Rare Kaons on the lattice

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Motivation

- Rare Kaon decays provide a rich environment for precision tests of the Standard Model
- Semileptonic processes - $K \to \pi\ell\bar{\ell}$, $K \to \pi\nu\bar{\nu}$
- Does Lattice QCD fit into this picture?
- Today's story: $K^+ \to \pi^+\ell^+\ell^-$
  $\to$ Long-distance dominated
  $\to$ Simplest lattice description
  $\to$ Excellent testing ground for lattice formalism

Motivation

- $K \to \pi\ell\bar{\ell}$ decays proceed *via* flavour-changing neutral current
  $\to$ Highly suppressed; sensitive to new physics
- CP-conserving processes dominated by virtual-γ-exchange[1]
  $\to$ Primarily long-distance quantities
  $\to$ Well-suited to lattice QCD techniques
- $K_S \to \pi^0\ell^+\ell^-$ very experimentally challenging
  $\to$ Focus lattice calculations on $K^+ \to \pi^+\ell^+\ell^-$
- Results anticipated for 2021-2024 NA62 run
  $\to$ $K^+ \to \pi^+\mu^+\mu^-$ recently published for 2017-2018 dataset[2]
  $\to$ Additional theoretical input is timely

[1] JHEP 08 (1998) 004 [arXiv:hep-ph/9808289]
[2] JHEP 11 (2022) 011 [arXiv:2209.05076]

Background

Background

- Theoretical framework for lattice computations of $K \to \pi\ell\bar{\ell}$ and $K \to \pi\nu\bar{\nu}$ first published by Isidori, Martinelli, and Turchetti (2006)[1]
- Extended for full evaluation by RBC-UKQCD collaborations
  $\to K^+ \to \pi^+\ell^+\ell^-$ (2015)[2]
  $\to K^+ \to \pi^+\nu^+\nu^-$ (2016)[3]
- Proof-of-concept: RBC-UKQCD Exploratory calculations
  $\to K^+ \to \pi^+\ell^+\ell^-$ (2016)[4]
  $\to K^+ \to \pi^+\nu^+\nu^-$ (2017)[5] [6], (2019)[7]
- Now: Production runs

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[1] Phys. Lett. B 633, 75 (2006) [arXiv:hep-lat/0506026]
[2] Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]
[3] Phys.Rev. D 93 (2016) 114517 [arXiv:1605:04442]
[4] Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]
[5] Phys.Rev.Lett. 118 (2017) 252001 [arXiv:1701.02858]
[6] Phys.Rev. D 98 (2018) 074509 [arXiv:1806.11520]
[7] Phys. Rev. D 100 (2019) 114506 [arXiv:1910.10644]

Background

- Long-distance Minkowski amplitude:

$$\mathcal{A}_\mu(q^2) = \int d^4x \langle \pi(\mathrm{p}) | T\left[J_\mu(0) H_W(x)\right] | K(\mathrm{k}) \rangle$$

- Re-expressed using EM gauge invariance[1] [2]:

$$\mathcal{A}_\mu(q^2) = -i\frac{G_F}{(4\pi)^2}\left[q^2\left(k+p\right)_\mu - \left(M_K^2 - M_\pi^2\right)q_\mu\right]V(z)$$

$$q_\mu = k_\mu - p_\mu,$$
$$z = q^2/M_K^2,$$
$$V(z) = a + bz + V^{\pi\pi}(z)$$

- Goal is to compute a, b

[1] JHEP 08 (1998) 004 [arXiv:hep-ph/9808289]

[2] Rev. Mod. Phys. 84, 399 (2012) [arXiv:1107.6001]

Background

- Long-distance Minkowski amplitude:

$$\mathcal{A}_\mu(q^2) = \int d^4x \langle \pi(\mathrm{p})| \, T\left[J_\mu(0) H_W(x)\right] |K(\mathrm{k})\rangle$$

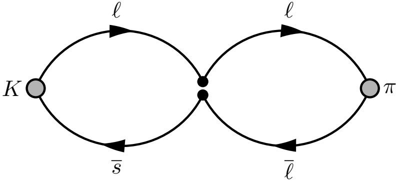
- $J_\mu(0)$ — Electromagnetic current
- $H_W(x)$ — $\Delta S = 1$ effective weak Hamiltonian density

$$H_W(x) = \frac{G_F}{2} V_{us}^* V_{ud} \sum_{j=1}^{2} C_j \left({Q_j}^u - {Q_j}^c \right)$$

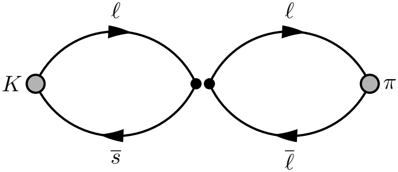
$$\begin{aligned} {Q_1}^q &= [\bar{s}\gamma_\mu^L d][\bar{q}\gamma^{L,\mu} q] \\ {Q_2}^q &= [\bar{s}\gamma_\mu^L q][\bar{q}\gamma^{L,\mu} d], \\ \gamma_\mu^L &= \gamma_\mu(1-\gamma_5) \end{aligned}$$

Weak Hamiltonian H_W generates four diagram classes:

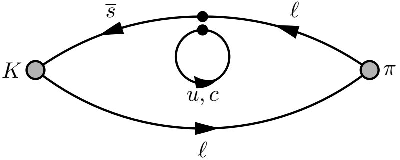
"Connected" (Q_1)



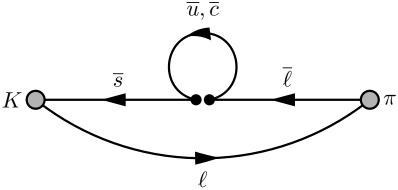
"Wing" (Q_2)



"Eye" (Q_1)

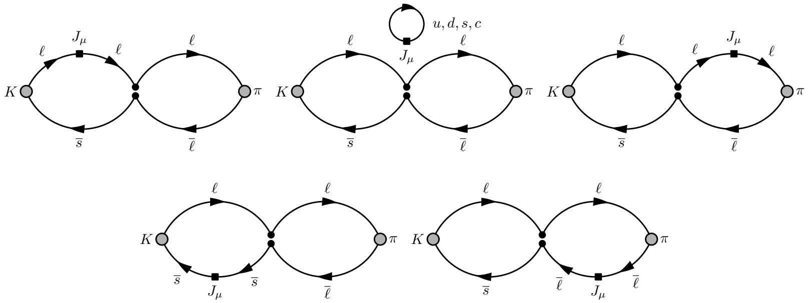


"Saucer" (Q_2)



Background

EM insertions J_μ for the "Connected" topology



- Same insertions exist for other 3 topologies
- 5 EM insertions $\times$ 4 topologies $=$ 20 diagrams total

Background

- Minkowski and Euclidean spectral representations:

$$\mathcal{A}_\mu(\mathbf{k},\mathbf{p}) = +i\int_0^\infty dE\,\frac{\rho(E)}{2E}\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E+i\epsilon} - i\int_0^\infty dE\,\frac{\rho_S(E)}{2E}\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})+i\epsilon}$$

$$A_\mu^E(\mathbf{k},\mathbf{p}) = \lim_{T_A,T_B\to\infty} I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}),$$

$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = -\int_0^\infty dE\,\frac{\rho(E)}{2E}\frac{\langle\pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k})-E}\left(1-e^{[E_K(\mathbf{k})-E]T_a}\right) + \int_0^\infty dE\,\frac{\rho_S(E)}{2E}\frac{\langle\pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E-E_\pi(\mathbf{p})}\left(1-e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- T_a, T_b come from integration of normalised 4pt function:

$$I_\mu(T_a,T_b,\mathbf{k},\mathbf{p}) = e^{-[E_\pi(\mathbf{p})-E_K(\mathbf{k})]t_J}\int_{t_J-T_a}^{t_J+T_b} dt_H\,\widetilde{\Gamma}_{\mathrm{4pt}}$$

Background

$$I_\mu(T_a, T_b, \mathbf{k}, \mathbf{p}) = -\int_0^\infty dE \frac{\rho(E)}{2E} \frac{\langle \pi(\mathbf{p})|J_\mu|E,\mathbf{k}\rangle\langle E,\mathbf{k}|H_w|K(\mathbf{k})\rangle}{E_K(\mathbf{k}) - E}\left(1 - e^{[E_K(\mathbf{k})-E]T_a}\right) + \int_0^\infty dE \frac{\rho_S(E)}{2E} \frac{\langle \pi(\mathbf{p})|H_W|E,\mathbf{p}\rangle\langle E,\mathbf{p}|J_\mu|K(\mathbf{k})\rangle}{E - E_\pi(\mathbf{p})}\left(1 - e^{-[E-E_\pi(\mathbf{p})]T_b}\right)$$

- Amplitude corresponds to limit $T_a, T_b \to \infty$
- First line: π, $\pi\pi$, and $\pi\pi\pi$ on-shell intermediate states enter the s=0 spectral density (for physical masses)
  $\to E_K > E_\pi, E_{\pi\pi}, E_{\pi\pi\pi}$: Causes the T_a exponential to diverge!
- Lattice - can't take $T_a, T_b \to \infty$
  $\to$ Must remove exponentially growing terms in T_a due to intermediate states

Background

- In addition to T_a divergence, there is a potentially quadratic divergence as $J_\mu(0)$ and $H_w(x)$ approach each other in the integral
- Using a conserved current for the electromagnetic current allows the quadratic divergence to be reduced to mass-independent logarithmic divergence *via* EM gauge invariance
- Using an explicit GIM mechanism exactly cancels the mass-independent logarthmic divergence
  $\rightarrow$ Difference of two diagrams differing only by loop quark flavour
- Alternatively, log divergence could be dealt with in a 3-flavour theory with an alternative renormalisation

Lattice Calculation

$K \to \pi\ell\bar{\ell}$

Theoretical proposal:

- Prospects for a lattice computation of rare kaon decay amplitudes: I, $K \to \pi\ell^+\ell^-$ decays
  RBC-UKQCD (2015)
  Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]

Existing results:

- First exploratory calculation of the long distance contributions to the rare kaon decay $K \to \pi\ell^+\ell^-$
  RBC-UKQCD (2016)
  Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]
- Simulating rare kaon decays $K \to \pi\ell^+\ell^-$ using domain wall lattice QCD with physical light quark masses
  RBC-UKQCD (2023)
  Phys. Rev. D 107, 114512 (2023) [arXiv:2202.08795]

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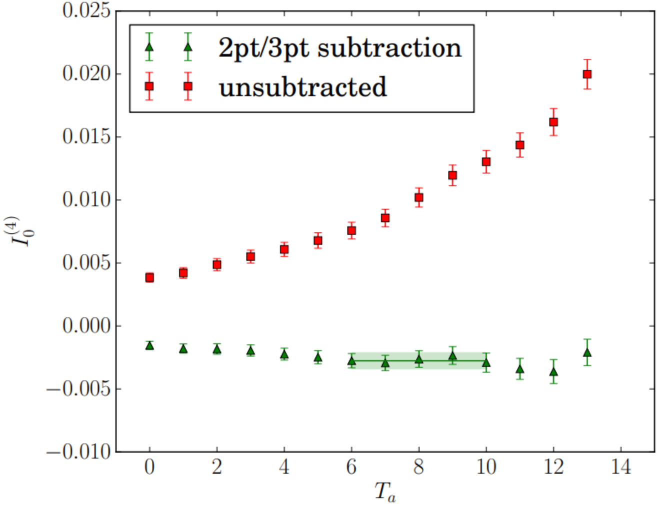
Exploratory Calculation (2016)

RBC-UKQCD Exploratory study (2016)[1]:

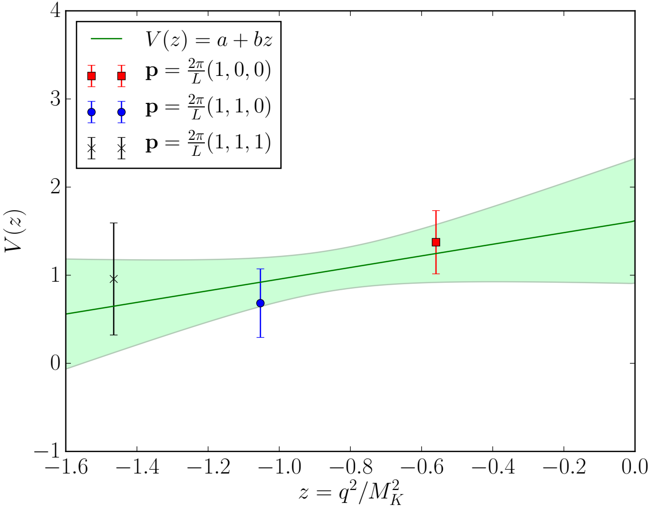
- $2+1$ flavour, $L^3 \times T = 24^3 \times 64$, $a^{-1} = 1.78$ GeV
- $\sim$430 MeV pion, $\sim$625 MeV Kaon
  $\rightarrow$ Only single-π intermediate state enters spectral density
- Shamir Domain Wall Fermions: good chiral symmetry
  $\rightarrow$ simplified renormalisation

[1] Phys.Rev. D 94 (2016) 114516 [arXiv:1608.07585]

Exploratory Calculation (2016)



Exploratory Calculation (2016)



Physical-Point Calculation (2023)

RBC-UKQCD physical-point calculation (2023)[1]:

- $2 + 1$ flavour, $L^3 \times T = 48^3 \times 96$, $a^{-1} = 1.73$ GeV
- Physical Pion and Kaon masses
  $\rightarrow$ Expensive calculation!
  $\rightarrow$ Energy budget allows π, $\pi\pi$, $\pi\pi\pi$ intermediate states
- Three non-physical charm masses
  $\rightarrow$ Physical charm not simulatable with the action used for light quarks (next slide)
  $\rightarrow$ **Critical** to use the same action for GIM cancellation
  $\rightarrow$ Therefore need to adopt non-physical charm masses
- Disconnected diagrams omitted: expected $\sim$10% systematic

[1] Phys. Rev. D 107, 114512 (2023) [arXiv:2202.08795]

Physical-Point Calculation (2023)

Variance reduction techniques:

- zMöbius Domain Wall Fermions
  $\rightarrow$ Significantly cheaper than Möbius DWF but requires a bias-correction step
  $\rightarrow$ Allows statistics to be accumulated on a cheaper estimator and then be shifted to the full Möbius action
- zMöbius deflated with 2000 low-modes
  $\rightarrow$ Further accelerates solve times
- All-Modes-Averaging (AMA) technique applied to zMöbius to improve stats : cost ratio
- Sparse Z_2 noise sources for loop estimators
  $\rightarrow$ Noise source reduces effect of local gauge fluctuation on loop propagators
  $\rightarrow$ Sparsening further improves stats : cost ratio

Physical-Point Calculation (2023)

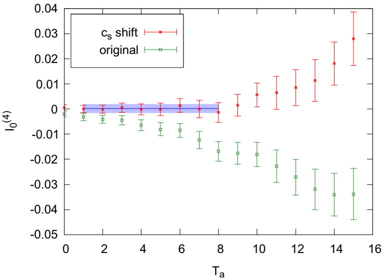
Intermediate states:

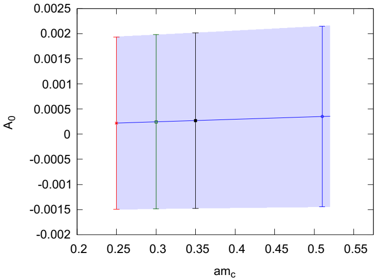
- π IS: Significant contribution, must be removed
  $\rightarrow$ Use same techniques employed in the exploratory study
- $\pi\pi$ IS: Introduced by lattice artefacts
  $\rightarrow$ At practical values of T_a, expected to be %-level effect[1]
  $\rightarrow$ Negligible contribution for now
- $\pi\pi\pi$ IS: Compare decay widths of $K_S \rightarrow \pi\pi$ to $K_{S,+} \rightarrow \pi\pi\pi$:
  $\rightarrow$ Factor $\sim \mathcal{O}(1/500)$ further suppressed beyond $\pi\pi$[1]
  $\rightarrow$ $\pi\pi\pi$ completely negligible for forseeable future

[1] Phys.Rev. D. 92 (2015) 094512 [arXiv:1507.03094]

Results

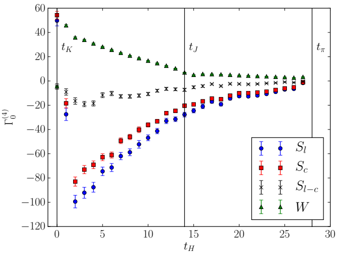
Results

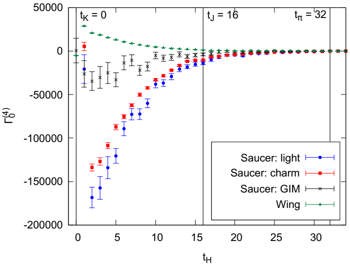




- $A_0 = 0.00035(180)$
- $V(z) = -0.87(4.44)$
- $V(z) \approx V(0) = a^+$ for our choice of kinematics
- Form factor unfortunately unresolved, but let's investigate why...

Results



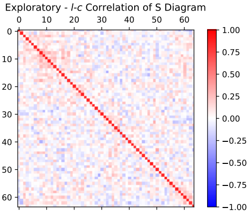


Exploratory Study

Physical-Point

- Plots show the GIM subtraction for the saucer diagram constructed from the *l* and *c* quark correlators
- GIM subtraction does not lead to a cancellation of errors with physical light masses

Results



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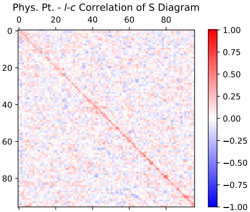

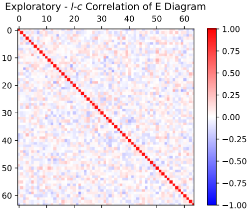

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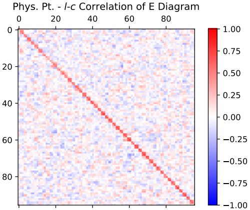


- Leading diagonal shows *l*-*c* timeslice cross-correlation
- **Much reduced** correlation between *l* and *c* loop quark diagrams at physical point due to large mass difference

Results

Statistical error cannot be overcome by square-root scaling of additional statistics alone in near future.

$\rightarrow$ Potential ways forward:

- Improvement of estimators for up- and charm-loop propagators
  $\rightarrow$ Similar to issues faced in disconnected diagrams
- Forgo explicit charm contribution to GIM loop and handle *via* different renormalisation procedure
  $\rightarrow$ Look to $K \rightarrow \pi\nu\bar{\nu}$ for lessons learned

$\rightarrow$ Combination of algorithmic improvements and next-generation computers makes a competitive lattice result appear feasible in the coming years.

$\rightarrow$ **Currently investigating improved loop estimators - stay tuned!**

Summary

Summary

- Viability of $K^+ \to \pi^+\ell^+\ell^-$ demonstrated at physical point
- Next steps identified: Gain control of GIM loop stochastic estimators (**in progress!**)
- Challenging calculation with physical kinematics achieved. Competitive errors in the next few years?

Backup Slides

Setup

Two approaches for the removal of on-shell single-π intermediate state:

- Reconstruct the single-π contribution and explicitly subtract it
  $\rightarrow$ Stable state, can be constructed from lattice QCD correlation functions
- Shift the weak Hamiltonian with an $\bar{s}d$ scalar current,
  $H'_w(x) = H_w(x) + c_s(\mathbf{k})\bar{s}(x)d(x)$
  $\rightarrow$ $C_s(\mathbf{k})$ tuned to condition $\langle\pi(\mathbf{k})|H'_W(0,\mathbf{k})|K(\mathbf{k})\rangle = 0$
  $\rightarrow$ Cancels the single-π intermediate state
  $\rightarrow$ Does not contribute to amplitude

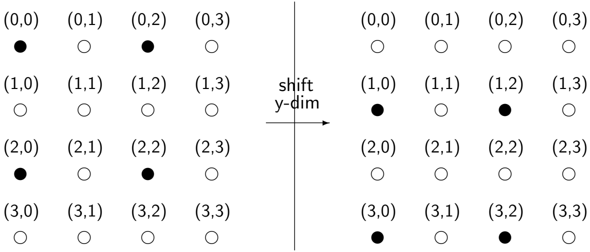
Setup

- Calculation performed with **Grid**[1], and the Grid-based workflow management software **Hadrons**[2]
- 87 configurations
- 6 time translations
- 10 noise sources
- Time translations and noise sources used in two-step AMA to bias-correct zMöbius propagators and accumulate statistics

[1] https://github.com/paboyle/Grid
[2] https://github.com/aportelli/Hadrons

Setup

- Sparse noise source partially covers lattice, with most sites set to 0[1]
- Cover full volume by computing source translations until full volume covered
- We require $2^4 = 16$ sparse sources to cover full volume



[1] [arXiv:1512.09054] Phys. Rev. Lett. 116, 232002

Setup

- Sparse noise has a clear advantage over full volume and time-diluted sources for estimating quark loops in terms of stats : cost ratio