AN ERGODIC APPROACH TO POLARIZATION LIFETIME IN AN ELECTRON STORAGE RING

With acknowledgments to NERSC

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Introduction (1/4)

Synchrotron radiation affects stored polarization, via two processes:

(i) Sokholov-Ternov self-polarization

$$\mathbf{P}(\mathbf{t}) = \mathbf{P}(\mathbf{0})\mathbf{e}^{-\mathbf{t}/\tau_{\mathbf{ST}}} + \mathbf{P}_{\mathbf{ST}}(\mathbf{1} - \mathbf{e}^{-\mathbf{t}/\tau_{\mathbf{ST}}})$$

- **which, essentially, causes electron spins to slowly self-polarize towards anti-$\vec{B}//$,**
- **and which I happily ignore in the simulations, to mostly keep in mind the time constant**

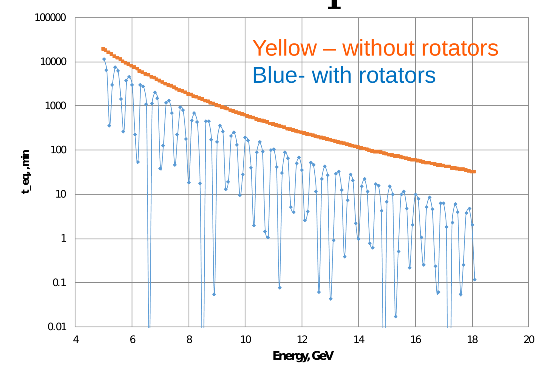
$$\tau_{\mathbf{ST}} = \left[\frac{\mathbf{5}\sqrt{\mathbf{3}}}{\mathbf{8}}\frac{\hbar\mathbf{r_e}}{\mathbf{m}}\frac{\gamma^{\mathbf{5}}}{\mathcal{C}} \times \oint_{\mathbf{dipoles}}\frac{\mathbf{ds}}{|\rho|^{\mathbf{3}}}\right]^{-\mathbf{1}} \xrightarrow{iso-B} 99\frac{\rho^2 R}{E^5_{[GeV]}}[sec] \overbrace{\overset{\substack{\rho\approx 250\,m \\ R=3835/2\pi\,m \\ 18\,\mathrm{GeV}}}{\approx} \quad 35\,\mathrm{minutes}}^{\mathbf{EIC}}$$

Introduction (2/4)
(ii) Spin diffusion
That's the focus in these slides

- **Stems from the stochastic change of the spin precession axis $\vec{n}_\delta$ upon emission of a photon: $\partial\vec{n}_\delta/\partial\gamma$**

◇ **in a similar way that the chromatic closed-orbit jumps when e emits a photon/loses energy**

◇ **The effect of spin rotators**

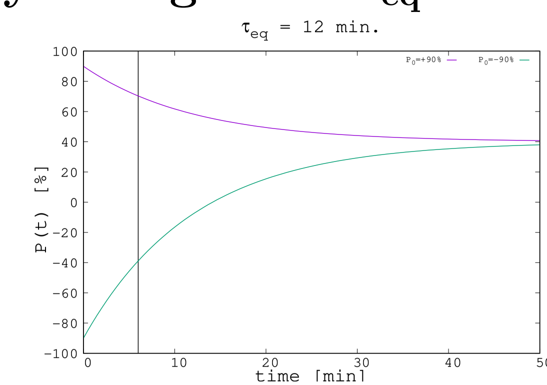


- **Causes polarization decay:**

$$\mathbf{P(t)/P(0) = \exp(-t/\tau_D)}$$

- **dominates bunch depolarization,**
- **shortens polarization time, $\tau_{eq} < \tau_{ST}$,**
- **reduces asymptotic polarization,**
- **scales as $\propto E^7$.**

◇ **Some EIC ESR setting, yielding here $P_{eq} \sim 40\%$**



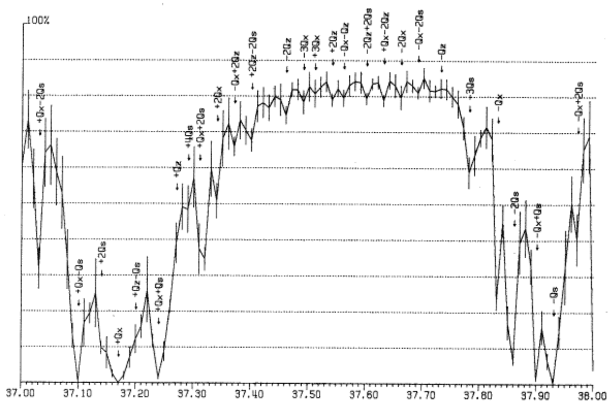
- **τ_D matters, as $P_{eq} = P_{ST} \times \frac{\tau_{eq}}{\tau_{ST}}$, with $\tau_{eq} = (1/\tau_{ST} + 1/\tau_D)^{-1}$.**

Introduction (3/4)
Assessment of polarization performance

- **Typically:**

As produced using SITROS,

◇ **at DESY *[J. Kewisch, DESY/83-032, 1983]***



HERA-e, linear, res. non-corrected

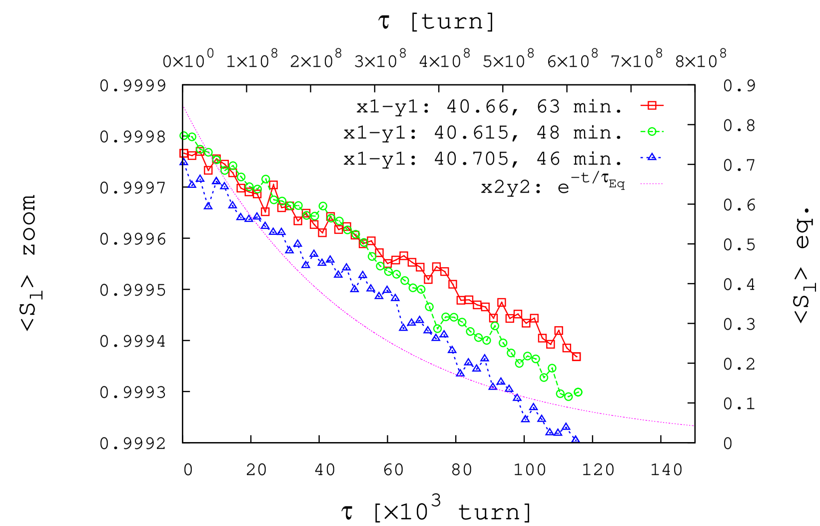
◇ **today by Eliana Gianfelice at EIC (this workshop)**

◇ **obtained by tracking a few hundred electrons per bin, and their spins, including SR, over a few damping times**

- **At EIC ESR today, typically, polarization decay with time**

240 damping times,

Polarization vs. time at store, 18 GeV



Diffusion time constant τ_D obtained from regression

$$P/P_0 = \exp(-t/\tau_D)$$

Introduction (4/4)
This is very much HPC resource hungry

- **Typically:**

◇ **tens of bunches (i.e., $a\delta\gamma_{ref}$ bins) spanning $\Delta a\gamma \sim 1$,**

◇ **hundreds of electrons per bunch,**

◇ **tracking is over several damping times,**

damping time:

- 500 turns at 18 GeV

- 3000 turns at 10 GeV

◇ **around a large ring - hundreds of optical elements (EIC ESR circumference 3.835 km).**

Try the ergodic hypothesis, instead

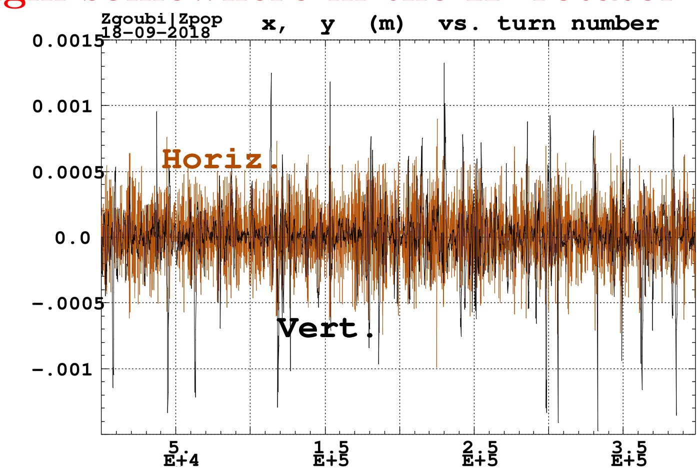
- As we know the dynamical system of an electron bunch in the presence of synchrotron radiation is ergodic. Let's see the equilibrium case for instance, here:

Watch 1 el. long enough that all phase-space is explored — **Take a photo of an electron bunch at time t**

$$\lim_{T\to\infty}\int_{t_0}^{t_0+T} f(\vec{X}(t))dt = \int f(\vec{X})\rho(\vec{X})d^N\vec{X}\Big|_{time=t}$$

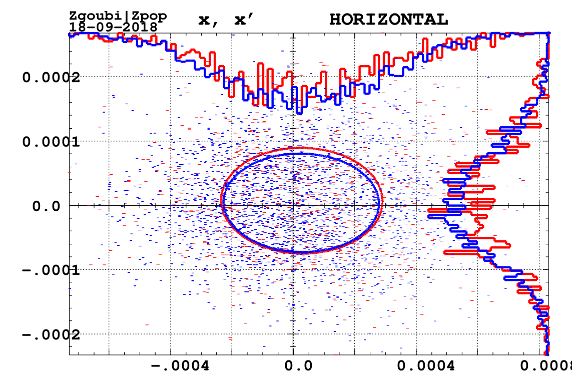
$\Downarrow$

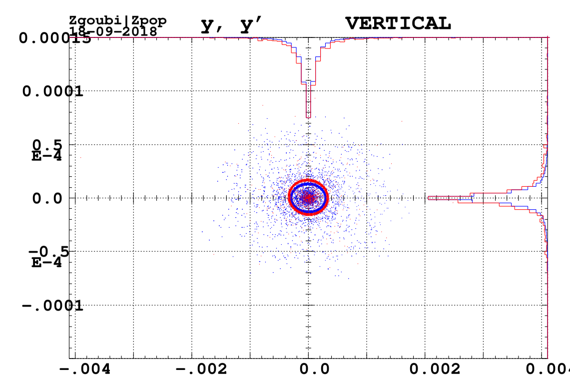
Electron excursion (4×10^5 turns in ESR ring):
(lattice is coupled, ~20% coupling, deliberate, origin somewhere in the IP rotator area).



TRANSVERSE PHASE SPACES

and projected densities:





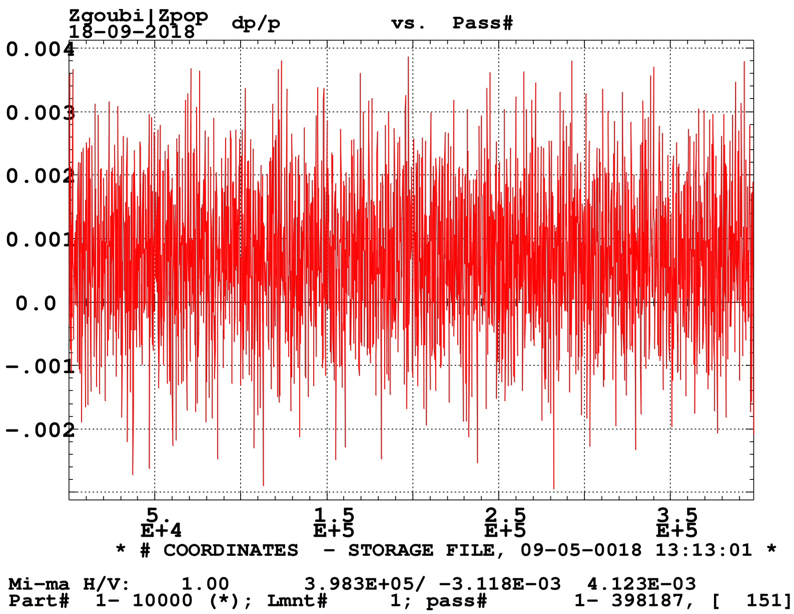
Phase-space two graphs on the right:

- Red dots: phase-space motion of 1! electron over a long time (as in the left figure);
- Blue dots: case of a 10^3 electron bunch, observed at arbitrary time t;

In both H and V phase-spaces, single-electron and bunch rms match ellipses as well as projected densities do coincide

LONGITUDINAL PHASE SPACE

Single-electron and bunch densities coincide as well:

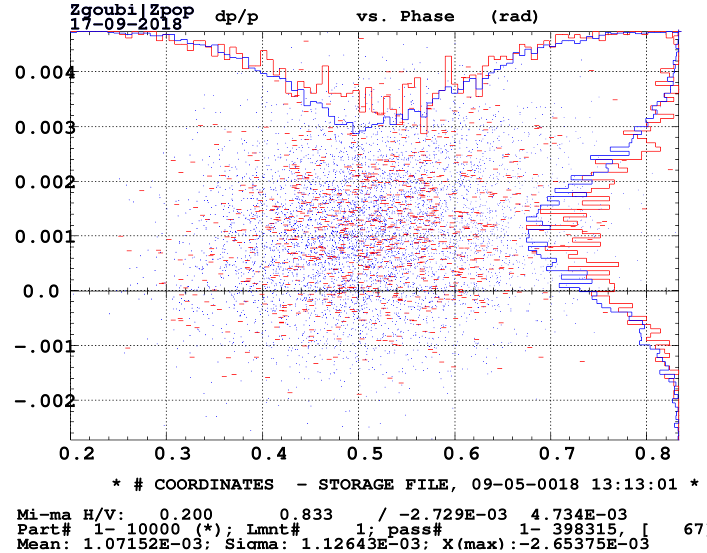


Stochastic energy excursion over time interval $t/\tau_{SR} : 1 \to 800$;

this yields:

$\langle\delta p/p\rangle = 1.11\,10^{-3}$ **and**

$\sigma_{\delta p/p} = 1.14\,10^{-3}$.

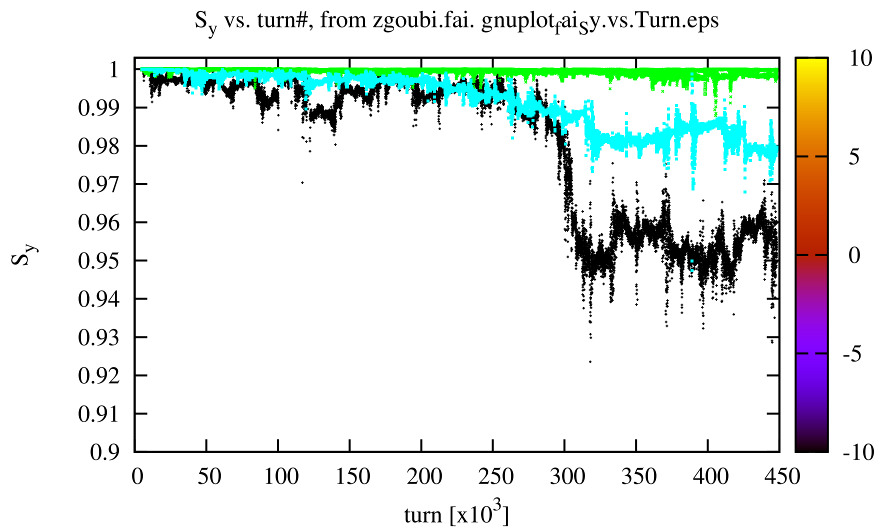


Red: projection of the single electron 4×10^5-turn motion;
$\langle\delta p/p\rangle = 1.11\,10^{-3}$, $\sigma_{\delta p/p} = 1.14\,10^{-3}$;
$\langle\phi\rangle = 0.519$, $\sigma_\phi = 0.091$.

Blue: case of a 10^3 electron bunch observed at arbitrary time t; densities appear to satisfy:
$\langle\delta p/p\rangle = 1.07\,10^{-3}$,
$\sigma_{\delta p/p} = 1.13\,10^{-3}$;
$\langle\phi\rangle = 0.519$, $\sigma_\phi = 0.091$;

Ergodicity and spin motion

- Stochastic spin motion, single electron, observed at IP8:



In passing:
tracking takes
$\approx$12hrs / 100$\times\tau_{SR}$
(50,000 turns)

- In regions of (possibly strong) resonances, polarization decay is fast:

$$\tau_D \sim (a\gamma_{Res.} - a\gamma)^2 \tau_{ST},$$
$$P_{eq} \sim (a\gamma_{Res.} - a\gamma) P_{ST},$$

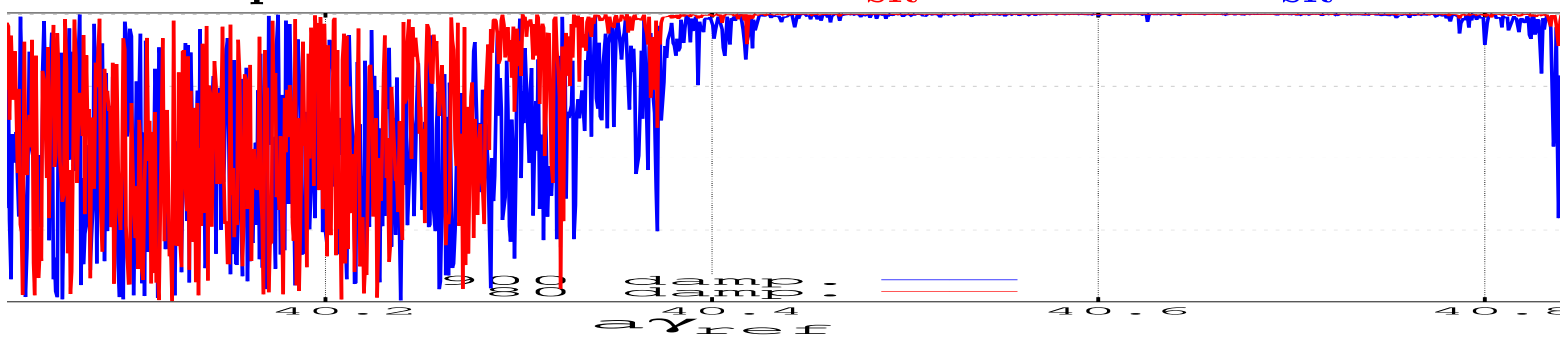
- and presumably the effect of a non viable lattice (error orbit and coupling, rotator mis-match, ...)

• I happily conclude from the latter consideration:
the sole ring settings that feature very slow polarization decay
are of interest ...

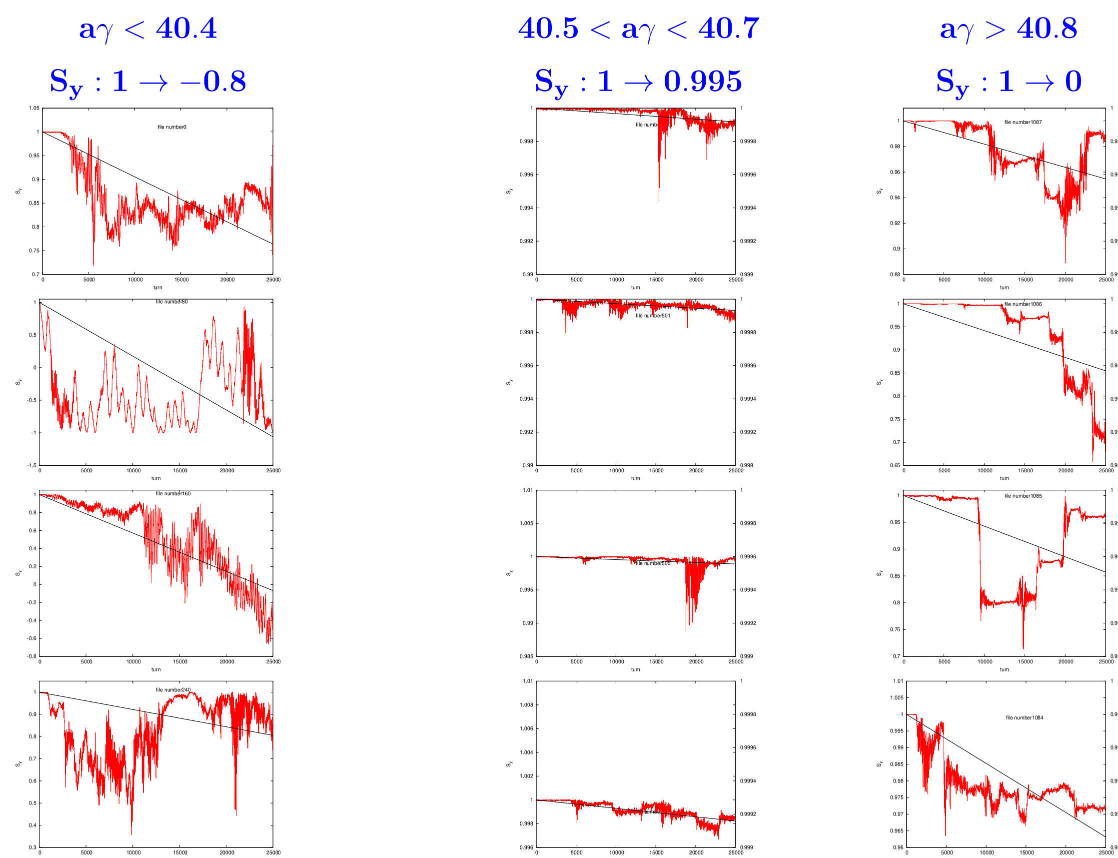
Numerical data in support to that hypothesis

Track a single electron per bin, 1 bin is 1 ring with specific rigidity ("$a\gamma_{ref}$")

Spin distributions at $80\times\tau_{SR}$ and at $900\times\tau_{SR}$:



Typical spin motion S_y(turn) in the 3 regions, and exponential regression:



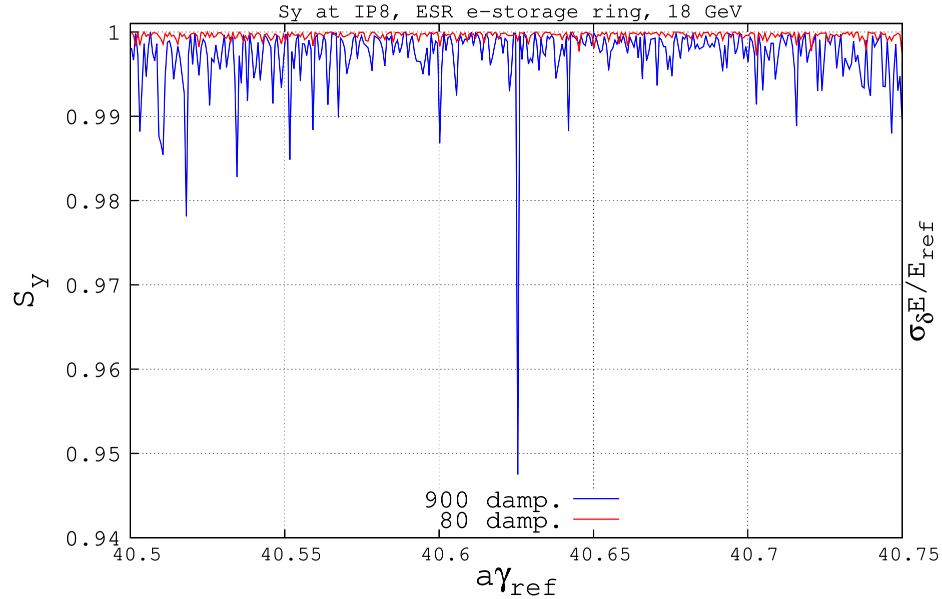
- In the central region the depolarization time constant τ_D is large, the jumps of $\vec{n}_\delta(s)$ are small most of the time.
- Assume that $S_y(t)$ takes it values (at each photon emission) near the average

$$\langle S_y \rangle (t) / \langle S_y \rangle (0) = \exp(-\frac{t}{\tau_D})$$

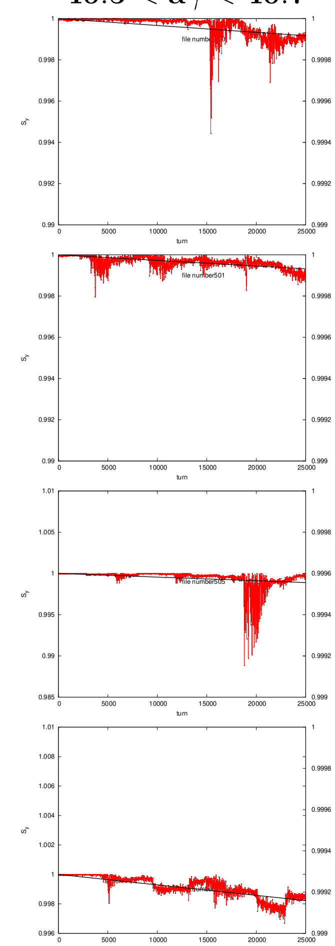
The stochastic variable here is $\delta S_y(t)$, the amplitude of the jump.

- Apply the ergodic hypothesis to $\delta S_y(t)$: Over a sufficiently long period of time T (to be determined), $\delta S_y(t)$ will have explored, a large number of times, all possible values (in the vicinity of the average $\overline{S}_y(t)$).
- My conclusion:

$\Rightarrow$ τ_D can be determined from a single particle sampling by applying the regression on $S_y(t : 0 \rightarrow T)$.



$40.5 < a\gamma < 40.7$



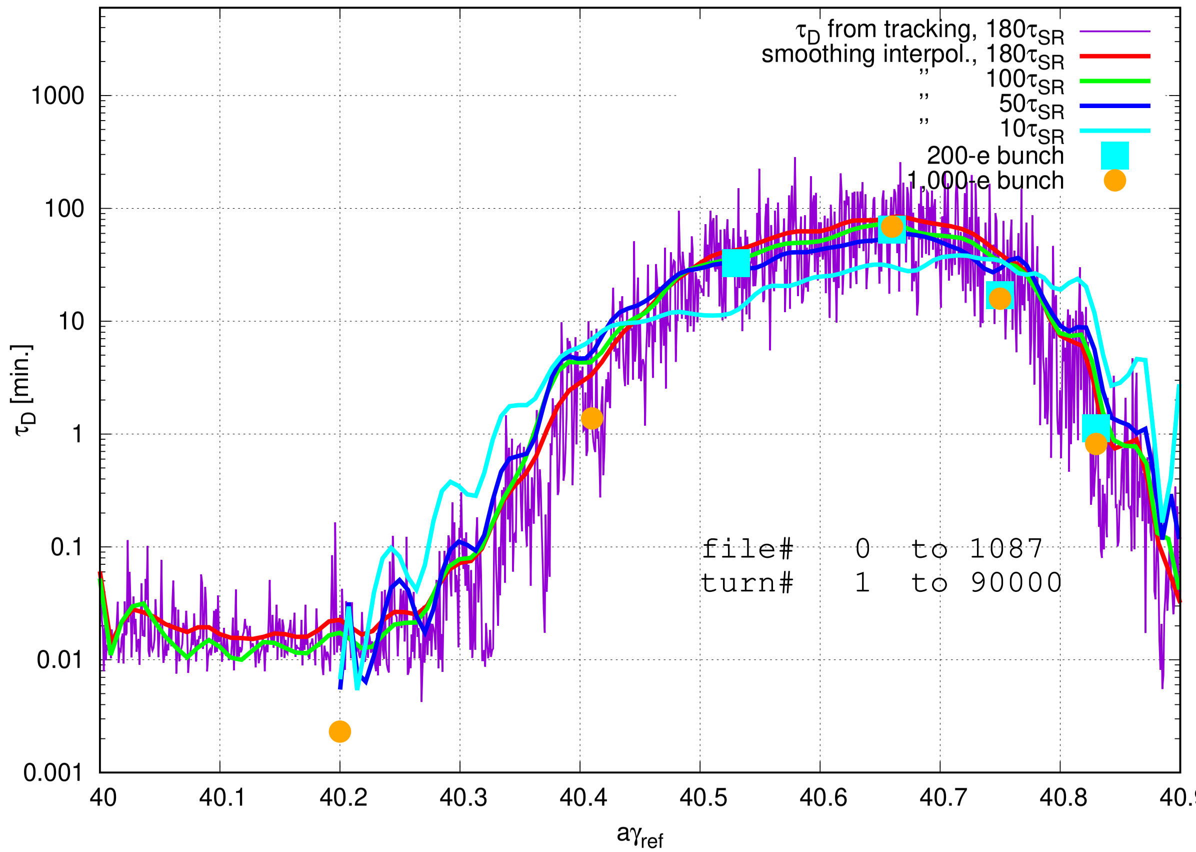
Convert to τ_D vs. $a\gamma_{ref}$

using the ad hoc regression

The graph gives indications about the convergence of the methods:

- red, green, blue, cyan smoothing interpolation curves, ergodic case, are from respectively 180, 100, 50 and $10 \times \tau_{SR}$ turns;

- Markers, electron bunch case, are for 200 or 1,000 electrons, 10 damping time tracking.



Fluctuat nec mergitur

Metric

A proper metric should allow comparison between rings with different settings (effects of errors, coupling, rotator mismatch, ...).

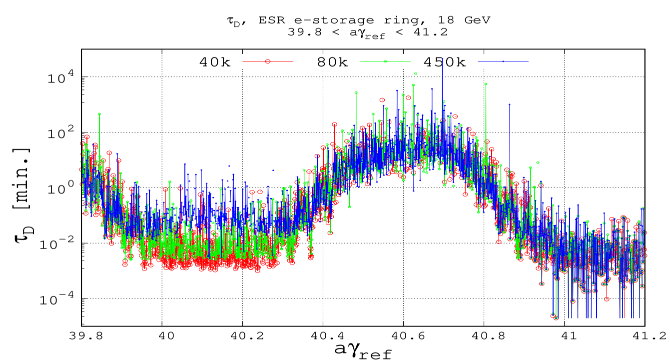
• A straightforward one: distance between interpolation curves, at the manner of the previous slide.
One point here is to determine an appropriate smoothing interpolation method for that stochastic $\tau_D(a\gamma_{ref})$ set.

• A possible additional ingredient: averaging $\tau_D(a\gamma_{ref})$ over a small $a\delta\gamma_{ref}$ interval, a few bins, greatly smooths the fluctuations.

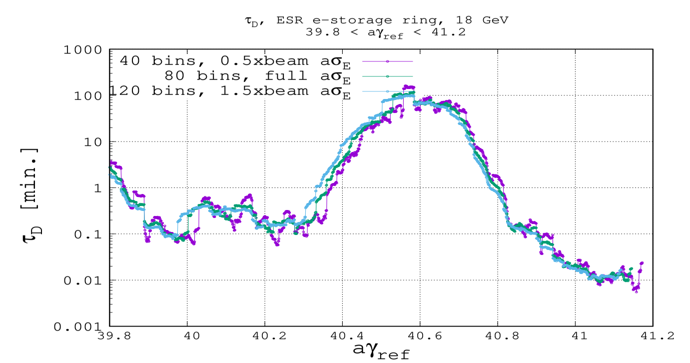
(i) assume for instance $a\Delta\gamma_{ref} = 1$ covered in 1,000 $a\gamma_{ref}$ bins,
1 electron explores $\sigma_\gamma/\gamma_{ref} \approx 10^{-3}$ (or $a\delta\gamma_{ref} = 0.04$), thus covering ∼40 bins,

(ii) thus, a set of a few bins can be considered to belong in the same ring,
⇒ averaging over a few bins is not very different from averaging over a few electrons in the same bin

• It can be a sliding averaging. The resulting distribution appears to evolve only weakly with number of bins N of the average interval (before, left; after, right):



⇒



• An additional possibilty to damp the τ_D fluctuations (?): rather than applying the regresison on a bare particle's $S_y(t)$ record, apply it on a cumulative averaging of $S_y(t)$ → liable to accelerate convergence to τ_D and/or reduce number of turns.

COMMENTS

- Assume the same $a\gamma_{ref}$ sampling (same number of reference rings in a given interval $a\Delta\gamma_{ref}$) and,

- using method 1: a few 100 electrons tracked a few SR damping times, or,

- using method 2: a single electron tracked over a few SR damping times.

In the present hypotheses (EIC ESR lattice, energy, etc.):

- method 1: the HPC volume is

  nRings $\times$ a few 10^2 [electrons/ring] $\times$ $\sim$10 SR damping times

- method 2: the HPC volume is

  nRings $\times$ 1 electron $\times$ a few 10s of SR damping times,

this is a two orders of magnitude difference.

- Larger HPC volume translates in one or the other of,

  - more processors, longer computing time, $\rightarrow$ longer HPC cluster queues, greater volume of I/Os, longer data analysis, ...

- Faster computation allows faster exploration of parameter space during design optimizations.

TO CONCLUDE

There is certainly more to learn and dig out from the ergodic hypothesis, beyond just the present brief considerations.

THANK YOU FOR YOUR ATTENTION

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