Fast First-Order Spin Propagation via the Bmad Library

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Bmad Library

- Open-source software library for accelerator simulations
- Primarily developed by David Sagan
- Well documented: ~600 pages and counting, not including full tutorial
- https://www.classe.cornell.edu/bmad/
- Supports many different orbital tracking methods:
  - Bmad tracking, Runge-Kutta integration, symplectic integration, more
- But what about spin?

Bmad Spin Tracking

- Bmad interfaces with PTC integration code
  - Enables accurate calculations of Taylor transfer maps to arbitrary order
  - But… very slow for big lattices
- Must make choice: speed vs accuracy
- Bmad: let the user choose on an element-by-element basis
  - Specify choice of tracking method in lattice file

"SPRINT" Spin Tracking

- Spin transport is governed by the Thomas-BMT equation:

$$\frac{d}{ds}\vec{S} = \vec{\Omega} \times \vec{S}$$

- PTC numerically integrates this equation
  – Requires resolving, even for identical elements
- For simple lattice elements, analytically solvable!
  – Drifts, dipoles, quadrupoles, combined function magnets, solenoids
- Solve differential equation once, reference later

Aside: Quaternions

- To save on computations, Bmad uses quaternions to represent rotations
  - Noncommutative, associative extension of imaginary numbers satisfying:

$$\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -\mathbf{1}$$

- A rotation of θ about the unit vector $\vec{e} = e_x\hat{x} + e_y\hat{y} + e_z\hat{z}$ looks like:

$$q = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)\left(e_x\boldsymbol{i} + e_y\boldsymbol{j} + e_z\boldsymbol{k}\right)$$

- This changes the Thomas-BMT equation to:

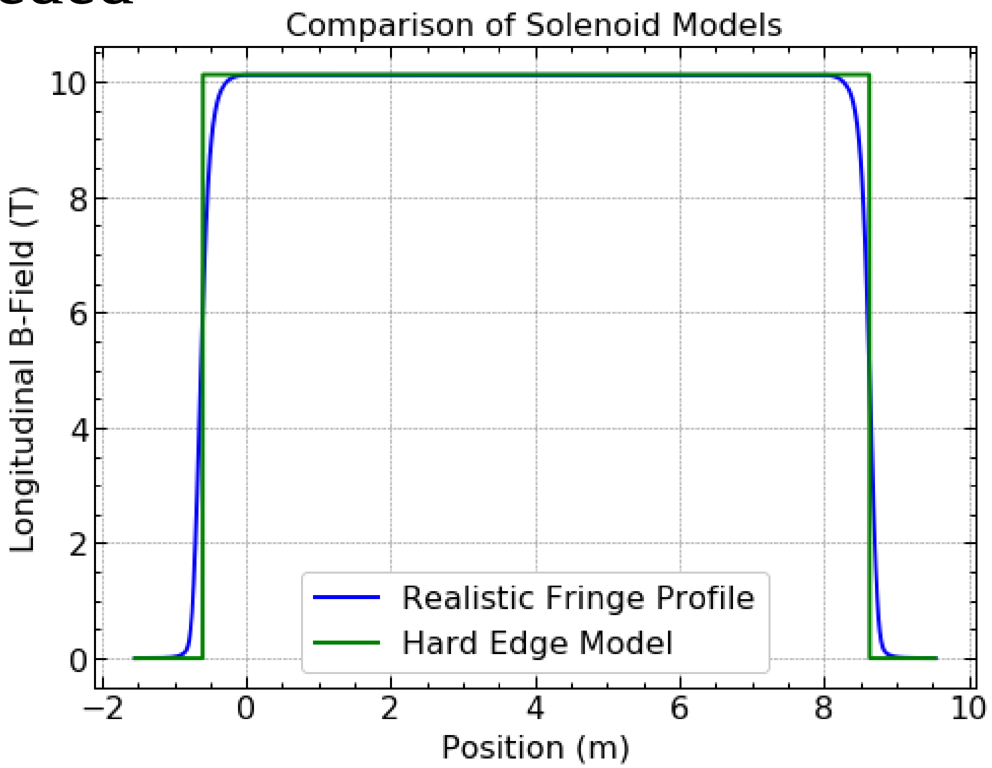
$$\frac{dq}{ds} = \frac{1}{2}\Omega q, \text{ with } \Omega = \Omega_x\boldsymbol{i} + \Omega_y\boldsymbol{j} + \Omega_z\boldsymbol{k}$$

Derivation, continued

- Split Ω into design components Ω_0 and perturbative components ω
- On design orbit, q_0 must satisfy $\frac{dq_0}{ds} = \frac{1}{2}\Omega_0 q_0$
  - This q_0 solution is easily solvable: rotation about Ω_0
- First order solution is of the form $q = (q_0 + \Delta q) = q_0(1 + \Delta \mathrm{b})$
- Must satisfy $\frac{d}{ds}\big(q_0(1 + \Delta b)\big) = \frac{1}{2}(\Omega_0 + \omega) q_0 (1 + \Delta b)$
- To first order, this becomes: $\Delta b' = \frac{1}{2} q_0^{-1} \omega q_0$

Fringe Effects

- Used a “hard-edge” model:
  - Approximate fringe as Dirac-Delta function
  - Historical calculations neglected this effect
- Bmad: let user enable/disable each fringe, as needed
- Bmad handles propagation of exit fringe



Results (Solenoid Example)

Table 1: Definitions of constants, with solenoid strength k_s and solenoid length l.

$s = a\, k_s\, l$	$t = (1+a)\, k_s l$	$\chi = 1 + a\,\gamma$
$c_s = \cos(s)$	$c_{t2} = \cos(\frac{t}{2})$	$\zeta = \gamma - 1$
$c_{s2} = \cos(\frac{s}{2})$	$s_{t2} = \sin(\frac{t}{2})$	

Table 2: Quaternion transfer map for hard-edge solenoid entrance fringe. To calculate the exit fringe, replace all field strengths k_s with $-k_s$, and propagate to the end of the element.

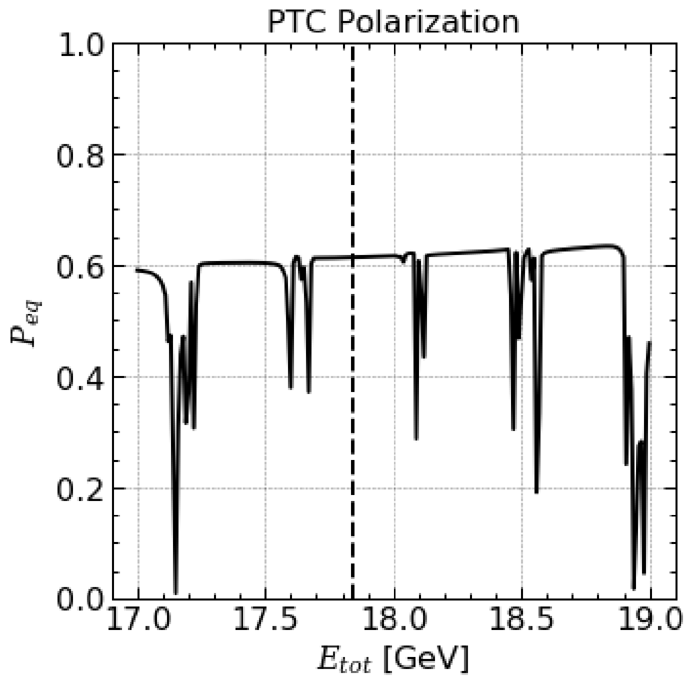
	q_0	q_x	q_y
1	1		
x		$\frac{1}{4}k_s\chi$	
y			$\frac{1}{4}k_s\chi$

Table 3: Quaternion transfer map for solenoid body. Quaternions must be normalized before use.

	q_0	q_x	q_y	q_z
1	c_{t2}			$-s_{t2}$
x		$\frac{1}{4}k_s\zeta((1-c_s)c_{t2} - s_s s_{t2})$	$\frac{1}{4}k_s\zeta((-1+c_s)s_{t2} - s_s c_{t2})$	
p_x		$\frac{1}{2}\zeta((1-c_s)s_{t2} + s_s c_{t2})$	$\frac{1}{2}\zeta((1-c_s)c_{t2} - s_s s_{t2})$	
y		$\frac{1}{4}k_s\zeta((1-c_s)s_{t2} + s_s c_{t2})$	$\frac{1}{4}k_s\zeta((1-c_s)c_{t2} - s_s s_{t2})$	
p_y		$\frac{1}{2}\zeta((-1+c_s)c_{t2} + s_s s_{t2})$	$\frac{1}{2}\zeta((1-c_s)s_{t2} + s_s c_{t2})$	
p_z	$\frac{1}{2}t s_{t2}$			$\frac{1}{2}t c_{t2}$

Comparisons

- First compared SPRINT Taylor map computations to PTC
- Once satisfied, compare equilibrium polarization on EIC ESR 5.3 lattice via Bmad routine – contains solenoids, quadrupoles, dipoles
- Nearly identical results produced ~6x faster via SPRINT!



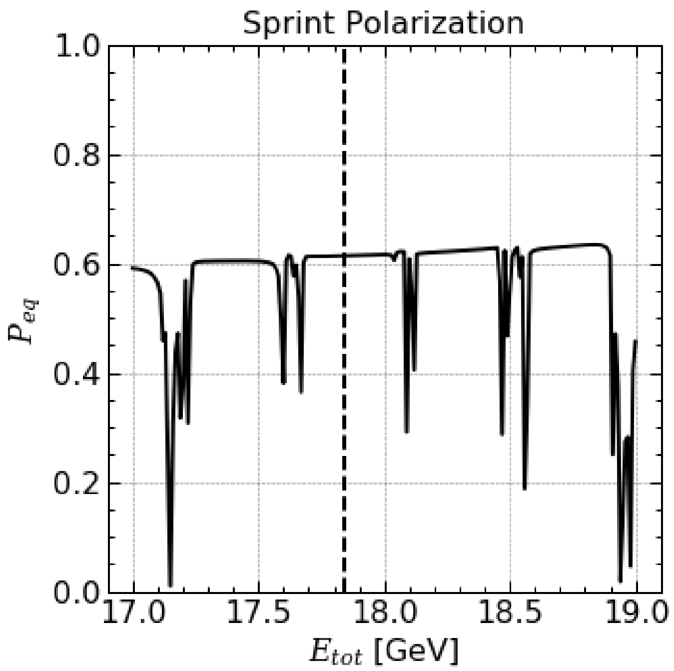


Figure 1-2: Equilibrium polarization calculations for ESR EIC 5.3 lattice. PTC requires ~2 hours to produce 200 data points for this plot, while SPRINT takes ~20 minutes.

Advantages

- PTC beneficial for long term tracking
  - Compute accurate, higher-order map once, then reuse
  - Initial time complexity of PTC becomes obsolete
- SPRINT useful for tuning of accelerators
  - Taylor maps need to be recalculated when tuning magnet strengths
  - Examples: tuning optics, spin matching

Conclusion

- Bmad now supports precalculated first-order spin maps
  - Supports drift, quadrupole, dipole, solenoid, and combined function magnets
- Includes fringe kick calculations
- Useful for quick polarization calculations
- Results are nearly identical to accepted PTC tracking
- Bmad now has a spin formalism corresponding to orbital matrix formalism.

Bmad Resources

- Curious to learn more?
- https://www.classe.cornell.edu/bmad/
- Free in-person Bmad school Oct. 7-9, following ERL 2022:
  - https://indico.classe.cornell.edu/event/2068/

Thank you!

Questions?