The impact of PDF uncertainties on W mass measurements

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The measurement of the W mass

- charged lepton transverse momentum p_t^l distribution
- missing transverse momentum p_t^ν distribution
- lepton pair transverse mass distribution

$$M_\perp^W = \sqrt{2 p_t^l p_t^\nu \left(1 - \cos\left(\phi^l - \phi^\nu\right)\right)}$$

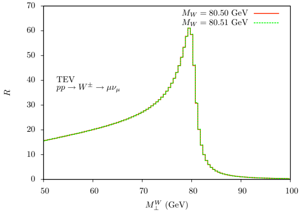
$\longrightarrow$ fit distributions with theoretical predictions (M_W free)

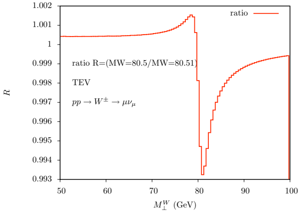
QCD and EW uncertainties already discussed: let's talk about PDFs!

PDF uncertainty $\longrightarrow$ ΔM_W=?

Sensitivity to shapes

- Transverse-mass distribution





- Total integrated cross-section

m_W (GeV)	80.368	80.378	80.388	80.398	80.408	80.418	80.428
$\sigma_{tot}(m_W)$ (pb)	368.72(6)	368.87(6)	369.03(6)	369.17(6)	369.32(6)	369.46(6)	369.61(6)

Born total cross sections, within acceptance cuts, as a function of m_W (Monte Carlo error in parenthesis)

$\longrightarrow$ a shift by 10 MeV of m_W yields a sensible change in shape

$\longrightarrow$ a shift by 10 MeV of m_W yields a change in σ at the 0.04% level

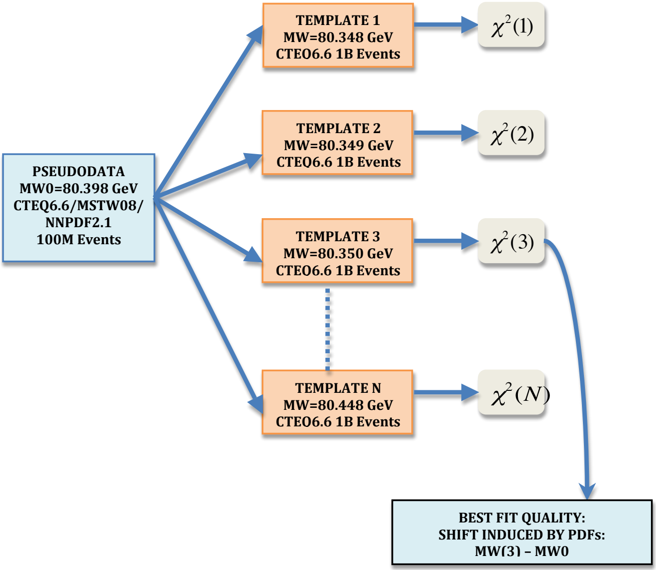
The fitting strategy

1. generate templates for a given fixed PDF set and for different values of m_w with very high statistics (1B events)
2. for each member of the PDF sets considered, generate pseudo-data with fixed $m_w^0 = 80.398$ GeV with lower statistics (100M events)
3. compute the χ^2 between the pseudo-data and each of the templates

$$\chi_j^2 = \frac{1}{N_{\text{bins}}} \sum_{i=1}^{N_{\text{bins}}} \frac{\left(O_i^j - O_i^{\text{data}}\right)^2}{(\sigma_i^{\text{data}})^2 + (\sigma_i^j)^2} \qquad j = 1, \dots, N_{\text{templates}}$$

4. the template with best χ^2 provides the information on Δm_w induced by this particular PDF set

The fitting strategy



Numerical setup

$$p\bar{p} \to \mu^+ + X \text{ at the Tevatron } (\sqrt{s} = 1.96 \text{ TeV})$$
$$pp \to \mu^\pm + X \text{ at the LHC } (\sqrt{s} = 7, 14 \text{ TeV})$$

$G_\mu = 1.16637\ 10^{-5}$ GeV^{-2}	$m_W = 80.398$ GeV	$m_Z = 91.1876$ GeV
$\Gamma_W = 2.141$ GeV	$\sin^2\theta_W = 1 - m_W^2/m_Z^2$	$m_H = 120$ GeV
$V_{cd} = 0.222$	$V_{cs} = 0.975$	$V_{cb} = 0$
$V_{ud} = 0.975$	$V_{us} = 0.222$	$V_{ub} = 0$
$V_{td} = 0$	$V_{ts} = 0$	$V_{tb} = 1$

Tevatron	LHC
$p_\perp^\mu \geq 25$ GeV	$p_\perp^\mu \geq 25$ GeV
$\not{E}_T \geq 25$ GeV	$\not{E}_T \geq 25$ GeV
$\lvert\eta_\mu\rvert < 1.0$	$\lvert\eta_\mu\rvert < 2.5$

We study the lepton pair transverse mass

Why the (normalized) lepton pair transverse mass?

$$\mathcal{O}\left(M_\perp^W\right) = \frac{d\sigma}{dM_\perp^W}\left(M_\perp^W\right), \qquad M_\perp^W = \sqrt{2p_t^l p_t^\nu\left(1 - \cos\left(\phi^l - \phi^\nu\right)\right)}$$

- QCD corrections quite moderate with respect to lepton p_T
- small QCD effects on the shape of the distribution
- PDF uncertainties induce similar effects w.r.t. other observables

$$\widetilde{\mathcal{O}}\left(M_\perp^W\right) = \frac{1}{\sigma^{\rm fit}}\frac{d\sigma}{dM_\perp^W}\left(M_\perp^W\right), \qquad \sigma^{\rm fit} = \int_{M_\perp^{W,\rm min}}^{M_\perp^{W,\rm max}} dM \frac{d\sigma}{dM_\perp^W}(M)$$

$$(M_\perp^{W,\rm min} = 50 \text{ GeV}, \; M_\perp^{W,\rm max} = 100 \text{ GeV})$$

- normalization greatly reduces the effect of PDF uncertainty

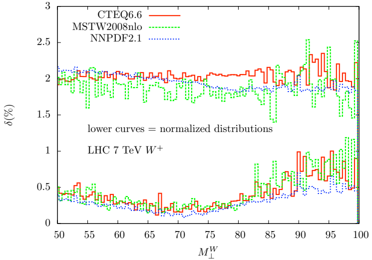
Codes and PDFs

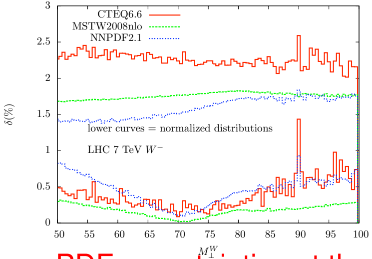
- Numerical codes
  1. Born: HORACE [Carloni Calame,Montagna,Nicrosini,Vicini(03-07)]
  2. NLO-QCD: DYNNLO [Catani,Cieri,Ferrera,de Florian,Grazzini(09)]
  3. NLO+NLL-QCD: ResBos [Balazs,Nadolsky,Yuan(93-98)]

- PDFs
  1. CTEQ6.6 [Nadolsky,Lai,Cao,Huston,Pumplin,Stump,Tung,Yuan(08)]
  2. MSTW2008 [Martin,Stirling,Thorne,Watt(09)]
  3. NNPDF2.1

     [Ball,Bertone,Cerutti,DelDebbio,Guffanti,Forte,Latorre,Rojo,Ubiali(11)]

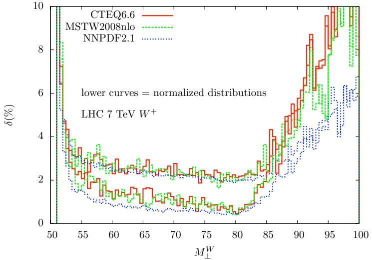
Born predictions

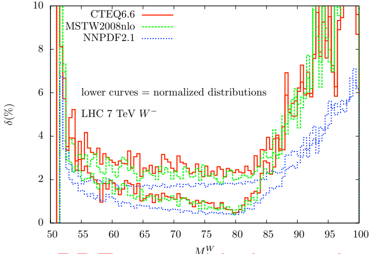




normalized distributions: PDF uncertainties at the permille level

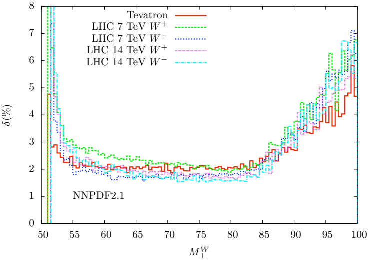
NLO-QCD predictions





normalized distributions: PDF uncertainties at the permille level

"Universality" of the effect



PDF uncertainty independent of the collider, energy and final state:
m_W determination at the LHC not more challenging than at the Tevatron

PDF uncertainty $\longrightarrow \Delta m_W$

1. apply the template fit procedure to transverse mass distributions
2. fit the W mass separately with each PDF set obtained with different replicas (NNPDF) or different eigenvectors (CTEQ, MSTW)
3. apply the corresponding prescriptions to compute the best estimate for m_W and the associated PDF uncertainty for that set
   - symmetric error for the Hessian approach (CTEQ,MSTW)

$$\Delta X = \frac{1}{2}\sqrt{\sum_{i=1}^{N}\left[X_i^+ - X_i^-\right]^2} \tag{1}$$

   - average over the ensemble of PDF replicas (NNPDF)

$$\langle \mathcal{F}[\{q\}]\rangle = \frac{1}{N_{rep}}\sum_{k=1}^{N_{rep}} \mathcal{F}[\{q^{(k)}\}] \tag{2}$$

$$\sigma_{\mathcal{F}} = \left(\frac{1}{N_{rep}-1}\sum_{k=1}^{N_{rep}}\left(\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}]\rangle\right)^2\right)^{1/2} \tag{3}$$

Normalized vs. non-normalized distributions

non-normalized Born distributions

collider,final state	CTEQ6.6		MSTW2008		NNPDF2.1	
	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$
Tevatron, $W^{\pm}$	80.398 $\pm$ 0.007	0	80.408 $\pm$ 0.007	+10	80.407 $\pm$ 0.008	+9
LHC 7 TeV W^{+}	80.398 $\pm$ 0.007	0	80.399 $\pm$ 0.006	+1	80.398 $\pm$ 0.005	0
LHC 7 TeV W^{-}	80.398 $\pm$ 0.004	0	80.401 $\pm$ 0.004	+3	80.399 $\pm$ 0.005	+1
LHC 14 TeV W^{+}	80.398 $\pm$ 0.008	0	80.393 $\pm$ 0.007	-5	80.388 $\pm$ 0.005	-10
LHC 14 TeV W^{-}	80.398 $\pm$ 0.005	0	80.399 $\pm$ 0.004	+1	80.391 $\pm$ 0.005	-7

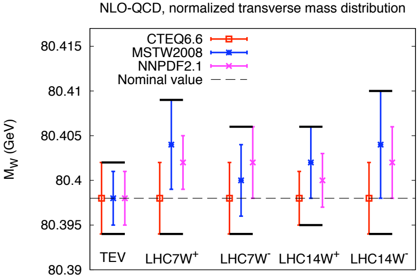
normalized Born distributions

collider,final state	CTEQ6.6		MSTW2008		NNPDF2.1	
	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$	$m_W \pm \delta_{\rm pdf}$	$\Delta_{\rm pdf}$
Tevatron, $W^{\pm}$	80.398 $\pm$ 0.004	0	80.399 $\pm$ 0.003	+1	80.399 $\pm$ 0.005	+1
LHC 7 TeV W^{+}	80.398 $\pm$ 0.003	0	80.404 $\pm$ 0.003	+6	80.401 $\pm$ 0.003	+3
LHC 7 TeV W^{-}	80.398 $\pm$ 0.002	0	80.396 $\pm$ 0.002	-2	80.400 $\pm$ 0.004	+2
LHC 14 TeV W^{+}	80.398 $\pm$ 0.003	0	80.402 $\pm$ 0.002	+4	80.399 $\pm$ 0.003	-1
LHC 14 TeV W^{-}	80.398 $\pm$ 0.002	0	80.398 $\pm$ 0.002	0	80.398 $\pm$ 0.005	0

$\delta_{\rm pdf}$ = spread due to PDF uncertainties, $\Delta_{\rm pdf}$ = shift in m_W compared to CTEQ6.6

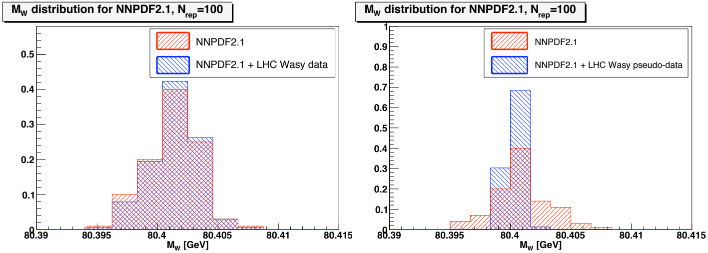
NLO-QCD results

m_W (GeV)	CTEQ6.6		MSTW2008		NNPDF2.1		
	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$m_W \pm \delta_{\rm pdf}$	$\langle\chi^2\rangle$	$\delta^{\rm tot}_{\rm pdf}$
Tevatron, $W^\pm$	80.398 ± 0.004	1.42	80.398 ± 0.003	1.42	80.398 ± 0.003	1.30	4
LHC 7 TeV W^+	80.398 ± 0.004	1.22	80.404 ± 0.005	1.55	80.402 ± 0.003	1.35	8
LHC 7 TeV W^-	80.398 ± 0.004	1.22	80.400 ± 0.004	1.19	80.402 ± 0.004	1.78	6
LHC 14 TeV W^+	80.398 ± 0.003	1.34	80.402 ± 0.004	1.48	80.400 ± 0.003	1.41	6
LHC 14 TeV W^-	80.398 ± 0.004	1.44	80.404 ± 0.006	1.38	80.402 ± 0.004	1.57	8



total (envelope) error at most 8 MeV + excellent agreement at Tevatron

W asymmetry data



- left: m_W distribution according to NNPDF 2.1. reweighted with LHC W asymmetry data (uncertainty $\sim$ 7%)
- right: m_W distribution according to NNPDF 2.1. reweighted with LHC W asymmetry pseudo-data (uncertainty $\sim$ 1%)

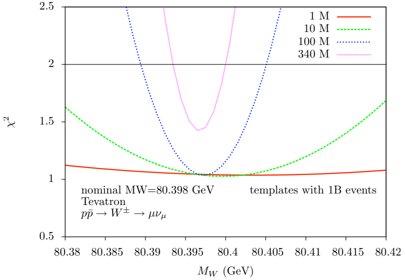
$\rightarrow$ clear narrowing effect, PDF error reduced by a factor of 2 or more!

Summary

- Born level: central values and PDF uncertainties agree between different sets and are independent of colliders, energies and final states
- NLO-QCD: same as Born with moderate increase of PDF uncertainty (gluon)
- use of normalized distributions allow for extremely precise measurements
- overall PDF uncertainty estimated to be less than 10 MeV
- further reduction of PDF uncertainty through the use of LHC data

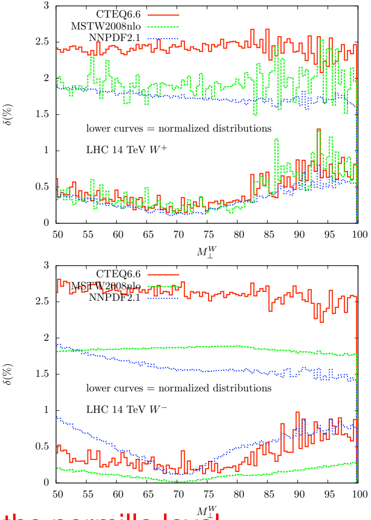
Back-up slides

Chi2 validation



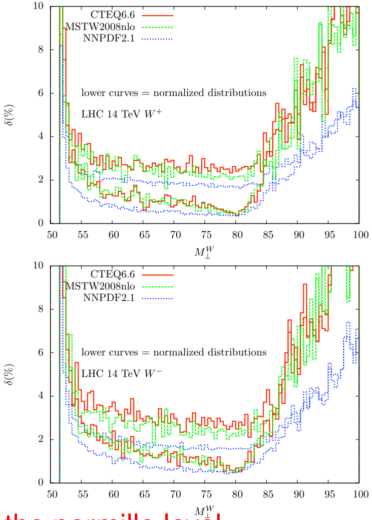
χ^2 distributions obtained fitting HORACE pseudodata with HORACE templates for the same PDF set at the Tevatron. The different curves correspond to different pseudodata samples each with different statistics. The $\Delta\chi^2 = 1$ rule indicates the resolution, at 68% C.L., on the W mass

Born @ LHC14

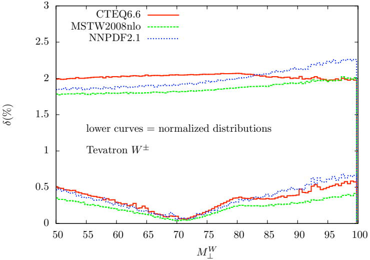


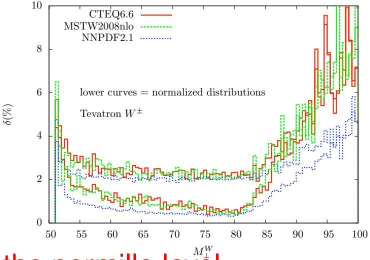
PDF uncertainties at the permille level

NLO-QCD @ LHC14



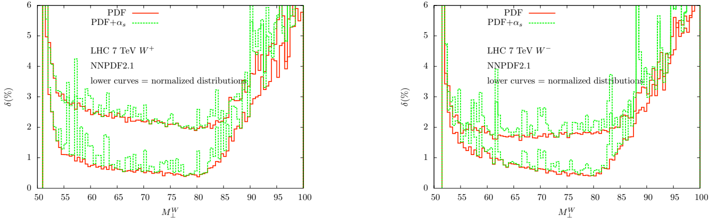
PDF uncertainties at the permille level





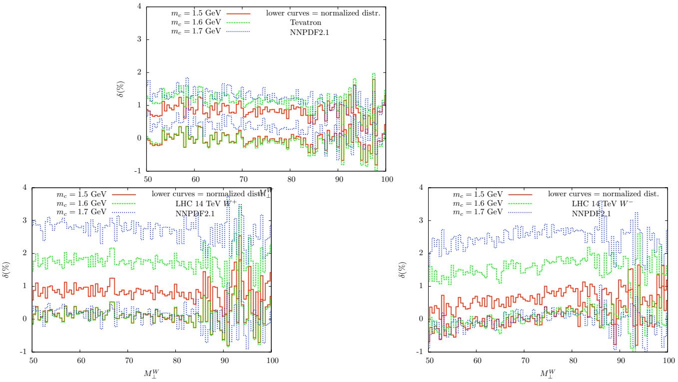
PDF uncertainties at the permille level

PDF+alphas



For NNPDF2.1 we show the differences between the PDF–only uncertainty only and the combined PDF+α_S uncertainty of the transverse mass distribution, expressed as relative deviation from the central NNPDF2.1 set with $\alpha_S\,(M_Z) = 0.119$. We assume that the uncertainty on the strong couping is $\delta_{\alpha_S} = 0.0012$ at the 68% confidence level. For simplicity we show only the distributions at the LHC 7 TeV, the distributions for Tevatron and LHC 14 TeV are quantitatively very similar

PDF+charm



For NNPDF2.1 we show the dependence on the charm quark mass of the transverse mass distribution, expressed as relative deviation from the central NNPDF2.1 set with m_c^2 =2 GeV^2. We show results both for the normalized and for the unnormalized transverse mass distributions

alphas table

	Tevatron	LHC7W+	LHC7W-	LHC14W+	LHC14W-
$\alpha_s(m_Z) = 0.118$	80.398	80.400	80.398	80.402	80.400
$\alpha_s(m_Z) = 0.119$ (ref)	80.398	80.402	80.402	80.400	80.402
$\alpha_s(m_Z) = 0.120$	80.398	80.400	80.398	80.402	80.402

Central value of the fit of m_W obtained with NNPDF2.1, using PDF sets that differ by the $\alpha_s(m_Z)$ value, for different colliders and energies. The fit has been done on normalized distributions and using normalized templates, and the distributions have been generated at NLO-QCD with DYNNLO

charm mass table

m_W (GeV)	Tevatron	LHC7W+	LHC7W-	LHC14W+	LHC14W-
$m_c = 1.414$ (ref)	80.398	80.402	80.402	80.400	80.402
$m_c = 1.5$	80.398	80.400	80.398	80.398	80.399
$m_c = 1.6$	80.398	80.400	80.400	80.398	80.399
$m_c = 1.7$	80.396	80.400	80.400	80.396	80.398

Central value of the fit of m_W obtained with NNPDF2.1 sets with different values of m_c for different colliders and energies. We include the default value in NNPDF2.1, $m_c^2 = 2$ GeV^2 as well