



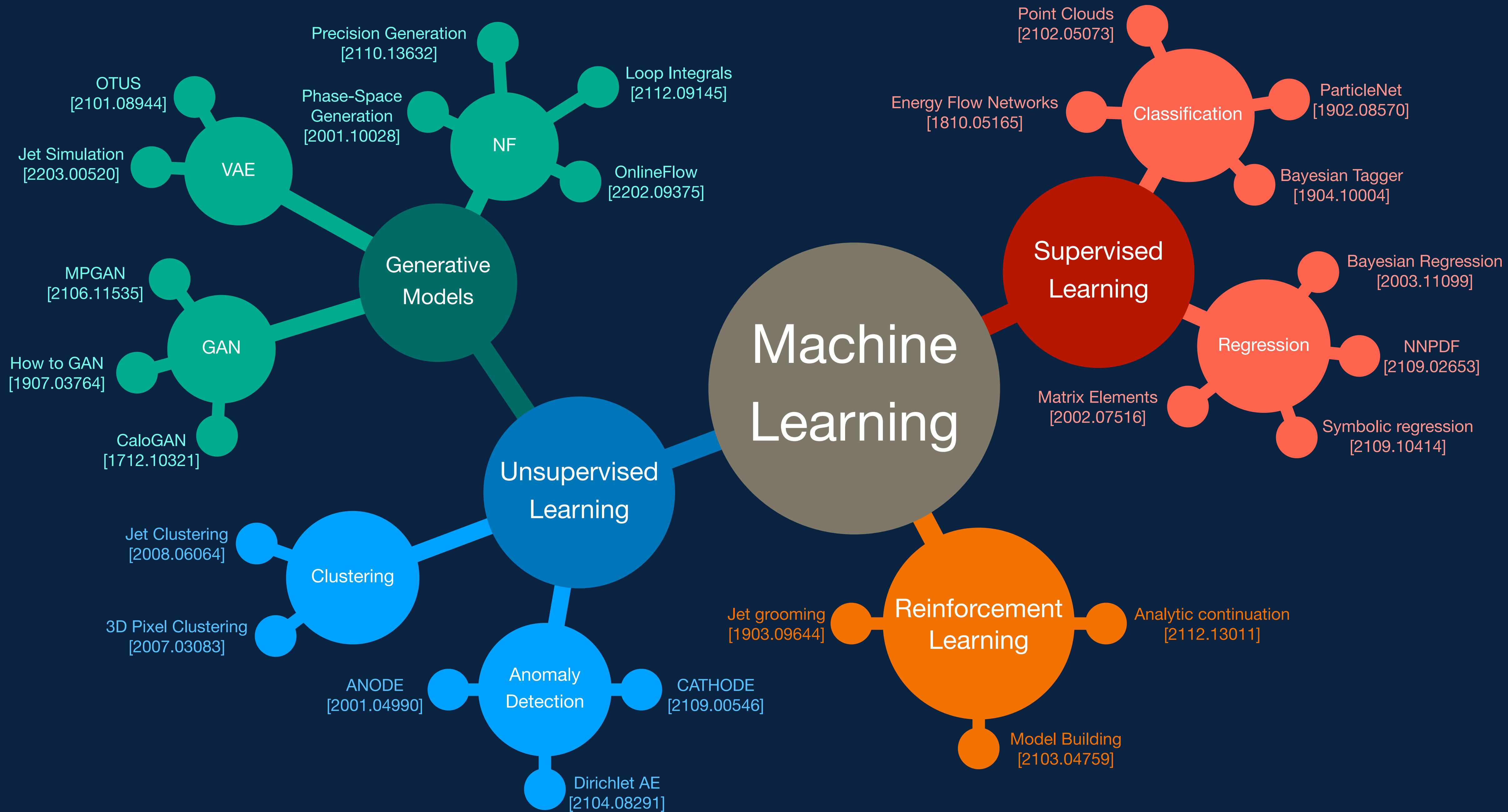
MadNIS

Neural networks for multi-channel
integration in MadGraph

Why talk about machine learning?

because

- rich **toolbox of algorithms** to develop **expressive and flexible models** for science
- fast development of **new methods and algorithms** in the past years
- promising applications in both **theory and experiment**
- large interest in **HEP community**: *IML, ML4Jets, MCnet, workshops,..*



Focus today
Event Generation



Theory predictions in HEP

$$\mathcal{A}_{\lambda,c,\dots}(p_a, p_b | p_1, \dots, p_n) : \mathbb{M} \rightarrow \mathbb{C}$$

Quantum numbers:
spin, colour charge etc.

Kinematics:
Momenta in Minkowski space, masses, etc.

$$\sigma = \frac{1}{\text{flux}} \sum_{a,b} \int dx_a dx_b f(x_a) f(x_b) \int d\Phi_n \langle |\mathcal{A}(p_a, p_b | p_1, \dots, p_n)|^2 \rangle$$

Cross section:
more generally,
differential observables*

PDFs:
convolution over all
possible initial state
configurations

**Phase-space
integral:**
over final state
kinematics

Squared amplitude:
summed over final states,
averaged over initial states

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Monte Carlo Integration

Standard Monte Carlo integration

$$I = \int_V d^d x f(x) \simeq \frac{1}{N} \sum_{j=1}^N f(x_j) = \langle f \rangle_x \quad \sigma_I \simeq \sqrt{\frac{\langle f^2 \rangle_x - \langle f \rangle_x^2}{N-1}}$$

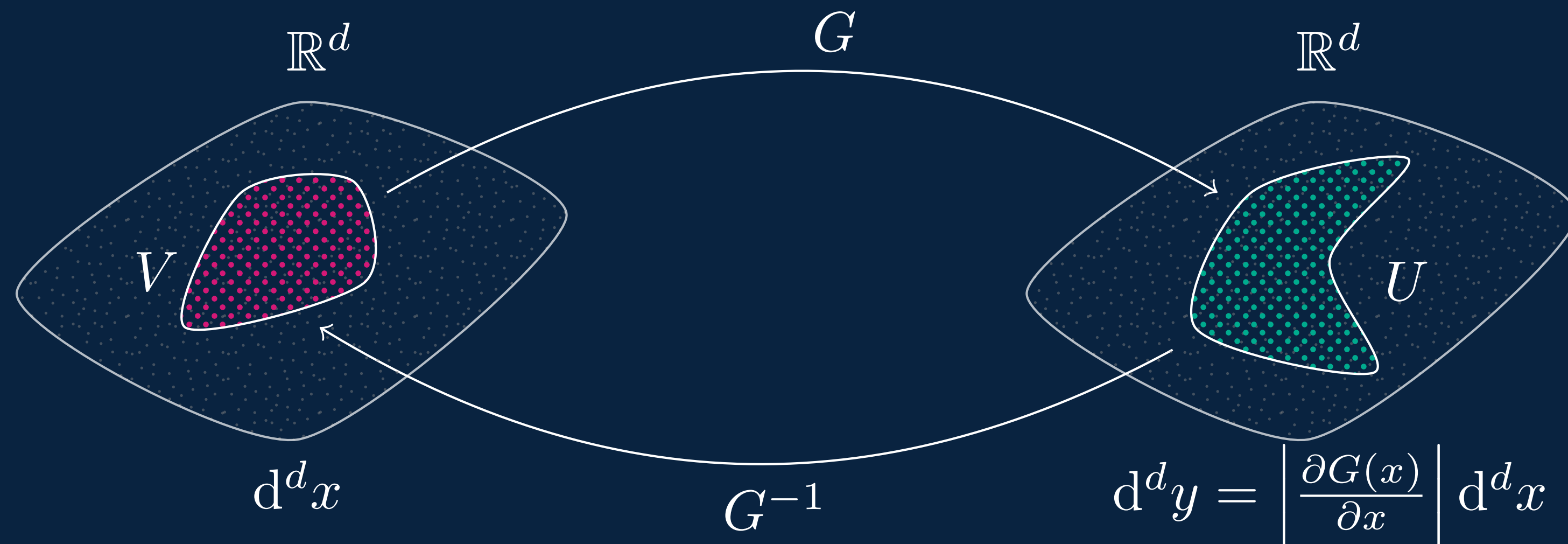
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$$y = G(x) \quad g(x) = \left| \frac{\partial G(x)}{\partial x} \right|$$



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Importance Sampling

$$I = \int_U d^d y \frac{f(x)}{g(x)} \Big|_{x=G^{-1}(y)} \simeq \langle f/g \rangle_y \quad \sigma_I \simeq \sqrt{\frac{\langle (f/g)^2 \rangle_y - \langle f/g \rangle_y^2}{N-1}}$$

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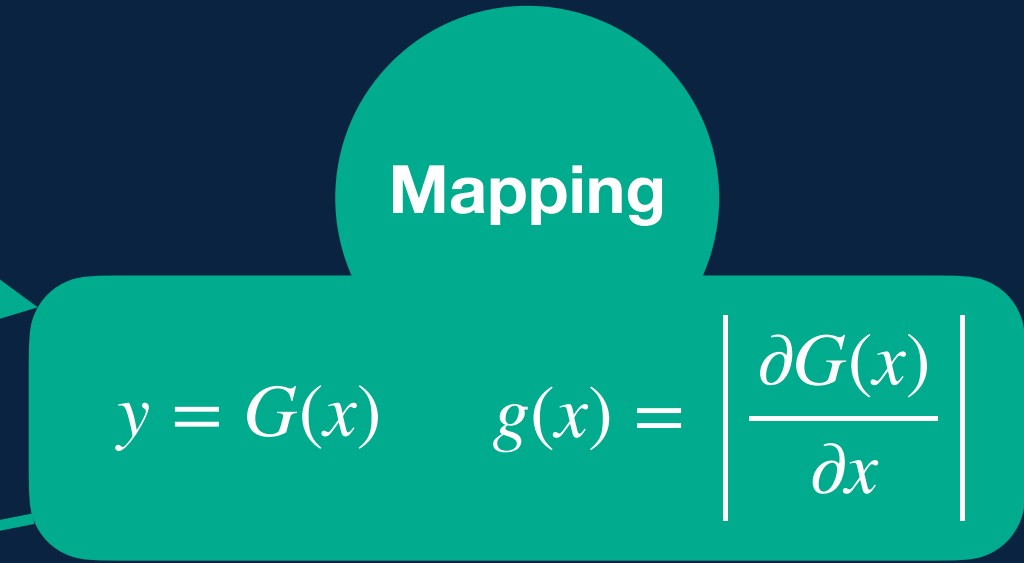
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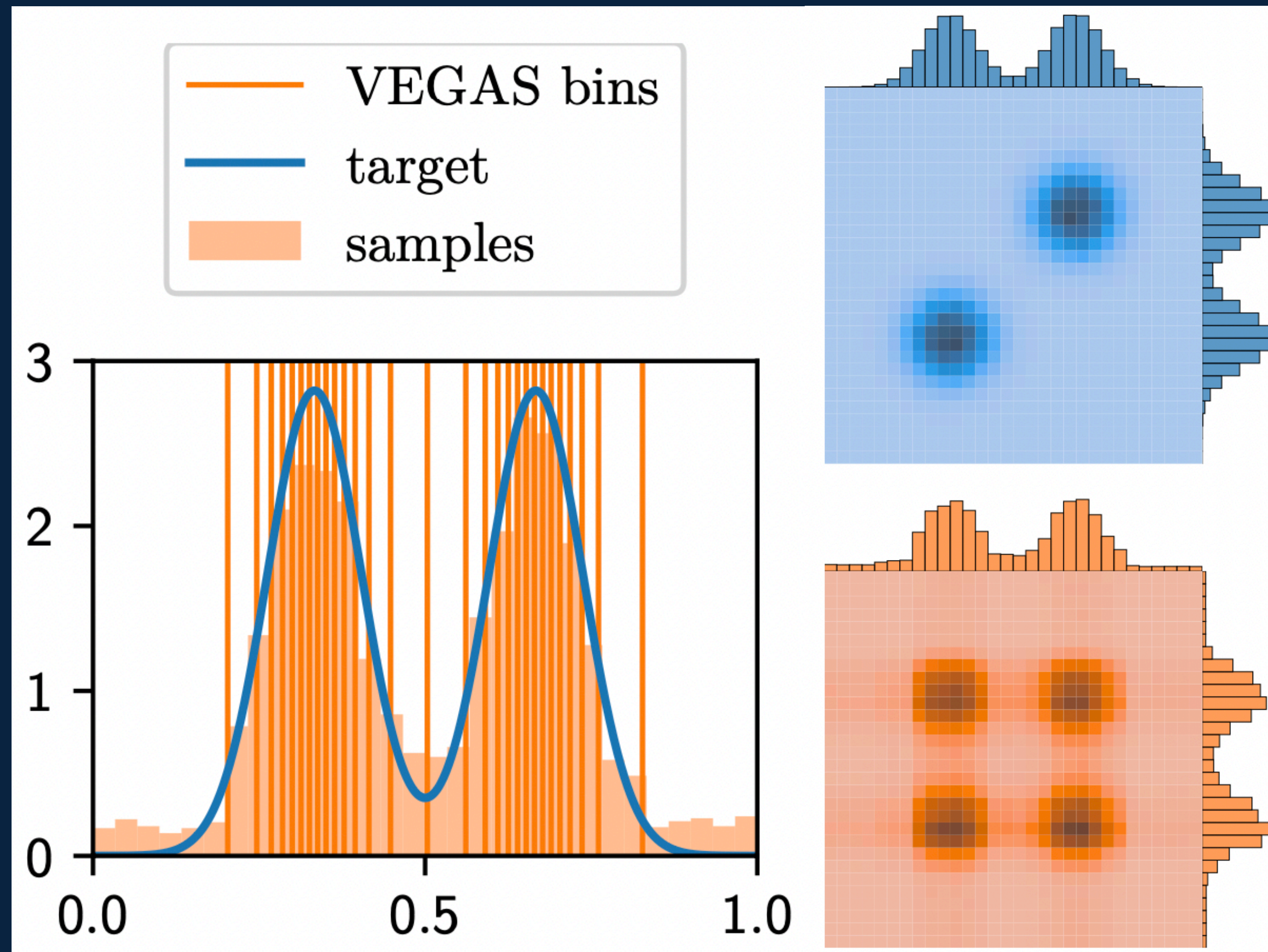
Wanted

$$\sigma_I \rightarrow 0$$

Needs

$$f/g \approx \text{const}$$

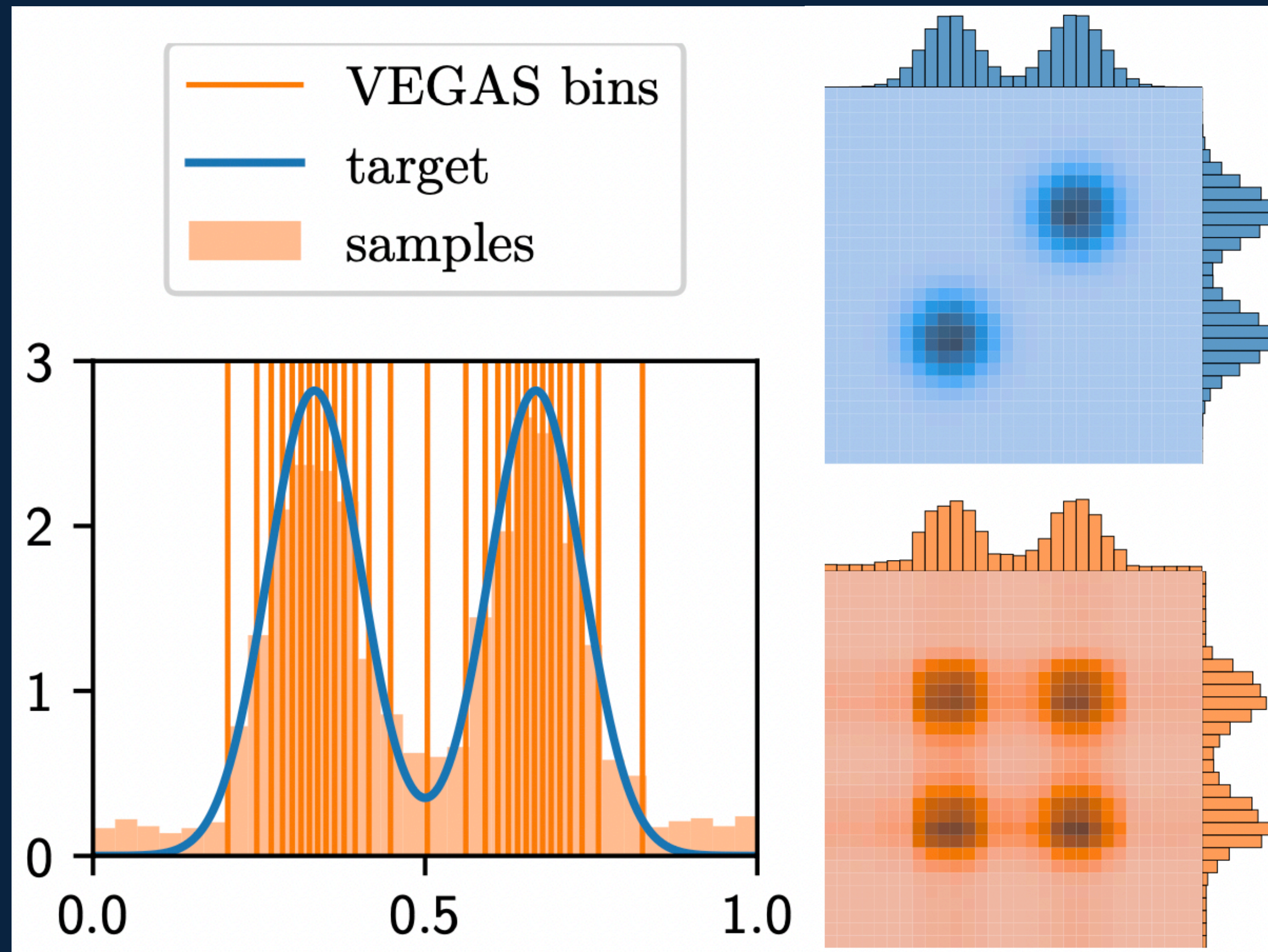
Importance sampling - VEGAS



Why not VEGAS for everything?

- High-dim and rich peaking
→ **slow convergence**
- If peaks are not aligned with grid axes
→ **"phantom peaks"**

Importance sampling - VEGAS



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Quality score?

- Unweighting efficiency

$$\varepsilon = \frac{\langle w \rangle}{w_{\max}}, \quad w = \frac{f}{g}$$

Importance sampling - NN

Using a NN

- Unbinned and no grids
 - no “phantom peaks”
- Bijectivity not guaranteed
 - training unstable
- Numerical Jacobians
 - slow training and evaluation

[1707.00028, 1810.11509, 2009.07819]

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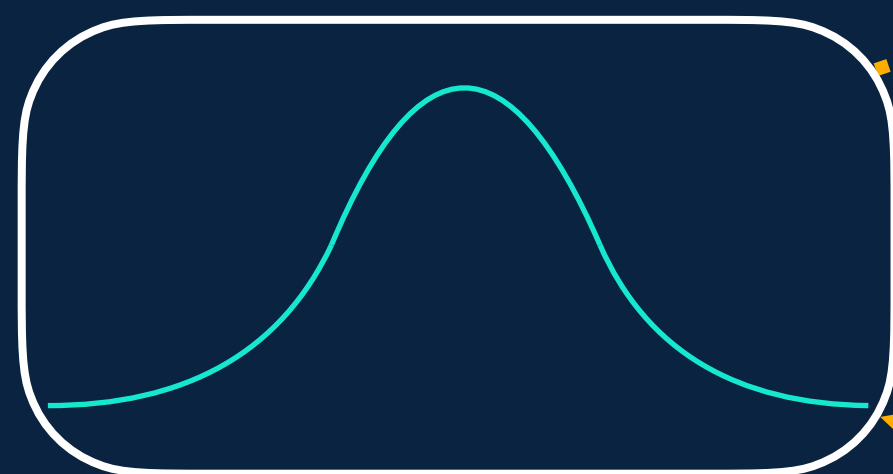


Using a Flow instead

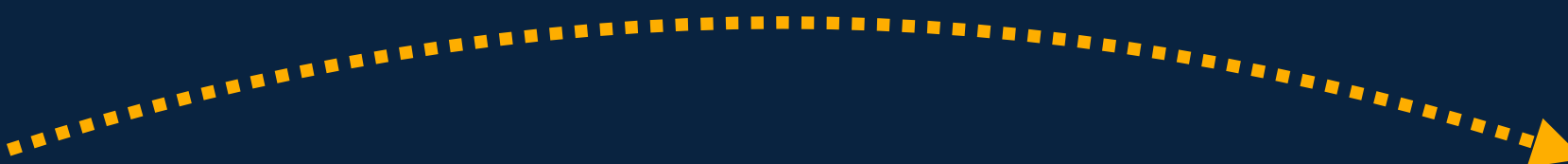
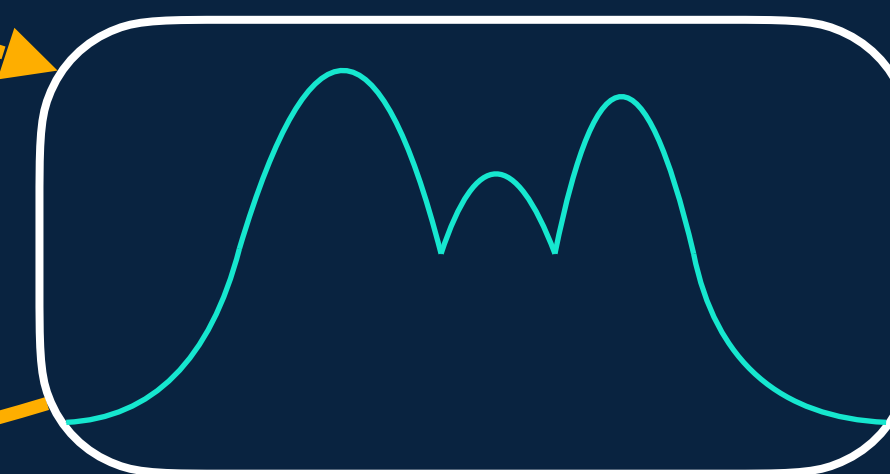
- Invertibility
→ bijective mapping
- tractable Jacobians
→ fast training and evaluation

[2001.05478, 2001.05486, 2001.10028, 2005.12719, 2112.09145]

Normalizing Flow



$$\log p_y(y) = \log p_x(x) + \log \left| \frac{\partial G(x)}{\partial x} \right|$$



Multi-Channel Monte Carlo

Standard Multi-Channel Integration $\longrightarrow g(x) = \sum_i \alpha_i g_i(x)$, with $\sum_i \alpha_i = 1$

$$I = \int_V d^d x f(x) = \sum_i \int_V d^d x \alpha_i g_i(x) \frac{f(x)}{g(x)} = \sum_i \int_{U_i} d^d y_i \alpha_i \frac{f(x)}{g(x)} \Big|_{x \equiv x(y_i)}$$

Channel Mappings

$$y_i = G_i(x) \quad g_i(x) = \left| \frac{\partial G_i(x)}{\partial x} \right|$$

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Connection

$$\beta_i(x) = \alpha_i \frac{g_i(x)}{g(x)}$$

Neural Multi-Channel Monte Carlo

Multi-Channel Importance Sampling

$$I = \sum_i \int_{\Omega} \alpha_i(x) \frac{f(x)}{g_i(x)} dG_i(x)$$

Neural Multi-Channel Monte Carlo

21

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n-dimensional remapping

- tractable Jacobian with **normalizing flow**

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→ *So far:* improvement factors of ~2-4 are achieved [2001.05486, 2001.05478, 2001.10028, 2112.09145]

Neural Multi-Channel Monte Carlo

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Possible
Improvements?

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k-dimensional regression

- with boundary condition

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Conditional Neural Channel Mappings

$$G_i^{\theta}(x) \rightarrow G^{\theta}(x|i), \quad g_i^{\theta}(x) \rightarrow g^{\theta}(x|i) = \left| \frac{\partial G^{\theta}(x|i)}{\partial x} \right|$$

n-dimensional remapping

- tractable Jacobian with **normalizing flow**
- **conditioned on channel**

→ So far: improvement factors of ~2-4 are achieved [2001.05486, 2001.05478, 2001.10028, 2112.09145]

Neural Multi-Channel Monte Carlo

Multi-Channel Importance Sampling

Further Improvements?

- Two-stage training
- Variance-weighted training

Neural Channel Splittings

- Overflow channels

$$\alpha_i(x) \rightarrow \alpha_i^\phi(x), \quad \sum \alpha_i^\phi(x) = 1$$

- Bayesian neural networks

Conditional Neural Channel Mappings

$$G_i^\theta(x) \rightarrow G^\theta(x|i), \quad g_i^\theta(x) \rightarrow g^\theta(x|i) = \left| \frac{\partial G^\theta(x|i)}{\partial x} \right|$$

- More expressive transformations: FFJORD, CubicSplines, LASER,...

• with boundary condition

[1810.0136]

[1906.02145]

le Jaco [2106.00792]

normalizing flow

• conditioned on channel

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Neural Multi-Channel Monte Carlo

Neural Channel Weights

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MadNIS

Neural Channel Splittings

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- k-dimensional regression
- with boundary condition

Additional Neural Channel Mappings

$$G_i^\theta(x) \rightarrow G^\theta(x|i), \quad g_i^\theta(x) \rightarrow g^\theta(x|i) = \left| \frac{\partial G^\theta(x|i)}{\partial x} \right|$$

- n-dimensional regression
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Two-Stage Training

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MadNIS

Neural Channel Weights

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Channel network

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MadGraph Channels

$$\beta_i(x) = \frac{|M_i(x)|^2}{\sum_j |M_j(x)|^2}$$

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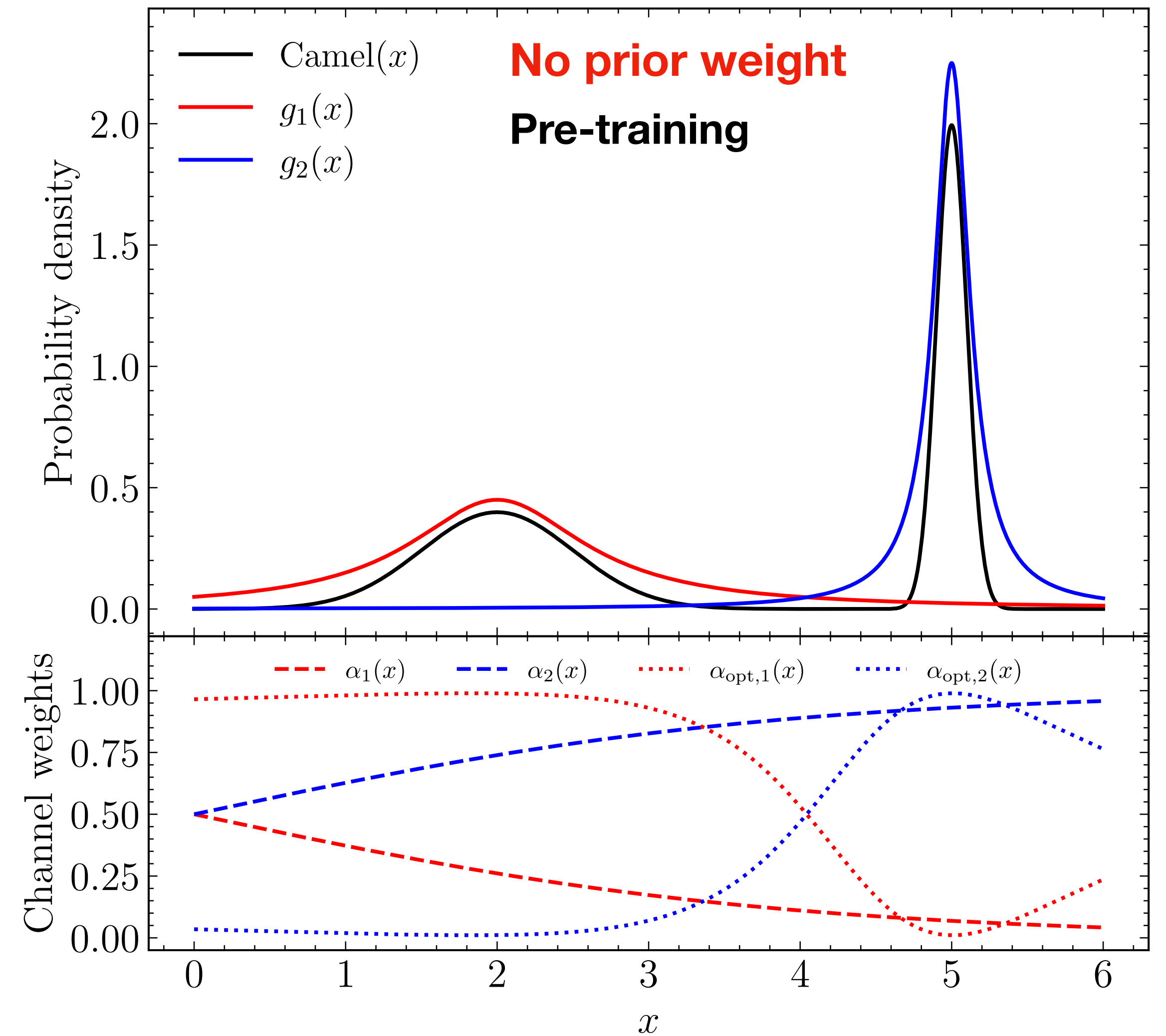
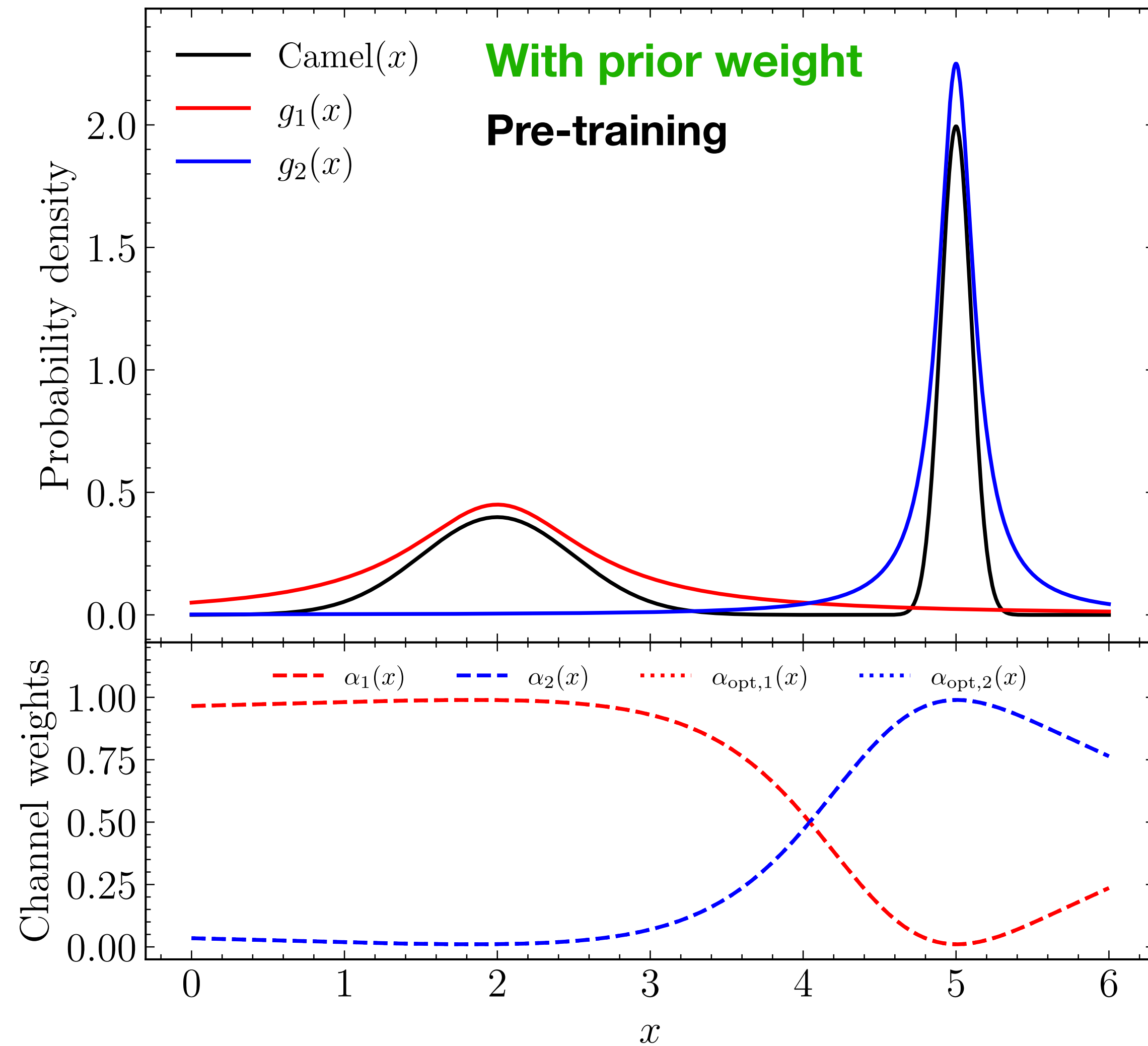
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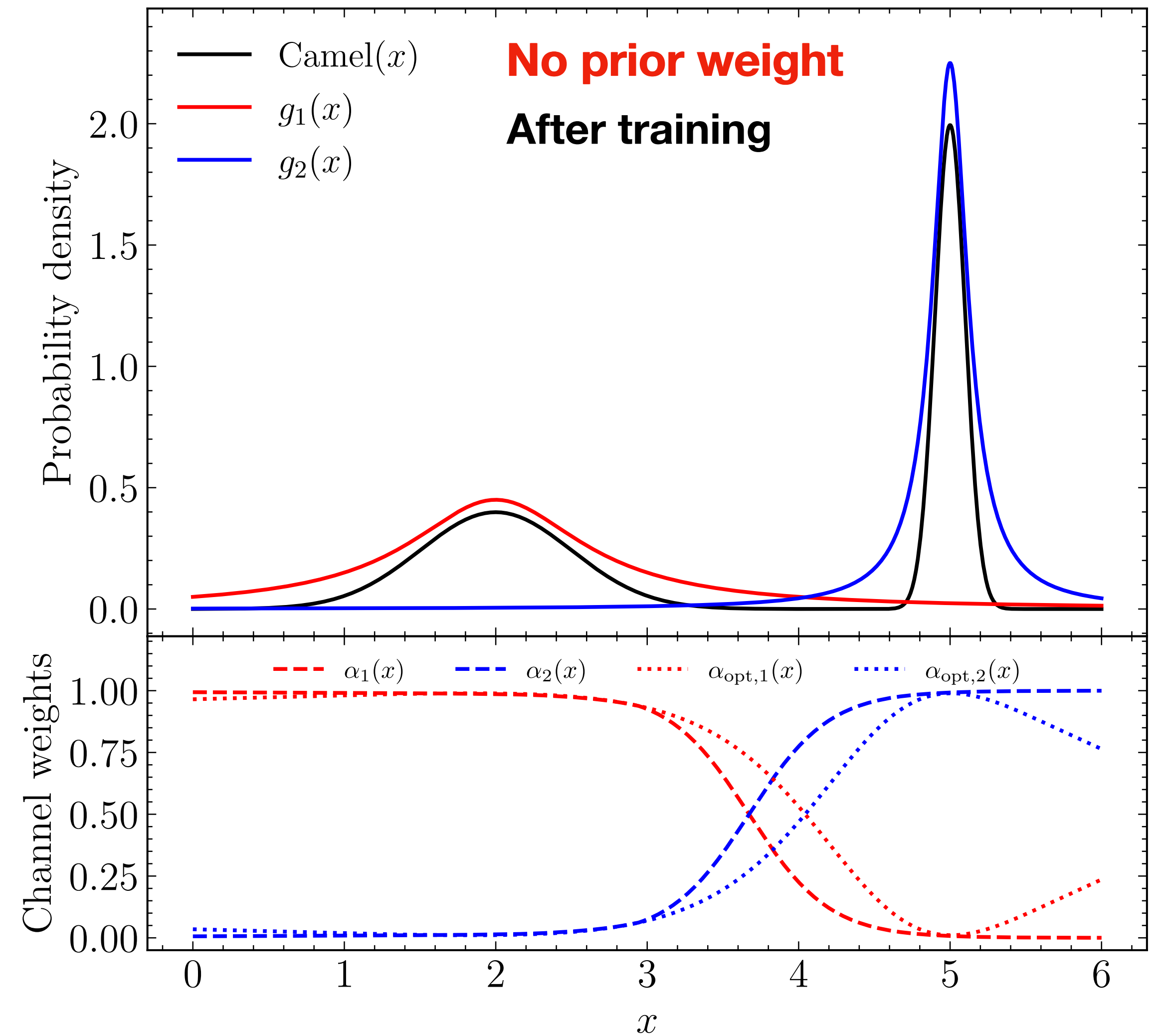
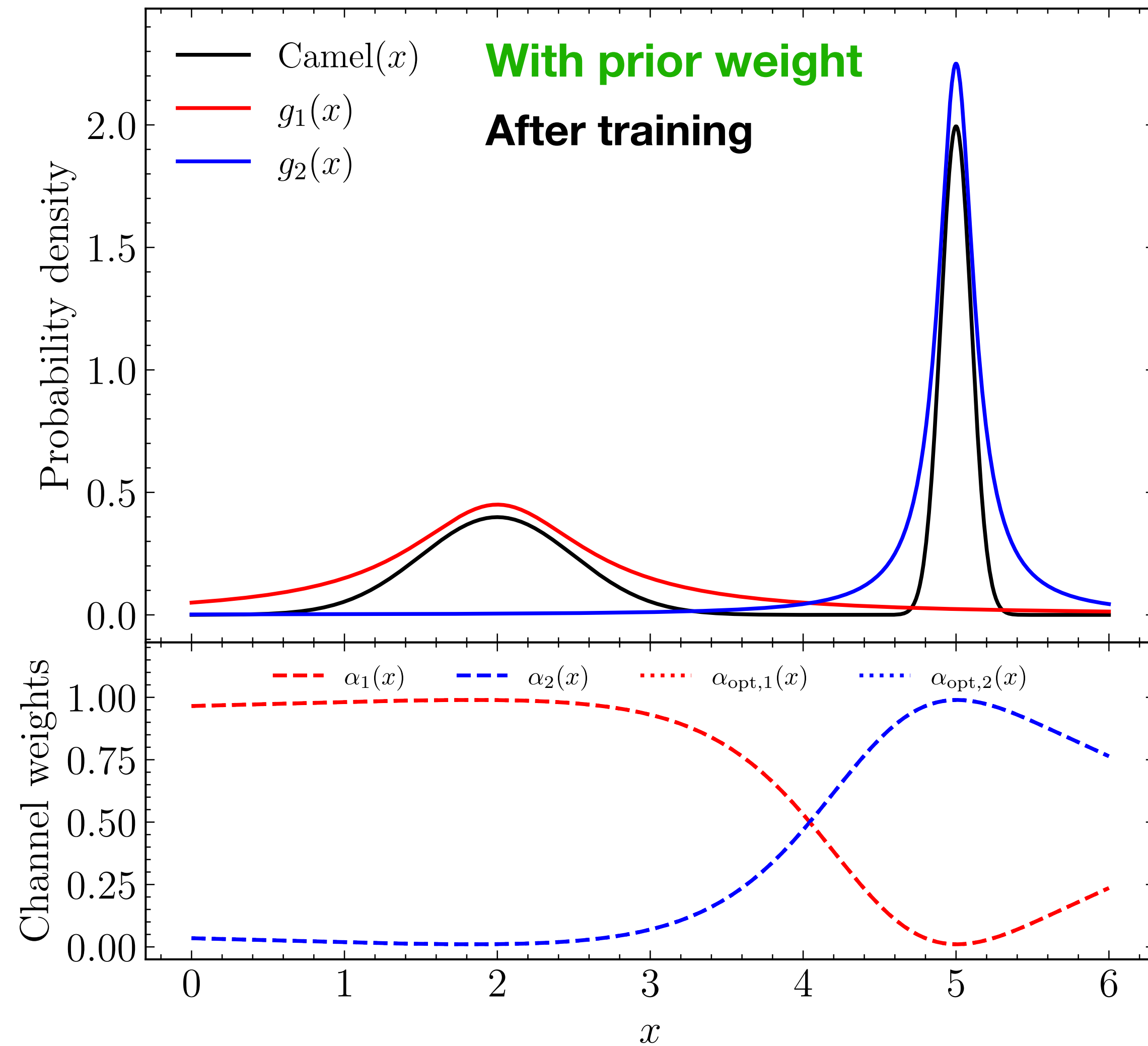
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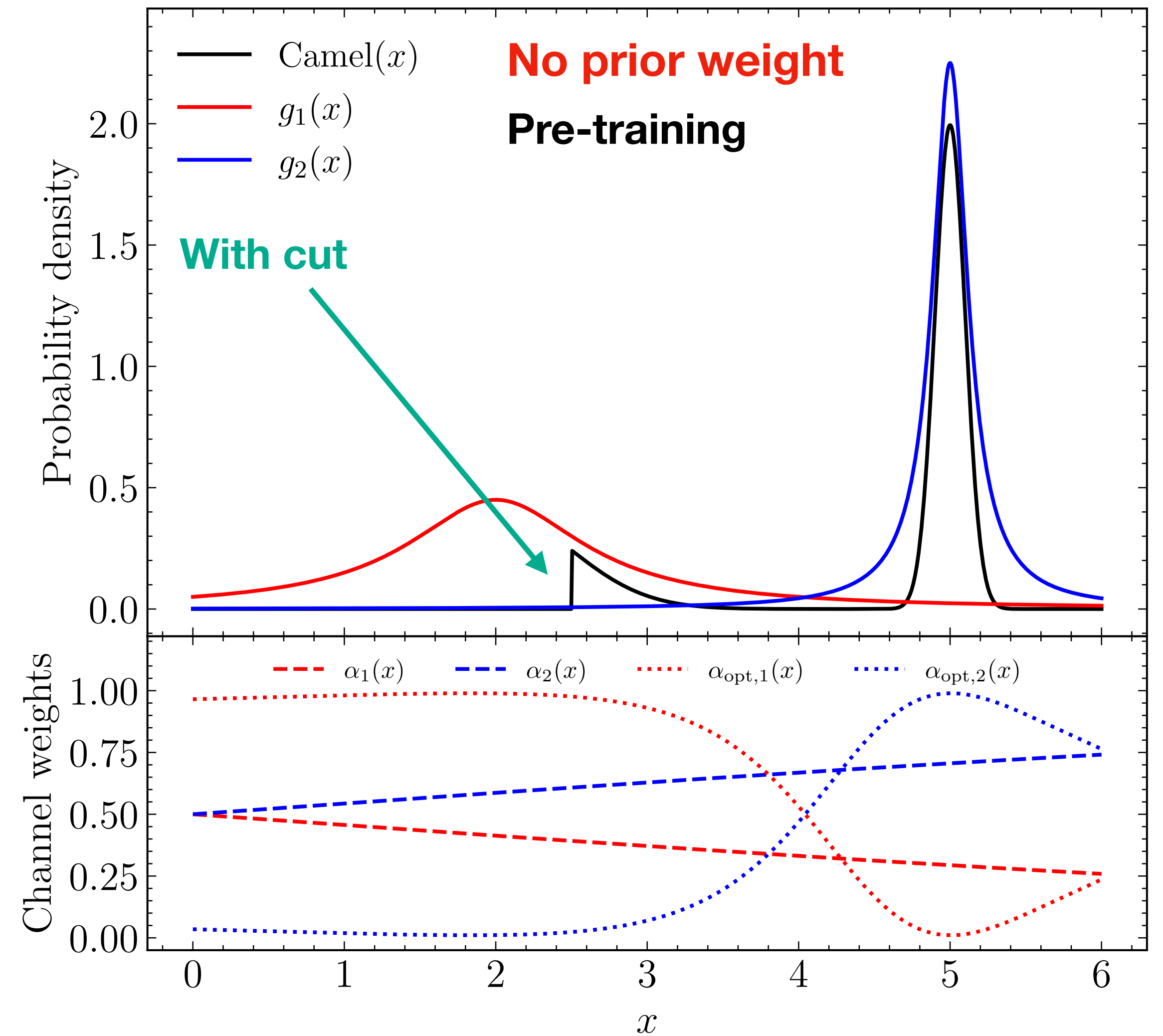
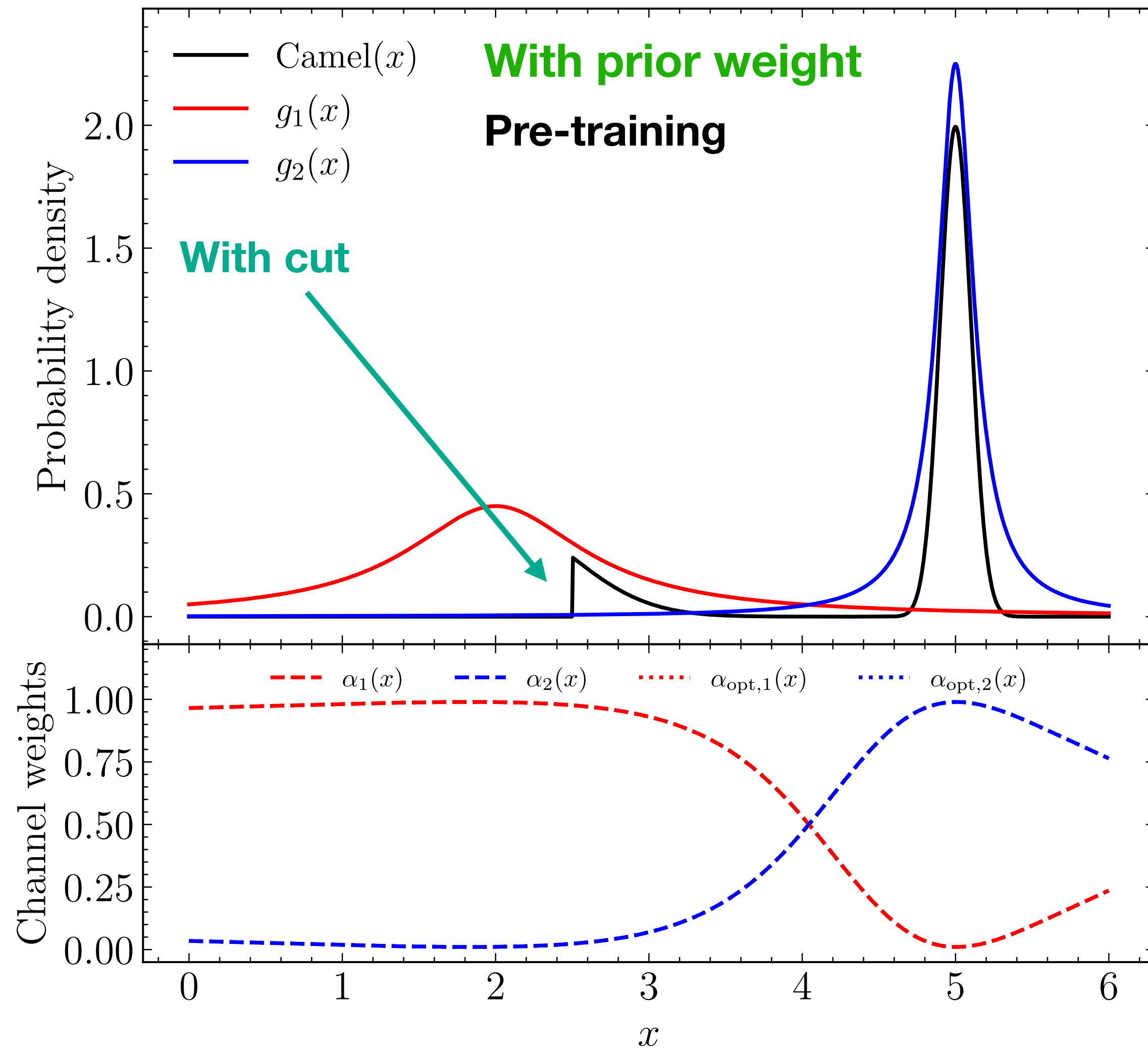
Neural Channel Weights



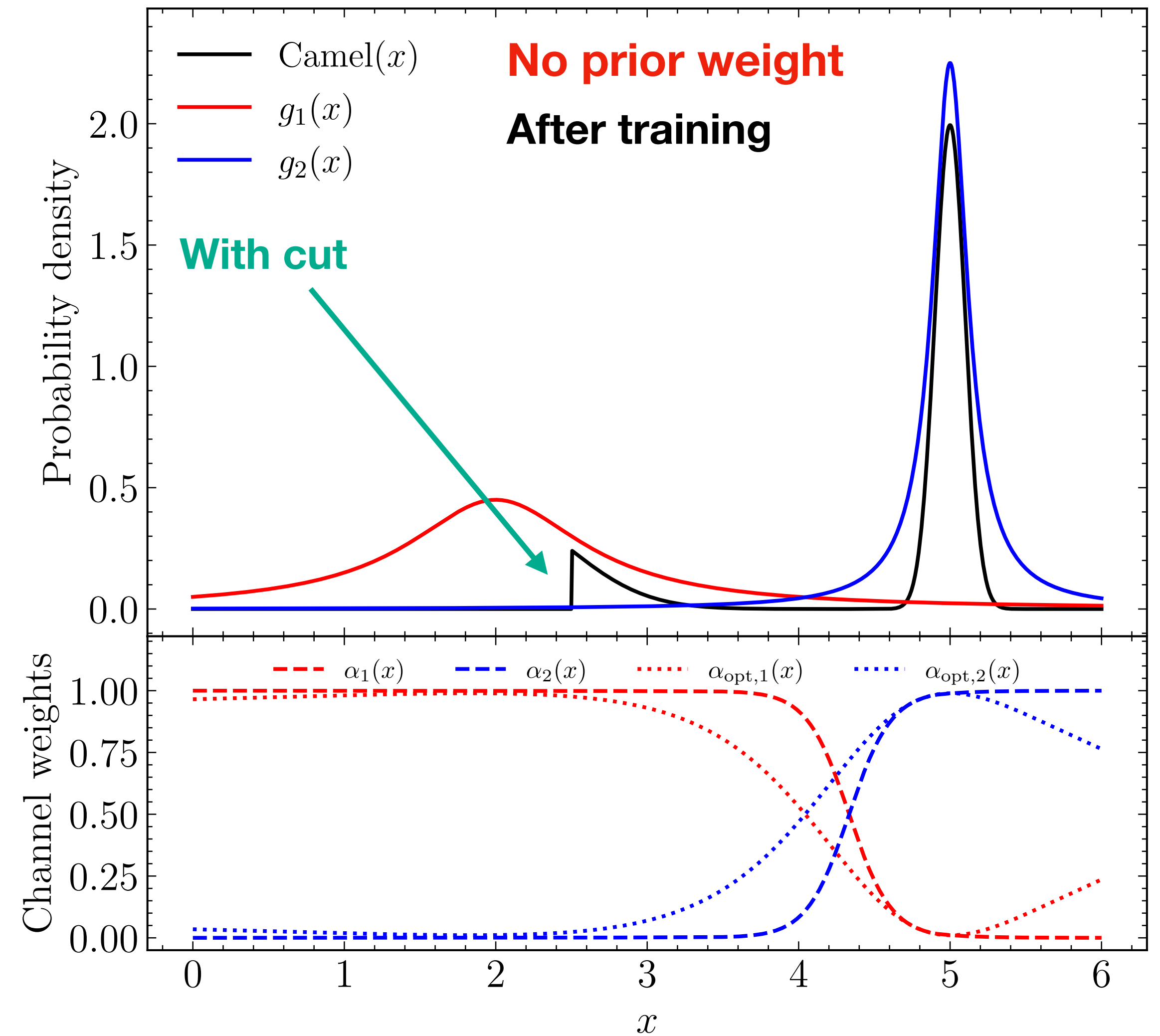
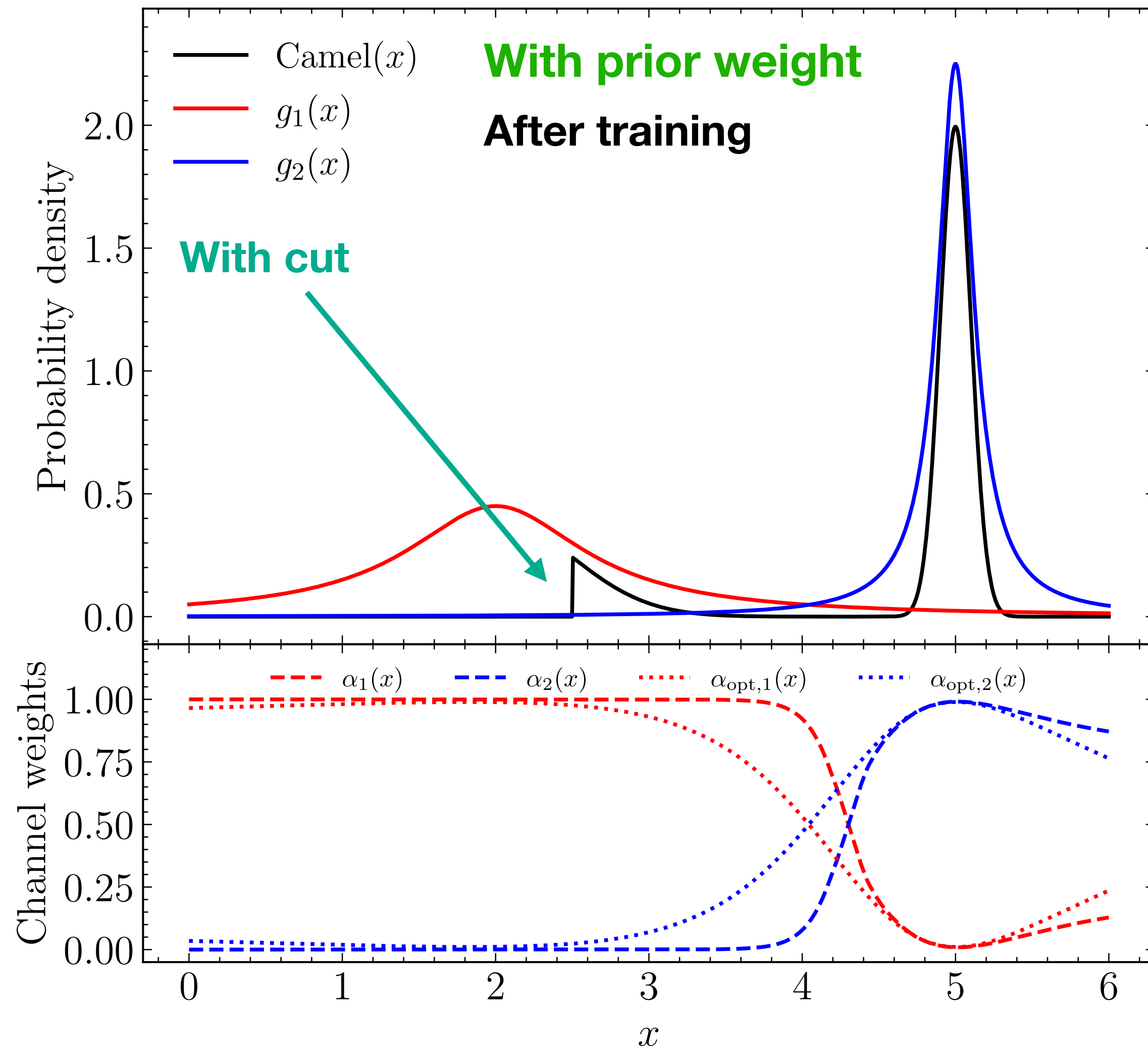
Neural Channel Weights



Neural Channel Weights



Neural Channel Weights



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Two-Stage Training

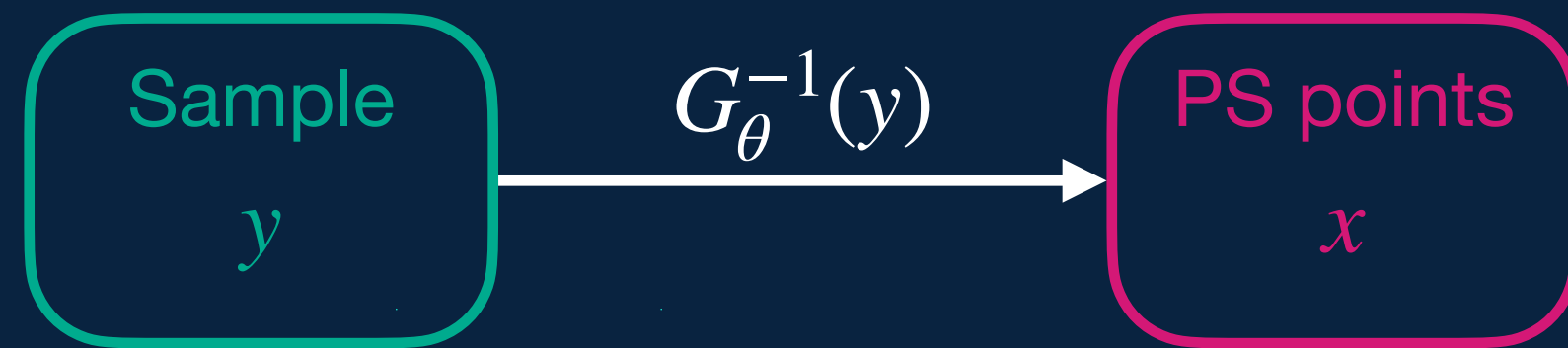
Generative training

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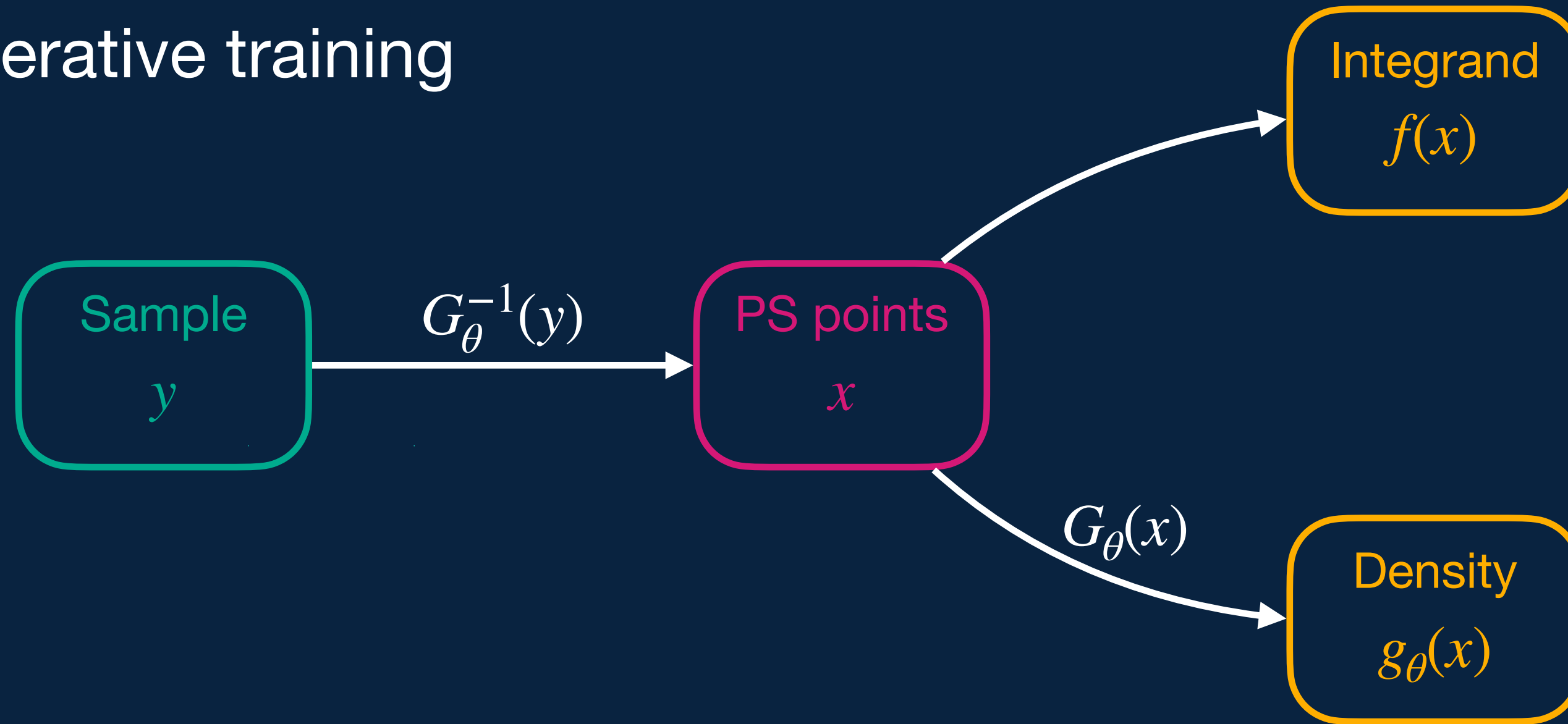
Sample

y

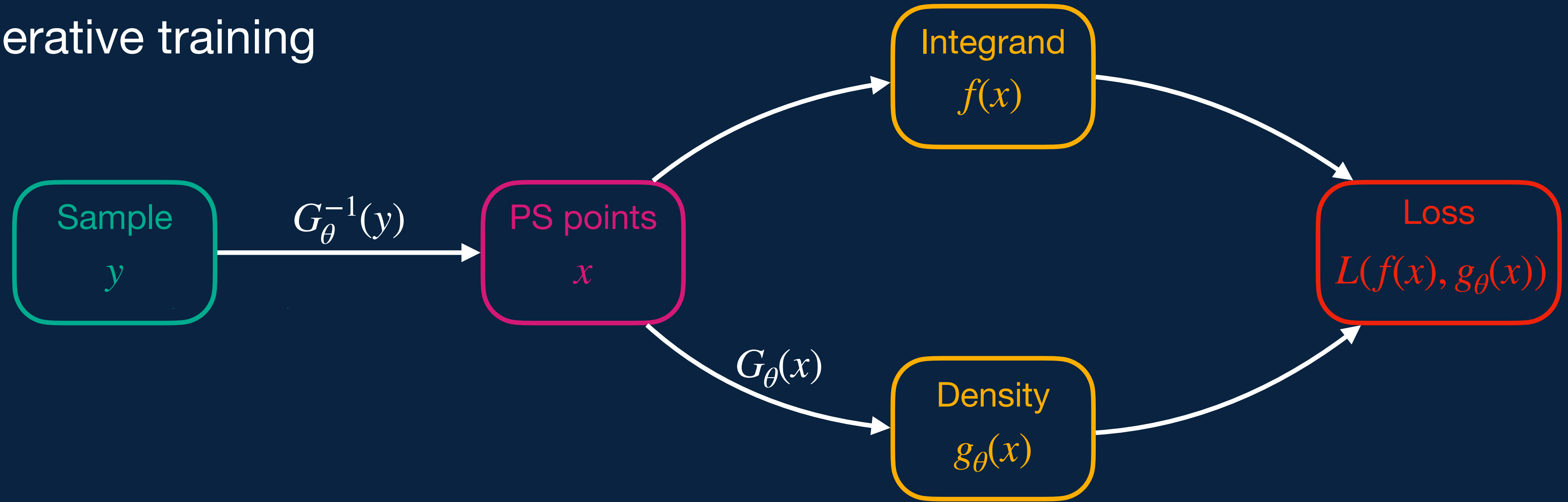
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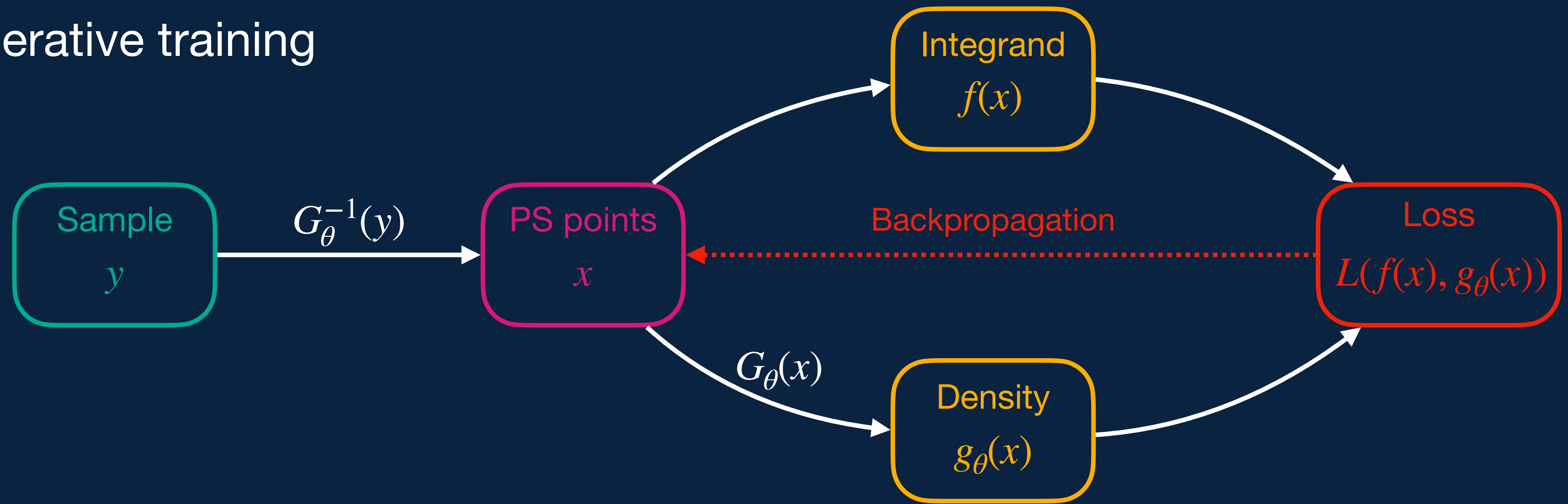
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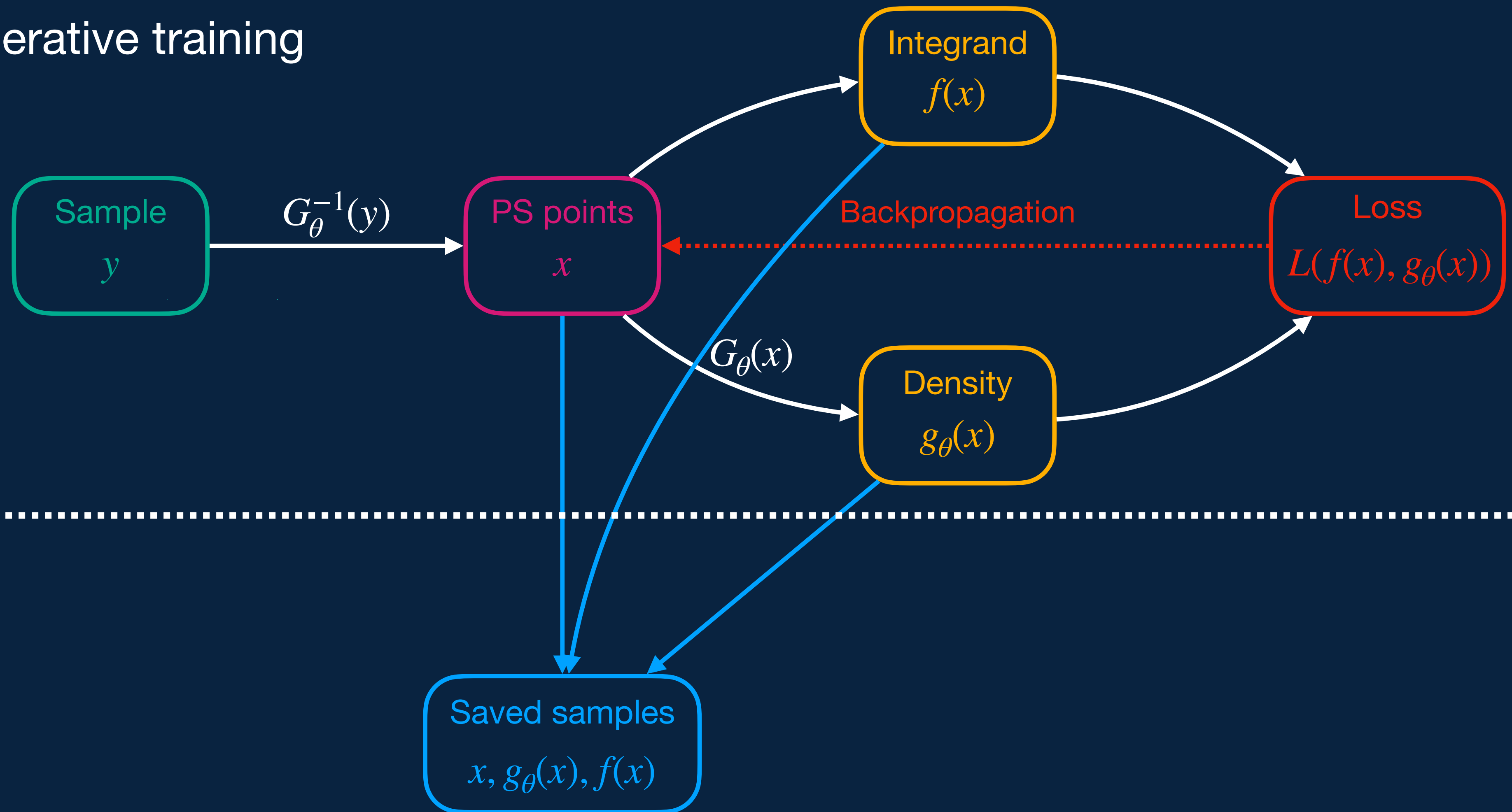
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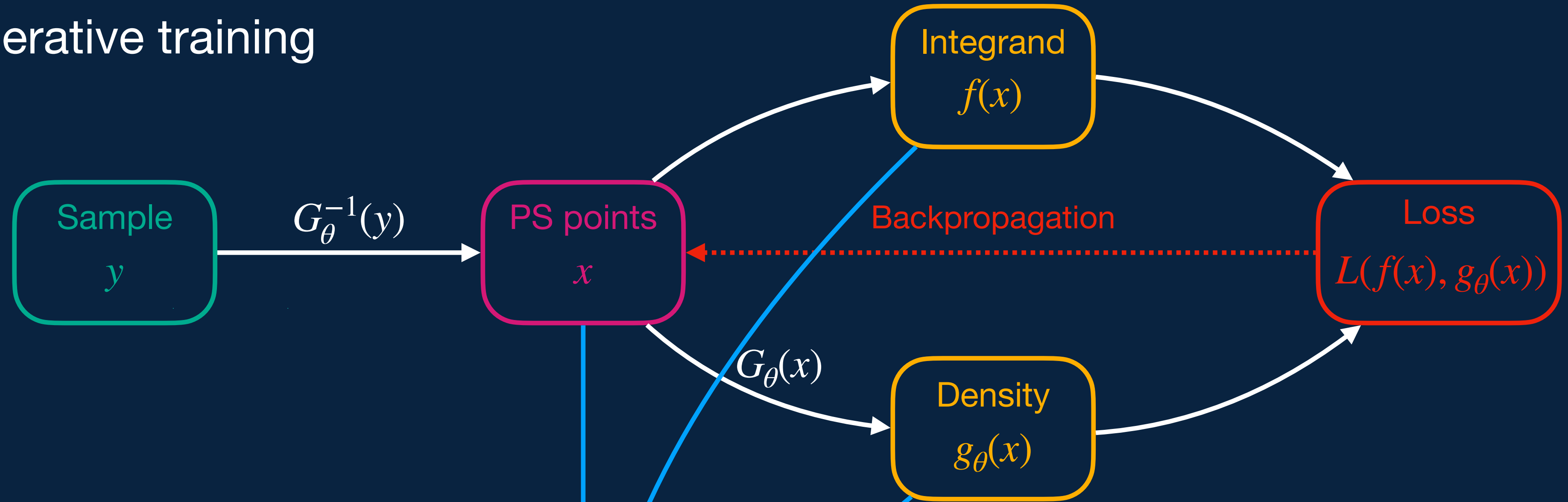
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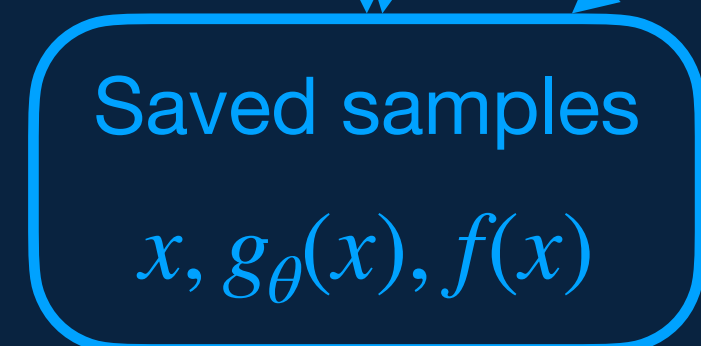
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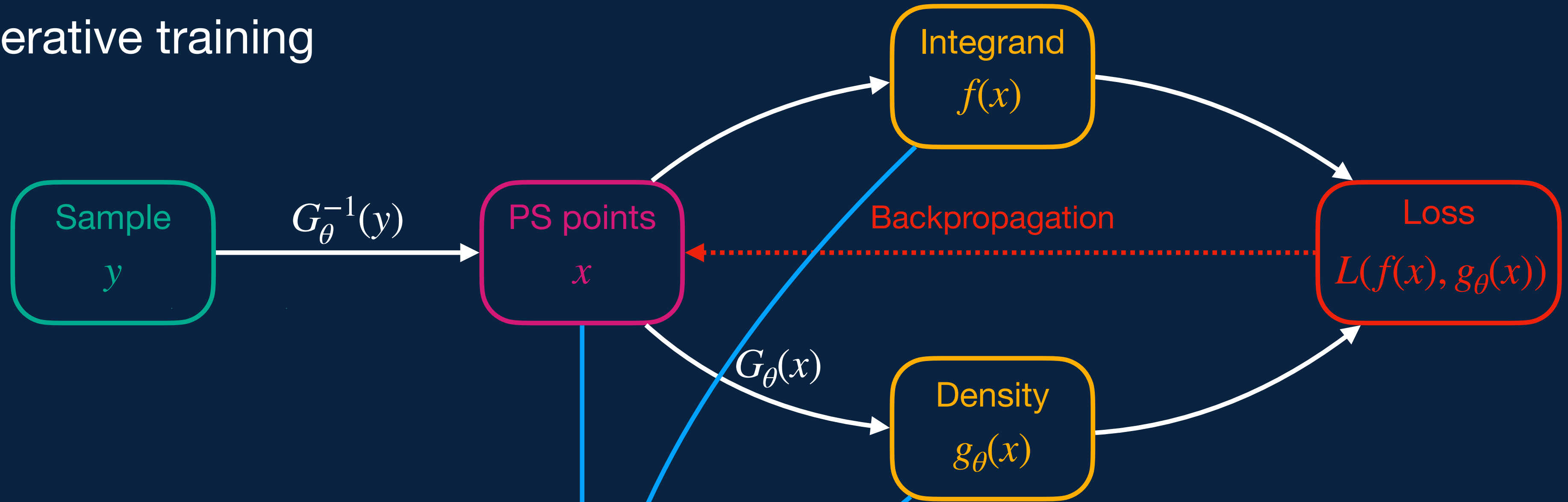
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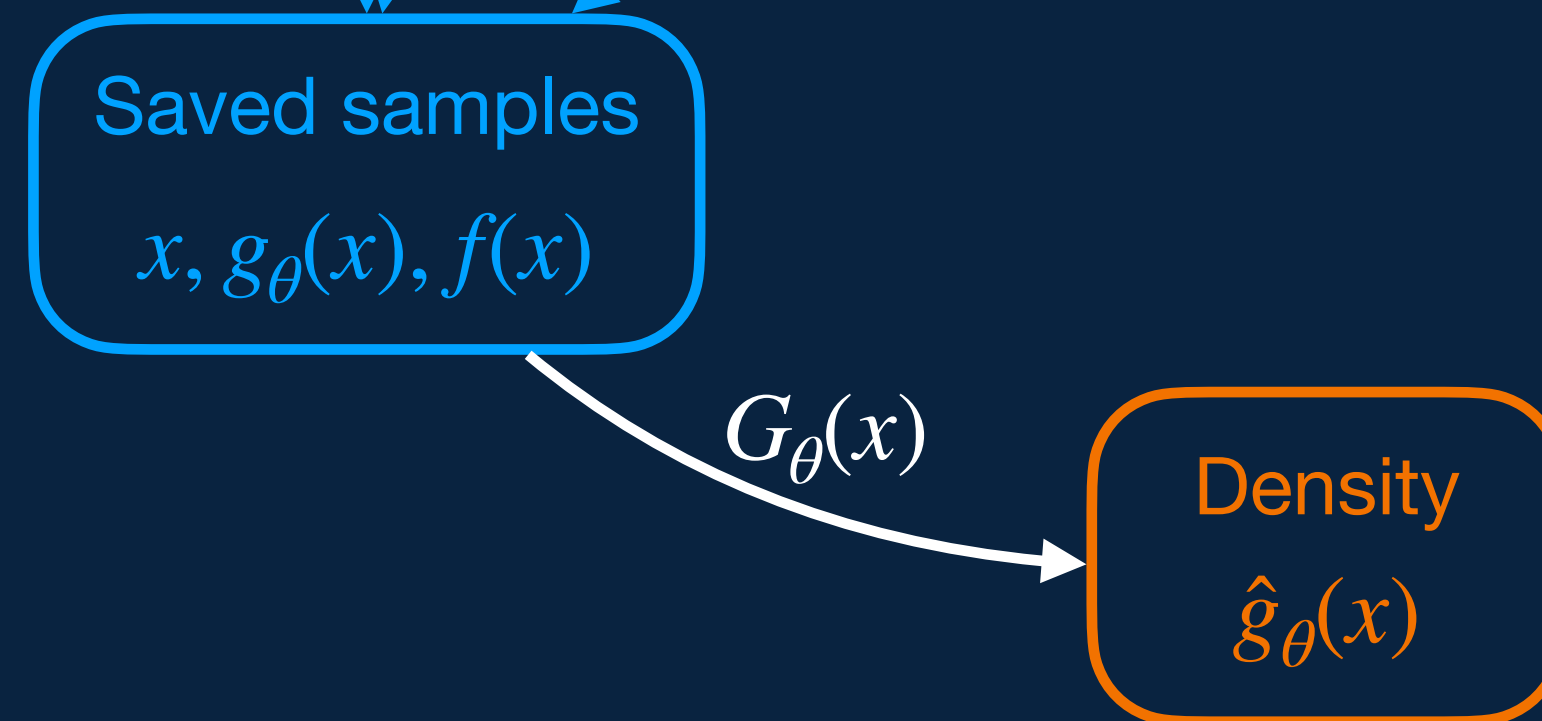
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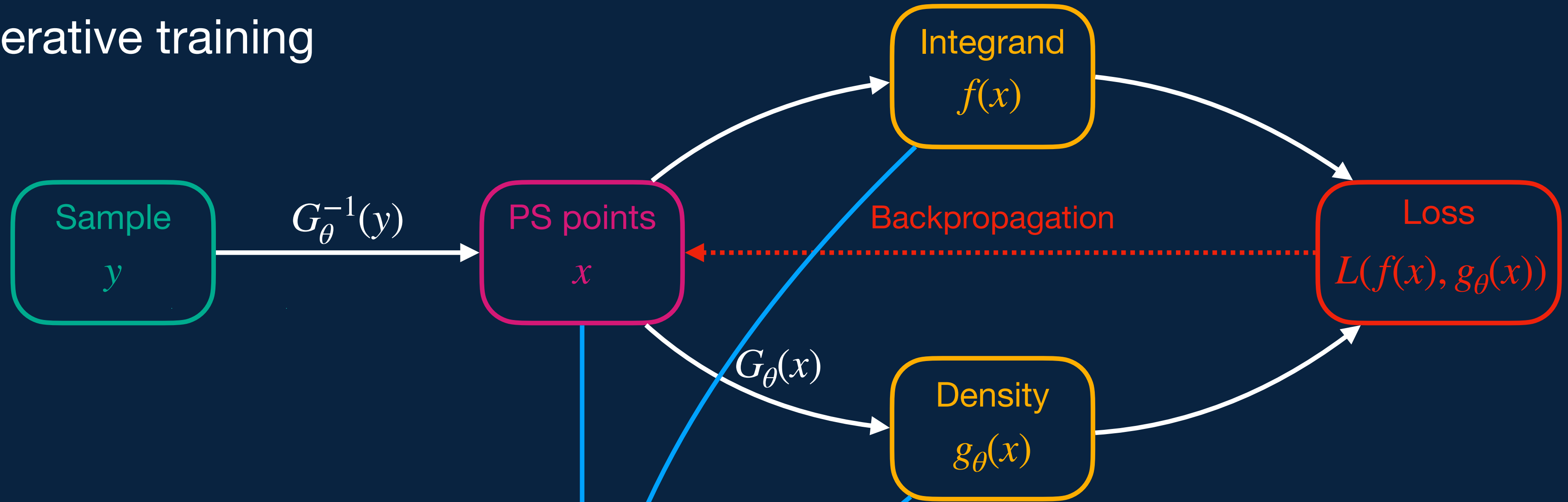
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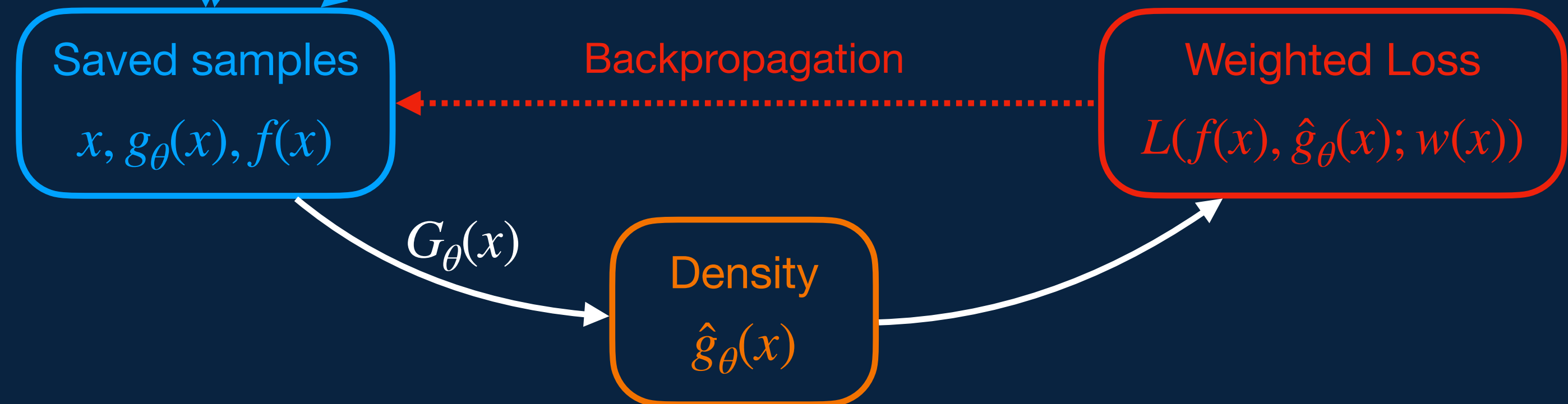
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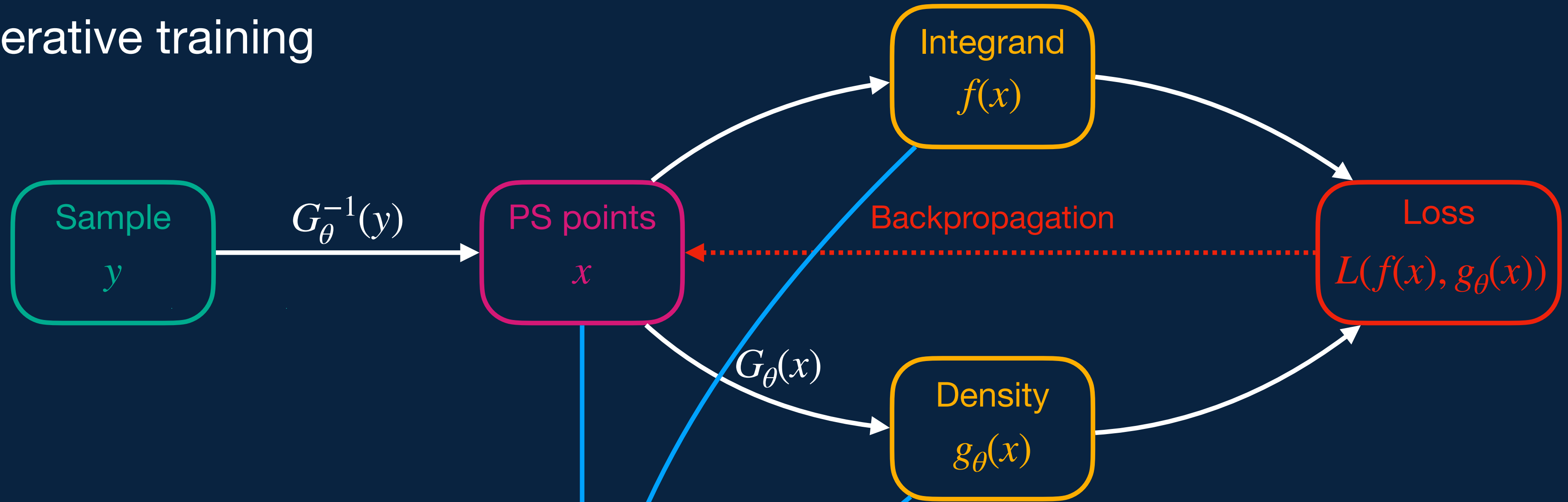
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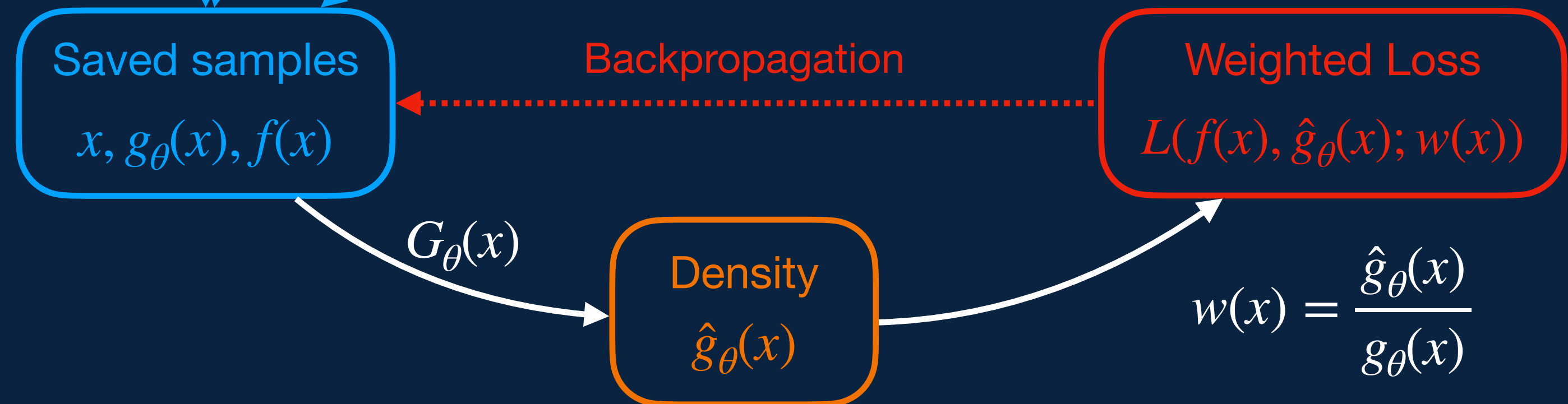
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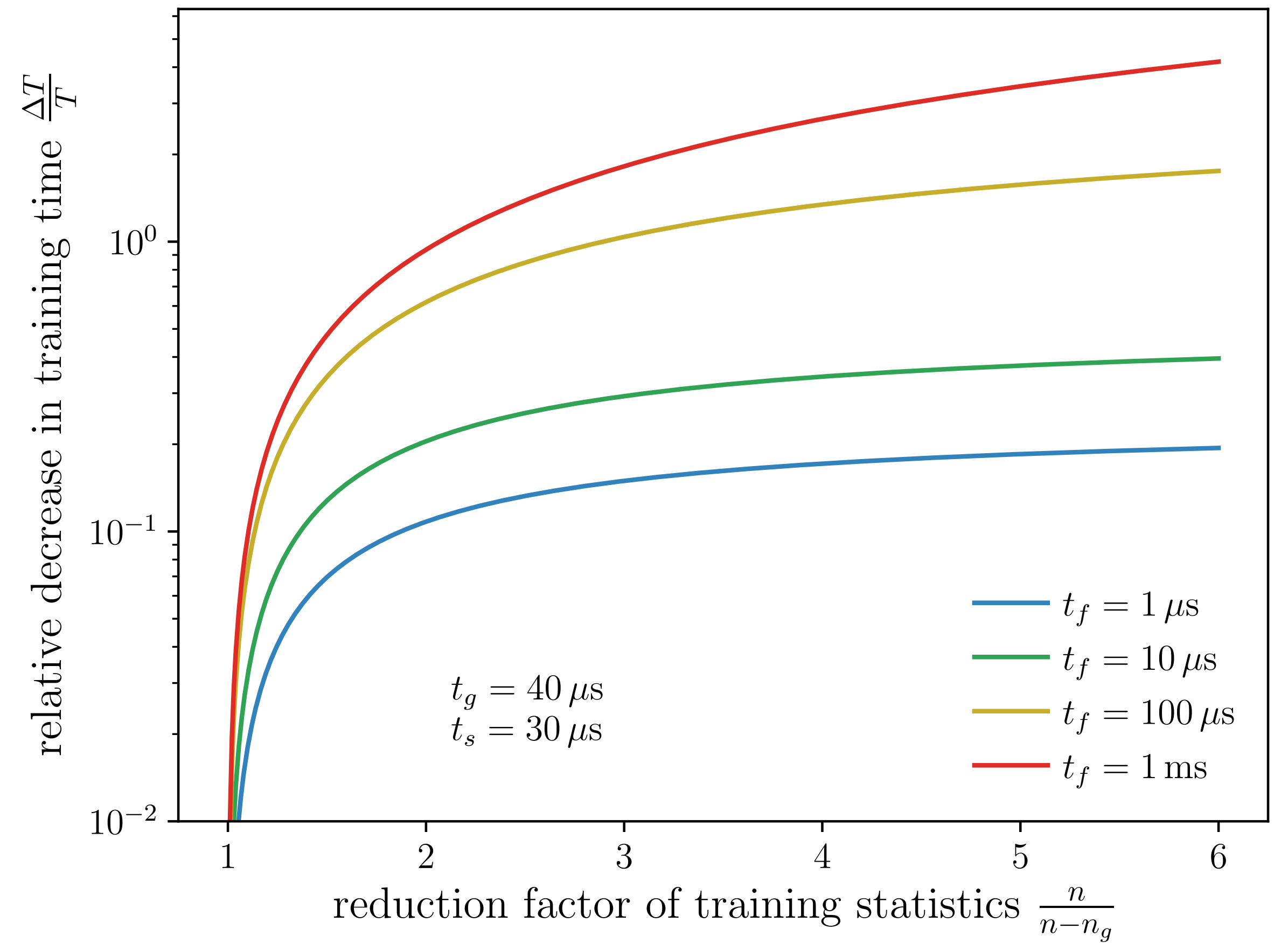
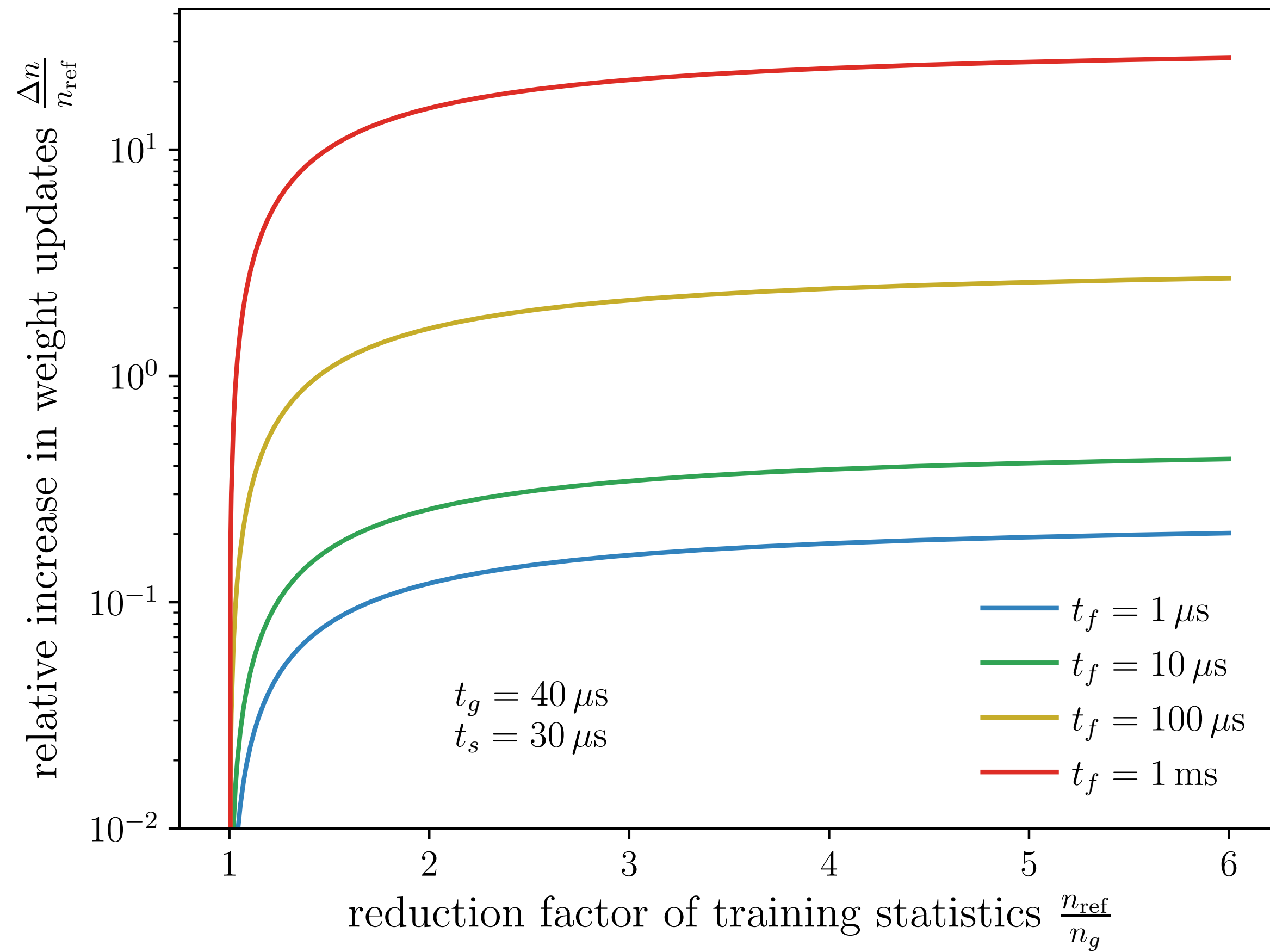


Sample training



Two-Stage Training

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Summary and Outlook

Summary

- ML can reduce the error in Monte Carlo Integration
- New developments can improve performance and reduce computational overhead

Summary and Outlook

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- ML can **reduce** the error in **Monte Carlo Integration**
- New developments can **improve performance** and **reduce computational overhead**
- Overview of ML in HEP:
<https://iml-wg.github.io/HEPML-LivingReview/>

HEPML-LivingReview

A Living Review of Machine Learning for Particle Physics

Modern machine learning techniques, including deep learning, is rapidly being applied, adapted, and developed for high energy physics. The goal of this document is to provide a nearly comprehensive list of citations for those developing and applying these approaches to experimental, phenomenological, or theoretical analyses. As a living document, it will be updated as often as possible to incorporate the latest developments. A list of proper (unchanging) reviews can be found within. Papers are grouped into a small set of topics to be as useful as possible. Suggestions are most welcome.

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The purpose of this note is to collect references for modern machine learning as applied to particle physics. A minimal number of categories is chosen in order to be as useful as possible. Note that papers may be referenced in more than one category. The fact that a paper is listed in this document does not endorse or validate its content - that is for the community (and for peer-review) to decide. Furthermore, the classification here is a best attempt and may have flaws - please let us know if (a) we have missed a paper you think should be included, (b) a paper has been misclassified, or (c) a citation for a paper is not correct or if the journal information is now available. In order to be as useful as possible, this document will continue to evolve so please check back before you write your next paper. If you find this review helpful, please consider citing it using `\cite{hepmlivingreview}` in HEPML.bib.



Summary and Outlook

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Summary

- ML can **reduce** the error in **Monte Carlo Integration**
- New developments can **improve performance** and **reduce computational overhead**
- Overview of ML in HEP:
<https://iml-wg.github.io/HEPML-LivingReview/>

Outlook

- Test performance of flow on **real LHC examples**: (eg. multi-leg, NLO, complicated cuts, ...)
- Possibly combine with other **methods** (eg. surrogate unweighting,...)
- Make matrix elements run on the **GPU and differentiable**

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