

# Old Thoughts about Monochromatization at FCC-ee

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# Preamble

The main ideas regarding parameter optimization for FCC-ee monochromatization at 62.5 GeV are set out in CDR ([Eur. Phys. J. Special Topics \*\*228\*\*, 261 - 623 \(2019\)](#)), paragraph 2.10.1, pp. 385-386.

Only a scheme with horizontal dispersion in the IP was considered there. But since then, some basic parameters of the collider have changed, so it makes sense to return to this topic again.

# Synchrotron Tune

The modulation of the spin precession frequency by synchrotron oscillations can be characterized by the synchrotron modulation index:

$$\zeta = \frac{\Delta\nu}{\nu_z} = \frac{\nu_0 \sigma_\delta}{\nu_z} = \frac{142.5 \cdot 0.000535}{\nu_z} \Rightarrow \nu_z = \frac{0.0762}{\zeta}$$

For a good determination of the spin frequency, it is necessary to fulfil the condition  $\zeta < 1.5$ , which means  $\nu_z > 0.05$ .

The total beam current  $I_{tot} \approx 0.36$  A, and this can impose restrictions on the number of RF cavities, and, accordingly, on the total RF voltage. Here we assume that RF frequency is 400 MHz, and  $U_{RF} = 1$  GV is the maximum possible value.

Arc cell	90°/90° short	90°/90° long	
$U_{RF}$ [GV]	1	0.6	1
$\nu_z$	0.0474	0.072	0.0936
$\sigma_z$ [mm]	1.2	3.1	2.35
$\varepsilon_x$ [nm]	0.174	1.33	
$\varepsilon_y$ [pm]	1	2.7	

The most realistic option: long arc cell. Then the bunch length will be  $\sim 3$  mm, and  $\beta_y^*$  should be about the same.

# Monochromatization Factor

## Similarity between dispersion at the IP and crossing angle

X-coordinate consists of betatron and synchrotron parts. The latter is proportional to either Z or  $\delta$ , which are shifted in phase of the synchrotron oscillations by  $\pi/2$ .

$$\lambda_m = \frac{\sigma_{xs}}{\sigma_{x\beta}} = \frac{\sigma_\delta \eta_x^*}{\sqrt{\varepsilon_x \beta_x^*}} \quad - \text{ analog of Piwinski angle } \phi$$

Modification of formulas for  $\xi_{x,y}$  and luminosity:

crossing angle

$$\sigma_x \Rightarrow \sigma_x \sqrt{1 + \phi^2}$$

dispersion

$$\sigma_x \Rightarrow \sigma_{x\beta} \sqrt{1 + \lambda_m^2}$$

Suggested name for  $\lambda_m$  – monochromatization parameter  
(used in articles in the 80s)

Monochromatization factor:  $\Lambda = \sqrt{1 + \lambda_m^2}$

But this formula is valid only without crossing angle...

In general case:

$$\Lambda = \sqrt{1 + \frac{\lambda_m^2}{1 + \phi^2 (1 + \lambda_m^2)}} < \sqrt{1 + \frac{1}{\phi^2}} \quad \phi = \frac{\sigma_z}{\sqrt{\sigma_{x\beta}^2 + \sigma_{xs}^2}} \tan\left(\frac{\theta}{2}\right)$$

How to decrease  $\phi$  if the crossing angle is fixed? 1) decrease in  $\sigma_z$  and increase in  $\sigma_x$  or 2) switch to crab crossing, which makes  $\phi = 0$ .

## Strategy for optimization:

- 1) Define the desired value for  $\Lambda$
- 2) Try to minimize  $\beta_x^*$ , then with the given  $\varepsilon_x$  we obtain  $\sigma_{x\beta}$
- 3) Find the required  $\sigma_{xs}$  (i.e. find  $\eta_x^*$  since  $\sigma_\delta$  is fixed)

Without crab crossing, larger dispersion is required for the same  $\Lambda$ .

# Luminosity and Beamstrahlung

$$L = \frac{\gamma}{2er_e} \cdot \frac{I_{tot} \xi_y}{\beta_y^*} \cdot R_{hg}$$

$I_{tot}$  is fixed by SR power of 50 MW per beam,  $\beta_y^* \approx \sigma_z$ , then  $R_{hg} \approx 0.86$

Luminosity is defined by  $\xi_y$

$$\xi_y = \frac{N_p r_e}{2\pi\gamma} \cdot \frac{\beta_y^*}{\sigma_y \sigma_x} = \frac{N_p r_e}{2\pi\gamma \sigma_x} \cdot \sqrt{\frac{\beta_y^*}{\kappa \epsilon_x}}$$

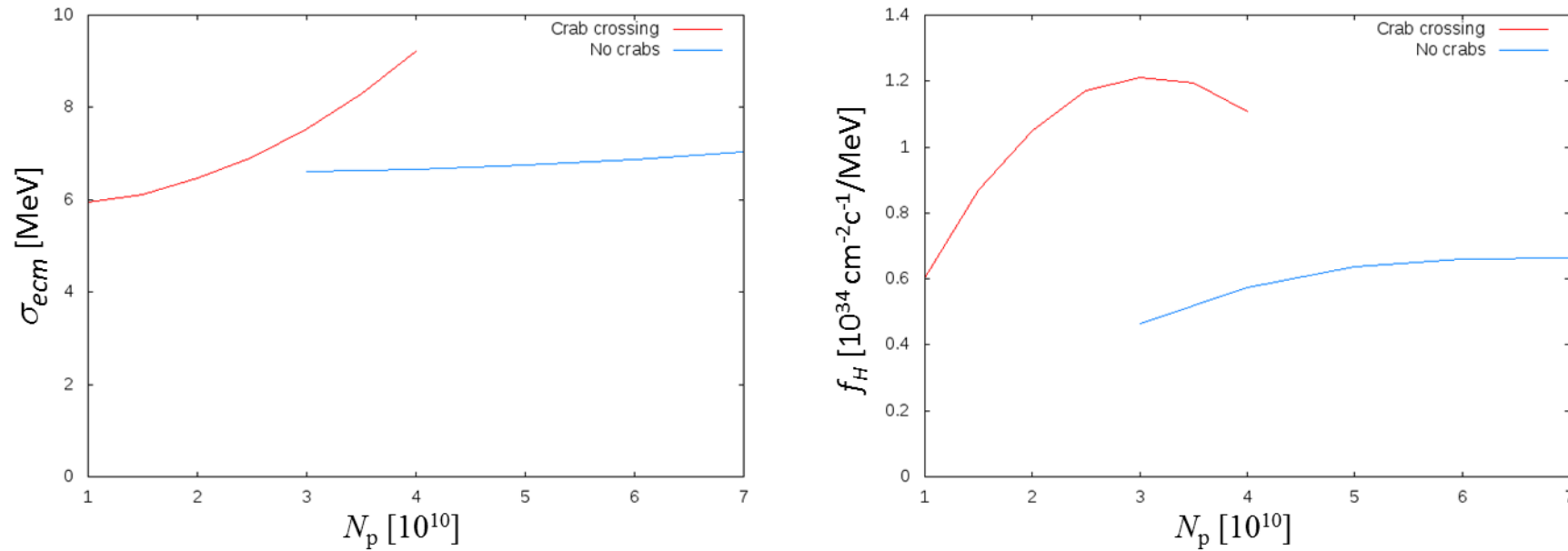
Beamstrahlung now leads to increase in  $\epsilon_x$ , not in  $\sigma_\delta$ . If  $\sigma_{xs} \gg \sigma_{x\beta}$  ( $\lambda_m \gg 1$ ), then  $\sigma_x$  is almost not affected. But  $\lambda_m$  and  $\xi_y$  will be affected (decreased).

Increase in  $U_{RF}$  makes the bunches shorter so that  $\phi$  decreases and  $\beta_y^*$  can be reduced. On the other hand, beamstrahlung is amplified for short bunches and its negative effects may outweigh the benefits, so this may not be the best choice.

Possible target function to maximize:  $f_H = \frac{L}{\sqrt{\Gamma_H^2 + \sigma_{ecm}^2}}$  (proportional to the Higgs event rate)  $\sigma_{ecm} = \frac{\sqrt{2}E_0\sigma_\delta}{\Lambda}$  – center-of-mass energy spread

The last step of optimization: chose (scan) the bunch population  $N_p$ . Then the number of bunches  $n_b \propto 1/N_p$ . At the very end, we need to make sure that  $n_b$  is acceptable. Probably, there will be no problems with electron clouds at this energy, so the bunch spacing will not be an issue.

# Example from CDR



The colors correspond to head-on collision with  $\eta_x^* = 15$  cm (red), and collision without crabbing and  $\eta_x^* = 50$  cm (blue).

These plots correspond to CDR with  $60^\circ/60^\circ$  arc cell,  $\beta_x^* = 20$  cm,  $\beta_y^* = 2$  mm, and  $\sigma_z = 2.4$  mm.

In the crab waist collision without monochromatization one can obtain  $f_H = 0.72$  with  $N_p = 3 \cdot 10^{10}$  and  $\sigma_{ecm} = 54$  MeV.

# Summary

## (Optimization Strategy)

- 1) Chose the arc cell lattice and RF parameters. This will determine the emittances and the bunch length. The latter determines  $\beta_y^*$ . What to watch out for: the synchrotron tune. The RF team and the depolarization team should be involved in the discussion.
- 2) Decide whether we will do crab crossing or not. This greatly affects the efficiency of monochromatization.
- 3) Try to achieve small  $\beta_x^*$  and large  $\eta_x^*$ . This is the key point, and it can be done independently of the previous two.
- 4) Perform beam-beam simulations in a simplified model: linear lattice without errors. What to watch out for: emittance growth due to beamstrahlung (both horizontal and vertical!). Make a scan of the bunch population to find the optimum.
- 5) Perform beam-beam simulations in a realistic model: nonlinear lattice with errors, misalignments and corrections, residual vertical dispersion at the IP, non-zero orbit at the IP, etc. Again, make a scan of the bunch population to find the optimum.