Explore 2022 Summer School

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Gamma-Ray Bursts and Relativistic Jets

Tutorial

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In this tutorial, we explore the internal shock model for prompt emission in gamma-ray bursts (GRBs) and the external forward shock model for the afterglow phase.

- All energies are isotropic-equivalent energies as defined in the lectures.
- The ultra-relativistic limit is taken everywhere, i.e., $\beta \sim 1 - 1/2\Gamma^2$.
- We use primes ($'$) to express quantities in the comoving frame.
- For simplicity, the effects of cosmological redshift are not included.

1. Internal shocks model for prompt emission in gamma-ray bursts

We seek to explore the diversity of GRB phenomenology in the internal shocks model of GRBs, where the prompt emission is due to the collision of shells of different velocities within the relativistic jet.

a. Internal shocks radius. Consider two relativistic shells ejected by the central source at times t_e and $t_e + t_{\rm var}$ with Lorentz factors Γ_1 and $\Gamma_2 > \Gamma_1$. Express the radius $R_{\rm is}$ and time $t_{\rm is}$ of the collision between the two shells (dubbed the "internal shock") as a function of t_e, $t_{\rm var}$, $\overline{\Gamma} = (\Gamma_1 + \Gamma_2)/2$ and $K = \Gamma_2/\Gamma_1$.

Recall the expressions of the photospheric and spreading radii in the fireball model. Compare these with the internal shocks radius.

b. Internal shocks: optically thin regime. In a plane $\log \dot{E}$ vs $\log \overline{\Gamma}$, plot the location of the region $R_{\rm is} > R_{\rm ph}$ for different values of K (2, 4, 10) and of the variability timescale $t_{\rm var}$ ($10^{-1}, 1, 10^{1}$ s). Throughout this tutorial, we will plot various physical quantities in the $\log \dot{E}$ vs $\log \overline{\Gamma}$ plane varying K and $t_{\rm var}$ as in this question.

This is the region considered in the internal shock model, where the emission is due to optically-thin synchrotron radiation of shock-accelerated electrons. Close to the frontier, the region where internal shocks occur below the photosphere can correspond to a "dissipative photospheric" model.

c. Internal shocks: energetics. Assume that the two shells have equal mass: $M_1 = M_2 = M$ and are cold before the collision. After the collision, they merge to produce a unique shell of mass $2M$ with Lorentz factor Γ_*, which has some internal energy E'_i (expressed in its comoving frame) due to the dissipation of kinetic energy by the shock. Using a Lorentz transformation, express the merged shell's 4-momentum in the fixed frame. Deduce Γ_* and $\epsilon'_* = E'_i/2M$ by energy-momentum conservation in the collision. Note that ϵ'_* is the comoving internal energy per unit mass in the merged shell.

Check that, except for $K \gg 1$, the shocked material is only mildly relativistic, i.e., $\epsilon'_* \lesssim c^2$.

Compute the efficiency f the collision, i.e., the fraction of initial kinetic energy that is dissipated to internal energy. Compute the numerical values for $K = 2, 4, 10$.

d. Internal shocks: luminosity. In the shocked region, a fraction ξ of electrons are accelerated to relativistic velocities and an entangled small-scale magnetic field is formed. Through which dissipation process will the merged shell shine? We denote by $\epsilon_e \sim 1/3$ the fraction of internal energy carried by the non-thermal population of electrons. Assuming that the electrons radiate very efficiently, express the luminosity $L_{\rm rad}$ radiated in the internal shock. As in question **b.**, plot constant-luminosity lines in the $\log \dot{E}$ vs $\log \overline{\Gamma}$ plane.

e. Internal shocks: density in the shocked region. Show that the comoving mass density ρ'_* in the shocked region can be estimated by:

$$\rho'_* \sim \alpha \frac{\dot{E}}{4\pi R_{\rm is}^2 \overline{\Gamma} \Gamma_* c^3} \tag{1}$$

with $\alpha \sim 1$ a dimensionless factor.

f. Internal shocks: peak energy. Denote by γ'_m the typical Lorentz factor of electrons in the shocked region. How would you estimate γ'_m using ξ, ϵ_e and ϵ'_*? The magnetic field carries a fraction $\epsilon_B \sim 10^{-3}$ of the internal energy in the shocked region. How would you estimate the magnitude B' of the magnetic field in the comoving frame? Through the synchrotron process, an electron with Lorentz factor γ_e moving in a magnetic field of magnitude B emits radiation with peak frequency (see, e.g., Rybicki & Lightman 1986, Chap. 6):

$$\nu_{\rm syn} = \frac{3e}{4\pi m_e c} B \gamma_e^2 \tag{2}$$

Express the spectral peak energy E_p of a pulse of prompt emission for the internal shocks synchrotron model as a proportionality relation between $\dot{E}$, $\overline{\Gamma}$, $t_{\rm var}$, and ϵ_e, ϵ_B and ξ. Assuming $\epsilon_e = 1/3$, $\epsilon_B = 10^{-3}$ and $\xi = 0.01$, plot constant peak-energy lines in the $\log \dot{E}$ vs $\log \overline{\Gamma}$ plane as in question **b.**.

g. Internal shocks: comparison with observations. Can the internal shocks model account for the following GRB phenomenology? Study the relevant parameter regions in the $\log \dot{E}$ vs $\log \overline{\Gamma}$ plane.

- A typical long GRB ($L_{\rm iso,\gamma} \sim 10^{52}$ erg/s, $E_p \sim 200$ keV).
- A typical short GRB ($L_{\rm iso,\gamma} \sim 10^{52}$ erg/s, $E_p \sim 1$ MeV).
- The hardness-duration correlation: shorter bursts tend to have higher peak energies.
- The Yonetoku relation observed in long GRB samples (see, e.g., Yonetoku et al. 2004, Fig. 1): $E_p \propto L_{\rm iso,\gamma}^{1/2}$.
- Low-luminosity GRBs are a distinct class of long GRBs with low luminosity ($L_{\rm iso,\gamma} \sim 10^{49}$ erg/s) and peak energy ($E_p \sim 10$ keV).

2. Afterglow phase from synchrotron emission in the forward shock

We describe the standard GRB afterglow model, in which the relativistic jet decelerates forming a strong forward shock where a non-thermal population of electrons is accelerated and shines through the synchrotron process.

a. External forward shock: dynamics. Recall the dynamics law $\Gamma(R)$ for the deceleration of the jet as a function of the radius. Denote by $E_0 \sim 10^{53}$ erg the jet's initial kinetic energy and $n_{\rm ext} \sim 1\,{\rm cm}^{-3}$ the density of the external medium. In the remainder, we will restrain to the deceleration phase $R_{\rm dec} \ll R \ll R_{\rm N}$.

Write down the expression for the arrival time $t_{\rm obs}$ of a photon emitted from the forward shock at radius R and time t for an observer at distance D. Shift the origin of time by D/c for observer times so that $t_{\rm obs} = 0$ corresponds to receiving a photon emitted at $R = 0$ at $t = 0$.

Deduce the evolution of the Lorentz factor of the forward shock as a function of $t_{\rm obs}$.

b. External forward shock: conditions in the shocked region. For an ultra-relativistic strong shock, jump conditions at the shock lead to the following relations for the comoving specific internal energy and mass density in the shocked region (e.g., Blandford & McKee 1976):

$$\epsilon'_* = \Gamma_* c^2 \tag{3}$$

and

$$\rho'_* = 4\Gamma_* n_{\rm ext} m_p \tag{4}$$

We denote once again γ'_m the typical Lorentz factor of electrons in the shocked region, and B' the magnetic field. Deduce the evolution of ϵ'_*, ρ'_*, B', γ'_m and ν_m as a function of $t_{\rm obs}$, where ν_m is the lab frame synchrotron frequency of electrons at γ_m (Eq. 2). As the forward shock is ultra-relativistic, we consider that all electrons in the shocked region are accelerated, and thus $\xi \sim 1$.

c. External forward shock: synchrotron regime. By which other process than synchrotron do the accelerated electrons cool? Show that this cooling occurs on a timescale $t'_{\rm dyn} = R/\Gamma_* c$, expressed in the comoving frame. To which timescale must we compare $t'_{\rm dyn}$ to know whether an electron cools efficiently through synchrotron radiation? The total synchrotron power radiated by an electron of Lorentz factor γ'_e is:

$$P'_{\rm syn} = \frac{\sigma_T c}{6\pi} B'^2 \gamma_e'^2 \tag{5}$$

where σ_T is the Thomson cross-section.

Deduce a lower-limit γ'_c on the Lorentz factor γ'_e of an electron in the shocked region for it to cool efficiently through synchrotron. Show that in the shocked medium, most electrons do not radiate efficiently. This regime is called the "slow cooling" regime in GRB theory. One can show that in the case of internal shocks (question **I f.**), $\gamma'_c \gg \gamma'_m$ and thus electrons radiate efficiently. This is the "fast cooling" regime.

d. External forward shock: peak frequency. In the slow cooling regime, most of the energy is radiated at the synchrotron frequency ν_c of electrons with Lorentz factor γ_c, rather than electrons at γ_m, which do not radiate efficiently. Compute ν_c and its evolution with $t_{\rm obs}$. Show that the afterglow peaks in X-rays at early times, and then successively in the visible and radio bands. Compare with the multi-wavelength sample of afterglows in Panaitescu & Kumar (2001), Fig. 1.

e. External forward shock: bolometric luminosity. When the forward shock has reached radius R, the number of electrons N_e in the shocked region is $N_e = 4\pi R^3 n_{\rm ext}/3$. After shock-acceleration, these electrons adopt a non-thermal distribution with slope $-p \sim -2.5$, as supported by observations and acceleration theory (e.g., Sironi et al. 2013):

$$\frac{{\rm d}N}{{\rm d}\gamma_e} = (p-1)\frac{N_e}{\gamma_m}\left(\frac{\gamma_e}{\gamma_m}\right)^{-p} \tag{6}$$

Only electrons above γ_c radiate efficiently. They lose their energy on the synchrotron timescale which is much shorter than the dynamical time of the system. Then, it can be shown[1] that the electron distribution averaged over a dynamical timescale is given by:

$$\frac{{\rm d}\overline{N}}{{\rm d}\gamma} = (p-1)\frac{N_e}{\gamma_m}\begin{cases} \left(\frac{\gamma}{\gamma_m}\right)^{-p} & \gamma_m < \gamma < \gamma_c \\ \left(\frac{\gamma_c}{\gamma_m}\right)\left(\frac{\gamma}{\gamma_m}\right)^{-p-1} & \gamma_c < \gamma \end{cases} \tag{7}$$

This distribution shows clearly than only electrons above γ_c cool efficiently. Then, the synchrotron power emitted by the shocked region is simply given by:

$$L_{\rm syn} \sim \Gamma_*^2 \int_{\gamma_c}^{\infty} \frac{{\rm d}\overline{N}}{{\rm d}\gamma} P_{\rm syn}(\gamma){\rm d}\gamma \tag{8}$$

where the pre-factor Γ_*^2 comes from the transformation from the comoving frame to the observer frame[2].

Show that the radiated power is given by:

$$L \sim \Gamma_*^2 \frac{\epsilon_e M_{\rm ext}\epsilon_*}{t'_{\rm dyn}}\left(\frac{\gamma_m}{\gamma_c}\right)^{p-2} \tag{9}$$

where $M_{\rm ext} = N_e m_p$ is the swept-up mass in the shocked region. It appears clearly that only a small fraction $\left(\frac{\gamma_m}{\gamma_c}\right)^{p-2}$ of the internal energy $\epsilon_e M_{\rm ext}\epsilon_*$ injected in non-thermal electrons is radiated over a timescale $t'_{\rm dyn}$. As expected, this luminosity shows a power-law decay. Compute the decay index and compare with the sample of X-ray afterglows in Panaitescu & Kumar (2001), Fig. 1.

[1] The evolution of the Lorentz factor of a single electron with an initial Lorentz factor γ_0 follows:

$$\frac{{\rm d}\gamma}{{\rm d}t'/t'_{\rm dyn}} = -\frac{\gamma^2}{\gamma_c} - \gamma$$

where the first term corresponds to the energy lost by synchrotron radiation and the second term to the adiabatic cooling on a timescale $t'_{\rm dyn}$. The first term dominates for $\gamma > \gamma_c$. This differential equation can be solved to get $\gamma(\gamma_0; t')$ and deduce the electron distribution at time t'. Eq. 7 is then obtained after time-averaging this solution over $t'_{\rm dyn}$.

[2] Assume monochromatic emission for simplicity: in the comoving frame, N photons of energy E' are emitted during $\Delta t'$, leading to a power $L' = NE'/\Delta t'$. The observer will measure an energy $E = \Gamma_* E'$ for these photons (relativistic Doppler boosting) and receive the corresponding energy $NE = N\Gamma_* E'$ during a time interval $\Delta t_{\rm obs} = (1-\beta_*)\Delta t = \Gamma_*(1-\beta_*)\Delta t' \sim \Delta t'/\Gamma_*$. The observer will then measure a power $L = \Delta E/\Delta t_{\rm obs} \sim \Gamma_*^2 L'$.

References

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