THE HENRYK NIEWODNICZAŃSKI
INSTITUTE OF NUCLEAR PHYSICS
POLISH ACADEMY OF SCIENCES

KrkNLO and ‘Lifting the Born’

James Whitehead

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Background

NLO: recap

Hadronic cross-sections:

$$\mathrm{d}\sigma_{AB\to X}(P_1, P_2) = \sum_{a,b} \int_{[0,1]^2} \mathrm{d}\xi_1 \mathrm{d}\xi_2 \; f_a^A(\xi_1)\, f_b^B(\xi_2)\; \mathrm{d}\hat{\sigma}_{ab\to X}(\xi_1 P_1, \xi_2 P_2)$$

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where

$$\begin{aligned}
\mathrm{d}\hat{\sigma}_{ab} &= \left(\frac{\alpha_s}{2\pi}\right)^k \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{B}} \\
&\quad + \left(\frac{\alpha_s}{2\pi}\right)^{k+1} \left[\mathrm{d}\hat{\sigma}_{ab}^{\mathrm{V}} + \mathrm{d}\hat{\sigma}_{ab}^{\mathrm{R}}\right] + \dots \\
&= \mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2)\; \left[\mathrm{B}(\Phi_m) + \mathrm{V}(\Phi_m) + \mathrm{d}\Phi_{+1} \sum \mathrm{S}(\Phi_m \cdot \Phi_{+1})\right] \\
&\quad + \mathrm{d}\Phi_{m+1}(\xi_1 P_1, \xi_2 P_2)\; \left[\mathrm{R}(\Phi_{m+1}) - \sum \mathrm{S}(\Phi_{m+1})\right]
\end{aligned}$$

Anatomy of a subtraction term

Built from dipoles[1] capturing singular limits:

- **final-final** (FF)   phase-space mapping projects onto $\mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2)$

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Anatomy of a subtraction term

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- **final-final** (FF) phase-space mapping projects onto $\mathrm{d}\Phi_m(\xi_1 P_1, \xi_2 P_2)$
- **initial-final, final-initial** (IF,FI)
- **initial-initial** (II)

In the latter two cases the momentum mapping involves a convolution: e.g.

$$\mathrm{d}\Phi_{m+1}(p_1, p_2) = \int_0^1 \mathrm{d}x \quad \mathrm{d}\Phi_m(p_1, x p_2) \quad \mathrm{d}\Phi_{+1}(p_2, x)$$

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Convoluted expressions

Residual convolution arising from integrated subtraction term:

$$\int_0^1 dx\, \hat{\sigma}_{ab}^{\mathrm{NLO}\,\{m\}}(x; xp_a, p_b, \mu_F^2)$$

$$= \sum_{a'} \int_0^1 dx \int_m \left[d\sigma^B_{a'b}(xp_a, p_b) \otimes (\boldsymbol{K} + \boldsymbol{P})^{a,a'}(x) \right]_{\epsilon=0}$$

$$= \sum_{a'} \int_0^1 dx \int d\Phi^{(m)}(xp_a, p_b) F_J^{(m)}(p_1, \ldots, p_m; xp_a, p_b)$$

$$\times_{m,a'b}\langle 1, \ldots, m; xp_a, p_b | \left(\boldsymbol{K}^{a,a'}(x) + \boldsymbol{P}^{a,a'}(xp_a, x; \mu_F^2) \right) | 1, \ldots, m; xp_a, p_b \rangle_{m,a'b}\,, \qquad (10.30)$$

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$$\boldsymbol{K}^{a,a'}(x) = \frac{\alpha_S}{2\pi} \left\{ \overline{K}^{aa'}(x) - K_{\mathrm{FS}}^{aa'}(x) + \delta^{aa'} \sum_i \boldsymbol{T}_i \cdot \boldsymbol{T}_a \frac{\gamma_i}{\boldsymbol{T}_i^2} \left[\left(\frac{1}{1-x} \right)_+ + \delta(1-x) \right] \right\} - \frac{\alpha_S}{2\pi} \boldsymbol{T}_b \cdot \boldsymbol{T}_{a'} \frac{1}{\boldsymbol{T}_{a'}^2} \widetilde{K}^{aa'}(x). \qquad (C.33)$$

KrkNLO

Krk PDFs

Idea:

$$f^{\overline{\mathrm{MS}}} \otimes ([\mathbf{K} + \mathbf{P}] \otimes \mathrm{B}) \equiv (f^{\overline{\mathrm{MS}}} \otimes [\mathbf{K} + \mathbf{P}]) \otimes \mathrm{B}$$

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- use freedom to choose factorisation scheme $\mathbf{K}_{\text{FS}}^{a,a'}$
- to define a new scheme to exactly cancel the convolution terms[2]
- pre-compute the PDFs by performing the convolution
- new PDFs apply to a class of processes
- on-the-fly convolution now unnecessary.

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Krk PDFs in practice: transformation

$$f_a^{\mathsf{FS}}(x;\mu_{\mathsf{F}}) = \sum_b \int_x^1 \frac{\mathrm{d}\xi}{\xi} \quad \mathbb{K}_{ab}^{\overline{\mathsf{MS}}\to\mathsf{FS}}\left(\frac{x}{\xi};\mu_{\mathsf{F}}\right) \; f_b^{\overline{\mathsf{MS}}}(\xi;\mu_{\mathsf{F}}),$$

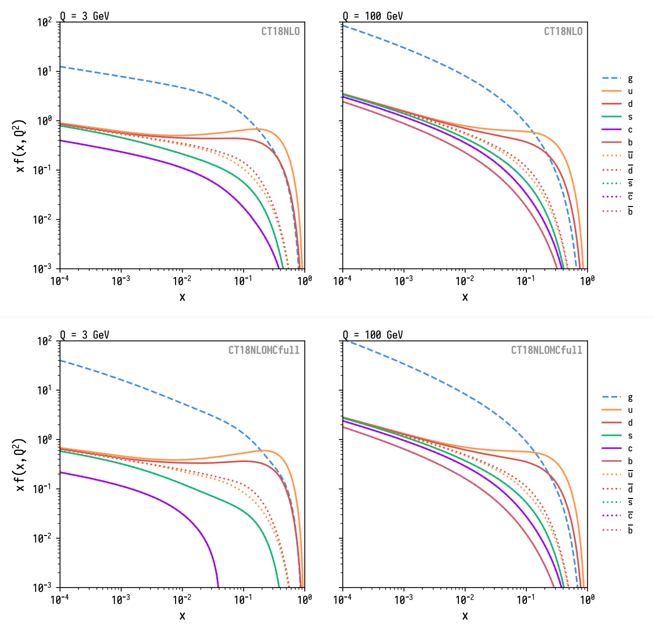
where

$$\mathbb{K}_{ab}^{\overline{\mathsf{MS}}\to\mathsf{FS}}(x;\mu) \equiv \delta_{ab}\,\delta(1-x) + \frac{\alpha_{\mathsf{s}}(\mu)}{2\pi}\,\mathsf{K}_{ab}^{\mathsf{FS}}(x;\mu) + \mathcal{O}\left(\alpha_{\mathsf{s}}^2\right).$$

Krk PDFs: kernels

$$
\begin{aligned}
K^{\mathrm{MC}}_{gq}(z) &= \frac{\alpha_s}{2\pi}\, C_F \left\{ \frac{1+(1-z)^2}{z} \ln \frac{(1-z)^2}{z} + z \right\}, \\
K^{\mathrm{MC}}_{gg}(z) &= \frac{\alpha_s}{2\pi}\, C_A \Bigg\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ + 2\left[\frac{1}{z} - 2 + z(1-z)\right] \\
&\qquad \times \ln \frac{(1-z)^2}{z} - 2\frac{\ln z}{1-z} - \delta(1-z) \\
&\qquad \times \left(\frac{\pi^2}{3} + \frac{341}{72} - \frac{59}{36}\frac{T_f}{C_A}\right) \Bigg\}, \\
K^{\mathrm{MC}}_{qq}(z) &= \frac{\alpha_s}{2\pi}\, C_F \Bigg\{ 4\left[\frac{\ln(1-z)}{1-z}\right]_+ - (1+z) \ln \frac{(1-z)^2}{z} \\
&\qquad -2\frac{\ln z}{1-z} + 1 - z - \delta(1-z)\left(\frac{\pi^2}{3} + \frac{17}{4}\right) \Bigg\}, \\
K^{\mathrm{MC}}_{qg}(z) &= \frac{\alpha_s}{2\pi}\, T_R \left\{ \left[z^2 + (1-z)^2\right] \ln \frac{(1-z)^2}{z} + 2z(1-z) \right\}, \\
K^{\mathrm{MC}}_{g\bar{q}}(z) &= K^{\mathrm{MC}}_{gq}(z), \quad K^{\mathrm{MC}}_{\bar{q}g}(z) = K^{\mathrm{MC}}_{qg}(z). \qquad (4.3)
\end{aligned}
$$

Krk PDFs in practice: examples



Application: NLO matching

Idea:

- dipole shower contributes II, IF, FI splittings
- these arise from the same dipoles
- NLO matching now only requires objects defined in a single copy of Φ_m, Φ_{m+1}

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¨ …**multiplicative weights**

Krk weights

$$\mathrm{d}\Phi_m\,\mathsf{B}\left(1+\frac{\mathsf{V}}{\mathsf{B}}+\frac{1}{\mathsf{B}}\int \mathrm{d}\Phi_{+1} S\right)\quad + \mathrm{d}\Phi_{m+1}\Big[\mathsf{R}-\mathsf{S}\Big]$$

leads to (NLO-accurate)

$$\mathrm{d}\Phi_m\,\mathsf{B}(\Phi_m)\Big[1+\Delta_{\mathsf{VS}}(\Phi_m)\Big]\mathcal{S}(t_{\rm cut},t_\Phi,\Phi_m)\left[\sum_d \int \mathrm{d}\Phi_1\, W_{\mathsf{R}}(\Phi_{m+1})\frac{D_d(\Phi_{m+1})}{\mathsf{B}(\Phi_m^d)}\right]$$

Example: Higgs production[3]

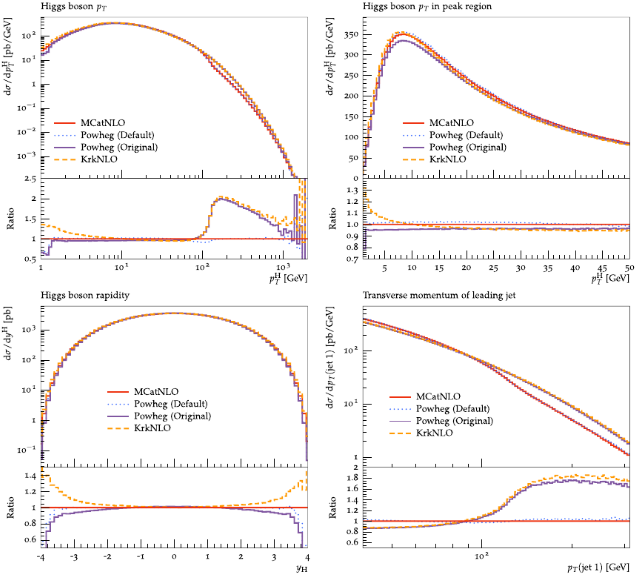

Fig. 5 Comparisons of the Higgs-boson transverse-momentum and rapidity distributions from the KrkNLO, MC@NLO and POWHEG methods implemented in Herwig 7 for Higgs-boson production in gluon–gluon fusion at the LHC; see text for details

[3] S. Jadach et al. "Monte Carlo simulations of Higgs-boson production at the LHC with the KrkNLO method". arXiv: 1607.06799

Ambitions

- more colourless FS processes (automate?)
- $X + j$
- investigate PDF universality, positivity
- combine with MC@NLO?

(KrK+MC@)NLO?[4]

$$
\begin{aligned}
&\mathrm{B}(\Phi_m)(1+\Delta_{\mathrm{VS}})\left[\mathcal{S}(t_{\mathrm{cut}}, t_\Phi; \Phi_m) \times \frac{\mathrm{S}(\Phi_{m+1})}{\mathrm{B}(\Phi_m)}\right] \times \left[\Theta[\mathrm{S} > \mathrm{R}]\frac{\mathrm{R}(\Phi_{m+1})}{\mathrm{S}(\Phi_{m+1})}\right] \mathrm{d}\Phi_1\,\mathrm{d}\Phi_m \\
&+ \mathrm{B}(\Phi_m)(1+\Delta_{\mathrm{VS}})\left[\mathcal{S}(t_{\mathrm{cut}}, t_\Phi; \Phi_m) \times \frac{\mathrm{S}(\Phi_{m+1})}{\mathrm{B}(\Phi_m)}\right]\Theta[\mathrm{S} < \mathrm{R}]\,\mathrm{d}\Phi_1\,\mathrm{d}\Phi_m \\
&+ (1+\Delta_{\mathrm{VS}})\Big[\mathrm{R} - \mathrm{S}\Big]\Theta[\mathrm{S} < \mathrm{R}]\,\mathrm{d}\Phi_1\,\mathrm{d}\Phi_m
\end{aligned}
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4 Paolo Nason and Gavin P. Salam. "Multiplicative-accumulative matching of NLO calculations with parton showers". arXiv: 2111.03553 [hep-ph].

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...positivity?

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‘Lifting the Born’

Idea

Final-final mappings:

$$\mathrm{d}\Phi_m(p_1, p_2) \quad \mathrm{B}(\Phi_m) \qquad \equiv \qquad \mathrm{d}\Phi_{m+1}(p_1, p_2)\, \frac{\mathrm{B}(\tilde{\Phi}_m)}{\mathrm{vol}\Phi_{+1}}$$

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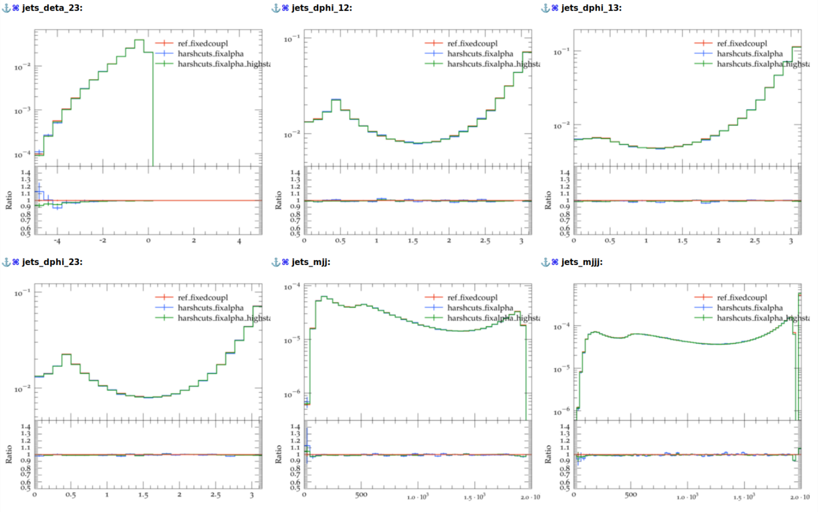
Possible for IF/FI, II? Work ongoing.

Example: $e^+e^- \to 3j$



Thank you!