Higher orders and resummation for DY precision physics

giuseppe bozzi

Università degli Studi di Milano
and
INFN Sezione di Milano

Working Group on EW precision measurements at the LHC
CERN, 04.04.2011

Outline

1. Fixed-order calculations: methods and associated uncertainties
2. Failure of fixed-order
3. Numerical resummation: methods and associated uncertainties
4. Analytical resummation: methods and associated uncertainties
5. Conclusions

PDF uncertainty not discussed!

Outline

A NNLO calculation

- For a general $2 \to n$ process we need
  - Two-loop amplitude for $2 \to n$
  - One-loop amplitude for $2 \to n+1$
  - Tree-level amplitude for $2 \to n+2$
- Each term has its own singularities
  - Ultraviolet (removed by renormalization)
  - Infrared (have to cancel among each other)

$\longrightarrow$ Much more difficult than NLO cancellation!

1. Fully inclusive quantities
   - analytical computation of contributions is possible
   - explicit cancellation of singularities
2. Fully exclusive quantities (real world!)
   - IR singularity structure at NNLO understood

     [Catani,Grazzini;Campbell,Glover;Bern,DelDuca,Kilgore,Schmidt; Kosower,Uwer;Sterman,Tejeda-Yeomans]
   - numerical integration still very difficult

   $\longrightarrow$ **Sector Decomposition**

   $\longrightarrow$ **Subtraction Method**

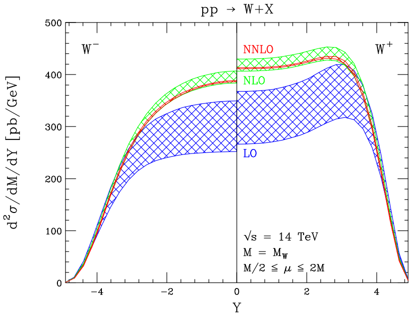
Sector Decomposition

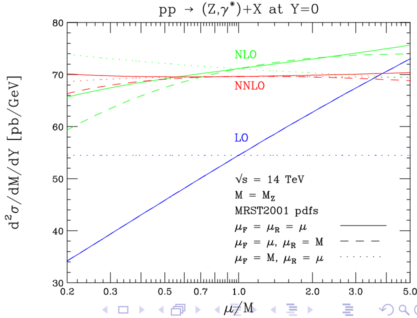
"Split the integration region into sectors, each containing a single singularity, and explicit the pole by expanding it into distributions"

Binoth,Heinrich[00,04];Anastasiou,Melnikov,Petriello[04]

AMP developed a fully automated procedure to compute pole coefficients and finite terms and applied it to

Higgs (*FEHiP*, 2005), W/Z (*FEWZ*, 2006)





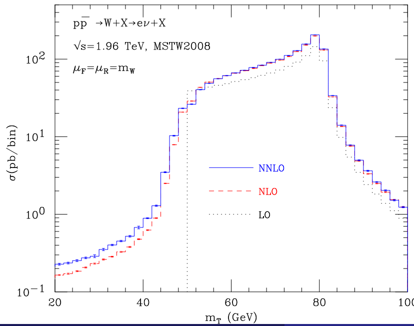
Subtraction Method

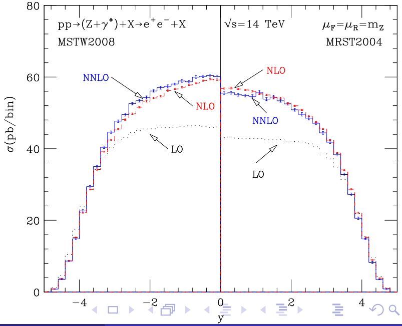
"Add and subtract a local counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton"

(NNLO):Kosower[03,05];Weinzierl[03];Frixione,Grazzini[04];
Gehrmann,Glover[05];Somogyi,Trocsanyi,DelDuca[05,07]

Applications: *HNNLO* (2007) and *DYNNLO* (2009)

H:Catani,Grazzini[07];W,Z:Catani,Cieri,DeFlorian,Ferrera,Grazzini[09]





NNLO uncertainty

Differences between the two prescriptions:
at the level of statistical precision

Theoretical uncertainty = scale variation

Outline

The need for resummation

Partonic cross section as a perturbative series

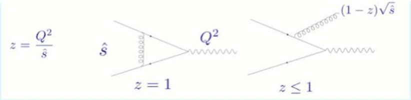
$$\begin{aligned}\sigma_{ab}^{part}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) &= \alpha_s^k(\mu_R)[\sigma_{LO}(p_1, p_2, Q, Q_i) \\ &+ \alpha_s(\mu_R)\sigma_{NLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) \\ &+ \alpha_s^2(\mu_R)\sigma_{NNLO}(p_1, p_2, Q, Q_i, \mu_R, \mu_F) + \ldots]\end{aligned}$$

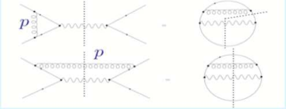
- The fixed-order result gives reliable result only when all the scales are of the same order of magnitude
- If $Q_i \gg Q$ or $Q_i \ll Q$, the appearance of $\alpha_s log(Qi/Q)$ terms could spoil the perturbative result: they need to be resummed!

Resummation: well-known examples

- $\log(Q/Q_0)$
  - evolution of pdfs from input scale Q_0 to hard scale Q
  - collinear radiation from colliding partons: single logs
  - systematically resummed by DGLAP equation
- $\log(Q/\sqrt{S})$
  - hadronic c.m. energy $\sqrt{S}$ much larger than hard scale Q
  - multiple radiation over wide rapidity range: single logs
  - systematically resummed by BFKL equation
- $\log(Q^2/q_T^2)$
  - systems with invariant-mass $Q \gg q_T$
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation
- $\log(1 - Q^2/S)$
  - hadronic c.m. energy $\sqrt{S}$ comparable to hard scale Q
  - soft and collinear gluon emission: single and double logs
  - treated by means of soft-gluon resummation

The origin of logs





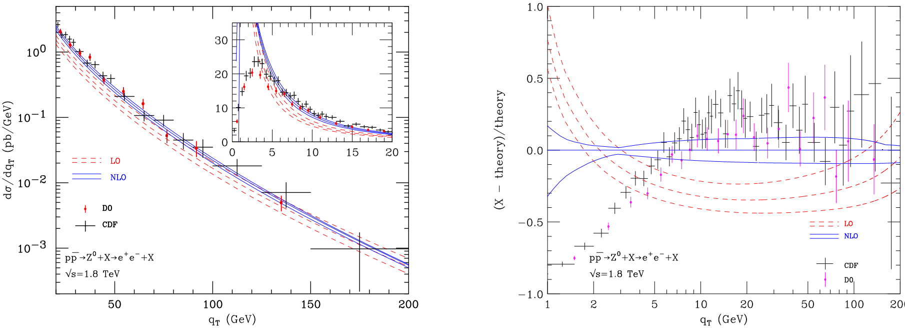
- Higher orders made up of real (z<1) and virtual (z=1) contributions (both IR divergent)
- Different cuts of the same diagram: cancellation of IR divergences (KLN theorem)
- Near "critical" regions of phase space we have kinematical unbalance (full virtual - partial real) $\rightarrow$ incomplete cancellation $\rightarrow$ large logs! $\alpha_S L^2$ (soft and collinear emission) $\alpha_s L$ (only soft emission)

An example: the small-q_T region ($q_T \ll Q$)

- Bulk of the events in the region $q_T \ll Q$
- Kinematical unbalance between real and virtual contributions

$\longrightarrow$ perturbative coefficients enhanced by $\alpha_S^n \log^m(\frac{Q^2}{q_T^2})$

$\longrightarrow$ convergence of perturbative result completely spoiled



Outline

Parton Shower vs. Matrix elements

Parton Shower Generator	**Matrix Element Generator**
Resums leading logs to all orders	Only go up to NLO
High multiplicity *hadrons* in final state	Low multiplicity *partons* in final state
Good for regions of low relative p_T	Good for regions of high relative p_T
Total rate accurate to LO	Total rate accurate to NLO

The perfect matching

- generates total rates accurate at NLO
- treats hard emission as in Matrix Element Generators
- treats soft/collinear emission as in Parton Shower Generators
- generates a set of fully exclusive events which can be interfaced with a hadronization model

NLO Matching

- **MC@NLO** [Frixione,Webber(02)]
  - add difference between exact(ME) NLO and approx.(PS) NLO
  - automatization (aMC@NLO) based on FKS subtraction @ NLO [Frederix,Frixione,Maltoni,Stelzer(09)]

  → dependent on the shower details

  → difference may be negative
- **POWHEG** [Nason(04)]
  - Generate the hardest emission at NLO accuracy (mod. Sudakov)
  - Angular-ordered showers: add truncated shower from hard scale

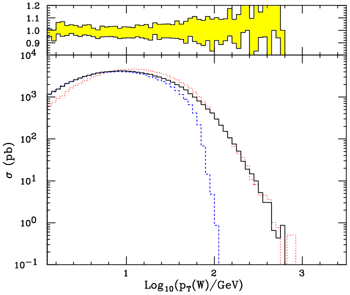
  → always positive weights

  → discrepancies with respect to MC@NLO thoroughly explained in several publications

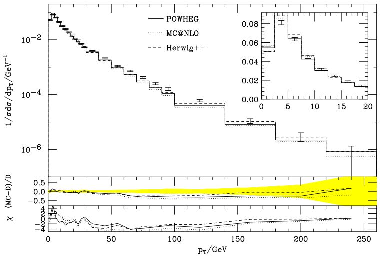
NLO matching uncertainties

Differences between matching procedures

MC@NLO/HW *vs.* MC@NLO/PY *vs.* PY [Frixione,Torrielli(10)]



POWHEG *vs.* MC@NLO *vs.* HERWIG *vs.* DATA [Hamilton,Richardson,Tully(08)]



Theoretical uncertainty = choice of method/shower, NP tuning

Outline

Analytical Resummation: the main idea

$\alpha_s L^2$	$\alpha_s L$	$\dots$	$\dots$	$\mathcal{O}(\alpha_s)$	(LO)
$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	$\mathcal{O}(\alpha_s^2)$	(NLO)
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$\alpha_s^n L^{2n}$	$\alpha_s^n L^{2n-1}$	$\alpha_s^n L^{2n-2}$	$\dots$	$\mathcal{O}(\alpha_s^n)$	(N^nLO)
LL	NLL	NNLL	$\dots$	$\dots$	

- Ratio of two successive rows: $\mathcal{O}(\alpha_s L^2)$
- improved expansion
  - *reorganization* of the terms into *towers of logs*
  - *all-order summation* of the terms in each class
- key-point: *exponentiation*

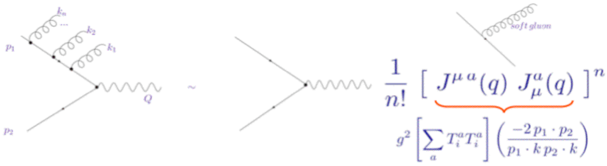
$$\sigma^{res} \sim \exp\left[L g_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \dots\right]$$

- Ratio of two successive columns: $\mathcal{O}(1/L)$

Exponentiation

The observable must fulfill factorization properties both for

- dynamics (matrix element)
  - $\longrightarrow$ in the soft limit, multigluon amplitudes fulfill *generalized factorization formulae* given in terms of *single gluon emission probability*



- kinematics (phase space)
  - $\longrightarrow$ usually factorizable working in *conjugate space*

$$\delta^{(2)}(q_T - q_{T1} - \cdots - q_{Tn}) \quad = \quad \int d^2b\; e^{ib\cdot q_T}\; \Pi_i\; e^{ib\cdot q_T}$$

$$\log(Q^2/q_T^2) \quad \to \quad \log(Q^2 b^2)$$

$\longrightarrow$ generalized exponentiation of single gluon emission

[Collins, Soper, Sterman (1985)]

Matching with fixed-order

The resummed result has to be properly matched with the fixed-order calculation to avoid double counting

$$\sigma = \sigma^{res} + \sigma^{fix} - \sigma^{asym}$$

where σ^{asym} = expansion of resummed result to same order

- $q_T \ll Q$: $\sigma^{fix} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{res}$
- $q_T > Q$: $\sigma^{res} \sim \sigma^{asym} \rightarrow \sigma = \sigma^{fix}$
- intermediate q_T: matching $\rightarrow \sigma$

Problems:

1) Resummation involves integration over b from 0 to ∞:

$\alpha_s(1/b)$ large when $b \rightarrow 1/\Lambda_{QCD}$

2) You must go back to the physical space!

Going back to the physical space

- Proposed solutions
  - return to p_T space (expansion of the exponent + inverse transformation performed analytically)

    [Ellis,Veseli(97);Frixione,Nason,Ridolfi(99);Kulesza,Stirling(99-03)]
  - integration over a complex b-plane to avoid singularities

    [Laenen,Sterman,Vogelsan(00);Kulesza,Sterman,Vogelsang(02)Bozzi,Catani,DeFlorian,Grazzini(05-09)]
  - extrapolation of perturbative results into large-b region [Qiu,Zhang(01)]
  - using Borel resummation [Bonvini,Forte,Ridolfi(08)]
- Improved matching [Bozzi,Catani,DeFlorian,Grazzini(05-09)]

$$\tilde{L} = \log(\frac{bQ}{b_0} + 1) \rightarrow \int dp_T \frac{d\sigma_{NLO}}{dp_T} = \sigma_{NNLO}$$

$\longrightarrow$introduction of resummation scale$\longleftarrow$

- Other approaches
  - joint resummation: resum both threshold and recoil logs

    [Laenen,Sterman,Vogelsang(00)]
  - resummation for double differential (p_T, y) distributions

    [Bozzi,Catani,DeFlorian,Grazzini(08)]

Non-perturbative effects

- Important non-perturbative (large-b) effects for q_T-distributions $\equiv$ intrinsic q_T of the partons, inside the hadrons.
- Resummation formula —> $\exp(S + F_{NP})$

$$F_{NP}(b, Q, x_a, x_b) = \exp[(-g_1 - g_2 \log(\frac{Q}{2Q_0}) - g_1 g_3 \log(100 x_a x_b)) b^2]$$

- NP form factor (g_1, g_2, g_3) obtained from experimental data:
  - Ladinsky,Yuan(94)
  - Brock-Landry-Nadolsky-Yuan(03)
  - Konyshev-Nadolsky (06)

Drell-Yan at NNLL+NLO [Bozzi,Catani,deFlorian,Ferrera,Grazzini(10)]

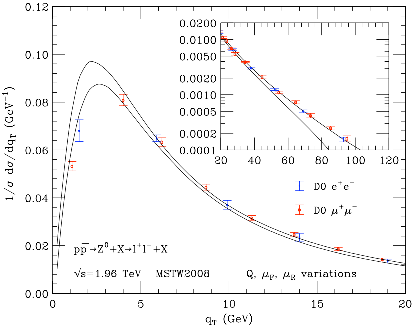
- Normalized q_T distribution
- Scales fixed to Z mass

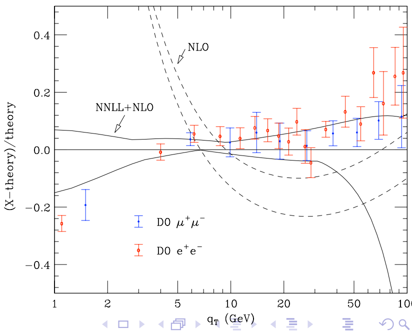
→ Uncertainty dominated by Q variation

→ Good agreement with Run II D0 data

→ Experimental errors are smaller than theoretical uncertainty

- most accurate QCD perturbative prediction for W and Z



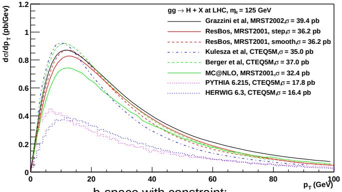


Analytical resummation uncertainties

Differences between resummation prescriptions: work in progress!

Higgs production via gluon fusion at the LHC

[*Balazs, Grazzini, Huston, AK, Puljak'04*]



NNLL+NLO	b-space with constraint: $\int dp_{\rm T} \frac{d\sigma^{\rm NLO}}{dp_{\rm T}} = \sigma^{\rm NNLO}$	[*Bozzi et al.'03'05*]
"Sudakov" NNLL + LO	b-space	[*Berger, Qiu'02*]
"Sudakov" NNLL + LO	joint	[*A.K., Sterman, Vogelsang'03*]
"Sudakov" NNLL + (N)LO	b-space	[*Balazs, Yuan'00*]
MC@NLO	LO $p_{\rm T}$-distribution + parton shower	[*Frixione, Webber'02*]
PYTHIA	with hard matrix el. corrections	
HERWIG	without hard matrix el. corrections	

A. Kulesza, p_T resummation for colour-singlet hadronic production – p. 24/28

Theoretical uncertainty = choice of prescription, NP input

Outline

Conclusions

There are MANY sources of theoretical uncertainties!

- factorization scale
- renormalization scale
- resummation scale
- type of resummation (shower vs. analytical (ResBos/DYqT))
- non-perturbative contributions
  (shower tuning procedure vs. use of different NP form factors)