Heavy-quark Langevin dynamics and single-electron spectra in AA collisions

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CERN, 4^{th} **March 2011**

Work done and in progress in Torino in collaboration with

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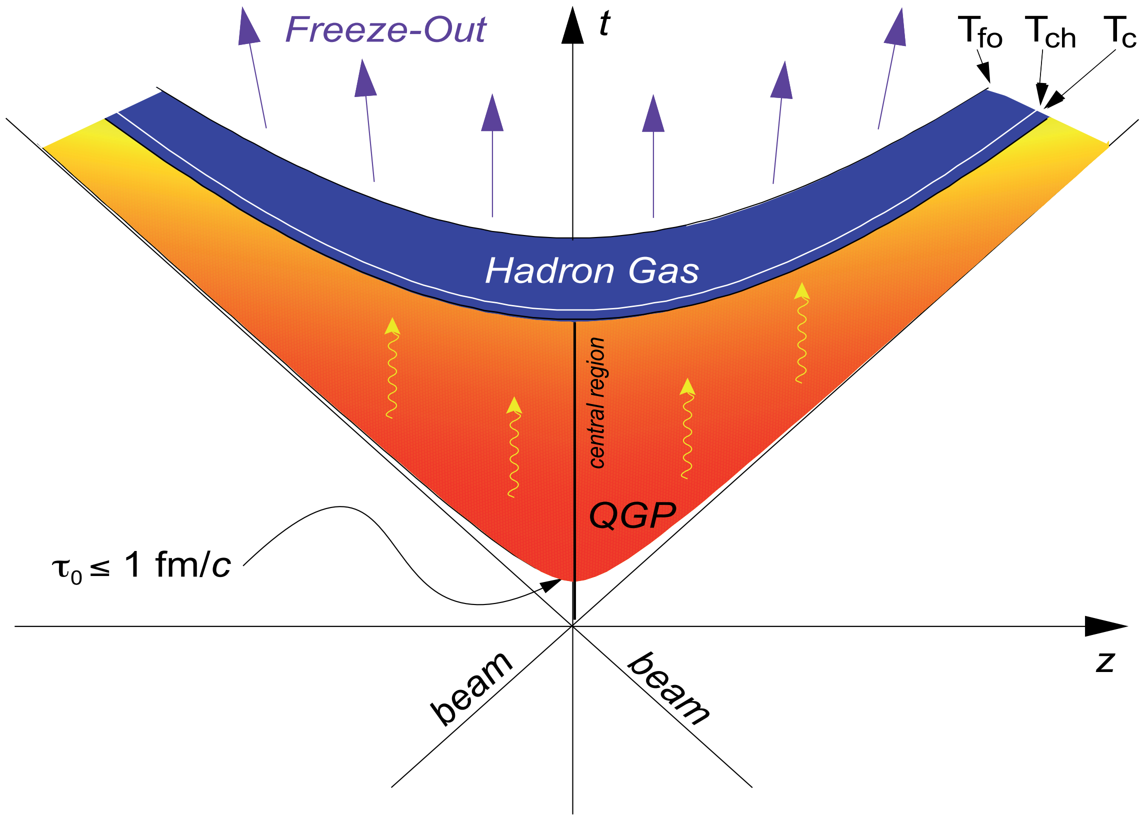
Nucl. Phys. A 831, 59 (2009) and arXiv:1101.6008 [hep-ph][a]

[a] Presented at "Hot Quarks 2010" (arXiv:1007.4170 [hep-ph]), "Jets in proton-proton and heavy-ion collisions" (arXiv:1009.2434 [hep-ph]), CERN Th. Institute "The first heavy ion collisions at the LHC" and "Hard Probes 2010" (arXiv:1011.0400 [hep-ph]).

Outline

- Heavy quarks as *hard* probes of the Quark Gluon Plasma
- Theoretical framework:
  - the relativistic Langevin equation in an expanding medium
  - evaluation of the transport coefficients
- Numerical results (for RHIC and LHC):
  from the initial $Q\overline{Q}$ production to the final e-spectra
  - (Invariant yields $E(dN/d^3p)$: pp vs AA)
  - Nuclear modification factor $R_{AA}(p_T)$
  - Elliptic flow coefficient $v_2(p_T)$

Space-time cartoon of AA collisions



Hard probes (high-p_T partons, heavy quarks and quarkonia): produced in hard pQCD processes in the *very early stages*, they cross the fireball allowing for a *tomography* of the matter.

Our tool to study the heavy-quark dynamics in the QGP: the relativistic Langevin equation

The relativistic Langevin equation...

$$\frac{\Delta p^i}{\Delta t} = -\underbrace{\eta_D(p)p^i}_{\text{determ.}} + \underbrace{\xi^i(t)}_{\text{stochastic}} \, ,$$

with the properties of the noise encoded in

$$\langle \xi^i(\boldsymbol{p}_t)\xi^j(\boldsymbol{p}_{t'})\rangle = b^{ij}(\boldsymbol{p}_t)\frac{\delta_{tt'}}{\Delta t} \qquad b^{ij}(\boldsymbol{p}) \equiv \kappa_\parallel(p)\hat{p}^i\hat{p}^j + \kappa_\perp(p)(\delta^{ij} - \hat{p}^i\hat{p}^j)$$

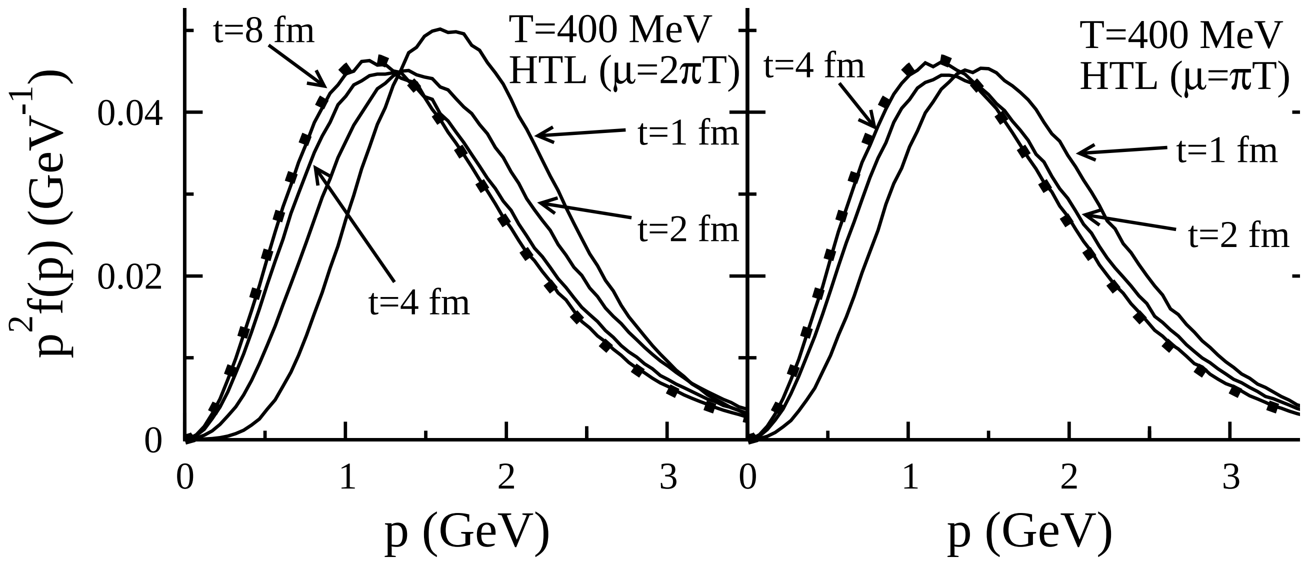
Transport coefficients to calculate:

- *Momentum diffusion* $\kappa_\perp \equiv \frac{1}{2}\frac{\langle \Delta p_\perp^2\rangle}{\Delta t}$ and $\kappa_\parallel \equiv \frac{\langle \Delta p_\parallel^2\rangle}{\Delta t}$;
- *Friction* term (dependent on the discretization scheme!)

$$\eta_D{}^{\rm Ito}(p) = \frac{\kappa_\parallel(p)}{2TE_p} - \frac{1}{E_p^2}\left[(1-v^2)\frac{\partial\kappa_\parallel(p)}{\partial v^2} + \frac{d-1}{2}\frac{\kappa_\parallel(p)-\kappa_\perp(p)}{v^2}\right]$$

fixed in order to insure the approach to equilibrium (Einstein relation):
Langevin eq. $\Leftrightarrow$ Fokker Planck eq. with steady solution $\exp(-E_p/T)$

...in a static medium



For $t \gg 1/\eta_D$ one approaches a relativistic Maxwell-Jüttner distribution[a]

$$f_{\rm MJ}(p) \equiv \frac{e^{-E_p/T}}{4\pi M^2 T\, K_2(M/T)}, \qquad \text{with} \int d^3p\, f_{\rm MJ}(p) = 1$$

(Test with a sample of c quarks with $p_0 = 2$ GeV/c)

[a] A.B., A. De Pace, W.M. Alberico and A. Molinari, NPA 831, 59 (2009)

...in an expanding fluid

The fields $u^\mu(x)$ and $T(x)$ are taken from the output of two longitudinally boost-invariant ("Hubble-law" longitudinal expansion $v_z = z/t$)

$$x^\mu = (\tau \cosh\eta, \boldsymbol{r}_\perp, \tau \sinh\eta) \quad \text{with} \quad \tau \equiv \sqrt{t^2 - z^2}$$

$$u^\mu = \bar{\gamma}_\perp (\cosh\eta, \bar{\boldsymbol{v}}_\perp, \sinh\eta) \quad \text{with} \quad \bar{\gamma} \equiv \frac{1}{\sqrt{1 - \bar{\boldsymbol{v}}_\perp^2}}$$

hydro codes[a].

- $u^\mu(x)$ used to perform the update each time in the fluid rest-frame;
- $T(x)$ allows to fix at each step the value of the transport coefficients.

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[a]P.F. Kolb, J. Sollfrank and U. Heinz, Phys. Rev. C **62** (2000) 054909
P. Romatschke and U.Romatschke, Phys. Rev. Lett. **99** (2007) 172301

Evaluation $\kappa_{\perp/\parallel}(p)$

Intermediate cutoff $|t|^* \sim m_D^2$ [a] *separating the contributions of*

- soft collisions ($|t| < |t|^*$): Hard Thermal Loop approximation
- hard collisions ($|t| > |t|^*$): kinetic pQCD calculation

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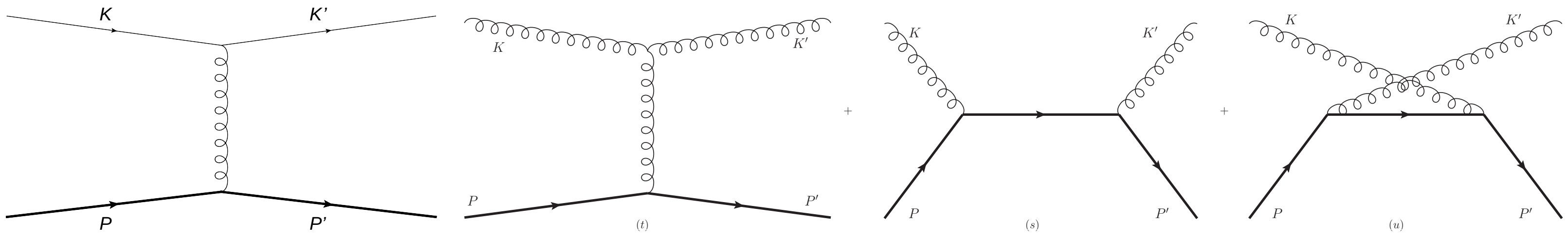
[a] Similar strategy for the evaluation of dE/dx in S. Peigne and A. Peshier, Phys.Rev.D77:114017 (2008).

$\kappa_{\perp/\parallel}(p)$: **hard contribution**



$$\kappa_{\perp}^{g/q(\text{hard})} = \frac{1}{2}\frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_{\perp}^2$$

$$\kappa_{\parallel}^{g/q(\text{hard})} = \frac{1}{2E}\int_k \frac{n_{B/F}(k)}{2k}\int_{k'}\frac{1 \pm n_{B/F}(k')}{2k'}\int_{p'}\frac{1}{2E'}\theta(|t|-|t|^*)\times$$
$$\times\,(2\pi)^4\delta^{(4)}(P+K-P'-K')\left|\overline{\mathcal{M}}_{g/q}(s,t)\right|^2 q_{\parallel}^2$$

$\kappa_{\perp/\|}(p)$: soft contribution

$$\kappa_\perp^{\rm soft} = \frac{1}{2}\,\frac{C_F g^2}{4\pi^2 v}\int_0^{|t|^*} d|t|\int_0^v dx\,\frac{|t|^{3/2}}{2(1-x^2)^{5/2}}\,\bar{\rho}(|t|,x)\left(1-\frac{x^2}{v^2}\right)\coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

$$\kappa_\|^{\rm soft} = \frac{C_F g^2}{4\pi^2 v}\int_0^{|t|^*} d|t|\int_0^v dx\,\frac{|t|^{3/2}}{2(1-x^2)^{5/2}}\,\bar{\rho}(|t|,x)\,\frac{x^2}{v^2}\coth\left(\frac{x\sqrt{\frac{|t|}{1-x^2}}}{2T}\right)$$

where

$$\bar{\rho}(|t|,x) \equiv \rho_L(|t|,x) + (v^2-x^2)\rho_T(|t|,x) \qquad (|t|\equiv q^2-\omega^2,\; x\equiv\omega/q)$$

The result is then expressed in terms of the *spectral functions of the resummed gluons exchanged* in the collisions *with the plasma particles*:

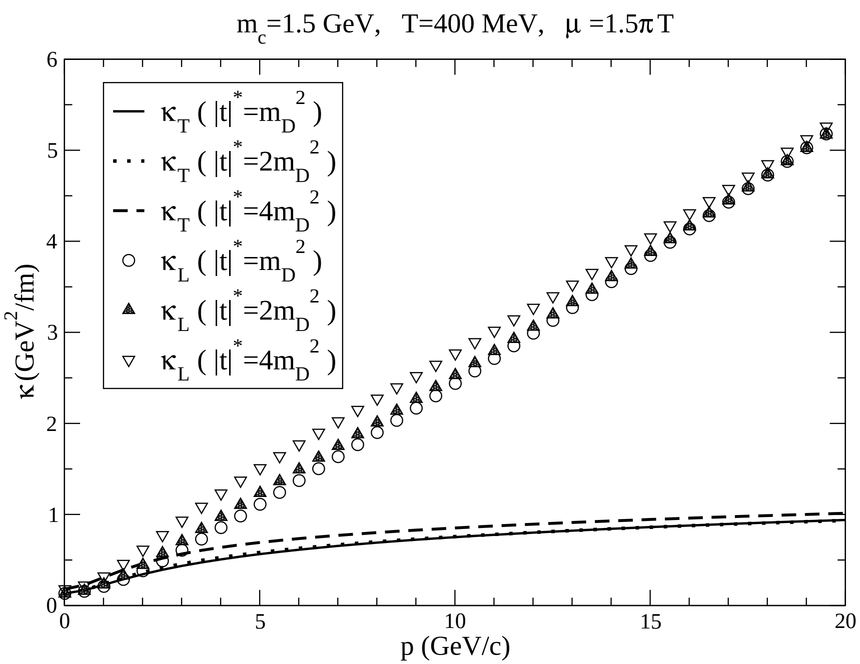
$$\rho_{L/T}(\omega,q) \equiv 2{\rm Im}\Delta_{L/T}(\omega+i\eta,q) \quad \text{where}$$

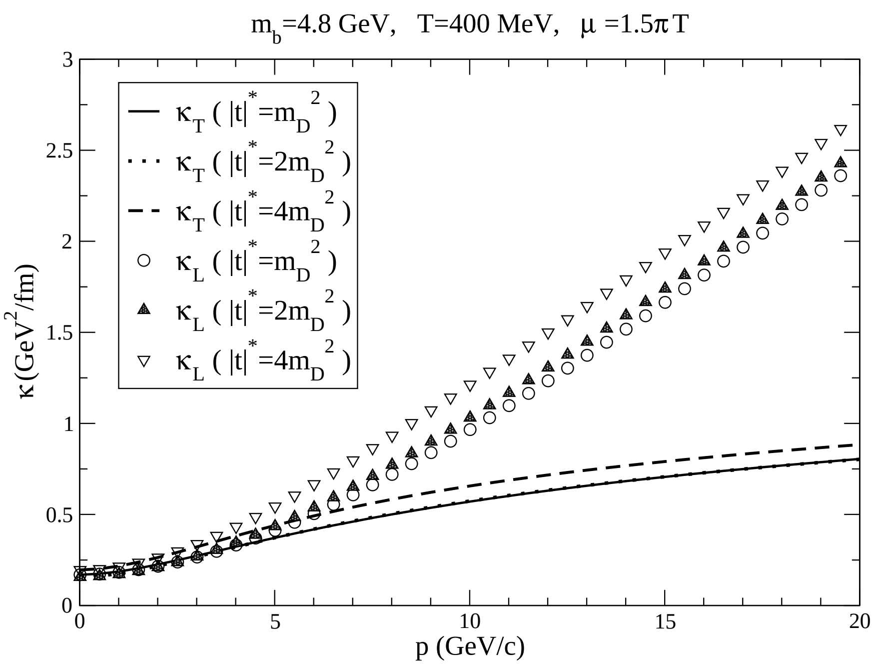
$$\Delta_L(z,q) = \frac{-1}{q^2+\Pi_L(z,q)}, \quad \Delta_T(z,q) = \frac{-1}{z^2-q^2-\Pi_T(z,q)}$$

NB: *medium effects* embedded in the HTL gluon self-energy!

$\kappa_{\perp/\parallel}(p)$: numerical results

Combining together the hard and soft contributions...





...the dependence on the intermediate cutoff $|t|^*$ is very mild!

We are ready to perform numerical simulations for a realistic case!

- Initial generation of $Q\overline{Q}$ pairs (POWHEG: pQCD@NLO);
- Langevin evolution in the QGP ($u^\mu(x)$ and $T(x)$ given by hydro);
- At T_c HQs hadronize (fragmentation with PDG branching ratios)
- and decay into electrons (PYTHIA decayer with PDG decay tables).

NB One has first of all to *check to be able to reproduce pp results*!

So far the experimental observables are

single-electron spectra from the decays of charm

$$D \to X \nu e$$

and bottom hadrons:

$$\begin{aligned} B &\to D\nu e \\ B &\to D\nu e \to X\nu e \nu e \\ B &\to DY \to X\nu e Y \end{aligned}$$

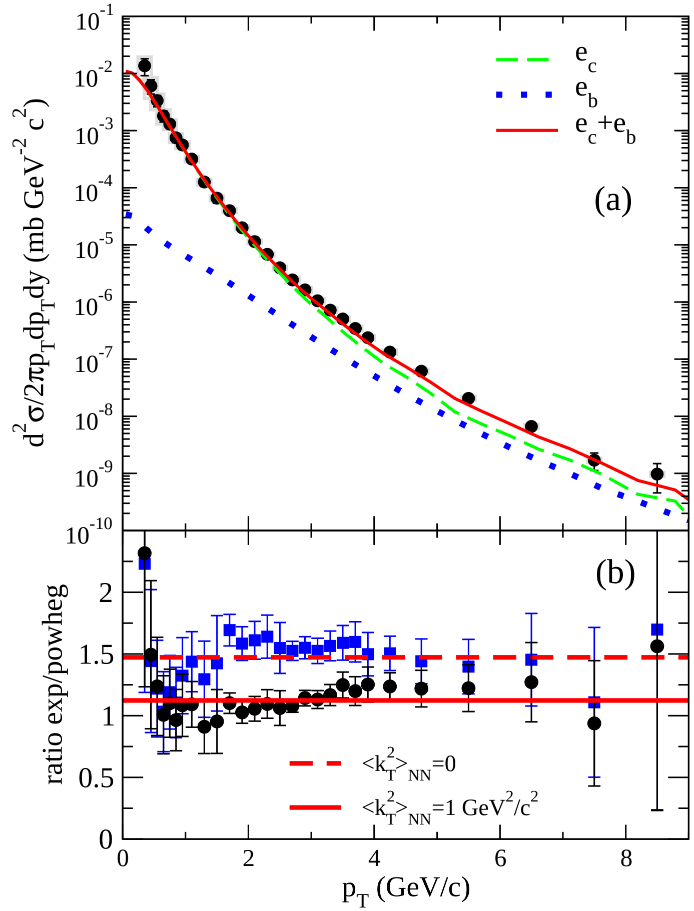
We plot the electrons falling into the PHENIX/ALICE acceptance ($|\eta| < 0.35/0.9$)

$$\eta \equiv \frac{1}{2} \ln \frac{p + p_z}{p - p_z} = -\ln \tan \frac{\theta}{2}$$

pp **collisions**

Comparison with PHENIX results @ $\sqrt{s} = 200$ GeV

Single-electron spectra: pp results



PHENIX results in pp collisions at $\sqrt{s}=200$ GeV are nicely reproduced!

(default POWHEG values $\mu_{R/F}=m_T$, $m_{c/b}$=1.5/4.8 GeV; CTEQ6M PDFs)

AA collisions:

Au-Au @ RHIC and Pb-Pb @ LHC

Initialization: hydro scenario ($0.1 < \tau_0 < 1$ fm)

	$\eta/s = 0$			$\eta/s = 0.08$		
	τ_0 (fm)	s_0 (fm^{-3})	T_0 (MeV)	τ_0 (fm)	s_0 (fm^{-3})	T_0 (MeV)
RHIC				0.1	8.4	666
	0.6	110	357	0.6	140	387
				1	84	333
LHC	0.1	2438	1000	0.1	1840	854
	0.45	271	482	1	184	420

initial $Q\overline{Q}$ production (from POWHEG)

$\sqrt{s}_{NN}$	$\sigma_{c\bar{c}}^{pp}$ (mb)	$\sigma_{c\bar{c}}^{AA}$ (mb)	$\sigma_{b\bar{b}}^{pp}$ (mb)	$\sigma_{b\bar{b}}^{AA}$ (mb)
200 GeV	0.254	0.236	1.77×10^{-3}	2.03×10^{-3}
5.5 TeV	3.015	2.288	0.187	0.169

NB huge *shadowing effects* (EPS09) for $c\bar{c}$ production in Pb-Pb @ LHC!

Final spectra: nuclear modification factor

$$R_{AA}(p_T) = \frac{1}{\langle N_{\rm coll}^{AA} \rangle} \frac{dN/dp_T|_{AA}}{dN/dp_T|_{pp}}$$

and elliptic flow coefficient

$$\frac{dN}{d\phi\, p_T dp_T} = \frac{dN}{2\pi\, p_T dp_T} \left(1 + 2 v_2(p_T) \cos[2(\phi - \psi_{\rm RP})] + ...\right)$$

$$v_2 \equiv \langle \cos[2(\phi - \psi_{RP})] \rangle$$

Some systematics

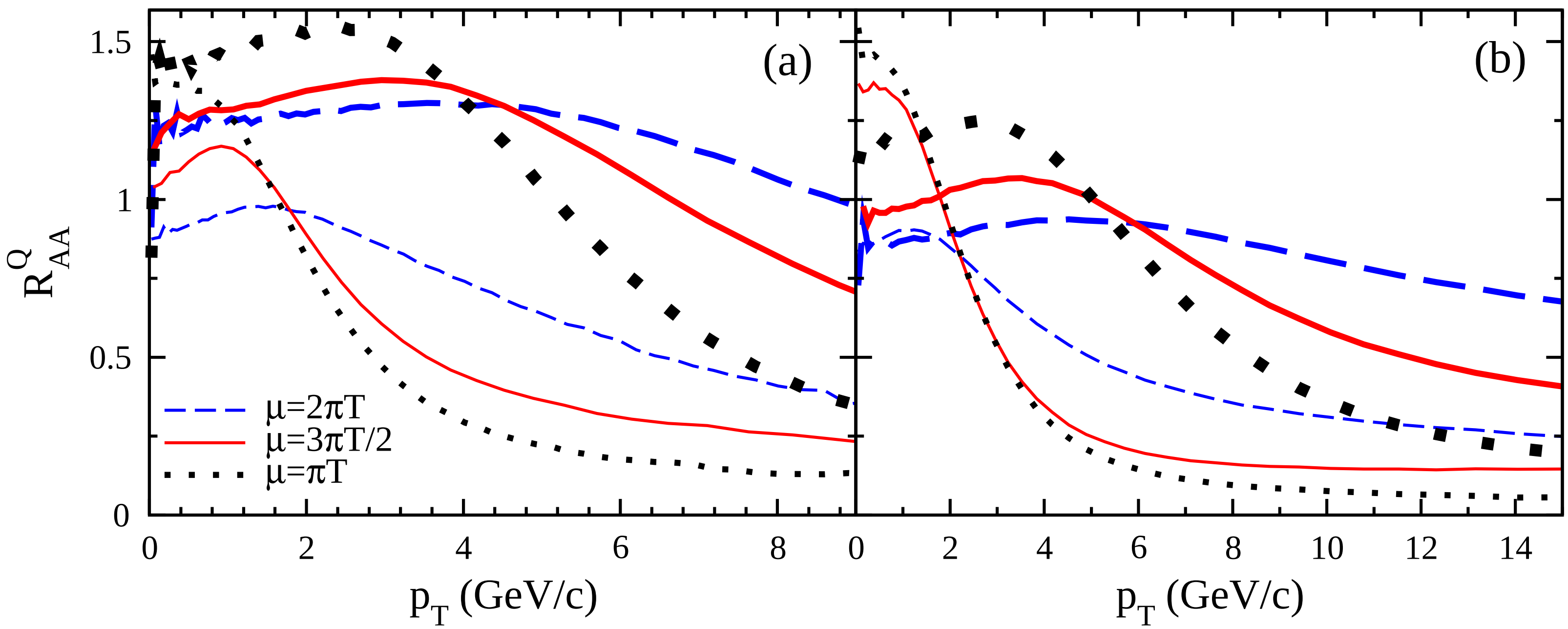
Dependence on

- Scale of the coupling;
- Hydro scenario;
- Hadronization and decays.

Heavy-quark R_{AA}: role of the coupling

RHIC (left panel) vs LHC (right panel)

(charm: thin lines, bottom: thick lines)



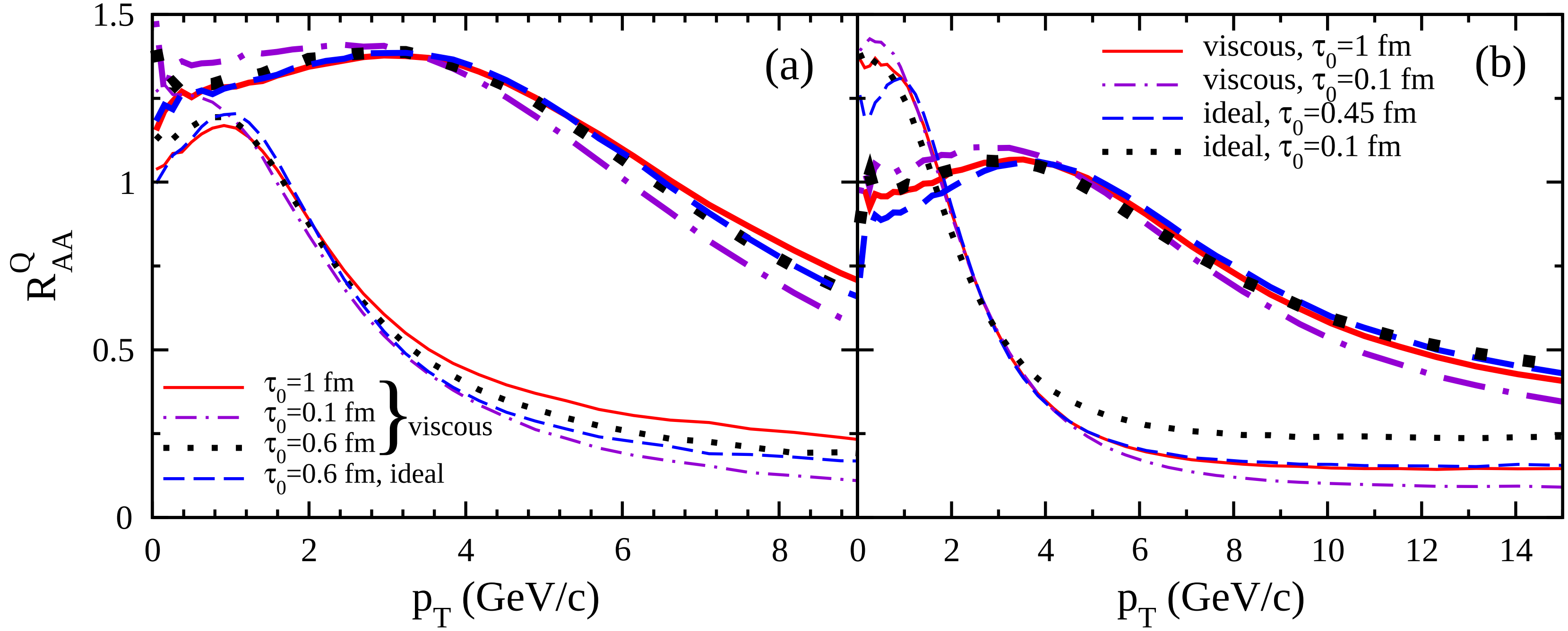
Strong dependence on the scale at which the coupling $\alpha_s(\mu)$ is evaluated ($\mu = 2\pi T$ and $\mu = \pi T$): at $T = 200$ MeV $\alpha_s \approx 0.34$ and 0.63

In the following we focus on $\mu = 1.5\pi T$.

Heavy-quark R_{AA}: role of hydrodynamics

RHIC (left panel) vs LHC (right panel)

(charm: thin lines, bottom: thick lines)

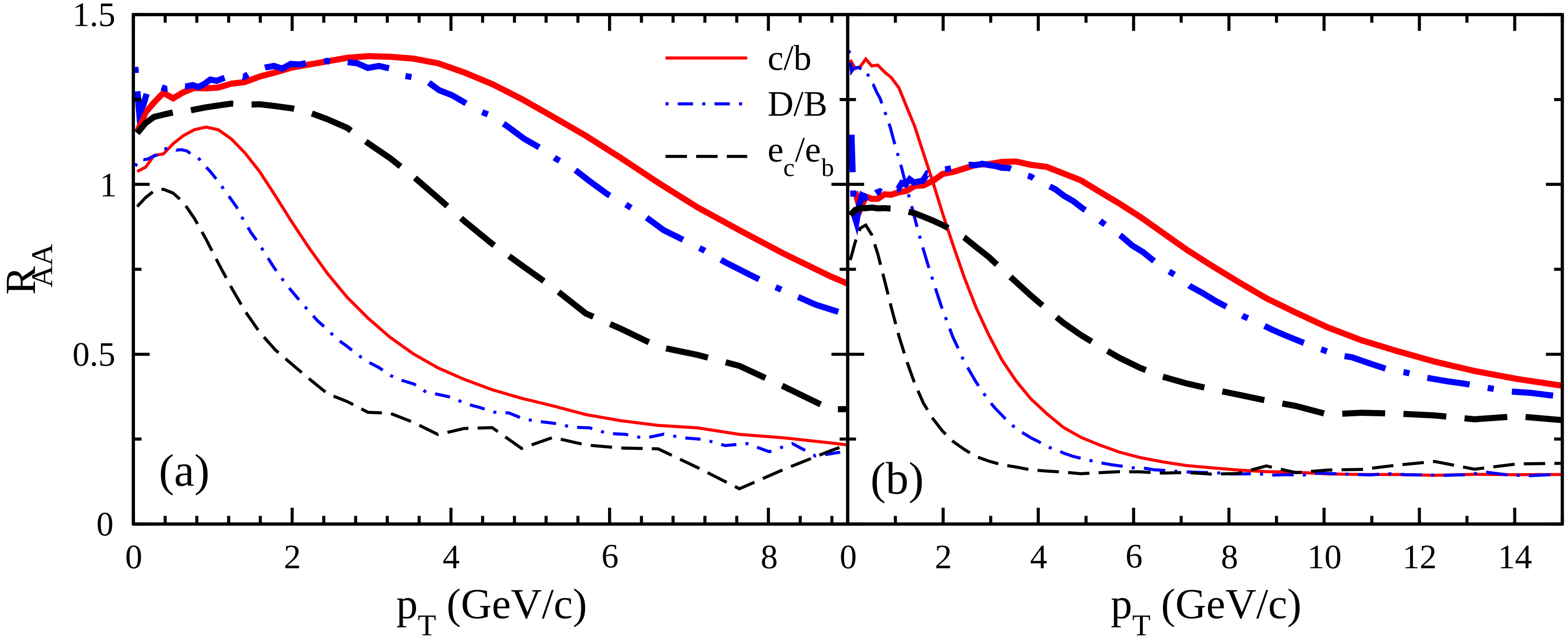


For minimum-bias collisions dependence on the hydrodynamical scenario is very mild.

Effects of fragmentation and decays: $h_{c/b}$ and $e_{c/b}$

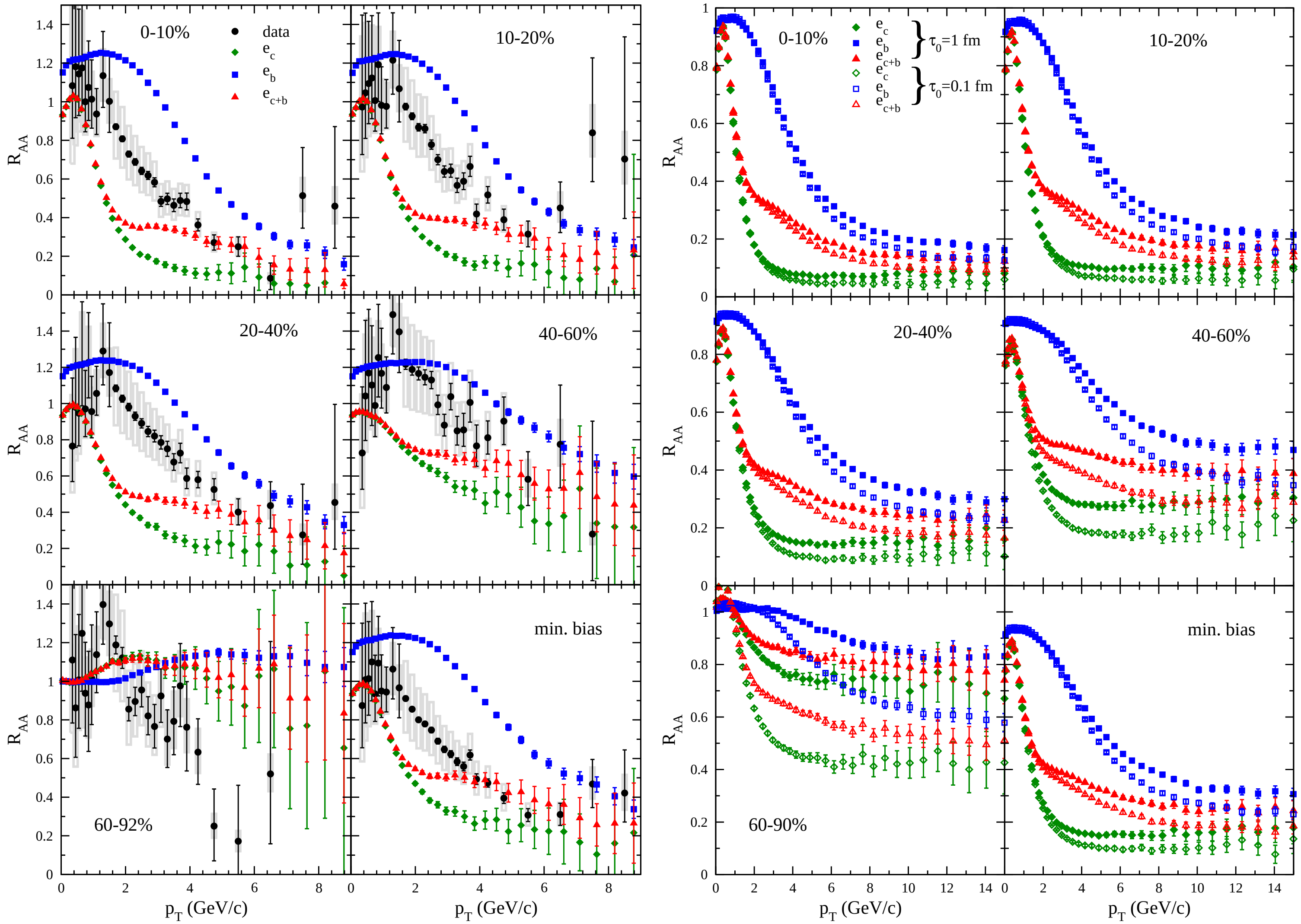
RHIC (left panel) vs LHC (right panel)

(charm: thin lines, bottom: thick lines)



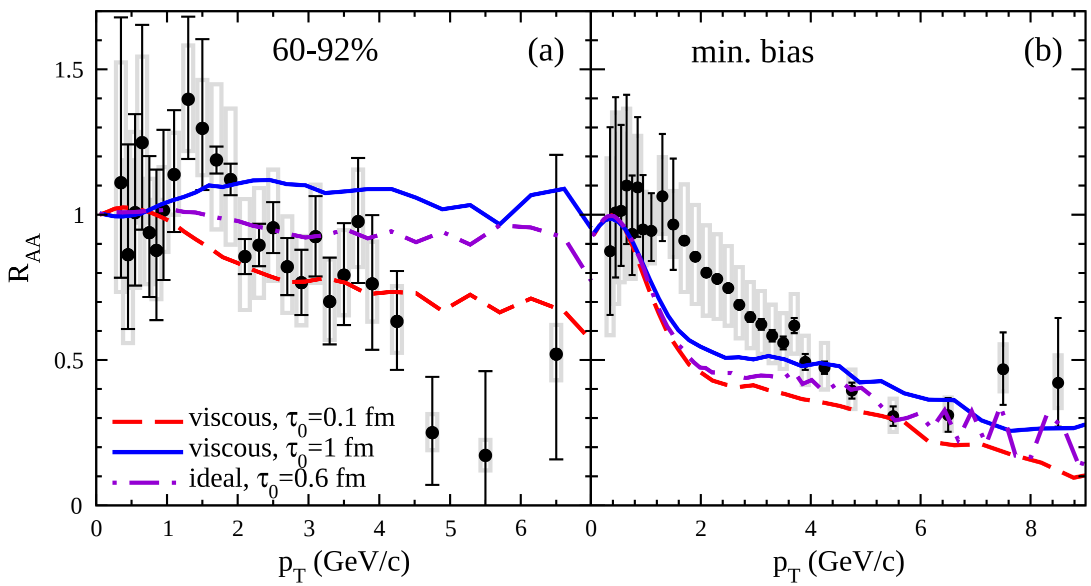
Fragmentation and semileptonic decays ($D \to X\nu e$) lead to a *quenching of* R_{AA}!

Single-electron spectra: RHIC (left) vs LHC (right)

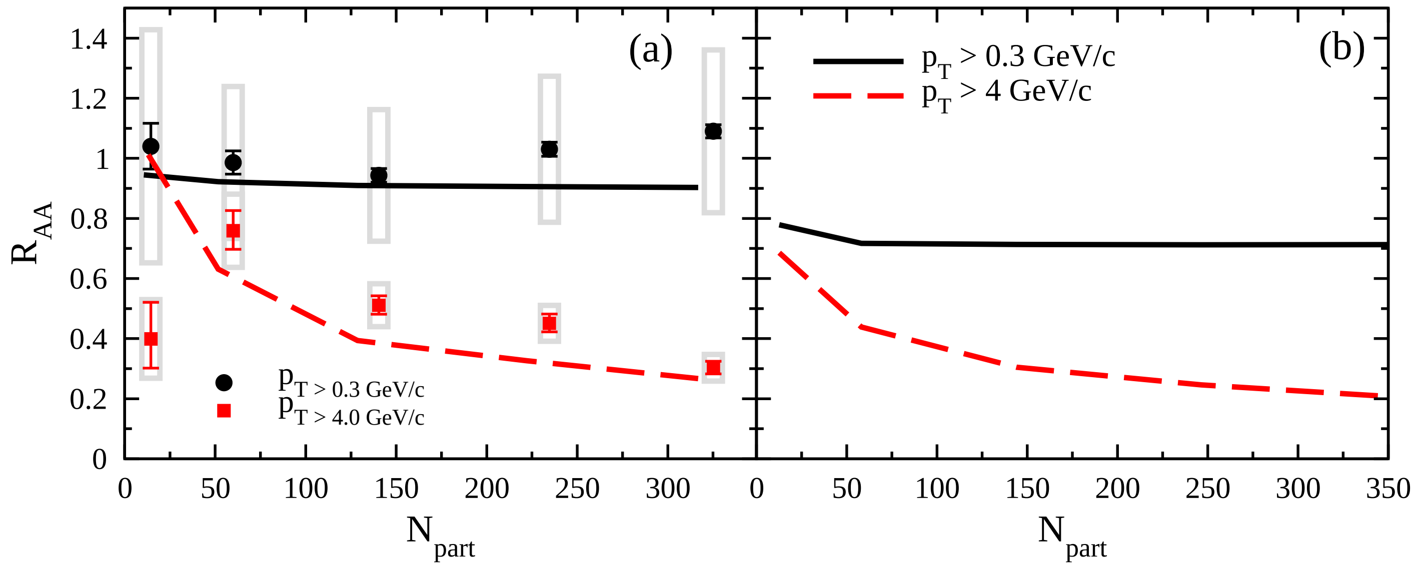


Some general comments:

- mild dependence on the hydro scenario (ideal/viscous) and the thermalization time τ_0 (in RHIC plots $\tau_0 = 1$ fm), *except for peripheral collisions*;
- high-p_T reproduced better with $\mu \sim 1.5 \Rightarrow \alpha = 0.32$ at $T = 300$ MeV
- intermediate-p_T spectra could get increased by coalescence;
- peripheral collisions represent a puzzle (accomodated by a smaller τ_0?).

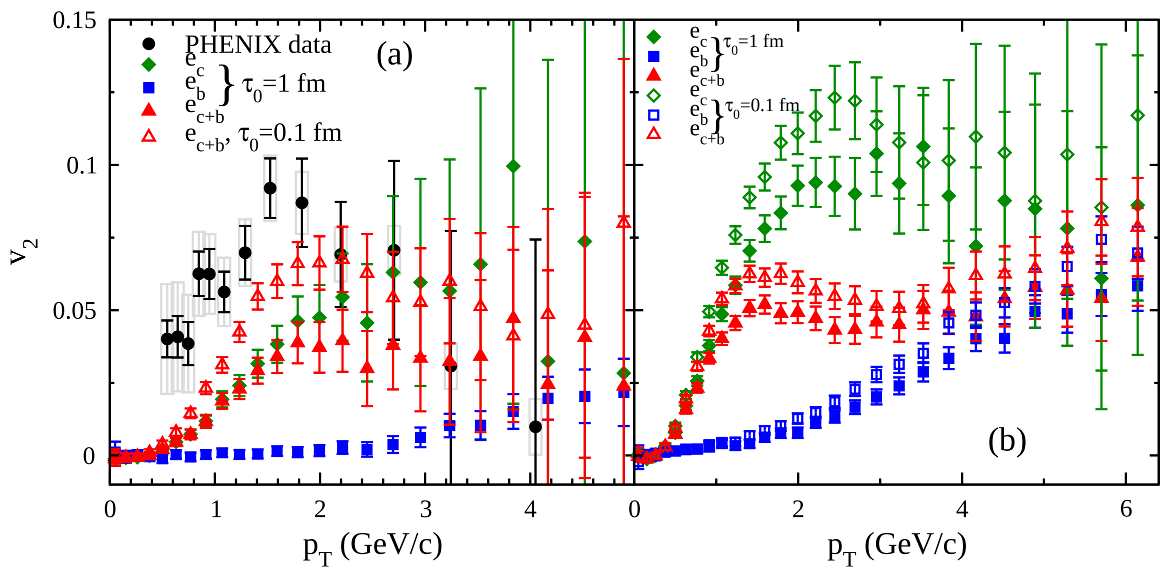


R_{AA} of heavy-flavor electrons vs centrality: RHIC (left panel) vs LHC (right panel)



- plots done using the *integrated yields*;
- parameter set: $\mu = 3\pi T/2$ and viscous hydro with $\tau_0 = 1$ fm;
- similar general trend (medium softens the spectrum conserving N_e^{tot})
  - $p_T > 0.3$ GeV/c: flat $R_{AA} \sim 1$ ($R_{AA} \neq 1$ at LHC due to nPDFs!)
  - $p_T > 4$ GeV/c: suppression increases with centrality.

Elliptic flow of heavy-flavor electrons: RHIC (left panel) vs LHC (right panel)



- v_2 with hot-QCD + fragmentation results a bit underestimated;
- Actually very good agreement with $\tau_0 = 0.1$ fm;
- v_2 could be increased by coalescence;
- Much larger flow of e_c @ LCH. Milder effect on $e_c + e_b$ due to role of b.

Conclusions and perspectives

- The relativistic Langevin equation is a powerful tool to study the HQ dynamics in the the QGP: it is an effective theory completely determined by the coefficients $\kappa_{T/L}(p)$ (*no matter their microscopic origin!*)
- $\kappa_{T/L}(p)$ have been evaluated considering only $2 \to 2$ collisions and distinguishing soft and hard scatterings
- For large p_T ($p_T \gtrsim 3$ GeV/c) it is possible to accommodate RHIC data for the single-electron spectra
- Coalescence could further improve the description at lower p_T, raising R_{AA} and v_2
- Preliminary simulations for LHC were attempted. In order to provide predictions at the current $\sqrt{s}_{NN} = 2.76$ GeV *we need a reliable hydrodynamical scenario*....

Back-up slides

The easiest algorithm

Going to the fluid rest-frame:

$$\Delta\bar{p}_n^i = -\eta_D(\bar{p}_n)\bar{p}_n^i\Delta\bar{t} + \xi^i(\bar{t}_n)\Delta\bar{t} \equiv -\eta_D(\bar{p}_n)\bar{p}_n^i\Delta\bar{t} + g^{ij}(\bar{\boldsymbol{p}}_n)\zeta^i(\bar{t}_n)\sqrt{\Delta\bar{t}},$$

$$\Delta\bar{\boldsymbol{x}}_n = \bar{\boldsymbol{p}}_n/\bar{E}_n\Delta\bar{t}$$

with $\Delta\bar{t}=0.02$ fm/c (*in the fluid rest-frame!*) and

$$g^{ij}(\boldsymbol{p}) \equiv \sqrt{\kappa_\parallel(p)}\hat{p}^i\hat{p}^j + \sqrt{\kappa_\perp(p)}(\delta^{ij} - \hat{p}^i\hat{p}^j) \quad \text{and} \quad \langle\zeta_n^i\zeta_{n'}^j\rangle = \delta^{ij}\delta_{nn'}$$

Hence one needs simply to:

- extract three independent random numbers ζ^i from a gaussian distribution with $\sigma=1$;
- update the momentum and position of the heavy quark;
- go back to the Lab-frame: $\boldsymbol{x}_{n+1}$ and $\boldsymbol{p}_{n+1}$.

The Hard Thermal Loop approximation

It is a one-loop gauge-invariant approximation allowing for the calculation of thermal corrections to vacuum propagators.

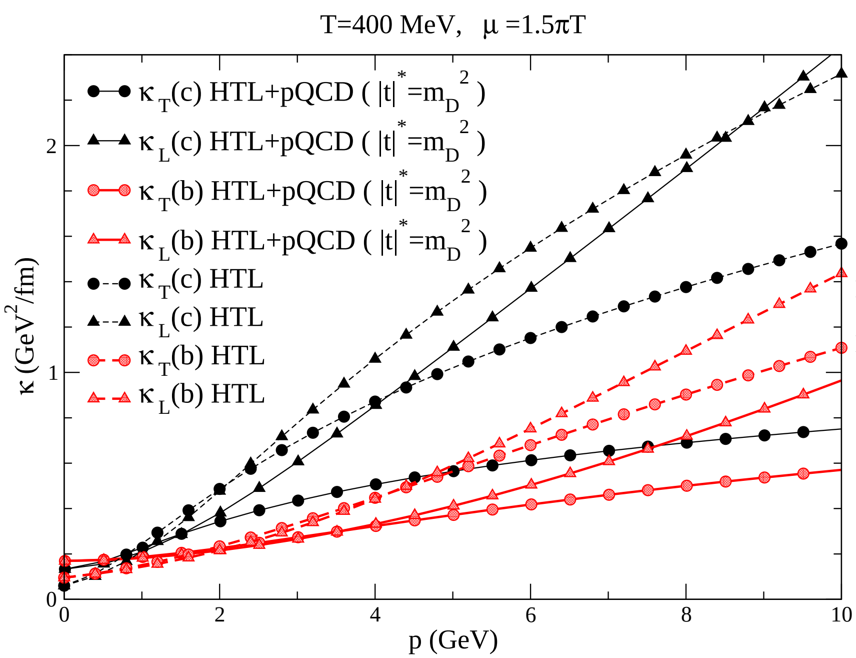
$$\Delta_L(q^0,q) = \frac{-1}{q^2 + \Pi_L(x)}, \quad \Delta_T(q^0,q) = \frac{-1}{(q^0)^2 - q^2 - \Pi_T(x)}$$

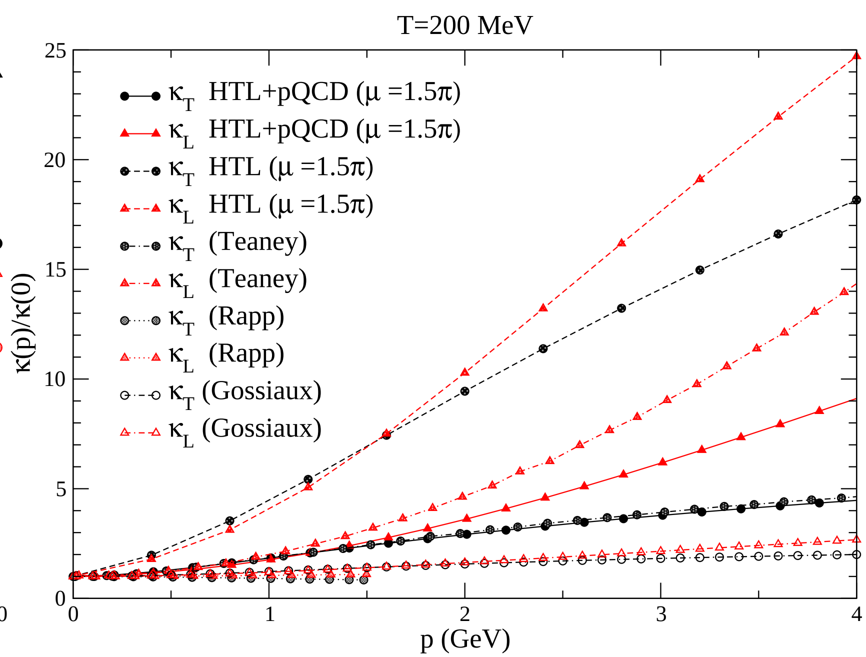
with $x \equiv q^0/q$ and

$$\Pi_L(x) = m_D^2 \left(1 - \frac{x}{2}\ln\frac{x+1}{x-1}\right),$$
$$\Pi_T(x) = \frac{m_D^2}{2}\left(x^2 + (1-x^2)\frac{x}{2}\ln\frac{x+1}{x-1}\right),$$

where $m_D \equiv gT\sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$ is the Debye mass, responsible for the screening of electro-static color fields.

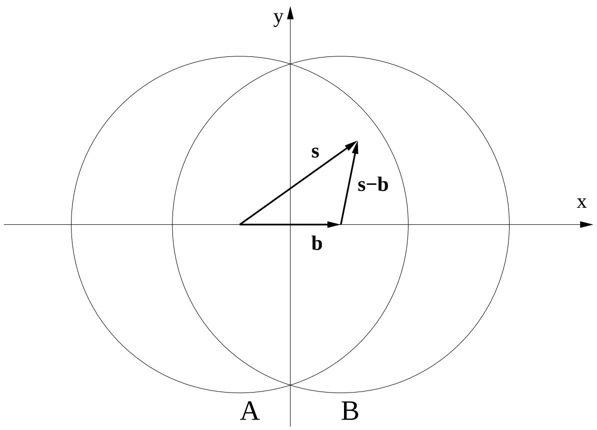
$\kappa_{\perp/\parallel}(p)$: comparison with previous results





- HTL: Hard Thermal Loop approximation applied to any collision;
- Teaney: $\sim$ shape given by pQCD with $\kappa(0)$ as a free parameter;
- Rapp: resonant scattering with formation of D-like mesons;
- Gossiaux: $\sim$ pQCD with m_D used as a free parameter.

Initial heavy-quark spectra

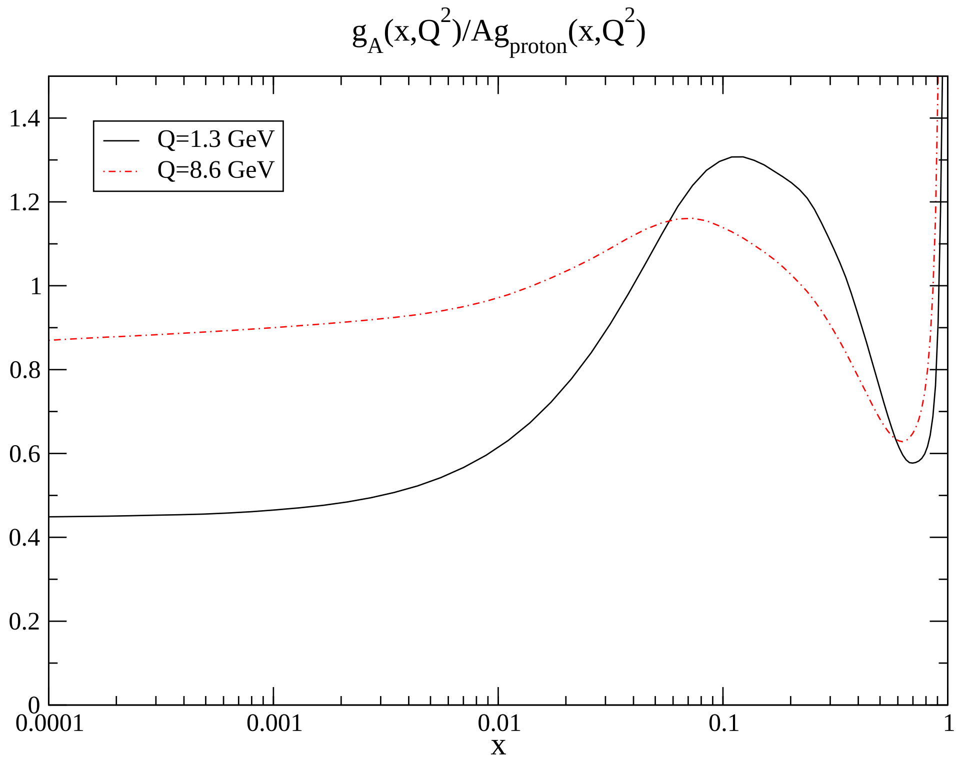


- Single inclusive HQ spectra $E(dN/d^3p)$ generated with POWHEG (pQCD @ NLO) using the PDF set CTEQ6M (+EPS09 in AA)
- HQ distributed in the transverse plane according to the nuclear overlap $dN/d\boldsymbol{x}_\perp \sim T_{AB}(x,y) \equiv T_A(x+b/2,y)T_B(x-b/2,y)$, with

$$T_{A/B}(\boldsymbol{x}_\perp) \equiv \int_{-\infty}^{+\infty} dz\, \rho_{A/B}(\boldsymbol{x}_\perp, z)$$

- Each quark is given a random $\boldsymbol{k}_\perp$ broadening extracted from a gaussian distribution, with $\langle k_\perp^2 \rangle = \langle k_\perp^2 \rangle_{pp} + \langle \delta k_\perp^2 \rangle_{AB}(\vec{b}, \vec{s})$.

Initial heavy-quark spectra: effects of nPDFs



The dominant process is $gg \to Q\bar{Q}$ in which

$$x_{1/2} = \frac{m_{Q\overline{Q}}}{\sqrt{s}_{NN}} e^{\pm y_{Q\overline{Q}}}$$

$\to$ role of nPDFs different for charm and beauty!

Glauber and $k_\perp$ broadening

Each HQ is given a $\boldsymbol{k}_\perp$-kick extracted from a gaussian distribution with

$$\langle k_\perp^2 \rangle_{AB}(\vec{b},\vec{s}) = \langle k_\perp^2 \rangle_{pp} + \frac{a_{gN}}{2} \left[\frac{\int dz_A \, \rho_A(\vec{s}, z_A) l_A(\vec{s}, z_A)}{T_A(\vec{s})} + \frac{\int dz_B \, \rho_B(\vec{s}-\vec{b}, z_B) l_B(\vec{s}-\vec{b}, z_B)}{T_B(\vec{s}-\vec{b})} \right]$$

due to the length crossed by the incoming partons in nucleus A/B before the hard event:

$$l_A(\vec{s}, z_A) \equiv \int_{-\infty}^{z_A} dz \, \rho_A(\vec{s}, z)/\rho_0 \quad \text{and} \quad l_B(\vec{s}-\vec{b}, z_b) \equiv \int_{z_B}^{+\infty} dz \, \rho_B(\vec{s}-\vec{b}, z)/\rho_0$$

We choose

a_{gN} (GeV^2/fm)	SPS	RHIC	LHC
c	0.072	0.092	0.153
b	0.197	0.252	0.420

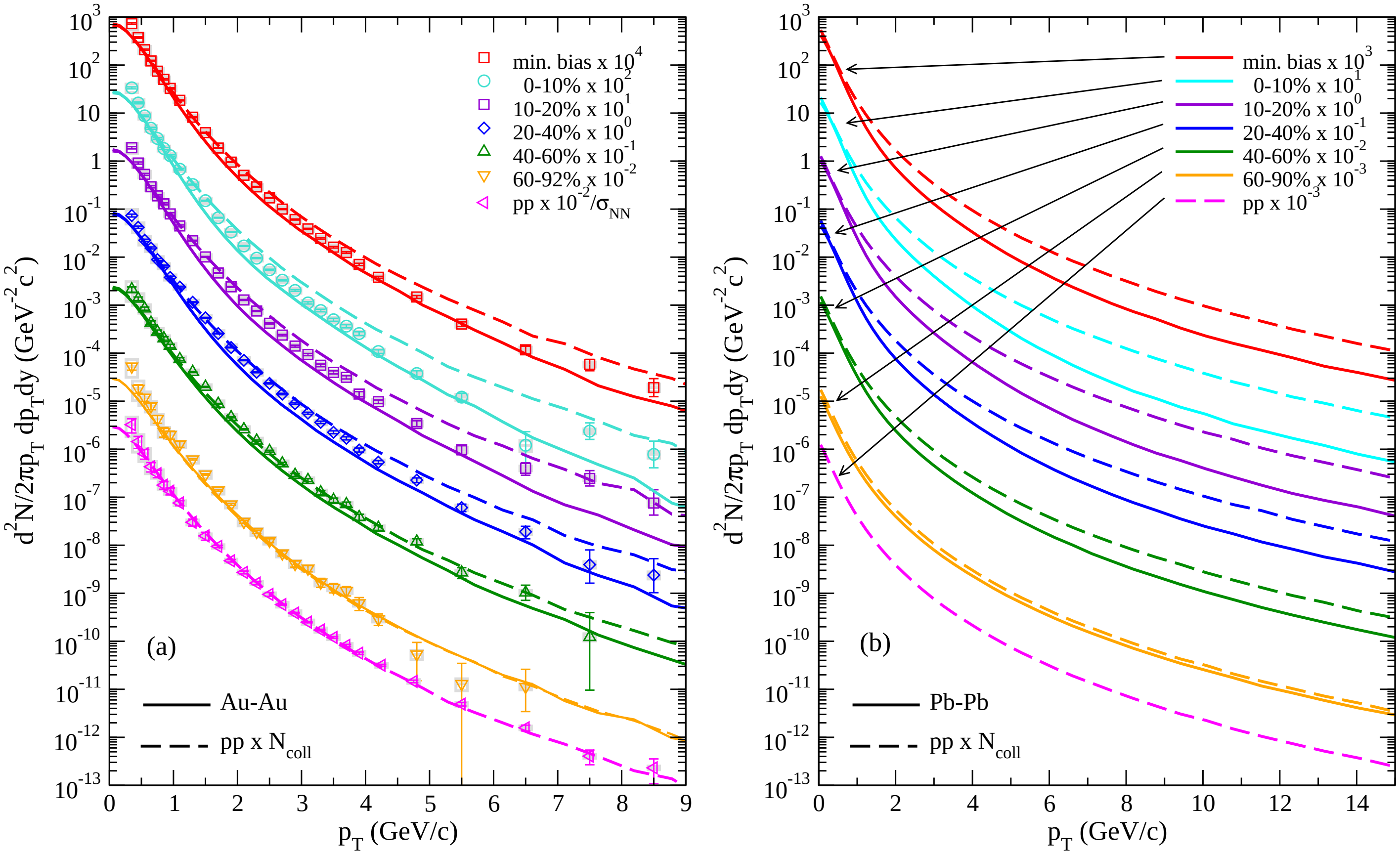
Single-electron spectra: procedure

- As an outcome of the initial generation (+ Langevin evolution in AA) one has two samples ($90 \cdot 10^6$) of c and b quarks;
- They are made fragment with Peterson FFs with the branching fractions given by the PDG;
- Each hadron is sent to the PYTHIA decayer (with updated PDG decay tables) till producing *at least* one electron in the final state (in case of no final electron the event is re-sampled);
- The two sources must be combined with their appropriate weight:

$$\left.\frac{dN}{dp_T}\right|_{e_c+e_b} \sim \sigma_{c\bar{c}} \sum_{h_c} \frac{N_{h_c}^{\mathrm{init}}}{N_{h_c}^{\mathrm{sampl}}} \left.\frac{dN}{dp_T}\right|_{e_c} + \sigma_{b\bar{b}} \sum_{h_b} \frac{N_{h_b}^{\mathrm{init}}}{N_{h_b}^{\mathrm{sampl}}} \left.\frac{dN}{dp_T}\right|_{e_b}$$

NB We plot the electrons falling into the PHENIX acceptance ($|\eta| < 0.35$), where $\eta \equiv \frac{1}{2} \ln \frac{p+p_z}{p-p_z} = -\ln \tan \frac{\theta}{2}$

Single-electron invariant yields: RHIC and LHC



- Dashed curves: pp result scaled by $\langle N_{\mathrm{coll}} \rangle$;
- Continuous curves: AA result after Langevin (viscous hydro, τ_0=1 fm).

Centrality classes

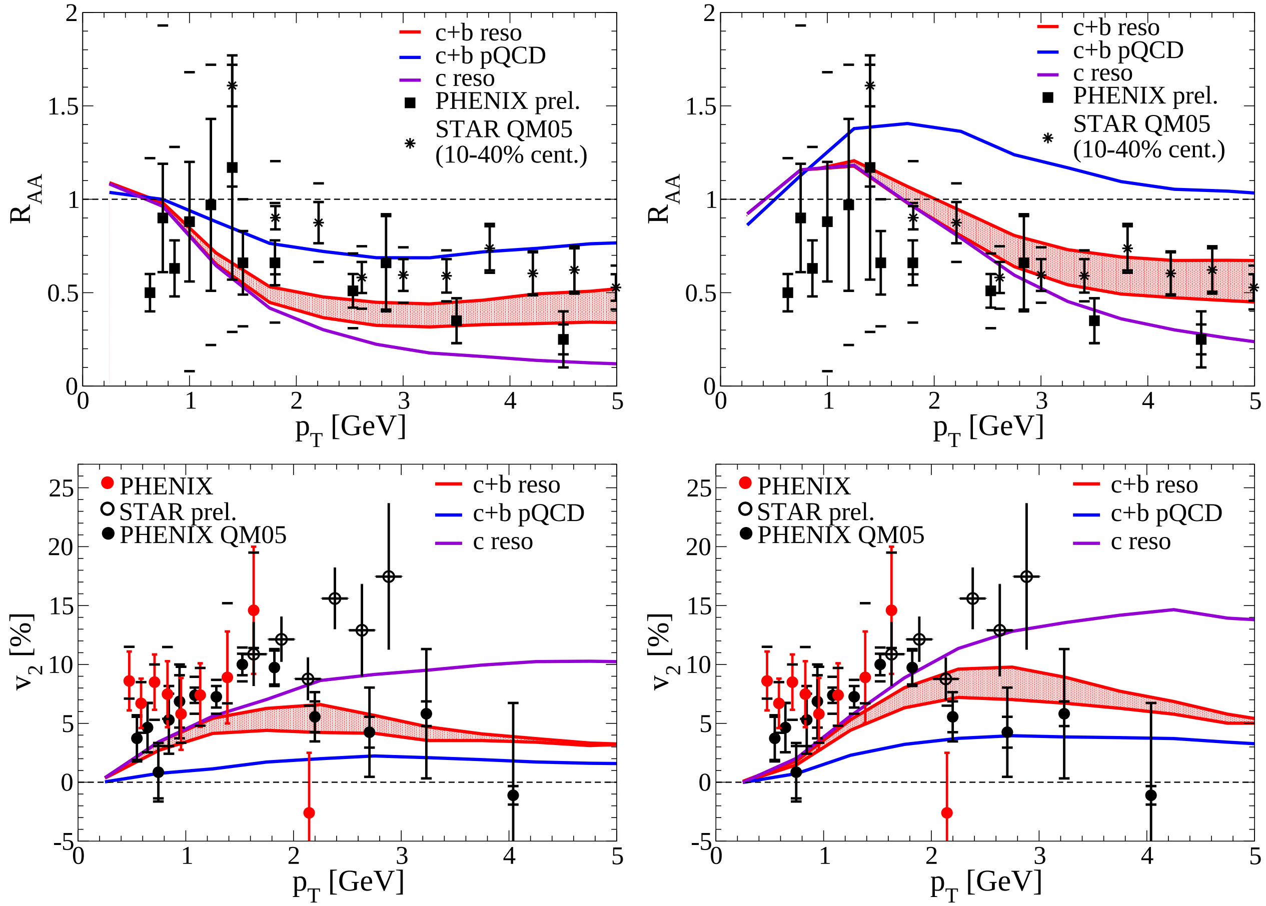
$$f_{C_1-C2} = \frac{\int_{b_1}^{b_2} db\, b[1-\exp(\sigma_{\rm NN} T_{AB}(b))]}{\int_0^{\infty} db\, b[1-\exp(\sigma_{\rm NN} T_{AB}(b))]},$$

where $T_{AB}(b) = \int d\boldsymbol{s} T_A(\boldsymbol{s}+\boldsymbol{b}/2) T_B(\boldsymbol{s}-\boldsymbol{b}/2)$.

Au-Au ($\sqrt{s}$ = 200 GeV)			Pb-Pb ($\sqrt{s}$ = 5.5 TeV)		
C_1-C_2	b (fm)	$N_{\rm coll}$	C_1-C_2	b (fm)	$N_{\rm coll}$
0-10%	3.27	963	0-10%	3.45	1698
10-20%	5.78	592	10-20%	6.11	1022
20-40%	8.12	282	20-40%	8.58	469
40-60%	10.51	81	40-60%	11.11	123
60-92%	12.80	10	60-90%	13.45	14
0-92%	8.44	247	0-90%	8.77	435

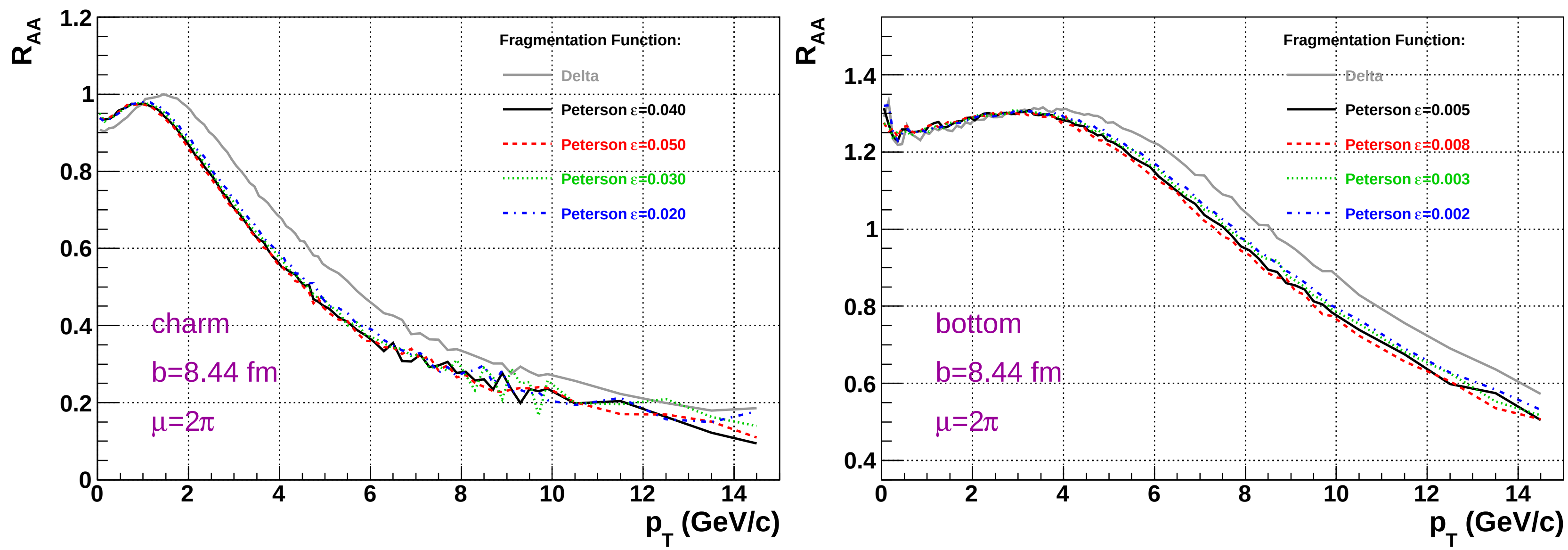
Which role could be played by coalescence?

Fragm. (left panels) vs Coalescence + Fragm.[a] (right panels)



[a]H. van Hees, V. Greco and R. Rapp, PRC 73, 034913 (2006)

Effects of fragmentation



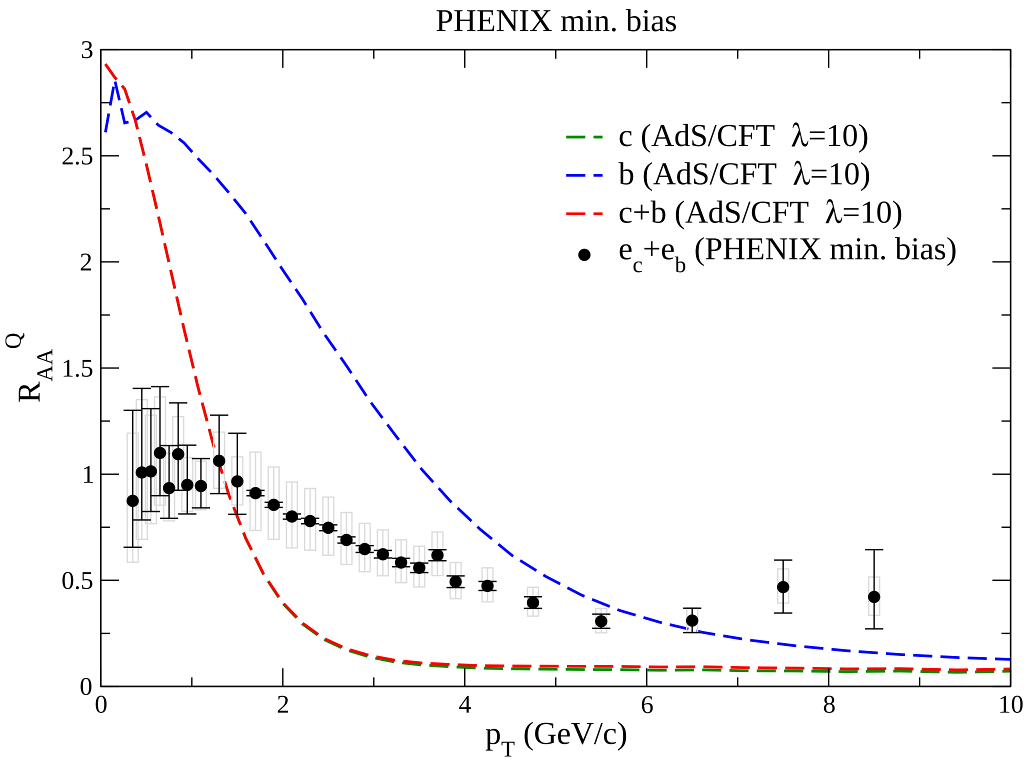
Fragmentation performed with Peterson FF tends to slightly suppress R_{AA}

- Mild dependence on the parameter ϵ
- $\epsilon = 0.04$ and 0.005 (for c and b) fixed in order to reproduce HQET FFs[a]

Fragmentation fractions taken from DESY results and PDG_2009

[a] E. Braaten, K. Cheung and T.C. Yuan, Phys. Rev. D 48, 5049 (1993)

Heavy-quark R_{AA}: AdS/CFT scenario



- Plot obtained setting $\lambda = 10$ in the transport coefficients

$$\kappa_T = \sqrt{\lambda}\pi T^3 \gamma^{1/2} \qquad \kappa_L = \sqrt{\lambda}\pi T^3 \gamma^{5/2}$$

- A more systematic study could be of interest.