Resonant depolarization precision, relation between spin tune and beam energy, beam energy and central mass energy for FCCee_z_213_nosol_18.seq

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Introduction: the goals and procedure

The goal of energy calibration is to determine central mass energy at the interaction point

Procedure:

1. spin precession frequency measurement
2. calculation of the ring average beam energy
3. calculation of the IP beam energy
4. calculation of the central mass energy

Introduction: spin precession frequency

Ω_0 is revolution frequency.
W is spin precession frequency.
Gyromagnetic ratio: $q = q_0 + q' = \frac{e}{mc} + q'$.

$$W = \frac{1}{2\pi} \oint \left(\frac{q_0}{\gamma} + q' \right) B_\perp(\theta) d\theta = \Omega_0 \cdot \left(1 + \frac{q'}{q_0} \frac{\langle B_\perp \rangle}{\langle B_\perp / \gamma \rangle} \right)$$
$$\approx \Omega_0 \cdot \left(1 + \langle \gamma \rangle \frac{q'}{q_0} \right) ,$$

$$\frac{q'}{q_0} = \frac{g-2}{2} = 1.1596521859 \cdot 10^{-3} \pm 3.8 \cdot 10^{-12} .$$

$$E[MeV] = 440.64846 \left(\frac{W}{\Omega_0} - 1 \right) .$$

Introduction: different energies

Design energy: $$E_0 \approx p_0 c = \frac{e}{2\pi} \oint B_\perp ds$$

Average energy: $$\langle E \rangle = \oint E(s) \frac{ds}{\Pi}$$

Measured energy: $$E_{meas} = \left(\frac{W_{spin}}{\Omega_0} - 1 \right) 440.64846 = \langle E \rangle + \Delta E$$

Energy at IP: $$E^{\pm}(IP) = \langle E^{\pm} \rangle + \Delta E$$

Invariant mass:
(central mass energy) $$M = (E^{-}(IP) + E^{+}(IP)) \cos\theta$$

Measured and average energy: momentum compaction chromaticity

Momentum compaction: $\delta = \frac{\Delta E}{E_0} = -\frac{1}{\alpha}\frac{\Delta\Omega}{\Omega_0}$, $\alpha = \alpha_0 + \alpha_1\delta$

Synchrotron oscillations: $\ddot{\delta} = -\omega_{syn}^2\delta - \omega_{syn}^2\frac{\alpha_1}{\alpha_0}\delta^2$

Average and RMS: $\langle\delta\rangle = -\frac{\alpha_1}{\alpha_0}\sigma_\delta^2$, $\langle\delta^2\rangle = \sigma_\delta^2$

Average W: $\langle W\rangle_\delta = \gamma_0\Omega_0\frac{q'}{q_0}\left(1 - \alpha_0\sigma_\delta^2 - \frac{\alpha_1}{\alpha_0}\sigma_\delta^2\right)$

Average energy: $\langle E\rangle = E_0\left(1 - \frac{\alpha_1}{\alpha_0}\sigma_\delta^2\right)$

Measured energy: $E_{meas} = E_0\left(1 - \frac{\alpha_1}{\alpha_0}\sigma_\delta^2 - \alpha_0\sigma_\delta^2\right)$

Measured and average energy: momentum compaction chromaticity

Beam calibrations are performed on noncolliding bunches.

Beamstrahlung increases beam energy spread 3.5 times.

$E_0 = 45.6$ GeV, $\alpha_0 = 1.5 \cdot 10^{-5}$, $\alpha_1 = 8.6 \cdot 10^{-6}$, $\sigma_\delta = 3.8 \cdot 10^{-4}$, $\sigma_{\delta,bs} = 1.32 \cdot 10^{-3}$

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Noncolliding bunch:
$$\frac{\langle E\rangle - E_{meas}}{E_{meas}} = \frac{\alpha_0\sigma_\delta^2}{1 - \frac{\alpha_1}{\alpha_0}\sigma_\delta^2 - \alpha_0\sigma_\delta^2} = 2 \cdot 10^{-12}$$

Colliding bunch:
$$\frac{\langle E\rangle_{col} - E_{meas}}{E_{meas}} = \frac{\alpha_0\sigma_\delta^2 + \frac{\alpha_1}{\alpha_0}(\sigma_\delta^2 - \sigma_{\delta,bs}^2)}{1 - \frac{\alpha_1}{\alpha_0}\sigma_\delta^2 - \alpha_0\sigma_\delta^2} = -9 \cdot 10^{-7}$$

Measured and average energy: longitudinal field compensation

Detector field is $B_0 = 2$ T.

Deviation of compensating field is $\Delta B_c = 0.1$ T.

Length of compensating solenoid is $L_c = 0.75$ m.

$B\rho = 152.105\,\mathrm{T}\cdot\mathrm{m}$, $E_0 = 45.6$ GeV, $\nu = 103.484$.

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$$\Delta\nu = \frac{\varphi^2}{8\pi}\cot(\pi\nu) \approx \frac{1}{8\pi}\cot(\pi\nu)\left(\frac{\Delta B_c}{B_0}\frac{2B_0 L_c}{B\rho}\right)^2 \approx 2\times 10^{-9}\,.$$

$$\frac{\Delta E}{E_0} = \frac{\Delta\nu \cdot 440.65}{E_0} \approx 2\times 10^{-11}\,.$$

Measured and average energy: vertical orbit

Spin tune shift (Kondratenko)

$$\Delta\nu = \frac{1}{2}\sum_k \frac{|\omega_k|^2}{\nu - k}$$

Spin harmonics

$$\omega_k = \frac{1}{2\pi}\int_0^{2\pi} \nu y'' \exp\left[-i(\Phi(\theta) - \nu\theta) - ik\theta\right] d\theta\,,$$

$$y'' = \frac{1}{R}\frac{d^2 y}{d\theta^2}\,, \quad \Pi = 2\pi R\,, \quad ds = Rd\theta\,,$$

$$\Phi(\theta) = \int_0^{\theta} \nu R K_0(\theta')d\theta'$$

Measured and average energy: vertical orbit distortions

Assumptions and definitions

- Spin tune $\nu = \frac{W_{spin}}{\Omega_0} - 1$
- No straight sections: $\Phi(\theta) = \nu\theta$
- Constant vertical beta function: $\beta_y = const = \langle\beta_y\rangle$
- Average over circumference $\langle\rangle$, average over orbits $\bar{\ }$

Results

$$\overline{\Delta\nu} = \frac{\nu^2}{2}\frac{\overline{\langle y^2\rangle}}{Q}\sum_{k=-\infty}^{\infty}\frac{k^4}{(\nu_y^2-k^2)^2(\nu-k)}$$

$$Q = \frac{\pi}{2\nu_y^3}\cot\pi\nu_y + \frac{\pi^2}{2\nu_y^2}\csc^2\pi\nu_y$$

$$\sigma_{\overline{\Delta\nu}} = \frac{\nu^2\sqrt{3}}{2}\frac{\overline{\langle y^2\rangle}}{Q}\sqrt{2\nu\sum_{k=-\infty}^{\infty}\frac{k^8}{(\nu_y^2-k^2)^4(\nu-k)^2(\nu+k)}}$$

Measured and average energy: vertical orbit distortions

E, GeV	45.6	78.65	81.3
$\sqrt{\langle y^2 \rangle}$, mm	0.6	0.28	0.27
ν_y	269.22	269.2	269.2
$\Delta E, keV$	-31	-54	-56
$\sigma_{\Delta E}, keV$	46	82	85
$\frac{\Delta E}{E}$	$-7 \cdot 10^{-7}$	$-7 \cdot 10^{-7}$	$-7 \cdot 10^{-7}$
$\frac{\sigma_{\Delta E}}{E}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-6}$	$1 \cdot 10^{-6}$

Beam energy shift needs to be added to the actual value of the beam energy, uncertainty is unavoidable and sets the minimum error.

Ring alignment: individual harmonics

Alignment harmonic

$$y = A_k \sin\left(k2\pi\frac{s}{\Pi}\right) = A_k \sin(k\theta)\,,$$

where A_k is harmonic amplitude.

Spin harmonics

assuming no straight sections ($\Phi(\theta) = \nu\theta$)

$$\omega_k = -\frac{1}{2i}\nu\frac{A_k}{R}k^2\,, \qquad \frac{\Delta\nu}{\nu} = \frac{1}{8}\frac{A_k^2}{R^2}\left(\frac{\nu k^4}{\nu - k} + \frac{\nu k^4}{\nu + k}\right) = \frac{1}{4}\frac{A_k^2}{R^2}\frac{\nu^2 k^4}{\nu^2 - k^2}$$

$$\omega_{-k} = \frac{1}{2i}\nu\frac{A_k}{R}k^2\,.$$

Measured and average energy: ring alignment

Beam energy $E = 45.6$ GeV, $\nu = 103.484$, $\Pi = 100$ km

$A_k = 15 \cdot 10^{-3}$ m

k	$\Delta\nu/\nu$	$\lvert\omega_k\rvert$
1	$2 \cdot 10^{-13}$	$5 \cdot 10^{-5}$
2	$4 \cdot 10^{-12}$	$2 \cdot 10^{-4}$
3	$2 \cdot 10^{-11}$	$4 \cdot 10^{-4}$
4	$6 \cdot 10^{-11}$	$8 \cdot 10^{-4}$
10	$2 \cdot 10^{-9}$	$5 \cdot 10^{-3}$
50	$2 \cdot 10^{-6}$	0.12
100	$3.5 \cdot 10^{-4}$	0.5
103	$2.8 \cdot 10^{-3}$	0.5

$A_k = 3 \cdot 10^{-4}$ m

k	$\Delta\nu/\nu$	$\lvert\omega_k\rvert$
1	$1 \cdot 10^{-16}$	$1 \cdot 10^{-6}$
2	$2 \cdot 10^{-15}$	$4 \cdot 10^{-6}$
3	$8 \cdot 10^{-15}$	$9 \cdot 10^{-6}$
4	$2 \cdot 10^{-14}$	$2 \cdot 10^{-5}$
10	$9 \cdot 10^{-13}$	$1 \cdot 10^{-4}$
50	$8 \cdot 10^{-10}$	$2 \cdot 10^{-3}$
100	$1.4 \cdot 10^{-7}$	$1 \cdot 10^{-2}$
103	$1.1 \cdot 10^{-6}$	$1 \cdot 10^{-2}$

Spin distribution width: synchrotron oscillations

Synchrotron oscillations: $\delta = \Delta E/E_0 = a \cdot \cos(\omega_{syn} t)$.

$$W = \Omega_0 \left(1 + \nu_0 - \alpha_0 \nu_0 \frac{a^2}{2}\right) + \Omega_0 \left(\nu_0(1 - \alpha_0) - \alpha_0\right) \sin(\omega_{syn} t) + \alpha_0 \Omega_0 \nu_0 \frac{a^2}{2} \cos(2\omega_{syn} t)$$

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Spin precession frequency distribution shifts and becomes wider by

$$\left\langle \frac{W - \Omega_0(1+\nu_0)}{\Omega_0(1+\nu_0)} \right\rangle = \left\langle -\frac{\alpha_0 \nu_0 \frac{a^2}{2}}{1+\nu_0} \right\rangle = -\frac{\alpha_0 \nu_0 \sigma_\delta^2}{1+\nu_0} = -2 \cdot 10^{-12}$$

$$\frac{\Delta E}{E_0} = -2 \cdot 10^{-14}$$

Spin distribution width: horizontal betatron oscillations

Ya.S. Derbenev, et al., "Accurate calibration of the beam energy in a storage ring based on measurement of spin precession frequency of polarized particles", Part. Accel. 10 (1980) 177-180

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Sextupole fields introduce additional $B_{\perp} \propto x^2$, $K2 = \frac{1}{B\rho}\frac{\partial^2 B_y}{\partial x^2}$.

Spin precession frequency distribution shifts and becomes wider by

$$\frac{\Delta\nu}{\nu} = -\frac{1}{2\pi}\oint \left(\varepsilon_x \beta_x(s) + \eta_x(s)^2 \sigma_\delta^2\right) K2(s) ds\,.$$

$$\frac{\Delta\nu}{\nu} = \frac{\Delta E}{E_0} = -7 \cdot 10^{-7}\,.$$

Beam energy: vertical magnetic fields (horizontal correctors)

One corrector with deflection χ: $\frac{\Delta E}{E_0} = -\frac{\chi \eta_x}{\alpha \Pi}, \quad \chi = \oint \frac{\Delta B_y}{B\rho} ds$.

RMS of energy shift: $\sigma\left(\frac{\Delta E}{E_0}\right) = \frac{2\sqrt{2}\sin(\pi\nu_x)}{\alpha\Pi}\frac{\langle\eta_x\rangle}{\langle\beta_x\rangle}\sqrt{\langle x^2\rangle}$

$\sqrt{\langle x^2\rangle}$ is RMS of horizontal orbit variation.

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$$\sigma\left(\frac{\Delta E}{E_0}\right) = -1.1 \cdot 10^{-3}[m^{-1}] \cdot \sqrt{\overline{\langle x^2\rangle}}[m],$$

$\sigma\left(\frac{\Delta E}{E_0}\right) = 10^{-6}$ demands stability of the horizontal orbit between calibrations

$$\sqrt{\langle x^2\rangle} = 0.9 \text{ mm}.$$

Beam energy: vertical magnetic fields (quadrupoles)

Shifted quadrupole: $\frac{\Delta E}{E_0} = -\frac{\chi \eta_x}{\alpha \Pi}, \quad \chi = K1L \cdot \Delta x \, , K1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}.$

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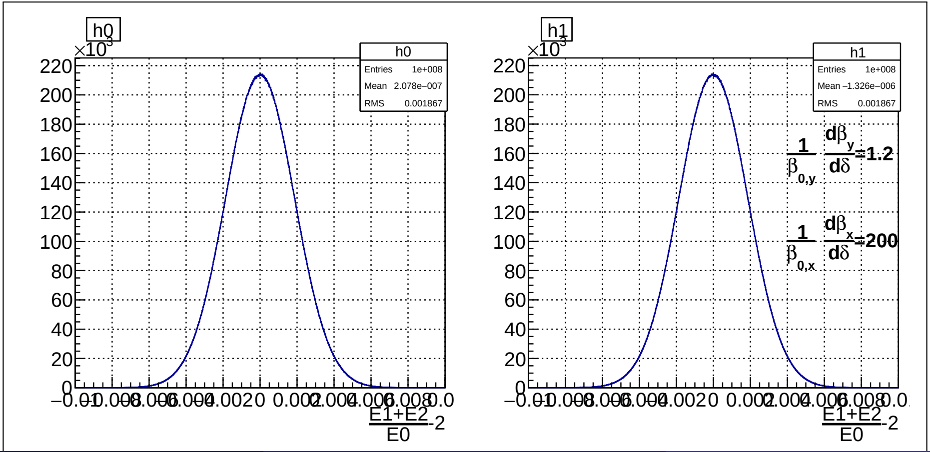
$\frac{\Delta E}{E_0} = 10^{-6}$ demands stability of quadrupoles position between calibrations (10 min)

Quadrupole	Δx, m
QC7.1:	$2 \cdot 10^{-4}$
QY2.1:	$6.5 \cdot 10^{-5}$
QFG2.4:	$1.6 \cdot 10^{-4}$
QF4.1:	$1.6 \cdot 10^{-4}$
QG6.1:	$5 \cdot 10^{-5}$
QF4:	$\Delta x / \sqrt{720} = 6 \cdot 10^{-6}$

Central mass energy: β chromaticity

Invariant mass: $M^2 = (E_1 + E_2)^2 \cos^2(\theta) + O(m_e^2) + O(\sigma_\alpha^2) + O(\sigma_E^2)$.
Beta function chromaticity at IP: $\beta_{x,y} = \beta_{0\,x,y} + \beta_{1\,x,y}\delta$, $\sigma_{x,y}^2 = \varepsilon_{x,y}\beta_{x,y}$.
Particles with energy deviation have higher collision rate.



Central mass energy: β chromaticity

FCCee_z_202_nosol_13: goal

$\frac{1}{\beta_x}\frac{d\beta_x}{d\delta}$	$\frac{1}{\beta_y}\frac{d\beta_y}{d\delta}$	ΔM, keV	$\frac{\Delta M}{E_0}$
0	1.2	-51 ± 8.5	$-1.1 \cdot 10^{-6} \pm 1.9 \cdot 10^{-7}$
200	0	-13 ± 8.5	$-2.8 \cdot 10^{-7} \pm 1.9 \cdot 10^{-7}$
200	1.2	-61 ± 8.5	$-1.3 \cdot 10^{-6} \pm 2 \cdot 10^{-7}$
200	12	-489 ± 8.5	$-1.1 \cdot 10^{-5} \pm 2 \cdot 10^{-7}$

Need to measure and adjust $\frac{1}{\beta_{0y}}\frac{d\beta_y}{d\delta}$.

Energy dependence on azimuth: full tapering

Two diametrically opposite RF cavities, U_0 — energy loss per revolution, $E(0)$ — after RF cavity. Full tapering — magnets fields are adjusted to keep design curvature, quadrupole strength etc.

$$\frac{dE}{ds} \propto E^4 , \quad E(s) = \frac{E(0)}{(1 + k \cdot s)^{\frac{1}{3}}} , \qquad k \approx \frac{3}{\Pi}\frac{U_0}{E(0)} + \frac{3}{\Pi}\frac{U_0^2}{E(0)^2} + O(U_0^3)$$

$$\langle E \rangle \approx E(0) - \frac{U_0}{4} - \frac{U_0^2}{12E(0)} , \qquad E(IP) = E(0) - \frac{U_0}{4}$$

$E(0)$, GeV	$E(IP) - \langle E \rangle$, keV	$\frac{E(IP) - \langle E \rangle}{\langle E \rangle}$
45.6	2.3	$5 \cdot 10^{-8}$
80.5	125	$1.5 \cdot 10^{-6}$

Energy dependence on azimuth: partial tapering

Partial tapering (ΔK_0) — fields of magnets groups are adjusted to keep approximately design curvature (K_0).

Equations of motion (canonical variables)

$$\begin{cases} \sigma' = -K_0 x\,, \\ p_t{}' = \left(-\frac{eV_0}{p_0 c}\right)\sin\left(\phi_s + \frac{2\pi}{\lambda_{RF}\sigma}\right)\delta(s-s_0) - \frac{2}{3}\frac{e^2\gamma^4}{p_0 c}K_0^2\sigma\,. \end{cases}$$

Solution: $p_t(s) = p_{0t} - f(s)$.

$$\sigma = 0 = -\int_0^\Pi K_0(s)x(s)ds = -p_{0t}\alpha\Pi + \Pi\left\langle (K_0 f + \Delta K_0)\eta\right\rangle_s\,.$$

$$p_{0t} = \frac{1}{\alpha}\left\langle (K_0 f + \Delta K_0)\eta\right\rangle_s\,.$$

Energy dependence on azimuth: partial tapering

For simple (symmetrical) cases we do need to know function $f(s)$, just at certain points.

Two RF cavities and symmetrical arcs

$$\begin{cases} \langle p_t \rangle = p_{0t} - \langle f \rangle = p_{0t} - \dfrac{U_0}{4E_0} = \dfrac{\langle E \rangle - E_0}{E_0}, \\ p_t(IP) = p_{0t} - f(IP) = p_{0t} - \dfrac{U_0}{4E_0} = \dfrac{E_{IP} - E_0}{E_0}, \end{cases}$$

$$\begin{cases} \langle E \rangle = E_0 + E_0 p_{0t} - \dfrac{U_0}{4}, \\ E_{IP} = E_0 + E_0 p_{0t} - \dfrac{U_0}{4}. \end{cases}$$

There is no difference between $\langle E \rangle$ and E_{IP} in the first order. Numerical calculations are needed for not symmetrical arcs, magnet misalignments.

Collective field of the own bunch

Electron in the field of own bunch will have potential energy

$$U[eV] = \frac{N_p e^2[Gs]}{\sqrt{2\pi}\sigma_s[cm]}\left(\gamma_e + \ln(2) - 2\ln\left(\frac{\sigma_x+\sigma_y}{r}\right)\right)\frac{10^{-7}}{e[C]},$$

$\gamma_e = 0.577$ Euler constant, $N_p = 4\cdot 10^{10}$ — bunch population, $r_{ip} = 15$ mm and $r_{arc} = 20$ mm — vacuum chamber radius at IP and in the arcs, $\sigma_{x,IP} = 6.2\cdot 10^{-6}$ m, $\sigma_{y,IP} = 3.1\cdot 10^{-8}$ m, , $\sigma_{x,arc} = 1.9\cdot 10^{-4}$ m, $\sigma_{y,arc} = 1.2\cdot 10^{-5}$ m.

$$\frac{U_{ip}}{E_0} = \frac{469keV}{45.6GeV} = 1\cdot 10^{-5},$$
$$\frac{U_{arc}}{E_0} = \frac{290keV}{45.6GeV} = 6\cdot 10^{-6}.$$

Potential energy at the center of the bunch $\{x, y, s, z = s - ct\} = \{0, 0, 0, 0\}$

$$U(x, y, s, ct) = -\frac{\gamma N_p r_e mc^2}{\sqrt{\pi}} \int_0^\infty dq \frac{\exp\left[-\frac{(x+s\cdot 2\theta)^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q} - \frac{\gamma^2(s+ct)^2}{2\gamma^2\sigma_s^2+q}\right]}{\sqrt{2\sigma_x^2+q}\sqrt{2\sigma_y^2+q}\sqrt{2\gamma^2\sigma_s^2+q}},$$

$$\frac{U(0,0,0,0)}{E_0} = -\frac{1\,MeV}{45.6\,GeV} = -2.3\cdot 10^{-5}.$$

Invariant mass in the external field

The four-momentum: $P^\mu = (E - e\varphi, \vec{p}) = (E - e\varphi, \vec{\mathcal{P}} - \frac{e}{c}\vec{A})$,

Energy-momentum relation: $(E - e\varphi)^2 = m^2c^4 + c^2(\vec{p})^2$.

Invariant mass:

$$M^2 = (P_1^\mu + P_2^\mu)^2 = 2E_1e_1\varphi + 2E_2e_2\varphi + 2E_1E_2 - (e_1\varphi)^2 - (e_2\varphi)^2 - 2\vec{p^{(1)}}\vec{p^{(2)}}.$$

Longitudinal momentum ($\delta = (E_i - E_0)/E_0$, $u = e_i\varphi/E_0$):

$$p_{i,s} = \sqrt{(E_i - e_i\varphi)^2 - p_{i,x}^2 - p_{i,y}^2} = E_0\sqrt{(1 + \delta_i - u)^2 - \left(\frac{p_{i,x}}{E_0}\right)^2 - \left(\frac{p_{i,y}}{E_0}\right)^2}.$$

Invariant mass

Average values

$$\left\langle M^2 \right\rangle = 4E_0^2 \cos^2(\theta)(1-u^2) - 2E_0^2\sigma_{px}^2\cos(2\theta) - 2E_0^2\sigma_{py}^2\cos(2\theta)$$

$$\langle M \rangle = 2E_0\cos(\theta)\left(1-\frac{u^2}{2}\right) - \frac{E_0}{2}\left(\sigma_\delta^2\cos(\theta) + \sigma_{px}^2\cos(\theta) + \sigma_{py}^2\frac{\cos(2\theta)}{\cos(\theta)}\right)$$

$$\left\langle M^2 \right\rangle - \langle M \rangle^2 = 2E_0^2\cos^2(\theta)\left(\sigma_\delta^2 + \sigma_{px}^2\tan^2(\theta)\right)$$

Invariant mass shift due to beam potentials

$$\frac{\langle M \rangle - 2E_0\cos(\theta)}{2E_0\cos(\theta)} = \left(1 - \frac{(e\varphi)^2}{2E_0^2}\right) \approx 3\times 10^{-10}$$

Summary

Largest corrections and errors

1. Momentum compaction factor chromaticity and different energy spreads for colliding and noncolliding bunches (correction, error) $\frac{\Delta E}{E} \lesssim -1 \times 10^{-6}$.
2. Vertical orbit distortions (error): $\frac{\sigma_{\Delta E}}{E} \simeq 1 \times 10^{-6}$ require $\sqrt{\langle y^2 \rangle} \leqslant 0.3$ mm.
3. Spin distribution width from horizontal betatron oscillations (error) $\frac{\Delta E}{E} \simeq 1 \times 10^{-6}$.
4. Horizontal correctors and shift of quadrupoles (error) $\sim 10^{-6}$ with position stability of arc quadrupoles $\Delta x < 6 \times 10^{-6}$ between calibrations (every 10 minutes).
5. IP energy difference from average at 80.5 GeV (correction) $\frac{\Delta E}{E} \simeq 1 \times 10^{-6}$.
6. Beta function chromaticity (correction, tunable) $< 1 \times 10^{-6}$.
7. Ring vertical alignment should be better than $3 \cdot 10^{-4}$ m over 1 km to have spin tune shift $\delta\nu/\nu < 10^{-6}$ for $E = 45.6$ GeV.