Beamstrahlung impacts on energy spread and emittance -analytical calculation

H. Jiang A. Faus-Golfe, F. Zimmermann, K. Oide, Z. Zhang

29 September, 2022

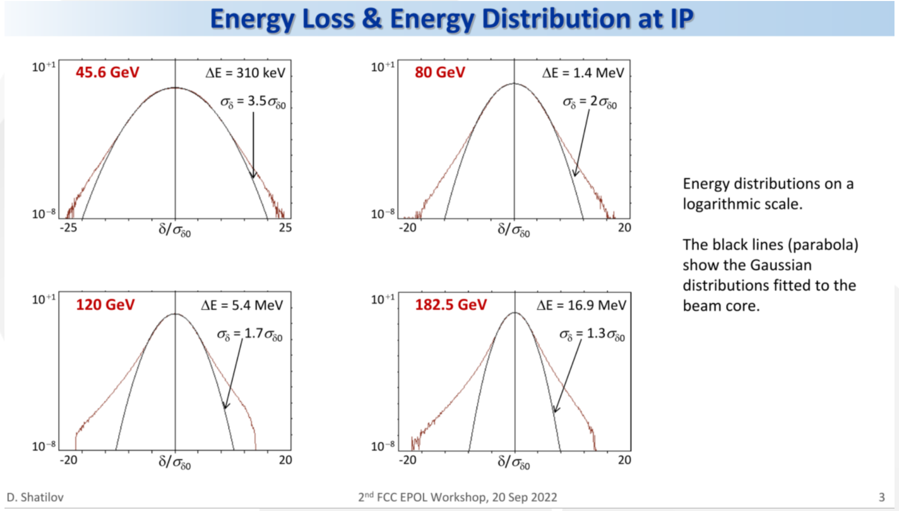
IJCLAB IN2P3-CNRS

Outline

- Introductions
- The impact on the energy spread
- The impact on the emittance
- Two cases for analytical calculation
  - The standard case
  - The monochromatization case
- Questions and conclusion

Introduction of Beamstrahlung

- Beamstrahlung = Beam + Bremsstrahlung
  - namely synchrotron radiation emitted during the beam-beam collision , can be an important effect for circular high-energy lepton colliders such as FCC-ee and CepC.
  - In those circular e+e− colliders, beamstrahlung can limit the beam lifetime and increase the energy spread and bunch length.
- The energy loss due to beamstrahlung[1]



[1]D. Shatilov Energy loss due to beamstrahlung and impact on local energy and on energy differences of colliding and non-colliding bunches, This workshop 2022

- The total energy spread[1]: $\sigma_{\delta,tot}$
  - The natural rms energy spread from Synchrotron Radiation(SR) : $\sigma_{\delta,SR}$
  - The additional energy spread due to beamstrahlung(BS): $\sigma_{\delta,BS}$

$$\boldsymbol{\sigma_{\delta,SR}^2 + \sigma_{\delta,BS}^2 = \sigma_{\delta,tot}^2} \textbf{(added in quadrature)}$$

  - The energy spread due to Beamstrahlung:

Conditions :

$\sigma_y \ll \sigma_x$

$\tau_{E,tot} \approx \tau_{E,SR}$

$U_{0,BS} \ll U_{0,SR}$

Z mode (0.62MeV<<40MeV)

$$\sigma_{\delta,BS}^2 = \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} n_\gamma \langle u^2 \rangle \approx 1.4 \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 \boldsymbol{\sigma_x^3}}$$

$$\text{Set: } A \equiv 1.4 \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_x^3}$$

$$\sigma_{\delta,BS}^2 = \frac{A}{\sigma_z^2}$$

n_{IP} :The number of IP
$\tau_{E,SR}$: Longitudinal damping time
α: Fine structure constant
N_b: Bunch population
γ: Relativistic Lorentz factor
σ_z: Bunch length
σ_x: Horizontal beam size
T_{rev}: Revolution period
n_γ:The average number of photon per collision
$\langle u^2 \rangle$: The average squared photon energy

  - The total energy spread(analytical expression):

$$\sigma_{\delta,tot} = \sqrt{\frac{1}{2}\sigma_{\delta,SR}^2 + \left(\frac{1}{4}\sigma_{\delta,SR}^4 + A\frac{\sigma_{\delta,SR}^2}{\sigma_{z,SR}^2}\right)^{1/2}}$$

[1]Ohmi K, Zimmermann F. FCC-ee/CepC beam-beam simulations with beamstrahlung[R]. 2014.

The total emittance

- The total emittance: $\epsilon_{x,tot}$
  - The emittance due to Synchrotron Radiation(SR) : $\epsilon_{x,SR}$
  - The emittance due to beamstrahlung(BS): $\epsilon_{x,BS}$

$$\epsilon_{x,tot} = \epsilon_{x,SR} + \epsilon_{x,BS}$$

- The emittance due to Beamstrahlung[1]:

$$\epsilon_{x,BS} = \frac{n_{IP}\tau_x}{4T_{rev}} n_\gamma \langle u^2 \rangle \mathcal{H}_x^*$$

  - The dispersion invariant:

$$\mathcal{H}_x^* \equiv \frac{(\beta_x^* D_x'^* + \alpha_x^* D_x^*)^2 + D_x^{*2}}{\beta_x^*}$$

[1]Garcia M A V, Zimmermann F. Effect of Beamstrahlung on Bunch Length and Emittance in Future Circular e+e− Colliders[J]. Proc. IPAC, 2016, 16: 2438.

- The total energy spread and emittance

$$\epsilon_{x,tot} = \epsilon_{x,SR} + \frac{n_{IP}\tau_x}{4T_{rev}} n_\gamma \langle u^2 \rangle \mathcal{H}_x^*$$

$$\sigma_{\delta,tot}^2 = \sigma_{\delta,SR}^2 + \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} n_\gamma \langle u^2 \rangle$$

- The relationship between emittance and energy spread due to the beamstrahlung:

$$\epsilon_{x,BS} = \frac{\tau_x}{\tau_{E,SR}} \boldsymbol{\sigma_{\delta,BS}^2} \mathcal{H}_x^*$$

- The Standard case: $(D_x^* = 0)$
  - meets condition: $D_x^* \sigma_{\delta,tot} \ll \sqrt{\beta_x^* \epsilon_x}$ (Beam size $\sigma_x = \sqrt{\left(D_x^* \sigma_{\delta,tot}\right)^2 + \beta_x^* \epsilon_{x,tot}}$)
    - $\mathcal{H}_x^* = 0$
  - The emittance growth due to beamstrahlung:
    - $\epsilon_{x,BS} = 0$, $\epsilon_{x,tot} = \epsilon_{x,SR}$
  - The energy spread due to the beamstrahlung:
    - $\sigma_{\delta,BS}^2 = \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} n_\gamma \langle u^2 \rangle \approx 1.4 \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 \boldsymbol{\sigma_x^3}}$
    - $\sigma_x = \sqrt{\left(D_x^* \sigma_{\delta,tot}\right)^2 + \beta_x^* \epsilon_{x,tot}} = \sqrt{\beta_x^* \epsilon_{x,SR}}$
- Simulation softwares: (simulate the energy spread and beam length with/without beamstrahlung)
  - FCC-ee: LIFETRAC (D. Shatilov)
  - CEPC: IBB(Y. Zhang)

FCC-ee CDR Simulation parameters with 2IPs

Mode	Z	S-channel	WW	ZH	$t\bar{t}$(182.5GeV)
Energy spread (%)(SR/BS)σ_δ	0.038 0.132	0.052 0.131	0.066 0.131	0.099 0.165	0.150 0.192
Bunch Length (mm) (SR/BS)σ_z	3.5 12.1		3.0 6.0	3.15 5.3	1.97 2.54
Bunch population (N_b: 10^{11})	1.7	0.6	1.5	1.8	2.3

The monochromatization case

- For the monochromatization case($D_x^*\sigma_{\delta,tot} \gg \sqrt{\beta_x^*\epsilon_x}$) ($\sigma_x = \sqrt{\left(D_x^*\sigma_{\delta,tot}\right)^2 + \beta_x^*\epsilon_{x,tot}}$)
  - The horizontal damping time: $\tau_x = 2\tau_{E,SR}$
  - $D_x'^*, \alpha_x^* = 0$
    - $\mathcal{H}_x^* \equiv \frac{\left(\beta_x^* D_x'^* + \alpha_x^* D_x^*\right)^2 + D_x^{*2}}{\beta_x^*} = \frac{D_x^{*2}}{\beta_x^*}$
  - The energy spread due to the beamstrahlung:
    - $\sigma_{\delta,BS}^2 = \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} n_\gamma \langle u^2 \rangle \approx 1.4 \frac{n_{IP}\tau_{E,SR}}{4T_{rev}} \frac{r_e^5 N_b^3 \gamma^2}{\alpha \sigma_z^2 \boldsymbol{\sigma_x^3}}$
    - $\sigma_x = \sqrt{\left(D_x^*\sigma_{\delta,tot}\right)^2 + \beta_x^*\epsilon_{x,tot}}$
    - **Reduce the $\boldsymbol{\sigma_{\delta,BS}^2}$**
  - The emittance growth due to the beamstrahlung:
    - $\epsilon_{x,BS} = \frac{\tau_x}{\tau_{E,SR}} \sigma_{\delta,BS}^2 \mathcal{H}_x^* = 2\, \sigma_{\delta,BS}^2 \frac{D_x^{*2}}{\beta_x^*}$
    - $\sqrt{\epsilon_{x,BS}\beta_x^*} = \sqrt{2}\sigma_{\delta,BS} D_x^*$ (beam size due to the dispersion on beamstrahlung energy spread)
    - Does this mean the emittance growth is due to the coupling between longitudinal and transverse motions?

The two equations are coupled

- The effective monochromatization factor
  - $\lambda_{eff} = \frac{\sigma_{\omega,D*=0}}{\sigma_{\omega,mc}} = \frac{\sigma_{\delta,tot}}{\sigma_{\delta,SR}} \left(\frac{D_x^{*2} \sigma_{\delta,SR}^2}{\varepsilon_{x,tot} \beta_x^*} + 1 \right)^{\frac{1}{2}}$

$$\varepsilon_{x,tot} = \epsilon_{x,SR} + \epsilon_{x,BS} = \epsilon_{x,SR} + 2\, \sigma_{\delta,BS}^2 \frac{D_x^{*2}}{\beta_x^*}$$

$$\lambda_{eff} = \frac{\sigma_{\omega,D*=0}}{\sigma_{\omega,mc}} = \frac{\sigma_{\delta,tot}}{\sigma_{\delta,SR}} \left(\frac{D_x^{*2} \sigma_{\delta,SR}^2}{\varepsilon_{x,SR} \beta_x^* + 2\sigma_{\delta,BS}^2 D_x^{*2}} + 1 \right)^{\frac{1}{2}}$$

Continuing work and questions

- The continuing work
  - New parameters for the standard case with 4 IPs
  - Numerical calculation of the monochromatization case with new parameters
  - Optimization of bunch population and bunch length
  - The guinea-pig simulation with new emittance and dispersion values with beamstrahlung
  - Re-plot the relation line between energy spread and luminosity in Significance contours, then find the best choice of dispersion.
- Why and how does the transverse emittance grow due to beamstrahlung?(0.52nm ----2.5nm)
  - If there is no transverse emittance growth or a smaller betaX ,a smaller dispersion at IP is needed....

Thanks for your attentions!

And welcome comments!