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# Speeding up SM Amplitude Calculations with Chirality Flow

**MCNET MEETING 2022 23 SEPTEMBER 2022 - ANDREW LIFSON**

**BASED ON HEP-PH:2003.05877 (EPJC), HEP-PH:2011.10075 (EPJC), AND HEP-PH:2203.13618 (EPJC)**

**IN COLLABORATION WITH JOAKIM ALNEFJORD, CHRISTIAN REUSCHLE, MALIN SJÖDAHL, AND ZENNY WETTERSTEN**



## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

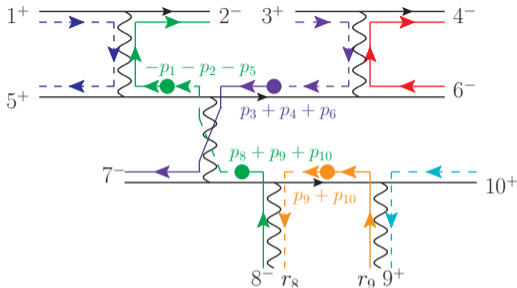
## Automation

Aim and method  
Results

## Conclusions

- 1 Introduction**
  - Spinor-helicity recap
  - Colour flow reminder
- 2 Chirality Flow**
  - Massless QED
  - Massless QCD
  - Massive Particles
- 3 Automation**
  - Aim and method
  - Results
- 4 Conclusions**

# Our Main Analytical Result



10-particle Feynman Diagram  
calculated in single slide

$$\begin{aligned}
 &= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{S_{1\ 2\ S_{3\ 4\ S_{7\ 8\ 9\ 10}}}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{S_{1\ 2\ 5\ S_{3\ 4\ 6\ S_{8\ 9\ 10\ S_{9\ 10}}}}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8r_8]\langle r_9 9 \rangle}}_{\text{polarization vectors}} [15]\langle 64 \rangle [10\ 9] \\
 &\times \left( \langle r_9 9 \rangle [9r_8] + \langle r_9 10 \rangle [10r_8] \right) \left( \underbrace{[33]\langle 37 \rangle + [34]\langle 47 \rangle + [36]\langle 67 \rangle}_0 \right) \\
 &\times \left( -\langle 89 \rangle [91]\langle 12 \rangle - \langle 89 \rangle [95]\langle 52 \rangle - \langle 8\ 10 \rangle [10\ 1]\langle 12 \rangle - \langle 8\ 10 \rangle [10\ 5]\langle 52 \rangle \right)
 \end{aligned}$$

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions



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# Our Main Numerical Result (so far) (hep-ph:2203.13618)

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

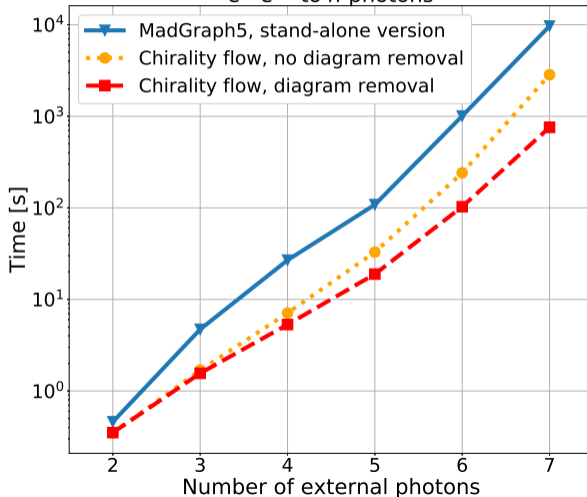
Aim and method  
Results

## Conclusions



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Evaluation time for 100 000 matrix elements for  $e^+e^-$  to  $n$  photons



# Spinor-Helicity: its Building Blocks

## Introduction

Spinor-helicity recap

Colour flow reminder

## Chirality Flow

Massless QED

Massless QCD

Massive Particles

## Automation

Aim and method

Results

## Conclusions



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Lorentz algebra  $so(3, 1) \cong su(2) \oplus su(2)$   
Consider massless particles: chirality  $\sim$  helicity

Spinors (use chiral basis):

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix} \quad u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = ( [p| \quad 0 ) \quad \bar{u}^-(p) = \bar{v}^+(p) = ( 0 \quad \langle p| )$$

- Amplitude written in terms of Lorentz-invariant spinor inner products

$$\langle ij \rangle = -\langle ji \rangle \equiv \langle i||j \rangle \quad \text{and} \quad [ij] = -[ji] \equiv [i||j]$$

- These are well known complex numbers,  $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

# Spinor-Helicity: Vectors and Removing $\mu$ Indices

## Introduction

Spinor-helicity recap

Colour flow reminder

## Chirality Flow

Massless QED

Massless QCD

Massive Particles

## Automation

Aim and method

Results

## Conclusions



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$$\text{Lorentz algebra } so(3, 1) \cong su(2) \oplus su(2)$$

Dirac matrices in chiral basis

$$\gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\tau^\mu \\ \sqrt{2}\bar{\tau}^\mu & 0 \end{pmatrix} \quad \sqrt{2}\tau^\mu = (1, \vec{\sigma}), \quad \sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma}),$$

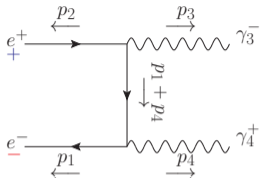
Remove vector indices with e.g.

$$\underbrace{\langle i | \bar{\tau}^\mu | j \rangle [k | \tau_\mu | l \rangle}_{\text{Fierz identity}} = \langle il \rangle [kj], \quad \underbrace{\sqrt{2}p^\mu \tau_\mu \equiv \not{p} = |p\rangle \langle p|}_{\text{Contraction with Pauli}}$$

Polarisation vectors ( $r \equiv$  gauge choice,  $r^2 = 0$ ,  $r \cdot p \neq 0$ ):

$$\not{\epsilon}_+(p, r) = \frac{|p\rangle \langle r|}{\langle rp \rangle}, \quad \not{\epsilon}_-(p, r) = \frac{|r\rangle \langle p|}{[pr]}$$

# An Illuminating Example: $e^+ e^- \rightarrow \gamma\gamma$



- $|p\rangle \equiv$  right-chiral spinor
- $|p] \equiv$  left-chiral spinor
- $\tau^\mu, \bar{\tau}^\mu \equiv$  Pauli matrices
- $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

## Spinor helicity: analytic

$$\begin{aligned}
 & \sim \langle p_1 | \bar{\tau}^\mu \underbrace{(|p_1\rangle\langle p_1| + |p_4\rangle\langle p_4|)}_{\not{p}_1 + \not{p}_4} \bar{\tau}^\nu | p_2 \rangle \underbrace{\frac{\langle r_3 | \bar{\tau}_\nu | p_3 \rangle}{\langle r_3 3 \rangle}}_{\epsilon_3^-} \underbrace{\frac{[r_4 | \tau_\mu | p_4 \rangle}{[4r_4]}}_{\epsilon_4^+} \\
 & = \frac{(\langle p_1 | \bar{\tau}^\mu | p_1 \rangle + \langle p_1 | \bar{\tau}^\mu | p_4 \rangle) [r_4 | \tau_\mu | p_4 \rangle (\langle p_1 | \bar{\tau}^\nu | p_2 \rangle + \langle p_4 | \bar{\tau}^\nu | p_2 \rangle) [p_3 | \tau_\nu | r_3 \rangle]}{\langle r_3 3 \rangle [4r_4]} \\
 & = \frac{\langle 1r_4 \rangle ([41]\langle 13 \rangle + [44]\langle 43 \rangle) [r_3 2]}{\langle r_3 3 \rangle [4r_4]} = \frac{\langle 1r_4 \rangle [41]\langle 13 \rangle [r_3 2]}{\langle r_3 3 \rangle [4r_4]} \\
 & \text{Fierz identities like } \langle i | \bar{\tau}^\mu | j \rangle [k | \tau_\mu | l \rangle = \langle il \rangle [kj] \qquad [ii] = 0
 \end{aligned}$$

### Introduction

Spinor-helicity recap

Colour flow reminder

### Chirality Flow

Massless QED

Massless QCD

Massive Particles

### Automation

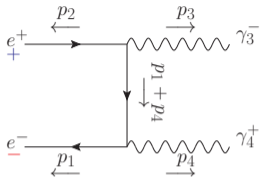
Aim and method

Results

### Conclusions



# An Illuminating Example: $e^+ e^- \rightarrow \gamma\gamma$



- $|p\rangle \equiv$  right-chiral spinor
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- $\tau^\mu, \bar{\tau}^\mu \equiv$  Pauli matrices
- $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$

## Spinor helicity: explicit matrix multiplication

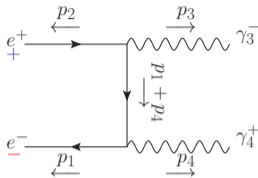
$$\sim [\bar{u}^-(p_1) \gamma^\mu \epsilon_\mu^+(p_4) (p_1^\nu + p_4^\nu) \gamma_\nu \gamma^\rho \epsilon_\rho^-(p_3) v^+(p_2)]$$

- Also cache and recycle various components
- Most common numerical method





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## Spinor helicity: explicit matrix multiplication

$$\sim [\bar{u}^-(p_1) \gamma^\mu \epsilon_\mu^+(p_4) (p_1^\nu + p_4^\nu) \gamma_\nu \gamma^\rho \epsilon_\rho^-(p_3) v^+(p_2)]$$

- Also cache and recycle various components
- Most common numerical method

Can we systematically remove need for algebra or matrix multiplication?

### Introduction

Spinor-helicity recap

Colour flow reminder

### Chirality Flow

Massless QED

Massless QCD

Massive Particles

### Automation

Aim and method

Results

### Conclusions



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# Chirality Flow Building Blocks

**Key idea:**  $su(2) = su(N)$  (hep-ph:2003.05877)

Draw & connect lines to directly obtain inner products  $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$   
Removes need to do algebra or matrix multiplication

## Define spinors as lines

$$\bar{u}_i^- = \bar{v}_i^+ = \langle i |^\alpha = \text{●} \longleftarrow i, \quad u_j^+ = v_j^- = |j\rangle_\alpha = \text{●} \longrightarrow j$$

$$\bar{u}_i^+ = \bar{v}_i^- = [i]_\beta = \text{●} \cdots\cdots\cdots i, \quad u_j^- = v_j^+ = |j]^\beta = \text{●} \cdots\cdots\cdots j$$

## Spinor inner products follow

$$\langle i |^\alpha |j\rangle_\alpha \equiv \langle ij \rangle = -\langle ji \rangle = i \longrightarrow j$$

$$[i]_\beta |j]^\beta \equiv [ij] = -[ji] = i \cdots\cdots\cdots j$$

## Define slashed momentum as dot

$$\not{p} \equiv \sqrt{2} p^\mu \tau_\mu^{\dot{\alpha}\beta} = \cdots\cdots\cdots \text{●} \longrightarrow p, \quad \bar{\not{p}} \equiv \sqrt{2} p_\mu \bar{\tau}^\mu_{\alpha\dot{\beta}} = \longrightarrow p \cdots\cdots\cdots$$

### Introduction

Spinor-helicity recap  
Colour flow reminder

### Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

### Automation

Aim and method  
Results

### Conclusions



# The Massless QED Flow Rules: External Particles

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions

Species	Feynman	Flow
$\bar{u}^-(p_i)$		
$v^-(p_j)$		
$v^+(p_j)$		
$\bar{u}^+(p_i)$		
$\epsilon_-^\mu(p_i, r)$		$\frac{1}{\langle ir \rangle}$ or $\frac{1}{\langle ir \rangle}$
$\epsilon_+^\mu(p_i, r)$		$\frac{1}{\langle ri \rangle}$ or $\frac{1}{\langle ri \rangle}$

Left-chiral  $\equiv$  dotted lines

right-chiral  $\equiv$  solid lines



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# The QED Flow Rules: Vertices and Propagators

## Introduction

Spinor-helicity recap  
Colour flow reminder

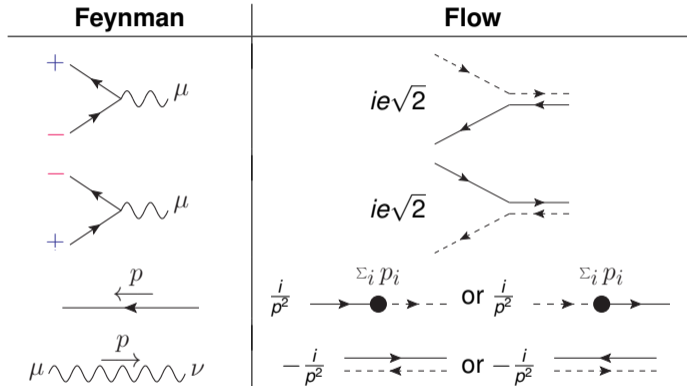
## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions



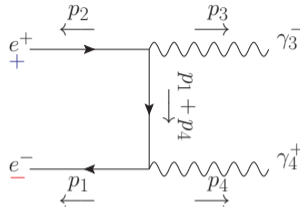
Left-chiral  $\equiv$  dotted lines

right-chiral  $\equiv$  solid lines



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# An Illuminating Example: $e^+ e^- \rightarrow \gamma\gamma$



**Spinor helicity:**

$$\begin{aligned}
 & \sim \langle p_1 | \bar{\tau}^\mu (|p_1\rangle \langle p_1| + |p_4\rangle \langle p_4|) \bar{\tau}^\nu | p_2 \rangle \underbrace{\frac{\langle r_3 | \bar{\tau}_\nu | p_3 \rangle}{\langle r_3 3 \rangle}}_{\epsilon_3^-} \underbrace{\frac{[r_4 | \tau_\mu | p_4 \rangle}{[4r_4]}}_{\epsilon_4^+} \\
 & = \frac{(\langle p_1 | \bar{\tau}^\mu | p_1 \rangle + \langle p_1 | \bar{\tau}^\mu | p_4 \rangle) [r_4 | \tau_\mu | p_4 \rangle (\langle p_1 | \bar{\tau}^\nu | p_2 \rangle + \langle p_4 | \bar{\tau}^\nu | p_2 \rangle) [p_3 | \tau_\nu | r_3 \rangle]}{\langle r_3 3 \rangle [4r_4]} \\
 & = \frac{\langle 1r_4 \rangle ([41] \langle 13 \rangle + [44] \langle 43 \rangle) [r_3 2]}{\langle r_3 3 \rangle [4r_4]} = \frac{\langle 1r_4 \rangle [41] \langle 13 \rangle [r_3 2]}{\langle r_3 3 \rangle [4r_4]} \\
 & \text{Fierz identities like } \langle i | \bar{\tau}^\mu | j \rangle [k | \tau_\mu | l \rangle = \langle il \rangle [kj] \qquad [ij] = 0
 \end{aligned}$$

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions



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## Introduction

- Spinor-helicity recap
- Colour flow reminder

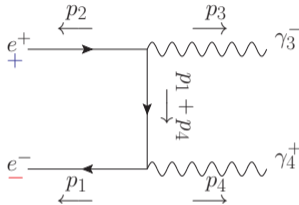
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- Massive Particles

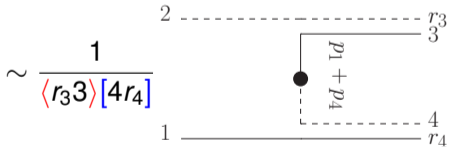
## Automation

- Aim and method
- Results

## Conclusions



Chirality flow:



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## Introduction

- Spinor-helicity recap
- Colour flow reminder

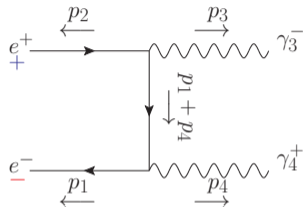
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- Massless QCD
- Massive Particles

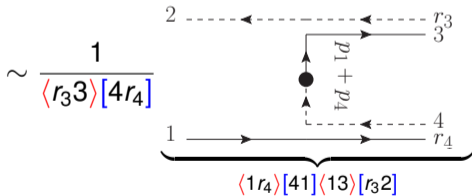
## Automation

- Aim and method
- Results

## Conclusions



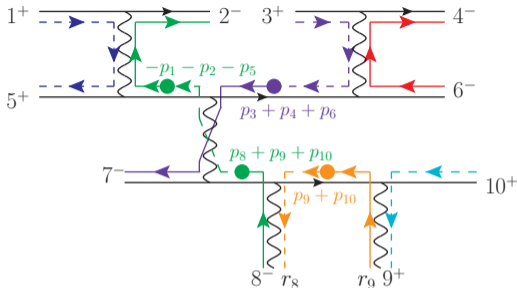
Chirality flow:



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# A complicated QED Example



Spinor-helicity analytic:

- 5 charge conjugation/Fierz + rearranging
- Not possible to fit on single slide!

$$\begin{aligned}
 &= \underbrace{(\sqrt{2}ei)^8}_{\text{vertices}} \underbrace{\frac{(-i)^3}{S_{12} S_{34} S_{78910}}}_{\text{photon propagators}} \underbrace{\frac{(i)^4}{S_{125} S_{346} S_{8910} S_{910}}}_{\text{fermion propagators}} \underbrace{\frac{1}{[8r_8] \langle r_9 9 \rangle}}_{\text{polarization vectors}} [15] \langle 64 \rangle [10 9] \\
 &\times \left( \langle r_9 9 \rangle [9r_8] + \langle r_9 10 \rangle [10r_8] \right) \left( \underbrace{[33] \langle 37 \rangle + [34] \langle 47 \rangle + [36] \langle 67 \rangle}_0 \right) \\
 &\times \left( - \langle 89 \rangle [91] \langle 12 \rangle - \langle 89 \rangle [95] \langle 52 \rangle - \langle 810 \rangle [10 1] \langle 12 \rangle - \langle 810 \rangle [10 5] \langle 52 \rangle \right)
 \end{aligned}$$

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

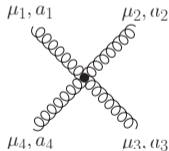
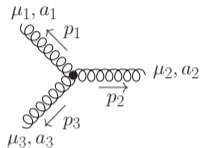
## Conclusions



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# The Non-abelian Massless QCD Flow Vertices

## Feynman



## Flow

$$-\frac{g_s f^{abc}}{2} \left( \underbrace{\text{Diagram 1}}_{g_{12}(p_1 - p_2)_3} + \underbrace{\text{Diagram 2}}_{g_{23}(p_2 - p_3)_1} + \underbrace{\text{Diagram 3}}_{g_{13}(p_3 - p_1)_2} \right)$$

$$ig_s^2 \sum_{Z(2,3,4)} f_{a_1 a_2 b} f_{b a_4 a_3} \left( \underbrace{\text{Diagram 4}}_{g_{14} g_{23}} - \underbrace{\text{Diagram 5}}_{g_{13} g_{24}} \right)$$

Arrow directions only consistently set within full diagram

### Introduction

Spinor-helicity recap  
Colour flow reminder

### Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

### Automation

Aim and method  
Results

### Conclusions



# QCD Example: $q_1 \bar{q}_1 \rightarrow q_2 \bar{q}_2 g$

## Introduction

Spinor-helicity recap  
Colour flow reminder

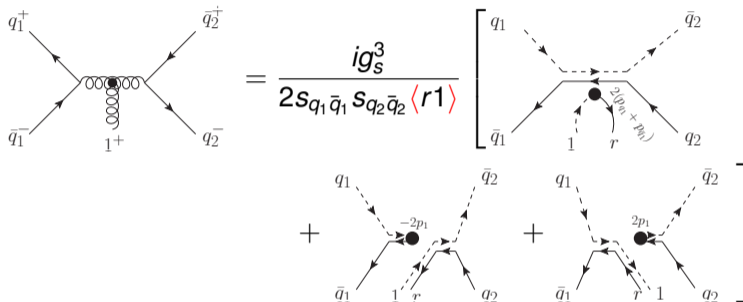
## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions



$$\left[ \dots \right] \equiv \left\{ 2[q_1 \bar{q}_2] \langle q_2 \bar{q}_1 \rangle ([1 q_1] \langle q_1 r \rangle + [1 \bar{q}_1] \langle 1 r \rangle) - 2[q_1 1] \langle 1 \bar{q}_1 \rangle \langle q_2 r \rangle [1 \bar{q}_2] + 2[q_1 1] \langle r \bar{q}_1 \rangle \langle q_2 1 \rangle [1 q_2] \right\}$$



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# Massive Chirality Flow (hep-ph:2011.10075)


## Decompose massive momentum into massless ones

$$p^\mu = p^b, \mu + \alpha q^\mu, \quad (p^b)^2 = q^2 = 0, \quad \alpha = \frac{p^2}{2p^b \cdot q}$$

- Spinors contain both chiralities, e.g.

$$\bar{v}^-(p) = \text{diagram} = \left( \text{diagram}, \frac{m}{\langle qp^b \rangle} \text{diagram} \right)$$

The diagram shows the decomposition of a massive spinor  $\bar{v}^-(p)$  into two massless spinors. On the left, a grey circle is connected to a black circle by a solid line with momentum  $p$  pointing right. This is equal to the sum of two terms in large parentheses. The first term is a grey circle connected to a black circle by a dashed line with momentum  $p^b$  pointing left. The second term is a grey circle connected to a black circle by a solid line with momentum  $q$  pointing left, multiplied by the factor  $\frac{m}{\langle qp^b \rangle}$ .

- Add new polarisation vector  $\not{\epsilon}_0 = \frac{1}{m\sqrt{2}}$  
- Need matrix structure in fermion propagators and vertices, e.g.

$$p^\mu \gamma_\mu - m \sim \begin{pmatrix} m \overset{\alpha}{\dashrightarrow} \overset{\beta}{\dashrightarrow} & \overset{\Sigma_i p_i}{\dashrightarrow} \bullet \overset{\Sigma_i p_i}{\dashrightarrow} \\ \overset{\Sigma_i p_i}{\dashrightarrow} \bullet \overset{\Sigma_i p_i}{\dashrightarrow} & m \overset{\alpha}{\dashrightarrow} \overset{\beta}{\dashrightarrow} \end{pmatrix}$$

The diagram shows the matrix structure of the fermion propagator  $p^\mu \gamma_\mu - m$ . It is represented as a 2x2 matrix of diagrams. The top-left element is a grey circle connected to a black circle by a dashed line with momentum  $\alpha$  on the left and  $\beta$  on the right. The top-right element is a black circle with momentum  $\Sigma_i p_i$  entering from the left and exiting to the right. The bottom-left element is a black circle with momentum  $\Sigma_i p_i$  entering from the left and exiting to the right. The bottom-right element is a grey circle connected to a black circle by a solid line with momentum  $\alpha$  on the left and  $\beta$  on the right.



# Massive Chirality Flow (hep-ph:2011.10075)

## Introduction

- Spinor-helicity recap
- Colour flow reminder

## Chirality Flow

- Massless QED
- Massless QCD
- Massive Particles

## Automation

- Aim and method
- Results

## Conclusions

## Main conclusion

Matrix structure unavoidable with massive fermions  
Proceed as before to calculate without algebra



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# A Massive *Illuminating* Example

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

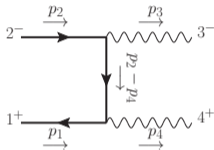
## Automation

Aim and method  
Results

## Conclusions

Consider the same diagram of  $f_1^+ \bar{f}_2^- \rightarrow \gamma_3^+ \gamma_4^-$  as before but include mass  $m_f$

- Obtain 3 new terms
- Simplify with choices of  $q_1, q_2, r_3, r_4$
- $e^{i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{\langle p_i^b q_i \rangle}$ ,  $e^{-i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^b]}$



$$= \frac{-2ie^2}{(s_{23} - m_f^2) \langle r_3 3 \rangle [4 r_4]} \left\{ \begin{array}{l} \begin{array}{c} p_2^b \text{---} \leftarrow \text{---} r_3 \\ \uparrow \quad \rightarrow \quad 3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad 4 \\ p_1^b \text{---} \rightarrow \text{---} r_4 \end{array} - \sqrt{\alpha_1 \alpha_2} e^{i(\varphi_2 - \varphi_1)} \begin{array}{c} q_2 \text{---} \leftarrow \text{---} 3 \\ \uparrow \quad \rightarrow \quad r_3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad r_4 \\ q_1 \text{---} \rightarrow \text{---} 4 \end{array} \end{array} \right.$$

$$+ m_f \left( \begin{array}{c} q_2 \text{---} \leftarrow \text{---} 3 \\ \uparrow \quad \rightarrow \quad r_3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad 4 \\ p_1^b \text{---} \rightarrow \text{---} r_4 \end{array} \sqrt{\alpha_2} e^{i\varphi_2} - \sqrt{\alpha_1} e^{-i\varphi_2} \begin{array}{c} p_2^b \text{---} \leftarrow \text{---} r_3 \\ \uparrow \quad \rightarrow \quad 3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad r_4 \\ q_1 \text{---} \rightarrow \text{---} 4 \end{array} \right)$$



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# MadGraph and the Automation of Chirality Flow

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions

## Summary

- So far: Numerical calculations use explicit multiplication rather than spin algebra analytically because quicker
- We have made the analytical spin algebra trivial
- Can we use this to make even faster numerics?



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# MadGraph and the Automation of Chirality Flow

## Introduction

Spinor-helicity recap  
Colour flow reminder

## Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

## Automation

Aim and method  
Results

## Conclusions



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## Summary

- So far: Numerical calculations use explicit multiplication rather than spin algebra analytically because quicker
- We have made the analytical spin algebra trivial
- Can we use this to make even faster numerics?

## Use MadGraph5\_aMC@NLO (MG5aMC) for proof of concept automation

- Make minimal changes to massless QED in MG5aMC
- Pro: any difference in speed from our changes  $\Rightarrow$  sound conclusions
- Con: MG5aMC not designed for chirality flow  $\Rightarrow$  not optimal implementation



# Sources of Expect Speed Gains

## 1 Simplified vertices and propagators

- We minimise matrix multiplication
- Each component of a calculation is simpler

### Introduction

Spinor-helicity recap  
Colour flow reminder

### Chirality Flow

Massless QED  
Massless QCD  
Massive Particles

### Automation

Aim and method  
Results

### Conclusions



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# Sources of Expect Speed Gains

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Results

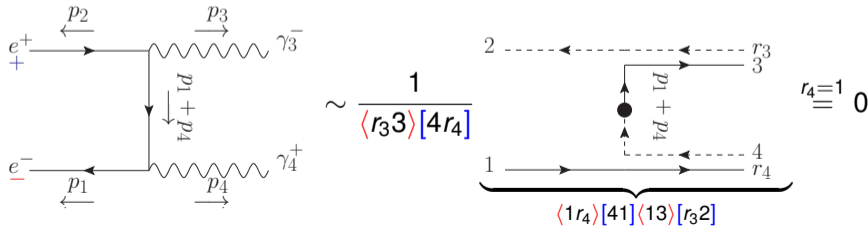
## Conclusions

### 1 Simplified vertices and propagators

- We minimise matrix multiplication
- Each component of a calculation is simpler

### 2 Gauge-based diagram removal

- Polarisation vectors contain arbitrary gauge-reference spinor of momentum  $r$
- Spinor inner products antisymmetric  $\Rightarrow \langle ii \rangle = [jj] = 0$
- Chirality-flow makes optimal choice of  $r$  obvious  $\Rightarrow$  remove diagrams!



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# Our Main Result (hep-ph:2203.13618)

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## Automation

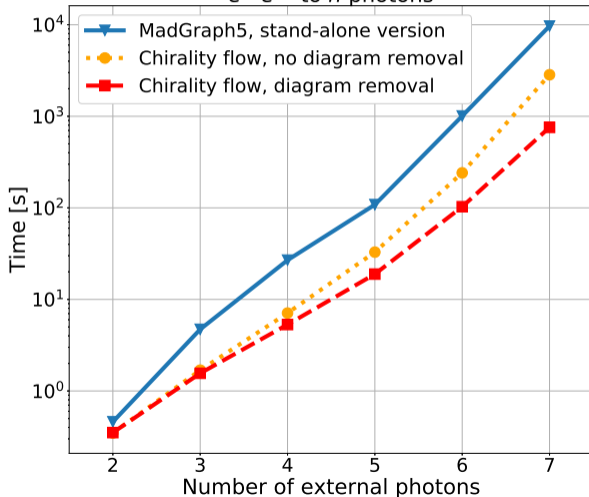
Aim and method  
Results

## Conclusions



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Evaluation time for 100 000 matrix elements for  $e^+e^-$  to  $n$  photons



# Conclusions and Outlook

## Introduction

Spinor-helicity recap  
Colour flow reminder

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Results

## Conclusions



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## Conclusions:

- Chirality flow is the shortest route from Feynman diagram to complex number
- We have flow rules for full SM at tree level
- We automatised it for massless QED, found significant gains in MadGraph

## Outlook and other work in this area:

- Simon Plätzer and Malin Sjödaahl used chirality flow as basis for resummation (hep-ph:2204.03258)
- Use method analytically to calculate loop amplitudes
  - Ongoing work by AL, Simon Plätzer, and Malin Sjödaahl,
- Automate for rest of (tree-level) Standard Model and tweak algorithm to use all possible features of chirality flow
  - Two current master students working to achieve this

# Reminder: Lorentz Group Representations

## Backup Slides

Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

## Massive Chirality Flow

Massive Examples

## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation



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Lorentz group elements:  $e^{i(\theta_i J_i + \eta_i K_i)}$   $J_i \equiv$  rotations,  $K_i \equiv$  boosts

- Lorentz group generators  $\simeq$  2 copies of  $su(2)$  generators

- $so(3, 1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

Group algebra defined by commutator relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} J_k$$

$$N_i^{\pm} = \frac{1}{2}(J_i \pm iK_i), \quad [N_i^-, N_j^+] = 0,$$

$$[N_i^-, N_j^-] = i\epsilon_{ijk} N_k^-, \quad [N_i^+, N_j^+] = i\epsilon_{ijk} N_k^+$$

- Representations

- $(0, 0)$  scalar particles
- $(\frac{1}{2}, 0)$  left-chiral and  $(0, \frac{1}{2})$  right-chiral Weyl (2-component) spinors.
- $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ , Dirac (4-component) spinors.
- $(\frac{1}{2}, \frac{1}{2})$  vectors, e.g. gauge bosons

# Spinor-Helicity: Gauge Bosons in Terms of Spinors

Lorentz algebra  $so(3, 1) \cong su(2) \oplus su(2)$   
Consider massless particles: chirality  $\sim$  helicity

Outgoing polarisation vectors:

$$\epsilon_+^\mu(p, r) = \frac{\langle r | \bar{\tau}^\mu | p \rangle}{\langle rp \rangle}, \quad \epsilon_-^\mu(p, r) = \frac{[r | \tau^\mu | p \rangle}{[pr]}$$

- $r$  is a (massless) arbitrary reference momentum ( $p \cdot r \neq 0$ )
- Different  $r$  choices correspond to different gauges

$$\epsilon_+^\mu(p, r') - \epsilon_+^\mu(p, r) = -p^\mu \frac{\langle r' r \rangle}{\langle r' p \rangle \langle rp \rangle}$$

- Gauge invariant quantities must be  $r$ -invariant
  - Choose  $r$  as conveniently as possible (remember  $\langle ij \rangle = -\langle ji \rangle$  s.t.  $\langle ii \rangle = 0$ ) (4-gluon amplitude: can make 20/21 terms vanish)
  - Variance under  $r \rightarrow r'$  good check of gauge invariance of (partial) amplitude

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Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

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Massive Examples

## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation



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# Spinor-Helicity: Vectors and Removing $\mu$ Indices

## Backup Slides

Spinor Helicity Reminder

Colour flow reminder

Massless QCD

## Massive Chirality Flow

Massive Examples

## Lorentz Group Details

## Spinor-hel details

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Lorentz algebra  $so(3, 1) \cong su(2) \oplus su(2)$   
Consider massless particles: chirality  $\sim$  helicity

Dirac matrices in chiral basis

$$\gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\tau^\mu \\ \sqrt{2}\bar{\tau}^\mu & 0 \end{pmatrix} \quad \sqrt{2}\tau^\mu = (1, \vec{\sigma}), \quad \sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma}),$$

Remove  $\tau/\bar{\tau}$  matrices in amplitude with

$$\underbrace{\langle i|\bar{\tau}^\mu|j\rangle\langle k|\tau_\mu|l\rangle}_{\text{Fierz identity}} = \langle il\rangle\langle kj\rangle, \quad \underbrace{\langle i|\bar{\tau}^\mu|j\rangle}_{\text{Charge Conjugation}} = [j|\tau^\mu|i]$$

Express (massless)  $p^\mu$  in terms of spinors

$$p^\mu = \frac{[p|\tau^\mu|p\rangle}{\sqrt{2}} = \frac{\langle p|\bar{\tau}^\mu|p\rangle}{\sqrt{2}}, \quad \sqrt{2}p^\mu\tau_\mu \equiv \not{p} = |p\rangle\langle p|, \quad \sqrt{2}p^\mu\bar{\tau}_\mu \equiv \bar{\not{p}} = |p\rangle[p|$$

# Spinor-Helicity: Gauge Bosons in Terms of Spinors

## Backup Slides

Spinor Helicity Reminder

Colour flow reminder

Massless QCD

## Massive Chirality Flow

Massive Examples

## Lorentz Group Details

## Spinor-hel details

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Lorentz algebra  $so(3, 1) \cong su(2) \oplus su(2)$   
Consider massless particles: chirality  $\sim$  helicity

Outgoing polarisation vectors ( $r \equiv$  gauge choice,  $r^2 = 0$ ,  $r \cdot p \neq 0$ ):

$$\epsilon_+^\mu(p, r) = \frac{\langle r | \bar{\tau}^\mu | p \rangle}{\langle rp \rangle}, \quad \epsilon_-^\mu(p, r) = \frac{[r | \tau^\mu | p \rangle}{[pr]}$$

$$p \cdot \epsilon_+(p, r) = \underbrace{\frac{\langle r | p^\mu \bar{\tau}_\mu | p \rangle}{\langle rp \rangle}}_{\text{Weyl eq. } p^\mu \bar{\tau}_\mu | p \rangle = 0} = 0, \quad p \cdot \epsilon_-(p, r) = \underbrace{\frac{[r | p^\mu \tau_\mu | p \rangle}{[pr]}}_{\text{Weyl eq. } p^\mu \tau_\mu | p \rangle = 0} = 0$$

$$\epsilon_+(p, r) \cdot (\epsilon_-)^*(p, r) = \underbrace{\frac{\langle r | \bar{\tau}^\mu | p \rangle}{\langle rp \rangle} \frac{[r | \tau_\mu | p \rangle}{[pr]}}_{\epsilon_\pm = (\epsilon_\mp)^*} = \frac{\langle rp \rangle [rp]}{\langle rp \rangle [pr]} = \underbrace{-1}_{[pr] = -[rp]}$$



# Colour Flow: a Quick Introduction

## Standard method in $SU(N)$ -colour calculations:

Write all objects in terms of  $\delta_{i\bar{j}} \equiv$  flows of colour (for simplicity  $T_R = 1$ )  
 Calculations done pictorially, not via indices

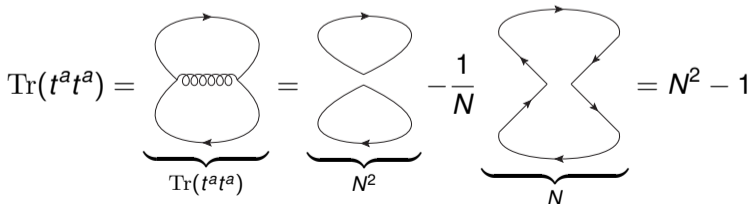
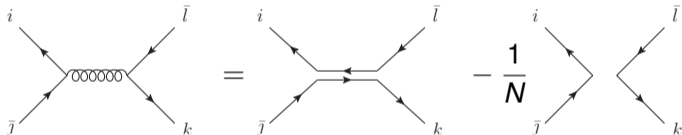
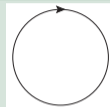
$$\begin{aligned}
 \delta_{i\bar{j}} &= \bar{j} \longrightarrow i \quad , \quad \sum_i \delta_{ii} = N = \text{circle} \quad , \quad t_{i\bar{j}}^a = \begin{array}{c} i \\ \searrow \\ \text{circle} \text{---} a \\ \nearrow \\ \bar{j} \end{array} \\
 if^{abc} &= \begin{array}{c} b \\ \text{circle} \\ \swarrow \quad \searrow \\ a \quad c \end{array} = \begin{array}{c} b \\ \text{circle} \\ \swarrow \quad \searrow \\ a \quad c \end{array} - \begin{array}{c} b \\ \text{circle} \\ \swarrow \quad \searrow \\ a \quad c \end{array} = \text{Tr}(t^a[t^b, t^c]) \\
 \underbrace{\begin{array}{c} i \quad \bar{l} \\ \swarrow \quad \searrow \\ \text{circle} \\ \nearrow \quad \nwarrow \\ \bar{j} \quad k \end{array}}_{t_{i\bar{j}}^a t_{k\bar{l}}^a} &= \underbrace{\begin{array}{c} i \quad \bar{l} \\ \swarrow \quad \searrow \\ \text{circle} \\ \nearrow \quad \nwarrow \\ \bar{j} \quad k \end{array}}_{\delta_{i\bar{l}} \delta_{k\bar{j}}} - \frac{1}{N} \underbrace{\begin{array}{c} i \quad \bar{l} \\ \swarrow \quad \searrow \\ \text{circle} \\ \nearrow \quad \nwarrow \\ \bar{j} \quad k \end{array}}_{\delta_{i\bar{j}} \delta_{k\bar{l}}}
 \end{aligned}$$



# Colour Flow: a Quick Introduction

Standard method in  $SU(N)$ -colour calculations:

Calculations done pictorially, not via indices  $\sum_i \delta_{ii} = N =$



## Backup Slides

Spinor Helicity Reminder

Colour flow reminder

Massless QCD

## Massive Chirality Flow

Massive Examples

## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation



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# The Non-abelian Massless QCD Flow Vertices

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Spinor Helicity Reminder

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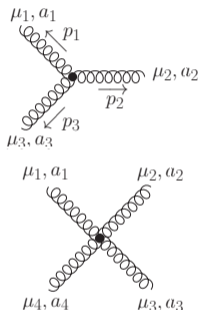
Massive Examples

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## Chirality-Flow Motivation

Feynman



Flow

$$-\frac{g_s f^{abc}}{2} \left( \begin{array}{c} 1 \\ \text{---} \\ \text{---} \\ 2 \\ \text{---} \\ \bullet \\ \text{---} \\ 1-2 \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ 2-3 \\ \bullet \\ \text{---} \\ 2 \\ \text{---} \\ 3 \end{array} + \begin{array}{c} 1 \\ \text{---} \\ \text{---} \\ 3 \\ \text{---} \\ \bullet \\ \text{---} \\ 3-1 \end{array} \right)$$

$$ig_s^2 \sum_{Z(2,3,4)} f_{a_1 a_2 b} f_{b a_4 a_3} \left[ \begin{array}{c} 1 \\ \text{---} \\ \text{---} \\ 4 \end{array} \quad \begin{array}{c} 2 \\ \text{---} \\ \text{---} \\ 3 \end{array} - \begin{array}{c} 1 \\ \text{---} \\ \text{---} \\ 4 \end{array} \quad \begin{array}{c} 2 \\ \text{---} \\ \text{---} \\ 3 \end{array} \right]$$

Arrow directions only consistently set within full diagram

Double line  $\equiv g_{\mu\nu}$ , momentum dot  $\equiv p_\mu$



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# QCD Example: $q_1 \bar{q}_1 \rightarrow q_2 \bar{q}_2 g$

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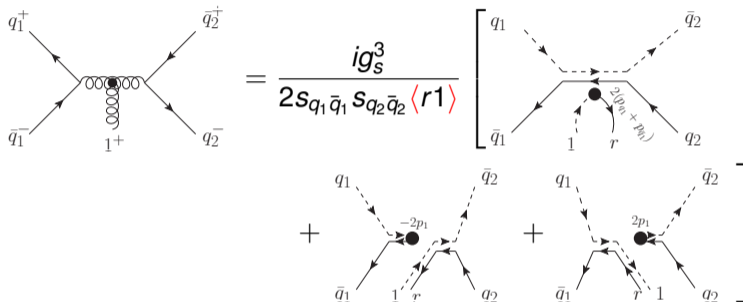
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$$\left[ \dots \right] \equiv \left\{ 2[q_1 \bar{q}_2] \langle q_2 \bar{q}_1 \rangle ([1 q_1] \langle q_1 r \rangle + [1 \bar{q}_1] \langle 1 r \rangle) - 2[q_1 1] \langle 1 \bar{q}_1 \rangle \langle q_2 r \rangle [1 \bar{q}_2] + 2[q_1 1] \langle r \bar{q}_1 \rangle \langle q_2 1 \rangle [1 q_2] \right\}$$



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# Incoming Massive Spinors in Chirality Flow

$$p^\mu = p^{b,\mu} + \alpha q^\mu, \quad (p^b)^2 = q^2 = 0, \quad e^{i\varphi} \sqrt{\alpha} = \frac{m}{\langle p^b q \rangle}, \quad e^{-i\varphi} \sqrt{\alpha} = \frac{m}{[qp^b]}$$

$$\text{Spin operator } -\frac{\Sigma^\mu s_\mu}{2} = \frac{\gamma^5 s^\mu \gamma_\mu}{2}, \quad s^\mu = \frac{1}{m}(p^{b,\mu} - \alpha q^\mu)$$

Spinor	Feynman	Flow
$\bar{v}^-(p)$		$\left( \text{grey circle} \xleftarrow{\text{dashed } p^b}, \sqrt{\alpha} e^{i\varphi} \text{grey circle} \xleftarrow{\text{solid } q} \right)$
$\bar{v}^+(p)$		$\left( -\sqrt{\alpha} e^{-i\varphi} \text{grey circle} \xleftarrow{\text{dashed } q}, \text{grey circle} \xleftarrow{\text{solid } p^b} \right)$
$u^-(p)$		$\left( \text{grey circle} \xrightarrow{\text{dashed } p^b}, \sqrt{\alpha} e^{i\varphi} \text{grey circle} \xrightarrow{\text{solid } q} \right)$
$u^+(p)$		$\left( -\sqrt{\alpha} e^{-i\varphi} \text{grey circle} \xrightarrow{\text{dashed } q}, \text{grey circle} \xrightarrow{\text{solid } p^b} \right)$

## Backup Slides

Spinor Helicity Reminder

Colour flow reminder

Massless QCD

## Massive Chirality Flow

Massive Examples

## Lorentz Group Details

## Spinor-hel details

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# Some Fermion Flow Rules

$$p^\mu = p^{b,\mu} + \alpha q^\mu, \quad (p^b)^2 = q^2 = 0, \quad \alpha = \frac{p^2}{2p \cdot q} \neq 0$$

Fermion-vector vertex

$$\begin{array}{c} \text{---} \\ \text{---} \end{array} \begin{array}{c} \nearrow \\ \searrow \end{array} \text{---}^\mu = ie(P_L C_L + P_R C_R) \gamma^\mu = ie\sqrt{2} \left( \begin{array}{cc} 0 & C_R \\ C_L & 0 \end{array} \right)$$

Fermion propagator

$$\frac{i}{p^2 - m_f^2} \begin{pmatrix} m_f \delta_{\dot{\alpha}\dot{\beta}} & \sqrt{2} p^{\dot{\alpha}\dot{\beta}} \\ \sqrt{2} \bar{p}_{\alpha\dot{\beta}} & m_f \delta_{\alpha\beta} \end{pmatrix} = \frac{i}{p^2 - m_f^2} \left( \begin{array}{cc} m_f \overset{\dot{\alpha}}{\text{---}} \overset{\dot{\beta}}{\text{---}} & \overset{\Sigma_i p_i}{\text{---}} \bullet \text{---} \\ \overset{\Sigma_i p_i}{\text{---}} \bullet \text{---} & m_f \overset{\alpha}{\text{---}} \overset{\beta}{\text{---}} \end{array} \right)$$

Left and right chiral couplings may differ



# A Massive *Illuminating* Example

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Colour flow reminder  
Massless QCD

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Massive Examples

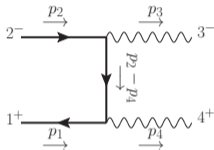
## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation

Consider the same diagram of  $f_1^+ \bar{f}_2^- \rightarrow \gamma_3^+ \gamma_4^-$  as before but include mass  $m_f$

- Obtain 3 new terms
- Simplify with choices of  $q_1, q_2, r_3, r_4$
- $e^{i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{\langle p_i^b q_i \rangle}$ ,  $e^{-i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^b]}$



$$= \frac{-2ie^2}{(s_{23} - m_f^2) \langle r_3 3 \rangle [4 r_4]} \left\{ \begin{array}{l} \begin{array}{c} p_2^b \text{---} \text{---} r_3 \\ \uparrow \quad \rightarrow \quad 3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad 4 \\ p_1^b \text{---} \text{---} r_4 \end{array} - \sqrt{\alpha_1 \alpha_2} e^{i(\varphi_2 - \varphi_1)} \begin{array}{c} q_2 \text{---} \text{---} 3 \\ \uparrow \quad \leftarrow \quad r_3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \rightarrow \quad r_4 \\ q_1 \text{---} \text{---} 4 \end{array} \end{array} \right. \\
+ m_f \left( \begin{array}{c} q_2 \text{---} \text{---} 3 \\ \uparrow \quad \leftarrow \quad r_3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \rightarrow \quad 4 \\ p_1^b \text{---} \text{---} r_4 \end{array} \sqrt{\alpha_2} e^{i\varphi_2} - \sqrt{\alpha_1} e^{-i\varphi_2} \begin{array}{c} p_2^b \text{---} \text{---} r_3 \\ \uparrow \quad \rightarrow \quad 3 \\ \bullet \quad p_4 - p_1^b - q_1 \\ \downarrow \quad \leftarrow \quad r_4 \\ q_1 \text{---} \text{---} 4 \end{array} \right) \left. \right\}$$



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# A Second Massive Example: $f_1 \bar{f}_2 \rightarrow W \rightarrow f_3 \bar{f}_4 h_5$

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Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

## Massive Chirality Flow

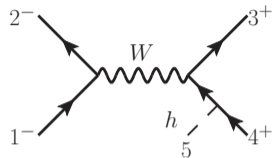
Massive Examples

## Lorentz Group Details

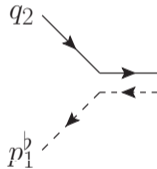
## Spinor-hel details

## Chirality-Flow Motivation

- W bosons simplifies ( $C_R = 0$ )
- Simplify with choices of  $q_1, \dots, q_5$
- $e^{i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{\langle p_i^b q_i \rangle}$ ,  $e^{-i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^b]}$
- Scalar has no flow line



Step 1: Draw fermion lines:  $\sim C_{L,12} \sqrt{\alpha_2} e^{i\varphi_2}$



$$\times C_{L,34} \sqrt{\alpha_3} (-e^{i\varphi_3}) \left[ \sqrt{\alpha_4} (-e^{i\varphi_4}) \begin{array}{c} q_3 \\ \leftarrow \\ \text{---} 4 \text{---} 5 \\ \bullet \\ \searrow q_4 \end{array} + m_4 \begin{array}{c} q_3 \\ \leftarrow \\ \text{---} \\ \searrow p_4^b \end{array} \right]$$



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# A Second Massive Example: $f_1 \bar{f}_2 \rightarrow W \rightarrow f_3 \bar{f}_4 h_5$

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Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

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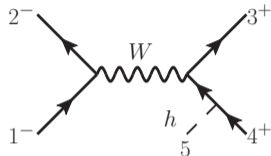
Massive Examples

## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation

- W bosons simplifies ( $C_R = 0$ )
- Simplify with choices of  $q_1, \dots, q_5$
- $e^{i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{\langle p_i^b q_i \rangle}$ ,  $e^{-i\varphi_i} \sqrt{\alpha_i} = \frac{m_i}{[q_i p_i^b]}$
- Scalar has no flow line



Step 2: Flip arrows and connect:  $C_{L,12} C_{L,34} \sqrt{\alpha_2 \alpha_3} e^{i(\varphi_2 + \varphi_3)}$

$$\times \left[ \begin{array}{c} \sqrt{\alpha_4} e^{i\varphi_4} \\ \begin{array}{c} q_2 \quad q_3 \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ n_1^b \quad q_4 \\ \text{---} \quad \text{---} \end{array} - m_4 \begin{array}{c} q_2 \quad q_3 \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ n_1^b \quad p_4^b \\ \text{---} \quad \text{---} \end{array} \end{array} \right]$$



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# Lorentz Group Representations

## Backup Slides

Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

## Massive Chirality Flow

Massive Examples

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Lorentz group elements:  $e^{i(\theta_i J_i + \eta_i K_i)}$   $J_i \equiv$  rotations,  $K_i \equiv$  boosts

- Lorentz group generators  $\simeq$  2 copies of  $su(2)$  generators

- $so(3, 1)_{\mathbb{C}} \cong su(2) \oplus su(2)$

Group algebra defined by commutator relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} J_k$$

$$N_i^{\pm} = \frac{1}{2}(J_i \pm iK_i), \quad [N_i^-, N_j^+] = 0,$$

$$[N_i^-, N_j^-] = i\epsilon_{ijk} N_k^-, \quad [N_i^+, N_j^+] = i\epsilon_{ijk} N_k^+$$

- Representations (i.e. realisations of  $N_i^{\pm}$ )

- $(0, 0)$  scalar particles
- $(\frac{1}{2}, 0)$  left-chiral and  $(0, \frac{1}{2})$  right-chiral Weyl (2-component) spinors.
- $(\frac{1}{2}, 0) \oplus (0, \frac{1}{2})$ , Dirac (4-component) spinors.
- $(\frac{1}{2}, \frac{1}{2})$  vectors, e.g. gauge bosons

# How to Calculate? Spinor-Helicity

Give each particle a defined helicity  $\Rightarrow$  amplitude now a number!

Spinors (in chiral basis):

$$u^+(p) = v^-(p) = \begin{pmatrix} 0 \\ |p\rangle \end{pmatrix}$$

$$u^-(p) = v^+(p) = \begin{pmatrix} |p] \\ 0 \end{pmatrix}$$

$$\bar{u}^+(p) = \bar{v}^-(p) = \langle p| \quad 0$$

$$\bar{u}^-(p) = \bar{v}^+(p) = 0 \quad \langle p|$$

$$\gamma^\mu = \begin{pmatrix} 0 & \sqrt{2}\tau^\mu \\ \sqrt{2}\bar{\tau}^\mu & 0 \end{pmatrix}$$

$$\sqrt{2}\tau^\mu = (1, \vec{\sigma}), \quad \sqrt{2}\bar{\tau}^\mu = (1, -\vec{\sigma}),$$

- Amplitude written in terms of Lorentz-invariant spinor inner products

$$\langle ij \rangle = -\langle ji \rangle \equiv \langle i||j \rangle \text{ and } [ij] = -[ji] \equiv [i||j]$$

- These are well known complex numbers,  $\langle ij \rangle \sim [ij] \sim \sqrt{2p_i \cdot p_j}$
- Remove  $\tau/\bar{\tau}$  matrices in amplitude with

$$\langle i|\bar{\tau}^\mu|j\rangle[k|\tau_\mu|l\rangle = \langle il\rangle[kj], \quad \langle i|\bar{\tau}^\mu|j\rangle = [j|\tau^\mu|i\rangle$$



# How to Calculate a Process

## Backup Slides

Spinor Helicity Reminder  
Colour flow reminder  
Massless QCD

## Massive Chirality Flow

Massive Examples

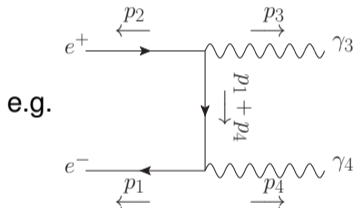
## Lorentz Group Details

## Spinor-hel details

## Chirality-Flow Motivation

Sum all Feynman diagrams, square, and integrate

Often spin structure is non-trivial



$$\sim \underbrace{[\bar{u}(p_1)\gamma^\mu (p_1^\nu + p_4^\nu) \gamma_\nu \gamma^\rho v(p_2)] \epsilon_\rho(p_3)\epsilon_\mu(p_4)}$$

A mathematical expression we have simplify and square

Most common method: use helicity basis

Each diagram is a complex number, easy to square

Can use algebra to simplify first, or brute force matrix multiplication



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# Define Problem

## Backup Slides

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## Chirality-Flow Motivation



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## Kinematic part of amplitude slowed by spin and vector structures

- Can we still improve on this?
  - Deriving spinor inner products  $\langle ij \rangle$ ,  $[kl]$  requires at least 2 steps
    - Re-write every object as spinors
    - Use Fierz identity  $\bar{\tau}_{\alpha\dot{\beta}}^{\mu} \tau_{\mu}^{\dot{\alpha}\beta} = \delta_{\alpha}^{\dot{\alpha}} \delta_{\dot{\beta}}^{\beta}$
  - Not intuitive which inner products we obtain
- In SU(N) use graphical reps for calculations
  - E.g. using the colour-flow method
  - (Also birdtracks etc.)
- Spinor-helicity  $\equiv su(2) \oplus su(2)$ 
  - Can we use graphical reps?

# Creating Chirality Flow: Building Blocks

A flow is a directed line from one object to another

$su(2)$  objects have dotted indices and  $su(2)$  objects undotted indices

- First step: Ansatz for spinor inner products (only possible Lorentz invariant)

$$\langle i^\alpha | j \rangle_\alpha \equiv \langle ij \rangle = -\langle ji \rangle = i \longrightarrow j$$

$$[i |_\beta j]^\beta \equiv [ij] = -[ji] = i \dashrightarrow j$$

- Spinors and Kronecker deltas follow

$$\langle i |^\alpha = \bullet \longleftarrow i \quad ,$$

$$|j \rangle_\alpha = \bullet \longrightarrow j$$

$$[i |_\beta = \bullet \dashleftarrow i \quad ,$$

$$|j]^\beta = \bullet \dashrightarrow j$$

$$\delta_\alpha^\beta \equiv \mathbb{1}_\alpha^\beta = \alpha \longrightarrow \beta \quad ,$$

$$\delta_{\dot{\alpha}}^{\dot{\beta}} \equiv \mathbb{1}_{\dot{\alpha}}^{\dot{\beta}} = \dot{\beta} \dashrightarrow \dot{\alpha}$$



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