

Perturbative Aspects of rare K and B decays

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Based on work with
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Sieja, Stamou, Yu

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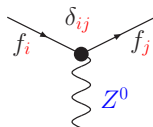
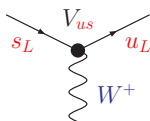
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- ▶ $B_s \rightarrow \mu\mu$ and $B \rightarrow X_s \ell^+ \ell^-$
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- ▶ $K \rightarrow \mu\mu$
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- ▶ Parametric uncertainties

Neutral & Charged Current Interactions

Mass \neq flavour eigenstates



SM: Only charged currents
change the flavour ($\propto V_{us}$)

SM: Neutral currents do not
change the flavour ($i=j$) at tree-level

$$V_{\text{CKM}} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho + i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- ▶ SM: Yukawas only source of flavour & CP violation
- ▶ CKM parametrises CP & flavour violation
- ▶ First row from tree-level semi-leptonic decays

Charged Current decays

- ▶ $K_{\ell 2}$ and $K_{\ell 3}$ extraction of $\lambda = |V_{us}|$

$$\Gamma(K^0 \rightarrow \pi^- \ell^+ \nu_\ell (\gamma)) = \frac{G_F^2 m_K^5}{128 \pi^3} |V_{us}|^2 S_{EW} |f_+^{K^0 \pi^-}(0)|^2 I_{K^0 \ell}^{(0)} (1 + \delta_{EM}^{K^0 \ell} + \delta_{SU(2)}^{K^0 \pi^-}).$$

- ▶ QED: χPT [Seng et.al.'2019, Cirigliano et.al.'23] and Lattice [Carrasco et.al.'15, DiCarlo et.al.'19]
- ▶ EW corrections in W-Mass scheme [Marciano, Sirlin]
- ▶ EFT Approach [Gorbahn et.al.'22, Cirigliano et.al.'23]
- ▶ $|V_{ud}|$, extracted from nuclear β decays [Hardy, Towner'20],
- ▶ $\Delta_{CKM} \equiv 1 - |V_{ud}|^2 - |V_{us}|^2 - \mathcal{O}(|V_{ub}|^2) = 0.$

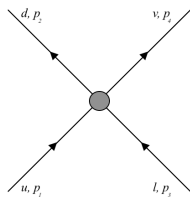
Effective Theory Calculation

- ▶ Decoupling Theorem (Renormalization [Collins]):

$$\langle \ell_3 | T | K \rangle = 4G_F / \sqrt{2} C(\mu_W) \langle \ell_3 | O | K \rangle(\mu_W) + \mathcal{O}(p_\ell^2 / M_W^2)$$

- ▶ Determine $C(\mu_W)$ in perturbation theory
- ▶ Use RGE to run $\langle \ell_3 | T | K \rangle =$
 $4G_F / \sqrt{2} C(\mu_W) U(\mu_W, \mu_{Lat}) \langle \ell_3 | O | K \rangle(\mu_{Lat}) + \mathcal{O}(p_\ell^2 / M_W^2)$
- ▶ Determine $\langle \ell_3 | O | K \rangle$ using symmetries and data or Lattice calculation
- ▶ Lattice: have to convert Lattice to continuum scheme
- ▶ Residual μ_W and μ_{Lat} dependence reduces at N^n LO

Lattice Renormalisation



- ▶ off-shell renormalisation conditions

- ▶ RI^(') – MOM: $p_1 = p_2 = p_3 = p_4 = p, \quad p^2 = -\mu^2$

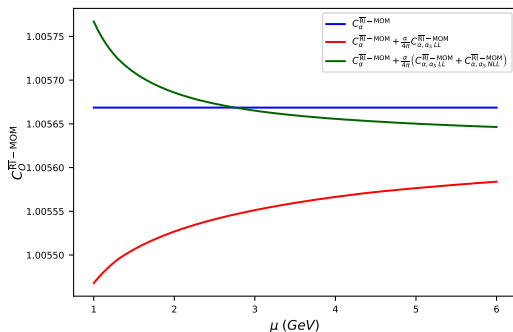
- ▶ RI – SMOM:

- $p_1 = p_3, \quad p_2 = p_4, \quad p_1^2 = p_2^2 = -\mu^2, \quad p_1 \cdot p_2 = -\frac{1}{2}\mu^2$

- ▶ Choose projectors so that $Z_O = 0 + O(\alpha)$ [2209.05289]

\overline{RI} and \overline{MS} Wilson coefficients

Including 2-loop EW matching and 3-loop RGE [MG, SJ, Moretti, EM]



For V_{ud} we have $C_O(m_c) = 1.00754$ [Cirigliano et.al.'23] \rightarrow
 $C_O(m_c) = 1.00794$

Rare $b \rightarrow s$ decays

- ▶ Up quark contribution CKM suppressed
- ▶ $V_{cb}^* V_{cs} \rightarrow -V_{tb}^* V_{ts} - \mathcal{O}(\lambda^4)$
- ▶ Only $V_{tb}^* V_{ts}$
- ▶ Operators:
 - ▶ $O_L = (\bar{s}\gamma_\nu b_L)(\bar{\mu}\gamma^\nu P_L \mu)$ generated at M_W
 - ▶ $O_V = (\bar{s}\gamma_\nu b_L)(\bar{\mu}\gamma^\nu \mu)$ generated at $\mathcal{O}(\alpha)$
- ▶ Pseudoscalar decay $B_s \rightarrow \mu^+ \mu^-$ has $\langle O_V \rangle = \mathcal{O}(\alpha)$

$$B_s \rightarrow \mu^+ \mu^-$$

- ▶ In the SM for $\alpha_{\text{QED}} = 0$ only one operator:
 - ▶ $\mathcal{L}_{\text{eff}} = V_{tb} V_{ts}^* G_F^2 M_W^2 \pi^{-2} \tilde{c}_{10} (\bar{s} \gamma_\alpha b_L) (\bar{\mu} \gamma^\alpha \gamma_5 \mu)$
 - ▶ ME: $\langle 0 | \bar{s} \gamma_\alpha \gamma_5 b | \bar{B}_s \rangle = i f_{B_s} p_\alpha$
 - ▶ Scalar contribution suppressed; $\langle O_9 \rangle = O(\alpha/4\pi)$
- ▶ NNLO QCD and two-loop EW corrections @ M_W plus $O(\alpha) \ln m_b/M_W$ [1311.0903, 1311.1347, 1311.1348]
 - ▶ Reduce perturbative uncertainties from 7% to 1%
 - ▶ Power enhanced non-local QED corrections
[1708.09152] gives $+1 \times (1 - 0.2)\%$ (resum [1908.07011])
 - ▶ Factorisation theorem: RGI improved O_7 contribution
[2211.04209]

Time integrated branching ratio

- ▶ Measured branching ratio in terms of time = 0

$$[1204.1737]: \overline{\mathcal{B}}_{s\mu} = (1 - y_s)^{-1} \mathcal{B}_{s\mu}^{[t=0]}$$

- ▶ $\overline{\mathcal{B}}_{s\mu} = (3.65 \pm 0.06) \times (1.008) \times R_{t\alpha} R_s \times 10^{-9}$ [1311.0903]
including QED 0.8% effect [1908.07011]

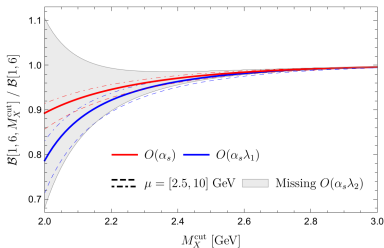
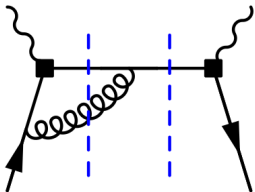
- ▶ Input f_{B_s} (Lattice), CKM, $m_t^{\overline{MS}}$ and α_s :

$$R_s = \left(\frac{f_{B_s} [\text{MeV}]}{227.7} \right)^2 \left(\frac{|V_{cb}|}{0.0424} \right)^2 \left(\frac{|V_{tb}^* V_{ts} / V_{cb}|}{0.980} \right)^2 \frac{\tau_H^s [\text{ps}]}{1.615}$$

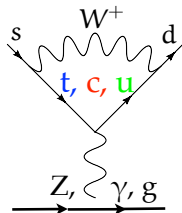
$$R_{t\alpha} = (m_t / (163.5 \text{ GeV}))^{3.02} (\alpha_s(M_Z) / 0.1184) \alpha^{0.032}$$

$b \rightarrow s\ell^+\ell^-$

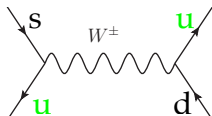
- ▶ $\mathcal{L}_{eff} = 2\sqrt{2}G_F V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i Q_i$ for $\bar{B} \rightarrow K^{(*)}\mu^+\mu^-$ pheno
- ▶ Alternative test $\bar{B} \rightarrow X_S\mu^+\mu^-$
 - ▶ SD Lagrangian known at NNLO + (EW)
 - ▶ Updated SM predictions [2007.04191]
 $\mathcal{B}[1,6]_{\mu\mu}^{SM} = (1.73 \pm 0.13) \times 10^{-6}$
 - ▶ High q^2 analysis using C_L & C_V [2305.03076]
 - ▶ Dependence on M_X cut@NLO [2306.03134]



Rare Decays: CKM Structure (Kaons)



Using the GIM mechanism, we can eliminate either $V_{cs}^* V_{cd}$ or $V_{us}^* V_{ud} \rightarrow -V_{cs}^* V_{cd} - V_{ts}^* V_{td}$



Z-Penguin and Boxes (high virtuality):

power expansion in: $A_c - A_u \propto 0 + \mathcal{O}(m_c^2/M_W^2)$

γ/g -Penguin (expand in mom.): $A_c - A_u \propto \mathcal{O}(\text{Log}(m_c^2/m_u^2))$

$$\text{Im}V_{ts}^* V_{td} = -\text{Im}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^5) \quad \text{Im}V_{us}^* V_{ud} = 0$$

$$\text{Re}V_{us}^* V_{ud} = -\text{Re}V_{cs}^* V_{cd} = \mathcal{O}(\lambda^1) \quad \text{Re}V_{ts}^* V_{td} = \mathcal{O}(\lambda^5)$$

- ▶ $K \rightarrow \pi \bar{\nu} \nu$ (from Z & Boxes): Clean and suppressed
- ▶ $K \rightarrow \mu \mu$ pollution from two photon contribution

$$K \rightarrow \mu^+ \mu^-$$

$$K \rightarrow \mu^+ \mu^-$$

SD: $Y_t + Y_{NNLO}$ from SM $\mathcal{L}_{\text{eff}}^{d=6}$ gives $\ell = 0$. $[O(\lambda_t, \frac{m_c^2}{M_W^2} \lambda_c)]$

LD: from γ - γ loop is **CP** conserving: $O(\frac{\alpha_{QED}}{4\pi})$

Setting ϵ_K to zero, K_L and K_S are CP eigenstates:

K_i^{CP}	BR_{exp}	$(\mu^+ \mu^-)_{\ell=0}^-$	$(\mu^+ \mu^-)_{\ell=1}^+$
$K_L^{(-)}$	$6.84(11) \times 10^{-9}$	$\lambda_t, \frac{m_c^2}{M_W^2} \lambda_c, \gamma\text{-}\gamma$	(CP) $\simeq 0$
$K_S^{(+)}$	$< 2.1 \times 10^{-10}$	CP: $\text{Im } \lambda_t$	$\gamma\text{-}\gamma$

ℓ is not experimentally accessible, but interference term in time dependent decay sensitive to **SD** [D'Ambrosio, Kitahara '17]

$$K(t) \rightarrow \mu^+ \mu^-$$

$$\frac{d\Gamma}{dt} \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2[C_{\sin} \sin(\Delta M t) + C_{\cos} \cos(\Delta M t)] e^{-\Gamma t}$$

$$C_{\sin/\cos} = \text{Im/Re} \left\{ (A_0^S)^* A_0^L + (A_1^S)^* A_1^L \right\}$$

$$\frac{BR(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}^{(pert)}}{BR(K_L \rightarrow \mu^+ \mu^-)} = \frac{\tau_S |A_0^S A_0^L|^2}{\tau_L |A_0^L|^4} = \frac{\tau_S C_{int}^2}{\tau_L C_L^2}$$

where we assumed $A_1^L = 0$, $\epsilon_K = 0$

- ▶ Measurement of $BR(K_S \rightarrow \mu^+ \mu^-)_{\ell=0}$ [Dery et.al.'21]

NNLO, EW and $\epsilon_K = 0$

Higher order corrections from $b \rightarrow s\ell^+\ell^-$ for 5 flavour

{NNLO QCD [Hermann et.al.'13]; 2-loop EW [Bobeth et.al.'13] RGE [Bobeth et.al.'03]}

$$BR(K_S \rightarrow \mu^+\mu^-)_{\ell=0}^{(pert)} = 1.70(02)_{QCD/EW}(01)_{f_K}(19)_{param.} \times 10^{-13}$$

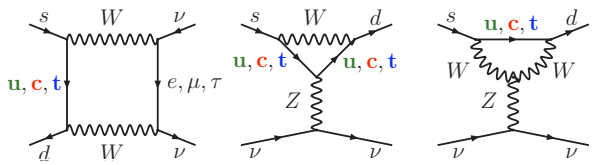
indirect CP violation mixes γ - γ : $A_0^S \rightarrow A_0^S + \epsilon_K A_0^L$

$$\frac{BR(K_S \rightarrow \mu^+\mu^-)_{\ell=0}}{BR(K_S \rightarrow \mu^+\mu^-)_{\ell=0}^{(pert)}} = 1 + \sqrt{2}\epsilon_K \frac{|A_0^L|}{|A_0^S|} (\cos \phi_0 - \sin \phi_0)$$

[Brod, Stamou'22] and can shift by $O(3\%)$, while ϕ_0 can be obtained from $K_L \rightarrow \mu^+\mu^-$ and $K_L \rightarrow \gamma\gamma$ [Dery et al.'22]

$$K \rightarrow \pi \bar{\nu} \nu$$

$K \rightarrow \pi \bar{\nu} \nu$ at M_W



$$x_i = \frac{m_i^2}{M_W^2}$$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td} (F(x_t) - F(x_u)) + V_{cs}^* V_{cd} (F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$

Matching (NLO + EW):

$$\lambda^5 \frac{m_t^2}{M_W^2}$$

$$\lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$$

$$\lambda \frac{\Lambda_{\text{QCD}}^2}{M_W^2}$$

Operator Mixing (RGE)

ChiPT & Lattice

$$Q_\nu = (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_L \gamma^\mu \nu_L)$$

- ▶ Below the charm: Only Q_ν , ME from K_{l3}
- ▶ semi-leptonic $(\bar{s} \gamma_\mu u_L) (\bar{\nu} \gamma^\mu \ell_L)$ operator: χ PT gives small contribution (10% of charm contribution)

Leading Effective Hamiltonian for $\mu < m_c$

SM: $\nu\bar{\nu}$ are only invisibles \Rightarrow no γ -Penguin \Rightarrow

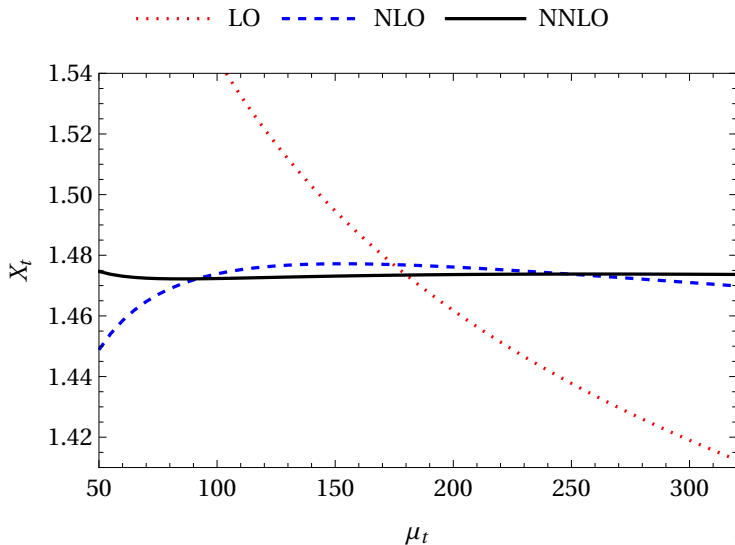
$$\mathcal{H}_{\text{eff}} = \frac{\sqrt{2}\alpha G_F}{\pi \sin^2 \theta_w} \sum_{\ell=e,\mu,\tau} (\lambda_c X^\ell + \lambda_t X_t) (\bar{s}_L \gamma_\mu d_L) (\bar{\nu}_{\ell L} \gamma^\mu \nu_{\ell L}) + \text{h.c.}$$

generated by highly virtual particles + tiny light quark contribution \Rightarrow clean & CKM suppressed ($\lambda_i = V_{is}^* V_{id}$).

- ▶ X_t known at NLO QCD [Buchalla,Buras;Misiak,Urban'99] and two-loop EW [Brod et.al.'10]:
 $X_t = 1.462 \pm 0.017_{\text{QCD}} \pm 0.002_{\text{EW}}$
- ▶ $P_c = (0.2255/\lambda)^4 \times (0.3604 \pm 0.0087)$ at NNLO QCD [Buras et.al.'05] + NLO EW [Brod et.al.'08]
 - ▶ χ PT matching logs: $\delta P_{c,u} = 0.04 \pm 0.02$ [Isidori et.al.'05]
 - ▶ Future Lattice: [Bai et.al.'18]

Scale Dependence @ NNLO

- ▶ Residual μ_t dependence estimates uncertainty
- ▶ Reduces from $\pm 1\%$ @NLO $\rightarrow \pm 0.1\%$ @NNLO



$K \rightarrow \pi \nu \bar{\nu}$ Branching Ratios

- ▶ Matrix elements from $K_{\ell 3}$ including strong and em iso-spin breaking [0705.2025] $\kappa_+, \kappa_L, \Delta_{EM}$

$$\kappa_+ = \frac{s_w^{-2} \lambda^8 \alpha (M_Z)^2}{7.5248 \cdot 10^{-9}} \times 0.5173(25) \times 10^{-10}, \quad \Delta_{EM} = -0.003$$

- ▶ indirect CP violation contribution given by r_{ϵ_K}

$$\text{Br}_{K^+} = \kappa_+ (1 + \Delta_{EM}) \left[\left(\frac{\text{Im} \lambda_t}{\lambda^5} X_t \right)^2 + \left(\frac{\text{Re} \lambda_c}{\lambda} (P_c + \delta P_{c,u}) + \frac{\text{Re} \lambda_t}{\lambda^5} X_t \right)^2 \right].$$

$$\text{Br}_{K_L} = \kappa_L r_{\epsilon_K} \left(\frac{\text{Im} \lambda_t}{\lambda^5} X_t \right)^2, \quad \kappa_L = \frac{s_w^{-2} \lambda^8 \alpha (M_Z)^2}{7.5248 \cdot 10^{-9}} \times 2.231(13) \times 10^{-10}$$

$K \rightarrow \pi \nu \bar{\nu}$ in the Standard Model

- ▶ 2105.02868 Standard Model Prediction

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 7.73(16)_{SD}(25)_{LD}(54)_{para.} \times 10^{-11},$$
$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.59(6)_{SD}(2)_{LD}(28)_{para.} \times 10^{-11}.$$

- ▶ Using 2022 PDG CKM fitter values

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 8.25(11)_{SD}(25)_{LD}(57)_{para.} \times 10^{-11},$$
$$\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 2.83(1)_{SD}(2)_{LD}(30)_{para.} \times 10^{-11}.$$

- ▶ V_{cb} dominates uncertainty: ϵ_K has similar V_{cb} dependence

- ▶ NA62 collaboration

$$\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (10.6_{-3.4}^{+3.4}|_{\text{stat}} \pm 0.9_{\text{syst}}) \times 10^{-11}$$

- ▶ JPARC-KOTO has $\text{BR}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 2.0 \times 10^{-9}$

ϵ_K

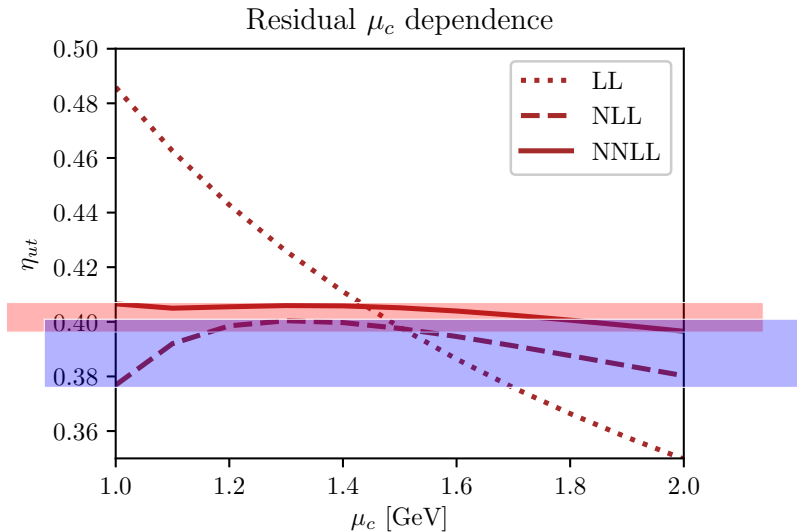
$\Delta S = 2$ Hamiltonian - Phase (In)Dependence

- ▶ Rephasing invariant: $\lambda_i \lambda_j^* = V_{id} V_{is}^* V_{jd}^* V_{js}$
- ▶ One Operator: $Q_{S2} = (\bar{s}_L \gamma_\mu d_L) \otimes (\bar{s}_L \gamma^\mu d_L)$

$$H_{f=3}^{\Delta S=2} = \frac{G_F^2 M_W^2}{4\pi^2 (\lambda_u^*)^2} Q_{S2} \left\{ f_1 C_{S2}^{uu}(\mu) + iJ \left[f_2 C_{S2}^{tt}(\mu) + f_3 C_{S2}^{ut}(\mu) \right] \right\} + \text{h.c.}$$

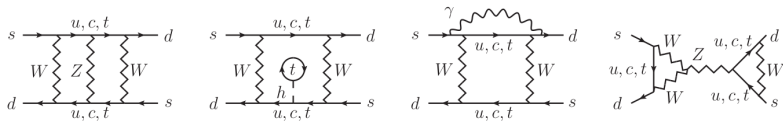
- ▶ $f_1 = |\lambda_u|^4$, $f_2 = 2\text{Re}(\lambda_t \lambda_u^*)$ and $f_3 = |\lambda_u|^2$
- ▶ NNLO QCD corrections [Brod et.al.'10, Brod et.al.'11] to C_{S2}^{ut} absorbed into η_{ut} [Brod et.al.'19]
- ▶ C_{S2}^{uu} is purely SD, while C_{S2}^{uu} does not contribute

Residual scale dependence



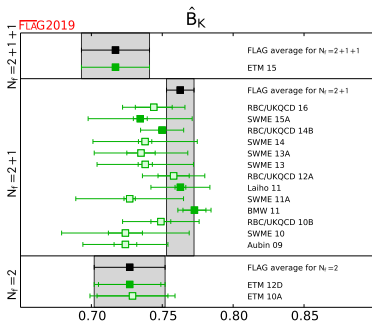
Further Improvements

- EW corrections to η_{tt} and η_{ut} [2207.07669,2108.00017]

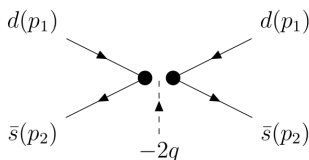


- LD effects can be calculated on the Lattice, but not at physical masses yet [2309.01193]

- $\hat{B}_K = \frac{3}{2f_K^2 M_K^2} \langle \bar{K}^0 | Q^{|\Delta S|=2} | K^0 \rangle u^{-1}(\mu_{\text{had}})$ from Lattice

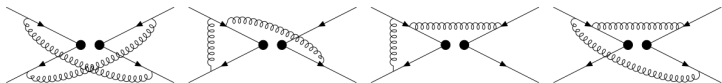


E.g. RBC UKQCD uses SMOM kinematics



- ▶ Define momentum-space subtraction schemes
- ▶ Projected renormalised Green's function $P_{(\gamma_\mu)}(\Lambda_R) \rightarrow$
- ▶ $Z_{QS2}^{(\gamma_\mu, \gamma_\mu)} = \left(Z_q^{(\gamma_\mu)} \right)^2 \frac{1}{P_{(\gamma_\mu)}(\Lambda_B)}$
- ▶ $Z_{QS2}^{(\gamma_\mu, \gamma_\mu)} / Z_{QS2}^{\overline{MS}}$ converts between Lattice and continuum

SMOM \hat{B}_K @ NNLO

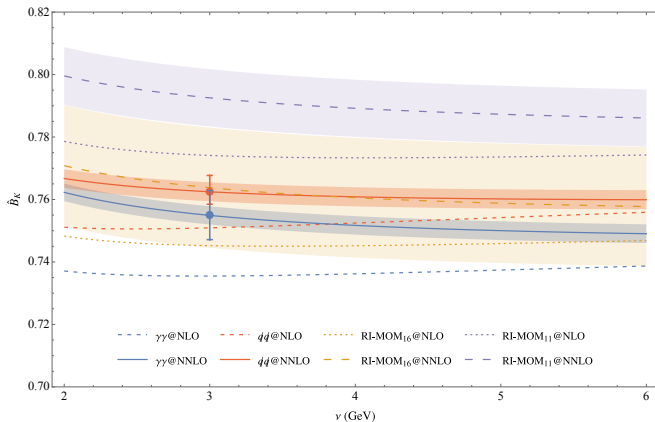


- ▶ Use projectors to find $\Lambda_{\alpha\beta\gamma\delta}^{ijkl}$ at 2-loops
- ▶ Integrals reduce to scalar off-shell 4-point functions
- ▶ The result should be independent of the matching scale

$$B_K^{(X,Y)}(|p|) C_{B_K}^{(X,Y)}(|p|, \mu) u^{-1}(\mu) u(\mu_0)$$

- ▶ study scale variation setting $\mu_0 = |p|$
- ▶ Compute \hat{B}_K for $f=3,4$ for SMOM, RIMOM, RI'MOM

\hat{B}_K at NNLO



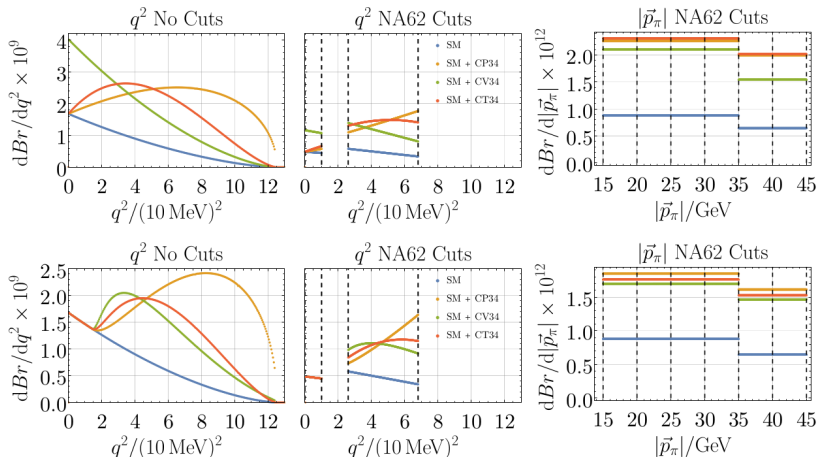
Numerics of NNLO result by [MG, Kvedaraitė, Jäger]

$K \rightarrow \pi\nu\bar{\nu}$ & ϵ_K

- ▶ $|\epsilon_K| = (2.170 \pm 0.065_{pt} \pm 0.076_{np} \pm 0.153_{par}) \times 10^{-3} = \kappa_\epsilon C_\epsilon \widehat{B}_K |V_{cb}|^2 \lambda^2 \bar{\eta} \times \left(|V_{cb}|^2 (1 - \bar{\rho}) \eta_{tttt}(x_t) - \eta_{utut}(x_c, x_t) \right)$
- ▶ CKM factors of ϵ_K and $K \rightarrow \pi\nu\bar{\nu}$ are quite similar
 - ▶ In fact $BR(K \rightarrow \pi\nu\bar{\nu})/|\epsilon_K|^{0.82}$ is effectively V_{cb} independent [Buras, Venturini '21] for current theory calculations
- ▶ This provides an excellent null test of the Standard Model
- ▶ Multiplying with $|\epsilon_K|^{0.82}$ results in a model prediction for $BR(K \rightarrow \pi\nu\bar{\nu})$ with 5% uncertainty (albeit ignoring other constraints)

$K \rightarrow \pi + \text{invisibles}$

- ▶ Sensitive to QCD Axion, Dark Photon . . . [2201.07805]
- ▶ Neutrino properties, such as Majorana vs. Dirac or sterile allows different NP effects [Gorbahn, Moldanarazova, Sieja, Stamou, Tabet, W.I.P.]



Gauge independent result for $K \rightarrow \pi\nu\bar{\nu}$

- $b \rightarrow s\ell\ell$ for generic theories [2104.10930] restricted to $K \rightarrow \pi\nu\bar{\nu}$ with only vectors and fermions:

$$\begin{aligned}
 C_{L\sigma}^{sd\nu} = & \sum_{\nu_1\nu_2f_1f_3} \frac{g_{\bar{\nu}_2\bar{s}f_1}^L g_{\nu_1\bar{f}_1d}^L}{M_{V_1}^2} g_{\nu_2\bar{\nu}f_3}^\sigma g_{\bar{\nu}_1\bar{f}_3\nu}^\sigma F_V^{\sigma,B'Z}(x_{\nu_1}^{f_0}, x_{\nu_1}^{f_1}, x_{\nu_2}^{\nu_1}, x_{\nu_1}^{f_3}) \\
 & + \sum_{Z\nu_1\nu_2f_1f_2} \frac{g_{Z\bar{\nu}\nu}^\sigma g_{\nu_1\bar{f}_1d}^L g_{\bar{\nu}_2\bar{s}f_2}^L}{M_Z^2} \left\{ \delta_{f_1f_2} g_{Z\bar{\nu}_1\nu_2} F_{V''}^Z(x_{\nu_1}^{f_0}, x_{\nu_1}^{f_1}, x_{\nu_2}^{\nu_1}) \right. \\
 & \left. + \delta_{\nu_1\nu_2} \left[g_{Z\bar{f}_2f_1}^L F_V^Z(x_{\nu_1}^{f_1}, x_{\nu_1}^{f_2}) + g_{Z\bar{f}_2f_1}^R F_{V'}^Z(x_{\nu_1}^{f_1}, x_{\nu_1}^{f_2}) \right] \right\},
 \end{aligned}$$

Extends the Penguin Box Coefficients to generic theories ($X_t \leftrightarrow F_V^{\sigma,B'Z}(0, x_W^t, 1, 0)$ & $F_{V^{(\nu)}}^Z(x, x) = F_{V''}^Z(x, y, 1) = 0$)

Conclusions

- ▶ Precise theory predictions for rare B and particular K decays.
- ▶ Can form null tests of the SM with ϵ_K .
- ▶ NNLO calculations for electroweak and lattice-continuum matching will increase precision.
- ▶ Measurement of $K \rightarrow \pi \bar{\nu} \nu$ can be compared with precise theory prediction.
- ▶ Precision test of the standard model (EFT)
 - ▶ but also of different light degrees of freedom