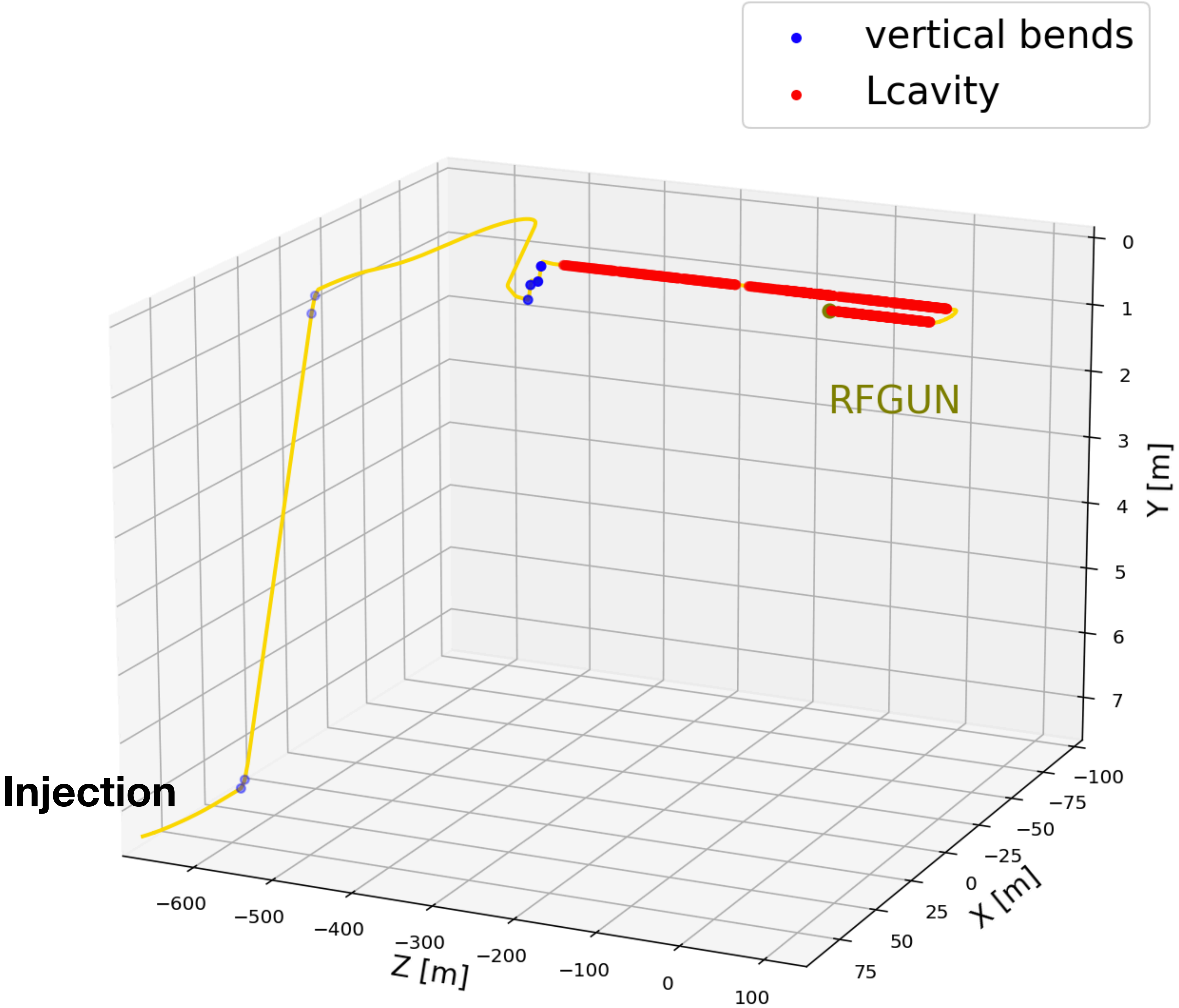


- electron polarized aligned with the B field at the injection point corresponds to longitudinal e-polarization at the IP
- electron polarized anti-aligned with B field at the injection point corresponds to anti-longitudinal e-polarization at the IP

KEK Injection Linac polarization studies

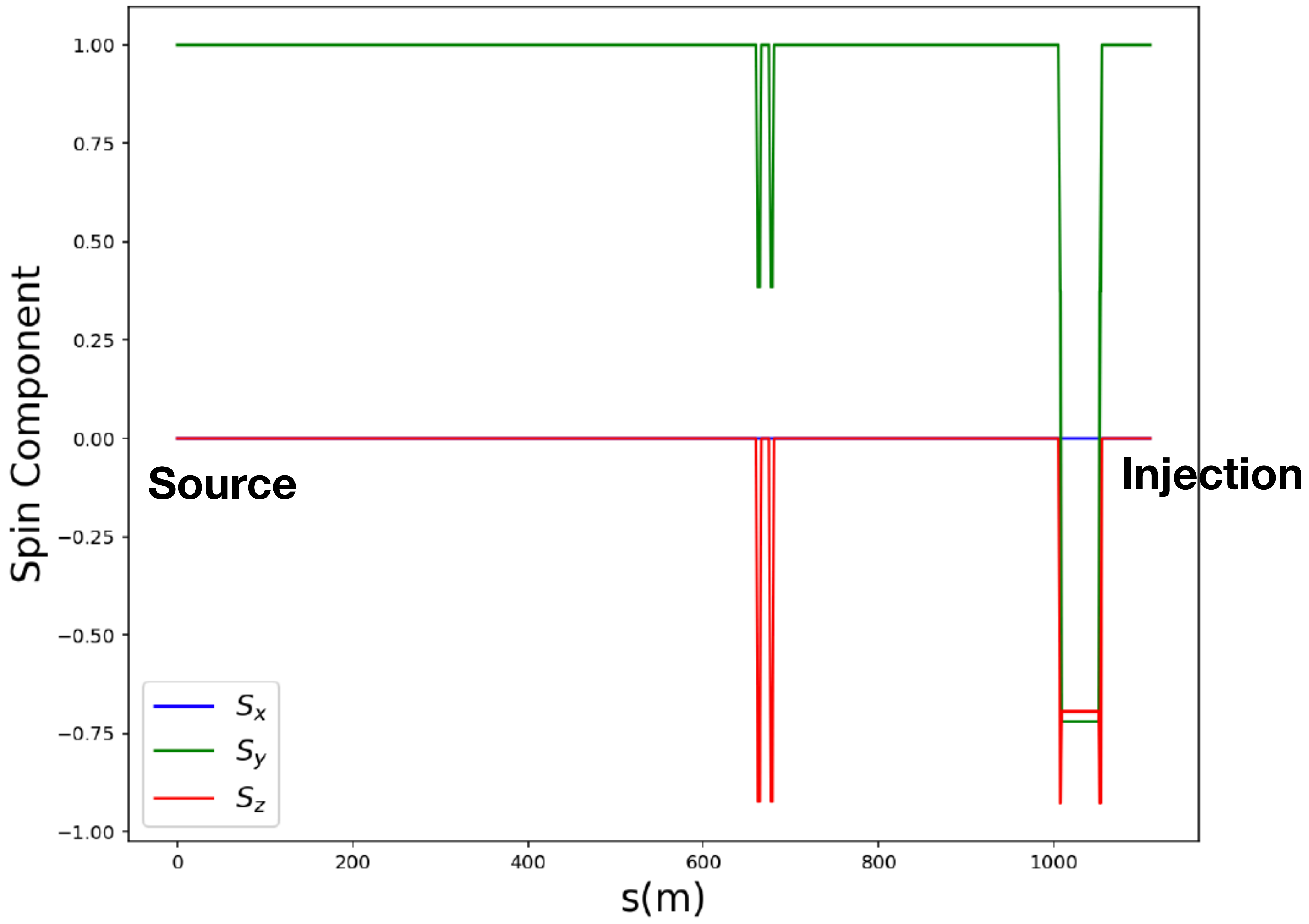


KEK Linac



Need to have transversely polarized beam at the injection point

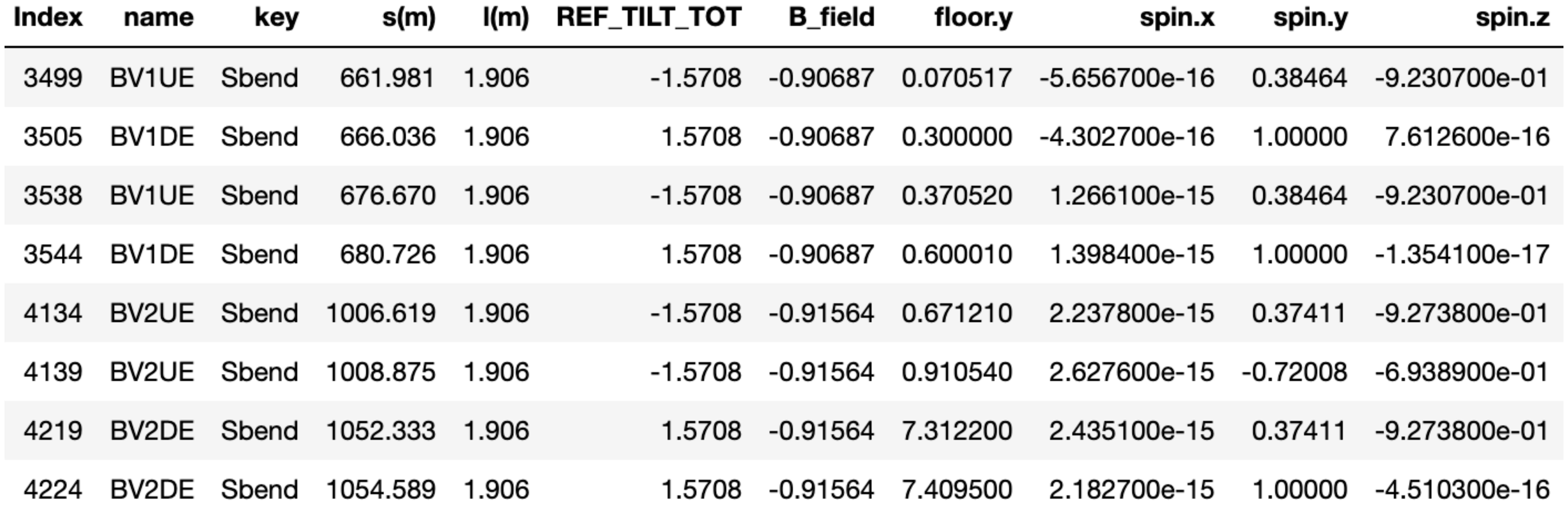
Spin motion in the Linac



The spin tracking result shows if the electron starts with vertical spin (0,1,0) at the source, it will end up with a vertical spin at the injection point

Underground section

Anti-symmetric structure of vertical bends



Index	name	key	s(m)	l(m)	REF_TILT_TOT	B_field	floor.y	spin.x	spin.y	spin.z
3499	BV1UE	Sbend	661.981	1.906	-1.5708	-0.90687	0.070517	-5.656700e-16	0.38464	-9.230700e-01
3505	BV1DE	Sbend	666.036	1.906	1.5708	-0.90687	0.300000	-4.302700e-16	1.00000	7.612600e-16
3538	BV1UE	Sbend	676.670	1.906	-1.5708	-0.90687	0.370520	1.266100e-15	0.38464	-9.230700e-01
3544	BV1DE	Sbend	680.726	1.906	1.5708	-0.90687	0.600010	1.398400e-15	1.00000	-1.354100e-17
4134	BV2UE	Sbend	1006.619	1.906	-1.5708	-0.91564	0.671210	2.237800e-15	0.37411	-9.273800e-01
4139	BV2UE	Sbend	1008.875	1.906	-1.5708	-0.91564	0.910540	2.627600e-15	-0.72008	-6.938900e-01
4219	BV2DE	Sbend	1052.333	1.906	1.5708	-0.91564	7.312200	2.435100e-15	0.37411	-9.273800e-01
4224	BV2DE	Sbend	1054.589	1.906	1.5708	-0.91564	7.409500	2.182700e-15	1.00000	-4.510300e-16

Due to the anti-symmetric structure(same B field magnitude but with opposite direction) of vertical bend pairs, the vertical spin is re-established at the injection point

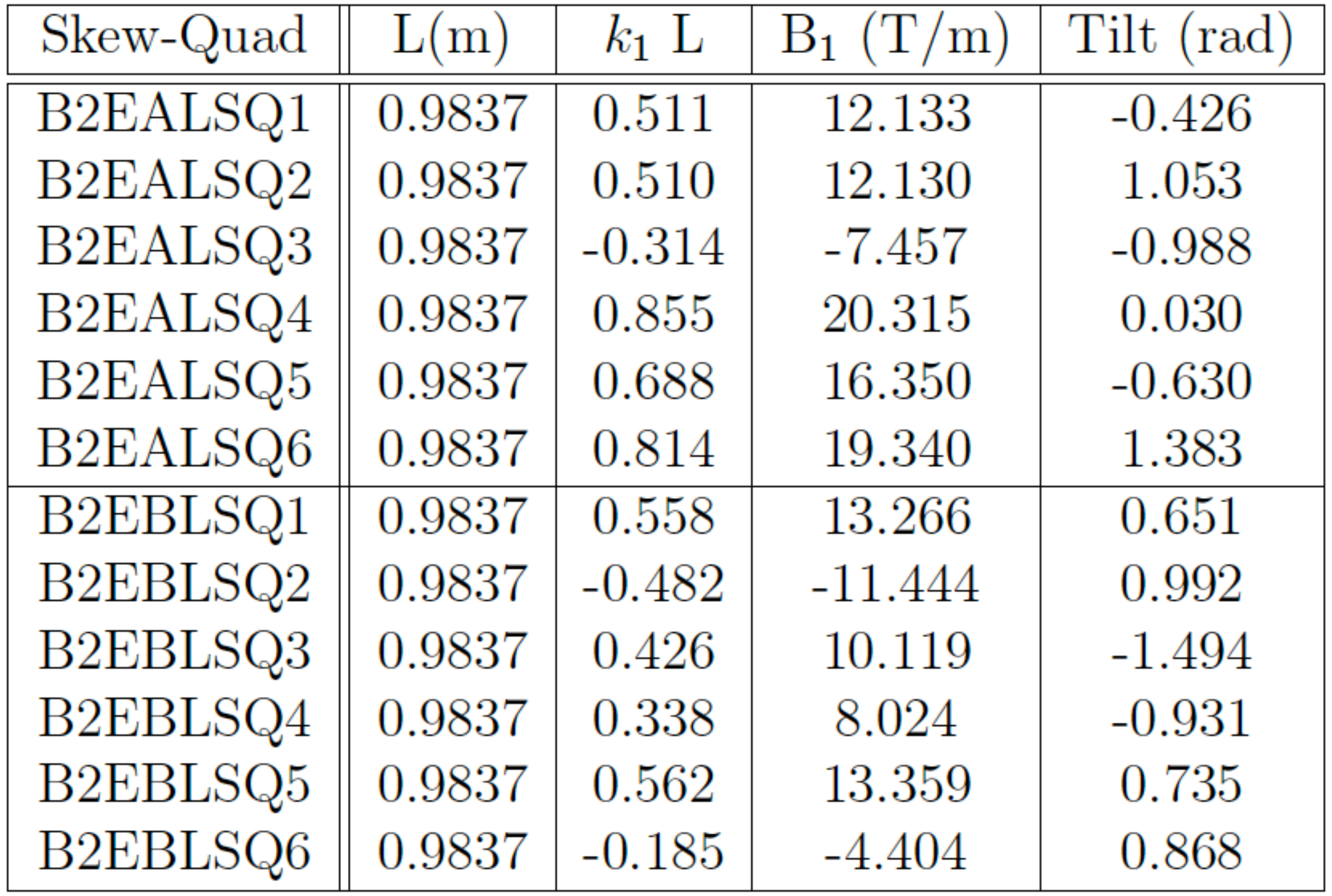
Current ongoing work

- Beam Tracking Studies in the HER with spin rotator (Long Term Tracking studies)
  - Investigate the dynamic aperture, and tune sextupoles to reach the maximum dynamic aperture
  - Determine the polarization lifetime and beam lifetime in the rotator ring with BMAD
- Transversely polarize the beam at the source

Appendix

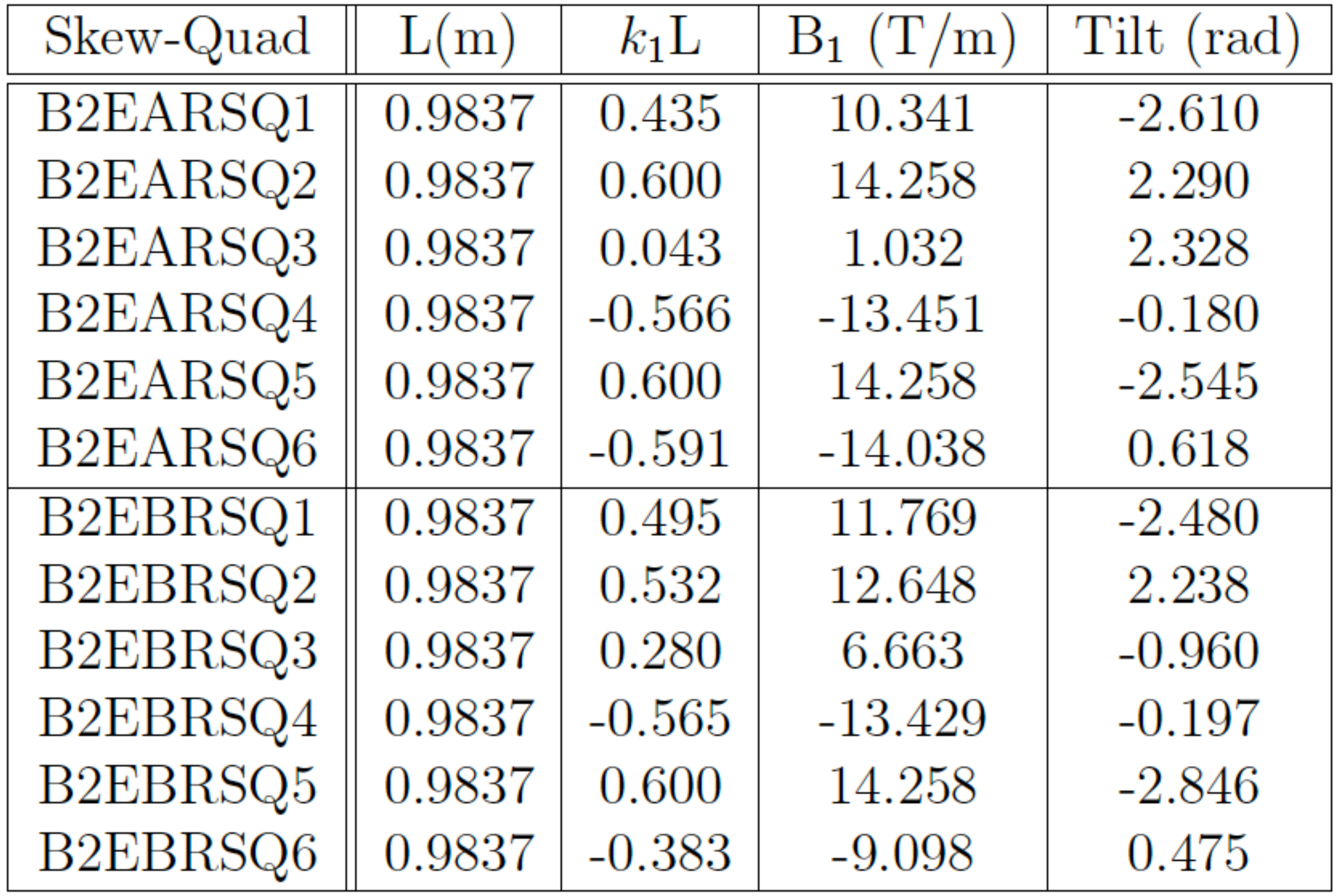
Open-geometry optimization result

Skew-Quad



Skew-Quad	L(m)	k_1 L	B_1 (T/m)	Tilt (rad)
B2EALSQ1	0.9837	0.511	12.133	-0.426
B2EALSQ2	0.9837	0.510	12.130	1.053
B2EALSQ3	0.9837	-0.314	-7.457	-0.988
B2EALSQ4	0.9837	0.855	20.315	0.030
B2EALSQ5	0.9837	0.688	16.350	-0.630
B2EALSQ6	0.9837	0.814	19.340	1.383
B2EBLSQ1	0.9837	0.558	13.266	0.651
B2EBLSQ2	0.9837	-0.482	-11.444	0.992
B2EBLSQ3	0.9837	0.426	10.119	-1.494
B2EBLSQ4	0.9837	0.338	8.024	-0.931
B2EBLSQ5	0.9837	0.562	13.359	0.735
B2EBLSQ6	0.9837	-0.185	-4.404	0.868

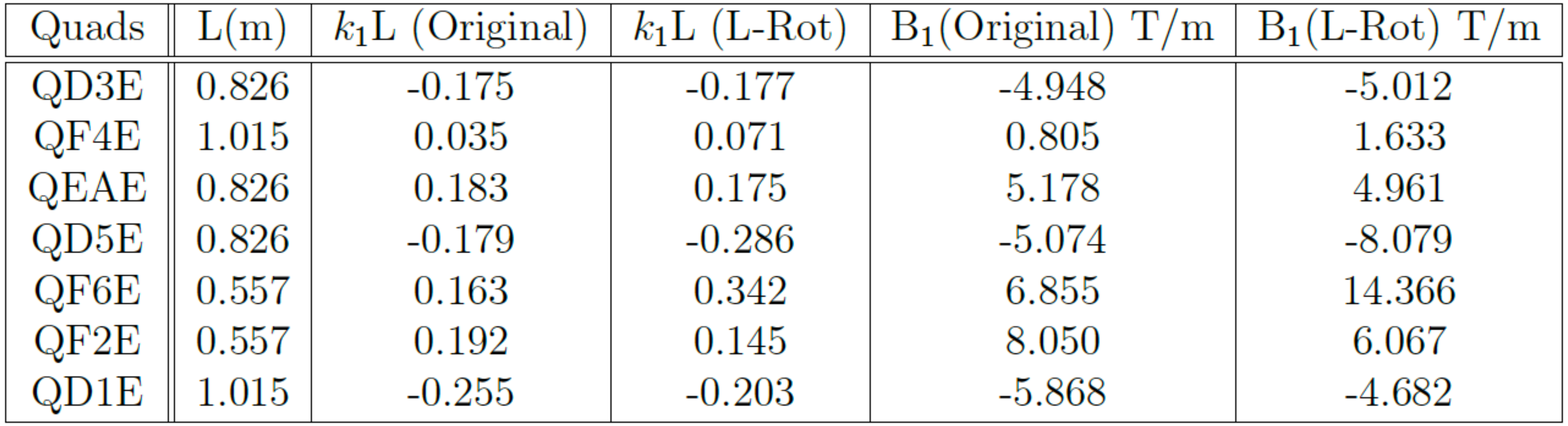
L-Rot



Skew-Quad	L(m)	k_1L	B_1 (T/m)	Tilt (rad)
B2EARSQ1	0.9837	0.435	10.341	-2.610
B2EARSQ2	0.9837	0.600	14.258	2.290
B2EARSQ3	0.9837	0.043	1.032	2.328
B2EARSQ4	0.9837	-0.566	-13.451	-0.180
B2EARSQ5	0.9837	0.600	14.258	-2.545
B2EARSQ6	0.9837	-0.591	-14.038	0.618
B2EBRSQ1	0.9837	0.495	11.769	-2.480
B2EBRSQ2	0.9837	0.532	12.648	2.238
B2EBRSQ3	0.9837	0.280	6.663	-0.960
B2EBRSQ4	0.9837	-0.565	-13.429	-0.197
B2EBRSQ5	0.9837	0.600	14.258	-2.846
B2EBRSQ6	0.9837	-0.383	-9.098	0.475

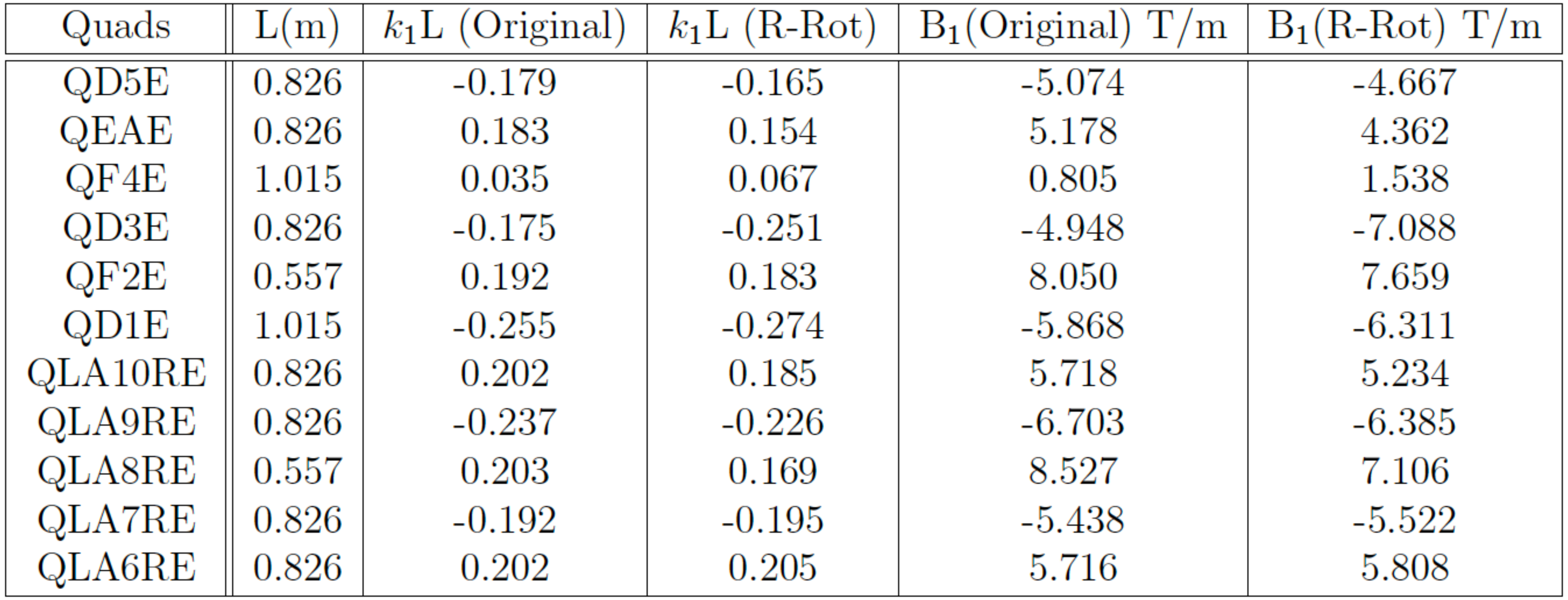
R-Rot

Quadrupoles at the Rotator tuning region



Quads	L(m)	k_1L (Original)	k_1L (L-Rot)	B_1(Original) T/m	B_1(L-Rot) T/m
QD3E	0.826	-0.175	-0.177	-4.948	-5.012
QF4E	1.015	0.035	0.071	0.805	1.633
QEAE	0.826	0.183	0.175	5.178	4.961
QD5E	0.826	-0.179	-0.286	-5.074	-8.079
QF6E	0.557	0.163	0.342	6.855	14.366
QF2E	0.557	0.192	0.145	8.050	6.067
QD1E	1.015	-0.255	-0.203	-5.868	-4.682

L-Rot

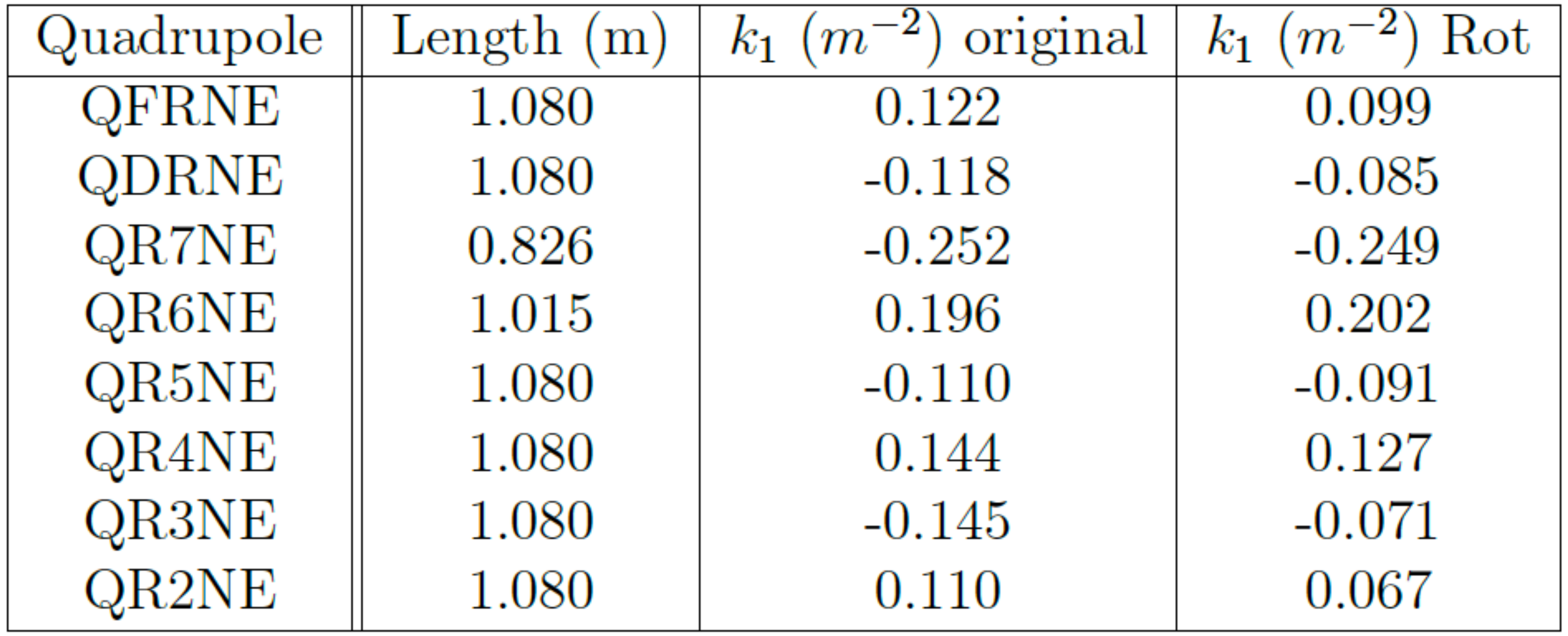


Quads	L(m)	k_1L (Original)	k_1L (R-Rot)	B_1(Original) T/m	B_1(R-Rot) T/m
QD5E	0.826	-0.179	-0.165	-5.074	-4.667
QEAE	0.826	0.183	0.154	5.178	4.362
QF4E	1.015	0.035	0.067	0.805	1.538
QD3E	0.826	-0.175	-0.251	-4.948	-7.088
QF2E	0.557	0.192	0.183	8.050	7.659
QD1E	1.015	-0.255	-0.274	-5.868	-6.311
QLA10RE	0.826	0.202	0.185	5.718	5.234
QLA9RE	0.826	-0.237	-0.226	-6.703	-6.385
QLA8RE	0.557	0.203	0.169	8.527	7.106
QLA7RE	0.826	-0.192	-0.195	-5.438	-5.522
QLA6RE	0.826	0.202	0.205	5.716	5.808

R-Rot

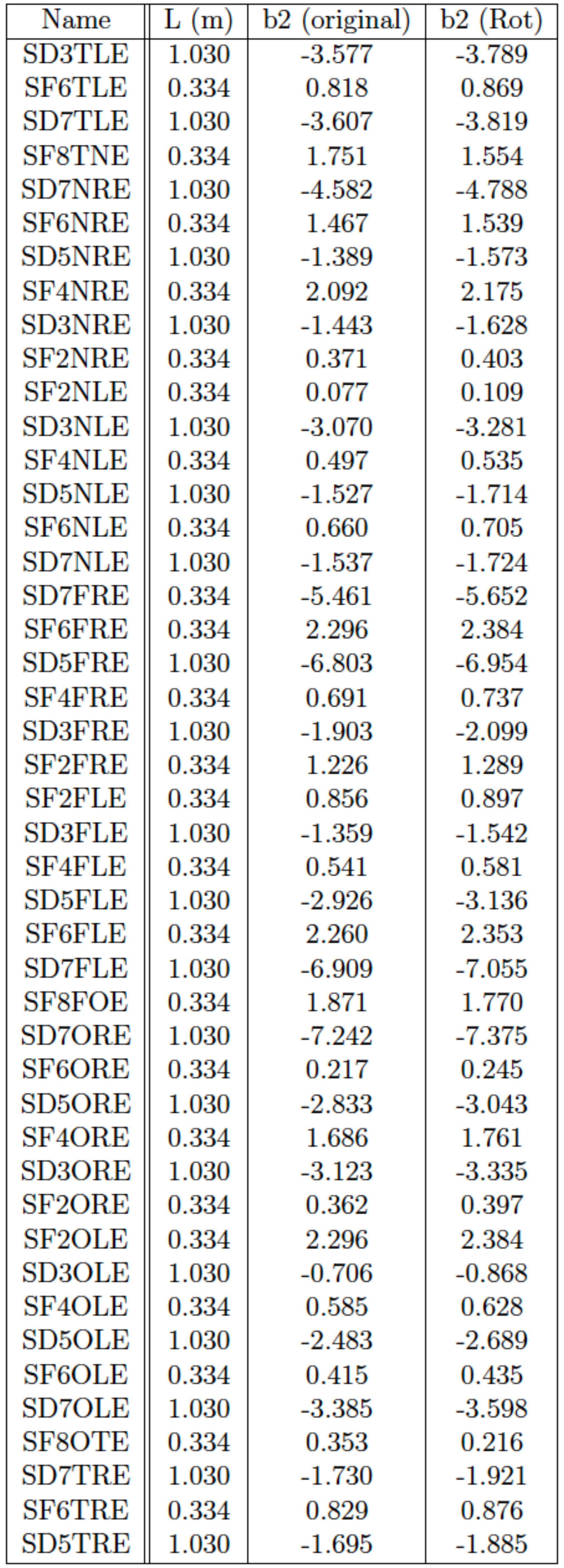
Closed-geometry optimization result

Quadrupole at “Nikko” Section



Quadrupole	Length (m)	k_1 (m^{-2}) original	k_1 (m^{-2}) Rot
QFRNE	1.080	0.122	0.099
QDRNE	1.080	-0.118	-0.085
QR7NE	0.826	-0.252	-0.249
QR6NE	1.015	0.196	0.202
QR5NE	1.080	-0.110	-0.091
QR4NE	1.080	0.144	0.127
QR3NE	1.080	-0.145	-0.071
QR2NE	1.080	0.110	0.067

Sextupoles adjusted to re-match the Chromaticity



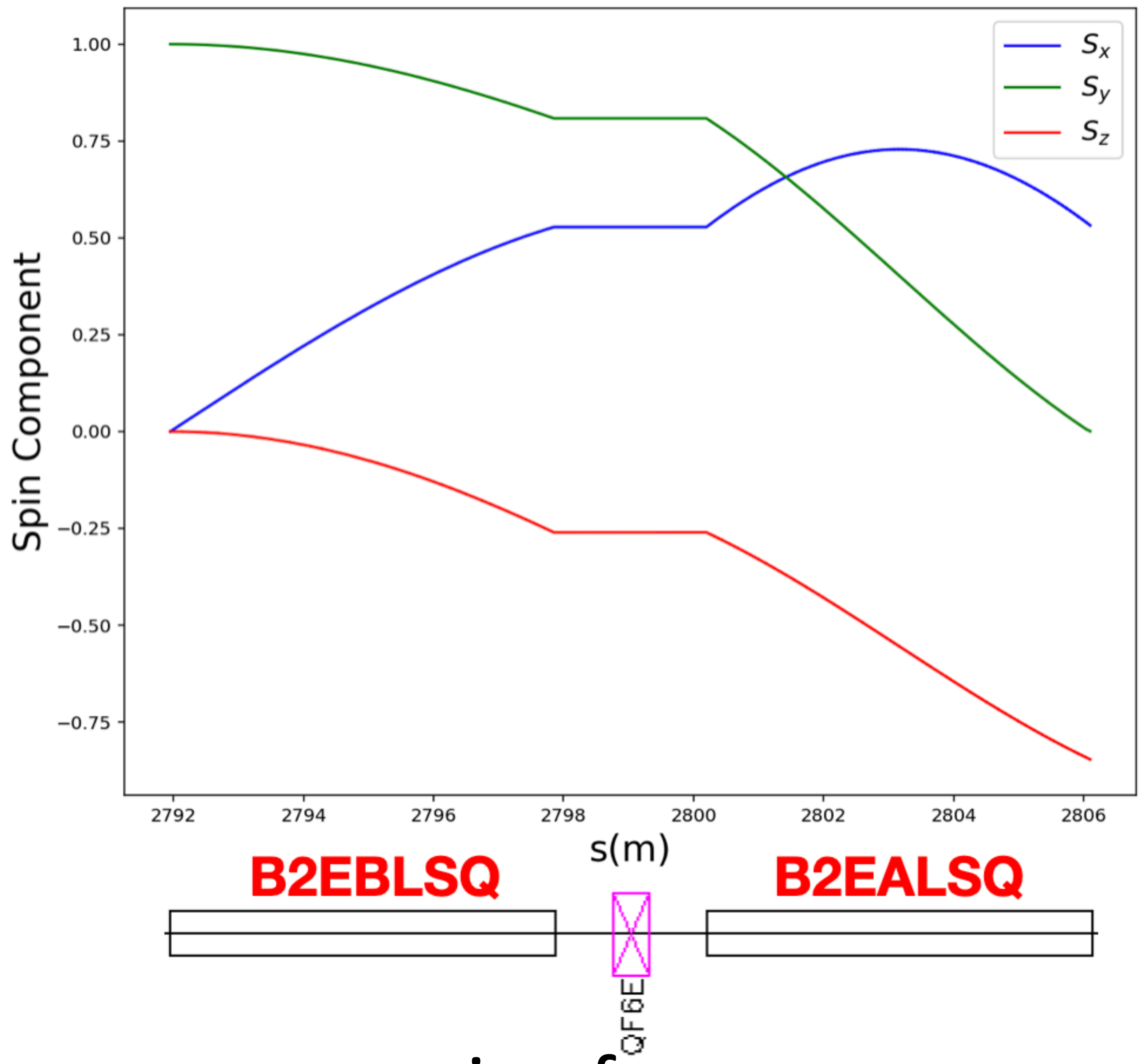
Name	L (m)	b2 (original)	b2 (Rot)
SD3TLE	1.030	-3.577	-3.789
SF6TLE	0.334	0.818	0.869
SD7TLE	1.030	-3.607	-3.819
SF8TNE	0.334	1.751	1.554
SD7NRE	1.030	-4.582	-4.788
SF6NRE	0.334	1.467	1.539
SD5NRE	1.030	-1.389	-1.573
SF4NRE	0.334	2.092	2.175
SD3NRE	1.030	-1.443	-1.628
SF2NRE	0.334	0.371	0.403
SF2NLE	0.334	0.077	0.109
SD3NLE	1.030	-3.070	-3.281
SF4NLE	0.334	0.497	0.535
SD5NLE	1.030	-1.527	-1.714
SF6NLE	0.334	0.660	0.705
SD7NLE	1.030	-1.537	-1.724
SD7FRE	0.334	-5.461	-5.652
SF6FRE	0.334	2.296	2.384
SD5FRE	1.030	-6.803	-6.954
SF4FRE	0.334	0.691	0.737
SD3FRE	1.030	-1.903	-2.099
SF2FRE	0.334	1.226	1.289
SF2FLE	0.334	0.856	0.897
SD3FLE	1.030	-1.359	-1.542
SF4FLE	0.334	0.541	0.581
SD5FLE	1.030	-2.926	-3.136
SF6FLE	0.334	2.260	2.353
SD7FLE	1.030	-6.909	-7.055
SF8FOE	0.334	1.871	1.770
SD7ORE	1.030	-7.242	-7.375
SF6ORE	0.334	0.217	0.245
SD5ORE	1.030	-2.833	-3.043
SF4ORE	0.334	1.686	1.761
SD3ORE	1.030	-3.123	-3.335
SF2ORE	0.334	0.362	0.397
SF2OLE	0.334	2.296	2.384
SD3OLE	1.030	-0.706	-0.868
SF4OLE	0.334	0.585	0.628
SD5OLE	1.030	-2.483	-2.689
SF6OLE	0.334	0.415	0.435
SD7OLE	1.030	-3.385	-3.598
SF8OTE	0.334	0.353	0.216
SD7TRE	1.030	-1.730	-1.921
SF6TRE	0.334	0.829	0.876
SD5TRE	1.030	-1.695	-1.885

The integrated sextupole strength is described by:

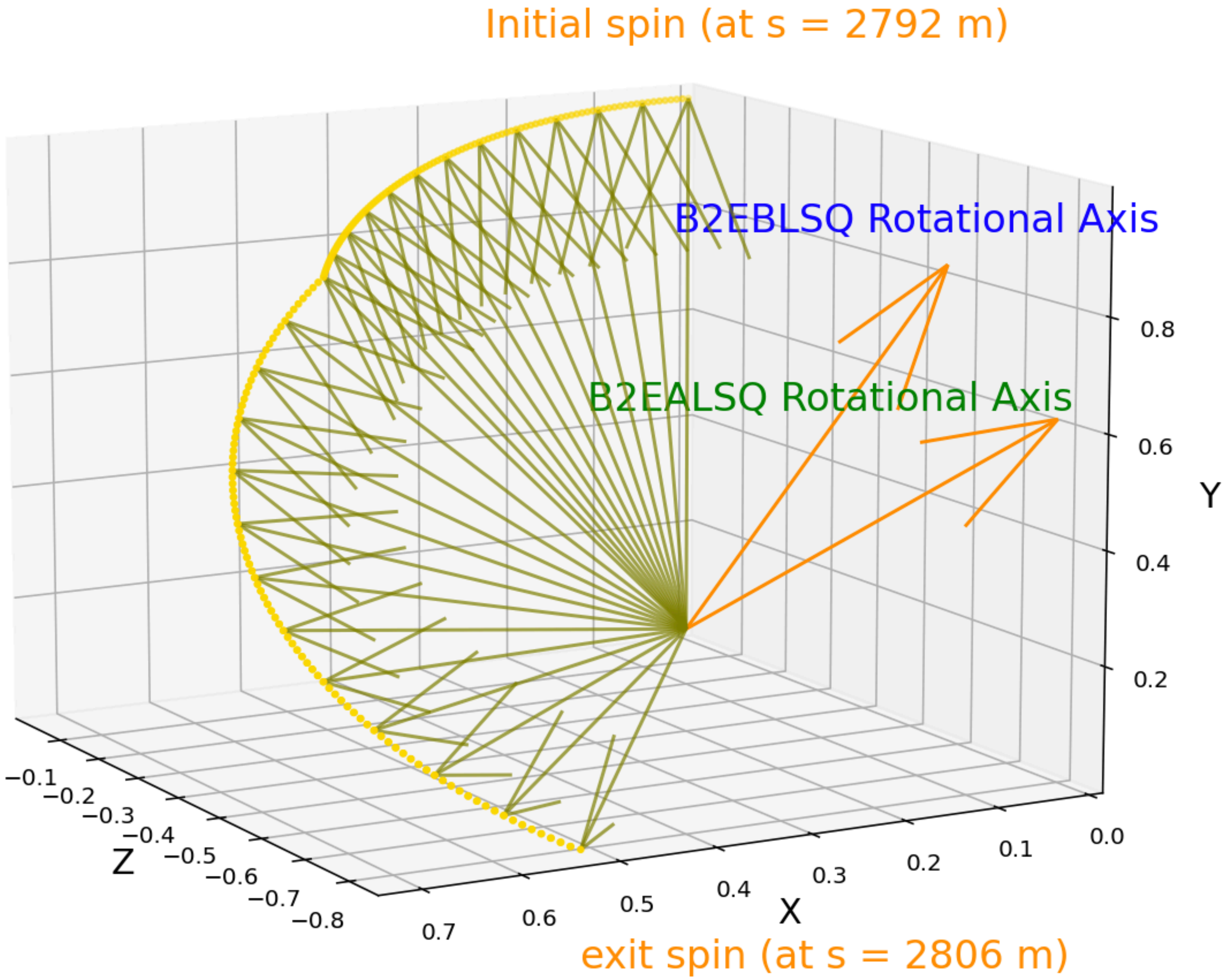
$$b_2 = \frac{k_2 L}{2}$$

Where k_2 is the sextupole strength, and L is the length

Spin motion in the L-Rot

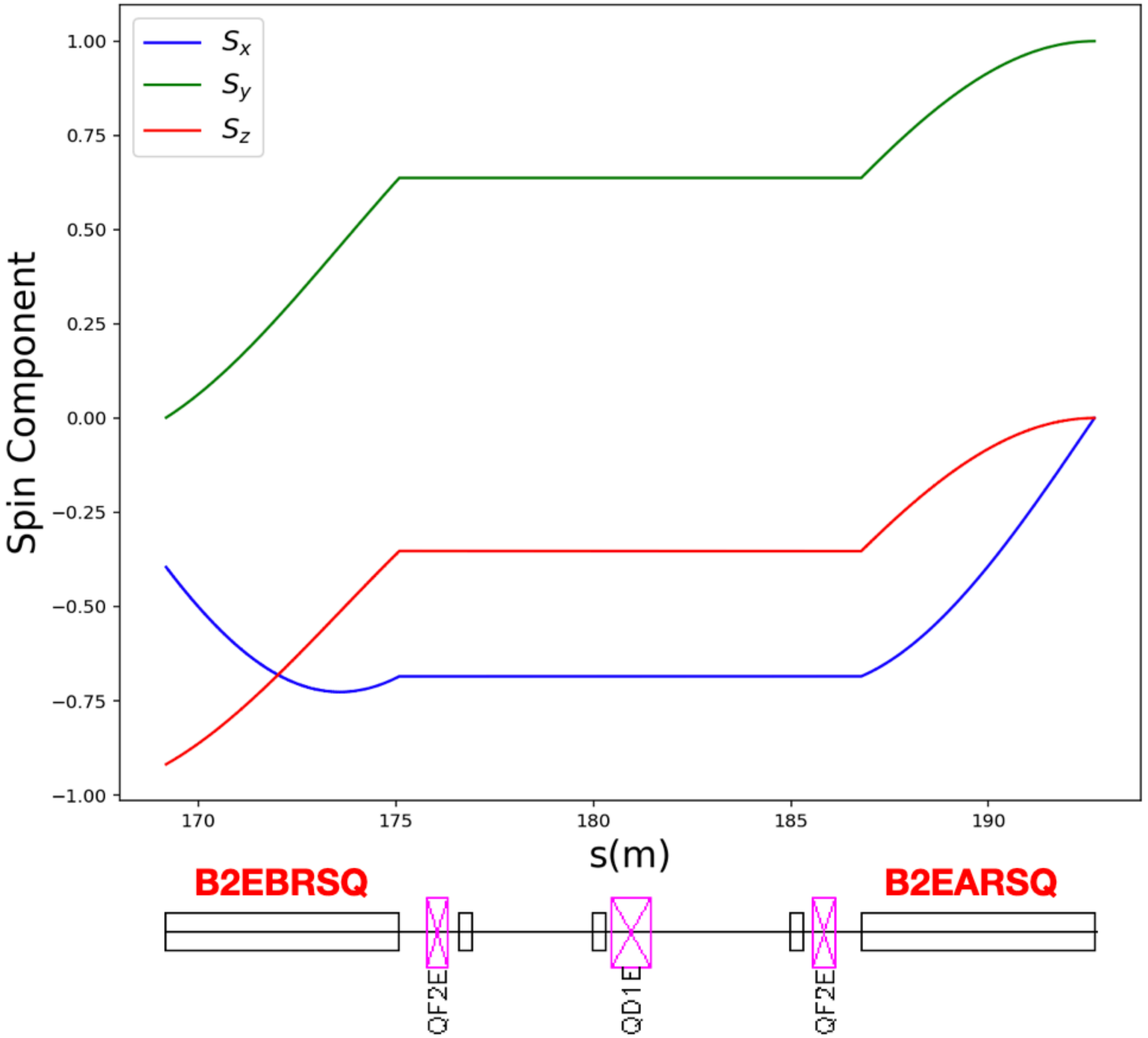


co-moving frame

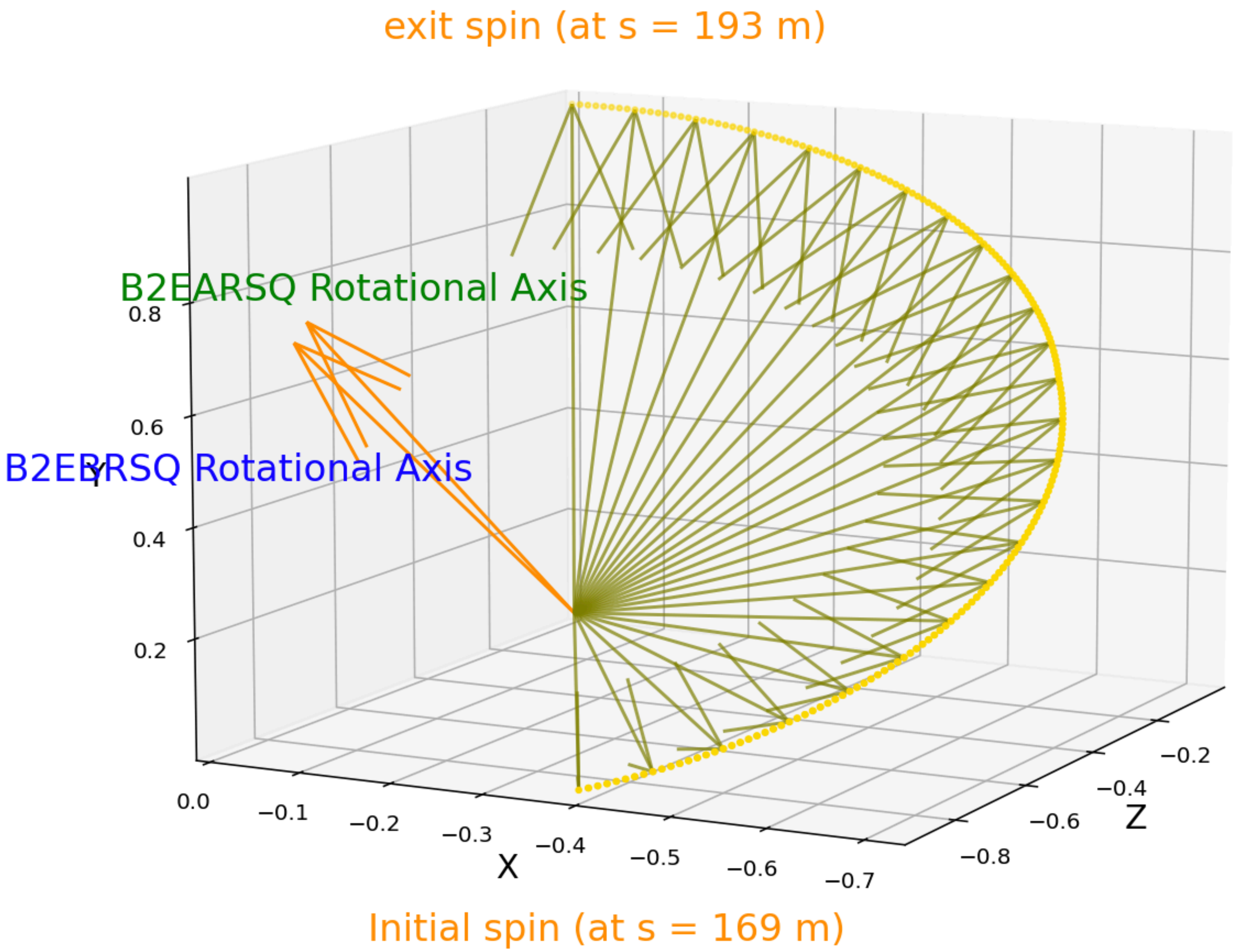


rest frame of the electron

Spin motion in the R-Rot



co-moving frame



rest frame of the electron

Spin motion between the Rot and the IP