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INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

DRESDEN concept

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Implementation of polarized cross sections for vector bosons in Sherpa

MCnet-Meeting Graz, September 23, 2022

Introduction: Motivation & Task

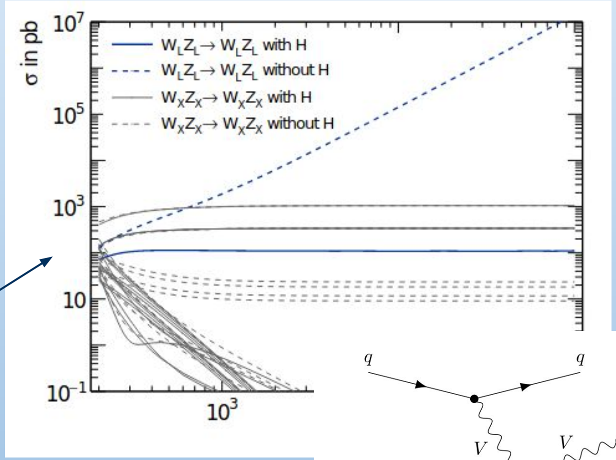
Why polarization?

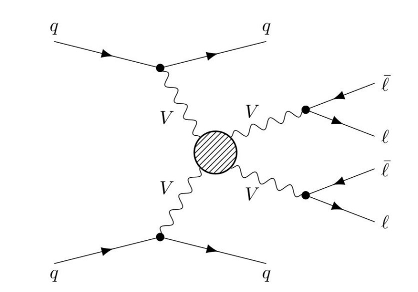
- longitudinal polarization: consequence of non-vanishing boson mass generated by electroweak symmetry breaking (EWSB) mechanism

➔ without Higgs-Boson: Unitarity-Breaking $\sigma(\mathrm{V_L V_L} \rightarrow \mathrm{V_L V_L}) \propto \mathrm{E}_{\mathrm{cm}}^2$

➔ sensitive to:

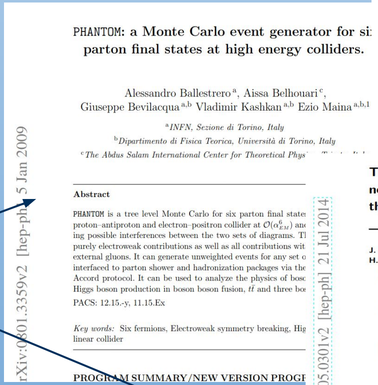
  - SM innermost gauge symmetry structure
  - concrete mechanism of EWSB
  - BSM physics





Current status

- only a few generators are available (Madgraph, Phantom, Whizard) which provide event simulation with polarization information
- Sherpa can not simulate polarized cross sections yet



arXiv:0801.3359v2 [hep-ph] 5 Jan 2009

PHANTOM: a Monte Carlo event generator for si
parton final states at high energy colliders.

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Giuseppe Bevilacqua[a,b] Vladimir Kashkan[a,b] Ezio Maina[a,b,1]

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[c]*The Abdus Salam International Center for Theoretical Phys*

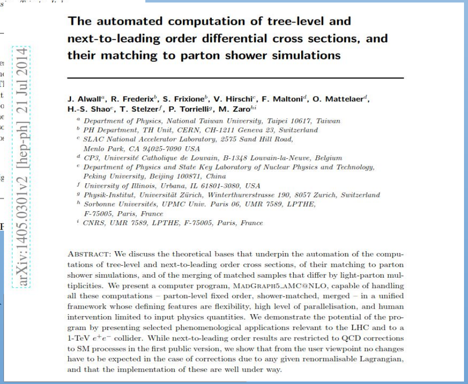
Abstract

PHANTOM is a tree level Monte Carlo for six parton final state
proton–antiproton and electron–positron collider at $\mathcal{O}(\alpha_{EM}^6)$ an
ing possible interferences between the two sets of diagrams. T
purely electroweak contributions as well as all contributions wit
external gluons. It can generate unweighted events for any set o
interfaced to parton shower and hadronization packages via the
Accord protocol. It can be used to analyze the physics of bos
Higgs boson production in boson boson fusion, $t\bar{t}$ and three bo

PACS: 12.15.-y, 11.15.Ex

Key words: Six fermions, Electroweak symmetry breaking, Hig
linear collider

PROGRAM SUMMARY/NEW VERSION PROG



arXiv:1405.0301v2 [hep-ph] 21 Jul 2014

The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations

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ABSTRACT: We discuss the theoretical bases that underpin the automation of the computations of tree-level and next-to-leading order cross sections, of their matching to parton shower simulations, and of the merging of matched samples that differ by light-parton multiplicities. We present a computer program, MADGRAPH5_AMC@NLO, capable of handling all these computations – parton-level fixed order, shower-matched, merged – in a unified framework whose defining features are flexibility, high level of parallelisation, and human intervention limited to input physics quantities. We demonstrate the potential of the program by presenting selected phenomenological applications relevant to the LHC and to a 1-TeV e^+e^- collider. While next-to-leading order results are restricted to QCD corrections to SM processes in the first public version, we show that from the user viewpoint no changes have to be expected in the case of corrections due to any given renormalisable Lagrangian, and that the implementation of these are well under way.

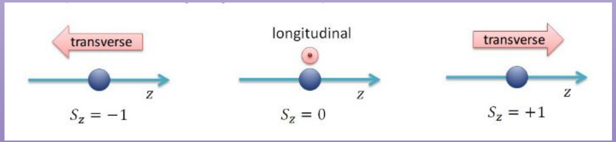
Definition of polarization for vector bosons

Definition of vector boson polarization

- free vector-boson field described by Proca equation: $\Box V^{\alpha}(x) + m_V^2 V^{\alpha}(x) = 0$
- solution: $V^{\alpha} = \int \frac{d^3 q}{(2\pi)^3 2q_0} \sum_{\lambda} \varepsilon^{\alpha}(q,\lambda)\hat{a}(q,\lambda)e^{-iqx} + \varepsilon^{*\alpha}(q,\lambda)\hat{b}^{\dagger}(q,\lambda)e^{iqx}$

Polarization vectors

- ❖ common definition in helicity basis:
  polarization vectors = eigenvectors of helicity operator $h = \frac{\vec{p}\cdot S}{|\vec{p}|}$
- ❖ possible eigenvalues:
  - ➢ h = ±1: transverse polarization
  - ➢ h = 0 : longitudinal polarization



Dr. Z. Zinonos: Tests of the Standard Model of Particles. https://www.mpp.mpg.de/~zinonos/material/lecture10.pdf

- **interacting theory:** external vector bosons described by polarization vector

Polarization of intermediate particles

- Problem: short living vector bosons not directly measurable
- Propagator terms: **no dependence** on polarization vectors

$$\frac{i\left(-g_{\mu\nu}+\frac{q_\mu q_\nu}{m_V{}^2}\right)}{q^2-m_V^2+i\Gamma_V m_V}$$

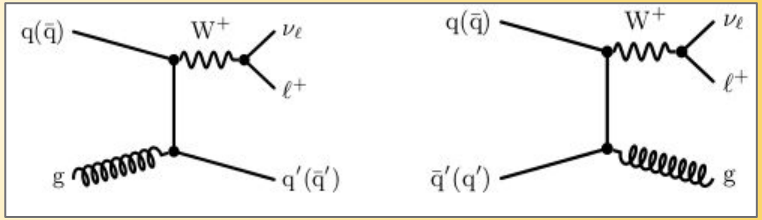
✓ **completeness relation**: $$\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{m_V^2}\right)=\sum_{\lambda=1}^{4}\varepsilon^\mu(q,\,\lambda)\varepsilon^{*\nu}(q,\,\lambda)$$

But: unphysical fourth polarization ⇨⇨⇨ vanishes for on shell particles

- definition of polarization only possible, if matrix element factorizes in production & decay

Example: Single W+j production and decay

$$\mathcal{M}\;=\mathcal{M}_\mu^{prod}\left(\frac{i\left(-g^{\mu\nu}+\frac{q^\mu\,q^\nu}{m_W^2}\right)}{q^2-m_W^2+i\Gamma_W m_W}\right)\mathcal{M}_\nu^{decay}$$



➔ also necessary for interpretation & separation of polarization

Polarization of intermediate particles - difficulties

Non-resonant contributions

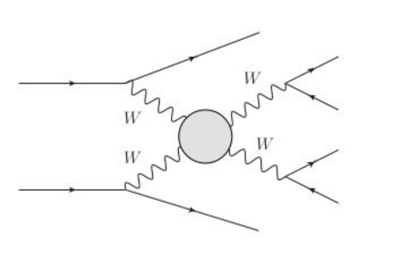
- no factorization possible
- necessary for gauge invariance

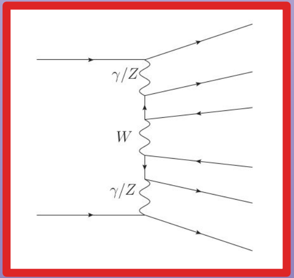
✓ suitable approximations:

- double-pole approximation (DPA)
- Sherpa: narrow-width approximation (NWA)

$$\frac{1}{\left(q^2 - m_V^2\right)^2 + \Gamma_V^2 m_V^2} \to \frac{\pi\delta\left(q^2 - m_V^2\right)}{\Gamma_V m_V}$$

example: ssWW-scattering





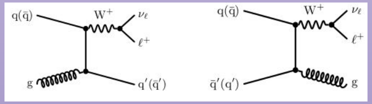
Interference between different polarizations

- off diagonal terms in matrix element tensor: matrix element & its complex conjugate have different helicities
- reason: helicity sum in matrix element

✓ zero for absence of lepton cuts

✓ analysis should be designed such that interference are small

example: W+j production



$$\mathcal{M} = \frac{\pi}{\Gamma_W m_W} \sum_{\lambda=1}^{3} \mathcal{M}_\mu^{prod} \, \varepsilon_\lambda^{*\mu} \varepsilon_\lambda^{\nu} \mathcal{M}_\nu^{decay} = \frac{\pi}{\Gamma_W m_W} \sum_{\lambda=1}^{3} \mathcal{M}_\lambda^{\mathcal{P}} \mathcal{M}_\lambda^{\mathcal{D}} := \sum_{\lambda=1}^{3} \mathcal{M}_\lambda^{\mathcal{F}}$$

$$\sigma \propto |\mathcal{M}|^2 = \sum_{\lambda} \left|\mathcal{M}_\lambda^{\mathcal{F}}\right|^2 + \sum_{\lambda,\,\lambda'} \mathcal{M}_\lambda^{\mathcal{F}*} \mathcal{M}_{\lambda'}^{\mathcal{F}}$$

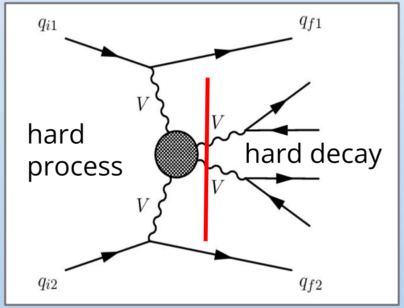
polarized cross section for VB helicity λ

interference terms

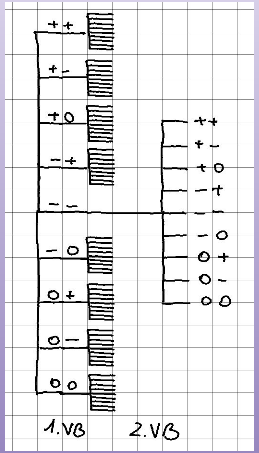
Extracting polarized cross sections from Sherpa

Status quo

- focus on hard interaction
- extended narrow-width approximation preserving spin correlations



$$|\mathcal{M}|^2 \propto \sum_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n} \mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{P}}_{\lambda'_1\ldots\lambda'_n}\mathcal{M}^{\mathcal{D}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{D}}_{\lambda'_1\ldots\lambda'_n}$$



Hard process

- vector boson production
- result: matrix element tensor

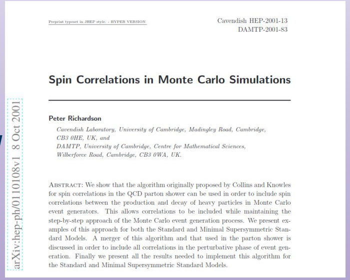
$$\left|\mathcal{M}^{\mathcal{P}}\right|^2_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n} = \mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{P}}_{\lambda'_1\ldots\lambda'_n}$$

structure: tree

- branch = one helicity combination $|\mathcal{M}|^2_{\lambda_1\lambda'_1\ldots\lambda_n\lambda'_n}$
- level = one particle
- number of starting branches / level = (helicity degrees of freedom)^2

Hard decays

- vector boson decays
- spin correlation algorithm from *P. Richardson. JHEP 0111 (2001) 029* implemented
- algorithm generates:
  - decay chain
  - decay matrix for each particle
- connection to the whole decay matrix element



Preprint typeset in JHEP style. - HYPER VERSION

Cavendish HEP-2001-13
DAMTP-2001-83

arXiv:hep-ph/0110108v1 8 Oct 2001

Spin Correlations in Monte Carlo Simulations

Peter Richardson

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DAMTP, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge, CB3 0WA, UK.

Abstract: We show that the algorithm originally proposed by Collins and Knowles for spin correlations in the QCD parton shower can be used in order to include spin correlations between the production and decay of heavy particles in Monte Carlo event generators. This allows correlations to be included while maintaining the step-by-step approach of the Monte Carlo event generation process. We present examples of this approach for both the Standard and Minimal Supersymmetric Standard Models. A merger of this algorithm and that used in the parton shower is discussed in order to include all correlations in the perturbative phase of event generation. Finally we present all the results needed to implement this algorithm for the Standard and Minimal Supersymmetric Standard Models.

$$\mathcal{D}_{\lambda_A\lambda'_A} = \frac{1}{\mathcal{N}_{\mathcal{D}}}\mathcal{M}^{\mathcal{D}}_{\lambda_A}\mathcal{M}^{\mathcal{D}*}_{\lambda'_A}$$

$$\mathcal{M}^{\mathcal{D}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{D}}_{\lambda'_1\ldots\lambda'_n} \propto \prod_{A=1}^{n}\mathcal{D}_{\lambda_A\lambda'_A}$$

contracting

➔ **result: unpolarized cross section in NWA**

Calculation of polarized cross sections

- *"Never change a running system"* : leave algorithm above unchanged

Steps for separating polarizations

$$\left|\mathcal{M}^{pol}\right|^2_{\lambda_1\ldots\lambda_n;\,\lambda'_1\ldots\lambda'_n} \propto \mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{P}}_{\lambda'_1\ldots\lambda'_n}\mathcal{M}^{\mathcal{D}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{D}}_{\lambda'_1\ldots\lambda'_n}$$

1. **Calculation of matrix elements**: production tensor and decay matrices are calculated by the algorithm above & saved for later before contracting them
2. **optional: Transformation**: Transformation of production tensor and decay matrices if change of polarization definition is desired (later)
3. **Multiplication**: Multiplying production tensor and decay matrices results in Amplitude tensor with all polarized matrix elements
4. **Labeling & Output**: Polarized cross section are handled & printed out as additional event weights

→ **polarized cross sections of all helicity combinations + interferences** between them easily accessible in a **single simulation run**

→ **interferences** between different polarizations are **calculated directly**

Change of polarization definition

Polarization definition

General: Spin basis

- massive VB characterized by momentum, integer spin value, spin projection onto an arbitrary axis
- common choice: axis // VB momentum k^μ **(=helicity basis)**

➔ common representation of polarization vectors:

$$\varepsilon^\mu_\pm(k) = \frac{e^{\mp i\phi}}{\sqrt{2}}(0, -\cos\theta\cos\phi \pm i\sin\phi, -\cos\theta\sin\phi \mp i\cos\phi, \sin\theta),$$

$$\varepsilon^\mu_0(k) = s^\mu_k = \frac{k^0}{m}\left(\frac{|\mathbf{k}|}{k^0}, \cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta\right),$$

- Weyl-van-der-Waerden formalism: spin basis connected with light-like reference vector

$$k^\mu = \alpha a^\mu + b^\mu \qquad \alpha = \frac{k^2}{2\, a\cdot k}$$

For helicity basis: Reference system

- helicity not lorentz covariant

$$\Lambda^\mu{}_\nu \varepsilon^\nu(k) \neq \varepsilon^\mu(\Lambda k)$$

➔ polarization vector depends on choice of frame for k^μ

Calculate ε^μ from k^μ in Lab ≠ Calculate ε^μ from k^μ in different frame + transformation to Lab

- Lab = frame for matrix element calculation

default polarization definition in Sherpa:

- no helicity basis
- laboratory frame: VB center of mass system & parton-parton frame = common frames in analysis

Change of polarization definition

- Two ways to change polarization definition in matrix elements:
  - a priori: change polarization definition directly in matrix element generator
  - **a posteriori: transformation of calculated production tensor, decay matrices**
- change of basis = basis transformation of polarization vectors

$$\begin{pmatrix} \tilde{\varepsilon}^0_+ & \tilde{\varepsilon}^0_- & \tilde{\varepsilon}^0_0 \\ \tilde{\varepsilon}^1_+ & \tilde{\varepsilon}^1_- & \tilde{\varepsilon}^1_0 \\ \tilde{\varepsilon}^2_+ & \tilde{\varepsilon}^2_- & \tilde{\varepsilon}^2_0 \\ \tilde{\varepsilon}^3_+ & \tilde{\varepsilon}^3_- & \tilde{\varepsilon}^3_0 \end{pmatrix} = \begin{pmatrix} \varepsilon^0_+ & \varepsilon^0_- & \varepsilon^0_0 \\ \varepsilon^1_+ & \varepsilon^1_- & \varepsilon^1_0 \\ \varepsilon^2_+ & \varepsilon^2_- & \varepsilon^2_0 \\ \varepsilon^3_+ & \varepsilon^3_- & \varepsilon^3_0 \end{pmatrix} \begin{pmatrix} a_{++} & a_{-+} & a_{0+} \\ a_{+-} & a_{--} & a_{0-} \\ a_{+0} & a_{-0} & a_{00} \end{pmatrix}$$

transformation coefficients

matrix of new polarization vectors

matrix of default polarization vectors

- Transformation of matrix elements

$$|\mathcal{M}|^2_{\lambda_1\lambda'_1\ldots\lambda_n\lambda'_n} = \sum_{\kappa_1\kappa'_1\ldots\kappa_n\kappa'_n} a^{\text{part}1}_{\lambda_1\kappa_1} a^{\text{part}1*}_{\lambda'_1\kappa'_1} \cdots a^{\text{part}n}_{\lambda_n\kappa_n} a^{\text{part}n*}_{\lambda'_n\kappa'_n} |\mathcal{M}|^2_{\kappa_1\kappa'_1\ldots\kappa_n\kappa'_n}$$

matrix element in new basis

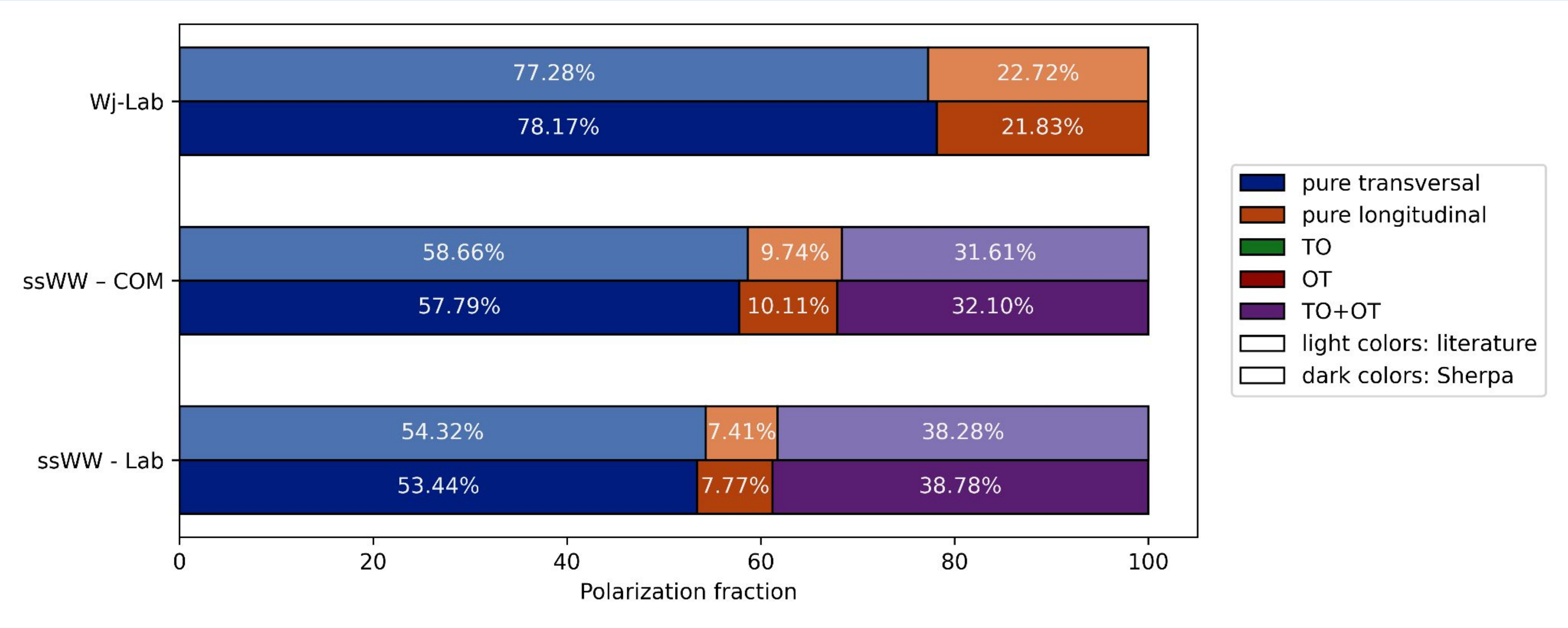
linear combination coefficients

matrix element in default basis

➔ **<u>Calculation of polarized cross sections for VB in different reference frames possible</u>**
currently laboratory frame, VB center of mass frame, parton-parton-frame, special user-defined frames

Validation

Polarization fractions



- **good agreement between literature results and Sherpa**

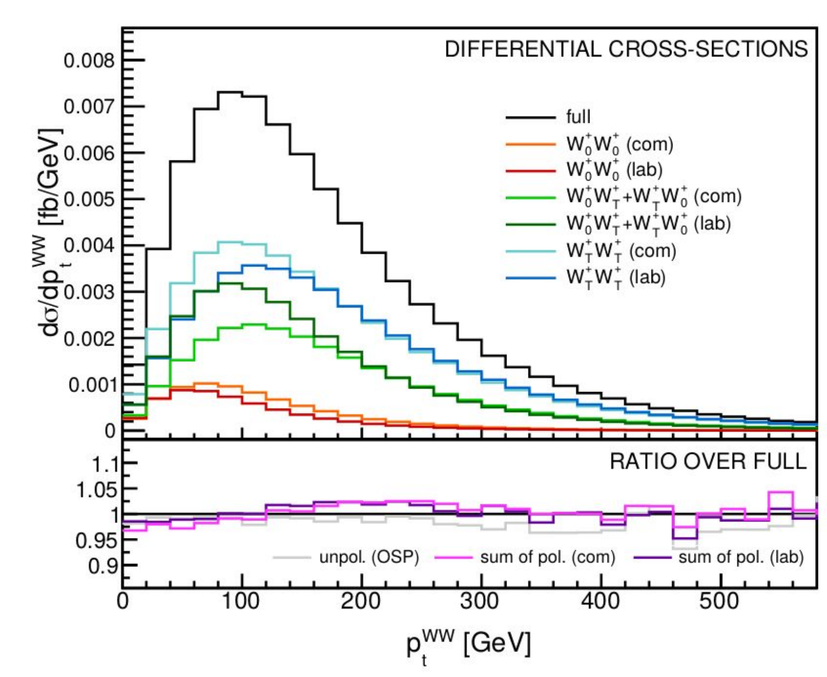
Literature:

- ***Wj:*** *M. Pellen et al., arXiv: 2109.14336 [hep-ph]*
  STRIPPER framework@NNLO, matrix element generation with AvH library, NWA, laboratory frame for polarization definition
- ***ssWW:*** *A. Ballestrero et al., arXiv: 2007.07133v2 [hep-ph]*
  Phantom Monte Carlo Event generator @LO, DPA, laboratory- & VB-COM frame for polarization definition

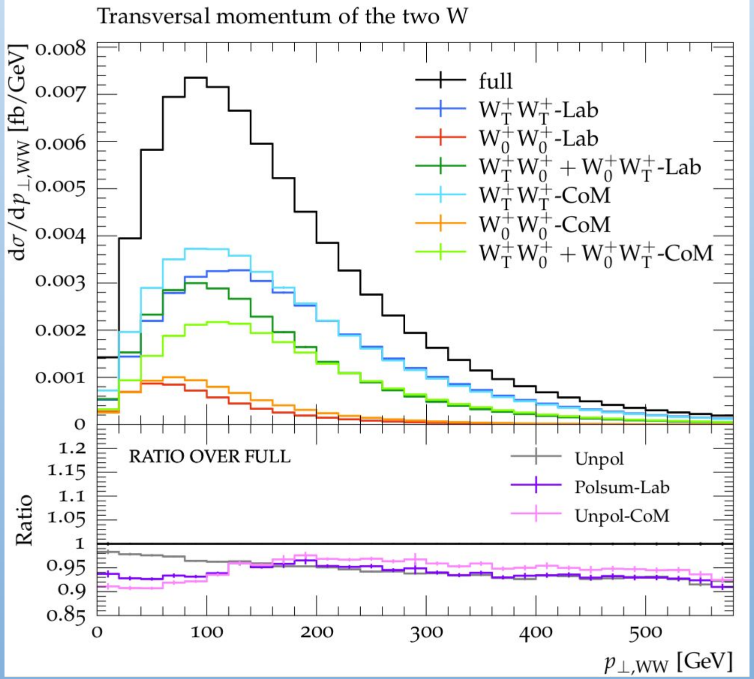
Differential cross sections : Same sign WW process

$p_\perp(WW)$

From paper



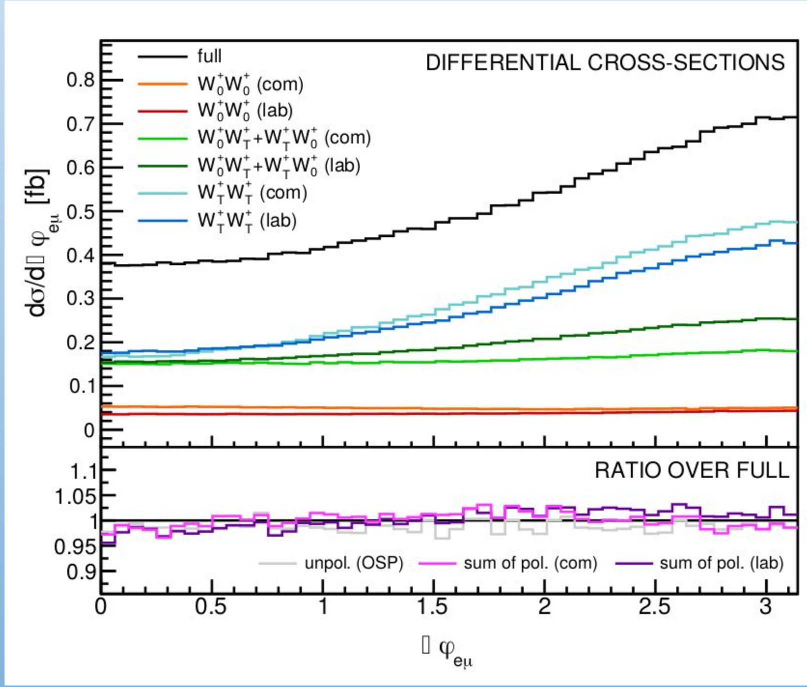
From my own implementation in Sherpa



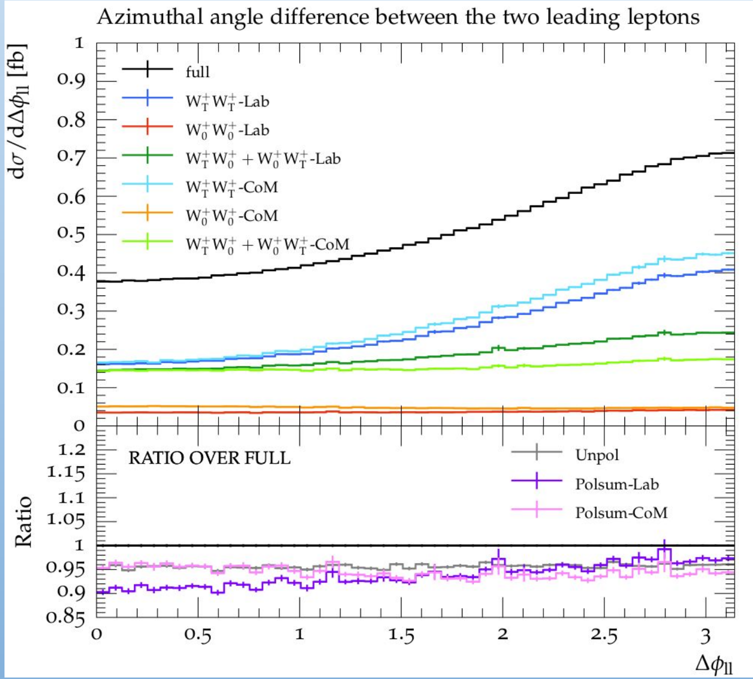
- Shapes, relative contributions of different polarizations to the full, contributions of same polarization with different polarization definitions relative to each other seem to be following the same trend
- Normalization does not fit exactly maybe due to different approximations DPA vs. NWA

$\Delta\phi(ll)$

From paper



From my own implementation in Sherpa

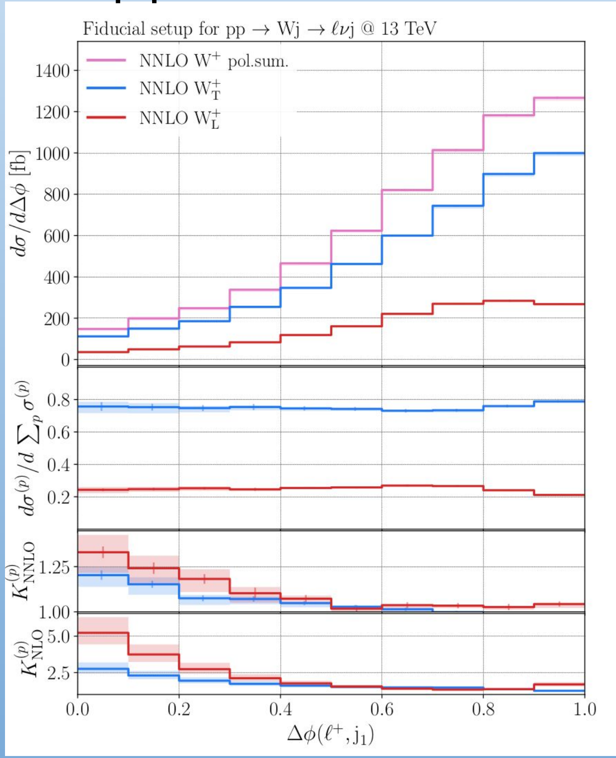


- Shapes, relative contributions of different polarizations to the full, contributions of same polarization with different polarization definitions relative to each other seem to be following the same trend
- Normalization does not fit exactly maybe due to different approximation DPA vs. NWA

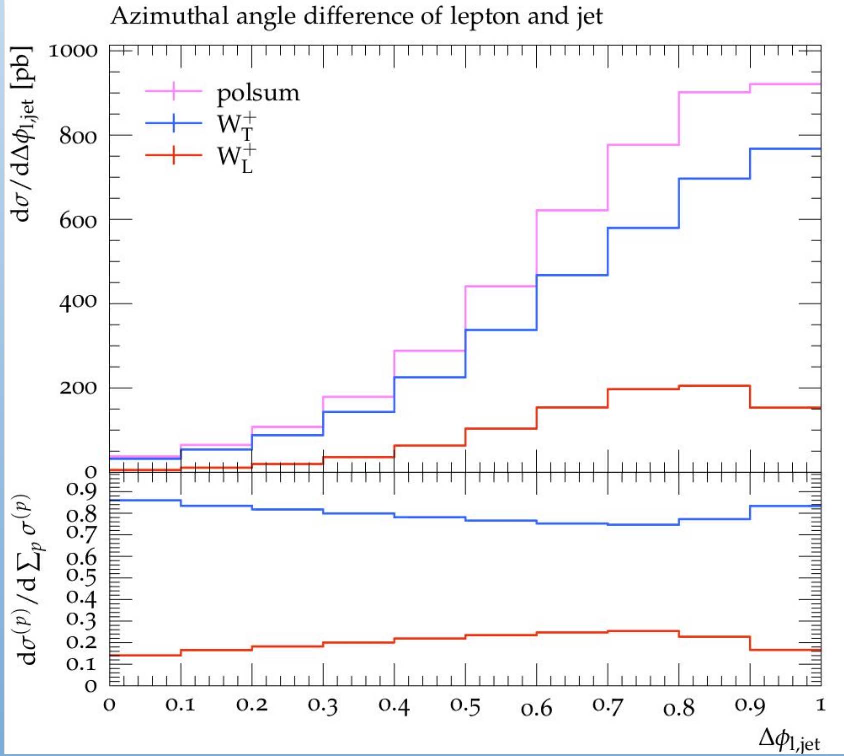
Differential cross sections: Single W process

$\Delta\phi_{\text{l, jet}}$

From paper



From my implementation

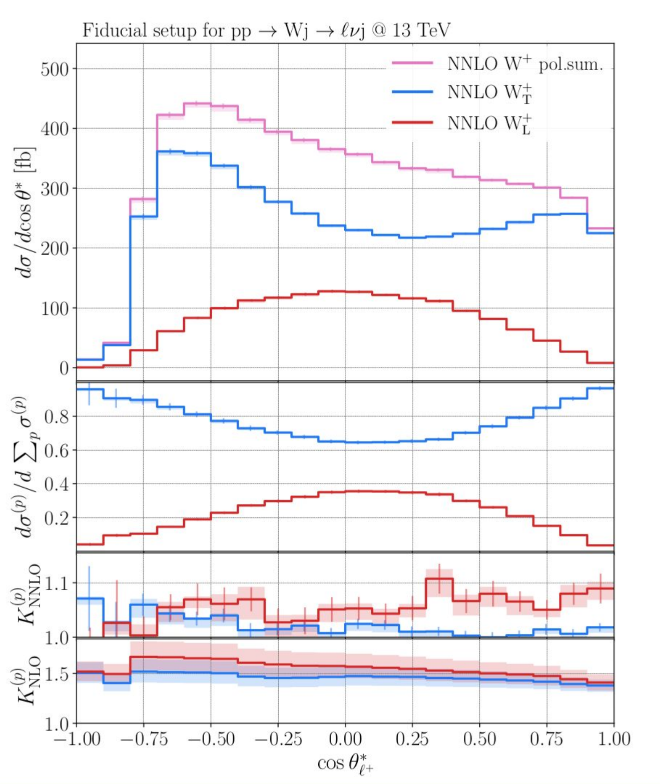


- Shapes are similar
- differences in relative contributions of different polarizations to the polsum and in shape maybe due to NNLO effects

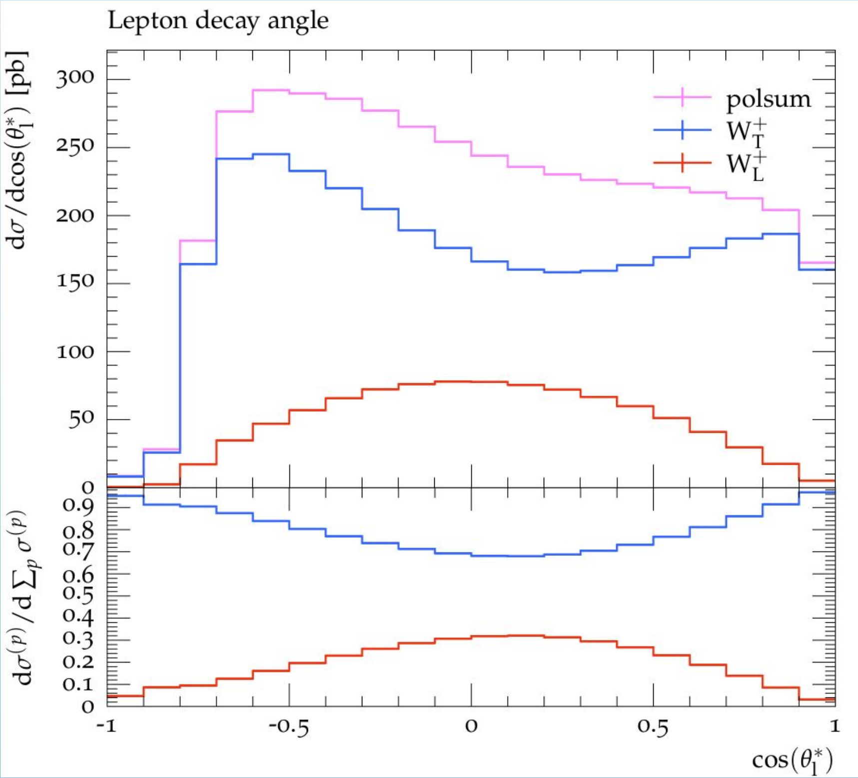
- Differential cross section difficult to compare due to higher order QCD effects

$\cos\theta_l^*$

From paper



From my implementation



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Summary and Outlook

What's already done …

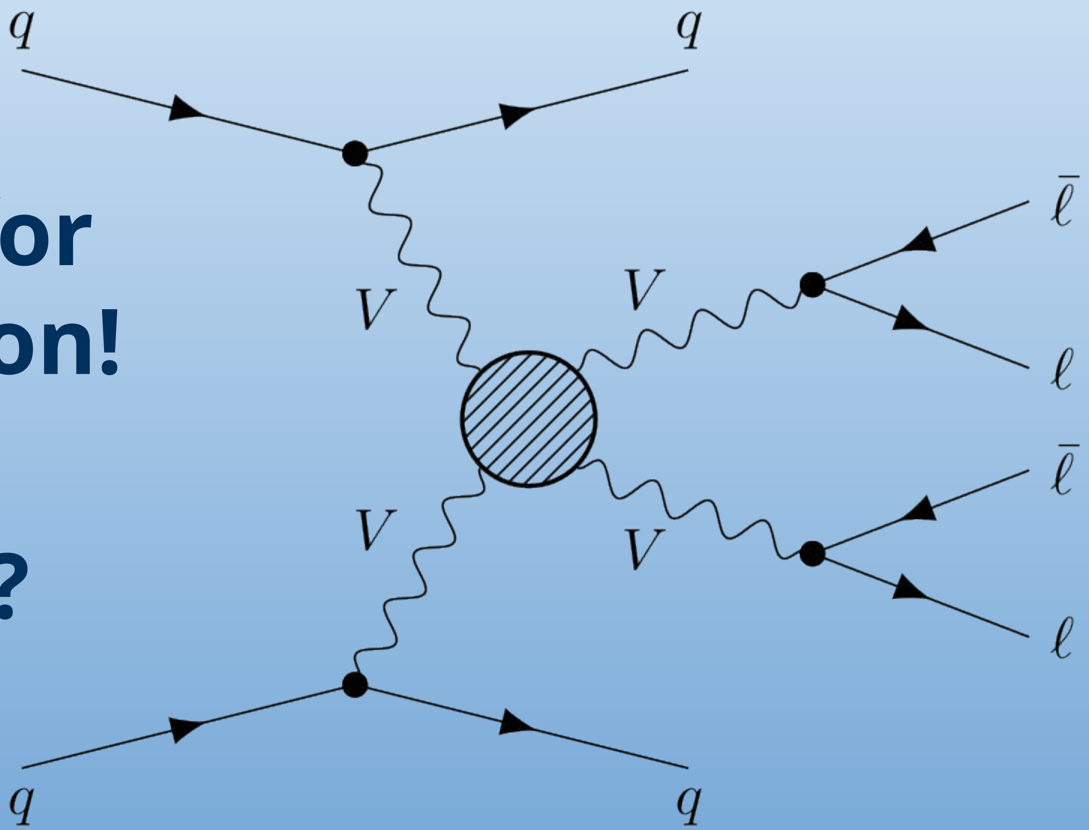
- ✓ **first implementation of polarized cross sections in Sherpa working**
- ✓ **Main features:**
  - all polarized cross sections in one simulation run
  - direct calculation of interference between different polarizations
  - provide several reference frames: laboratory frame, center-of-mass system of the VB, parton-parton frame, frames defined from custom combinations of hard process particles

What comes next …

- ❑ Validation with further vector boson production processes: WZ(jj), W+W-(jj), ZZ(jj)
- ❑ Validation with samples from Madgraph
- ❑ first applications in phenomenological studies e.g. off shell effects or NLO-QCD calculation
- ❑ Preparation for Release

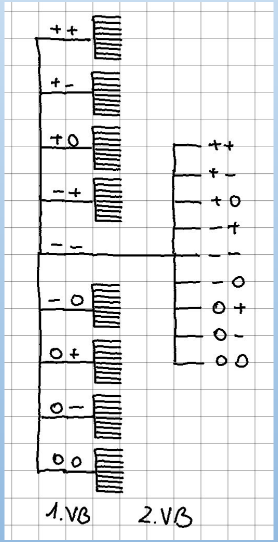
Thank you for your attention!

Questions?



Backup

Calculation of polarized cross sections



hard process

output of matrix element generator (COMIX):

$$\left|\mathcal{M}^{\mathcal{P}}\right|^2_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n} = \mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{P}}_{\lambda'_1\ldots\lambda'_n}$$

structure: tree

- branch = one helicity combination
- level = one particle
- number of starting branches / level = (helicity degrees of freedom)^2
- end of branch: $|\mathcal{M}|^2_{\lambda_1\lambda'_1\ldots\lambda_n\lambda'_n}$

hard decay

spin correlation algorithm generates

★ decay chain
★ decay matrix for each particle

$$\mathcal{D}_{\lambda_\mathcal{A}\lambda'_\mathcal{A}} = \frac{1}{\mathcal{N}_\mathcal{D}}\mathcal{M}^{\mathcal{D}}_{\lambda_\mathcal{A}}\mathcal{M}^{\mathcal{D}*}_{\lambda'_\mathcal{A}}$$

connection with whole decay matrix element:

$$\mathcal{M}^{\mathcal{D}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{D}}_{\lambda'_1\ldots\lambda'_n} \propto \prod_{\mathcal{A}=1}^{n}\mathcal{D}_{\lambda_\mathcal{A}\lambda'_\mathcal{A}}$$

Calculation of full onshell matrix element $\left|\mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n}\right|^2\left|\mathcal{M}^{\mathcal{D}}_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n}\right|^2$ out of $\left|\mathcal{M}^{\mathcal{P}}\right|^2_{\lambda_1\ldots\lambda_n;\lambda'_1\ldots\lambda'_n} = \mathcal{M}^{\mathcal{P}}_{\lambda_1\ldots\lambda_n}\mathcal{M}^{*\mathcal{P}}_{\lambda'_1\ldots\lambda'_n}$ & $\mathcal{D}_{\lambda_\mathcal{A}\lambda'_\mathcal{A}} = \frac{1}{\mathcal{N}_\mathcal{D}}\mathcal{M}^{\mathcal{D}}_{\lambda_\mathcal{A}}\mathcal{M}^{\mathcal{D}*}_{\lambda'_\mathcal{A}}$

1. Search level of current VB A in production matrix element
2. For each branch $\lambda_A\lambda'_A$ starting at this level multiply all matrix elements at its end with same entry of $D_{\lambda_A,\lambda_{A'}}$

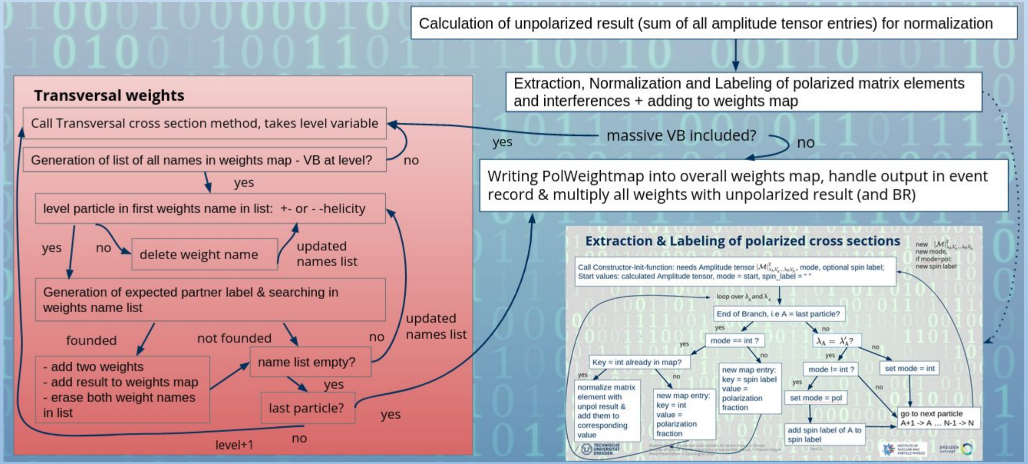
Result: Amplitude tensor with all polarized matrix elements

Output of polarized cross sections

- polarized cross sections of all helicity combinations + interferences between them easily available at same time

➔ only one simulation run necessary

➔ different to e.g. Madgraph



What's missing?

1. separation between polarized cross sections & interferences between different polarizations
2. labeling of polarized matrix elements
3. for massive VBs: add + and - contribution to transversal weight

Output

- polarized cross section (& interference) handled & printed out as additional weights
- weightnames: particle1.helicity1_particle2.helicity2... (e.g. W+.+_W+.-)

Spin-Correlation Algorithm

Spin-Correlation Algorithm

- **originally invented (and implemented?) for unstable strong interacting particles**

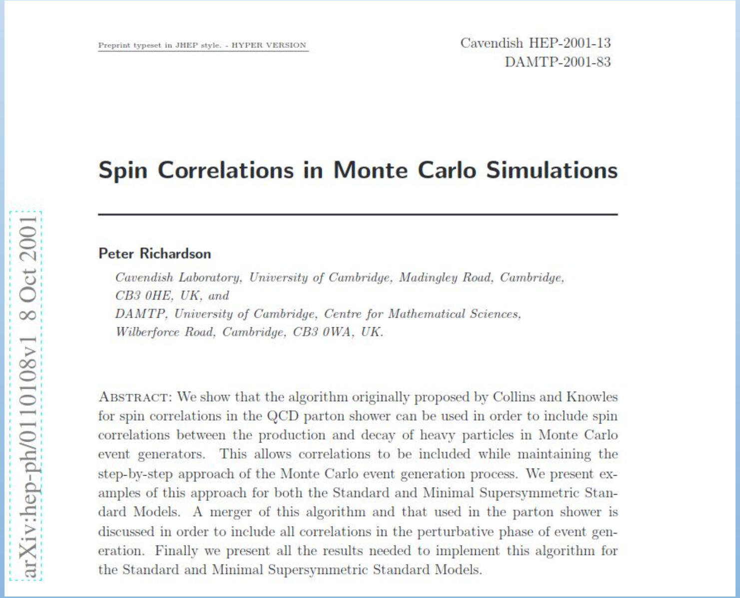
➔ production and decay should be splitted to simulate QCD radiation before decay

➔ generate right kinematics

- **but can also be used to access polarization**
- **implemented in the Hard_Decays-Modul of Sherpa, more concrete Decay_Handler_Base class**
- **algorithm starts after calculating production matrix element**

➔ **starting point: Amplitude2-Tensor**

$$\left|\mathcal{M}^{prod}\right|^2_{\lambda_1\lambda_1'\ldots\lambda_n\lambda_n'}$$



Preprint typeset in JHEP style. - HYPER VERSION

Cavendish HEP-2001-13
DAMTP-2001-83

arXiv:hep-ph/0110108v1 8 Oct 2001

Spin Correlations in Monte Carlo Simulations

Peter Richardson

Cavendish Laboratory, University of Cambridge, Madingley Road, Cambridge, CB3 0HE, UK, and
DAMTP, University of Cambridge, Centre for Mathematical Sciences, Wilberforce Road, Cambridge, CB3 0WA, UK.

Abstract: We show that the algorithm originally proposed by Collins and Knowles for spin correlations in the QCD parton shower can be used in order to include spin correlations between the production and decay of heavy particles in Monte Carlo event generators. This allows correlations to be included while maintaining the step-by-step approach of the Monte Carlo event generation process. We present examples of this approach for both the Standard and Minimal Supersymmetric Standard Models. A merger of this algorithm and that used in the parton shower is discussed in order to include all correlations in the perturbative phase of event generation. Finally we present all the results needed to implement this algorithm for the Standard and Minimal Supersymmetric Standard Models.

P. Richardson: Spin Correlations in Monte Carlo Simulations. J. High Energy Phys. 11 (2001). DOI: 10.1088/1126-6708/2001/11/029, arXiv: hep-ph/0110108

Spin-Correlation Algorithm

- **here only for VB decaying into stable leptons**

hard process final state particles & production (2→n) matrix element tensor $\left|\mathcal{M}^{\mathcal{P}}\right|^2_{\lambda_1 \dots \lambda_n \lambda'_1 \dots \lambda'_n}$

choose one outgoing particle A randomly

↓

Spin density matrix $\rho_{\lambda_j \lambda'_j}(A) = \frac{1}{N_\rho} \mathcal{M}^{\mathcal{P}}{}_{\kappa_1\kappa_2;\lambda_1 \dots \lambda_j \dots \lambda_n} \mathcal{M}^{\mathcal{P}*}_{\kappa_1\kappa_2;\, \lambda'_1 \dots \lambda'_j \dots \lambda'_n} \prod_{i \neq j} \mathcal{D}^i_{\lambda_i \lambda'_i}$

with $\mathcal{D}^i_{\lambda_i \lambda'_i} = \frac{1}{n_{hel}} \delta_{\lambda_i \lambda'_i}$ if particle not chosen yet

choose decay channel of A according to branching ratios

↓

Generate momenta of A's decay products according to $\rho_{\lambda_A \lambda'_A} \mathcal{M}^{\mathcal{D}}_{\lambda_A;\lambda_1 \dots \lambda_n} \mathcal{M}^{\mathcal{D}*}_{\lambda'_A;\, \lambda_1 \dots \lambda_n}$

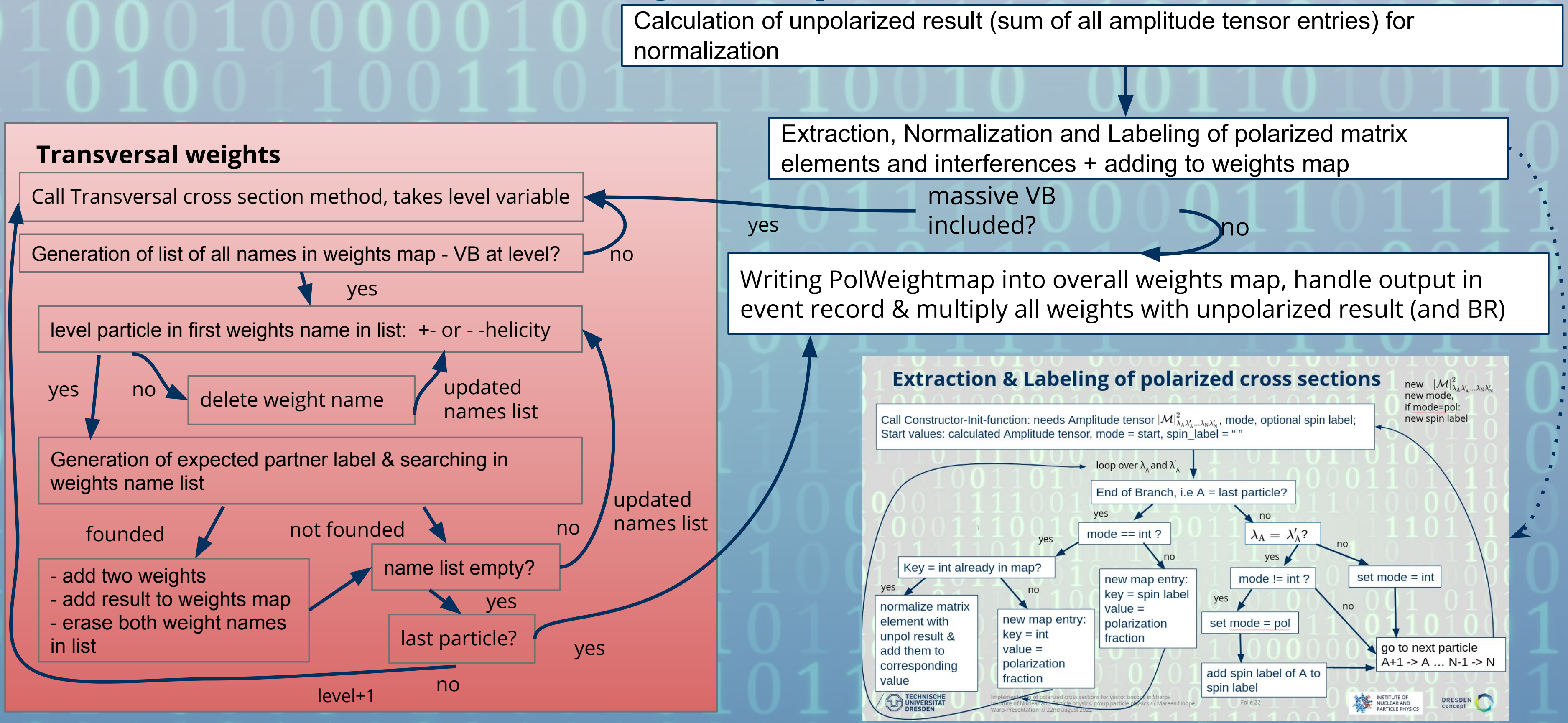
all decay products stable

↓

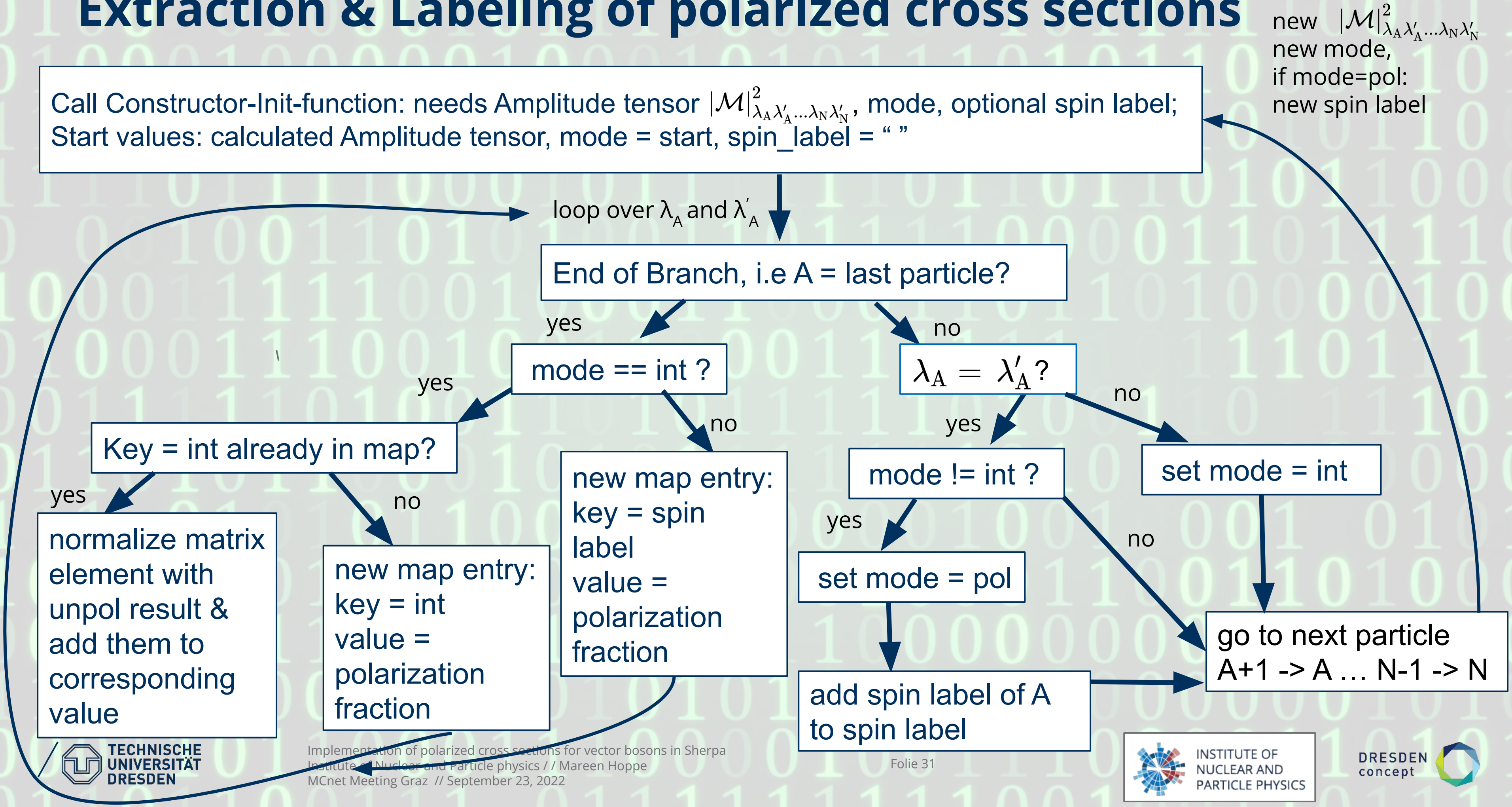
Calculate A's decay matrix $\mathcal{D}_{\lambda_A \lambda'_A} = \frac{1}{\mathcal{N}_{\mathcal{D}}} \mathcal{M}^{\mathcal{D}}_{\lambda_A;\lambda_1 \dots \lambda_n} \mathcal{M}^{\mathcal{D}*}_{\lambda'_A;\lambda_1 \dots \lambda_n}$

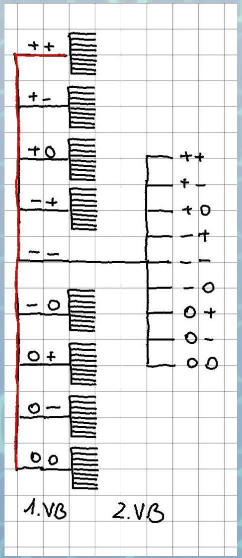
PolWeightMap - Extraction of polarized cross sections out of Amplitude tensor

Generation of PolWeightMap



Extraction & Labeling of polarized cross sections



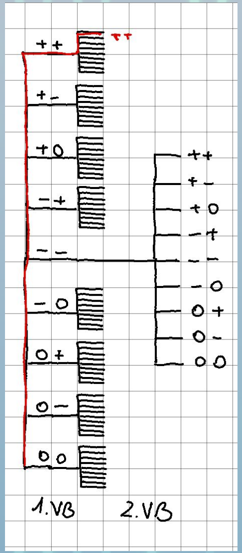


new call

$|\mathcal{M}|^2_{++\lambda_2\lambda_2'}$

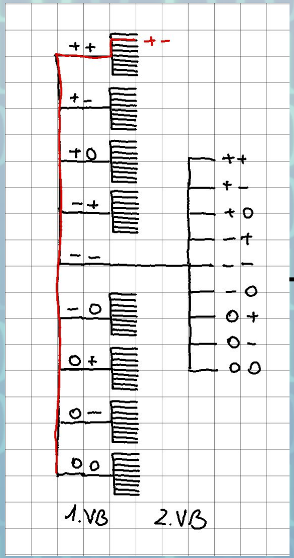
mode=po

spinlabel = VB1.+



key-value-pair:
key: VB1.+_VB2.+
value: $\frac{|\mathcal{M}|^2_{++++}}{|\mathcal{M}|^2_{unpol}}$

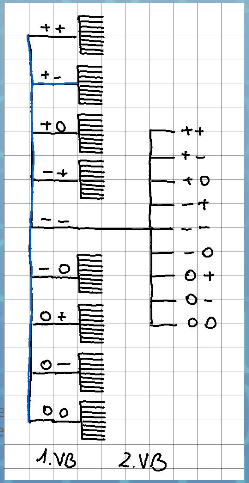
next step in loop



interference contribution

loop over all combinations with VB1 ++ finished

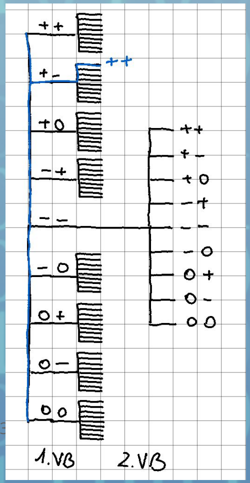
next VB1 branch / helicity combination



new call

$|\mathcal{M}|^2_{+-\lambda_2\lambda_2'}$

mode=int



interference contribution

Weyl-van-der-Waerden formalism

How Sherpa calculates polarization vectors - Weyl-van-der Waerden formalism

- matrix elements contain **different mathematical objects:** spinors, lorentz vectors …

➔ **difficult to calculate** especially for many final state particles

➔ group theory: all matrix element objects can be **described by same mathematical object: Weyl-van-der-Waerden-Spinors** (2D fundamental irreduzible representations of Lorentz group D(½,0) & D(0, ½))
   - ◆ covariant: ψ_A
   - ◆ contravariant: $\psi^{\dot{A}}$

➔ simplifies calculation

➔ discrete symmetries: number of independent matrix elements decreases

Weyl–van-der-Waerden formalism for helicity amplitudes of massive particles

Stefan Dittmaier

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Abstract:

The Weyl–van-der-Waerden spinor technique for calculating helicity amplitudes of massive and massless particles is presented in a form that is particularly well suited to a direct implementation in computer algebra. Moreover, we explain how to exploit discrete symmetries and how to avoid unphysical poles in amplitudes in practice. The efficiency of the formalism is demonstrated by giving explicit compact results for the helicity amplitudes of the processes $\gamma\gamma \to f\bar{f}$, $f\bar{f} \to \gamma\gamma\gamma$, $\mu^-\mu^+ \to f\bar{f}\gamma$.

How Sherpa calculates polarization vectors - Weyl-van-der Waerden formalism

- four vectors: belong to spinor representation D(½,½) = D(½,0) ⊗ D(0,½):

$$K_{\dot{A}B} = k^\mu \sigma_{\mu,\dot{A}B} = \begin{pmatrix} k^0 + k^3 & k^1 + ik^2 \\ k^1 - ik^2 & k^0 - k^3 \end{pmatrix}$$

→ not a spinor decomposition yet

- factorize (not light-like) four vector k^μ into two light-like four vectors:

$$k^\mu = \alpha a^\mu + b^\mu \text{ with } \alpha = \frac{k^2}{2\, a \cdot k}$$

arbitrary choice, reference vector

- transformation to spinor representation:

$$K_{\dot{A}B} = \alpha a_{\dot{A}} a_B + b_{\dot{A}} b_B \quad \text{with} \quad b_A = -\frac{K_{\dot{B}A} a^{\dot{B}}}{\sqrt{K_{\dot{C}D} a^{\dot{C}} a^D}} \quad \text{and} \quad \alpha = \frac{k^2}{K_{\dot{C}D} a^{\dot{C}} a^D}$$

- for polarization vectors:

$$\varepsilon_{+,\dot{A}B}(k) = \frac{\sqrt{2} a_{\dot{A}} b_B}{\langle ab \rangle^{\cdot}} \quad \varepsilon_{-,\dot{A}B}(k) = \frac{\sqrt{2} b_{\dot{A}} a_B}{\langle ab \rangle} \quad \varepsilon_{0,\, \dot{A}B}(k) = \frac{1}{m}\left(b_{\dot{A}} b_B - \alpha a_{\dot{A}} a_B\right)$$

Implementation of Basis-Transformation

Implementation of Basis-Transformation

- Two ways to get Amplitude tensor with polarization vectors defined in new basis:
  - a priori: change polarization definition directly in matrix element generator COMIX
  - **a posteriori: transformation of calculated Amplitude tensor during decay generation**
- change of basis = basis transformation of polarization vectors

➔ polarization vectors with new reference vector = linear combination of default polarization vectors

- Transformation of Amplitude tensor
- $$|\mathcal{M}|^2_{\lambda_1\lambda_1'\ldots\lambda_n\lambda_n'} = \sum_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'} a^{\text{part}1}_{\lambda_1\kappa_1} a^{\text{part}1*}_{\lambda_1'\kappa_1'} \cdots a^{\text{part}n}_{\lambda_n\kappa_n} a^{\text{part}n*}_{\lambda_n'\kappa_n'} |\mathcal{M}|^2_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'}$$

I. Determination of transformation coefficients

- solving system of equations:

$$\begin{pmatrix} \tilde{\varepsilon}^0_+ & \tilde{\varepsilon}^0_- & \tilde{\varepsilon}^0_0 \\ \tilde{\varepsilon}^1_+ & \tilde{\varepsilon}^1_- & \tilde{\varepsilon}^1_0 \\ \tilde{\varepsilon}^2_+ & \tilde{\varepsilon}^2_- & \tilde{\varepsilon}^2_0 \\ \tilde{\varepsilon}^3_+ & \tilde{\varepsilon}^3_- & \tilde{\varepsilon}^3_0 \end{pmatrix} = \begin{pmatrix} \varepsilon^0_+ & \varepsilon^0_- & \varepsilon^0_0 \\ \varepsilon^1_+ & \varepsilon^1_- & \varepsilon^1_0 \\ \varepsilon^2_+ & \varepsilon^2_- & \varepsilon^2_0 \\ \varepsilon^3_+ & \varepsilon^3_- & \varepsilon^3_0 \end{pmatrix} \begin{pmatrix} a_{++} & a_{-+} & a_{0+} \\ a_{+-} & a_{--} & a_{0-} \\ a_{+0} & a_{-0} & a_{00} \end{pmatrix}$$

matrix of new polarization vectors

matrix of default polarization vectors

transformation coefficients

Implementation of Basis-Transformation

- **zeroth component not independent due to** $p^{\mu}\epsilon_{\mu} = 0$

➔ need to consider only spatial components

- **solving equation by inverting matrix of default polarization vectors: using explicit formula for 3x3 invertible matrices**

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} = \frac{1}{\det A} \cdot \begin{pmatrix} ei - fh & ch - bi & bf - ce \\ fg - di & ai - cg & cd - af \\ dh - eg & bg - ah & ae - bd \end{pmatrix}$$

- determination of transformation coefficients must be done for each particle in Amplitude tensor
- result:

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{++} & a^{\text{part1}}_{-+} & a^{\text{part1}}_{0+} \\ a^{\text{part1}}_{+-} & a^{\text{part1}}_{--} & a^{\text{part1}}_{0-} \\ a^{\text{part1}}_{+0} & a^{\text{part1}}_{-0} & a^{\text{part1}}_{00} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{++} & a^{\text{part2}}_{-+} & a^{\text{part2}}_{0+} \\ a^{\text{part2}}_{+-} & a^{\text{part2}}_{--} & a^{\text{part2}}_{0-} \\ a^{\text{part2}}_{+0} & a^{\text{part2}}_{-0} & a^{\text{part2}}_{00} \end{pmatrix} \\ \dots \end{pmatrix}$$

- coefficients only calculated for incoming particles

➔ coefficients for outgoing particles & antiparticles: complex conjugate

Implementation of Spinbasis-Transformation

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{++} & a^{\text{part1}}_{-+} & a^{\text{part1}}_{0+} \\ a^{\text{part1}}_{+-} & a^{\text{part1}}_{--} & a^{\text{part1}}_{0-} \\ a^{\text{part1}}_{+0} & a^{\text{part1}}_{-0} & a^{\text{part1}}_{00} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{++} & a^{\text{part2}}_{-+} & a^{\text{part2}}_{0+} \\ a^{\text{part2}}_{+-} & a^{\text{part2}}_{--} & a^{\text{part2}}_{0-} \\ a^{\text{part2}}_{+0} & a^{\text{part2}}_{-0} & a^{\text{part2}}_{00} \end{pmatrix} \\ \cdots \end{pmatrix}$$

II. Transformation of Amplitude tensor

$$|\mathcal{M}|^2_{\lambda_1\lambda_1'\ldots\lambda_n\lambda_n'} = \sum_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'} a^{\text{part1}}_{\lambda_1\kappa_1} a^{\text{part1}*}_{\lambda_1'\kappa_1'} \cdots a^{\text{part}n}_{\lambda_n\kappa_n} a^{\text{part}n*}_{\lambda_n'\kappa_n'} |\mathcal{M}|^2_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'}$$

→ transformation method is recursive to handle arbitrary number of propagators

Call transformation method, actions steered by level variable

level < # particles

loop over all branches starting at level level of tensor & for each branch delete all coefficients of current particle which does not match helicity combination assigned to current branch

level +1; modifie d coeff matrix

level = # particles

end of branch: calculate new matrix element for helicity combination of this branch

sum up all components of resulting tensor to get new matrix element

level +1; current coeff matrix

level > # particles

loop over all branches starting at level - # particles -1 of old tensor and for each branch delete all coefficients of current particles does not match helicity combination assigned to current branch

level = 2 # particles +1

end of branch of old tensor: multiply all remaining coefficients to current matrix element

level +1; modified coeff matrix

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{\lambda_1+} & a^{\text{part1}}_{\lambda_1-} & a^{\text{part1}}_{\lambda_1 0} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{\lambda_2+} & a^{\text{part2}}_{\lambda_2-} & a^{\text{part2}}_{\lambda_2 0} \end{pmatrix} \\ \cdots \end{pmatrix}^{\mathrm{T}}$$

$$\begin{pmatrix} a_{\lambda_1\lambda_1'} \\ a_{\lambda_2\lambda_2'} \\ \cdots \end{pmatrix}$$

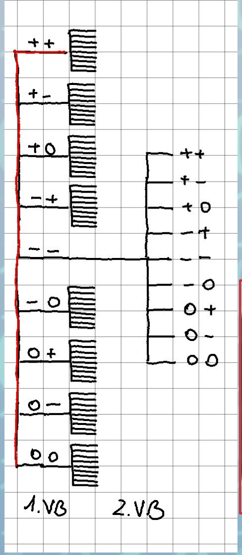
$$|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'} = \sum_{\kappa_1\kappa_1'\kappa_2\kappa_2'} a^{\text{part1}}_{\lambda_1\kappa_1} a^{\text{part1}*}_{\lambda_1'\kappa_1'} a^{\text{part2}}_{\lambda_2\kappa_2} a^{\text{part2}*}_{\lambda_2'\kappa_2'} |\mathcal{M}|^2_{\kappa_1\kappa_1'\kappa_2\kappa_2'}$$

first call

$|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

level=0

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{++} & a^{\text{part1}}_{-+} & a^{\text{part1}}_{0+} \\ a^{\text{part1}}_{+-} & a^{\text{part1}}_{--} & a^{\text{part1}}_{0-} \\ a^{\text{part1}}_{+0} & a^{\text{part1}}_{-0} & a^{\text{part1}}_{00} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{++} & a^{\text{part2}}_{-+} & a^{\text{part2}}_{0+} \\ a^{\text{part2}}_{+-} & a^{\text{part2}}_{--} & a^{\text{part2}}_{0-} \\ a^{\text{part2}}_{+0} & a^{\text{part2}}_{-0} & a^{\text{part2}}_{00} \end{pmatrix} \end{pmatrix}$$

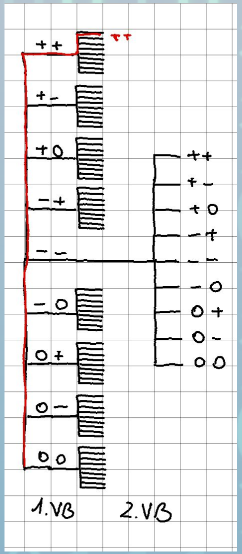


new call

$|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

level=1

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{++} & a^{\text{part1}}_{-+} & a^{\text{part1}}_{0+} \\ a^{\text{part1}}_{+-} & a^{\text{part1}}_{--} & a^{\text{part1}}_{0-} \\ a^{\text{part1}}_{+0} & a^{\text{part1}}_{-0} & a^{\text{part1}}_{00} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{++} & a^{\text{part2}}_{-+} & a^{\text{part2}}_{0+} \\ a^{\text{part2}}_{+-} & a^{\text{part2}}_{--} & a^{\text{part2}}_{0-} \\ a^{\text{part2}}_{+0} & a^{\text{part2}}_{-0} & a^{\text{part2}}_{00} \end{pmatrix} \end{pmatrix}$$



new call

$|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

level=2

$$\begin{pmatrix} \begin{pmatrix} a^{\text{part1}}_{++} \\ a^{\text{part1}}_{+-} \\ a^{\text{part1}}_{+0} \end{pmatrix} \\ \begin{pmatrix} a^{\text{part2}}_{++} \\ a^{\text{part2}}_{+-} \\ a^{\text{part2}}_{+0} \end{pmatrix} \end{pmatrix}$$

end of branch, calculating new matrix element by calling transformation method at $|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

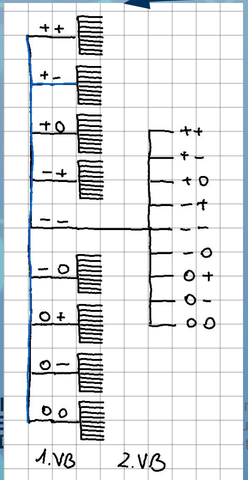
sum over all entries of $|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

$|\mathcal{M}|^2_{\lambda_1\lambda_1'\lambda_2\lambda_2'}$

new call

level=3

...

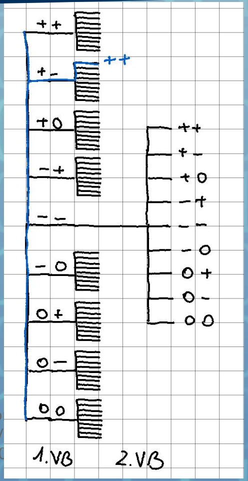


new call

$|\mathcal{M}|^2_{+-\lambda_2\lambda_2'}$

level = level+1

$$\begin{pmatrix} \left(a^{\text{part1}}_{+-}\right) \\ \begin{pmatrix} a^{\text{part2}}_{++} \\ a^{\text{part2}}_{+-} \\ a^{\text{part2}}_{+0} \end{pmatrix} \end{pmatrix}$$



end of branch, multiply $|\mathcal{M}|^2_{+-\lambda_2\lambda_2'}$ by remaining coefficients $\begin{pmatrix} \left(a^{\text{part1}}_{+-}\right) \\ \left(a^{\text{part2}}_{++}\right) \end{pmatrix}$

From spinbasis trafo to reference system trafo

- Iterate through Amplitude2_Tensor and for each particle (layer in tensor) do:

I. **Determination of desired and current polarization vectors**

polarization vectors defined in default frame

Particle's momentum in laboratory frame

polarization vectors defined in new frame

1. Lorentz transformation of momentum in new frame

if spin basis = helicity basis: calculation of reference vector

new polarization short:

$$\varepsilon^{\mu}_{Lab} = \Lambda^{-1\mu}{}_{\nu}\varepsilon^{\nu}_{ref}(\Lambda p_{Lab})$$

3. 2. 1.

Calculation of polarization vectors with momentum in Lab

2. Calculation of new polarization vectors with momentum in new frame

3. Lorentz transformation of polarization vectors back to Lab

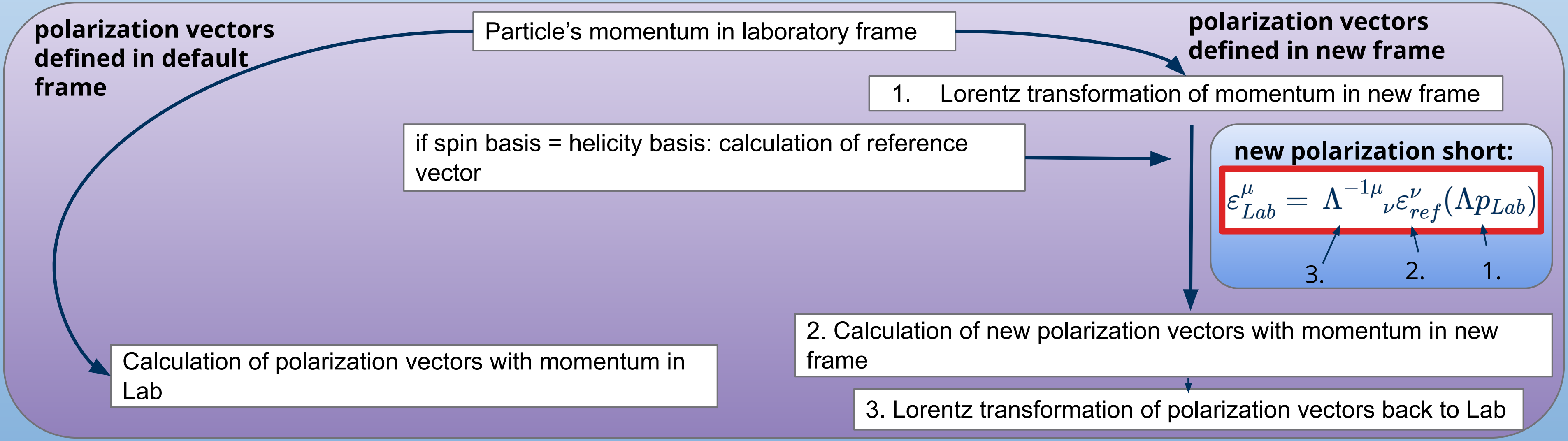
II. **Determination of transformation coefficients** equivalent to transformation of spin basis

- **Transformation of Amplitude tensor** equivalent to transformation of spin basis

Steps towards basis transformation

I. **For each particle in Amplitude tensor:**

1. **Determination of desired and current polarization vectors**
   - changing spin basis: calculation of polarization vector with default & new reference vector
   - changing reference system:



2. **Determination of transformation coefficients**

II. **Transformation of Amplitude tensor**

$$|\mathcal{M}|^2_{\lambda_1\lambda_1'\ldots\lambda_n\lambda_n'} = \sum_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'} a^{\text{part}1}_{\lambda_1\kappa_1} a^{\text{part}1*}_{\lambda_1'\kappa_1'} \cdots a^{\text{part}n}_{\lambda_n\kappa_n} a^{\text{part}n*}_{\lambda_n'\kappa_n'} |\mathcal{M}|^2_{\kappa_1\kappa_1'\ldots\kappa_n\kappa_n'}$$

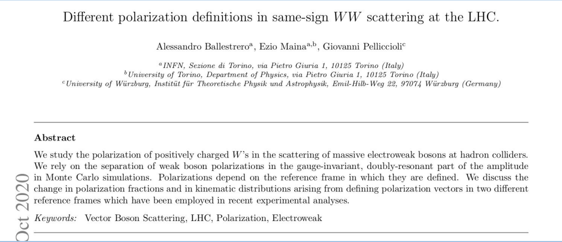
Simulation details

Current status: Validation

- **First implementation finished**
- **Currently testing / validation with different processes / generators**
- **first comparison with**

A. Ballestrero et al. : Different polarization definitions in same-sign WW scattering at the LHC. Physics Letters B, Volume 811, 135856 (2020). DOI: 10.1016/j.physletb.2020.135856, arXiv: 2007.07133v2 [hep-ph]

- ❑ Phantom Monte Carlo Event generator at LO
- ❑ On-Shell projection technique
- ❑ parton level
- ❑ fiducial phase space
- ❑ only $W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$ decays
- ❑ Laboratory- & VB-COM frame for polarization definition


Different polarization definitions in same-sign WW scattering at the LHC.

Alessandro Ballestrero[a], Ezio Maina[a,b], Giovanni Pelliccioli[c]

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Abstract

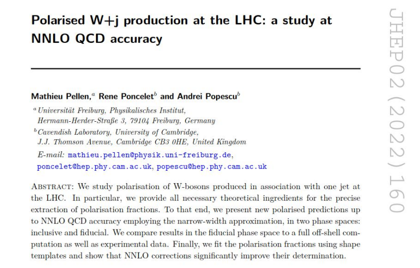
We study the polarization of positively charged W's in the scattering of massive electroweak bosons at hadron colliders. We rely on the separation of weak boson polarizations in the gauge-invariant, doubly-resonant part of the amplitude in Monte Carlo simulations. Polarizations depend on the reference frame in which they are defined. We discuss the change in polarization fractions and in kinematic distributions arising from defining polarization vectors in two different reference frames which have been employed in recent experimental analyses.

Keywords: Vector Boson Scattering, LHC, Polarization, Electroweak

Oct 2020

M. Pellen et al.: polarized W+j production at the LHC: a study at NNLO QCD accuracy. Journal of High Energy Physics (2022), 160. DOI: 10.1007/JHEP02(2022)160, arXiv: 2109.14336 [hep-ph]

- ❑ STRIPPER framework, matrix element generation with AvH library, OPENLOOPS 2, own work at NNLO QCD
- ❑ narrow-width approximation
- ❑ inclusive and fiducial phase space
- ❑ W-decays to e, μ; all results for one decay channel
- ❑ only laboratory frame for polarization definition


Polarised W+j production at the LHC: a study at NNLO QCD accuracy

Mathieu Pellen,[a] Rene Poncelet[b] and Andrei Popescu[b]

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ABSTRACT: We study polarisation of W-bosons produced in association with one jet at the LHC. In particular, we provide all necessary theoretical ingredients for the precise extraction of polarisation fractions. To that end, we present new polarised predictions up to NNLO QCD accuracy employing the narrow-width approximation, in two phase spaces: inclusive and fiducial. We compare results in the fiducial phase space to a full off-shell computation as well as experimental data. Finally, we fit the polarisation fractions using shape templates and show that NNLO corrections significantly improve their determination.

JHEP02(2022)160

Simulation Setup

- **Simulation:** parton level, LO, narrow-width approximation
- **Hard process:** $jj \to W^+W^+jj$
- **Hard decay:** $W^+W^+ \to e^+\nu_e\mu^+\nu_\mu$ $\quad W^+W^+ \to e^+\nu_e e^+\nu_e$ $\quad W^+W^+ \to \mu^+\nu_\mu\mu^+\nu_\mu$
- **PDF-Set:** NNPDF30_lo_as_0130
- **EW-parameter:** G_μ-scheme: m_W= 80.358; m_Z= 91.153; G_F = 1.16637 x 10^{-5} GeV^2
- **Factorization/Renormalization scale:** $\sqrt{p_\perp(j_1)p_\perp(j_2)}$

Phasespace definition

- **Inclusive jet cuts** directly during simulation
  - minimum transversal jet momentum $p_\perp(j) \geq 20\,GeV$
  - maximum jet pseudorapidity $\eta(j) \leq 5$
  - minimum jet-jet-mass $m(jj) \geq 500\,GeV$
  - minimum jet-jet-pseudorapidity separation $|\Delta\eta(jj)| \geq 2.5$
- **Fiducial lepton cuts** implemented in modified Rivet analysis from Carsten Bittrich
  - minimum MET: 40 GeV
  - minimum transverse momentum: 20 GeV
  - maximum pseudorapidity: 2.5

```
BEAMS: 2212
BEAM_ENERGIES: 6500

MI_HANDLER: None  # Amisic
SHOWER_GENERATOR: None
FRAGMENTATION: None
ME_QED: {ENABLED: false}
ME_GENERATORS:
- Comix
BEAM_REMNANTS: false

PDF_LIBRARY: LHAPDFSherpa
PDF_SET: NNPDF30_lo_as_0130
ALPHAS: {USE_PDF: 1}

EW_scheme: Gmu
GF: 0.0000116637
GMU_CMS_AQED_CONVENTION: 4

EVENTS: 500k
EVENT_GENERATION_MODE: Weighted
SCALES: VAR{PPerp(p[4])*PPerp(p[5])} #oder M_W

PARTICLE_DATA:
  24:
    Width: 0
    Mass: 80.358
  23:
    Mass: 91.153

YFS:
   MODE: None

HARD_DECAYS:
  Enabled: true
  Mass_Smearing: 1
  Channels:
    24,12,-11: {Status: 2}
    24,14,-13: {Status: 2}
  QED_Corrections: 0

POL_CROSS_SECTION:
  Enabled: true
  Spinbasis: Helicity
  Referencesystem: Lab

PROCESSES:
- 93 93 -> 24 24 93 93:
    Order: {QCD: 0, EW: 4}

SELECTORS:
- FastjetSelector:
    Expression: Mass(p[4]+p[5])>500
    Expression: abs(Eta(p[4])-Eta(p[5]))>2.5
    Algorithm: antikt
    N: 2
    PTMin: 20.0
    EtaMax: 5.0

ANALYSIS: Rivet
RIVET:
  --analyses:
     - MC_polssWW_Analysis
     - MC_polssWW_Analysis:PHASESPACE=FIDUCIAL
  --ignore-beams: 1
```

Simulation Setup

- **Simulation:** parton level, LO
- **Approximation:** narrow-width without mass smearing
- **Hard process:** $jj \to W^+ j$
- **Hard decay:** average of $W^+ \to e^+ \nu_e \quad W^+ \to \mu^+ \nu_\mu$
- **PDF-Set:** NNPDF31_lo_as_0118, $n_f = 5$
- **EW-parameter:** G_μ-scheme: $m_W = 80.3520$; $m_Z = 91.1535$; $G_F = 1.16638 \times 10^{-5}$ GeV^2
- **Factorization/Renormalization scale:**
  $$\mu = \frac{1}{2}\left(\sqrt{M_W^2 + p_{\perp,W}^2} + p_{\perp,j}\right)$$
- **Inclusive jet cuts** directly during simulation
  - minimum transversal jet momentum $p_\perp(j) > 30\,\mathrm{GeV}$
  - maximum jet rapidity $|y(j)| < 2.4$
- **Fiducial charged lepton & jet cuts** implemented in my own Rivet analysis
  - minimum Delta R: $\Delta R(l,\, j) > 0.4$
  - minimum transverse momentum: $p_\perp(l) > 25\,\mathrm{GeV}$
  - maximum pseudorapidity: $|\eta_l| < 2.5$
  - minimum transverse W mass: $M_\perp(W) > 50\,\mathrm{GeV}$

$$M_\perp(W) = \sqrt{M_W^2 + p_{\perp,W}^2} = \sqrt{2 p_{\perp,l} \cdot p_{\perp,\nu}(1 - \cos\Delta\phi)}$$

with

$$\Delta\phi = \min(|\phi_l - \phi_\nu|,\, 2\pi - |\phi_l - \phi_\nu|)$$

```
BEAMS: 2212
BEAM_ENERGIES: 6500

MI_HANDLER: None  # Amisic
SHOWER_GENERATOR: None
FRAGMENTATION: None
ME_QED: {ENABLED: false}
ME_GENERATORS:
- Comix
BEAM_REMNANTS: false
YFS:
  MODE: None
PDF_LIBRARY: LHAPDFSherpa
PDF_SET: NNPDF31_lo_as_0118
SCALES: VAR{0.25*(sqrt(PPerp2(p[2])+Abs2(p[2]))+PPerp(p[3]))*(sqrt(PPerp2(p[2])+Abs2(p[2]))+PPerp(p[3]))}

EW_SCHEME: Gmu
GF: 0.0000116638
GMU_CMS_AQED_CONVENTION: 4

PARTICLE_DATA:
  24:
    Mass: 80.3520
    Width: 0
  23:
    Mass: 91.1535

HARD_DECAYS:
  Enabled: true
  Mass_Smearing: 1
  Channels:
    24,12,-11: {Status: 2}
    24,14,-13: {Status: 2}
  QED_Corrections: 0
POL_CROSS_SECTION:
    Enabled: true
    Spinbasis: Helicity

PROCESSES:
- 93 93 -> 24 93:
    Order: {QCD: 1, EW: 1}

SELECTORS:
- FastjetFinder:
    Algorithm: antikt
    N: 1
    PTMin: 30.0
    YMax: 2.4

ANALYSIS: Rivet
RIVET:
  --analyses:
     - MC_singleW_Analysis
     - MC_singleW_Analysis:PHASESPACE=FIDUCIAL
  --ignore-beams: 1
```

ssWW - total cross sections

Total cross section - inclusive phase space

	$\sigma_{paper} \cdot 2\,[fb]$ *	Ratio [%] *	$\sigma_{Sherpa}[fb]$	Ratio [%] *
full	6.370 ± 0.006		6.3697 ± 0.0024	
unpol	6.334 ± 0.004		6.2225 ± 0.0024	
Lab				
polsum	6.3262 ± 0.0032	100	6.231 ± 0.015	100
int			-0.008 ± 0.015	
0-0	0.5146 ± 0.0006	8.134 ± 0.010	0.5132 ± 0.0026	8.24 ± 0.05
0-T + T-0	2.4796 ± 0.0024	39.20 ± 0.04	2.450 ± 0.005	39.32 ± 0.12
T-T	3.332 ± 0.002	52.67 ± 0.04	3.268 ± 0.005	52.45 ± 0.15
WW-CoM				
polsum	6.3274 ± 0.0029	100	6.370 ± 0.005	100
int			0.001 ± 0.005	
0-0	0.6550 ± 0.0008	10.352 ± 0.014	0.6625 ± 0.0009	10.40 ± 0.016
0-T + T-0	2.0324 ± 0.0020	32.121 ± 0.035	2.0606 ± 0.0021	32.35 ± 0.04
T-T	3.640 ± 0.002	57.53 ± 0.04	3.6469 ± 0.0032	57.25 ± 0.07

Comparing my obtained cross section with the values on the paper (multiplied by a factor 2) we obtain an agreement of better than

Lab: ≤ 2% (cross section)
< 1.5% (ratio)
CoM: ≤ 1.5% (cross section)
< 1% (ratio)

interference contribution compatible with zero

*errors calculated according to gaussian error propagation; polsum calculated

INSTITUTE OF NUCLEAR AND PARTICLE PHYSICS

DRESDEN concept

Total cross section - inclusive phase space

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Comparing my obtained cross section with the values on the paper (multiplied by a factor 2) we obtain an agreement of better than

Lab: ≤ 2% (cross section)
< 1.5% (ratio)
CoM: ≤ 1.5% (cross section)
< 1% (ratio)

interference contribution compatible with zero

*errors calculated according to gaussian error propagation; polsum calculated

Total cross section - fiducial phase space

	$\sigma_{paper} \cdot 2\,[fb]$ *	Ratio [%] *	$\sigma_{Sherpa}[fb]$	Ratio [%] *
full	3.186 ± 0.004		3.1826 ± 0.0012	
unpol	3.144 ± 0.004		3.0718 ± 0.0016	
Lab				
polsum	3.1998 ± 0.0022	100	3.0239 ± 0.0030	100
0-0	0.2370 ± 0.0002	7.407 ± 0.010	0.2349 ± 0.0004	7.768 ± 0.015
0-T + T-0	1.2248 ± 0.0012	38.28 ± 0.06	1.1728 ± 0.0012	38.78 ± 0.06
T-T	1.7380 ± 0.0018	54.32 ± 0.09	1.6160 ± 0.0018	53.44 ± 0.08
WW-CoM				
polsum	3.1880 ± 0.0022	100	3.0268 ± 0.0031	100
0-0	0.3104 ± 0.0004	9.737 ± 0.016	0.3059 ± 0.0005	10.106 ± 0.019
0-T + T-0	1.0076 ± 0.0012	31.61 ± 0.05	0.9717 ± 0.0010	32.10 ± 0.05
T-T	1.8700 ± 0.0018	58.66 ± 0.08	1.7493 ± 0.0021	57.79 ± 0.09

*errors calculated according to gaussian error propagation; polsum calculated

Comparing my obtained cross section with the values on the paper (multiplied by a factor 2) we obtain an agreement of better than
Lab: ≤ 5.5% (cross section)
≤ 5% (ratio)
CoM: < 6.5% (cross section)
≤ 4% (ratio)

Literature:
A. Ballestrero et al. : Different polarization definitions in same-sign WW scattering at the LHC. arXiv: 2007.07133v2 [hep-ph]

- Phantom Monte Carlo Event generator at LO
- double-pole approximation
- Laboratory- & VB-COM frame for polarization definition

Single W

Total cross section - one lepton decay channel

*errors calculated according to gaussian error propagation; polsum calculated
** calculated from simulation electrons + muons

Lab	σ_{paper} [pb] *	Ratio [%] *	σ_{Sherpa} [pb] **	Ratio [%] *
full	408.69 ± 0.03		431.40 ± 0.09	
unpol	413.83 ± 0.03	100.150 ± 0.010	420.30 ± 0.09	96.872 ± 0.030
polsum	413.21 ± 0.03	100	433.87 ± 0.10	100
L	93.898 ± 0.005	22.7240 ± 0.0020	94.711 ± 0.032	21.829 ± 0.009
T	319.31 ± 0.03	77.275 ± 0.009	339.16 ± 0.08	78.171 ± 0.026

Comparing my obtained cross section with the values on the paper (multiplied by a factor 2) we obtain an agreement of better than
cross section: ≤ 6.5 %
ratio: ≤ 4 %

Open questions

- why full calculation does not fit?
- why is the interference so big?
- why are there differences in polarized cross sections?

Literature:

M. Pellen et al.: polarized W+j production at the LHC: a study at NNLO QCD accuracy, arXiv: 2109.14336 [hep-ph]

- ❏ STRIPPER framework, matrix element generation with AvH library, OPENLOOPS 2, own work at NNLO QCD
- ❏ narrow-width approximation
- ❏ only laboratory frame for polarization definition

Total cross section - one lepton decay channel

*errors calculated according to gaussian error propagation; polsum calculated
** calculated from simulation electrons + muons

Lab	$\sigma_{\text{paper}}\,[\text{pb}]$ *	Ratio [%] *	$\sigma_{\text{Sherpa}}\,[\text{pb}]$ **	Ratio [%] *
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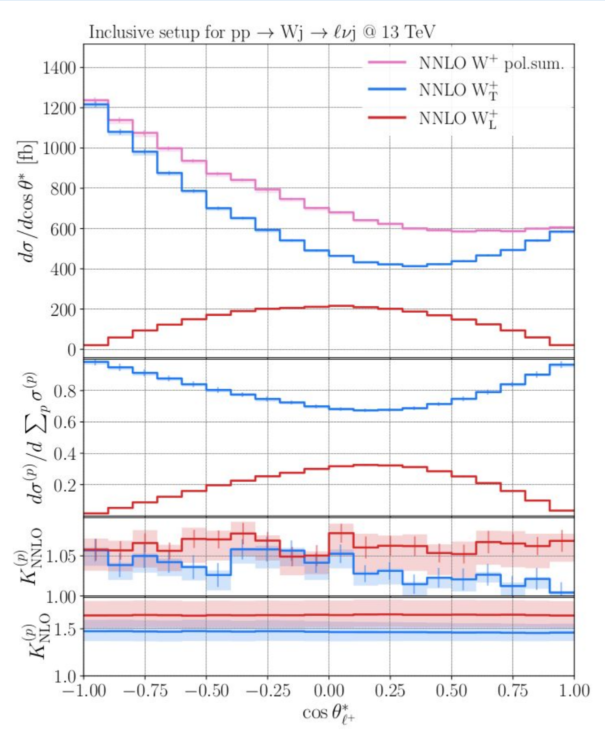
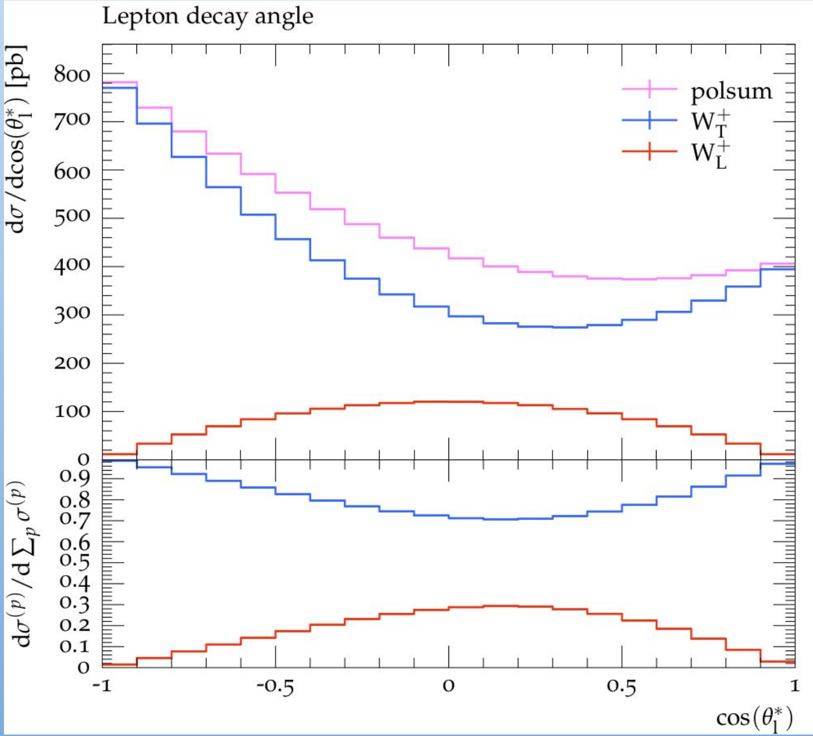
- STRIPPER framework, matrix element generation with AvH library, OPENLOOPS 2, own work at NNLO QCD
- narrow-width approximation
- only laboratory frame for polarization definition

Single W
inclusive phase space

$\cos\theta_l^*$

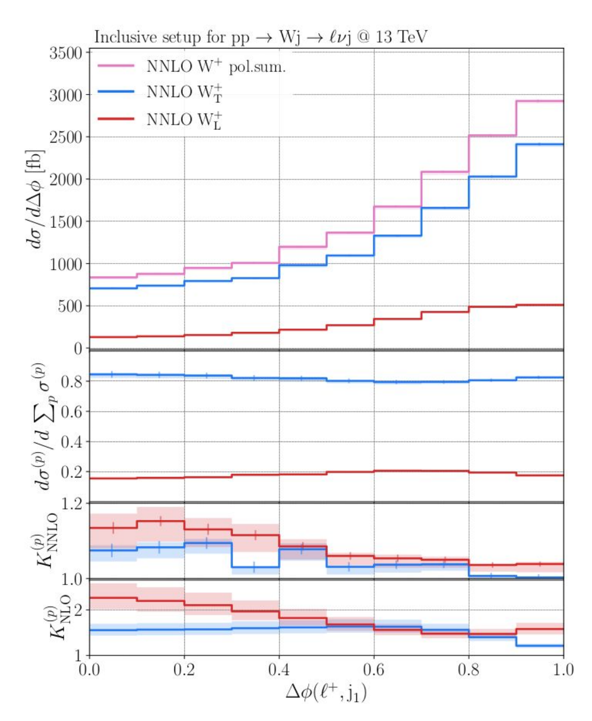


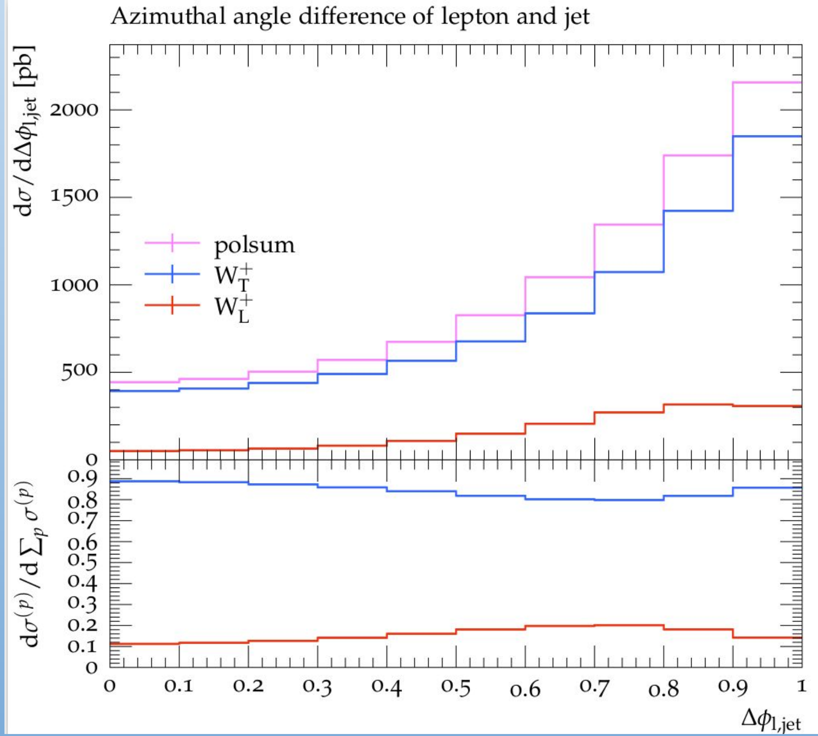
Inclusive setup for pp → Wj → ℓνj @ 13 TeV
NNLO W+ pol.sum.
NNLO W+T
NNLO W+L
dσ/dcos θ* [fb]
dσ(p)/d Σp σ(p)
K(p)NNLO
K(p)NLO
cos θ*ℓ+



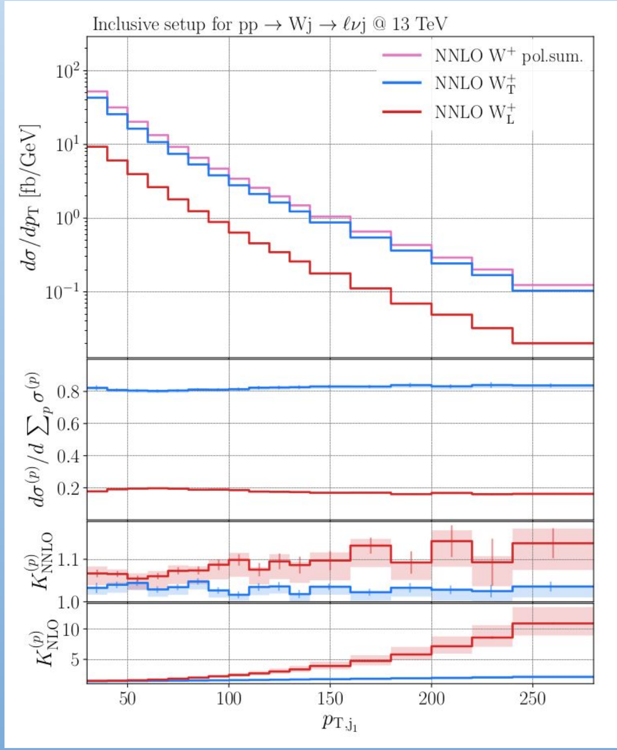
Lepton decay angle
polsum
W+T
W+L
dσ/dcos(θ*l) [pb]
dσ(p)/d Σp σ(p)
cos(θ*l)

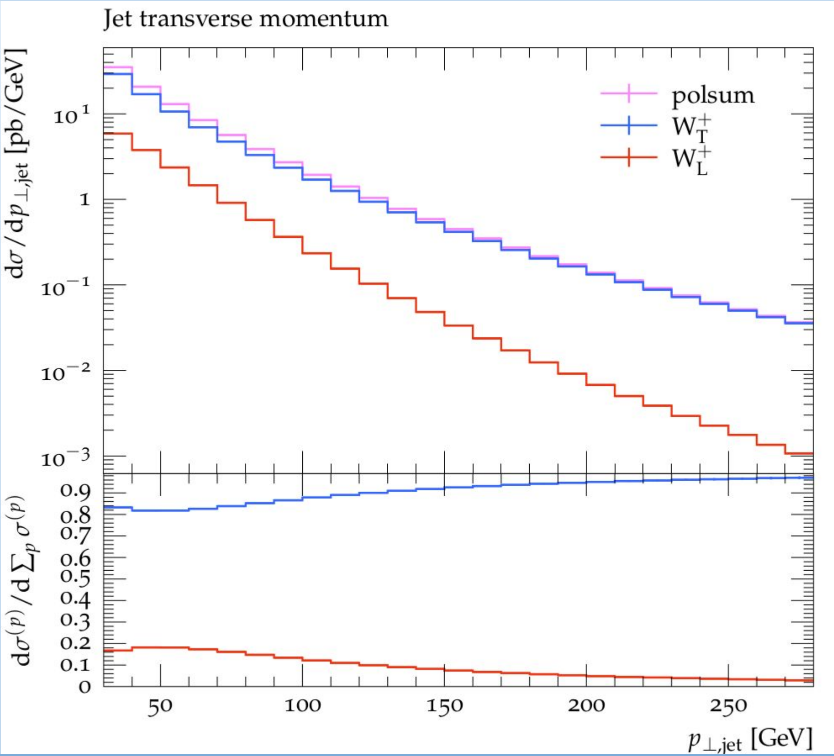
$\Delta\phi_{\mathrm{l,\,jet}}$



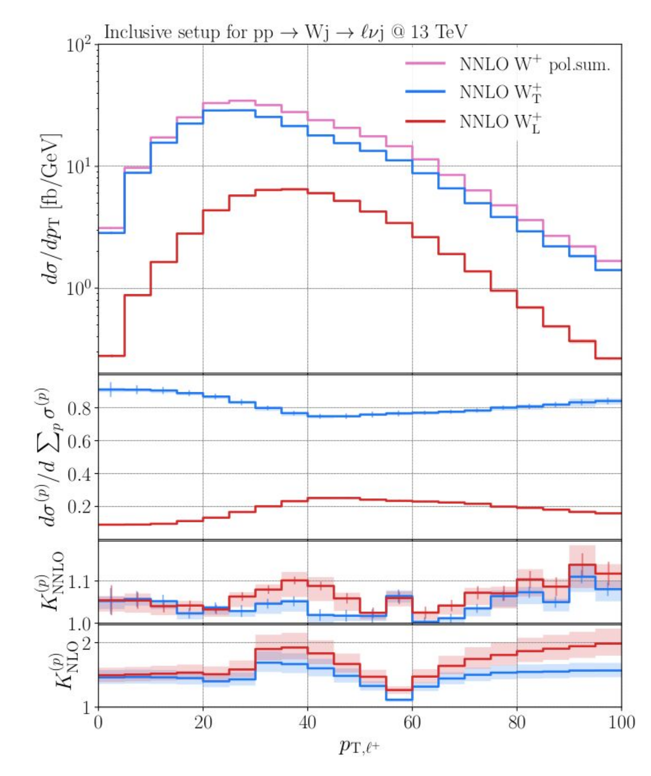


$p_{\perp,\text{jet}}$

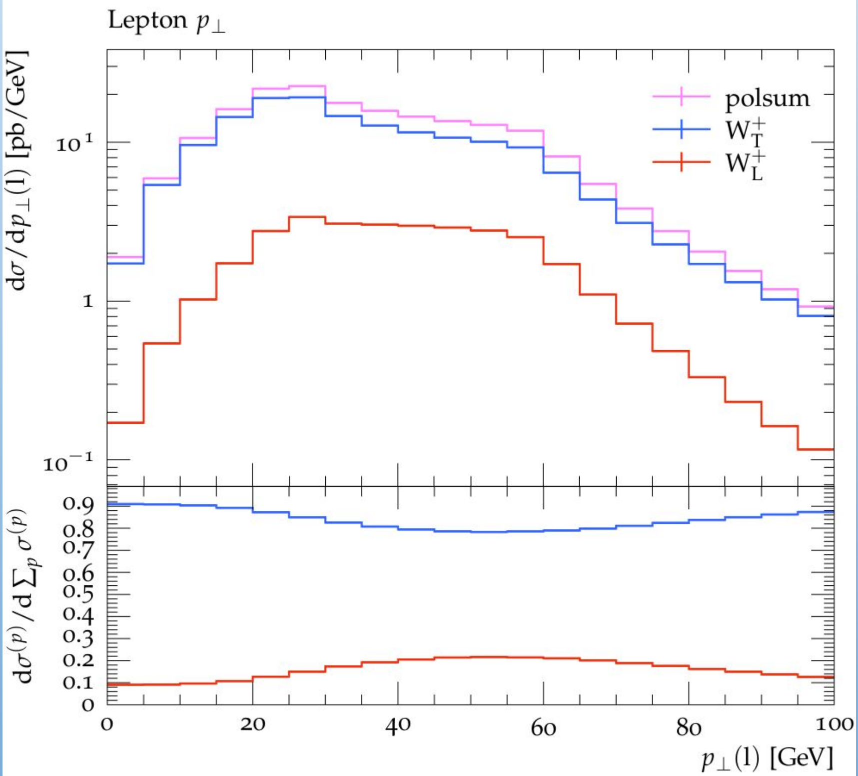




$p_{\perp,1}$

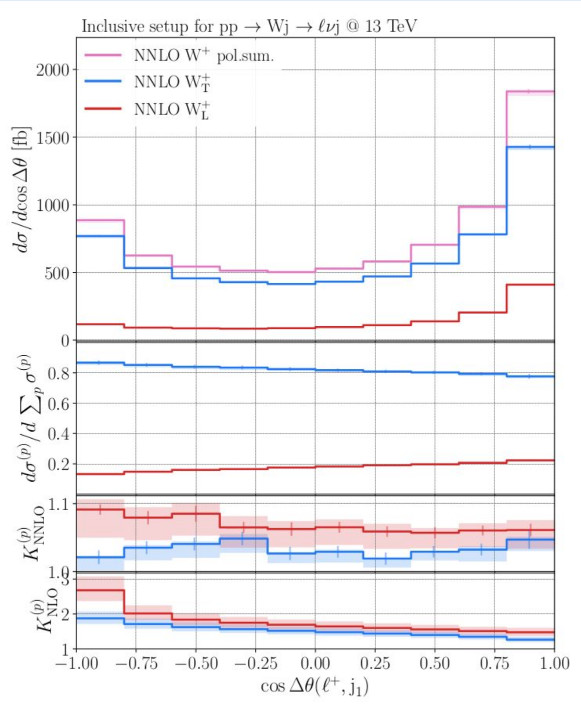


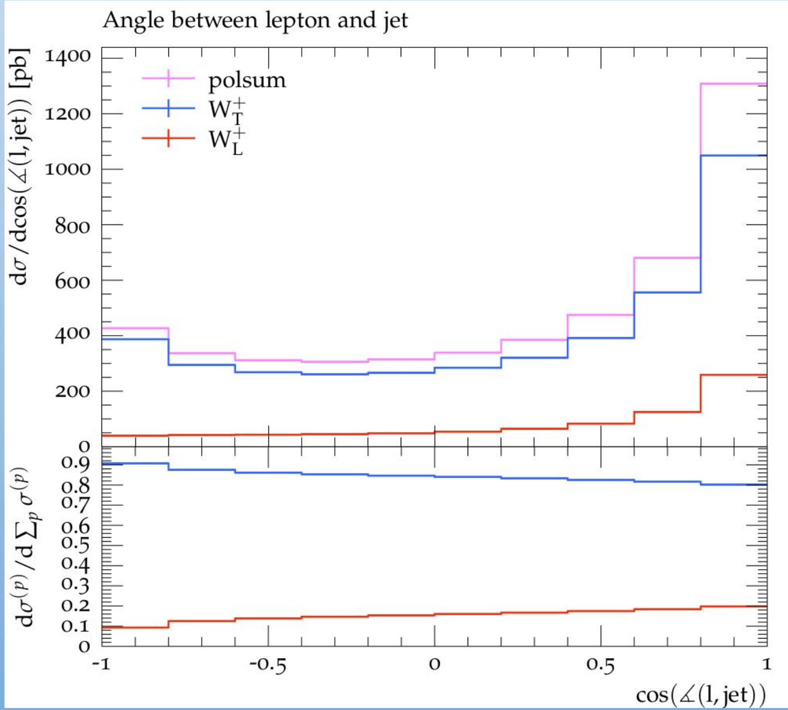
Inclusive setup for pp → Wj → ℓνj @ 13 TeV
NNLO W+ pol.sum.
NNLO W+T
NNLO W+L
dσ/dpT [fb/GeV]
dσ(p)/d Σp σ(p)
K(p)NNLO
K(p)NLO
pT,ℓ+



Lepton p⊥
polsum
W+T
W+L
dσ/dp⊥(l) [pb/GeV]
dσ(p)/d Σp σ(p)
p⊥(l) [GeV]

$\cos(\Delta\theta_{\mathrm{l,jet}})$

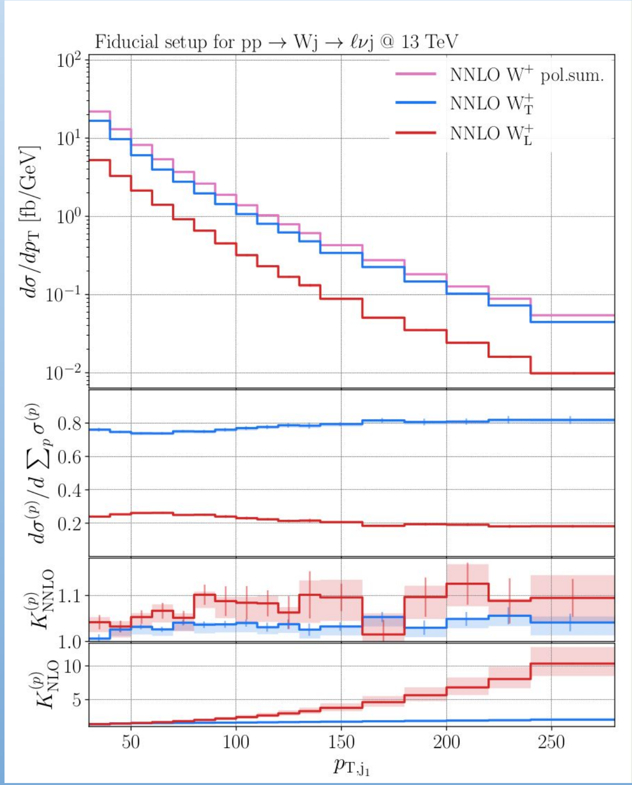


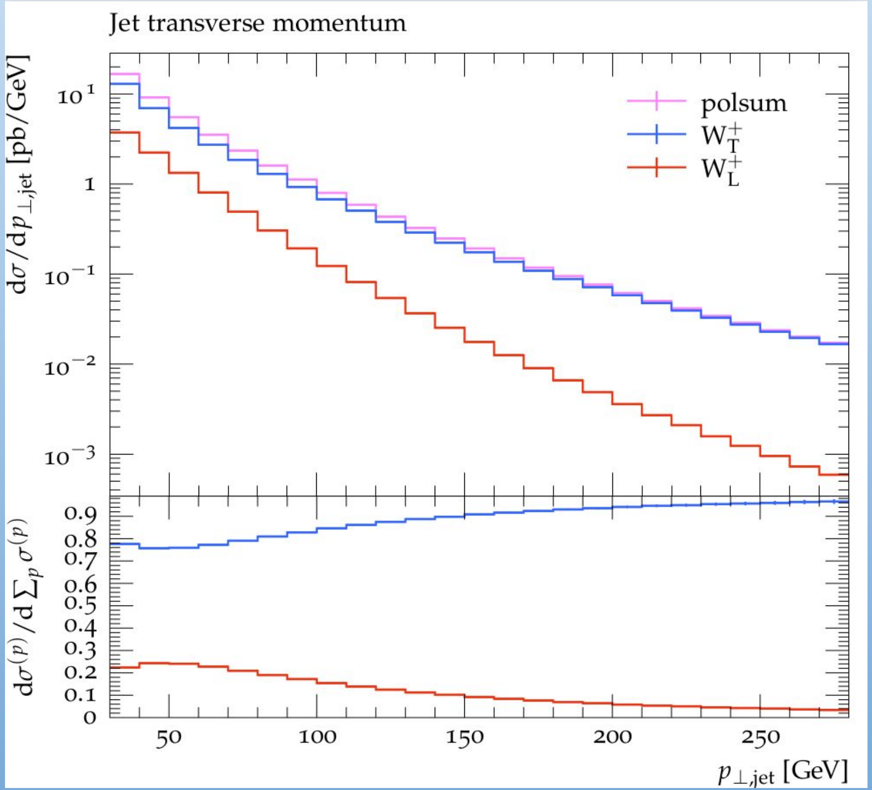


Single W

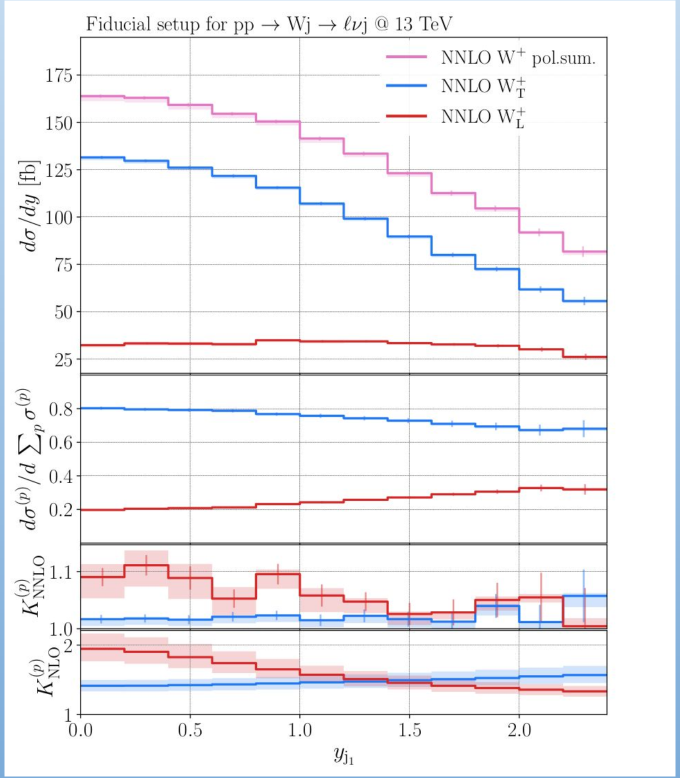
fiducial phase space -
further observables

$p_{\perp,\mathrm{jet}}$

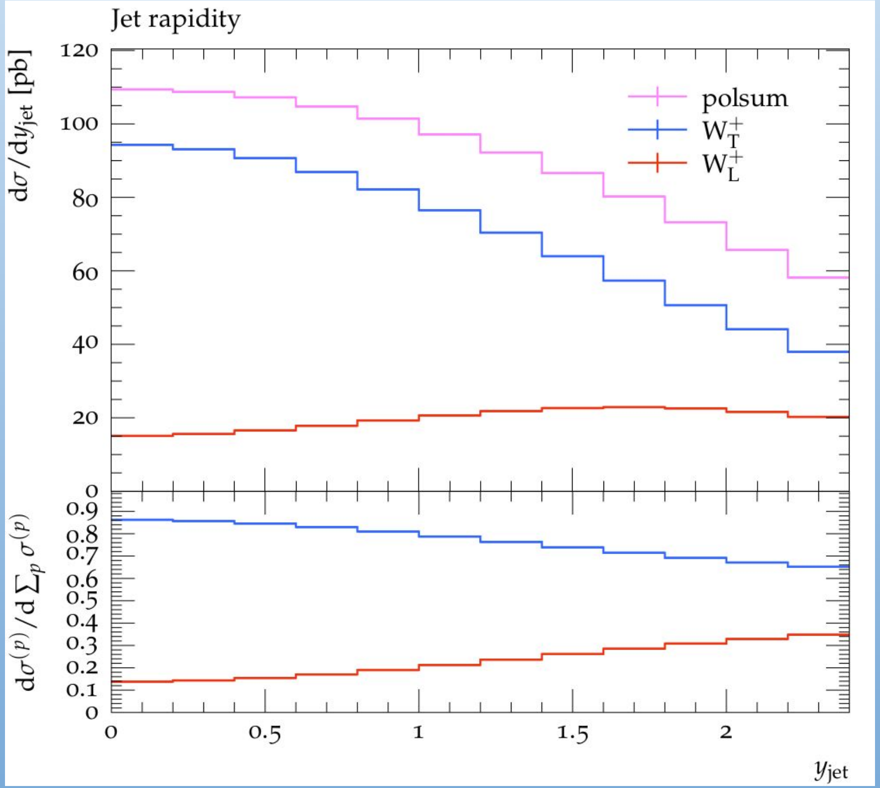




y_j

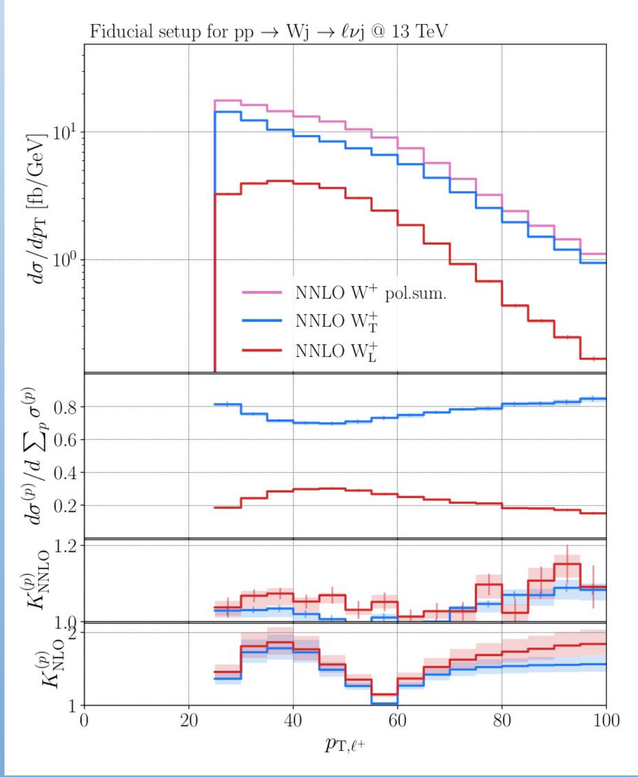


Fiducial setup for pp → Wj → ℓνj @ 13 TeV
NNLO W+ pol.sum.
NNLO W+T
NNLO W+L
dσ/dy [fb]
dσ(p)/d Σp σ(p)
K(p)NNLO
K(p)NLO
yj1

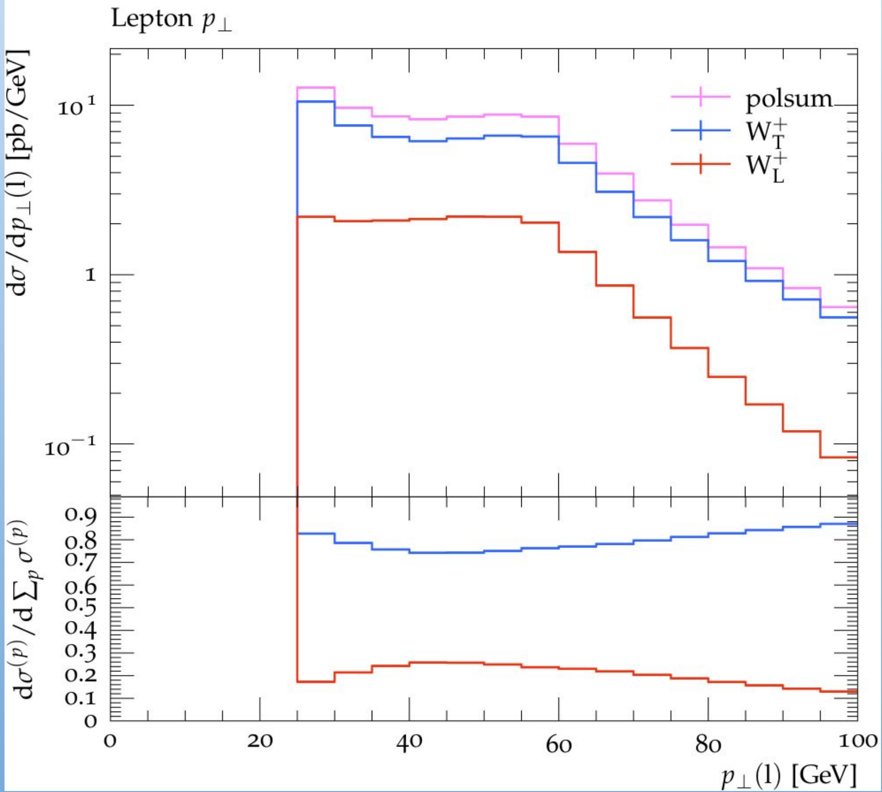


Jet rapidity
polsum
W+T
W+L
dσ/dyjet [pb]
dσ(p)/d Σp σ(p)
yjet

$p_{\perp, l}$

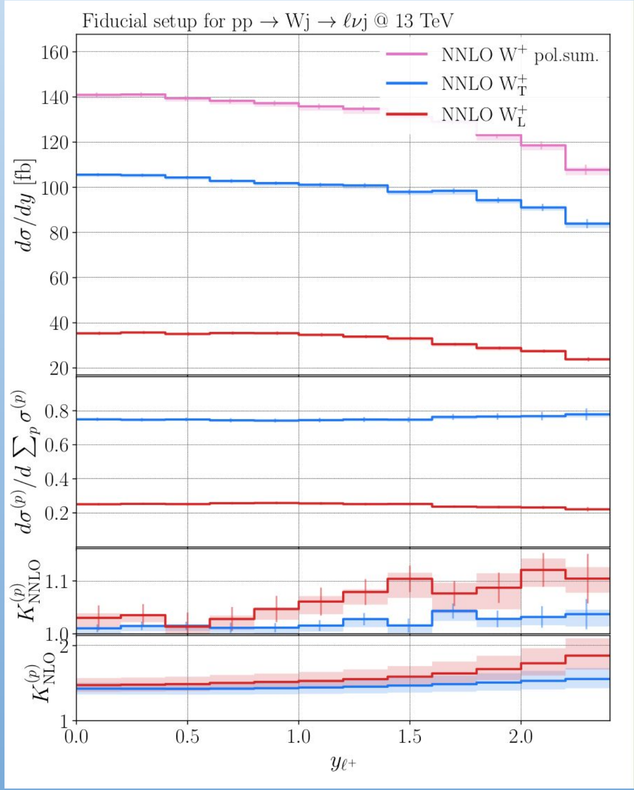


Fiducial setup for pp → Wj → ℓνj @ 13 TeV
NNLO W+ pol.sum.
NNLO W+T
NNLO W+L
dσ/dpT [fb/GeV]
dσ(p)/d ∑p σ(p)
K(p)NNLO
K(p)NLO
pT,ℓ+

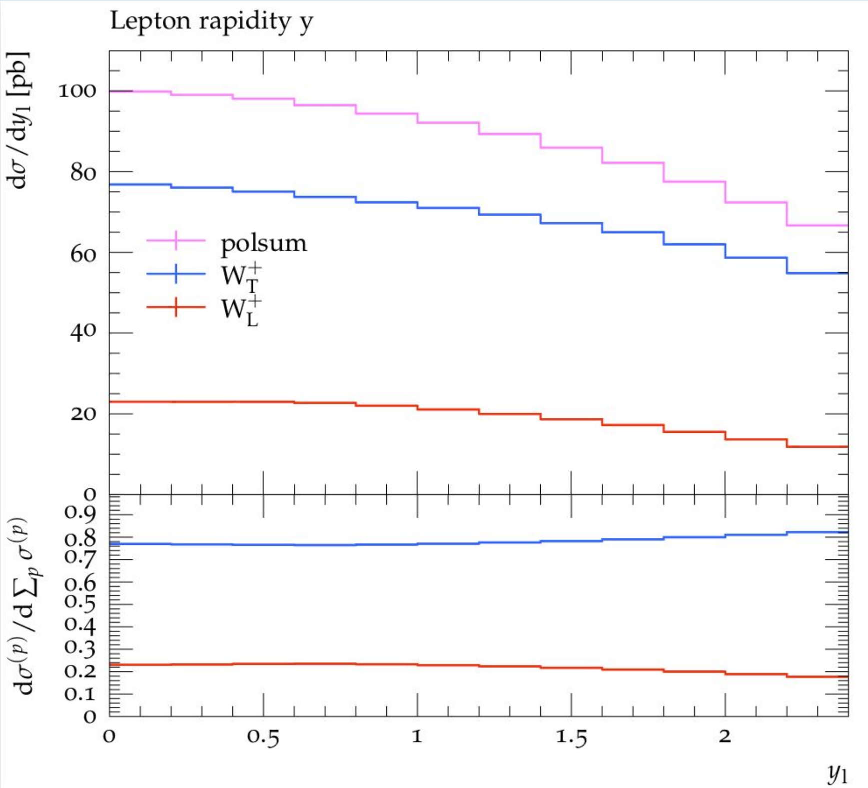


Lepton p⊥
polsum
W+T
W+L
dσ/dp⊥(l) [pb/GeV]
dσ(p)/d ∑p σ(p)
p⊥(l) [GeV]

y_l



Fiducial setup for pp → Wj → ℓνj @ 13 TeV
NNLO W+ pol.sum.
NNLO W+T
NNLO W+L
dσ/dy [fb]
dσ(p)/d Σp σ(p)
K(p)NNLO
K(p)NLO
yℓ+



Lepton rapidity y
dσ / dyl [pb]
polsum
W+T
W+L
dσ(p) / d Σp σ(p)
yl

$\cos(\Delta\theta_{\mathrm{l,jet}})$