Dispersive Analysis of $B \to K^{(*)}$ and $B_s \to \varphi$ Form Factors

CKM 2023 – Santiago de Compostela

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Based on: Gubernari, MR, van Dyk, Virto 2305.06301

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IP3

Motivation

- Hadronic effects dominate the theory uncertainties in most of the flavour observables

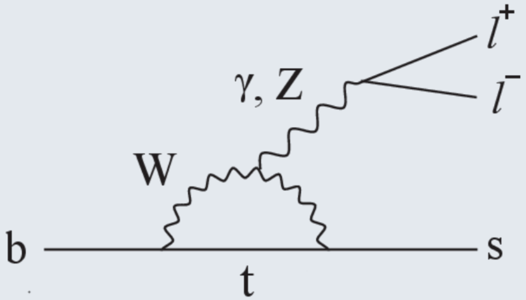
  $\rightarrow$ They are a **blocker** for the extraction of both SM and BSM parameters

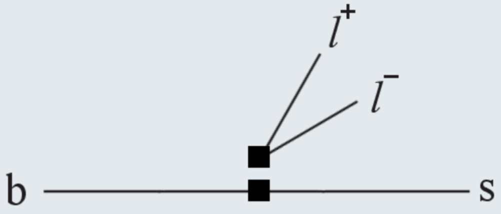
- Here we focus on **b → s transitions**, but the method applies equally well to
  - b → d [Bordone, Reboud, Virto, w.i.p.]
  - b → {u, c} [Flynn, Jüttner, Tsang '23; Bolognani, van Dyk, Vos '23]

    → see Carolina's talk
  - charm physics…

Weak Effective Theory

- These processes take place at a scale $m_b < m_W, m_t$





- Allows for a model independent interpretation of the anomalies

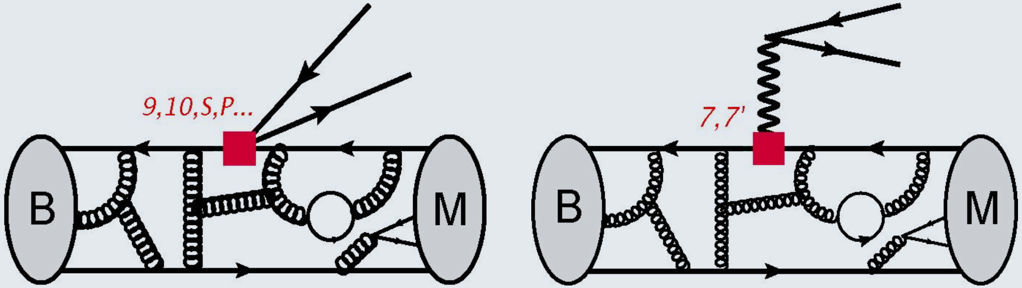
$$\mathcal{H}(b \to s\ell\ell) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu)$$

$$\mathcal{O}_{9(10)} = \frac{e^2}{16\pi^2}\left(\bar{s}_L \gamma_\mu b_L\right)\left(\bar{\ell}\gamma^\mu(\gamma_5)\ell\right)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2}\left(\bar{s}_L \sigma_{\mu\nu} b_R\right) F^{\mu\nu}$$

- Avoids the appearance of large logarithm in the calculations of observables

QCD in b → sℓℓ



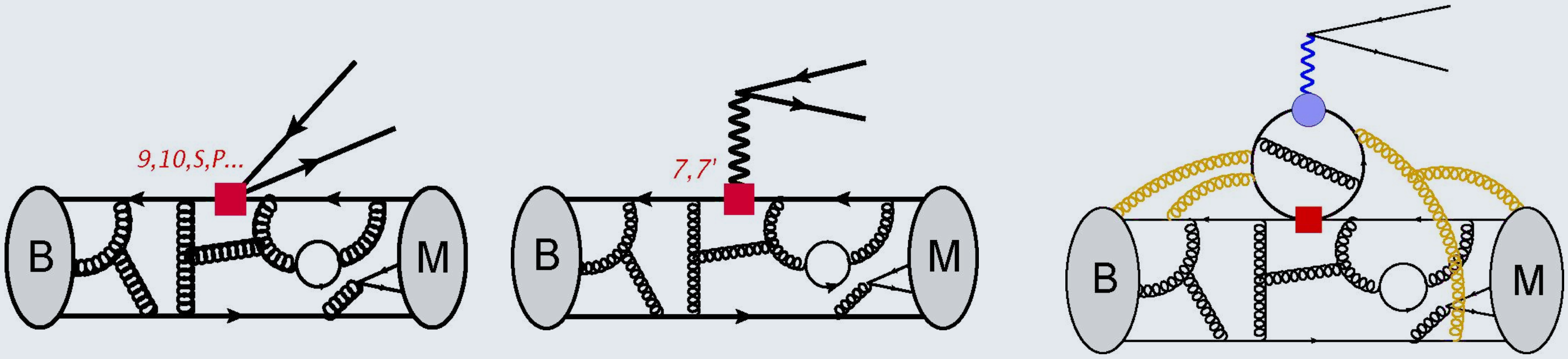
$$\mathcal{A}_\lambda^{L,R}(B \to M_\lambda \ell\ell) = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) \qquad \right] \right\}$$

- $B \to K^{(*)} \mu\mu$
- $B_s \to \varphi \mu\mu$
- $\Lambda_b \to \Lambda^{(*)} \mu\mu$

Local form-factors, involves e.g.

$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s} \gamma_\mu b_L | \bar{B}(q+k) \rangle$$

Not in this talk, see Andrea's talk



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$$\mathcal{H}_\mu(k,q) = i\int d^4x\, e^{iq\cdot x}\langle \bar{M}(k)|T\{\mathcal{J}_\mu^{\rm em}(x), \mathcal{C}_i\mathcal{O}_i\}|\bar{B}(q+k)\rangle$$

Non-local form-factors

→ Main contributions: the **"charm-loops"** $\mathcal{O}_{2(1)}^c = (\bar{s}_L\gamma_\mu(T^a)c_L)(\bar{c}_L\gamma^\mu(T^a)b_L)$

→ Under control far from charmonium? [Gubernari, MR *et al*, '22; Ciuchini *et al*, '23]

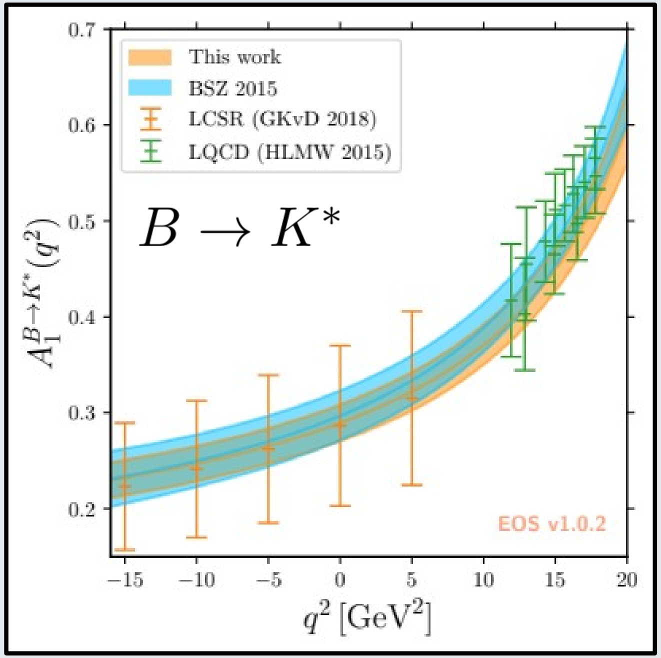
Local form factors

- Conceptually easy, but still the dominant source of uncertainties
- **2 main approaches**
  - Lattice QCD → most feasible at **large q^2**
  - Light-cone sum rules → most feasible at **small q^2,** 2 possible LCSRs

→ **Interpolation/Extrapolation**, depending on the use case

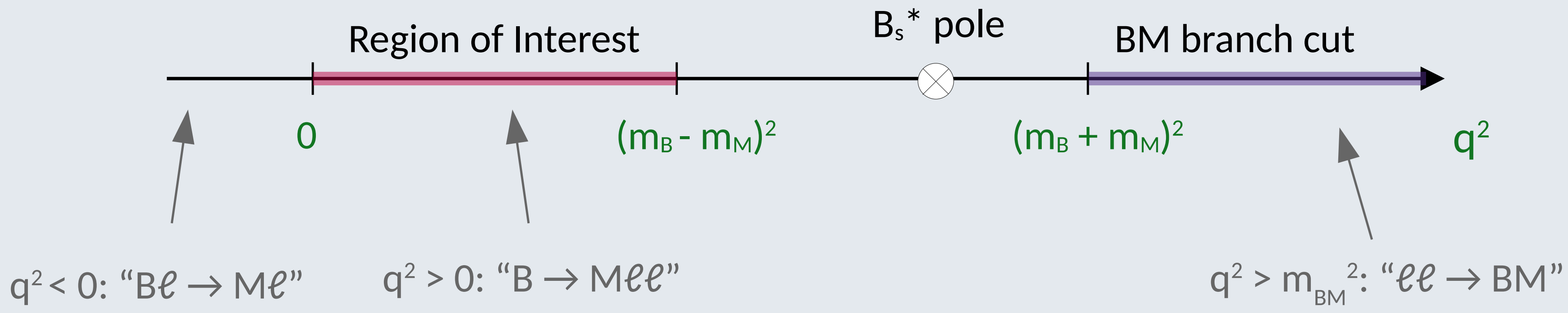
→ How to control extrapolation uncertainties?

→ What can we learn from theory?



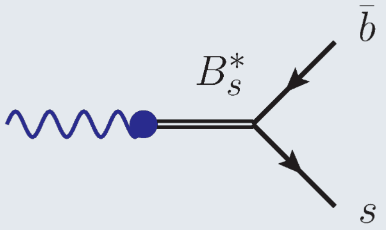
Form Factor Properties

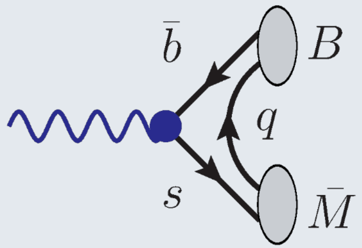
$$\mathcal{F}_\mu(k,q) = \langle \bar{M}(k) | \bar{s}\gamma_\mu b_L | \bar{B}(q+k) \rangle$$



Analytic properties of the form factors:

- Pole due to **$\bar{b}$s bound state**
- **Branch cut** due to on-shell BM production





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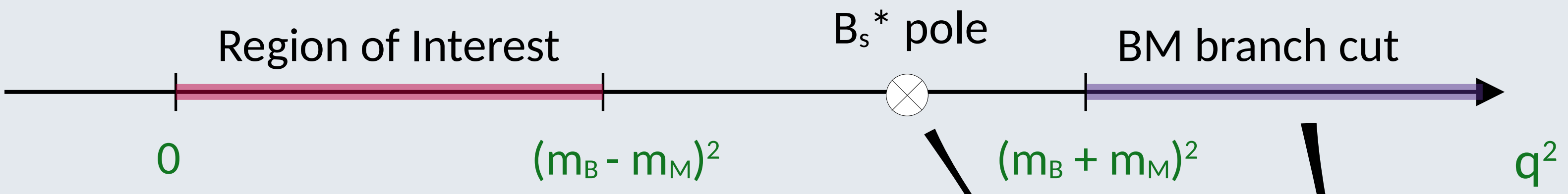

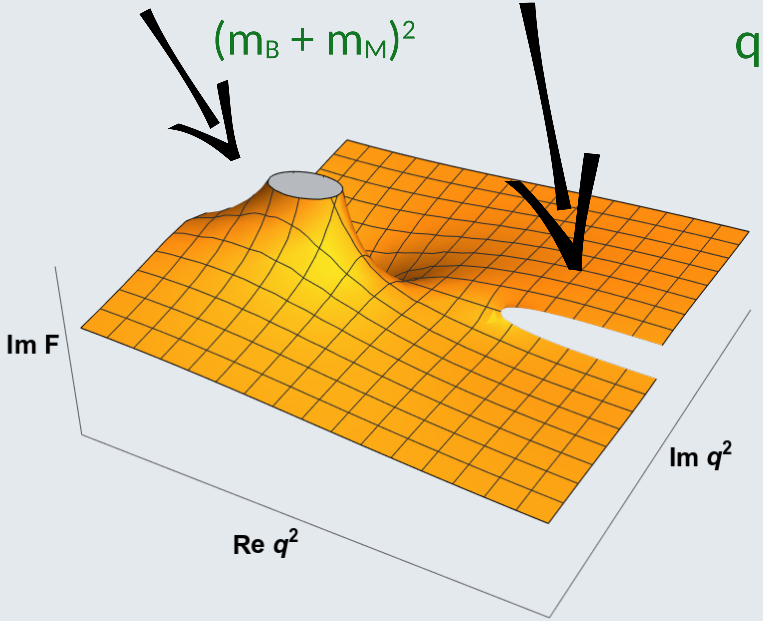

Analytic properties of the form factors:

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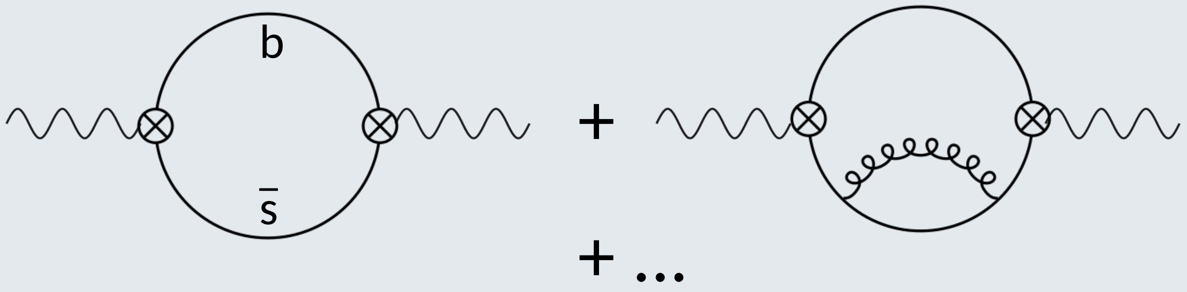
Dispersive bounds

- **Main idea:** Compute the inclusive $e^+e^- \to \bar{b}s$ cross-section and relate it to the form factors [Bharucha, Feldmann, Wick '10]

$$\Pi_\Gamma^{\mu\nu}(q) \equiv i\int d^4x\, e^{iq\cdot x}\, \langle 0|\mathcal{T}\left\{J_\Gamma^\mu(x) J_\Gamma^{\dagger,\nu}(0)\right\}|0\rangle$$

<u>1) Partonic calculation</u>

Insertion of a scalar, vector or tensor current →



Dispersive bounds

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<u>2) Relation to form factors</u>

Integration over the entire phase space

$$\mathrm{Im}\,\Pi_I^X(q^2) = \frac{1}{2}\sum_\Gamma \int d\rho_\Gamma\,(2\pi)^4\,\delta^4(q - p_\Gamma)\,P_I^{\mu\nu}\,\langle 0|\,j_\mu^X\,|\Gamma\rangle\langle\Gamma|\,j_\nu^{\dagger X}\,|0\rangle$$

Sum over all the $\bar{s}b$ states: $\bar{B}_s$, $\bar{B}K$, $\bar{B}K^*$, $\bar{B}K\pi$, ...

~ |form factor|^2

Dispersive bounds

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$\sim \int_{\text{threshold}} |\text{form factor}|^2$

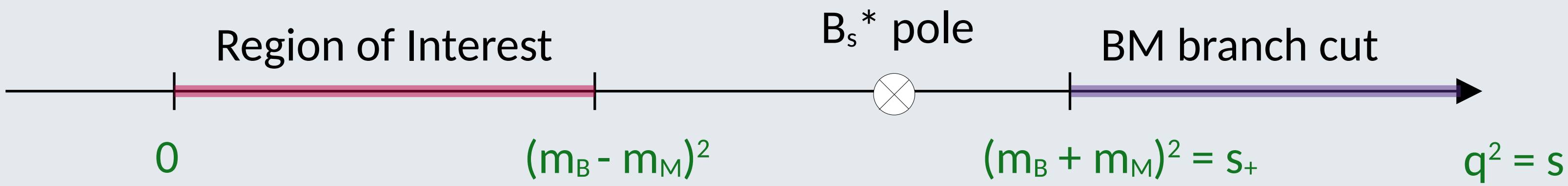
- Assuming global quark-hadron duality we have

$$\chi_\Gamma^{(\lambda)}\big|_{\text{OPE}} = \chi_\Gamma^{(\lambda)}\big|_{\text{1pt}} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}K^*} + \chi_\Gamma^{(\lambda)}\big|_{\bar{B}_s\phi} + \dots .$$

Known terms | Sum of positive quantities

- Any new terms *strengthens* the bounds, we can already add $\Lambda_b \to \Lambda^{(*)}$ [Amhis, Bordone, MR '22; Blake *et al* '22]
- We accounted for the finite width of the K* [Descotes-Genon *et al* '19]
  → see also the talk by Florian on Tuesday

Form Factor Parametrization



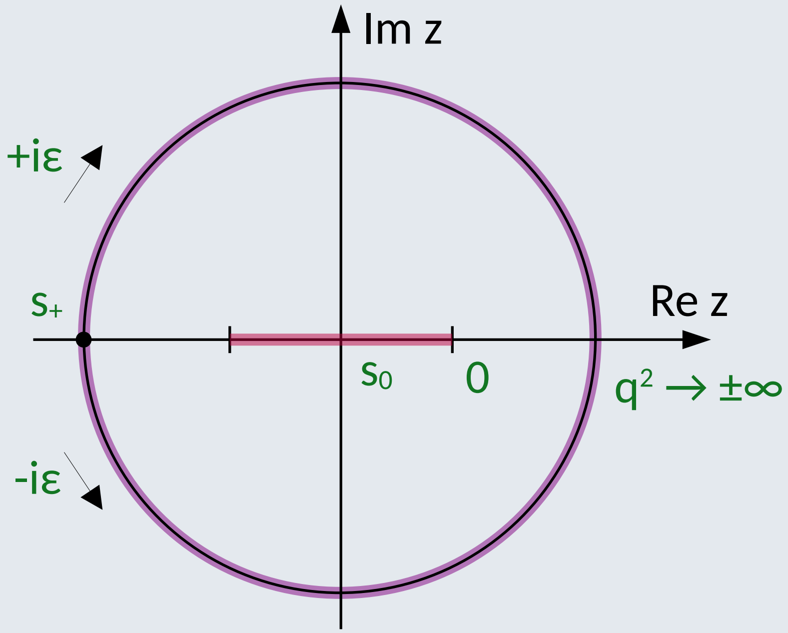
Conformal mapping [Boyd, Grinstein, Lebed '97]

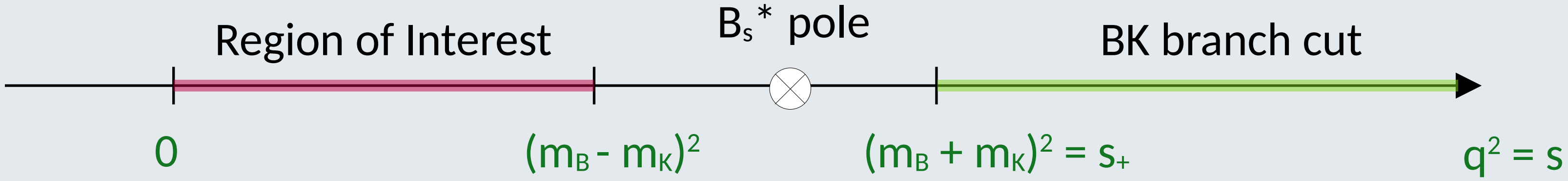
$$z(s) \equiv \frac{\sqrt{s_+ - s} - \sqrt{s_+ - s_0}}{\sqrt{s_+ - s} + \sqrt{s_+ - s_0}}$$

Simplified Series expansion [Bourrely, Caprini, Lellouch, '08; Bharucha, Feldmann, Wick '10]

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{q^2 - m_{B_s^*}^2} \sum_{k=0}^{N} \alpha_{\lambda,k} z^k$$

What is the uncertainty due to the truncation order N?

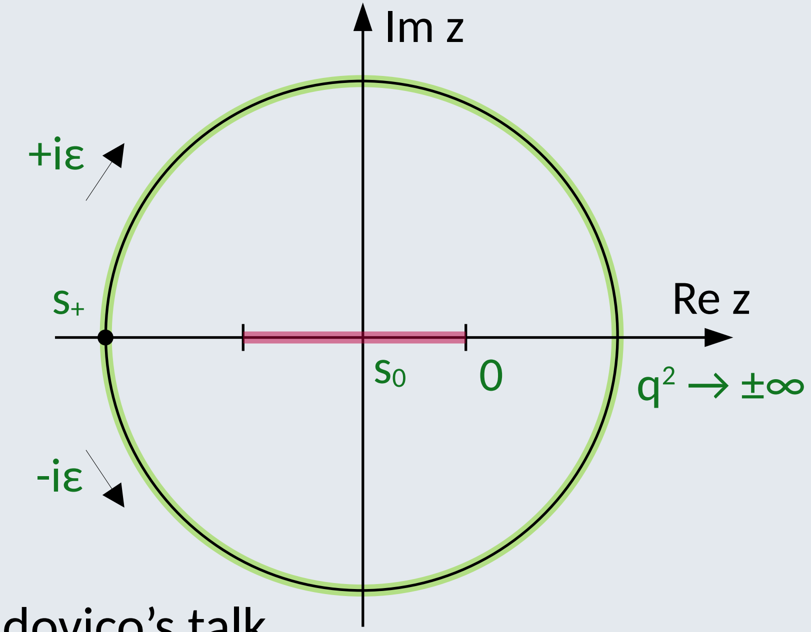




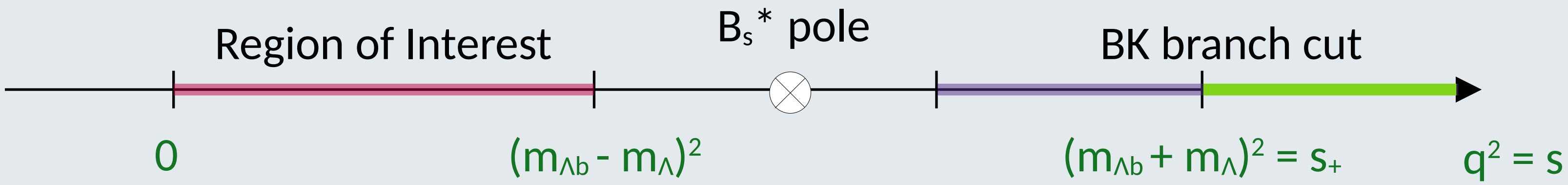
- The branch cut starts **at** the pair production threshold
- The monomial z^k are **orthogonal** on the unit circle

$$\mathcal{F}^{B\to K} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k z^k$$

$$\chi_\Gamma^{(\lambda)}|_{\bar{B}K} = \sum_{k=0}^{N} |\alpha_k|^2$$

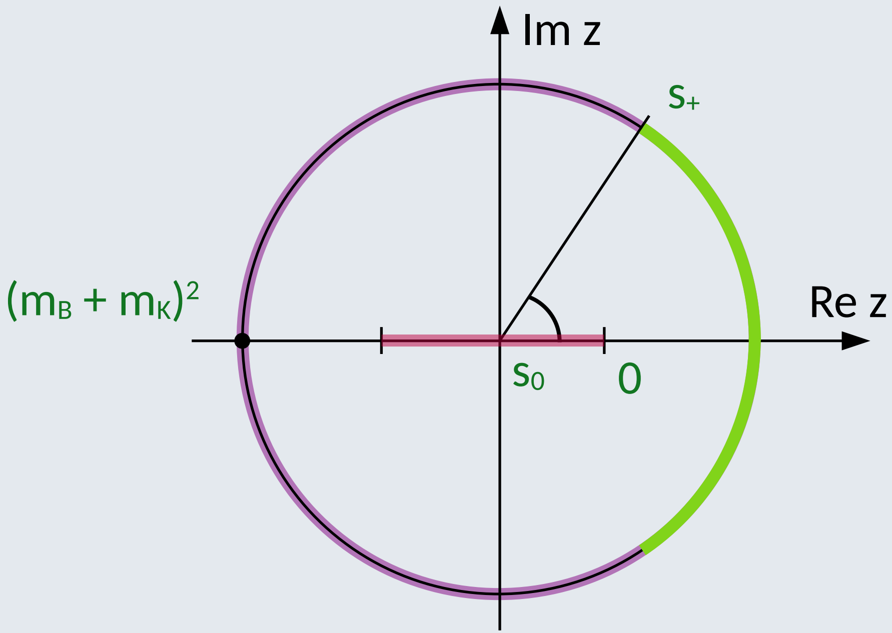


- Equivalent to the dispersive matrix formalism, see Ludovico's talk



- The first branch cut (BK) starts **before** the pair production threshold
- Introduce orthonormal polynomials of the **arc of the unit circle**

$$\mathcal{F}^{\Lambda_b \to \Lambda} = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{N} \alpha_k p_k(z)$$



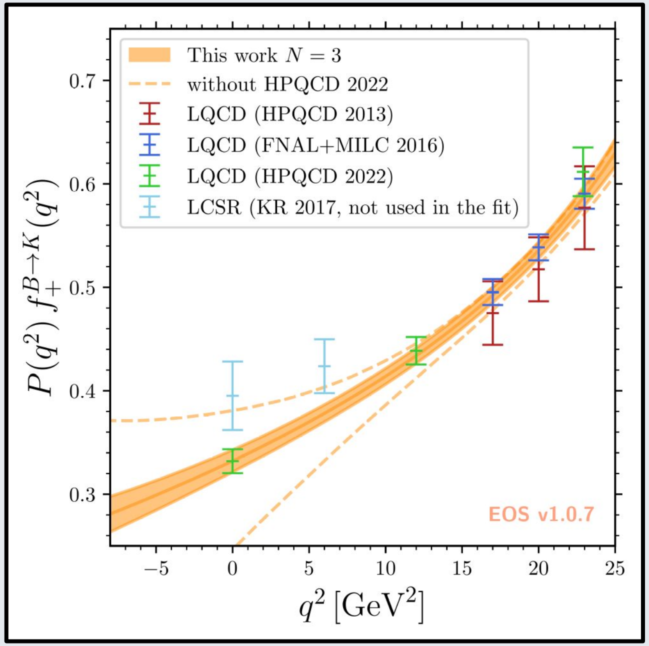
- (Or still expand in z and deal with a more complicated bound [Flynn, Jüttner, Tsang '23])

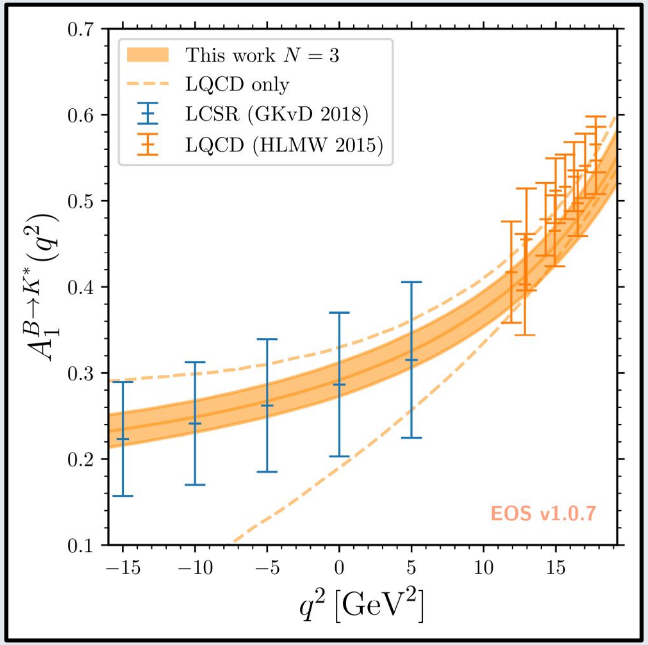
Local form factors fit

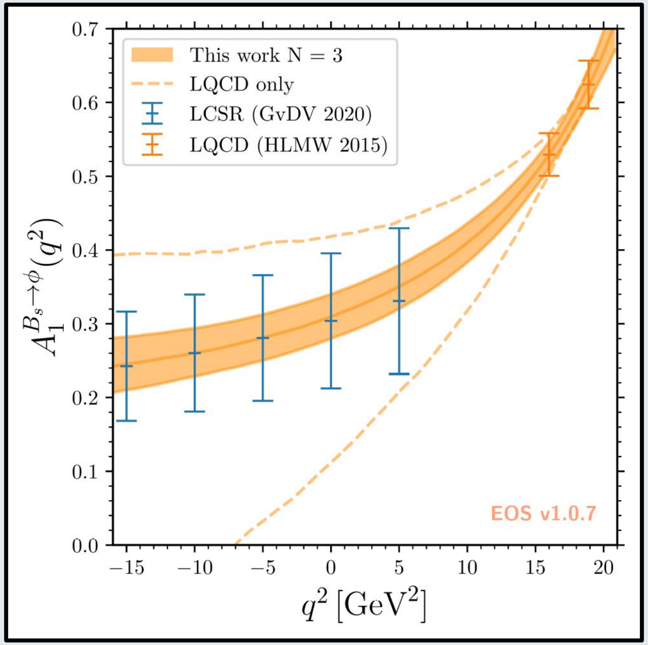
- With this framework we perform a **combined fit** of $B \to K$, $B \to K^*$ and $B_s \to \varphi$ LCSR and lattice QCD inputs:
  - $B \to K$:
    - [HPQCD ’13 and ’22; FNAL/MILC ’17]
    - ([Khodjamiriam, Rusov ’17]) → large uncertainties, not used in the fit
  - $B \to K^*$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, Kokulu, van Dyk ’18] (B-meson LCSRs)
  - $B_s \to \varphi$:
    - [Horgan, Liu, Meinel, Wingate ’15]
    - [Gubernari, van Dyk, Virto ’20] (B-meson LCSRs)
- Adding $\Lambda_b \to \Lambda^{(*)}$ form factors is possible and desirable

Main conclusions:

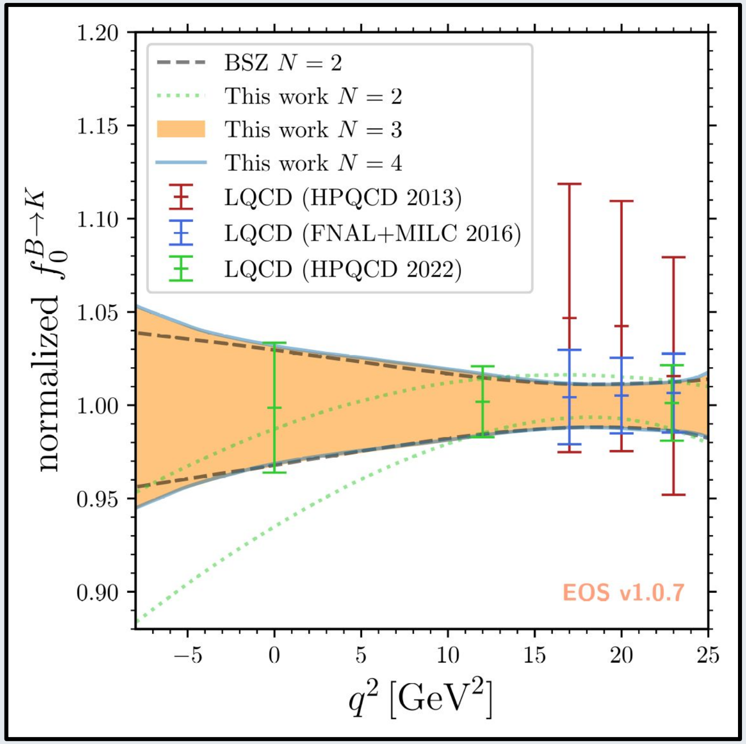
- Fits are very good already at N = 2 (p-values > 77%)
- LCSR and LQCD combine nicely and still dominate the uncertainties
- Progresses in LQCD will gradually replace LCSR

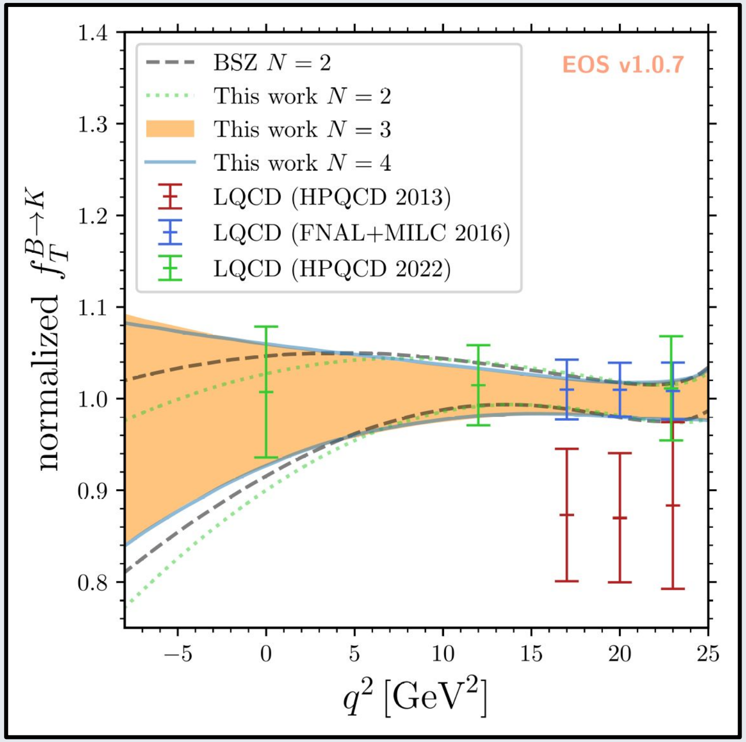






Comparison plots for B → K





- Normalizing the form factors to the N = 3 best fit point allows for a model comparison
- All the plots are available here: https://doi.org/10.5281/zenodo.7919635

Conclusion

Discussing BSM models requires a solid understanding of the hadronic physics:

- **Local form factors** uncertainties can be controlled and reduced by using improved dispersive bound and a *appropriate* parametrization
  - This is the first global analysis of $b \to s$ form factors
  - It is reassuring as it confirms channel-specific analyses…
  - … and promising as dispersive effects start to be visible
- Understanding of **non-local form factors** is essential to distinguish BSM from SM interpretation of the measurements, but still requires theory inputs.

  → In both cases:
  - Uncertainties are still large, but controlled by dispersive bounds
  - Our approach is systematically improvable

Back-up

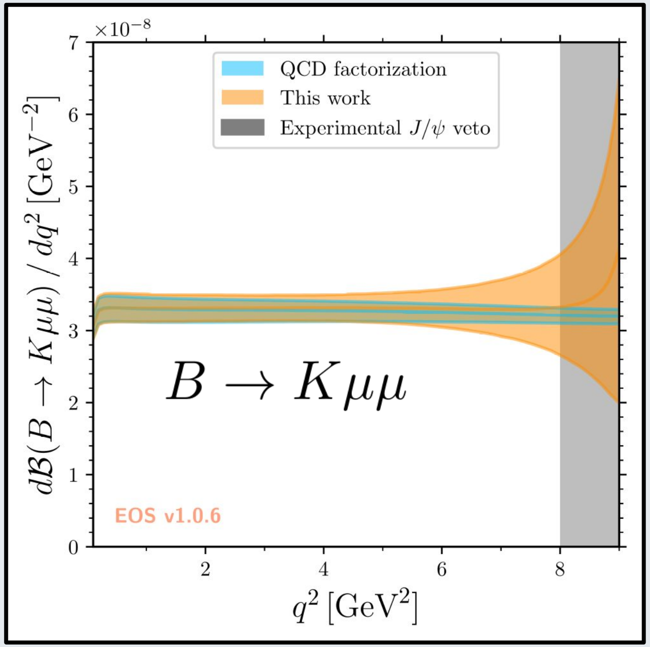
Where to find our results

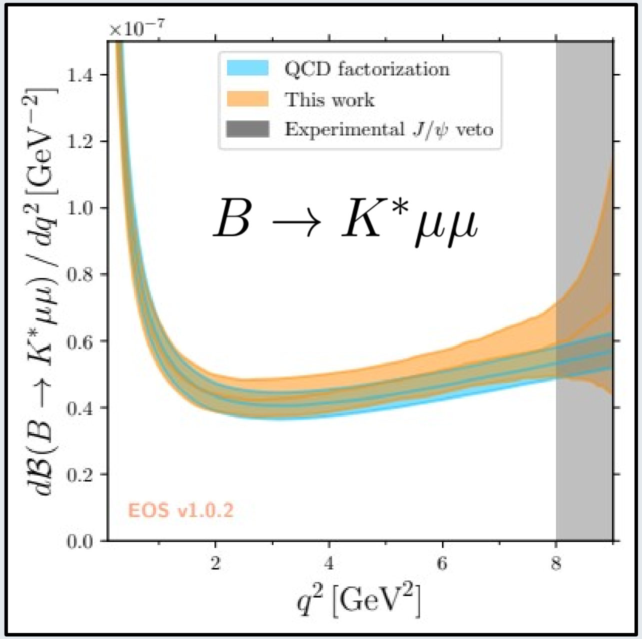
- All the plots are available here: https://doi.org/10.5281/zenodo.7919635
- We also added
  - the updated posterior distributions for N = 2 in our parametrization and using a SSE as YAML files
  - All the tools/documentation to reproduce our results
- These results are also available in **EOS v1.0.7**:
  - /eos/constraints/B-to-P-form-factors.yaml
  - /eos/constraints/B-to-V-form-factors.yaml

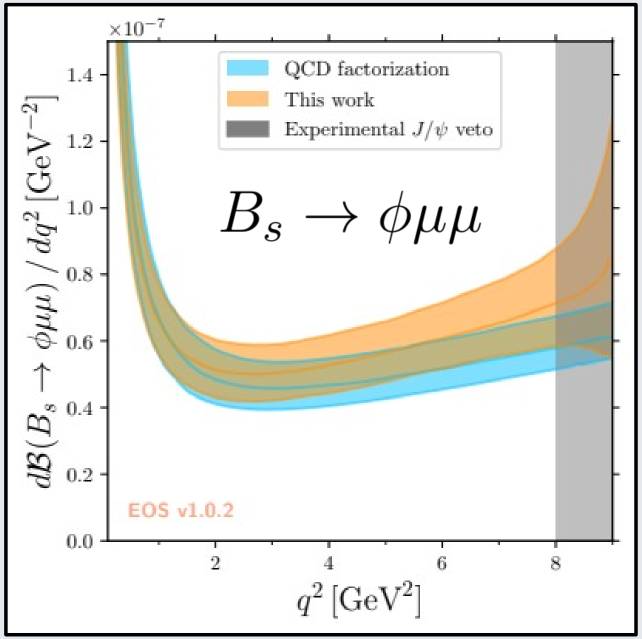
SM predictions

- Good overall agreement with previous theoretical approaches [Beneke, Feldman, Seidel '01 & '04]
  - Small deviation in the slope of $B_s \to \phi\mu\mu$
- Larger but controlled uncertainties especially near the J/ψ

  → The approach is **systematically improvable** (new channels, ψ(2S) data…)



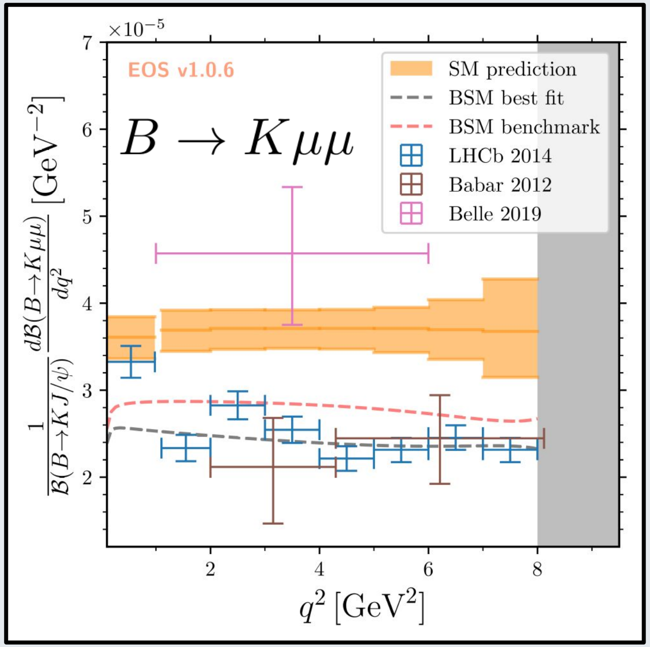


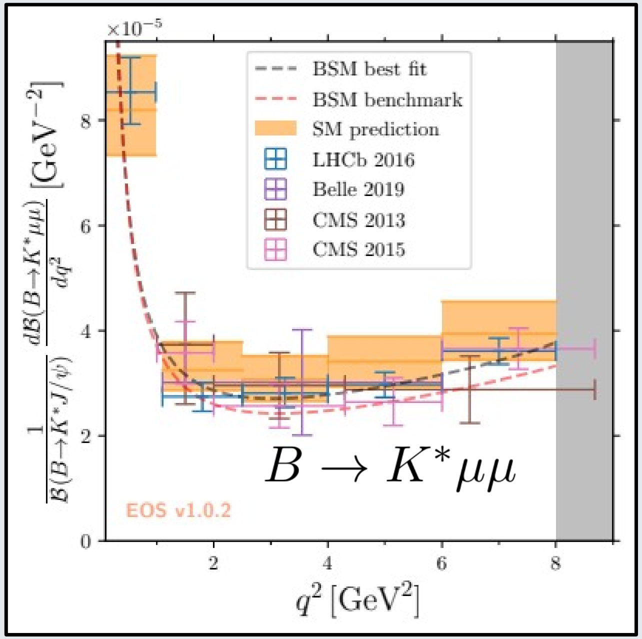


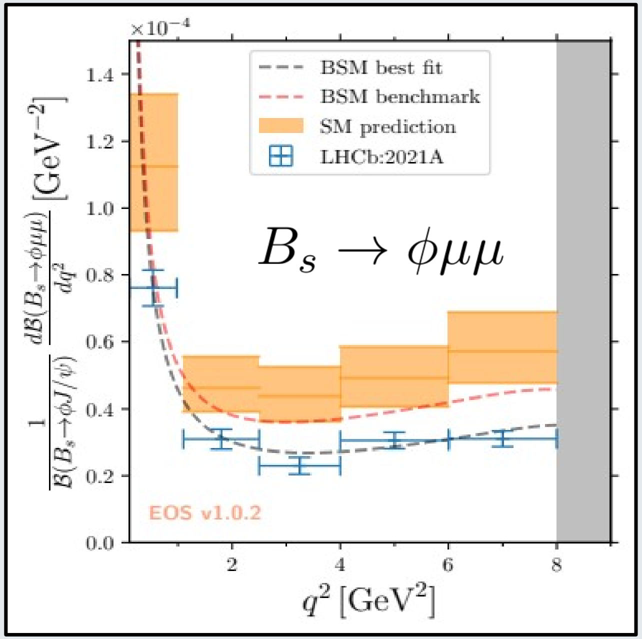
Comparison with data

- Conservatively accounting for the non-local form factors does not solve the b → sμμ anomalies
- The largest source of theoretical uncertainty at low q^2 still comes from local form factors

Experimental results:
[Babar: 1204.3933; Belle: 1908.01848, 1904.02440; ATLAS: 1805.04000, CMS: 1308.3409, 1507.08126, 2010.13968, LHCb: 1403.8044, 2012.13241, 2003.04831, 1606.04731, 2107.13428]



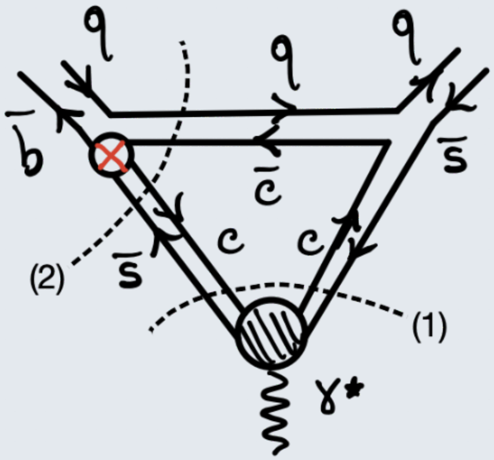


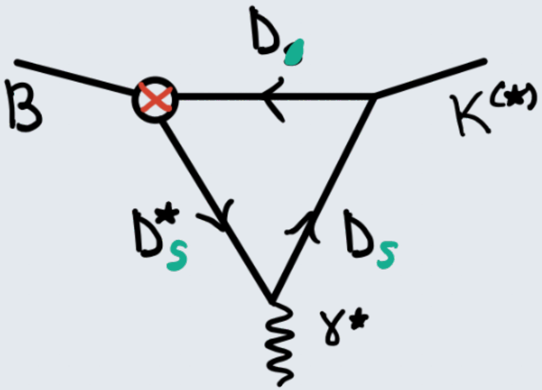


Additional plots can be found in the paper: 2206.03797

Additional effects

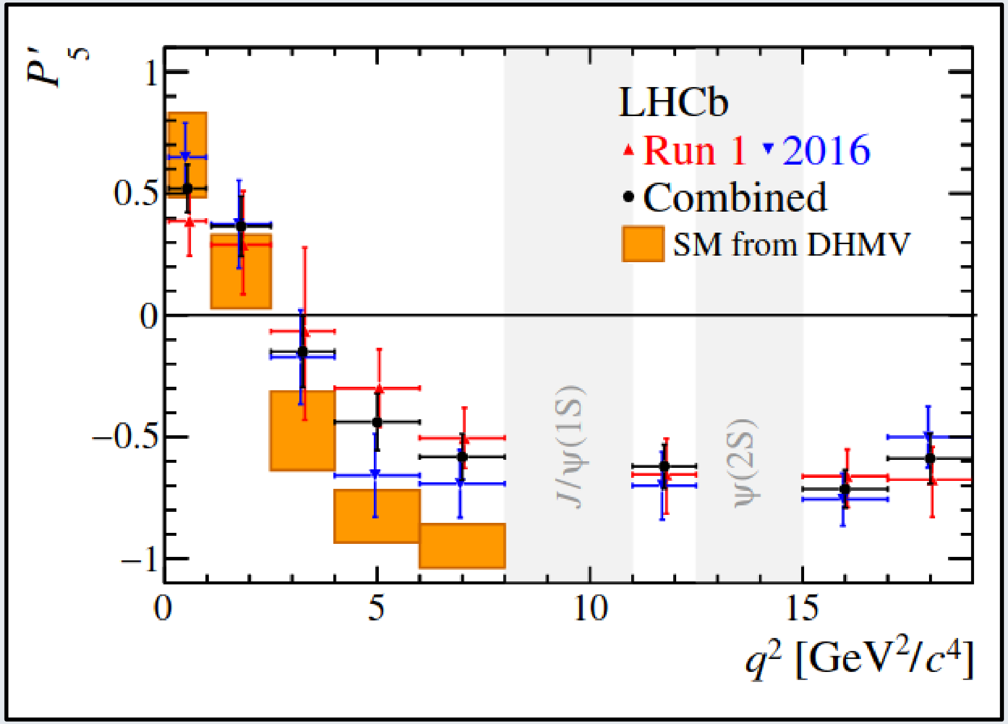
- Rescattering of intermediate hadronic states might spoil the analytic structure of the non-local form-factors [Ciuchini, *et al*, '22]

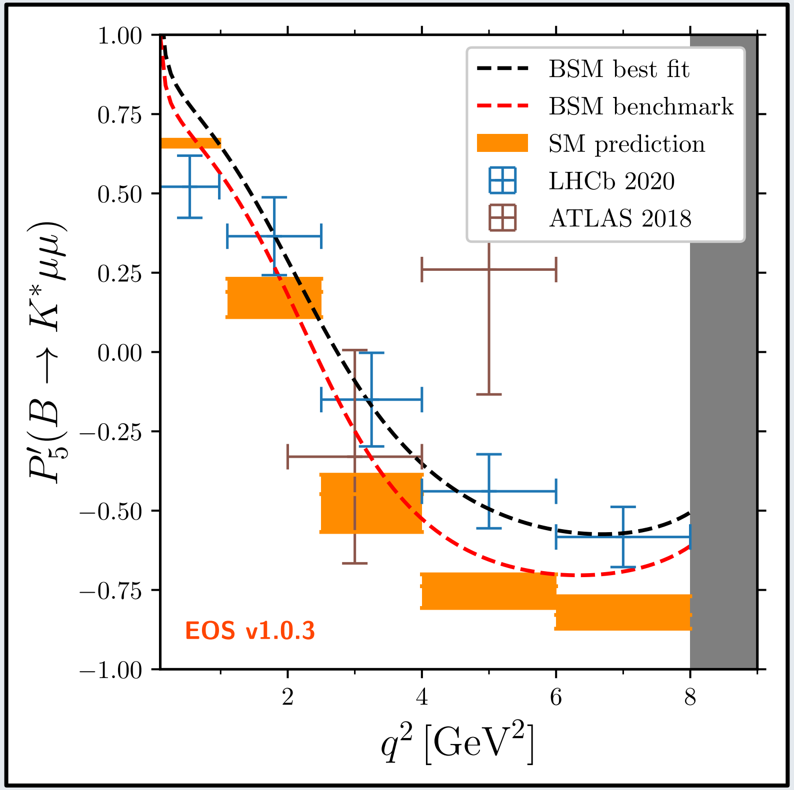




- The effects of the finite width of the K^* amount to a ~10% shift and are accounted for in the fit [Descotes-Genon, Khodjamirian, Virto, '19]

$B \to K^* P'_5$


LHCb
Run 1
2016
Combined
SM from DHMV
J/ψ(1S)
ψ(2S)
P'_5
q^2 [GeV^2/c^4]


BSM best fit
BSM benchmark
SM prediction
LHCb 2020
ATLAS 2018
EOS v1.0.3
$P'_5(B \to K^* \mu\mu)$
q^2 [GeV^2]