$K \to \mu^+\mu^-$ in the continuum

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A lot of recent activity in rare kaon decays

Theory

- D'Ambrosio Kitahara, 1707.06999
- Dery Ghosh Grossman StS, 2104.06427
- Buras Venturini, 2109.11032
- Dery Ghosh, 2112.05801
- D'Ambrosio Iyer Mahmoudi Neshatpour 2206.14748
- Brod Stamou, 2209.07445
- Dery Ghosh Grossman Kitahara StS, 2211.03804

Experiment

- NA62, 2103.15389: 3.4σ evidence for $K^+ \to \pi^+ \nu\bar{\nu}$ at NA62.
- KOTO, 2012.07571: Improved upper limit on $K_L \to \pi^0 \nu\bar{\nu}$.
- LHCb, 2001.10354: Upper limit on $K_S \to \mu^+\mu^-$.
- LHCb, KAON'22: Upper limits on $K_{S,L} \to 2(\mu^+\mu^-)$.
- HIKE LoI, 2211.16586: New ideas for Kaons at CERN.

$K \to \mu^+\mu^-$ is exciting!

[D'Ambrosio Kitahara 1707.06999, Dery Ghosh Grossman StS, 2104.06427, Brod Stamou 2209.07445]

The new idea

- We can very cleanly measure $\mathrm{Im}(V_{td}^* V_{ts})$ (or η) from $K \to \mu^+\mu^-$.
- We can do so employing time-dependent interference effects.

- Third golden channel alongside:
  $K^+ \to \pi^+ \nu\bar{\nu}$ gives $|V_{td} V_{ts}|$ NA62
  $K_L \to \pi^0 \nu\bar{\nu}$ gives $\mathrm{Im}(V_{td}^* V_{ts})$ (or η) KOTO
- Determine the unitarity triangle purely with kaon decays.
  ➥ Crucial intergenerational consistency check of the SM.
- New ways to probe for new physics.

The three golden channels

$K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

"Theoretically clean, experimentally hard"

$K \to \mu^+\mu^-$, common lore

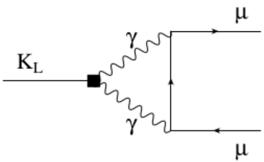
"Theoretically not clean, experimentally not hard."

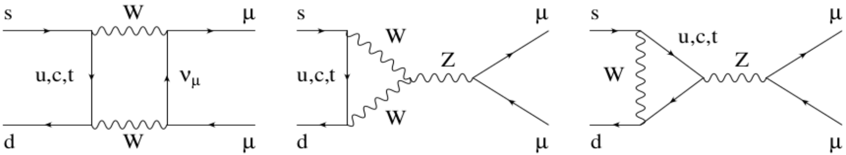
$K \to \mu^+\mu^-$, this talk

"Theoretically clean, experimentally hard."

- f_K is the main hadronic uncertainty.
- Challenging to measure time-dependent interference effects.

Long-Distance and Short-Distance Physics





[Isidori Unterdorfer hep-ph/0311084]

Can we overcome soft QCD?

- To get a theoretically clean method we need a theory error of $\lesssim 1\%$.
- We are currently not able to achieve theory precision of long distance (LD) effects in $K \to \mu^+\mu^-$ below $\sim 10\%$.
- We know short-distance (SD) physics at desired precision.
- How can we measure the SD physics?

Basics of $K \to \mu^+\mu^-$

Approximation

- In this talk we neglect CP-violating in mixing ε_K.
  Can be incorporated into analysis [Brod Stamou 2209.07445].

Angular momentum conservation: Only $(\mu\mu)_{l=0}$ or $(\mu\mu)_{l=1}$

- CP-conserving decays

K_L	$\to$	$(\mu\mu)_{l=0}$	K_S	$\to$	$(\mu\mu)_{l=1}$
CP-odd		CP-odd	CP-even		CP-even

- CP-violating decays

K_S	$\to$	$(\mu\mu)_{l=0}$	K_L	$\to$	$(\mu\mu)_{l=1}$
CP-even		CP-odd	CP-odd		CP-even

$K \to \mu^+\mu^-$ in the Standard Model

To good approximation:

- LD effects are CP conserving. $\Rightarrow$ CP violating amplitudes are purely SD.

Short-distance (SD) and long-distance (LD) physics

- CP-conserving decays: SD and LD

K_L	$\to$	$(\mu\mu)_{l=0}$		K_S	$\to$	$(\mu\mu)_{l=1}$
CP-odd		CP-odd		CP-even		CP-even

- CP-violating decays: Only SD

K_S	$\to$	$(\mu\mu)_{l=0}$		K_L	$\to$	$(\mu\mu)_{l=1}$
CP-even		CP-odd		CP-odd		CP-even

$K \to \mu^+\mu^-$ in the Standard Model

- SM: SD operator does not generate $(\mu\mu)_{l=1}$ state (CPT).

Short-distance (SD) and long-distance (LD) physics

- CP-conserving decays: SD and LD

K_L	$\to$	$(\mu\mu)_{l=0}$	K_S	$\to$	$(\mu\mu)_{l=1}$
CP-odd		CP-odd	CP-even		CP-even

- CP-violating decays: Only SD

K_S	$\to$	$(\mu\mu)_{l=0}$	K_L	$\to$	$(\mu\mu)_{l=1}$
CP-even		CP-odd	CP-odd		CP-even
				$= \mathbf{0}$	

Counting of Theory Parameters

- CP-conserving decays: SD and LD

$$|A(K_L \to (\mu\mu)_{l=0})| \qquad\qquad |A(K_S \to (\mu\mu)_{l=1})|$$

CP-odd CP-odd CP-even CP-even

- CP-violating decays: Only SD

$$|A(K_S \to (\mu\mu)_{l=0})| \qquad\qquad |A(K_L \to (\mu\mu)_{l=1})| = \mathbf{0}$$

CP-even CP-odd CP-odd CP-even

- Phases

$$\varphi_0 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=0})\mathcal{A}(K_L \to (\mu\mu)_{l=0})\right)$$

$$\varphi_1 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=1})\mathcal{A}(K_L \to (\mu\mu)_{l=1})\right) = \mathbf{0}$$

- A priori: 6 parameters: 4 magnitudes, 2 phases.
- In SM/large class of NP models: Reduction to 4, 1 of which is pure SD.

$K_S \to (\mu\mu)_{l=0}$ is the key to SD physics

- We can cleanly calculate it in the SM.

$$\mathcal{B}(K_S \to (\mu\mu)_{l=0}) = 1.7 \cdot 10^{-13} \times \left(\frac{A^2 \lambda^5 \bar{\eta}}{1.3 \times 10^{-4}}\right)$$

[Inami Lim 1981, Isidori Unterdorfer hep-ph/0311084, Dumm Pich hep-ph/9801298, Brod Stamou 2209.07445]

- Hadronic uncertainties from $f_K < 1\%$.
- Way to extract η theoretically clean.
- We can also calculate $\mathcal{B}(K_S \to (\mu\mu)_{l=0})$ cleanly in NP models.

In practice we measure incoherent sum

- Muon states with specific angular momentum $(\mu\mu)_{l=0}$ and $(\mu\mu)_{l=1}$:
  Not available to us: We cannot separate $l = 0$ and $l = 1$.

- Instead, we measure the incoherent sum:

$$\Gamma(K_S \to \mu^+\mu^-)_{\text{meas.}} = \Gamma(K_S \to (\mu^+\mu^-)_{l=0}) + \Gamma(K_S \to (\mu^+\mu^-)_{l=1})$$

$$\Gamma(K_L \to \mu^+\mu^-)_{\text{meas.}} = \Gamma(K_S \to (\mu^+\mu^-)_{l=0}) + \Gamma(K_S \to (\mu^+\mu^-)_{l=1})$$

$\Rightarrow$ "So what are you talking about?"

Solution: Look at time dependence

- Generic time dependence of K decay:

$$\left(\frac{d\Gamma}{dt}\right) \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

- $\Gamma = (\Gamma_S + \Gamma_L)/2$. Δm: Kaon mass difference.
- The 4 Cs are the observables:
  - C_L is related to K_L decay rate.
  - C_S is related to K_S decay rate.
  - C_{sin} and C_{cos} are due to interference.
- We can calculate the 4 Cs in terms of the 4 theoretical parameters.

We can completely solve the system.

- For pure K^0 beam:

$$C_L = |A(K_L)_{l=0}|^2$$
$$C_S = |A(K_S)_{l=0}|^2 + |A(K_S)_{l=1}|^2$$
$$C_{cos} = \mathrm{Re}\left(A(K_S)_{l=0} \times A^*(K_L)_{l=0}\right)$$
$$C_{sin} = \mathrm{Im}\left(A(K_S)_{l=0} \times A^*(K_L)_{l=0}\right)$$

- We can get the clean amplitude from the observable combination

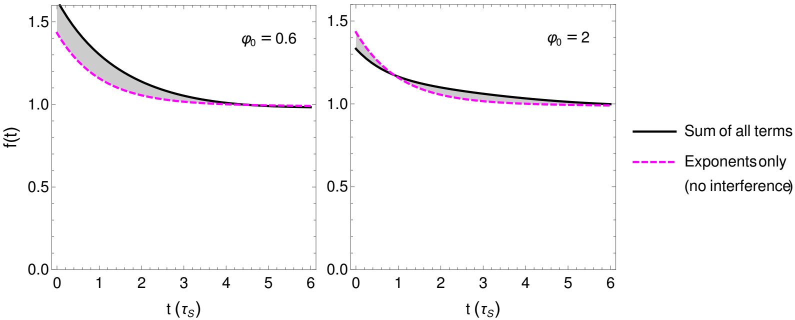
$$|A(K_S)_{l=0}|^2 = \frac{C_{cos}^2 + C_{sin}^2}{C_L} \,.$$

- We can rewrite this as:

$$\mathcal{B}(K_S \to (\mu^+\mu^-)_{l=0}) = \mathcal{B}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \frac{C_{cos}^2 + C_{sin}^2}{C_L^2}$$

- Compare with calculation of $\mathcal{B}(K_S \to (\mu^+\mu^-)_{l=0}) \Rightarrow$ extract η.
- We need the interference terms!

Demonstration of Interference Effect



$$\left(\frac{d\Gamma}{dt}\right) \propto f(t) \equiv C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

- Using estimates, not showing large hadronic uncertainties for long-distance contributions.
- As examples, two ad-hoc values for the phase.
- All parameters can be determined from experiment.

Experimental Considerations

- Experimentally, not easy to have pure K^0 or $\overline{K}^0$ beam.
- NA62: charged kaons. KOTO: pure K_L. LHCb: almost equal mix.
- In these limits no sensitivity to interference term.
- Employ mixed beam. Need non-zero production asymmetry.
  - Regeneration of K_S in K_L beam through matter effects.
  - Charged exchange targets: turn charged K^+ beams into K^0 beams.
  - Post-selection using tagging (?)
- Future new kaon facility (?)
- Interference terms are then diluted by dilution factor D:

$$D = \frac{N_{K^0} - N_{\overline{K}^0}}{N_{K^0} + N_{\overline{K}^0}} \quad C_{cos} \mapsto DC_{cos} \qquad C_{sin} \mapsto DC_{sin}$$

  N_K: Number of incoherent mixture of kaons/anti-kaons at $t = 0$.
- Pure $K^0/\overline{K}^0$: $D = \pm 1$.

How many kaons are needed to do the measurement?

- We have $\mathcal{B}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \cdot 10^{-9}$.
- Only 1% of the K_L decay in region of interest $t \lesssim 6\tau_S$
- Fraction of useful events: $\sim\ 10^{-10}$.
- For $\mathcal{O}(1000)$ events we need $\mathcal{O}(10^{13})\ K^0$ to start with.

First Experimental Studies: Next-generation Kaon Experiments

[Marchevski 2301.06801]

- First possible experimental setup presented:
  Modification of NA62 $K^+ \Rightarrow K^0$ at CERN SPS.
- Need sample of $\mathcal{O}(10^{14} - 10^{15})$ K_L and K_S decays.
- Need to suppress background from $K_S \to \pi^+\pi^-$ $(\mathcal{B} \sim 70\%)$
  and radiative $K_L \to \mu^+\mu^-\gamma$ $(\mathcal{B}(\sim 3.6 \cdot 10^{-7}))$.
- Requires excellent kinematic resolution + efficient photon detection

Relating $\Gamma(K \to \mu^+\mu^-)(t)$ and $\mathcal{B}(K_L \to \gamma\gamma)/\mathcal{B}(K_L \to \mu^+\mu^-)$

[Dery Ghosh Grossman Kitahara StS 2211.03804]

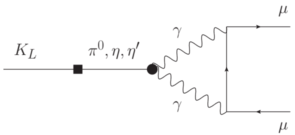
- We know more about the phase shift in the oscillating rate.

$$\varphi_0 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=0})\mathcal{A}(K_L \to (\mu\mu)_{l=0})\right)$$

$$\frac{1}{\mathcal{N}}\frac{d\Gamma(K^0 \to \mu^+\mu^-)}{dt} = C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2C_{\text{int.}}\cos(\Delta M_K t - \varphi_0)e^{-\frac{\Gamma_L+\Gamma_S}{2}t}$$

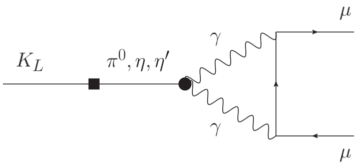
We find the precision relation

$$\cos^2\varphi_0 = (\text{known QED factor}) \times \frac{\mathcal{B}(K_L \to \gamma\gamma)}{\mathcal{B}(K_L \to \mu^+\mu^-)}$$



Relating $\Gamma(K \to \mu^+\mu^-)(t)$ and $\mathcal{B}(K_L \to \gamma\gamma)/\mathcal{B}(K_L \to \mu^+\mu^-)$

[Dery Ghosh Grossman Kitahara StS 2211.03804]



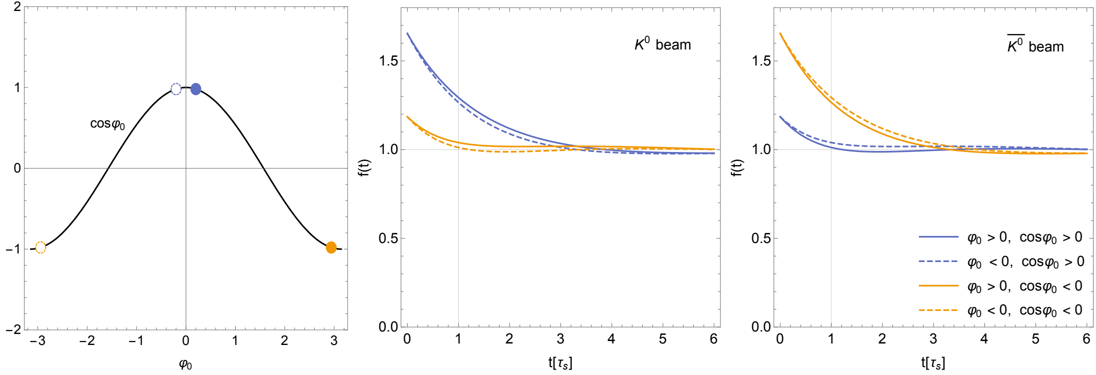
Result (model-independent)

$$\cos^2 \varphi_0 = 0.96 \pm 0.02_{\text{exp.}} \pm 0.02_{\text{th}}$$

- Exp. error: From BR measurements in R_{K_L}.
- Th. error 1: Higher order QED corrections $\sim \alpha \sim 1\%$.
- Th. error 2: Contribution of additional intermediate on-shell contributions $(3\pi, \pi\pi\gamma)$, also estimated as $\sim 1\%$. [Martin De Rafael Smith 1970]

Discrete Ambiguities (model-independent)

[Dery Ghosh Grossman Kitahara StS 2211.03804]



$$\frac{1}{\mathcal{N}}\frac{d\Gamma(K^0 \to \mu^+\mu^-)}{dt} = C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2C_{\text{int.}} \cos(\Delta M_K t - \varphi_0) e^{-\frac{\Gamma_L+\Gamma_S}{2} t}$$

Beyond the Standard Model

How much room is there for NP?

[Dery Ghosh 2112.05801]

- 2020 measurement of LHCb [LHCb, 2001.10354]

$$\mathcal{B}(K_S \to \mu^+\mu^-) < 2.1 \cdot 10^{-10}$$

- Sum of contributions with different CP (no interference):

$$\mathcal{B}(K_S \to \mu^+\mu^-) = \mathcal{B}(K_S \to \mu^+\mu^-)_{l=0} + \mathcal{B}(K_S \to \mu^+\mu^-)_{l=1}$$

- Conservative interpretation:
  Use as bound solely for the $l = 0$ (SD) contribution

$$\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0} \leq 2.1 \cdot 10^{-10} .$$

- $\Rightarrow$ A lot of room for NP in the SD amplitude:

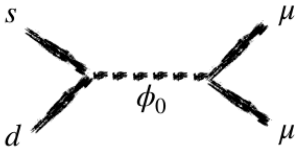
$$R(K_S \to \mu^+\mu^-)_{l=0} \equiv \frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}}{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}^{\text{SM}}} \lesssim 10^3 .$$

How much room is there for NP?

[Dery Ghosh 2112.05801]

- Scalar Leptoquarks:

- 2HDM:



- Both can saturate bound, consistent with existing constraints.
- Updated bounds from LHCb important to constrain the model space further.

[Diagrams courtesy Avital Dery]

Another very clean SM test

[Buras Venturini 2109.11032]

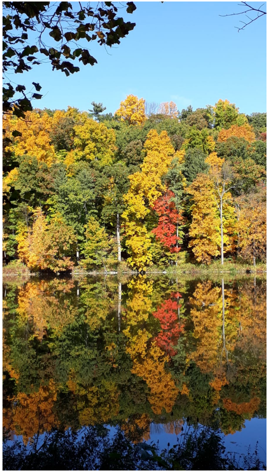
- Combination of $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\nu\bar{\nu}$:

$$\frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}}{\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})} = 1.55 \times 10^{-2} \left(\frac{\lambda}{0.225}\right)^2 \left(\frac{Y(x_t)}{X(x_t)}\right)^2$$

- Depends only on Wolfenstein-λ ($|V_{us}|$) and m_t.
- Does not depend on $|V_{cb}|$.
- $K \to \mu^+\mu^-$ and $K_L \to \pi^0\nu\bar{\nu}$ sensitive to different NP operators.

Conclusion

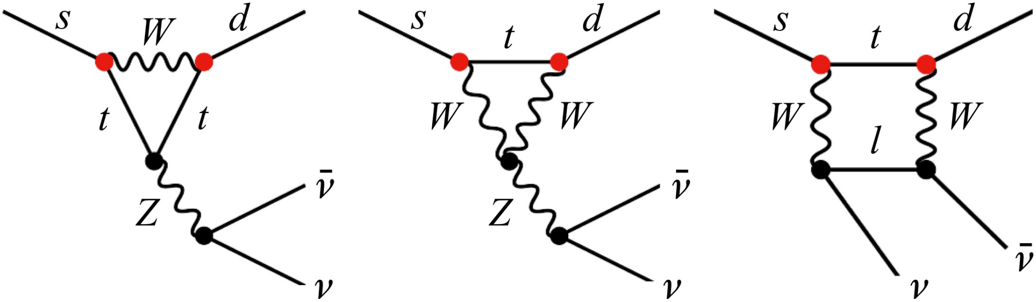
- Within SM, $K(t) \to \mu^+\mu^-$ gives same info as $K_L \to \pi^0\nu\bar{\nu}$.
- Complementary NP sensitivity: Combination distinguishes models.
- Time-dependence of $K \to \mu^+\mu^-$: 2 independent SM tests:
  - Coefficient of interference term $\Rightarrow \mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}$.
    In SM, $\propto |V_{ts}V_{td}\sin(\beta+\beta_s)|$.
  - Interference term phase shift φ_0 predicted up to 4-fold ambiguity.
- "Theoretically clean, experimentally hard": Can we do it?



BACK-UP

$K \to \pi\nu\bar{\nu}$: Very clean SM prediction.

- Probing FCNC $s \to d\nu\bar{\nu}$.
- Loops dominated by top quark contribution.
- Hadronic matrix elements from $K \to \pi e\nu$.



[Snowmass white paper 2204.13394]

Current Status of $K \to \pi\nu\bar{\nu}$

SM prediction

[Buras et al, 1503.02693]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = (8.39 \pm 0.30) \cdot 10^{-11} \left(\frac{|V_{cb}|}{0.0407}\right)^{2.8} \left(\frac{\gamma}{73.2^\circ}\right)^{0.74}$$

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) = (3.36 \pm 0.05) \cdot 10^{-11} \left(\frac{|V_{ub}|}{3.88 \times 10^{-3}}\right)^2 \left(\frac{|V_{cb}|}{0.0407}\right)^2 \left(\frac{\sin\gamma}{\sin 73.2^\circ}\right)^2$$

NA62

[NA62, 2103.15389]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \left(10.6^{+4.0}_{-3.4} \pm 0.9\right) \times 10^{-11}$$

KOTO

[KOTO, 1810.09655]

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu}) \leq 3 \times 10^{-9}$$

Resolving Discrete Ambiguities (model-dependent): 4-fold $\Rightarrow$ 2-fold

- Sign of $\cos\varphi_0$ can be determined using ChPT and/or Lattice QCD.
- In large N_C limit, it was found destructive interference between SD and LD contributions.

[Isidori/Unterdorfer hep-ph/0311084, Dumm/Pich hep-ph/9801298]

We find that destructive interference implies:

$\mathrm{sgn}\left(\cos\varphi_0\right) = \mathrm{sgn}\left(\tan\theta_{SD}\right)\,.$ θ_{SD}: weak phase of SD $K^0 \to (\mu^+\mu^-)_{l=0}$ amplitude.

In the Standard Model

$$\tan\theta_{SD}^{\mathrm{SM}} = \tan\left(\arg\left(-\frac{V_{ts}^* V_{td} + V_{cs}^* V_{cd} Y_{NL}/Y_t}{V_{cs}^* V_{cd}}\right)\right) < 0$$

$$\Rightarrow \left[\cos\varphi_0\right]^{\mathrm{SM}}_{\text{large } N_C} = -0.98 \pm 0.02\,.$$

- Remaining ambiguity $\mathrm{sgn}(\sin\varphi_0)$ cannot be resolved: large uncertainty of LD contributions.

Model Independent Effective Operator Analysis

$$\mathcal{H}_{eff}^{\Delta S=1} = \sum_i C_i O_i \,.$$

[Dery Ghosh 2112.05801]

- Vectorial:

$$O_{VLL} = \left(\overline{Q}_L \gamma^\mu Q_L\right)\left(\overline{L}_L \gamma_\mu L_L\right)\,, \qquad O_{VLR} = \left(\overline{Q}_L \gamma^\mu Q_L\right)\left(\bar{e}_R \gamma_\mu e_R\right)\,,$$

$$O_{VRL} = \left(\overline{d}_R \gamma^\mu d_R\right)\left(\overline{L}_L \gamma_\mu L_L\right)\,, \qquad O_{VRR} = \left(\overline{d}_R \gamma^\mu d_R\right)\left(\bar{e}_R \gamma_\mu e_R\right)\,.$$

- Scalar:

$$O_{SLR} = \left(\overline{Q}_L d_R\right)\left(\bar{e}_R L_L\right)\,, \qquad O_{SRL} = \left(\bar{d}_R Q_L\right)\left(\bar{L}_L e_R\right)\,.$$

- SM: $K_S \to (\mu^+\mu^-)_{l=0}$ and $K_L \to \pi^0\bar{\nu}\nu$ from O_{VLL}.
- Current bound sensitive to NP scales:
  $\Lambda \sim 40$ TeV (vectorial) and $\Lambda \sim 130$ TeV (scalar).

Complementarity of $K_S \to (\mu^+\mu^-)_{l=0}$ and $K_L \to \pi^0\nu\bar{\nu}$

SM

- $\bar{\eta}$ also accessible at KOTO using $K_L \to \pi^0\nu\bar{\nu}$.
  Prospect: "KOTO step 2" aims to measure $\bar{\eta}$ with precision of $\sim 14\%$.

[J-PARC white paper 2110.04462]

NP

[Dery Ghosh 2112.05801]

- In units of SM prediction: $R(X) = X/X_{SM}$:

$$R(K_S \to \mu^+\mu^-)_{l=0} = 1 + \text{function}\left(C^{NP}_{SLR}, C^{NP}_{SRL}, C^{NP}_{VLL}, C^{NP}_{VLR}, C^{NP}_{VRL}, V^{NP}_{VRR}\right)$$

$$R(K_L \to \pi^0\bar{\nu}\nu) = 1 + \text{function}\left(C^{NP}_{VLL}, C^{NP}_{VRL}\right) .$$

- $K_S \to (\mu^+\mu^-)_{l=0}$: sensitivity to RH leptonic currents + scalar operators.
- Complementary NP sensitivity.