

Status and progress of lattice QCD

KEK, Sokendai, Nagoya Univ. KMI

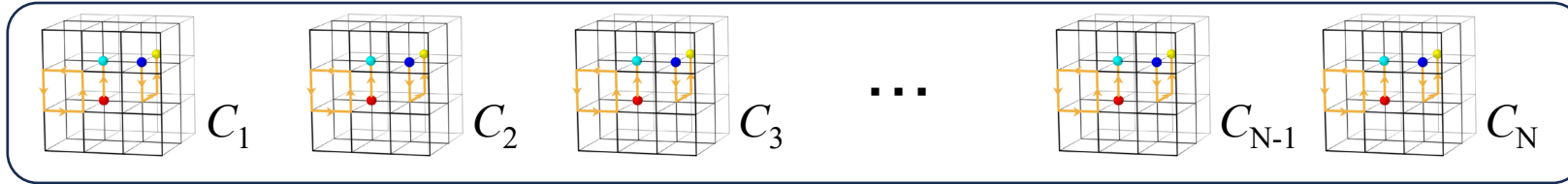
Takashi Kaneko



The 12th International Workshop on CKM Unitarity Triangle,
Santiago de Compostela, Sep 18, 2023

lattice QCD simulation

on finite-volume

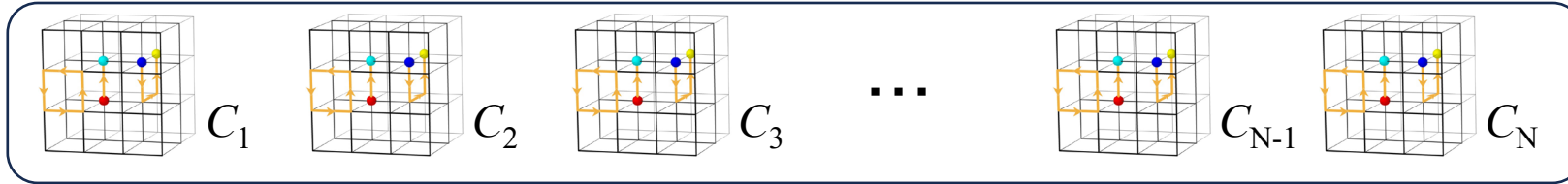


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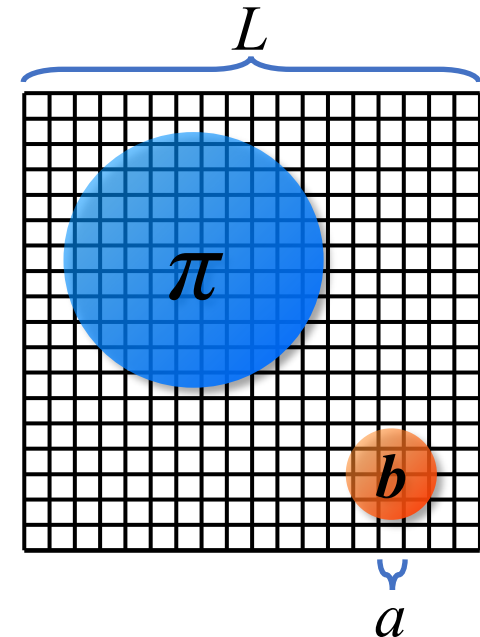


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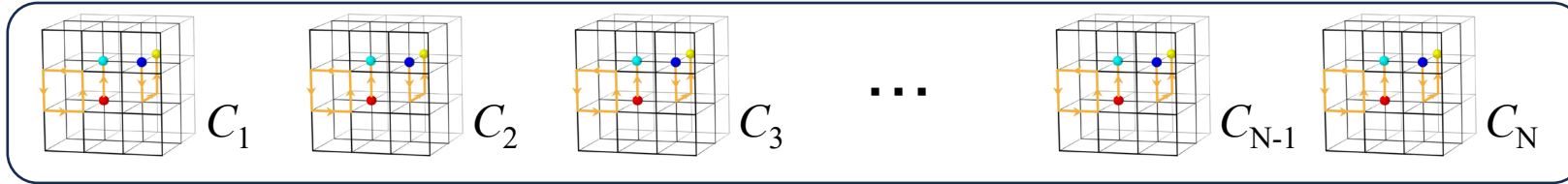
w/ bottom quarks

- multi-scale problem : $a \ll m_b^{-1} \ll M_\pi^{-1} \ll L \Rightarrow L/a \geq \mathcal{O}(100) \Leftrightarrow \text{cost} \propto (L/a)^7$
- relativistic action w/ unphysically small m_b , or effective field theory action for b



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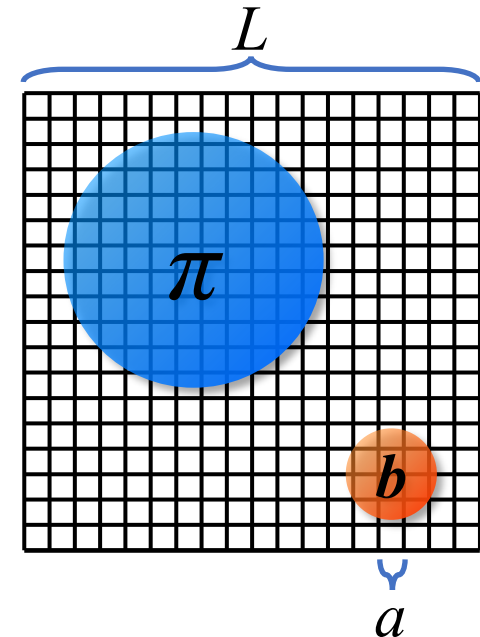


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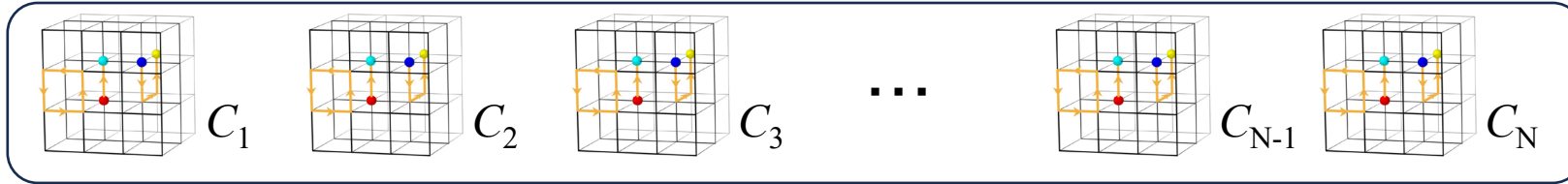
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- need a method to connect finite L Euclidean MEs to infinite L Minkowski ones



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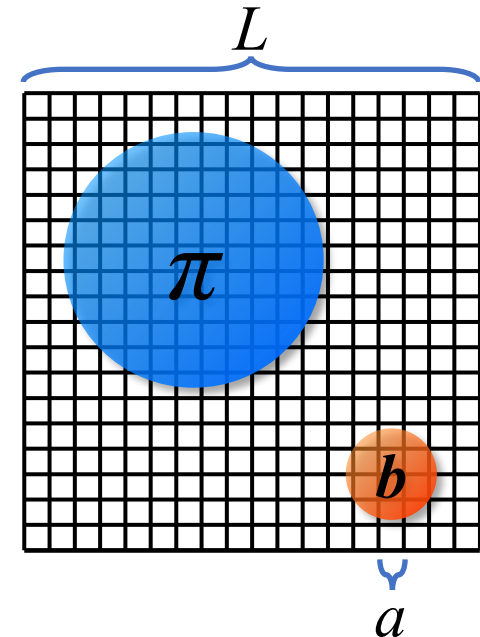


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status and progress w/ powerful computers and newly-developed methods

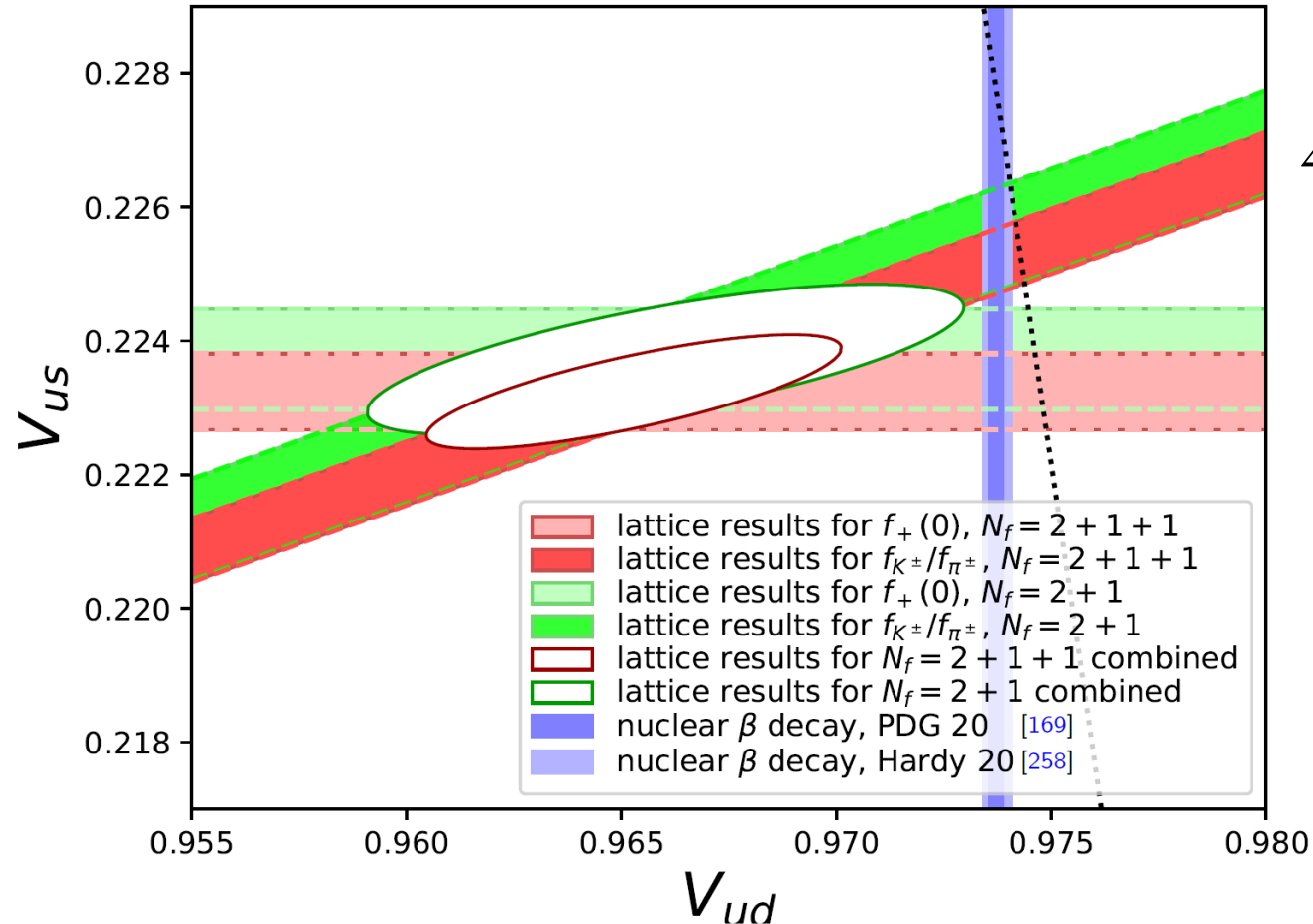
at this workshop

- Elvira Gamiz, “Overview of D form factors and decay constants (lattice)”, WG1 Tue 10:00-
- Felix Erben, “Input to V_{us} from lattice QCD including the progress on hyperon decays”, WG1 Wed 10:00-
- Mikhail Gorchtein, “Improved radiative corrections for $K_{\ell 3}$ decays and superallowed β decays”, WG1 Thu 11:30-
- Ryan Kellermann, “Updates on inclusive charmed and bottomed meson decays from lattice”, WG1&2 Tue 12:00-
- Alejandro Vaquero, “ $B_{(s)} \rightarrow D^{(*)}_{(s)}$ from FNAL/MILC”, WG2 Tue 18:03-
- Ludovico Vittorio, “Unitarity constraints and the dispersive matrix”, WG2 Wed 9:00-
- Carolina Bolognani, “Combining lattice and sum rules to determine $|V_{ub}|/|V_{cb}|$ ”, WG2 Wed 9:24-
- Luka Leskovec, “Lattice outlook on $B \rightarrow \rho$ and $B \rightarrow K^*$ ”, WG2&3 Thu 17:50-
- Chris Bouchard, “Rare $B \rightarrow \pi$ and $B \rightarrow K$ decays on the lattice”, WG2&3 Thu 18:20-
- Brian Colquhoun, “ $B \rightarrow \pi, B \rightarrow D^{(*)}$ from JLQCD”, WG2&3 Thu 18:50-
- En-Hung Chao, “ $K \rightarrow \mu\mu$ on the lattice”, WG3 Mon 14:45-
- Ryan Hill, “Rare kaons on the lattice”, WG3 Tue 14:45-
- Felix Erben, “Update on SU(3)-breaking ratios and bag parameters for $B_{(s)}$ mesons”, WG4

Cabibbo angle anomaly

Flavour Lattice Averaging Group (FLAG) Web Update '23

FLAG2023



new entry : ETM '21 $f_K/f_\pi(N_f=4)$ ~ FLAG'21 ave

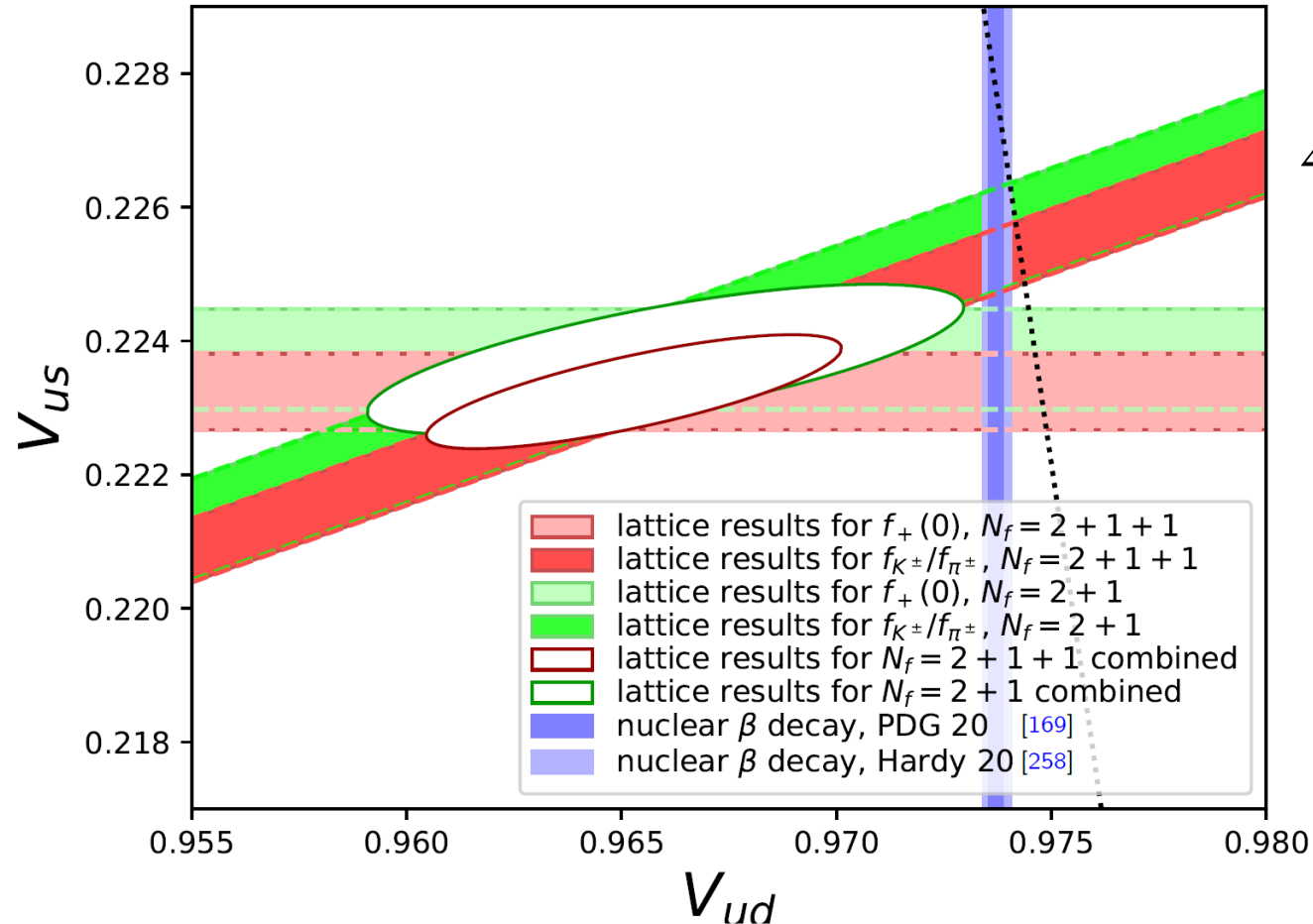
$$f_{K^\pm}/f_{\pi^\pm} = 1.1932(21) \Rightarrow 1.1934(19) \text{ [0.16\% err]}$$

$$\Delta_{\text{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00113(67) \text{ [1.7}\sigma\text{]}$$

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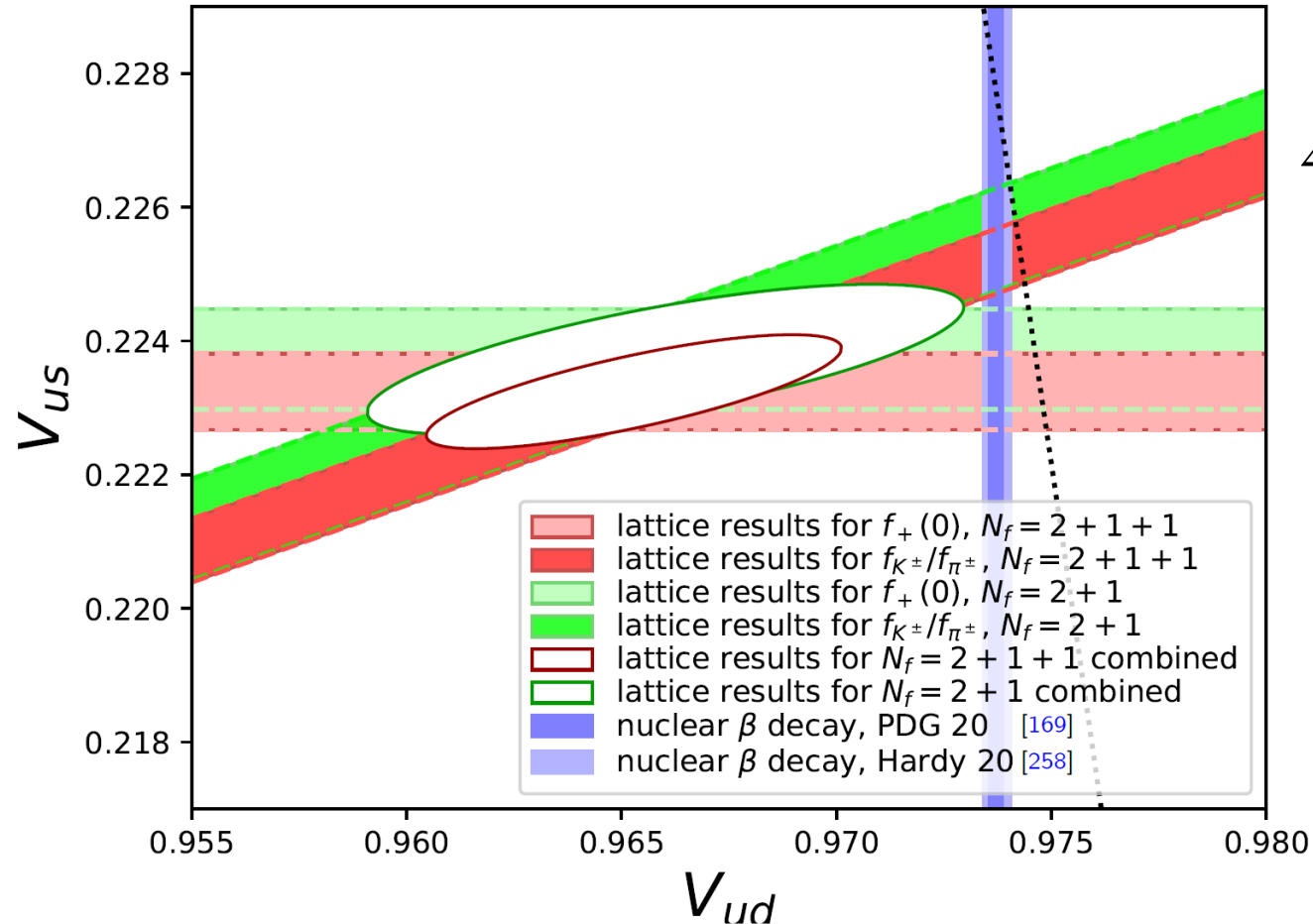
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new entry for $f_+(0)$?

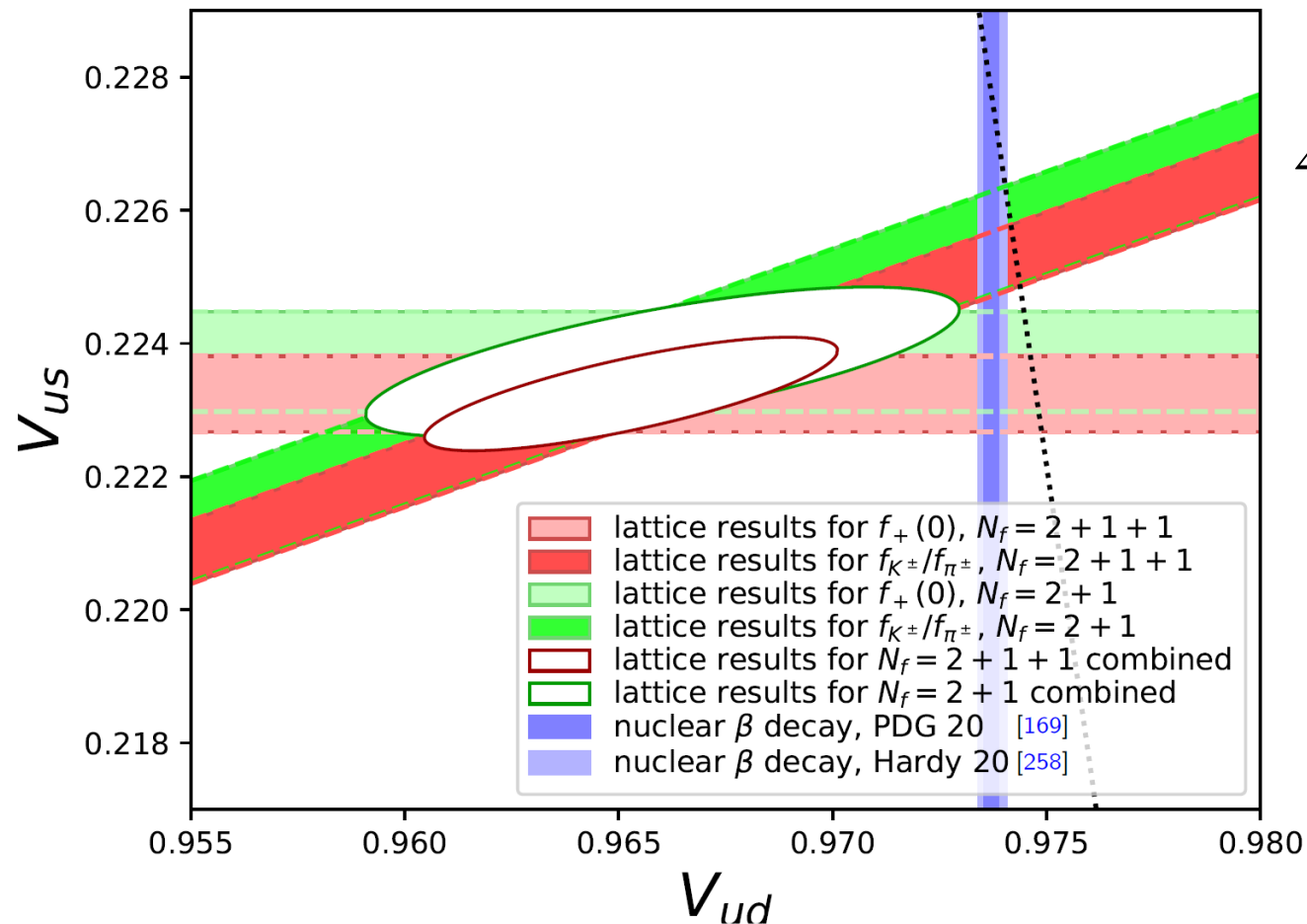
FLAG2023

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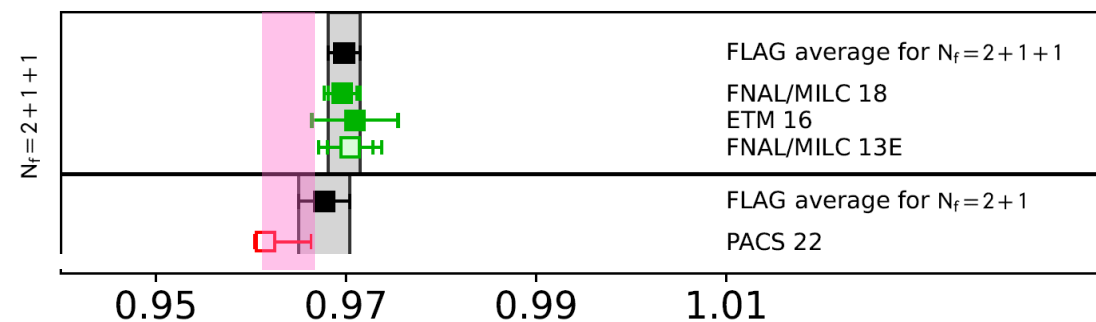
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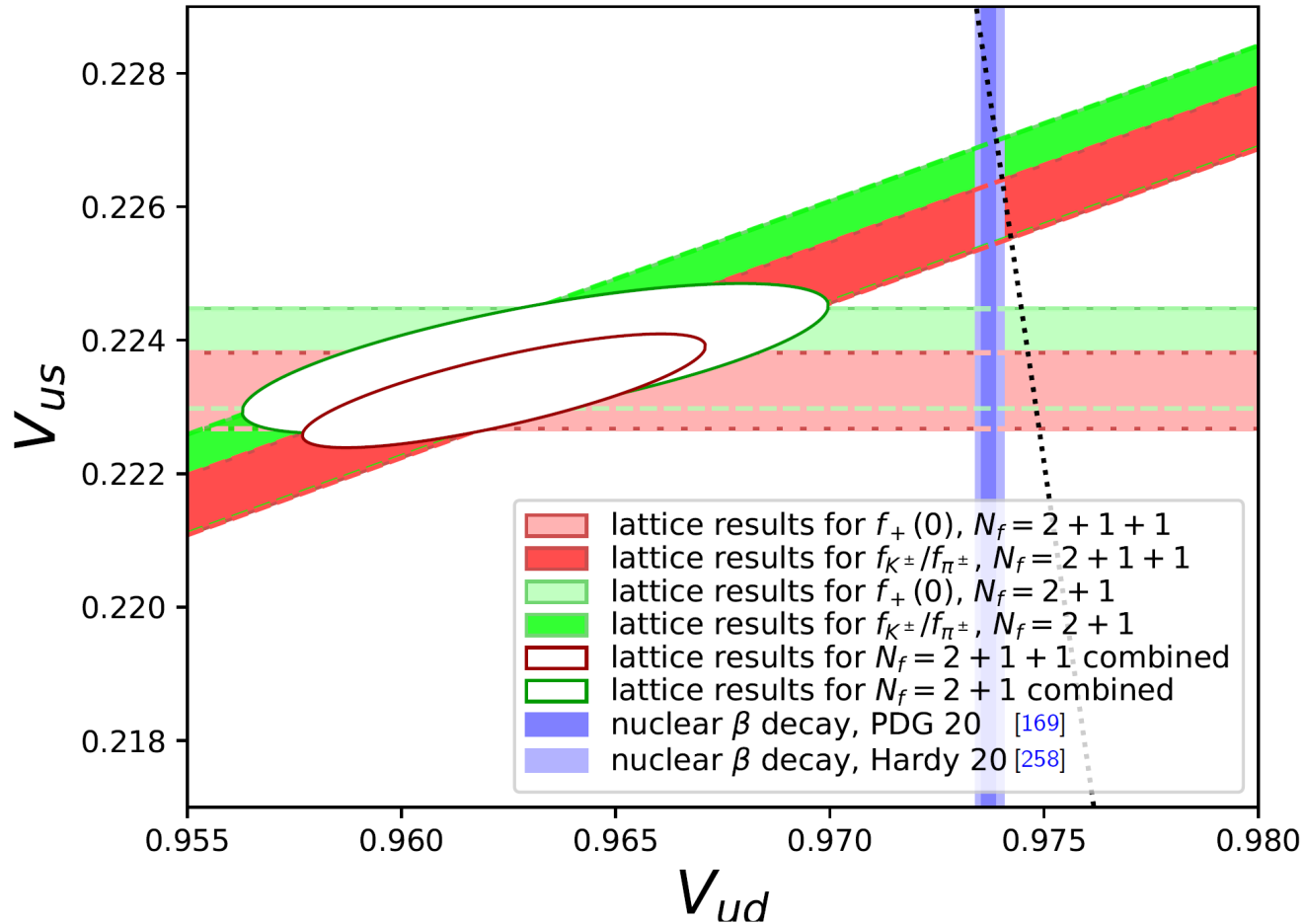
PACS @ Lattice 2023; $N_f=3$, 3 a 's, $L \sim 10\text{fm}$

$$f_+(0) = 0.9634(24)_{\text{stat}} \Leftrightarrow 0.9696(19) \quad (\text{FNAL/MILC})$$

Cabibbo angle anomaly

radiative corrections

FLAG2023



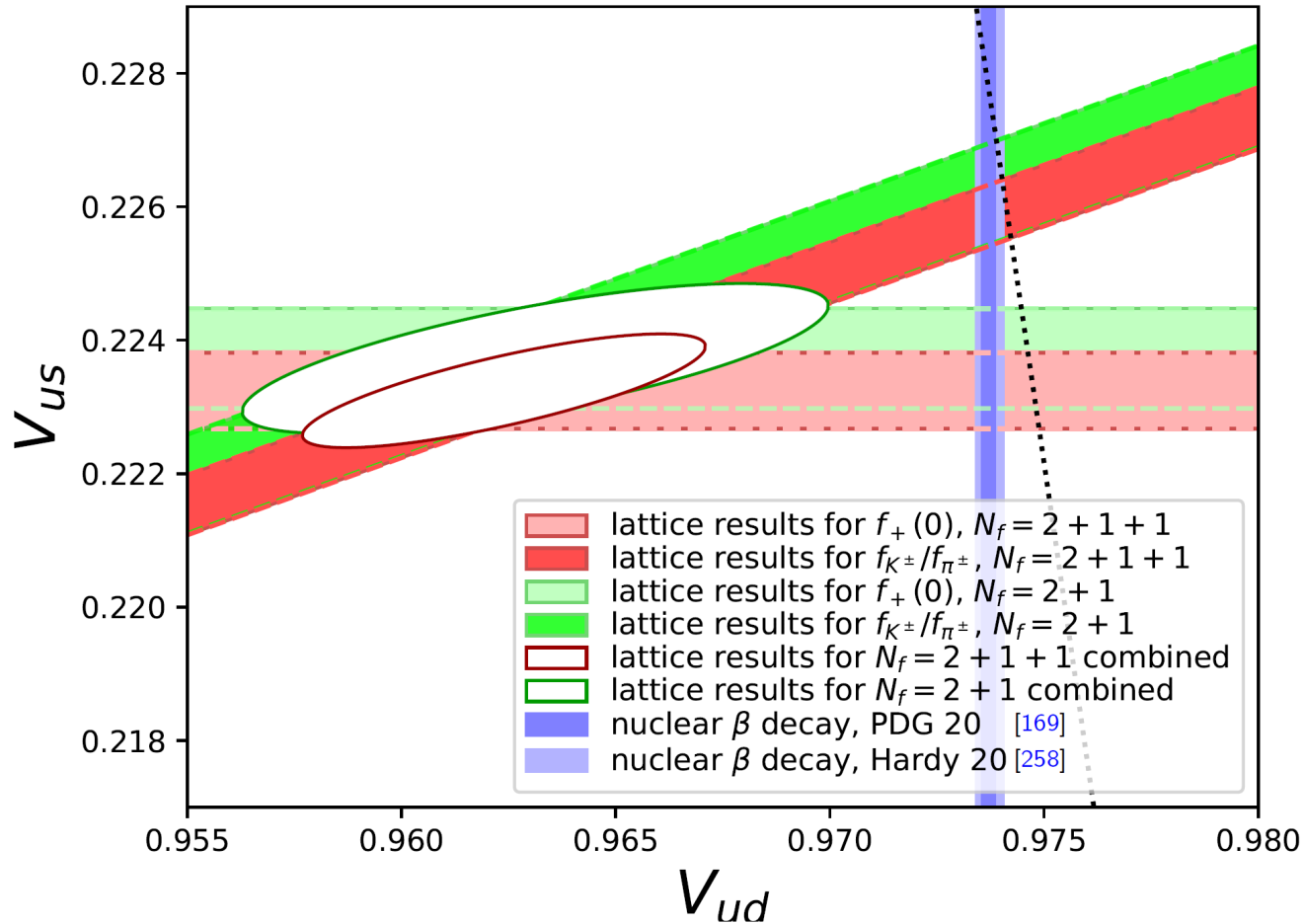
isospin correction to $\Gamma(K \rightarrow \ell \nu)/\Gamma(\pi \rightarrow \ell \nu)$

- ChPT $\Rightarrow \delta_{\text{EM}+\text{SU}(2)}/2 = -0.56(11)\%$ [LEC, higher orders]
- RM123+SOTON '19 : $\delta_{\text{EM}+\text{SU}(2)}/2 = -0.63(7)\%$
 - + smaller $|\Delta_{\text{CKM}}|$ [1.2σ] with $P_{\ell 2} + \beta$ decay
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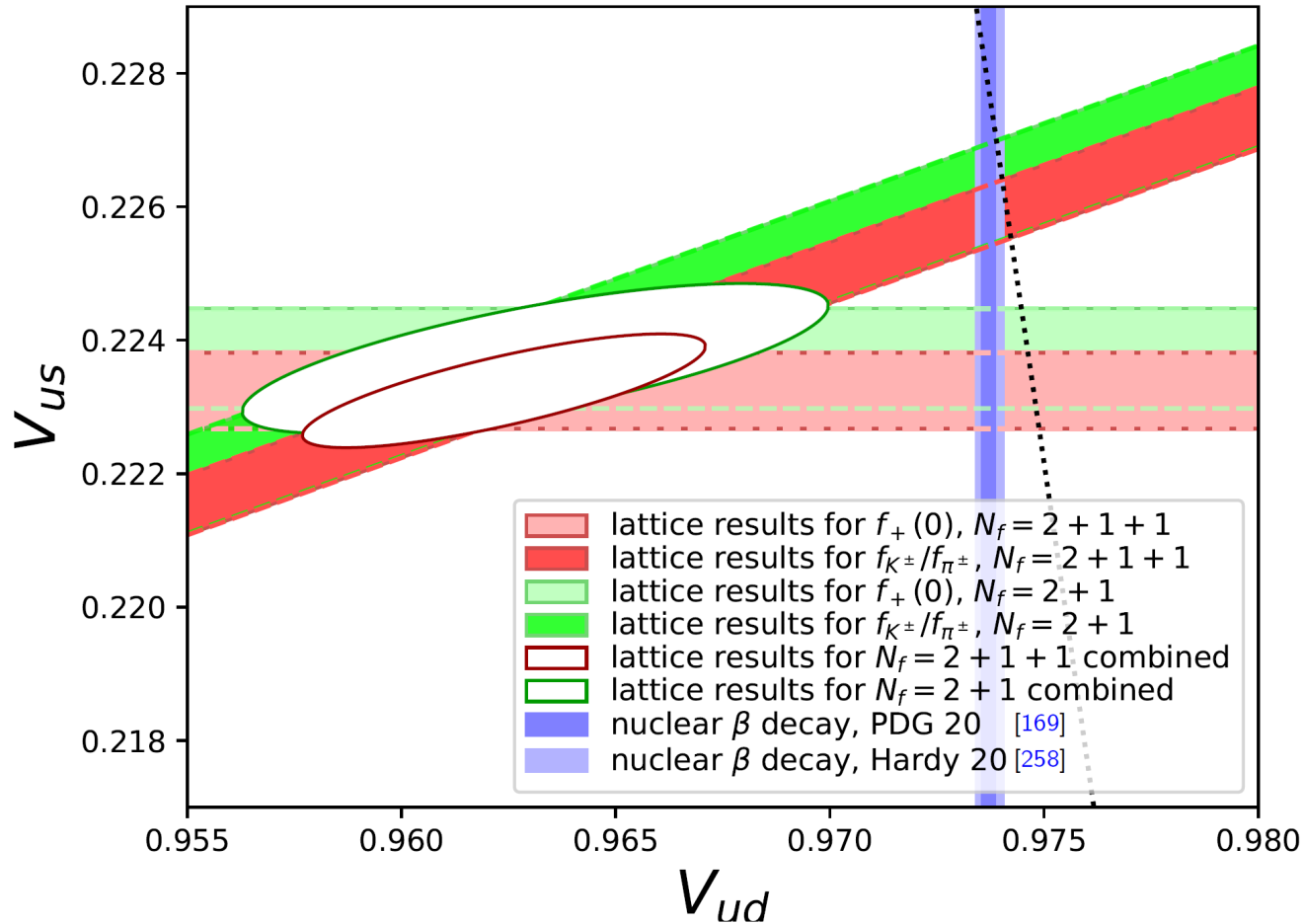
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- RBC/UKQCD 2211.12865: $\delta_{\text{EM}+\text{SU}(2)}/2 = -0.43(21)\%$
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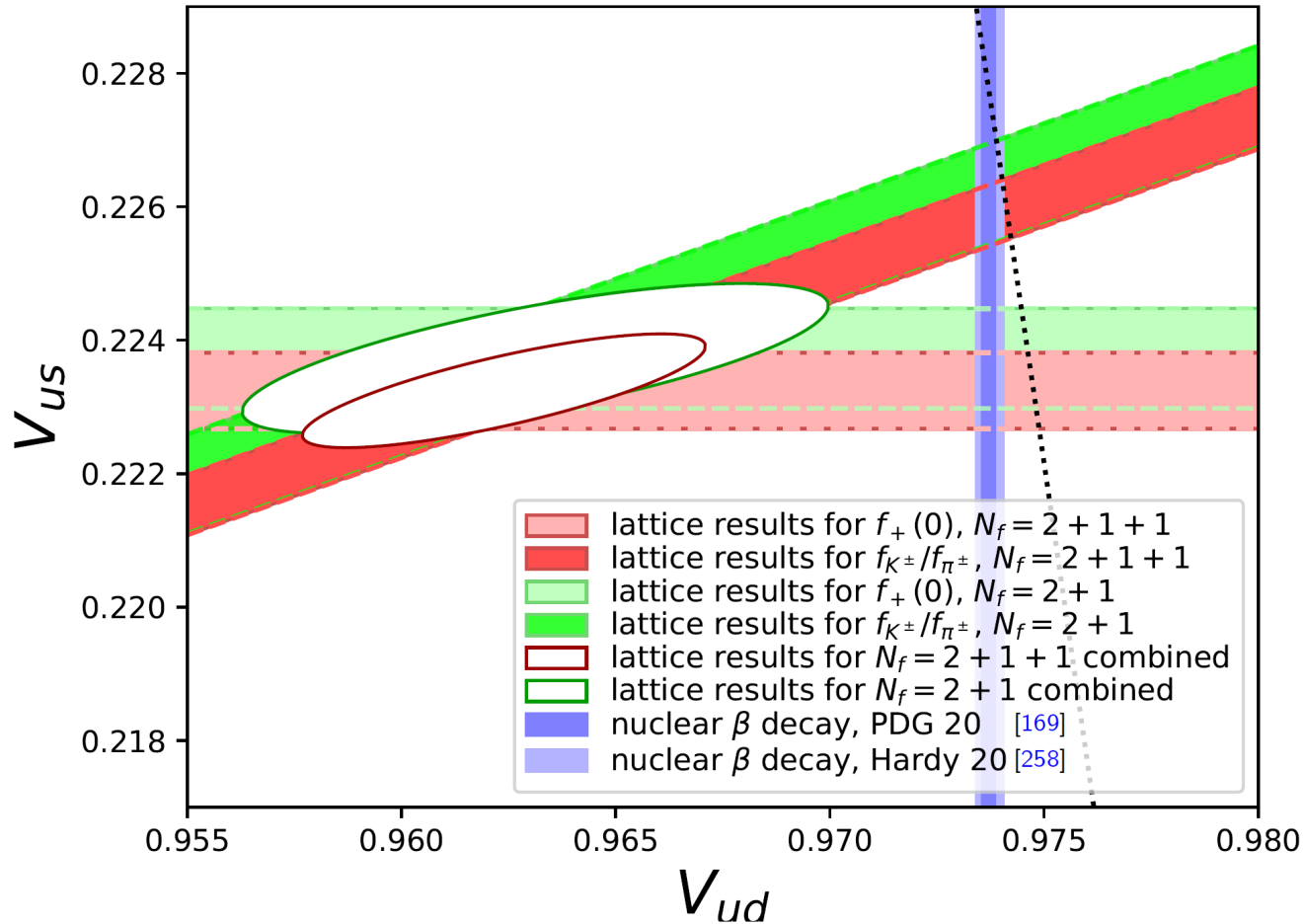
γW box diagram to β decay

- dispersive analysis by Seng+ '18 \Rightarrow anomaly
- + exp + pheno + pQCD $\Rightarrow \langle p | T [J_{\text{EM}} J_{W,A}] | n \rangle$

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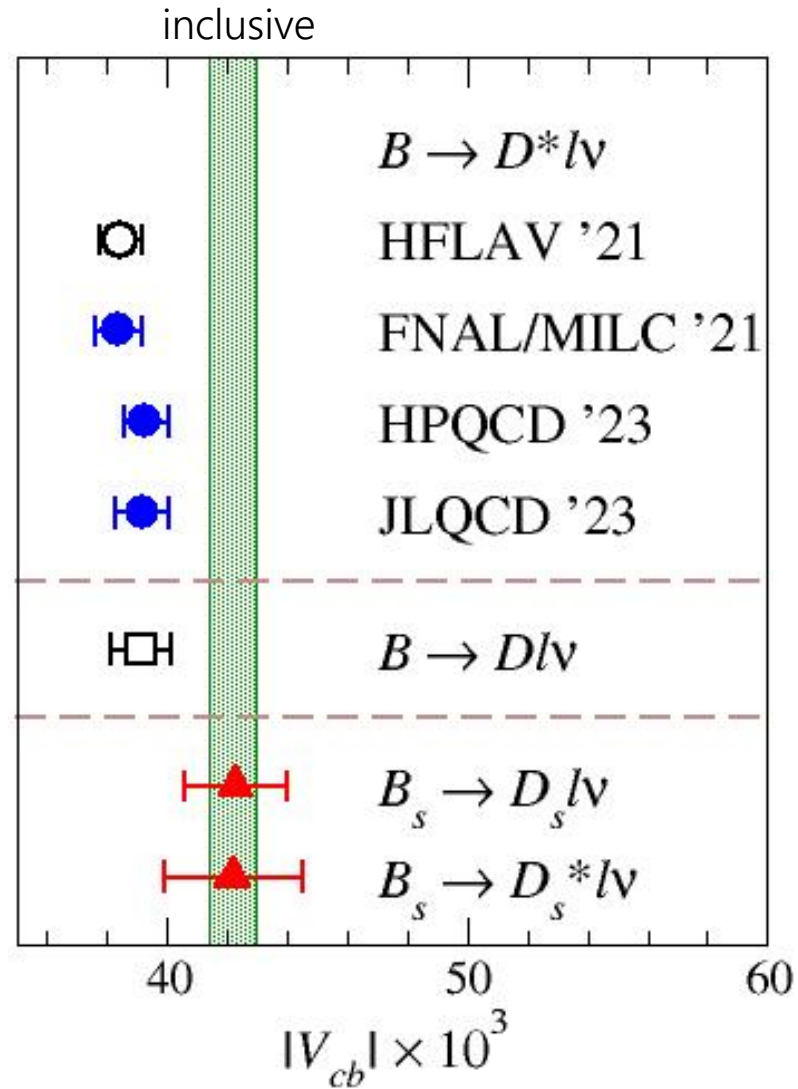
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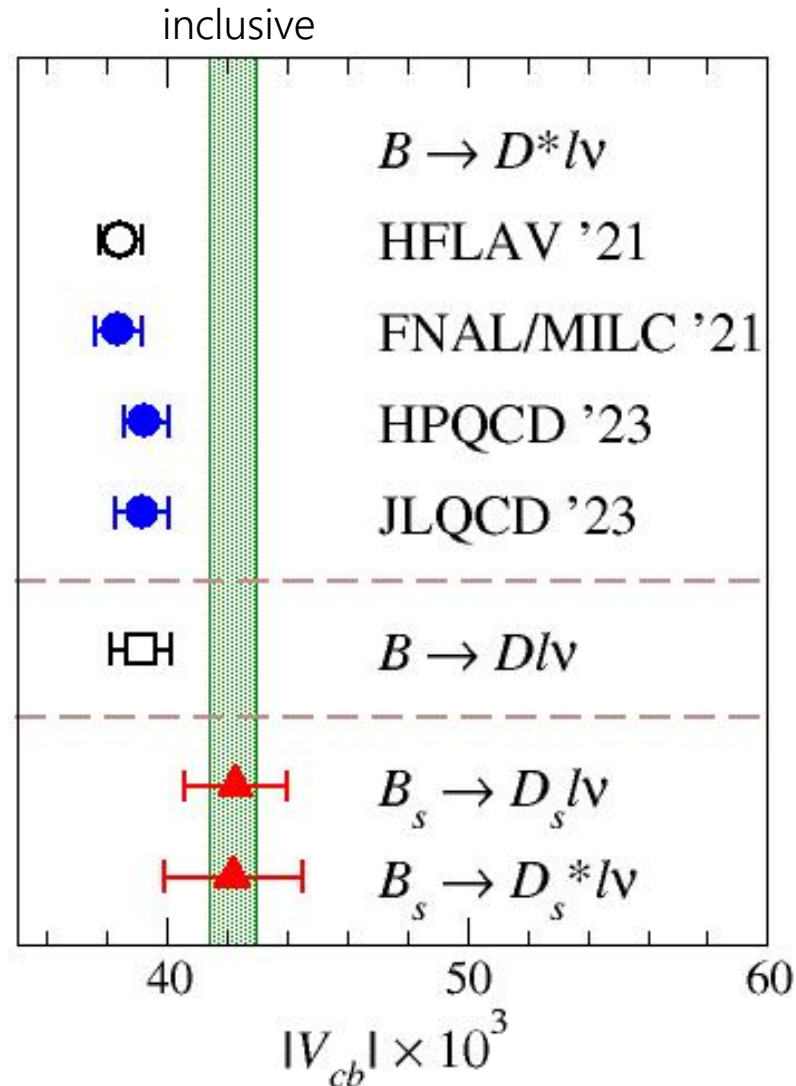
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- Ma+ 2308.16755 : $\langle p | T [J_{\text{EM}} J_{W,A}] | n \rangle$ on the lattice
 - $\square_{\gamma W} (\leq 2\text{GeV}) \times 10_3 = 1.62(10) \rightarrow 1.49(8)$
 - $|V_{ud}| = 0.97386(31)$, $\Delta_{\text{CKM}} = 2.1\sigma \rightarrow 1.8\sigma$
 - independent calc : Yoo+ @ Lattice 2023

$|V_{cb}|$

$|V_{cb}|$ from $B_{(s)}$ decays



HFLAV -'21 : $h_{A1}(w=1)$ from lattice QCD (Fermilab/MILC '14, HPQCD '17)

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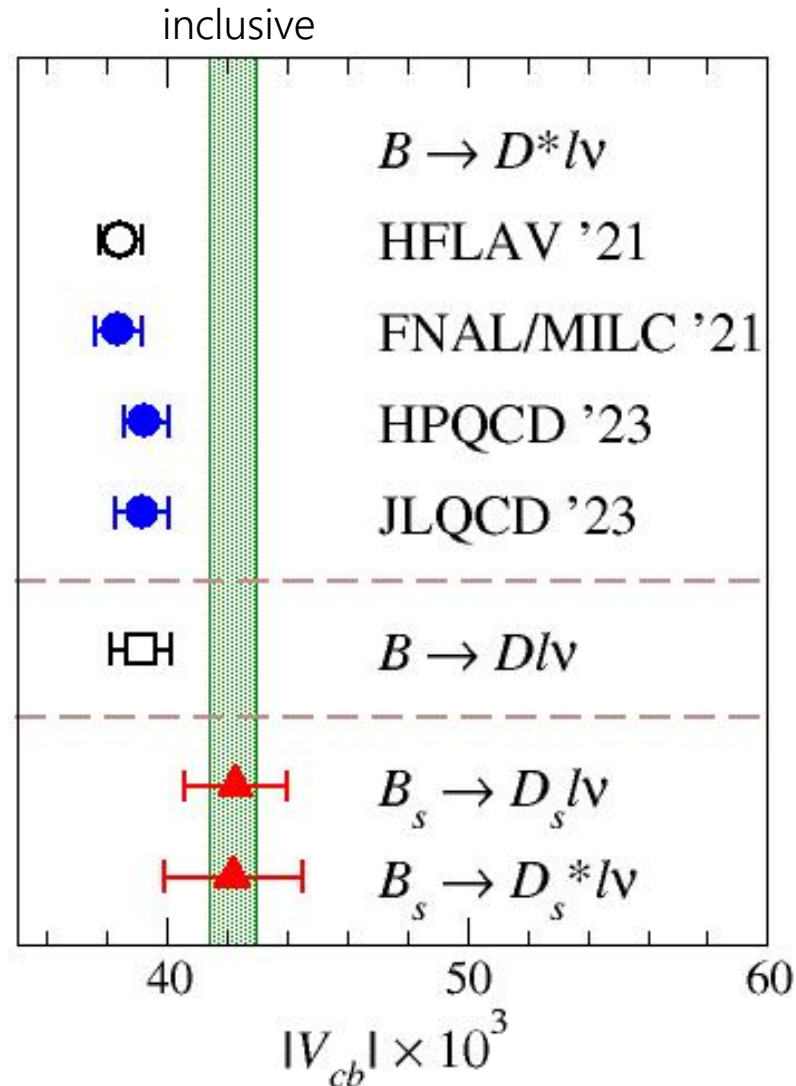
3 lattice calculations of all $B \rightarrow D^* \ell \nu$ FFs also @ non-zero recoils !!

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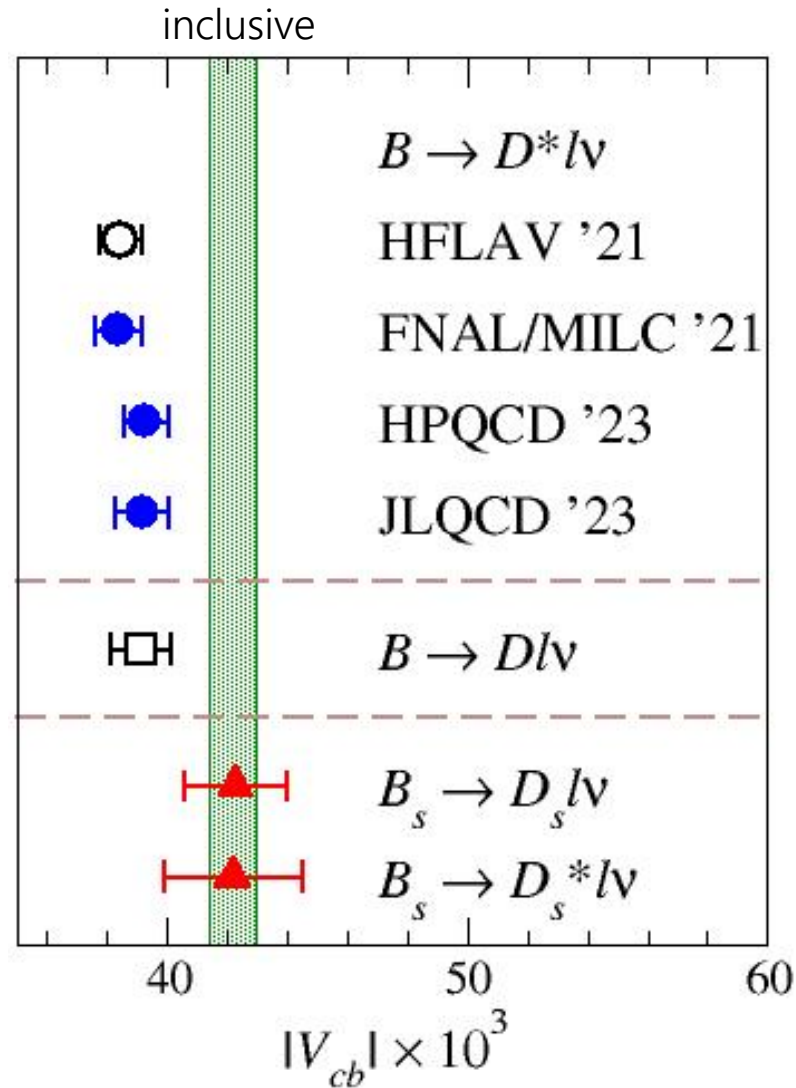
- w/ Belle (+BaBar) data \Rightarrow consistent $|V_{cb}|$'s (2% err) < inclusive

+ Gambino+Jung, Flavour@TH: JLQCD $|V_{cb}| \times 10^3 = 39.2(9) \Rightarrow 40.8(+1.8/-2.3)$

D'Agostini bias Nucl. Inst. Meth. Phys. Res. A 346 (1994) 306

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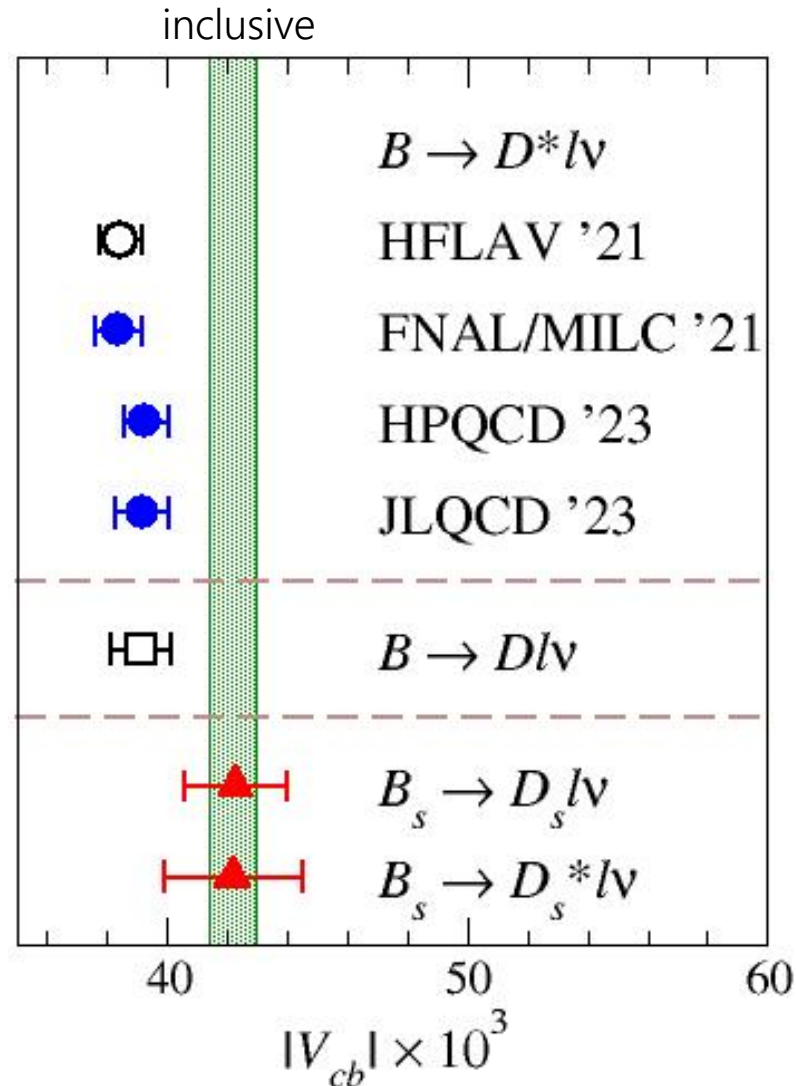
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HPQCD $B_s \rightarrow D_s^{(*)} \ell \nu$ FFs [1906.00701, 2105.11433] + LHCb data \Rightarrow 5% err

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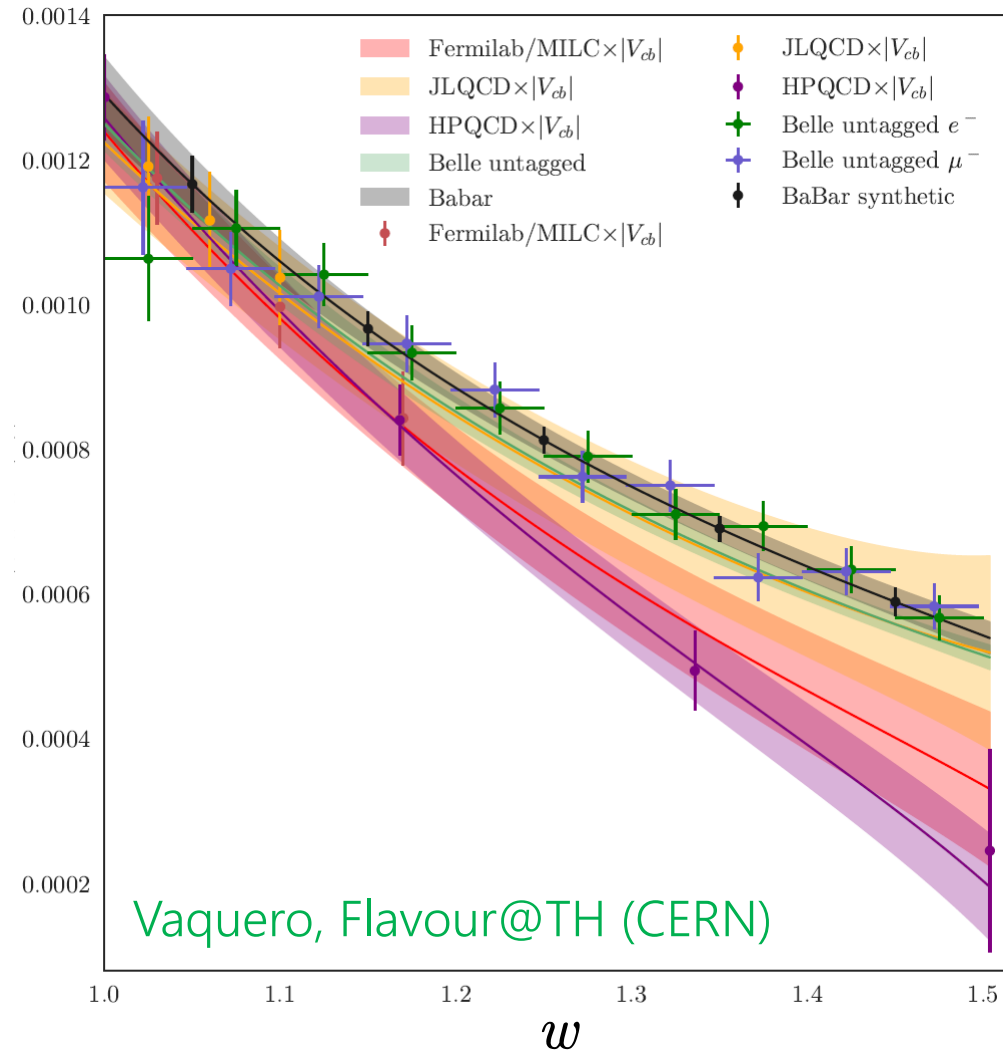
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quoted error \approx comparable to exp: stat., $a \neq 0$, $[m_b < m_{b,\text{phys}}]$

@ Lattice '23: ($B \rightarrow D^*$) LANL-SWME ($B_s \rightarrow D_s$) Fermilab/MILC, ...

tension on $B \rightarrow D^* \ell \nu$

$$|\eta_{EW} V_{cb} \mathcal{F}|^2 \quad \mathcal{F}^2 \propto \left[2 \frac{1 - 2wr + r^2}{(1 - r)^2} \left\{ 1 + \frac{w - 1}{w + 1} R_1^2 \right\} + \left\{ 1 + \frac{w - 1}{1 - r} (1 - R_2) \right\}^2 \right] h_{A_1}^2 \quad R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{h_{A_3} + r h_{A_2}}{h_{A_1}}$$

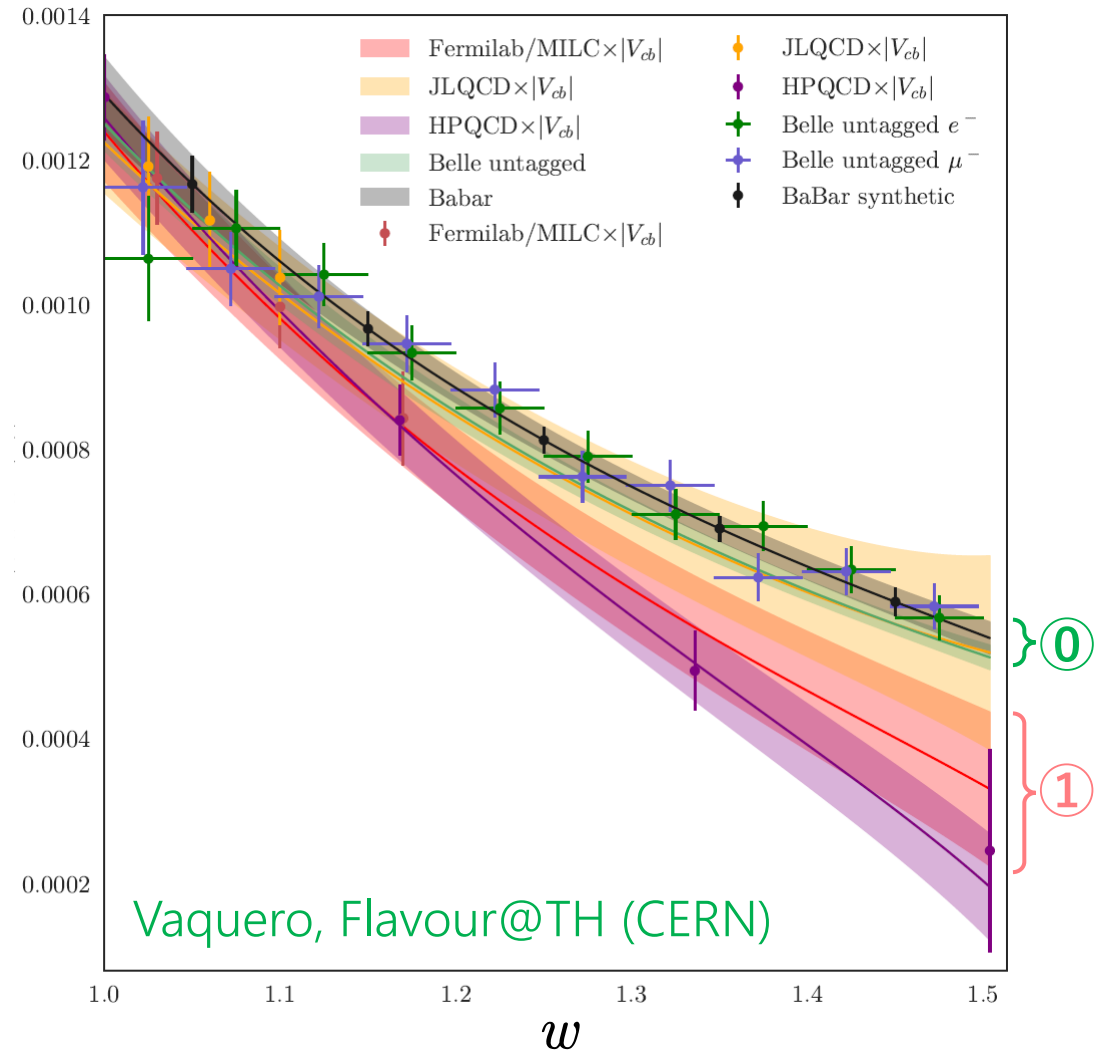


① Belle and BaBar data

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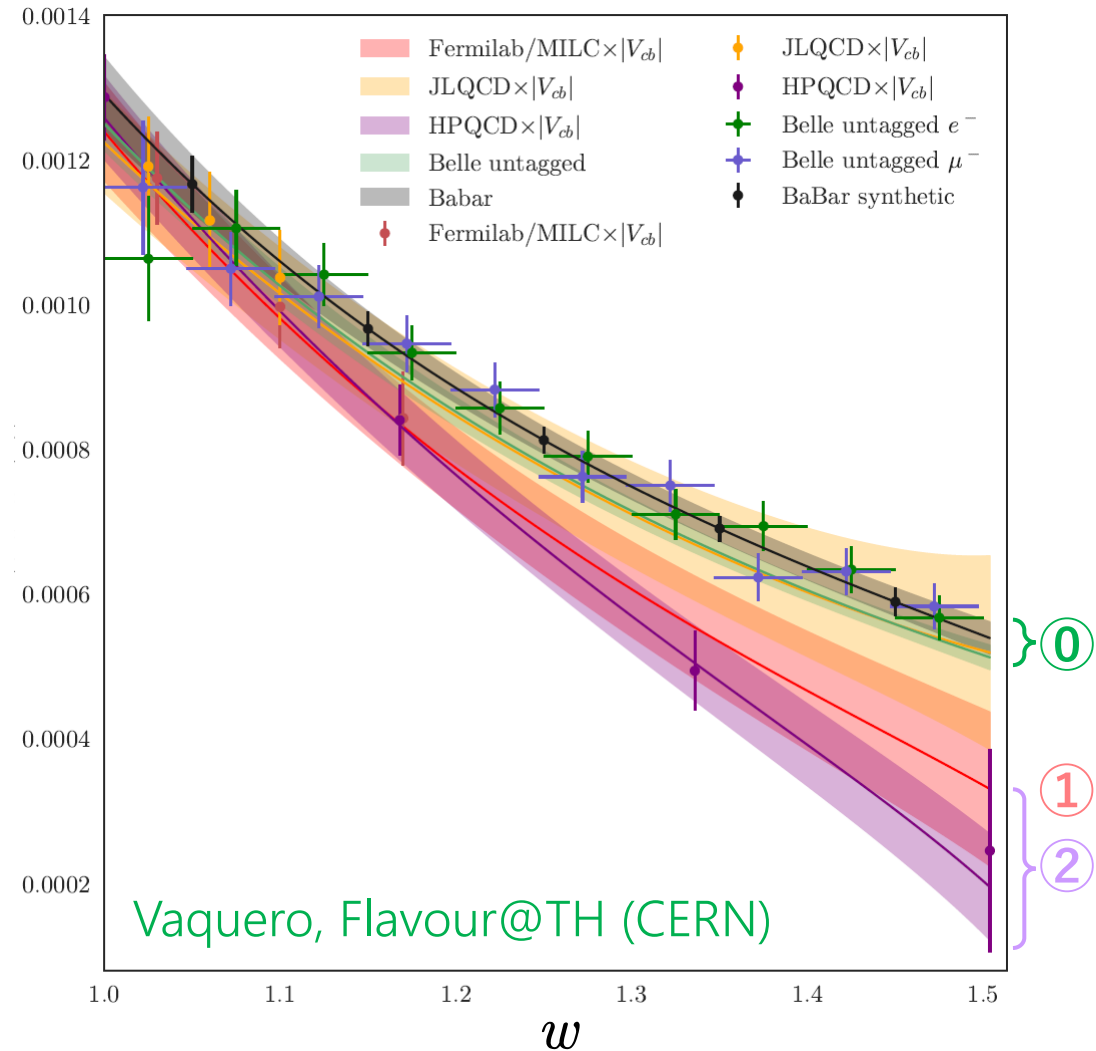
0 Belle and BaBar data

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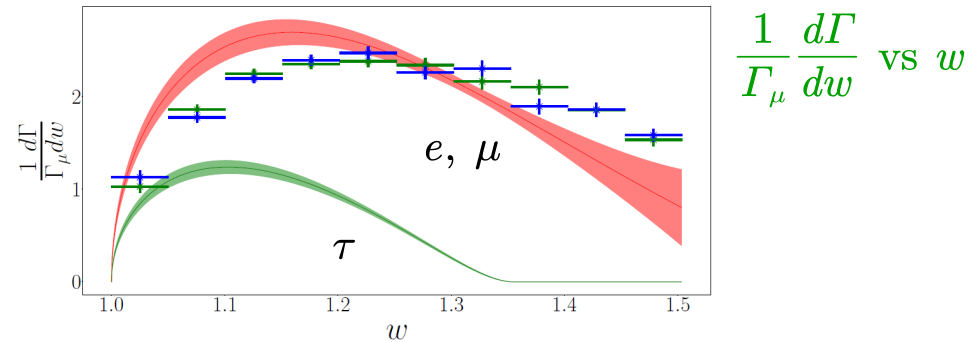
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2 HPQCD : even steeper slope !

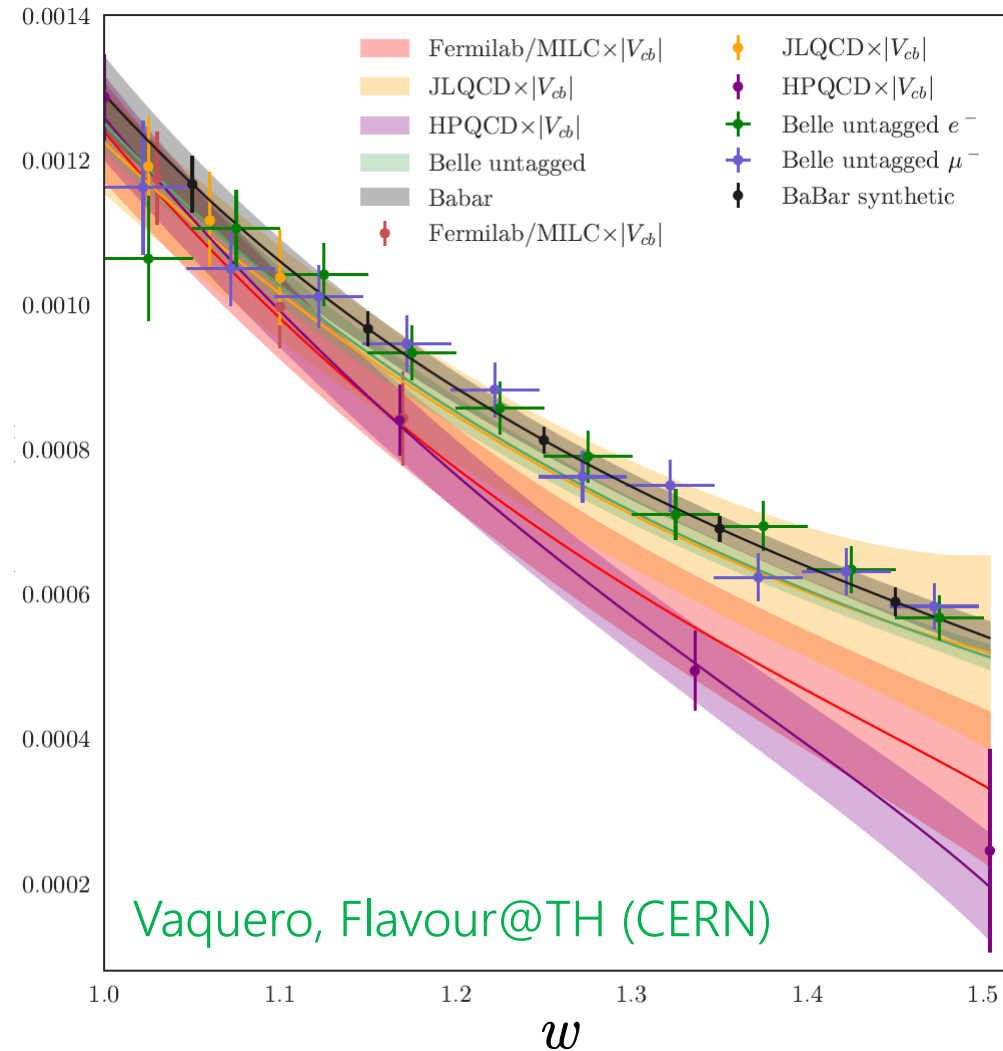
+ significant tension with exp ($\ell=e, \mu$) at medium/large w

+ $|V_{cb}| = 44.2(1.8) \times 10^{-3}$ from total Γ



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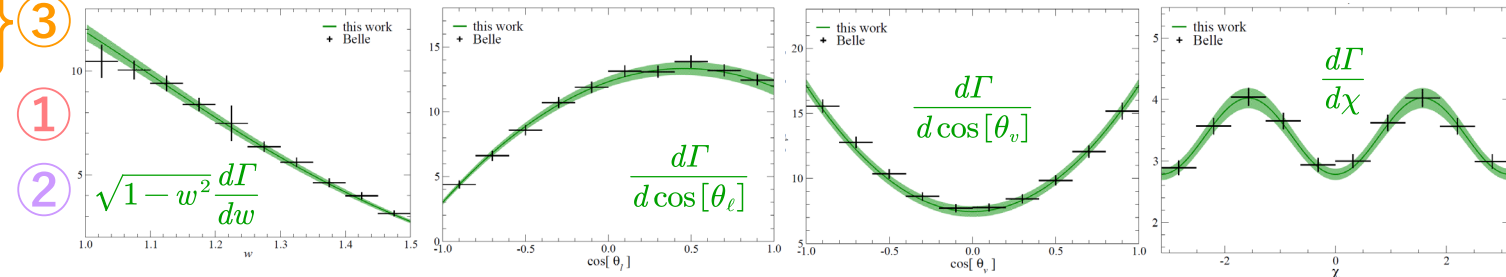
+ $\chi^2/\text{dof} \sim 1.5$ to fit w/ exp data

2 HPQCD : even steeper slope !

+ significant tension with exp ($\ell=e, \mu$) at medium/large w

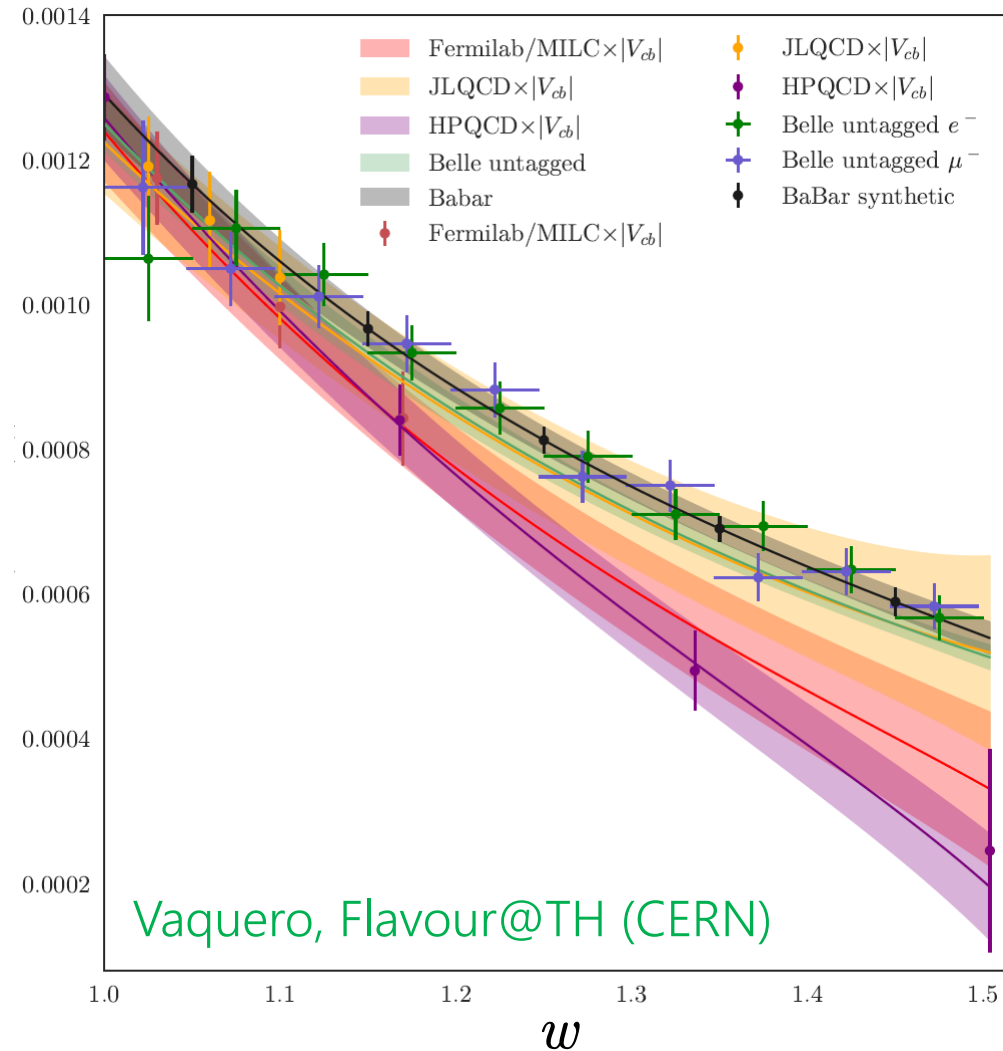
+ $|V_{cb}| = 44.2(1.8) \times 10^{-3}$ from total Γ

3 JLQCD : good consistency w/ exp



tension on $B \rightarrow D^* \ell \nu$

$$|\eta_{EW} V_{cb} \mathcal{F}|^2 \quad \mathcal{F}^2 \propto \left[2 \frac{1 - 2wr + r^2}{(1 - r)^2} \left\{ 1 + \frac{w - 1}{w + 1} R_1^2 \right\} + \left\{ 1 + \frac{w - 1}{1 - r} (1 - R_2) \right\}^2 \right] h_{A_1}^2 \quad R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{h_{A_3} + r h_{A_2}}{h_{A_1}}$$



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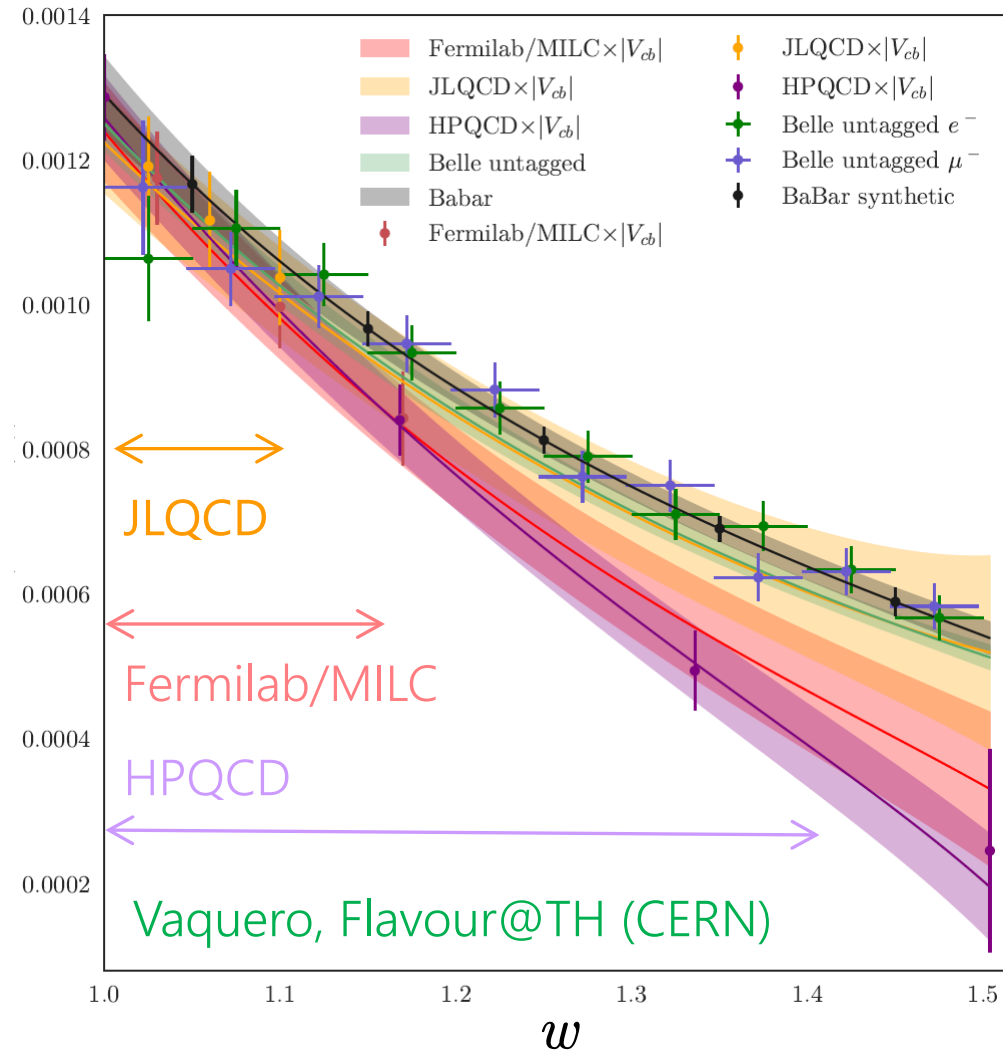
0
3
1
2

1 - tension on R_2 (?) [Belle 2301.07529, Jung Flavour@TH]

Vaquero, Flavour@TH (CERN)

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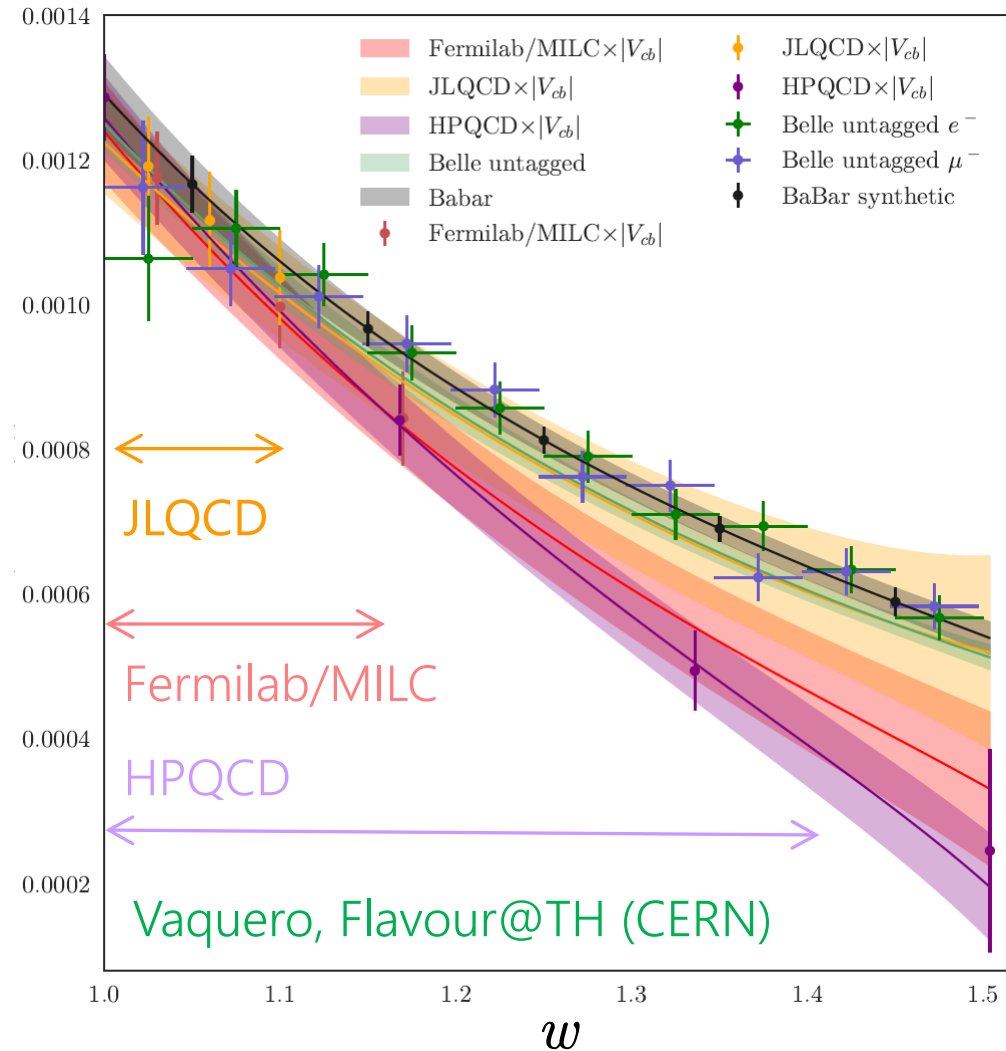
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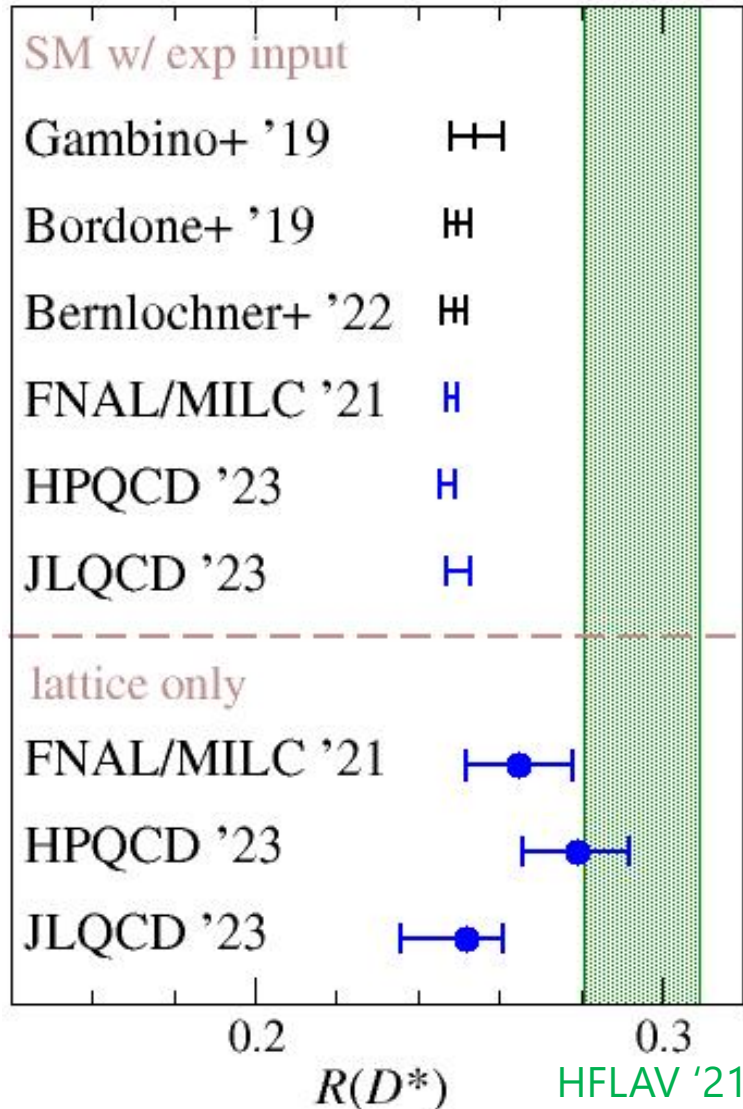
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\Rightarrow "safe" extension to large w : JLQCD; Fermilab/MILC $a^{-1} \sim 6.6 \text{ GeV}$

$R(D^*)$

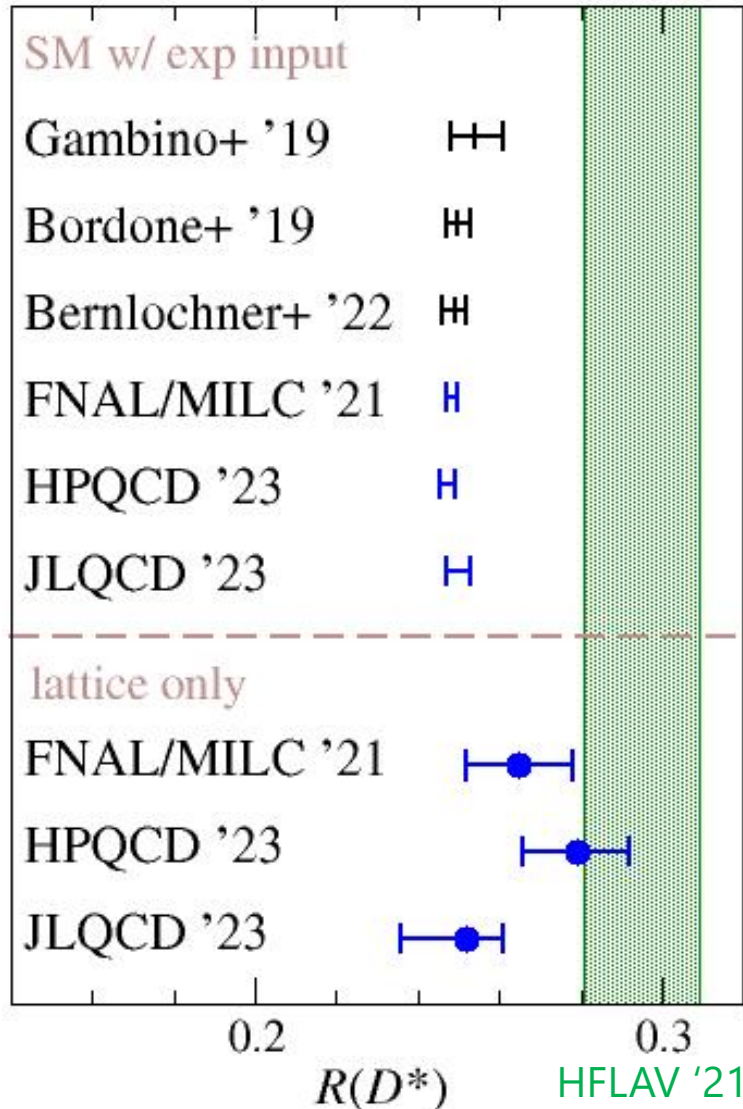
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– BGL, HQET analyses of exp data + $h_{A1}(1)_{\text{lattice}} \Rightarrow \gtrsim 1\%$ accuracy



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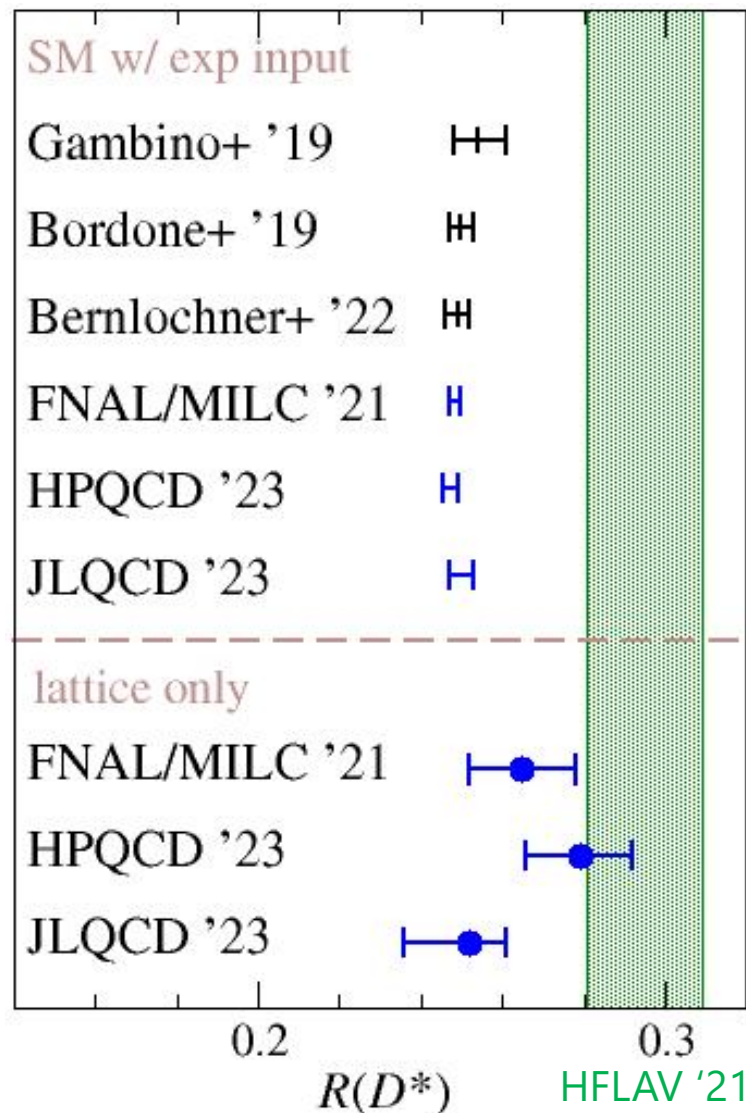
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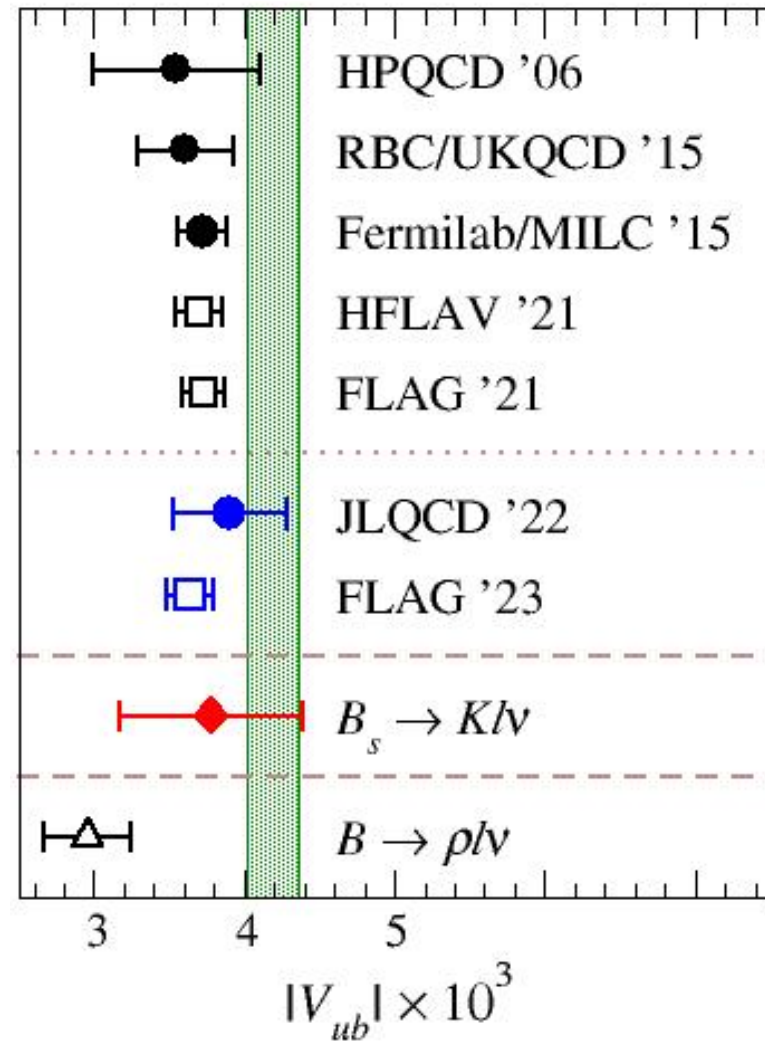
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- only from lattice data $\Rightarrow 5\%$
+ robust: reasonable ($\leq 1.7\sigma$) consistency among 3 studies
+ phase factor $(w^2-1)^{1/2}$, $w_{\text{max},\tau} < w_{\text{max},\ell} \Leftrightarrow$ extension to large w

$|V_{ub}|$
 $|V_{ub}|$ from $B_{(s)}$ decays

-CKM'21 : 3 realistic simulations, WA dominated by Fermilab/MILC '15



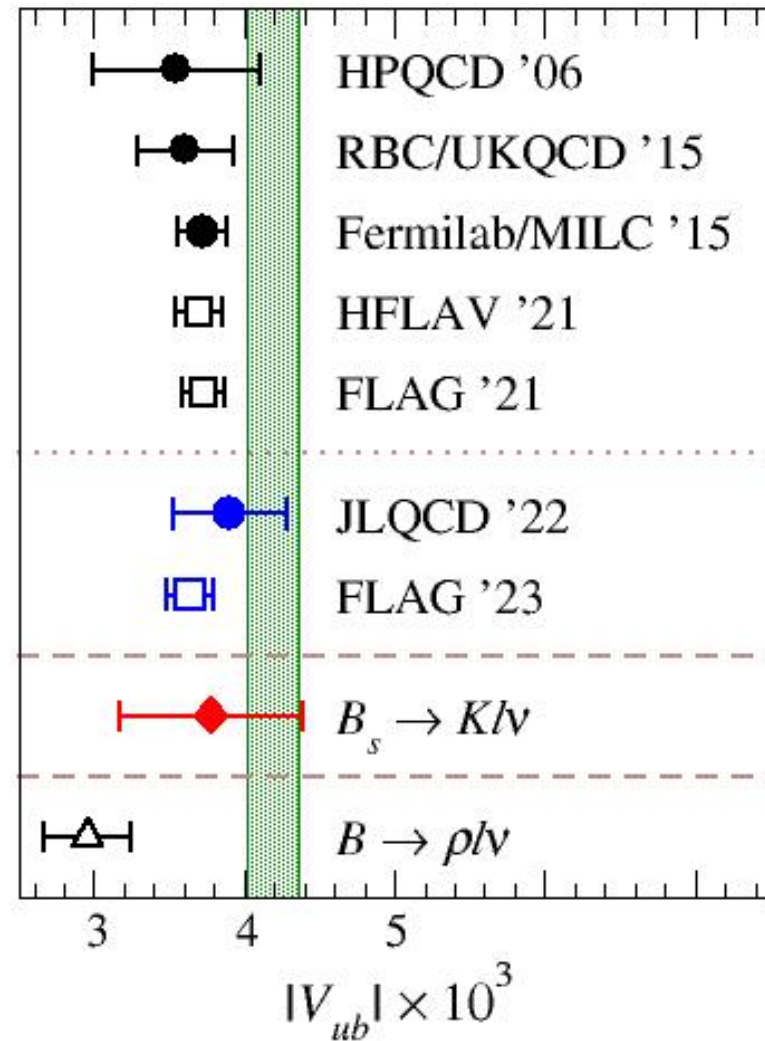
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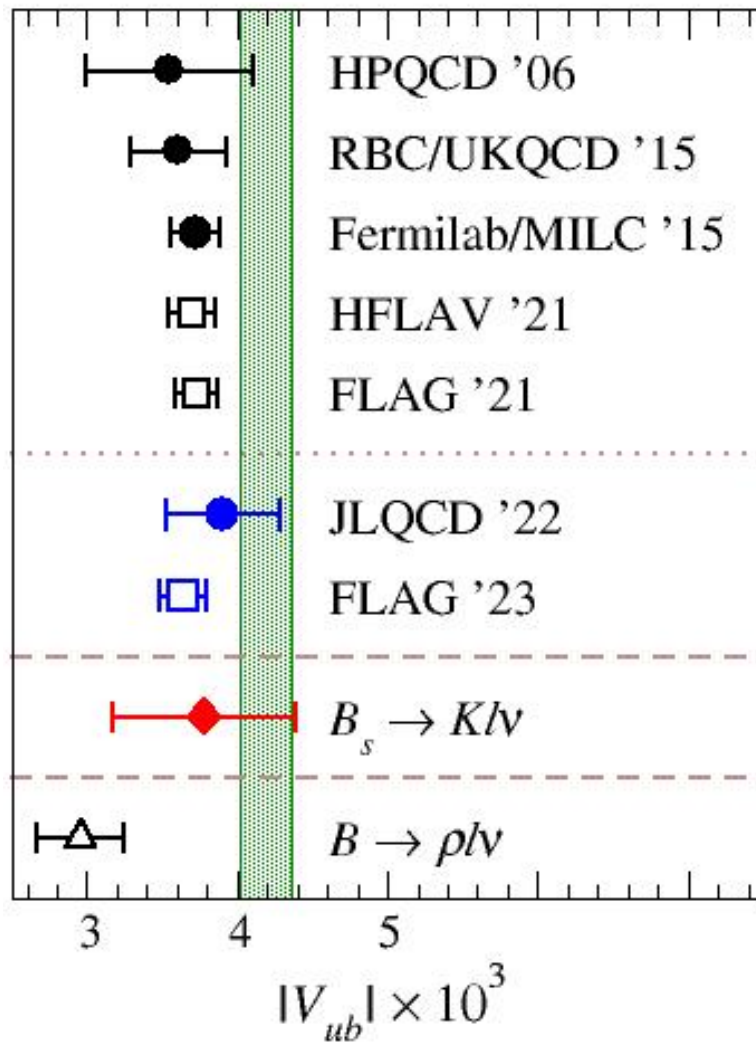
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JLQCD 2203.04938

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- largest uncertainties : **statistics, chiral extrap. to $M_{\pi,\text{phys}}$ [chiral log.]**
- near future improvement?: **MILC, RBC/UKQCD confs @ $M_{\pi,\text{phys}}$** ...



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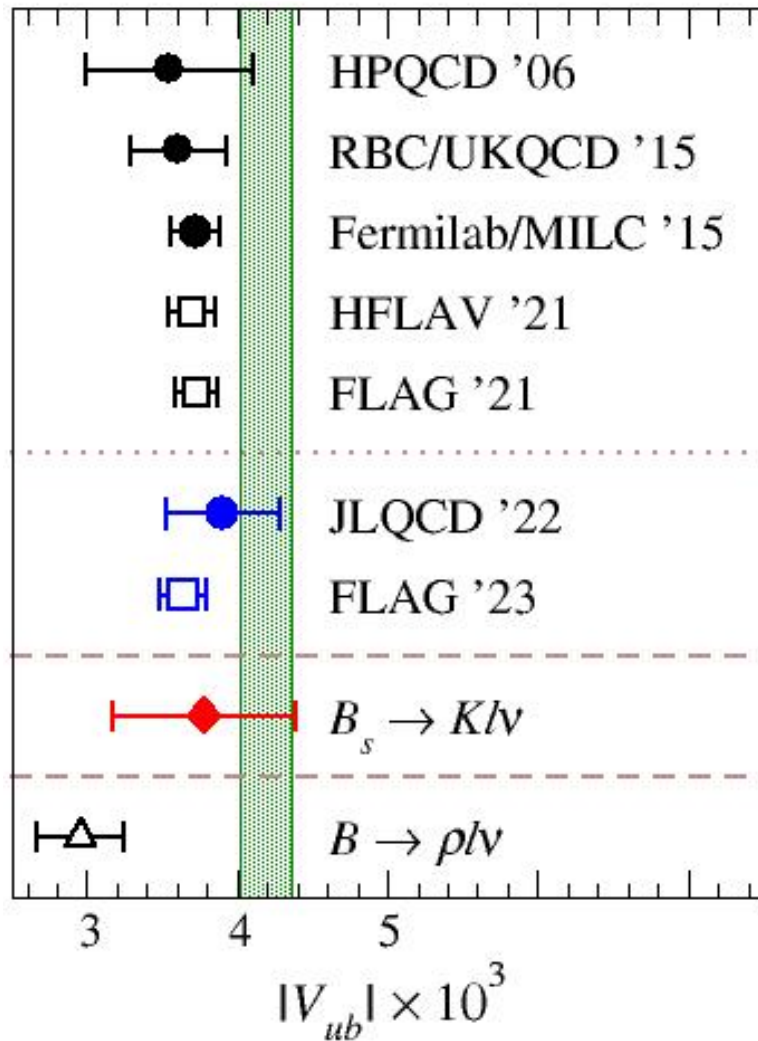
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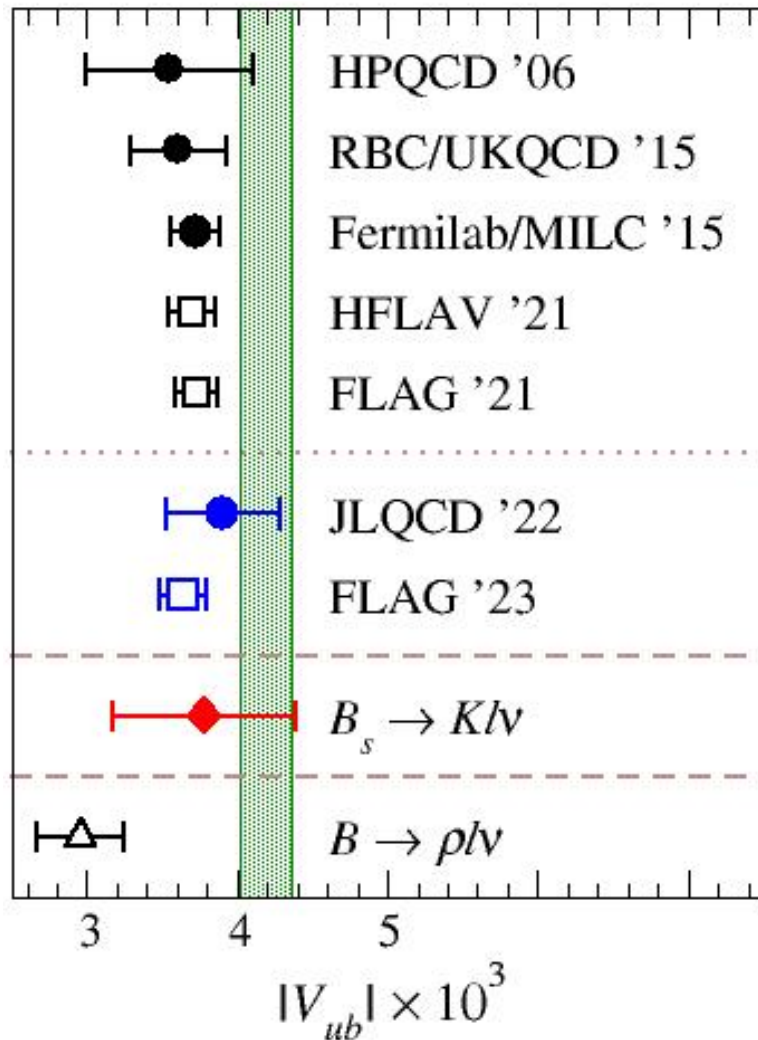
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$B \rightarrow \rho \ell \nu$: Bernlocknler+ 2104.05739 w/ LCSR for FFs

- lattice approach with a finite volume framework \Rightarrow Leskovec [WG2]
- \Rightarrow **$B \rightarrow K^* \ell \ell$?**

inclusive semileptonic decays

$$\frac{d\Gamma(B \rightarrow X_c \ell \nu)}{d\mathbf{q}^2 dq^0 dE_\ell} = \frac{G_F^2}{8\pi^3} |V_{cb}|^2 L^{\mu\nu} W_{\mu\nu} \quad W_{\mu\nu} \sim \sum_{X_c} \langle B | J_\mu^\dagger | X_c \rangle \langle X_c | J_\nu | B \rangle = \text{im} \langle B | J_\mu^\dagger \otimes J_\nu | B \rangle$$

hadronic tensor optical theorem

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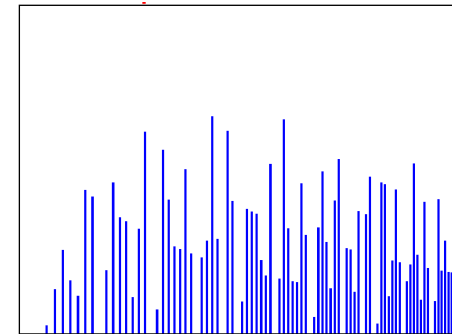
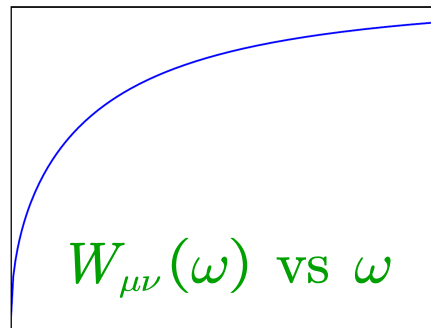
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$C_{\mu\nu}(t) \Rightarrow W_{\mu\nu}(\omega)$ as a function of ω – ill-posed inverse problem

- deformed $W_{\mu\nu}(\omega)$
- limited input / non-exact $C_{\mu\nu}(t)$



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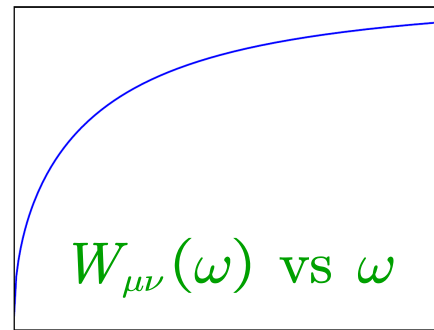
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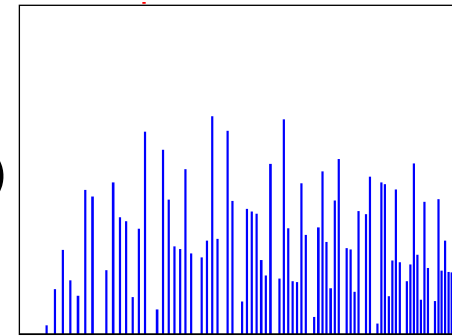
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\Leftrightarrow smeared / integrated $W_{\mu\nu}(\omega) \int d\omega K(\omega)$



$\sim \int d\omega K_{L,\sigma}(\omega)$
?



Hanesen+ '17, Gambino-Hashimoto '20

$$\frac{d\Gamma}{d\mathbf{q}^2} = \frac{G_F^2}{24\pi^3} |V_{cb}|^2 \sqrt{\mathbf{q}^2} \bar{X}(\mathbf{q}^2) \quad \bar{X}(\mathbf{q}^2) = \int_0^\infty d\omega K_{\mu\nu,\sigma}(\omega, \mathbf{q}^2) W_{\mu\nu,L}(\omega, \mathbf{q}^2)$$

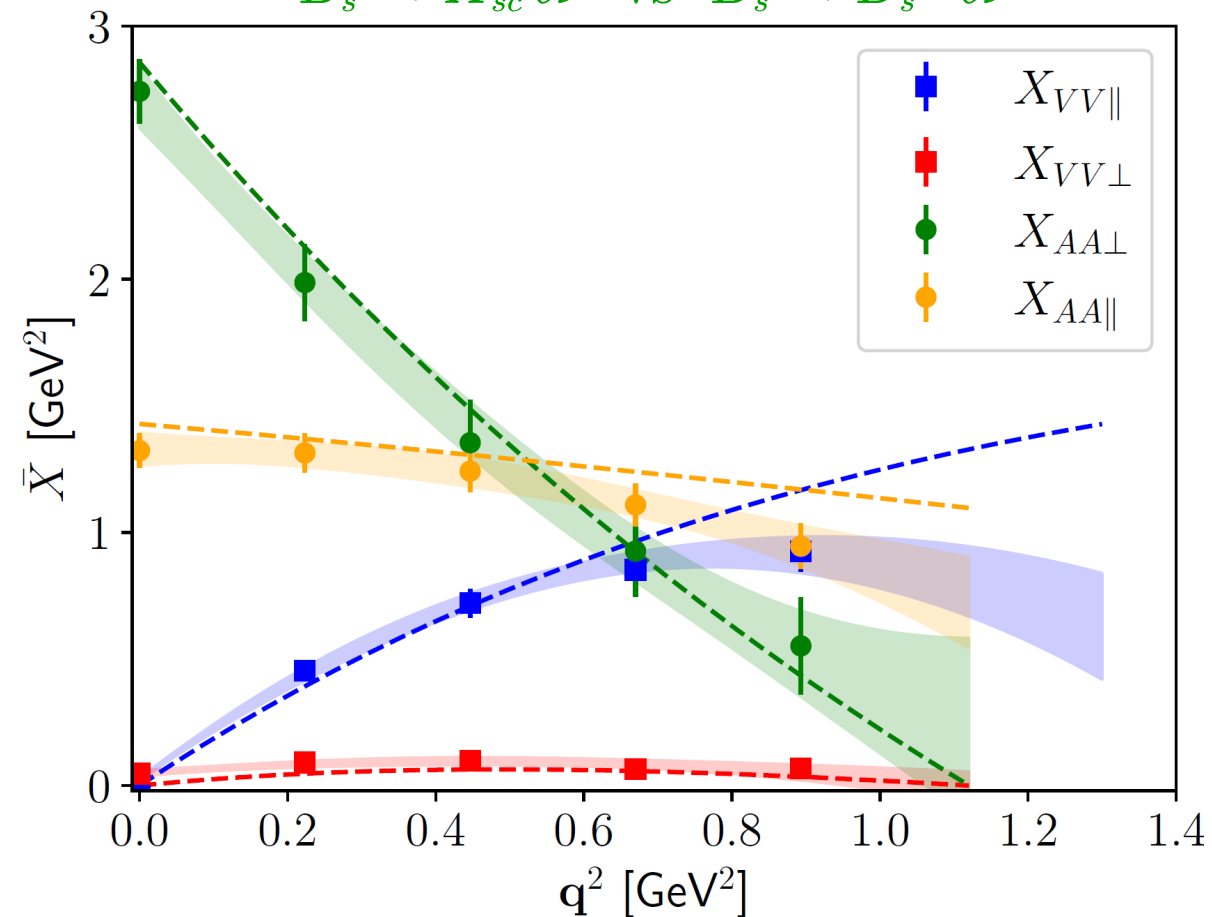
feasibility on the lattice

Gambino+ 2203.11762

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- (too) good consistency w/ exclusive
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$B_s \rightarrow X_{sc} \ell \nu$ vs $B_s \rightarrow D_s^{(*)} \ell \nu$



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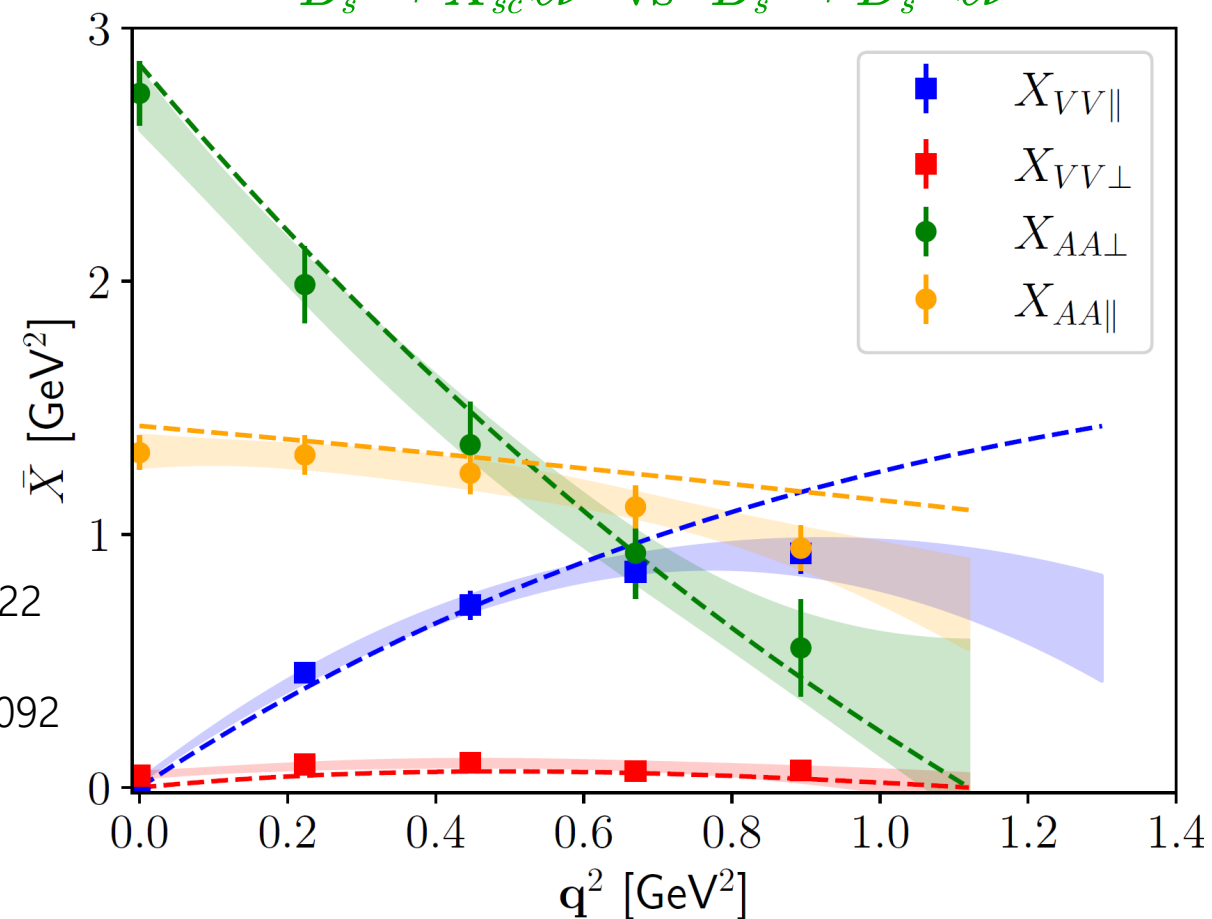
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studies of systematics

Barone-Kellerman-Hashimoto-Jüttner-TK

- upper limit of $d\omega$ $K_{\mu\nu,\sigma}(\omega, \mathbf{q}^2) \propto \theta(\omega_{\max} - \omega)$ Baron+ Lat'22
- expression in $C_{\mu\nu}(t)$ $K_{\mu\nu,\sigma}(\omega, \mathbf{q}^2) = \sum_t k_{\mu\nu}(\mathbf{q}^2) e^{-\omega t}$ 2305.14092
- finite volume effects → Kellermann [WG1+2]
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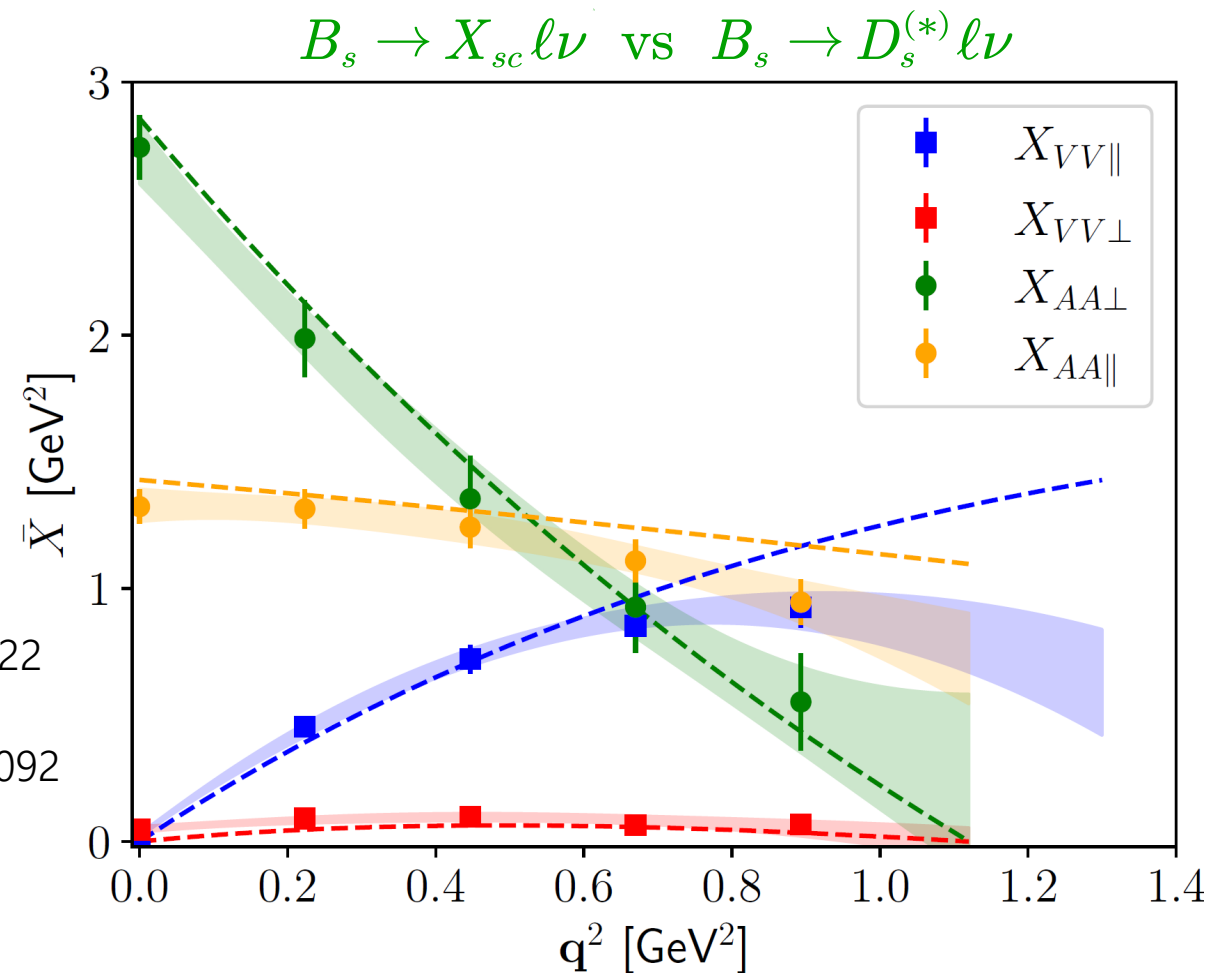
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– ...

⇒ inclusive vs exclusive analyses on the same lattice ⇒ tension in $|V_{cb}|$, $|V_{ub}|$

⇒ CCQE $N\nu \rightarrow X\ell$, γW box for $0^+ \rightarrow 0^+$ β decays, ... (Fukaya-Hashimoto-TK-Ohki 2010.01253)

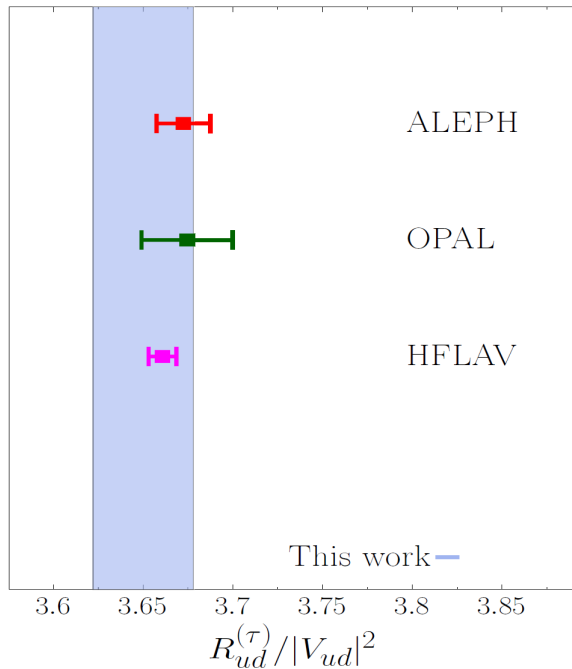


inclusive τ decays

$$\Gamma(\tau \rightarrow X_D \nu_\tau) = \frac{G_F^2}{4m_\tau} |V_{uD}|^2 \int \frac{d^3 q}{(2\pi)^3 2E_\nu} L_{\mu\nu}(p_\tau, p_\nu) W_B^{\mu\nu}(q) \quad (D = d, s) \quad W_{\mu\nu}(q) \Leftrightarrow \langle 0 | J_\mu^\dagger(-\mathbf{q}) e^{-Ht} J_\mu^\dagger(\mathbf{q}) | 0 \rangle$$

$|V_{ud}|$ from $\tau \rightarrow X_d \nu_\tau$ ETM 2308.03125

$$R_d = \Gamma(\tau \rightarrow X_d \nu_\tau) / \Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau)$$



$L = 5.1, 7.6 \text{ fm}$
 \Rightarrow FVEs
 α^{-1}
 $= 2.5 - 3.5 \text{ GeV}$
 $\Rightarrow a = 0$

$$\Rightarrow |V_{ud}| = 0.9752(39) \quad (0.4\%)$$

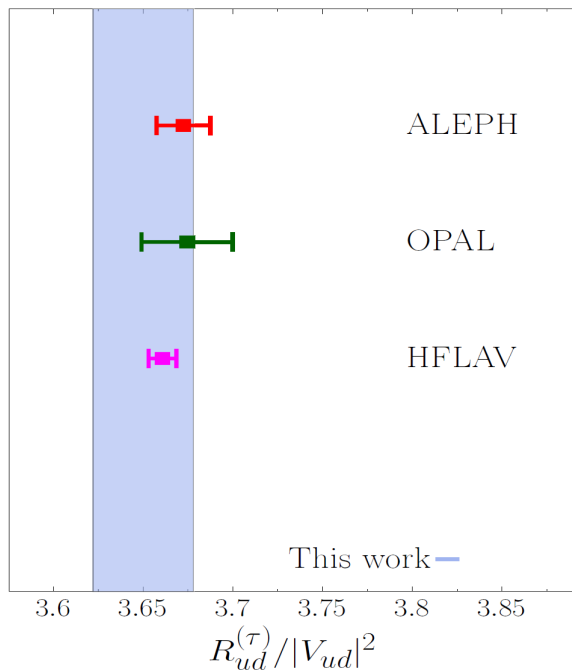
$$\Leftrightarrow 0.97373(31) \quad (0^+ \rightarrow 0^+ \beta \text{ decays})$$

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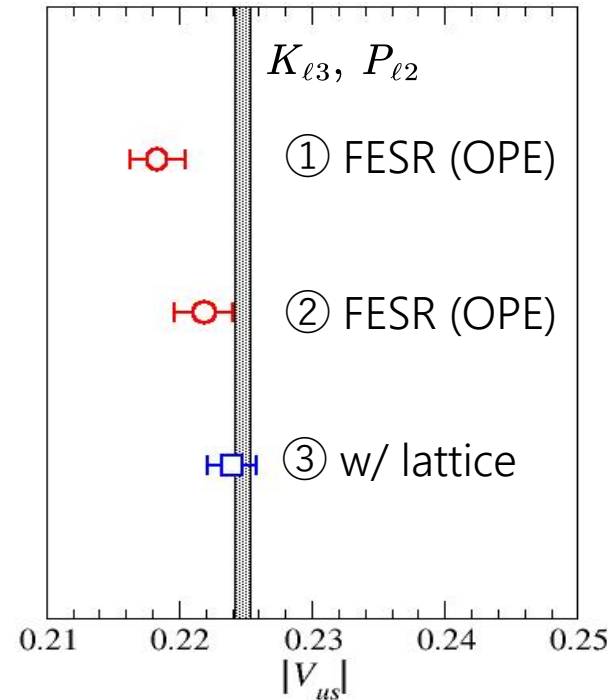
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$|V_{us}|$ from $\tau \rightarrow X_s \nu_\tau$

R_s w/ generic weight for decay channels



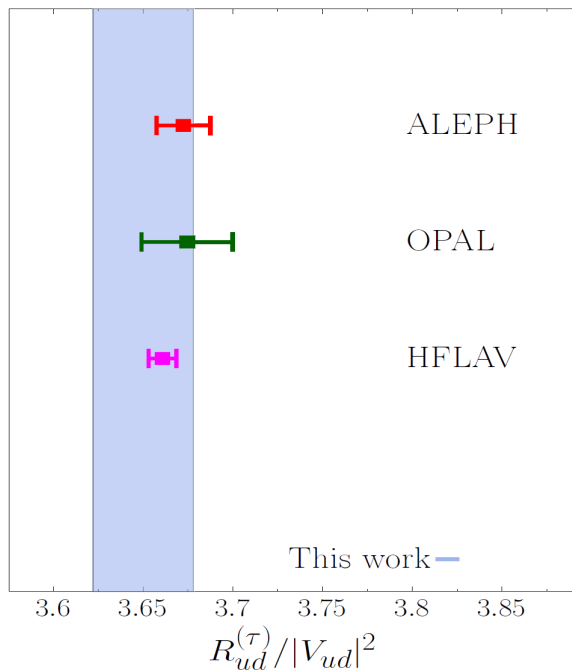
- ① Gamiz+ hep-ph/0612154
- ② Maltman+ '19
 - new treatment HO in OPE
 - partly different exp. input
- ③ RBC/UKQCD 1803.07228
 - HVPF from lattice
 - large weights for $K\nu, K\pi\nu$
 - \Rightarrow mostly rely on these ch.s

inclusive τ decays

$$\Gamma(\tau \rightarrow X_D \nu_\tau) = \frac{G_F^2}{4m_\tau} |V_{uD}|^2 \int \frac{d^3 q}{(2\pi)^3 2E_\nu} L_{\mu\nu}(p_\tau, p_\nu) W_B^{\mu\nu}(q) \quad (D = d, s) \quad W_{\mu\nu}(q) \Leftrightarrow \langle 0 | J_\mu^\dagger(-\mathbf{q}) e^{-Ht} J_\mu^\dagger(\mathbf{q}) | 0 \rangle$$

$|V_{ud}|$ from $\tau \rightarrow X_d \nu_\tau$ ETM 2308.03125

$$R_d = \Gamma(\tau \rightarrow X_d \nu_\tau) / \Gamma(\tau \rightarrow e \bar{\nu}_e \nu_\tau)$$

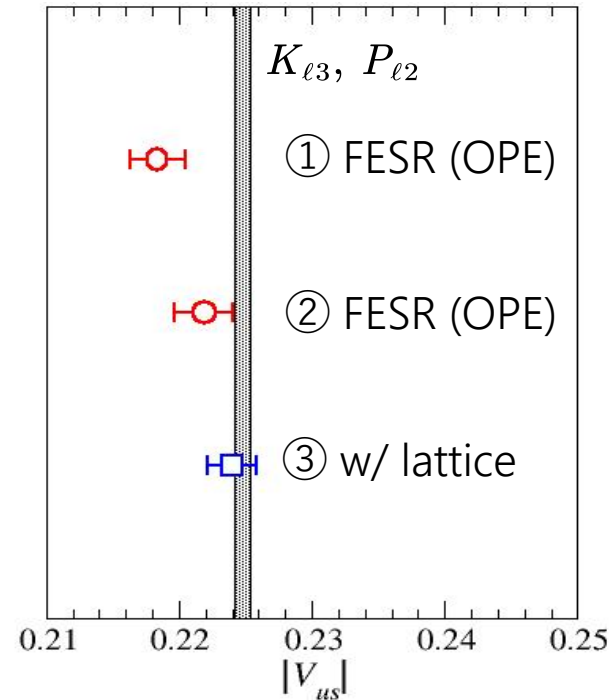


$L = 5.1, 7.6 \text{ fm}$
 \Rightarrow FVEs
 a^{-1}
 $= 2.5 - 3.5 \text{ GeV}$
 $\Rightarrow a = 0$

$\Rightarrow |V_{ud}| = 0.9752(39) \text{ (0.4\%)}$
 $\Leftrightarrow 0.97373(31) \text{ (} 0^+ \rightarrow 0^+ \text{ } \beta \text{ decays)}$

$|V_{us}|$ from $\tau \rightarrow X_s \nu_\tau$

R_s w/ generic weight for decay channels



- ① Gamiz+ hep-ph/0612154
- ② Maltman+ '19
 - new treatment HO in OPE
 - partly different exp. input
- ③ RBC/UKQCD 1803.07228
 - HVPF from lattice
 - large weights for $K\nu, K\pi\nu$
 - \Rightarrow mostly rely on these ch.s

\Rightarrow independent lattice study to clarify the situation

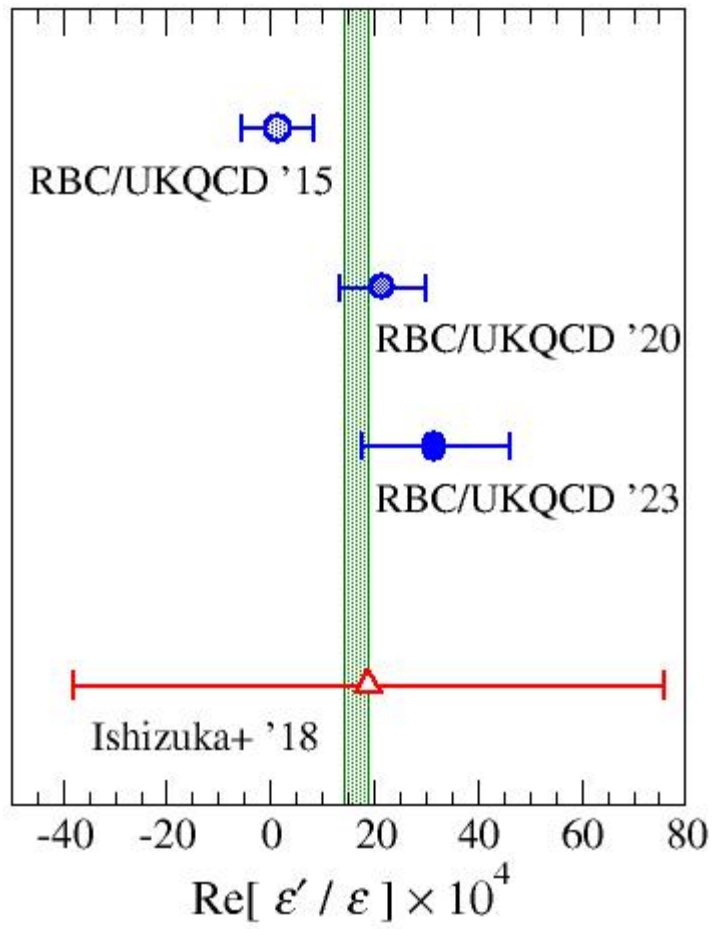
at this workshop

- Elvira Gamiz, “Overview of D form factors and decay constants (lattice)”, WG1 Tue 10:00-
- Felix Erben, “Input to V_{us} from lattice QCD including the progress on hyperon decays”, WG1 Wed 10:00-
- Mikhail Gorchtein, “Improved radiative corrections for $K_{\ell 3}$ decays and superallowed β decays”, WG1 Thu 11:30-
- Ryan Kellermann, “Updates on inclusive charmed and bottomed meson decays from lattice”, WG1&2 Tue 12:00-
- Alejandro Vaquero, “ $B_{(s)} \rightarrow D^{(*)}_{(s)}$ from FNAL/MILC”, WG2 Tue 18:03-
- Ludovico Vittorio, “Unitarity constraints and the dispersive matrix”, WG2 Wed 9:00-
- Carolina Bolognani, “Combining lattice and sum rules to determine $|V_{ub}|/|V_{cb}|$ ”, WG2 Wed 9:24-
- Luka Leskovec, “Lattice outlook on $B \rightarrow \rho$ and $B \rightarrow K^*$ ”, WG2&3 Thu 17:50-
- Chris Bouchard, “Rare $B \rightarrow \pi$ and $B \rightarrow K$ decays on the lattice”, WG2&3 Thu 18:20-
- Brian Colquhoun, “ $B \rightarrow \pi, B \rightarrow D^{(*)}$ from JLQCD”, WG2&3 Thu 18:50-
- En-Hung Chao, “ $K \rightarrow \mu\mu$ on the lattice”, WG3 Mon 14:45-
- Ryan Hill, “Rare kaons on the lattice”, WG3 Tue 14:45-
- Felix Erben, “Update on SU(3)-breaking ratios and bag parameters for $B_{(s)}$ mesons”, WG4

$K \rightarrow \pi\pi$

sensitive to NP

challenging on the lattice : $\pi\pi$ state, disconnected digrams



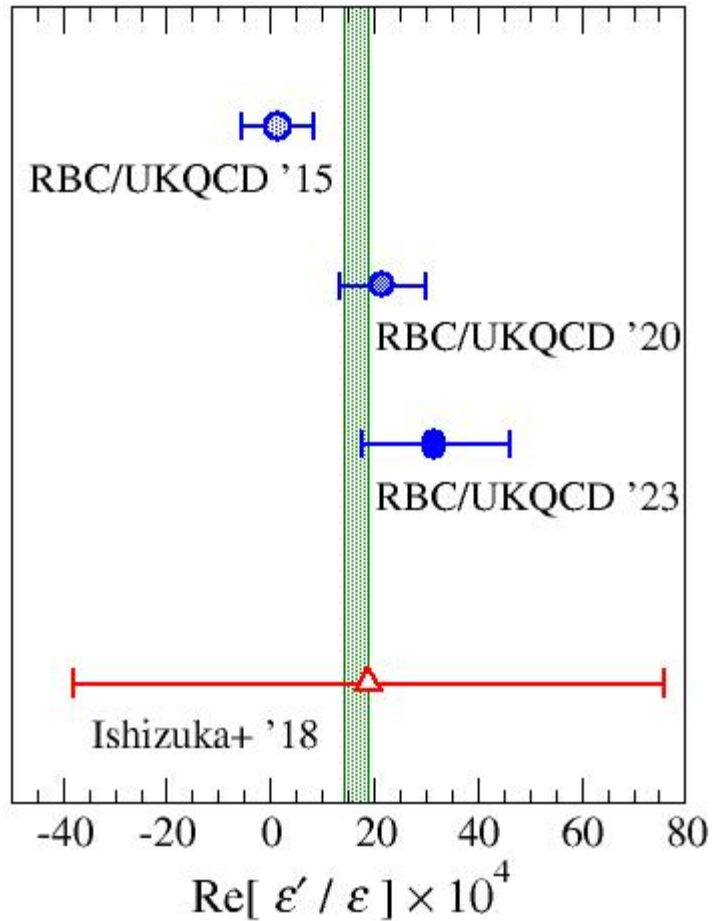
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challenging on the lattice : $\pi\pi$ state, disconnected digrams

RBC/UKQCD w/ "G parity boundary condition"

- G parity BC : flavor rotation
 - + avoid exponentially large off-shell contributions
 - + need dedicated gauge ensembles
- $\text{Re}[\varepsilon'/\varepsilon] \times 10^4 = 21.7(2.6)_{\text{stat}} (5.0)_{\text{EM+iso}} (6.2)_{\text{sys-else}}$
- continued efforts to control $a \neq 0$ error [Kelly @ Lattice 2023]
- independent calculations are welcome!! [cf. Ishizuka+ '18 w/ Wilson quarks]



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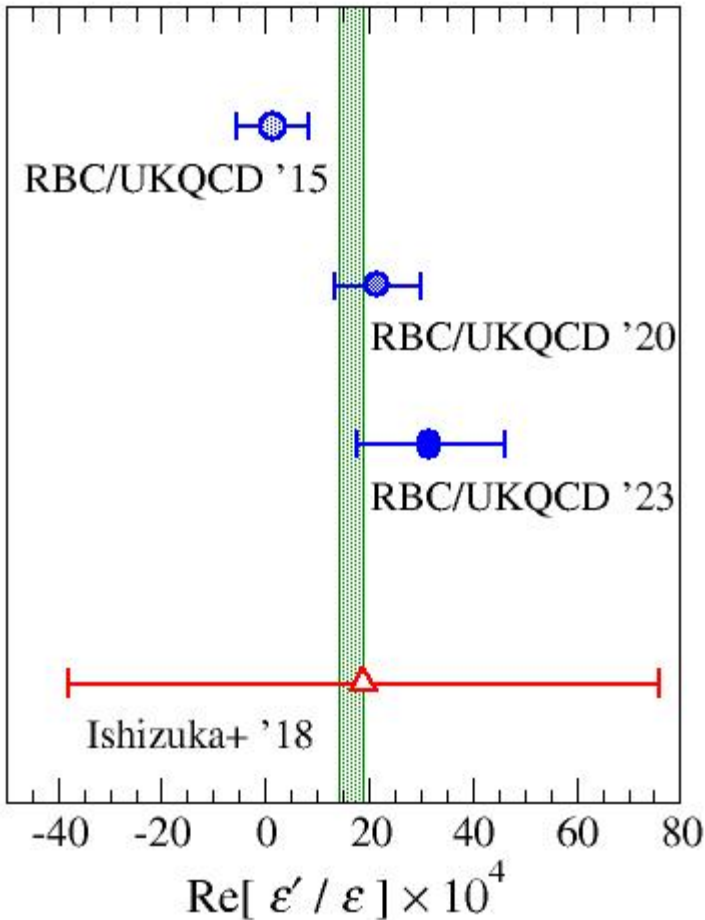
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RBC/UKQCD w/ periodic boundary condition

- + Generalized Eigen Value Prob. to extract "excited state contributions"
- + configurations shared w/ other subjects
- + straightforward for future control of the largest isospin corrections
- $\text{Re}[\varepsilon'/\varepsilon] \times 10^4 = 31.8(6.3)_{\text{stat}} (5.0)_{\text{EM+isospin}} (11.8)_{\text{sys-else}}$
- largest errors from $a \neq 0$, EM+isospin, Wilson coefficients to be controlled



summary

good progress from lattice QCD

for leptonic / semileptonic decays & mixing

- independent calculations with different setups by different group also for B physics
- tensions among precision lattice studies \Rightarrow more realistic simulations
- tension in $|V_{cb}|, |V_{ub}| \Rightarrow$ interplay w/ experiments & phenomenology

for processes involving unstable / multi-particle states

- new applications w/ newly-developed methods: inclusive, $B \rightarrow \rho \ell \nu$, ...
- studies of systematics towards precise calculation
- future subjects: resonance contribution to $B \rightarrow K^{(*)} \ell \ell$, D mixing, ...