Energy Measurement with undulator

September 22st, 2022

U.S. DEPARTMENT OF ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

As discussed with Joel Chavanne (ESRF), a possible scheme would be to have an
undulator with at least 100-1000 periods, with a period around 10-100cm.
The K probably should be very small <<1.
A possibility would be to have a K such as the rate of photons (gammas)
emitted is of the order of 10-1000KHz. Joel (or any expert in the filed) could dimension the undulator accordingly,
possibly we could use third harmonic gammas for the beam energy < 100GeV and the ones at the fundamental
for Higgs/TOP operations, in this way the photons could have much similar energies for all cases.
The energy measurement of the gammas needs to be deeply studied (and we should involve
experienced people for this),
We are considering energies in the range of 0.1-10MeV and we should define the Undulator parameters
according to the best (and fastest) energy resolution measurement.
If we produce gammas just above the pair production threshold we
could gain in accuracy since we have to measure just the kinetic energy of the pair (a trick
similar to the one used for the polarization where a large part of the energy is given by physics laws),
for instance if the photon energy is 1.122MeV we have to accurately measure just 50KeV
electrons (and positrons), however the converter should be very thin or we could have a sandwich/multilayer CCD-type
detector in a weak Bfield…
It also should be desirable that the system could be calibrated at least for two different energies with the depolarization
measurement…

We need to know the longitudinal motion β_s of the electron in the undulator to bring more consistence to our initial "naïve" device:

Since γ is constant so is $\beta_e^2 = \beta_x^2 + \beta_s^2 = 1 - \frac{1}{\gamma^2}$ $\quad (\beta_x(s) = \frac{K}{\gamma}\sin(\frac{2\pi s}{\lambda_0}))$

$$\beta_s(s) \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2} + \frac{K^2}{4\gamma^2}\cos(\frac{4\pi s}{\lambda_0})$$

Average longitudinal relative velocity: $\hat{\beta}_s \approx 1 - \frac{1}{2\gamma^2} - \frac{K^2}{4\gamma^2}$

Angle dependent emitted wavelength: $\lambda(\theta) \approx \frac{\lambda_0}{2\gamma^2}(1 + \frac{K^2}{2} + \gamma^2\theta^2)$

γ	λ_0[cm]	B[T]	K	$\lambda(0)$
11820	2	1	1.87	1.96 Å
11820	2	0.1	0.187	0.72 Å

We have now a field dependent wavelength

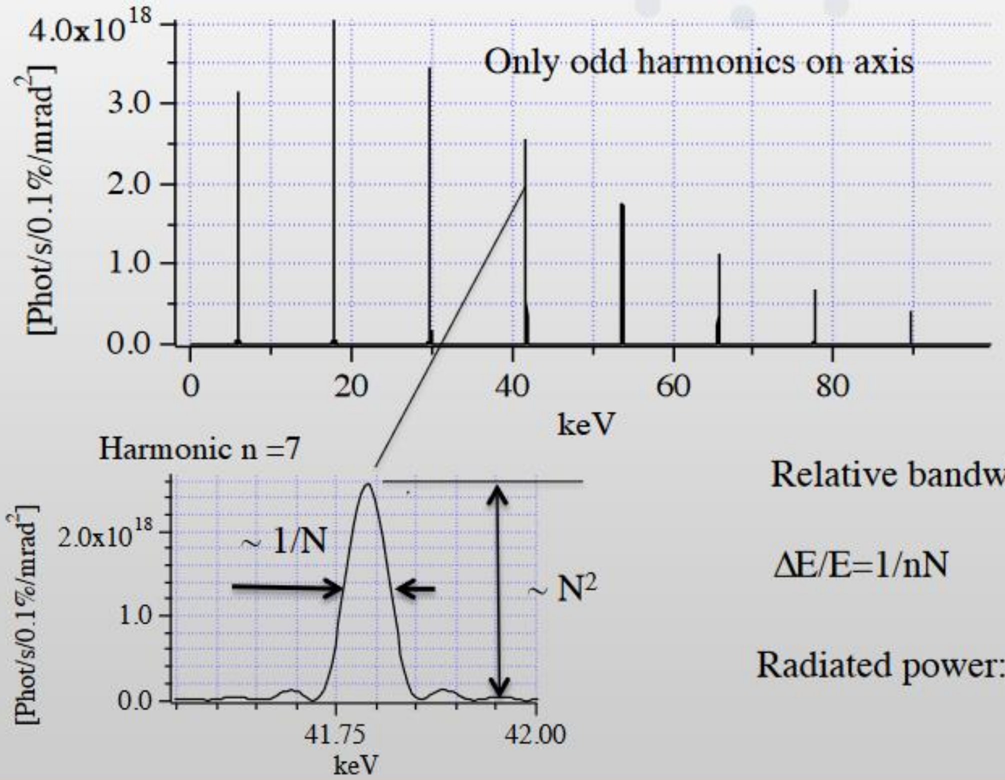
On axis angular spectral flux

Spectral photon flux units: Watts/eV can be translated into photons/sec/relative bandwidth

Ex: 1 phot/s/0.1%bw= 1.602e-16 W/eV

Angular spectral flux: photon flux/unit solid angle   Usual unit is phot/sec/0.1%/mrad^2:

Ideal on axis angular spectral flux with filament electron beam (zero emittance)



Undulator:
Period λ_0= 22 mm
Number of period **N**=90
K=1.79

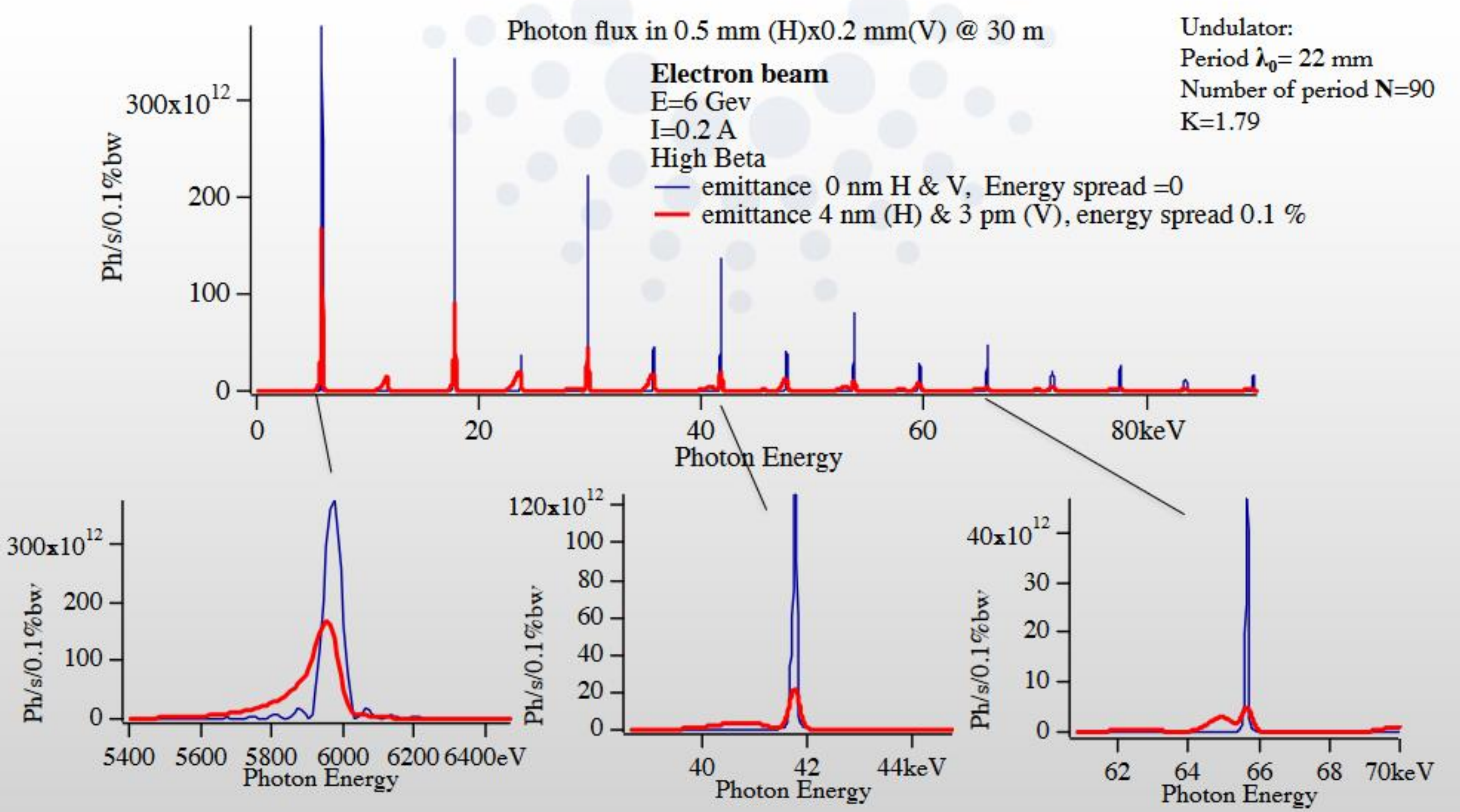
Relative bandwidth at harmonic n:

$\Delta E/E=1/nN$

Radiated power: ~ $N^2/N=N$ proportional to N

Undulator spectra with actual beam

A light for Science



Spectral performances dominated by horizontal emittance and energy spread at high harmonics

~ additional off axis contribution due to electron beam size and divergence $\left(\lambda_n(\theta) = \frac{\lambda_0}{2n\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)\right)$

Unfortunately the beam emittance is large wrt to the photon one, so the spectrum is far from being monochromatic.
In this case a precision measurement should rely on a high statistic and possibly a full recostruction of the spectrum from the undulator. The photon rate should be close to the deadtime of the detector.

Another possibility would be to have a very big Undulator period (>200cm), in this case the spectrum is much more Monochromatic, the photon energies will be in the range of 10KeV, and there are many detectors readly available. Most likely the undulator will be composed of air-coils properly placed in a long straight section, interleaved with quadrupoles.

In order to have a spectrum that is not affected by the emittance (supposing Ex~1nm) the period should be at least 1m (photon energies around 1Kev). It helps to have very large beta-functions (>100m) across the undulator.

Suppose we have:

@ 45.5 GeV

500 period 0.5m undulator with K<<<1 (air coils possibility winding around the beam pipe)

1KeV photons with a rate of 10MHz (supposing a detector so fast does exists)

1 photon gives a measure of the center of the Xray peak with a resolution of 10-3

10^8 photons measure the center od the peak with a resolution of 10-7 (10-3 *sqrt(10^8))

The measure will take about 10sec.

Systematic errors must be addressed: for instance the absolute response of the detectors and their stability,
Possibly some calibration system should be envisaged (e.g with known sources (spectral lines from atoms))
The overall length of the undulator should be known and stable within 10-7 (interferometer/laser tracker in the tunnel?)

….