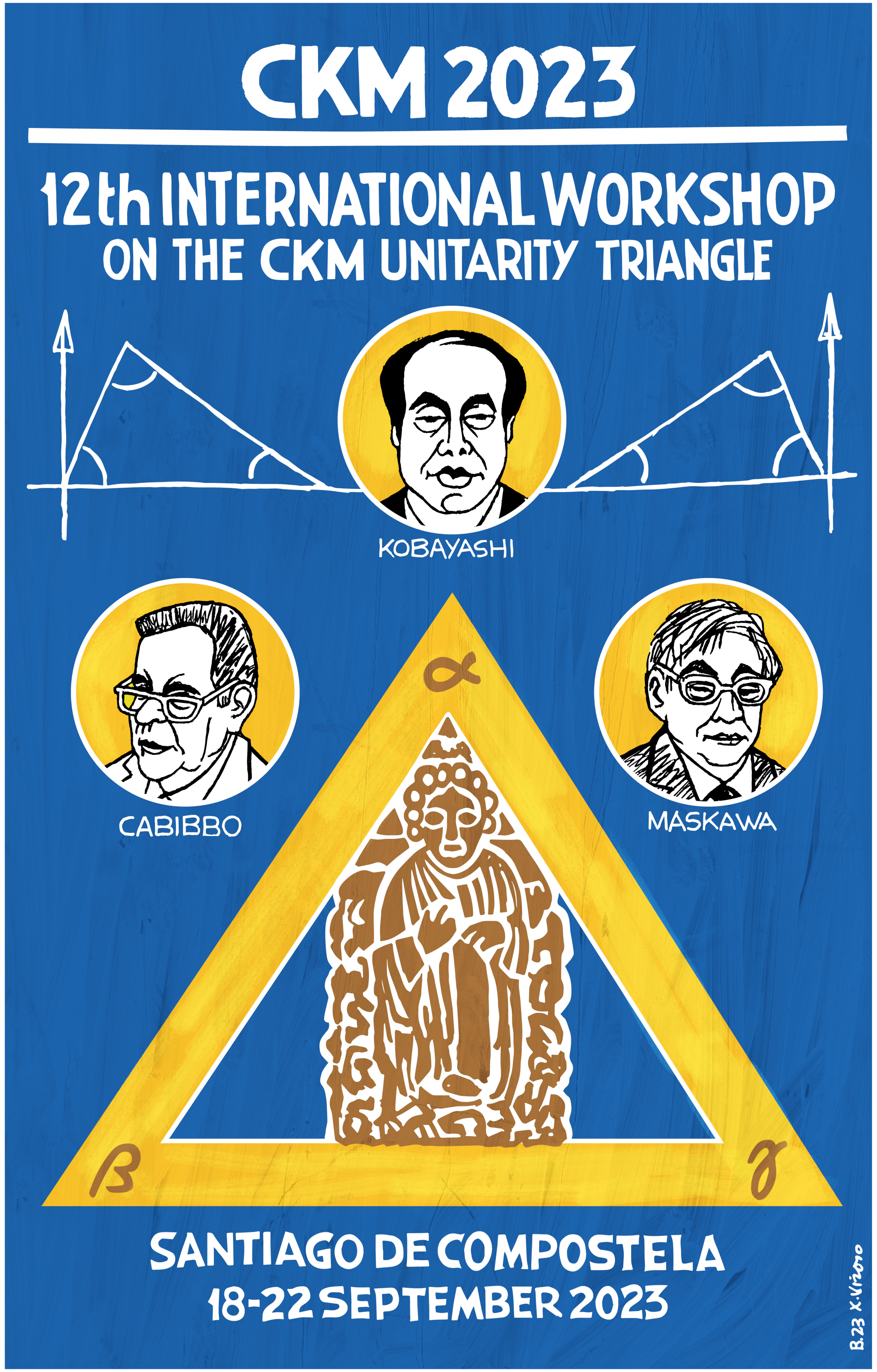
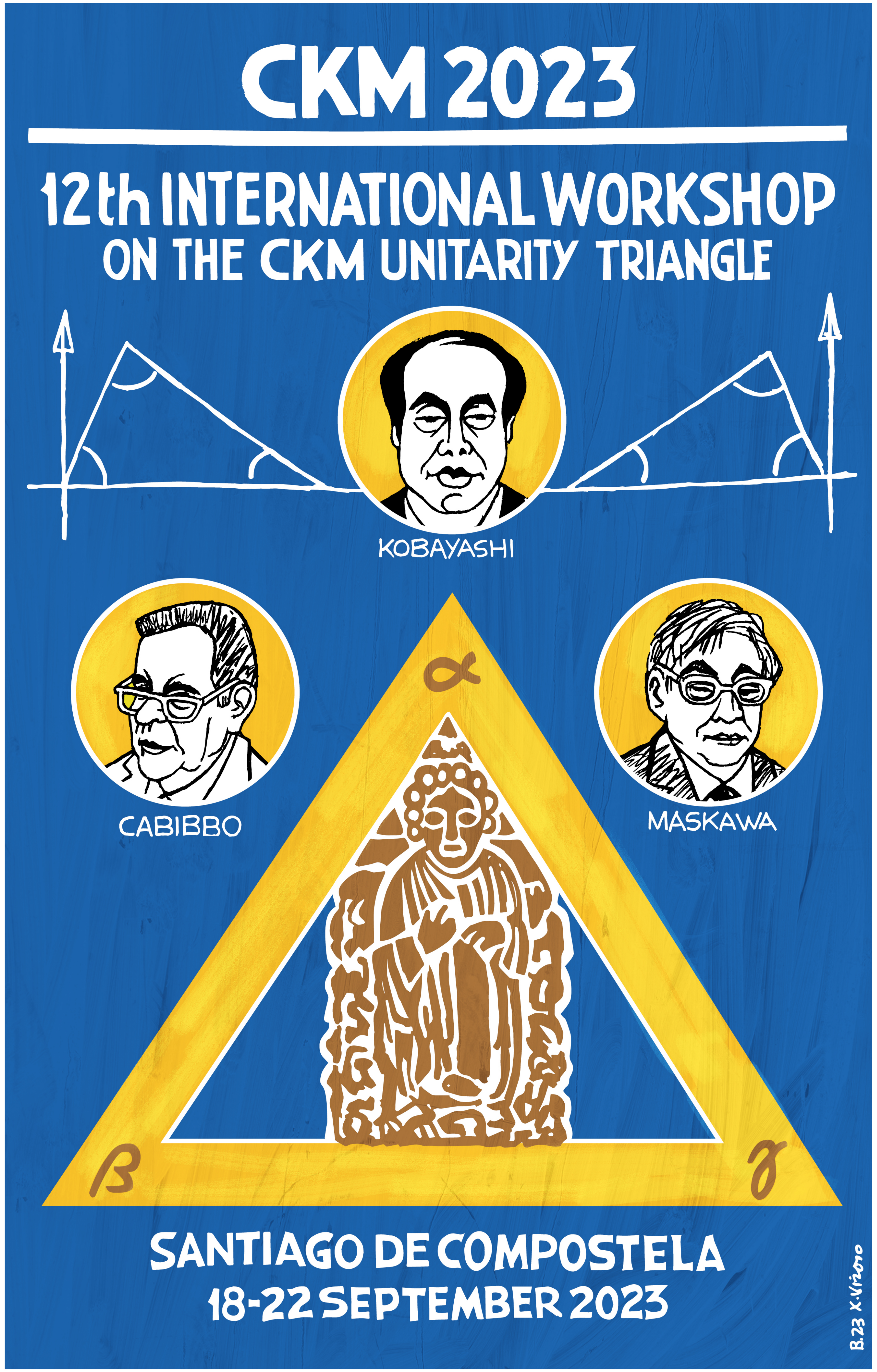


Rare $B \to \pi, K$ decays from lattice QCD

Chris Bouchard
University of Glasgow



I. motivation

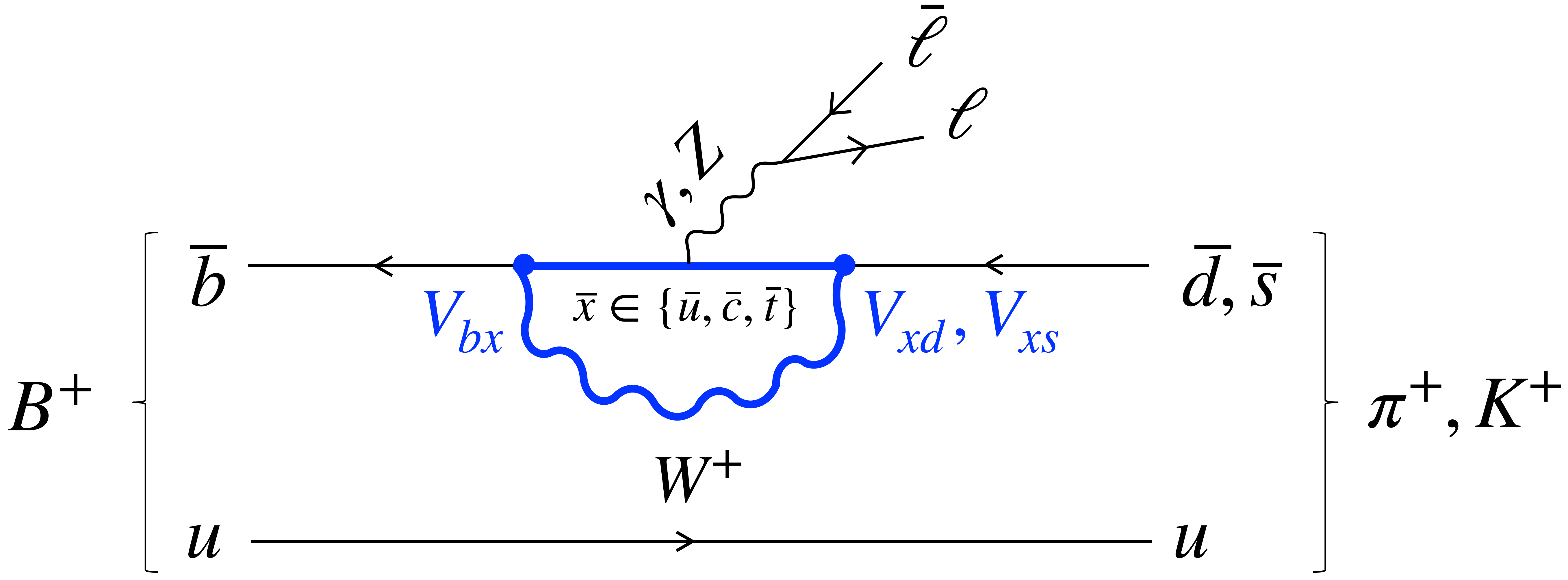
II. lattice form factors

III. $B \to \pi$

IV. $B \to K$

V. outlook

Motivation: small SM contribution



- loop and CKM suppression of SM makes new physics effects potentially visible
- BaBar, Belle, Belle II, and LHCb measurements

$B \to \pi\mu\bar{\mu}$

(LHCb) Aaij et al., JHEP 1212, 125 (2012)
(LHCb) Aaij et al., JHEP 10 (2015) 034

$B \to K\ell\bar{\ell}$

…, (BaBar) Lees et al., PRL 118, 031802 (2017)
…, (Belle) Choudhury et al., JHEP 03, 105 (2019)
…, (LHCb) Aaij et al., Nature Phys. 18, 277 (2022)

$B \to K\nu\bar{\nu}$

Belle II (2023)

Motivation: phenomenology, e.g., $B \to \pi\ell\bar{\ell}$

- measured differential decay rate compared to SM prediction

$$\frac{d\Gamma(B \to \pi\ell\bar{\ell})}{dq^2} = 2a_\ell + \frac{2}{3}c_\ell$$

- SM prediction depends on $F_{P,A,V}$ - functions of form factors and Wilson coefficients

$$a_\ell = \mathscr{C}\left[q^2|F_P|^2 + \frac{\lambda(q, M_B, M_\pi)}{4}(|F_A|^2 + |F_V|^2) + 4m_\ell^2 M_B^2 |F_A|^2 + 2m_\ell(M_B^2 - M_\pi^2 + q^2)\mathrm{Re}(F_P F_A^*)\right]$$

$$c_\ell = -\mathscr{C}\frac{\lambda(q, M_B, M_\pi)\beta_\ell^2}{4}(|F_A|^2 + |F_V|^2)$$

Motivation: phenomenology, e.g., $B \to \pi \ell \bar{\ell}$

$$F_P = -m_\ell C_{10}\left[f_+ - \frac{M_B^2 - M_\pi^2}{q^2}(f_0 - f_+)\right]$$

$$F_A = C_{10} f_+$$

$$F_V = C_9^{\text{eff}} f_+ + \frac{2m_b^{\overline{\text{MS}}}(\mu_b)}{M_B + M_\pi} C_7^{\text{eff}} f_T(\mu_b)$$

- $C_{7,9}^{\text{eff}}$ include $\mathcal{O}(\alpha_s)$ perturbative QCD and estimates of nonfactoriazable corrections
- (for this talk) ignore nonlocality and discuss lattice calculation of short distance form factors $f_{0,+,T}$

Lattice form factors

- form factors parametrize hadronic matrix elements

$$\langle \pi | S_{\text{latt}} | B \rangle = \frac{M_B^2 - M_\pi^2}{m_b - m_{u,d}} f_{0,\text{latt}}(q^2)$$

$$Z_V \langle \pi | V^\mu_{\text{latt}} | B \rangle = f_{+,\text{latt}}(q^2) \left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2} q^\mu \right) + f_{0,\text{latt}}(q^2) \frac{M_B^2 - M_\pi^2}{q^2} q^\mu$$

$$Z_T(\mu_b) \langle \pi | T^{\mu\nu}_{\text{latt}} | B \rangle = 2 \frac{p_B^\mu p_\pi^\nu - p_B^\nu p_\pi^\mu}{M_B + M_\pi} f_{T,\text{latt}}(\mu_b, q^2)$$

- matrix elements extracted from lattice 2pt and 3pt correlation functions
- if necessary, lattice matrix elements matched to continuum
- lattice form factors extrapolated to continuum, infinite volume, and physical quark masses
- q^2 dependence determined after (or as part of physical) extrapolation

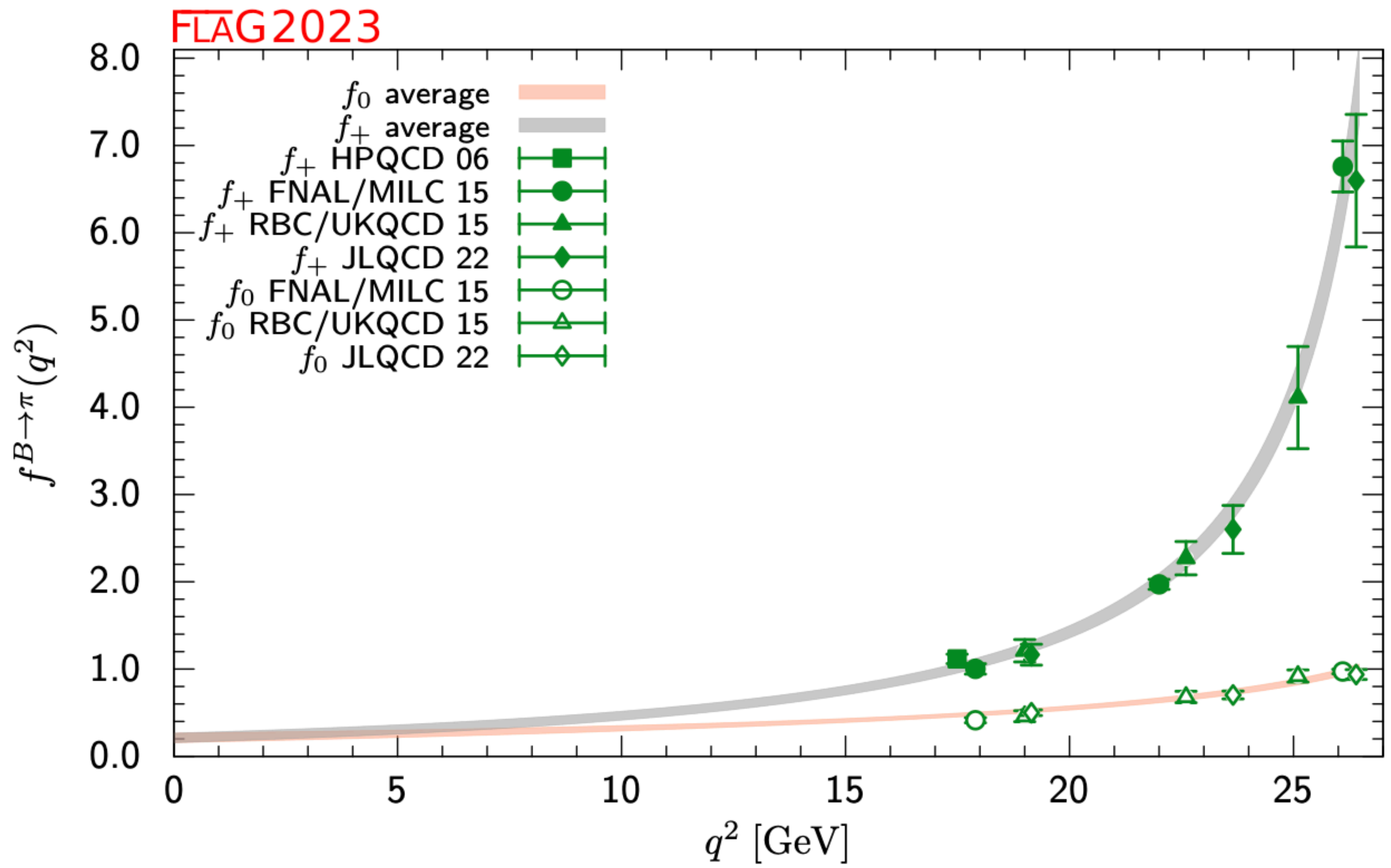
$B \to \pi$: current status

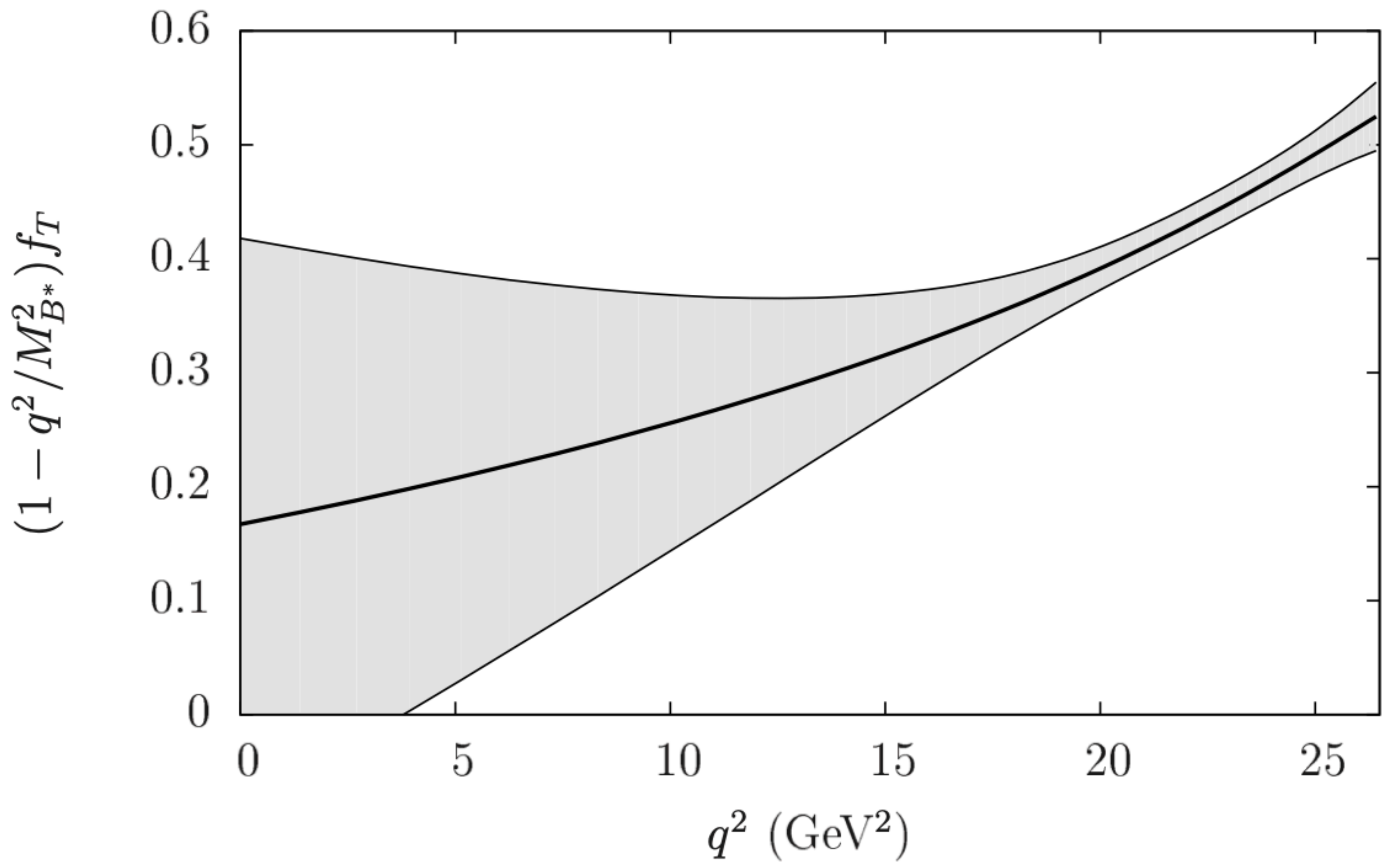
currently dominated by FNAL/MILC 2015a ($f_{0,+}$), FNAL/MILC 2015b (f_T)

$f_{0,+}$: HPQCD 2006
FNAL/MILC 2015a
RBC/UKQCD 2015
} updates underway
JLQCD 2022 (see Brian Colquhoun's talk)

f_T : FNAL/MILC 2015b

- FNAL/MILC update underway
- calculation by other groups underway





$B \to \pi$

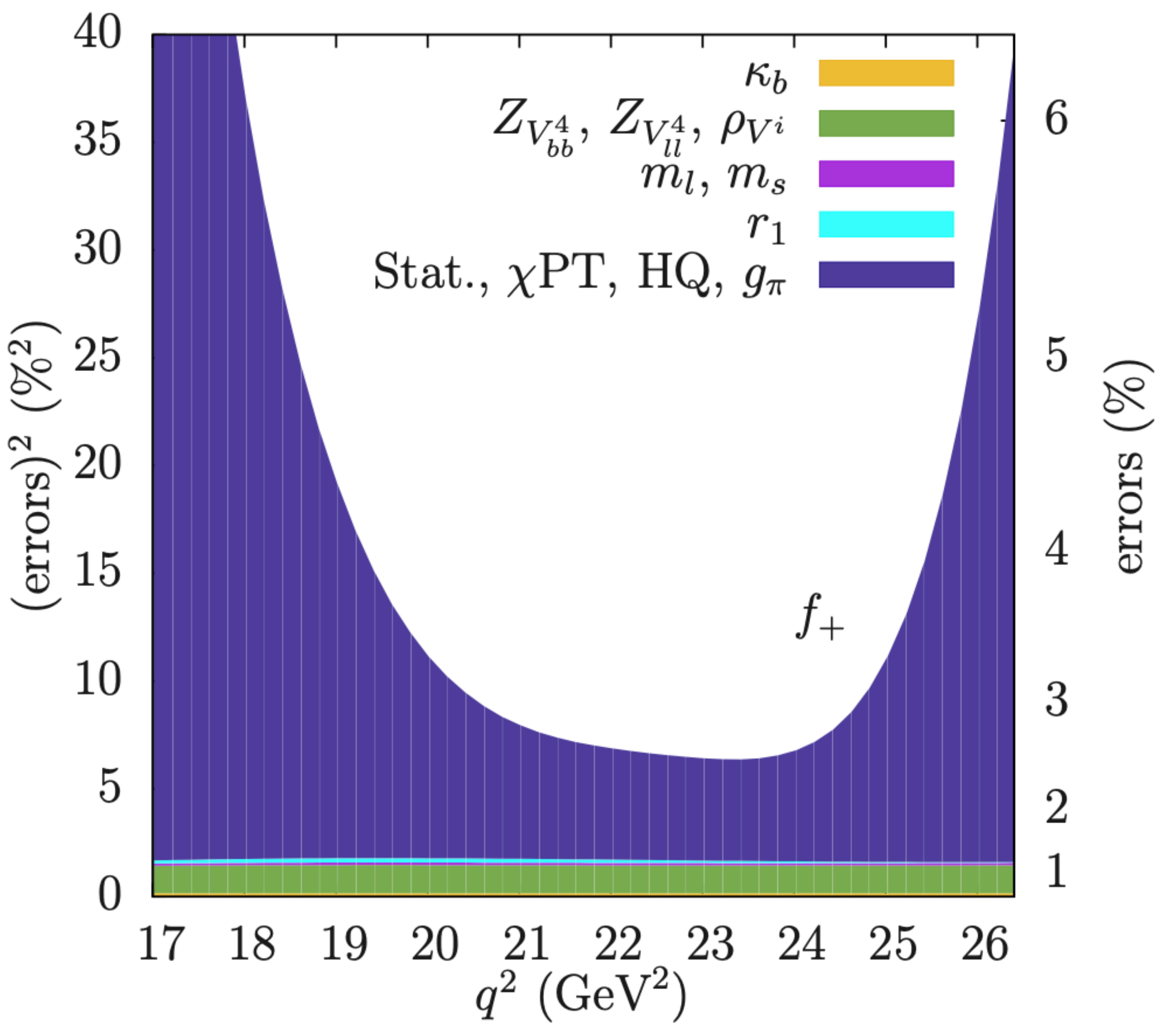
FNAL/MILC 2015a ($f_{0,+}$), FNAL/MILC 2015b (f_T)

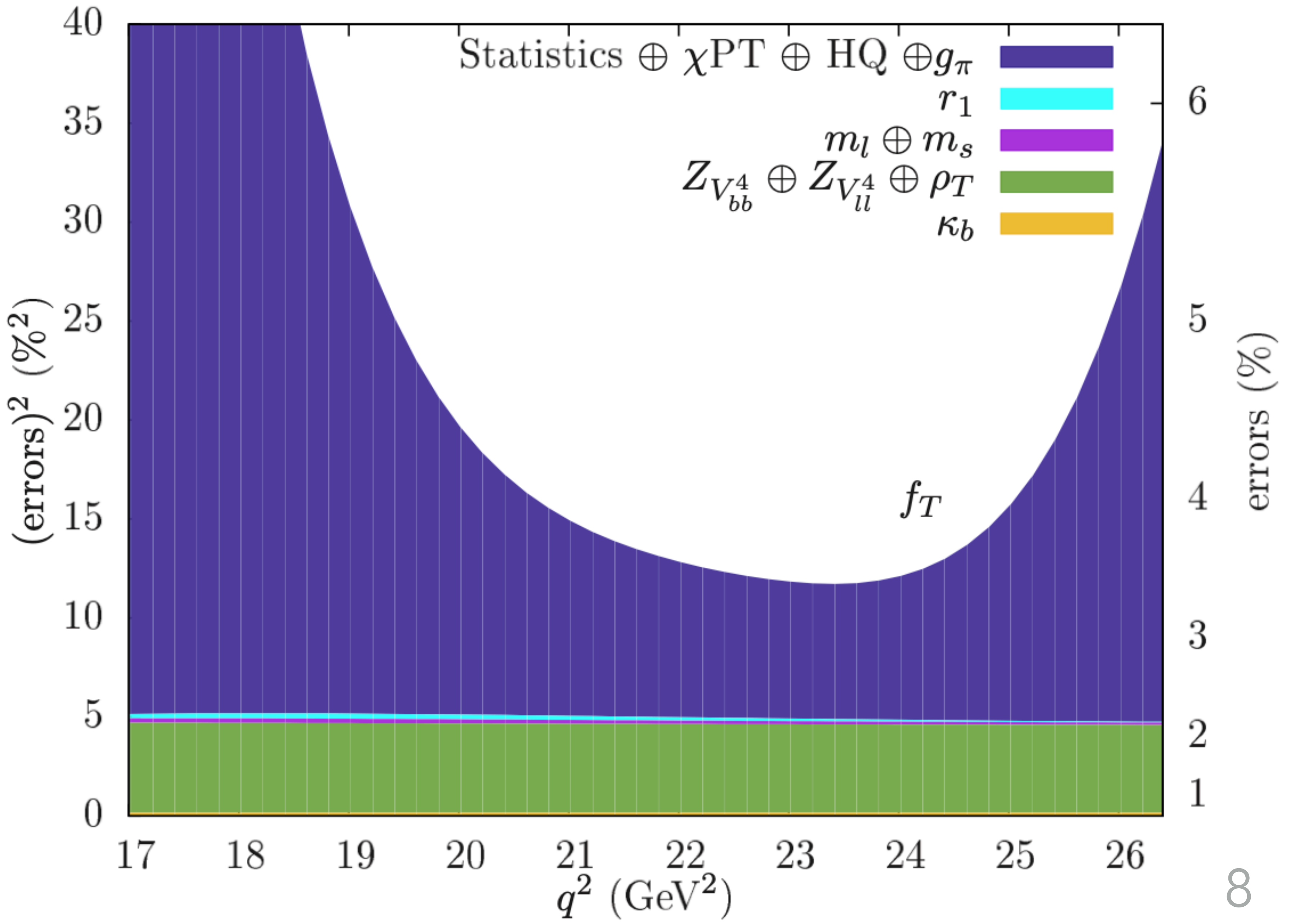
FNAL/MILC 2015a: Bailey et al., PRD 92 (2015) 014024 [1503.07839]
FNAL/MILC 2015b: Bailey et al., PRL 115 (2015) 152002 [1507.01618]

- MILC asqtad $n_f = 2 + 1$ flavor ensembles
- Fermilab b quark (must match to QCD)
- asqtad light valence quarks
- 4 lattice spacings, from 0.12 - 0.045 fm

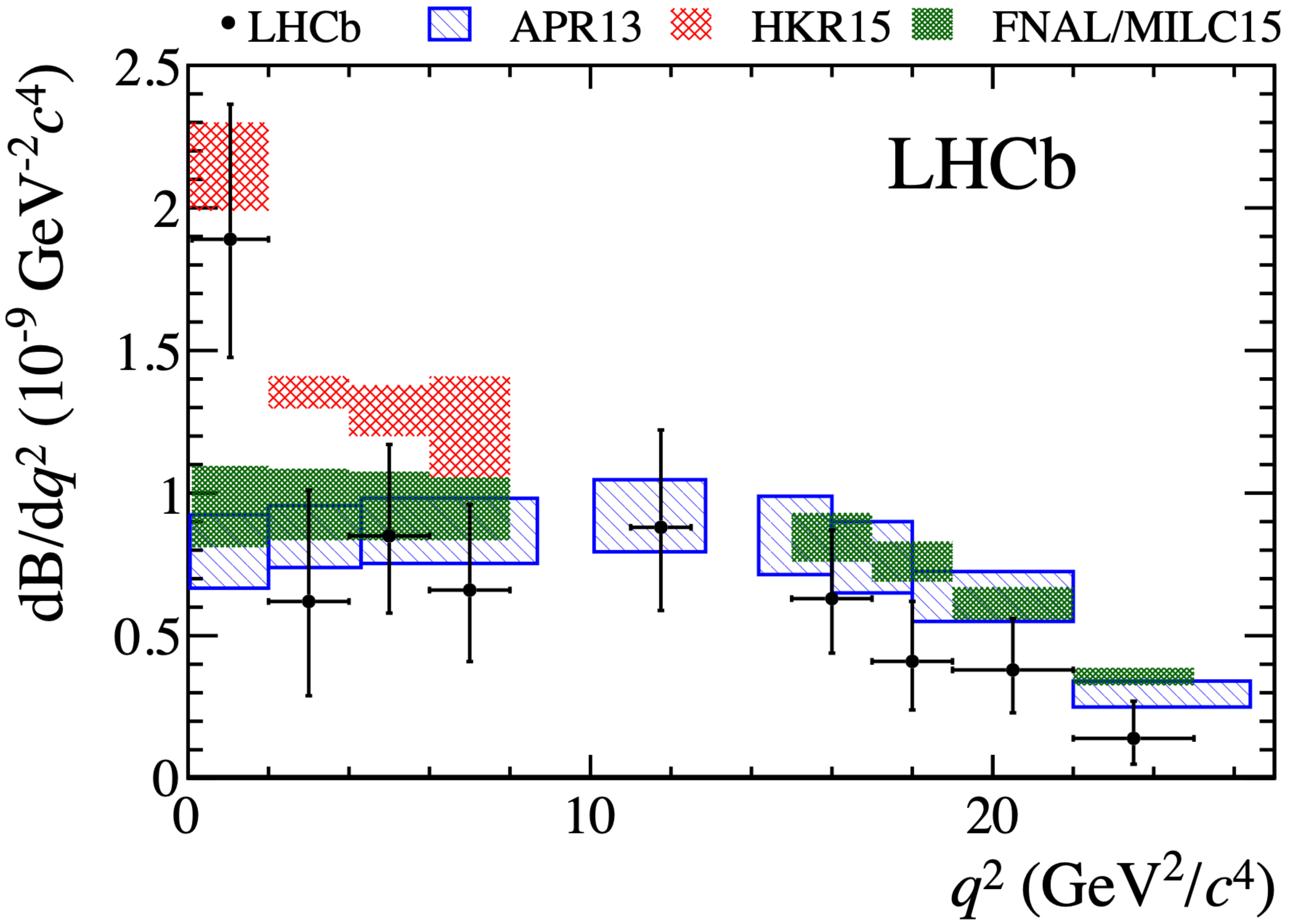
dominant errors: statistics, chiral extrapolation, discretization

- most reduced by MILC's HISQ $n_f = 2 + 1 + 1$ ensembles
- relativistic treatment of b quark would address HQ





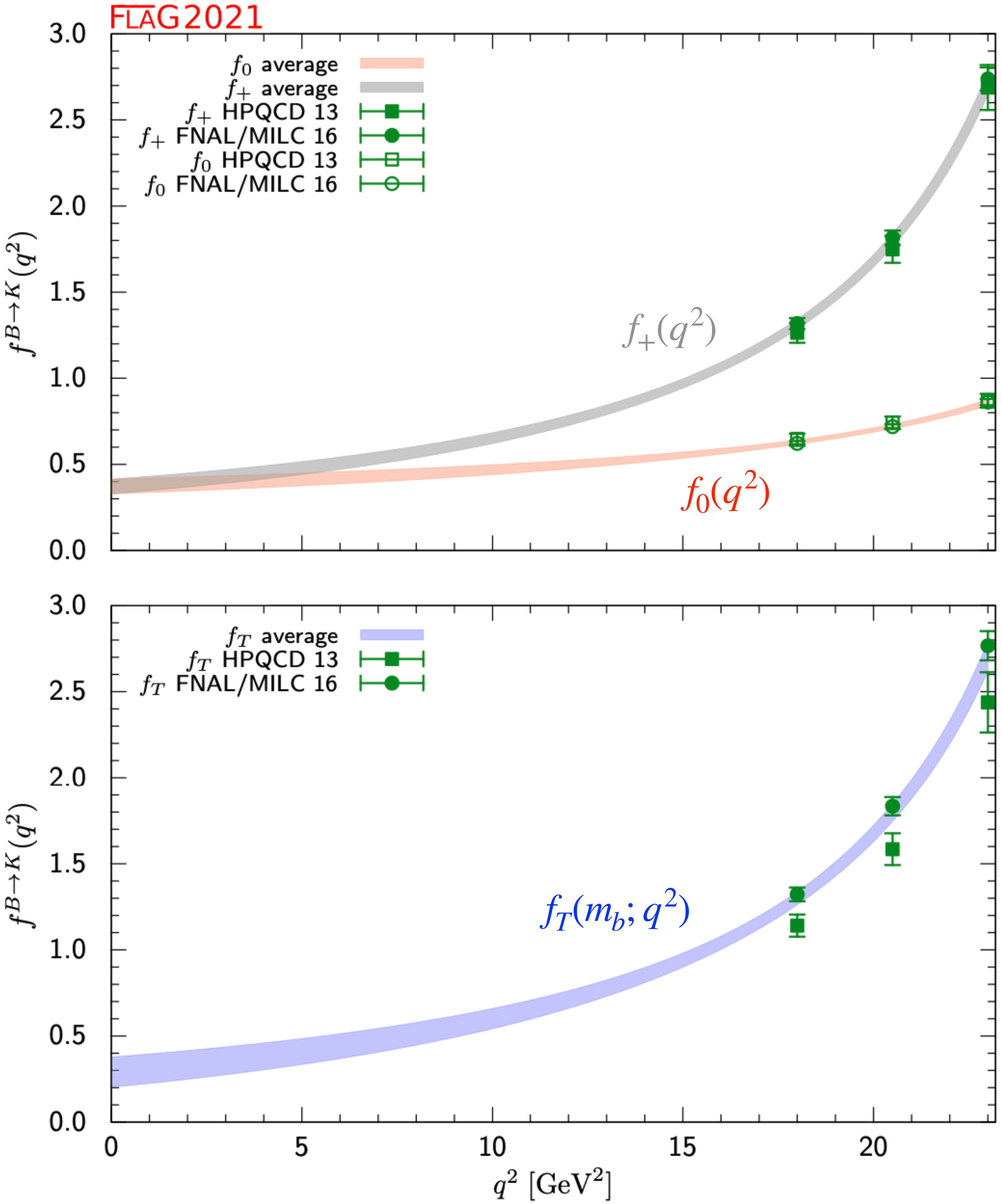
$B \to \pi$

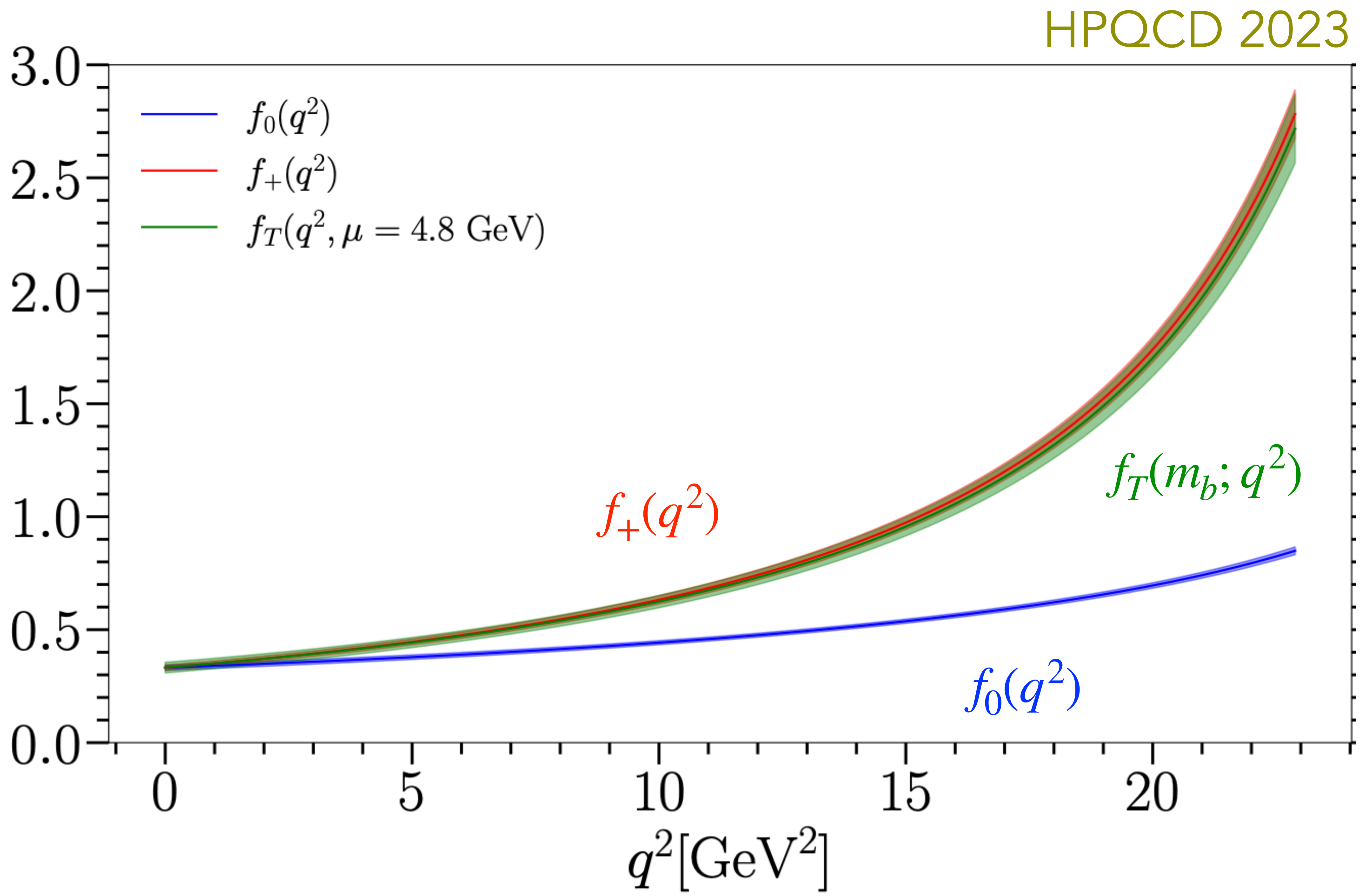


LHCb, Aaij et al., JHEP 10 (2015) 034

- APR13 uses lattice form factors with SU(3) breaking ansatz Ali, Parkhomenko, Rusov, PRD 89 (2014) 094021
- HKR15 uses light cone sum rules Hambrock, Khodjamirian, Rusov, PRD 92 (2015) 7, 074020
- $f_{0,+,T}$ will improve, need to revisit how to handle long distance effects

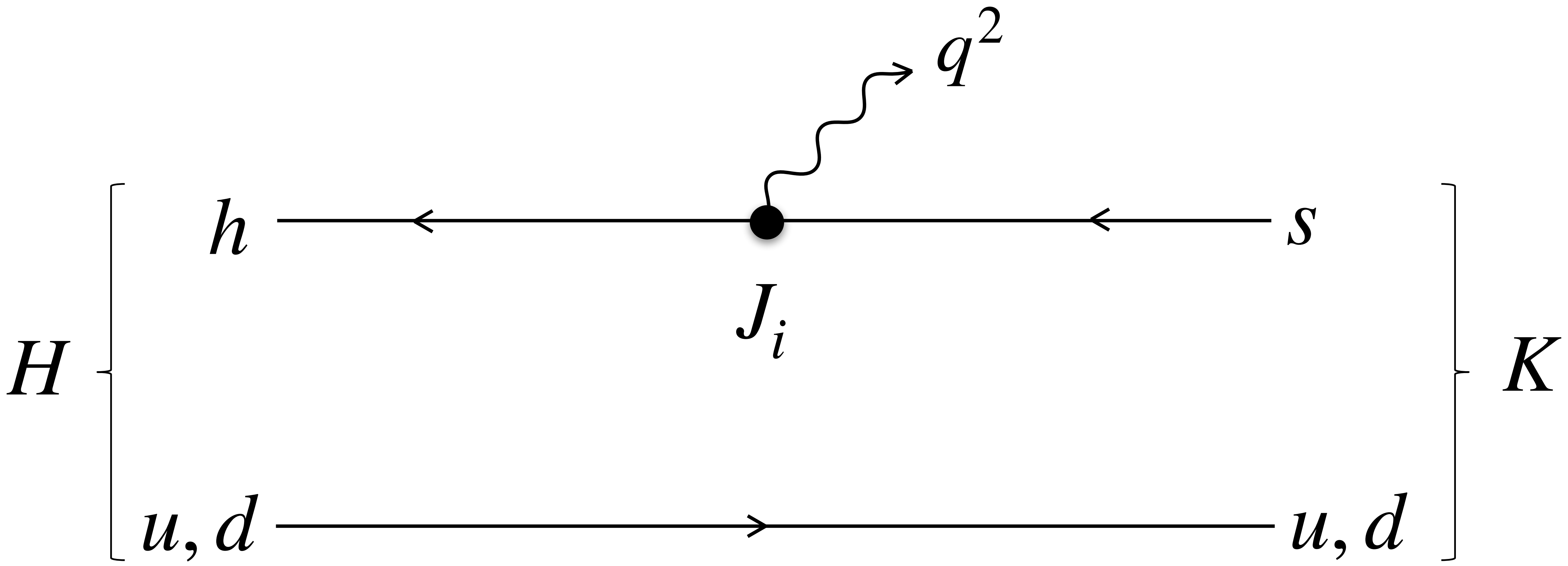
$B \to K$: current status





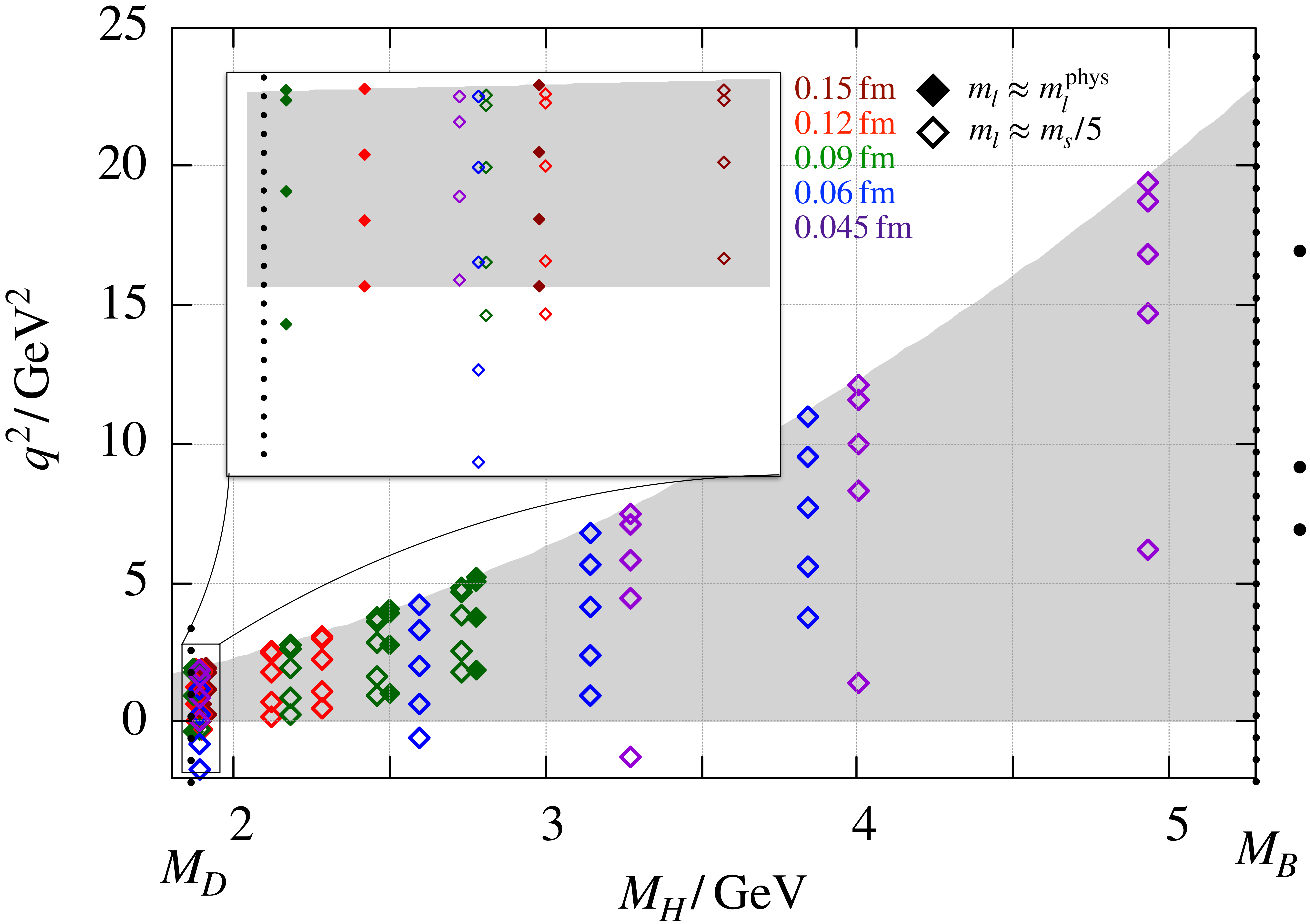
- HPQCD 2023, first using fully relativistic b quark
  Parrott, Bouchard, Davies (HPQCD), PRD 107 (2023) 1, 014510
- heavy HISQ removes matching EFT b quark to QCD
- better coverage of kinematic range
- at least 3x more precise at $q^2 = 0$

$B \to K$ with heavy HISQ



- h a heavy HISQ quark with $m_c \leq m_h \lesssim m_b$; HISQ s and u, d
- simulate a range of m_h
- guided by HQET, extrapolate m_h from $m_c \to m_b$
- $M_D \leq M_H \leq M_B$, obtain results for both B and D decays

$B \to K$: kinematic coverage



- MILC HISQ $n_f = 2 + 1 + 1$ ensembles
  Bazavov et al., PRD 82, 074501 (2010);
  Bazavov et al., PRD 87, 054505 (2012)
- for large range of M_H, cover q^2
- near M_B on finest lattice

$B \to K$: matching matrix elements

- form factors parametrize matrix elements

$$\langle K|S|H\rangle = \frac{M_H^2 - M_K^2}{m_h - m_s} f_0(q^2)$$

$$Z_T(\overline{\text{MS}}, M_H)\,\langle K|T^{jo}|H\rangle = \frac{2iM_H p_K^j}{M_H + M_K} f_T(\overline{\text{MS}}, M_H; q^2)$$

$$Z_V\,\langle K|V^\mu|H\rangle = f_+(q^2)\left(p_H^\mu + p_K^\mu - \frac{M_H^2 - M_K^2}{q^2} q^\mu\right) + f_0(q^2)\,\frac{M_H^2 - M_K^2}{q^2} q^\mu$$

- Z_T calculated via RI-SMOM at 2 GeV (accounting for nonperturbative contributions)

Hatton, Davies, Lepage, Lytle, PRD 102, 094509 (2020)

- Z_V calculated via PCVC relation, $Z_V = \left.\frac{m_h - m_s\langle K|S|H\rangle}{(M_H - M_K)\langle K|V^0|H\rangle}\right|_{\vec{p}_K=0}$

Na, Davies, Follana, Lepage, PRD 82, 114506 (2010)

$B \to K$: modified z-expansion

- form factors at simulated a, $m_{\rm quarks}$, V and q^2
- extrapolate to $a \to 0$, $m_{\rm quarks} \to m_{\rm quarks}^{\rm phys}$ and $V \to \infty$ using modified z-expansion

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}} \quad ; \quad t_+ = (M_H + M_K)^2 \quad , \quad \text{we choose} \quad t_0 = 0$$

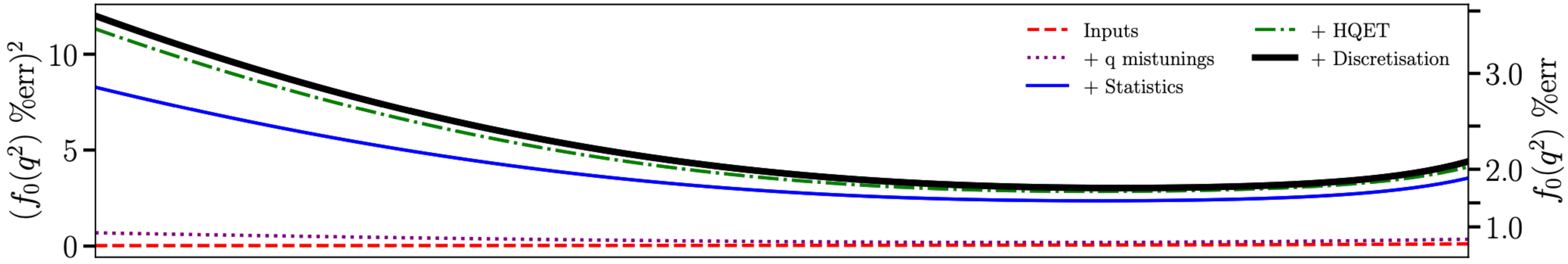
$$f_{+,T}(q^2) = \frac{\mathcal{L}(V)}{1 - q^2/M_{H_s^*}^2} \sum_{n=0}^{N-1} a_n^{+,T}\left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right) \ , \quad f_0(q^2) = \frac{\mathcal{L}(V)}{1 - q^2/M_{H_s^0}^2} \sum_{n=0}^{N-1} a_n^0 z^n$$

- $\mathcal{L}(V)$ are hard pion ChPT logs including (small) FV corrections Bijnens, Jemos, NPB 846, 145-166 (2011)
- a_n contains mistuning, heavy quark expansion, discretization, and analytic chiral terms

$B \to K$: extrapolation results

- bands show continuum, infinite volume, physical quark mass ($m_h = m_b$) form factors
- large f_+ errors at large q^2, when using V^0
  - using spatial component V^k fixes this *

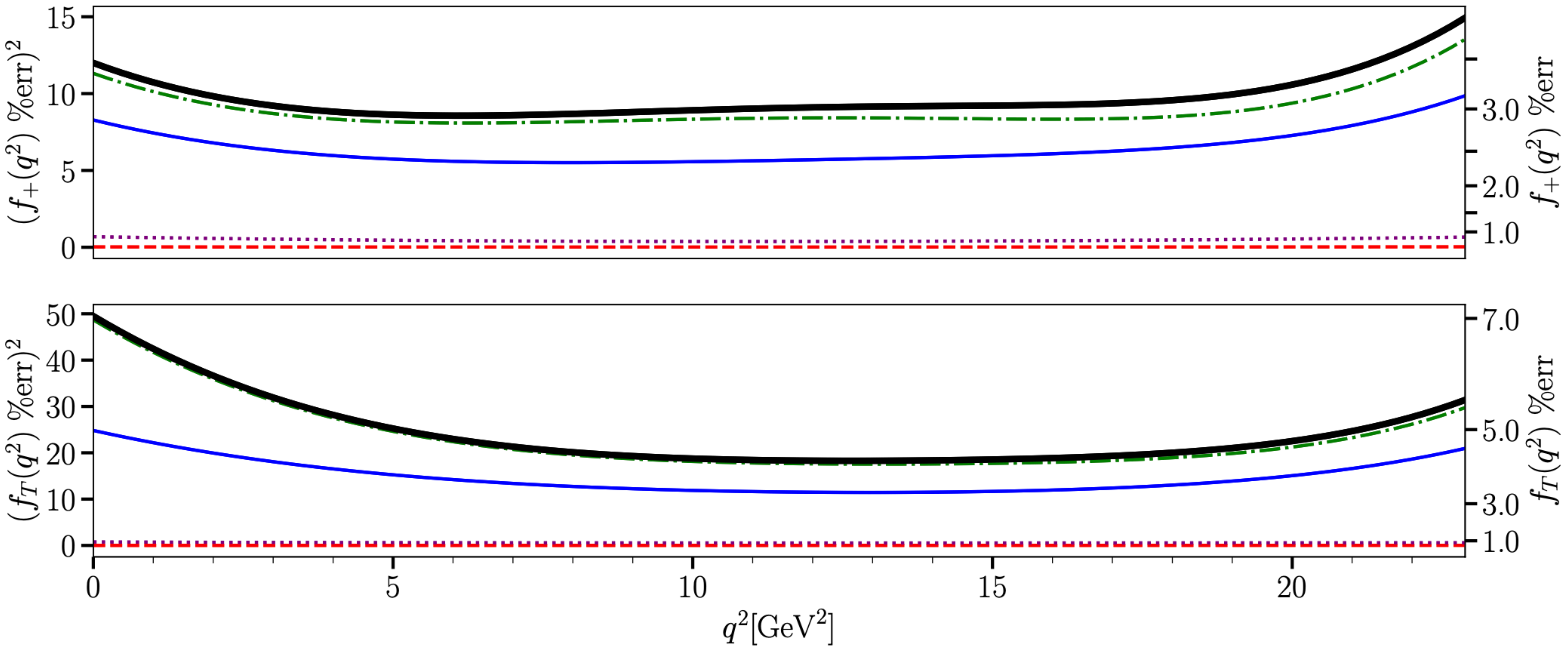
$B \to K$: error budget vs q^2



error budget (stacked variances)

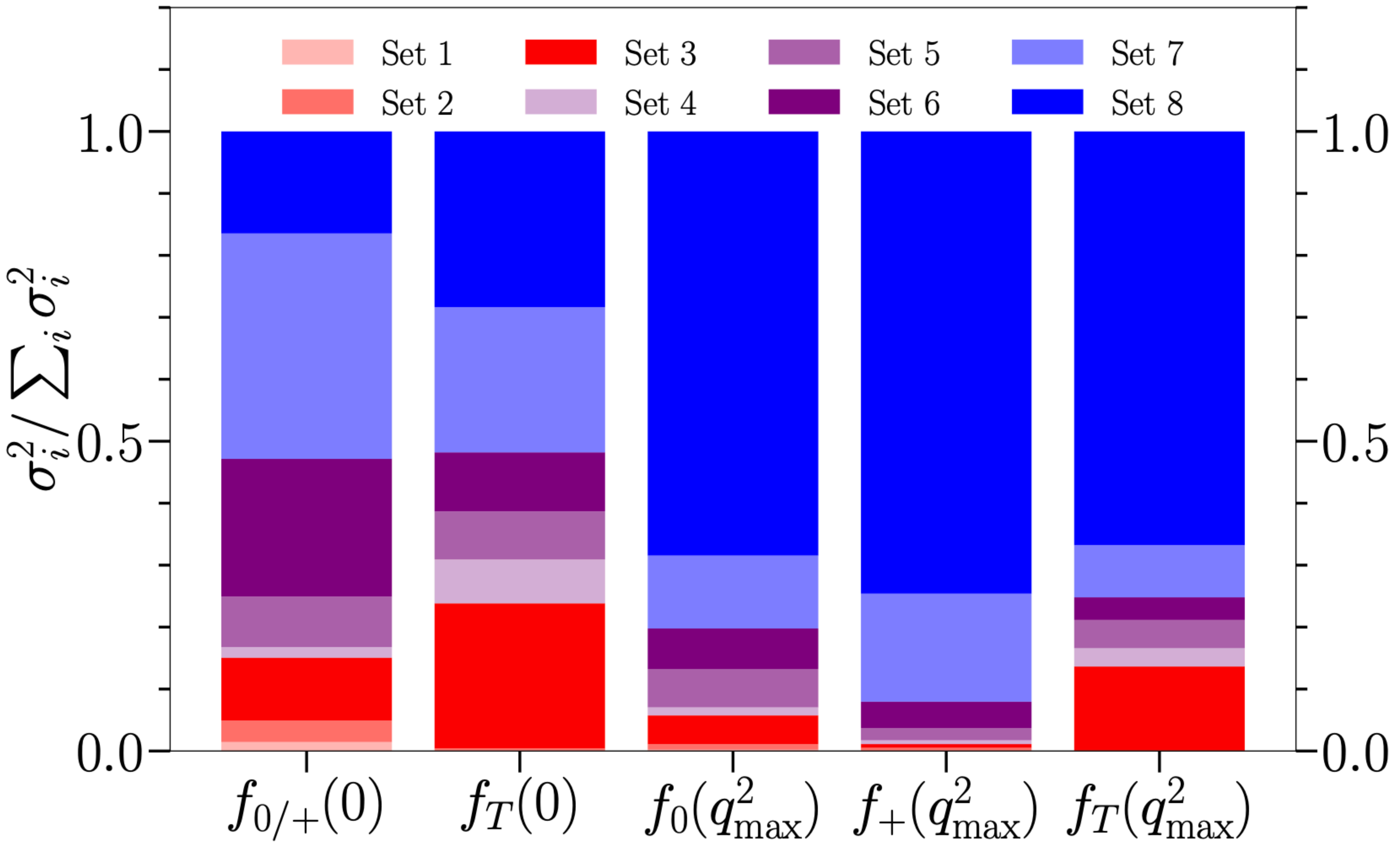
- "Inputs" from errors of input quantities, e.g. meson masses
- "q mistunings" mistuned simulation quark masses and chiral effects - ϵ_n and $L(m_l)$ terms
- "Statistics" from finite ensemble size in Monte Carlo evaluation of path integral
- "HQET" from extrapolation $m_h \to m_b$
- "Discretisation" from uncertainty in extrapolation $a \to 0$

$B \to K$: error budget vs q^2



- improved precision, especially at low q^2, where it is needed
- statistics dominated, so improvement straightforward

$B \to K$: error budget by ensemble



- blue are lattices with finest lattice spacing, needed to reach m_b
- red are lattices with physical light quark mass

Phenomenology: $B \to K\ell\bar{\ell}$

- differential decay rate Γ (or branching fraction $\mathscr{B} = \tau_B\Gamma$) is measured

$$\frac{d\Gamma(B \to K\ell\bar{\ell})}{dq^2} = 2a_\ell + \frac{2}{3}c_\ell$$

- prediction depends on $F_{P,A,V}$ - functions of form factors and Wilson coefficients

$$a_\ell = \mathscr{C}\left[q^2|F_P|^2 + \frac{\lambda(q, M_B, M_K)}{4}(|F_A|^2 + |F_V|^2) + 4m_\ell^2 M_B^2 |F_A|^2 + 2m_\ell(M_B^2 - M_K^2 + q^2)\mathrm{Re}(F_P F_A^*)\right]$$

$$c_\ell = -\mathscr{C}\frac{\lambda(q, M_B, M_K)\beta_\ell^2}{4}(|F_A|^2 + |F_V|^2)$$

Phenomenology: $B \to K\ell\bar{\ell}$

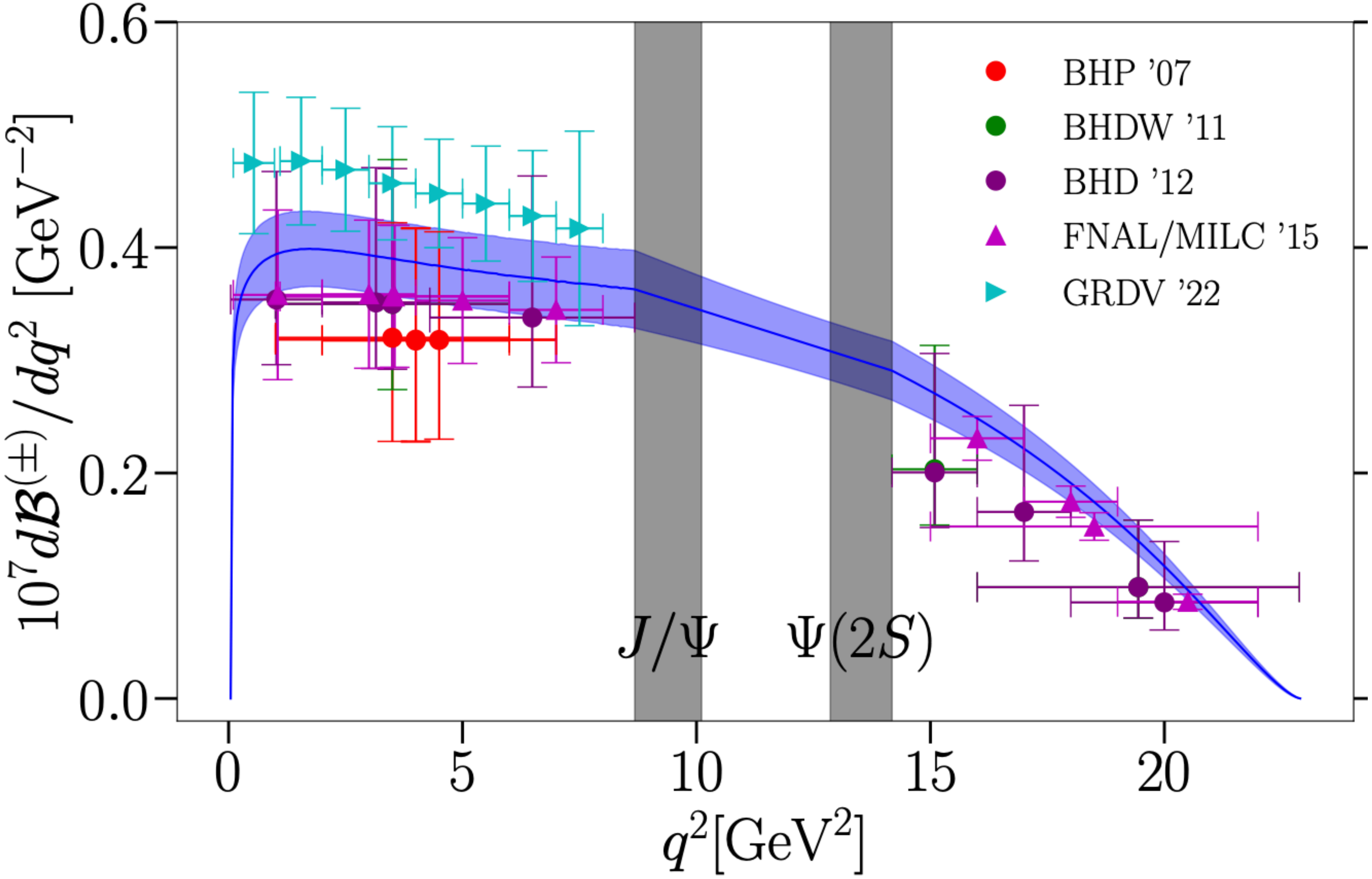
$$F_P = -m_\ell C_{10}\left[f_+ - \frac{M_B^2 - M_K^2}{q^2}(f_0 - f_+)\right]$$

$$F_A = C_{10} f_+$$

$$F_V = C_9^{\text{eff},1} f_+ + \frac{2m_b^{\overline{\text{MS}}}(\mu_b)}{M_B + M_K} C_7^{\text{eff},1} f_T(\mu_b)$$

- $C_9^{\text{eff},1}$ includes $\mathcal{O}(\alpha_s)$ perturbative QCD and estimates of nonfactoriazable corrections
- $C_7^{\text{eff},1}$ includes $\mathcal{O}(\alpha_s)$ corrections FNAL/MILC, PRD 93, 034005 (2016)
  - these corrections give $< 1\sigma$ shift, slightly reducing tension with expt
- QED effect from final state radiation: 2% (5%) in $d\mathcal{B}/dq^2$ for $\mu(e)$; 1% in ratio R_K
- other small uncertainties included (e.g. scale dependence of Wilson coefficients, $m_u \neq m_d$)

Phenomenology: $B \to K\ell\bar{\ell}$ vs other theory



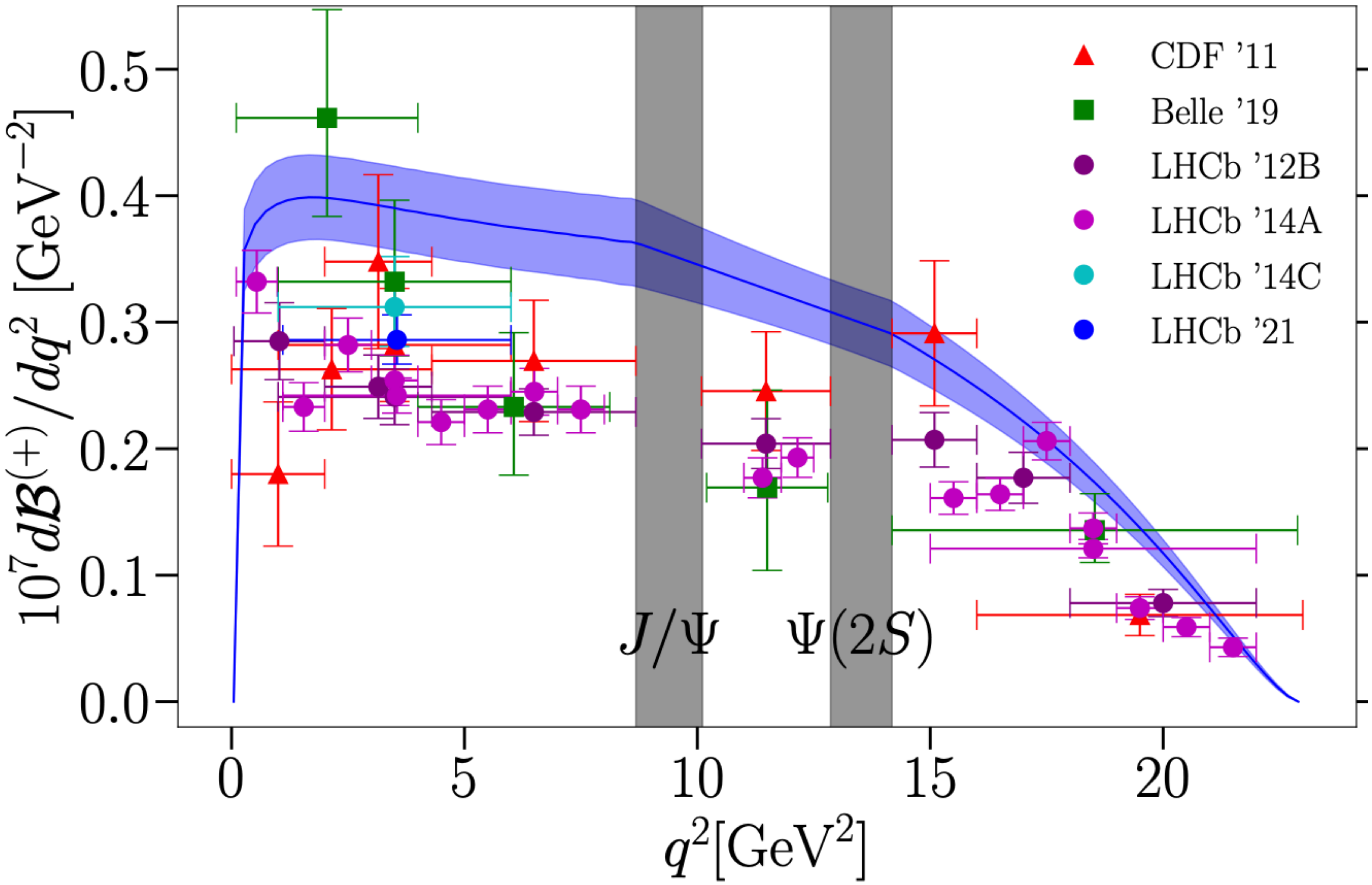
Bobeth, Hiller, and Piranishvili

Bobeth, Hiller, van Dyk, and Wacker

Bobeth, Hiller, and van Dyk

Gubernari, Reboud, van Dyk, and Virto

Phenomenology: $B \to K\ell\bar{\ell}$ vs experiment

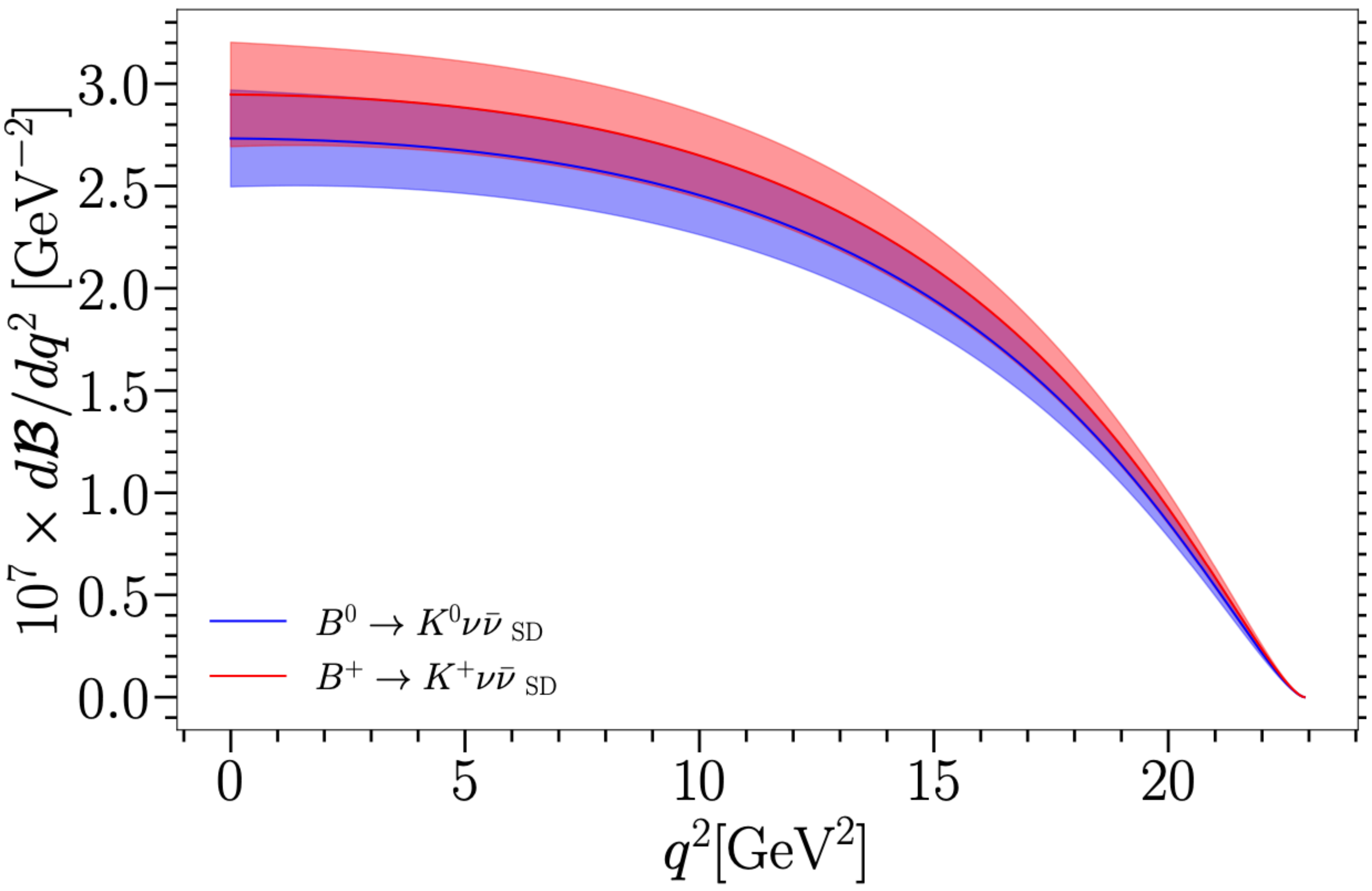


Focus on two well-behaved regions:

- $1.1 \le q^2/\text{GeV}^2 \le 6$: below $c\bar{c}$ resonances; improved precision and increased tension
- $15 \le q^2/\text{GeV}^2 \le 22$: above (dominant) $c\bar{c}$ resonances, include 2% uncertainty for broad resonances

LHCb, Eur. Phys. J. C 77, 161 (2017)

Phenomenology: $B \to K\nu\bar{\nu}$

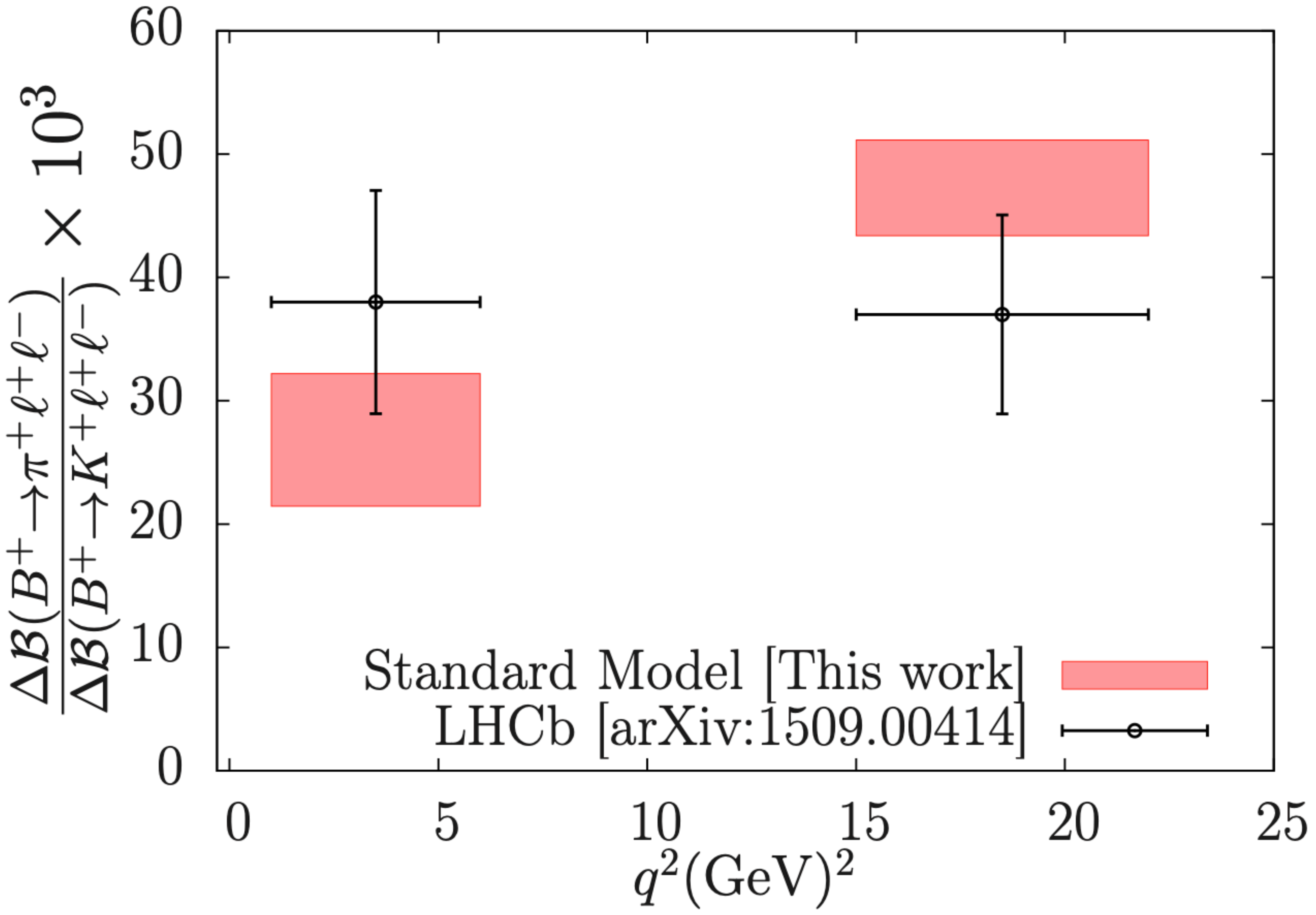


Decay	$\mathcal{B} \times 10^6$	Reference	
$B^0 \to K_S^0\nu\bar{\nu}$	< 13 (90% CL) Exp.	[32]	Belle '17
$B^0 \to K^0\nu\bar{\nu}$	< 49 (90% CL) Exp.	[34]	BaBar '13
	4.01(49)	[9]	FNAL '16
	$4.1^{+1.3}_{-1.0}$	[37]	Wang, Xiao '12
	4.60(34)	HPQCD '22	
$B^+ \to K^+\nu\bar{\nu}$	< 16 (90% CL) Exp.	[34]	BaBar '13
	< 19 (90% CL) Exp.	[32]	Belle '17
	< 41 (90% CL) Exp.	[33]	Belle II '21
	5.10(80)	[79, 81]	Altmanshoffer et al '09, Kamenik, Smith '09
	$4.4^{+1.4}_{-1.1}$	[37]	Wang, Xiao '12
	3.98(47)	[45]	Buras et al '14
	4.94(52)	[9]	FNAL '16
	4.53(64)	[86]	Buras, Venturini '21
	4.65(62)	[87]	Buras, Venturini '22
	5.58(37)	HPQCD '22	

24(7) Belle II $2.6\,\sigma$

- modest improvement in precision
- cleaner theoretically; no resonances or nonfactorizable contributions

Phenomenology: $B \to \pi$ with $B \to K$



$B \to \pi$

(FNAL/MILC) Bailey et al., PRD 92 (2015) 014024

(FNAL/MILC) Bailey et al., PRL 115 (2015) 152002

$B \to K$

(FNAL/MILC) Bailey et al., PRD 93 (2016) 2, 025026

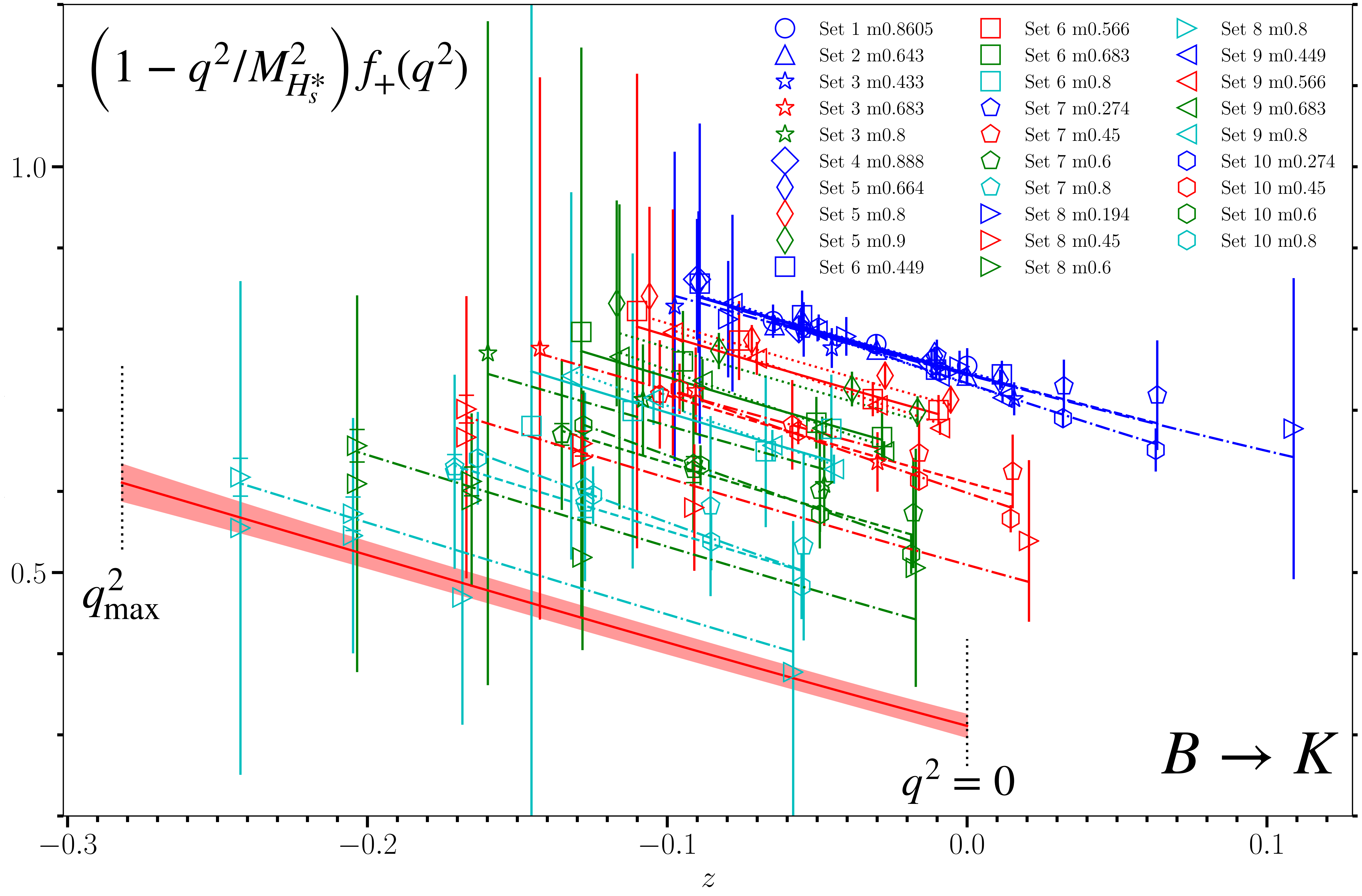
- FNAL/MILC combined phenomenological analysis on $B \to \pi, K$

(FNAL/MILC) Du et al., PRD 93 (2016) 3, 034005

- capitalises on correlations in lattice calculations
- both calculations (have been/are being) improved with heavy-HISQ

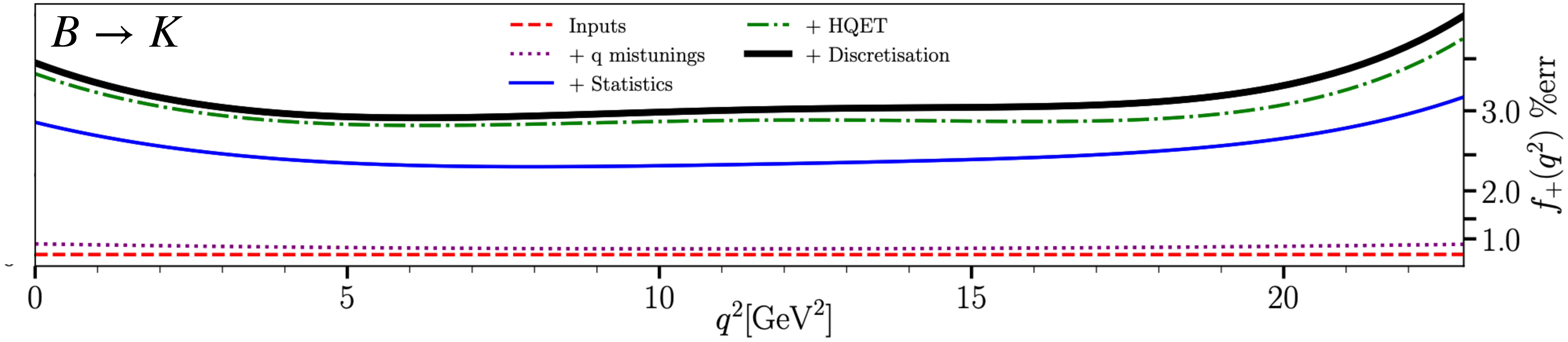
Outlook

- fully relativistic b quark removes EFT matching error
- improved q^2 coverage changes the story that LQCD is only applicable at large q^2
- others also using fully relativistic treatments of the b quark, e.g., RBC/UKQCD and JLQCD using DWF



Outlook

- heavy HISQ $B \to K$ form factors most precise to date at low q^2
  - statistics limited, improvement straightforward

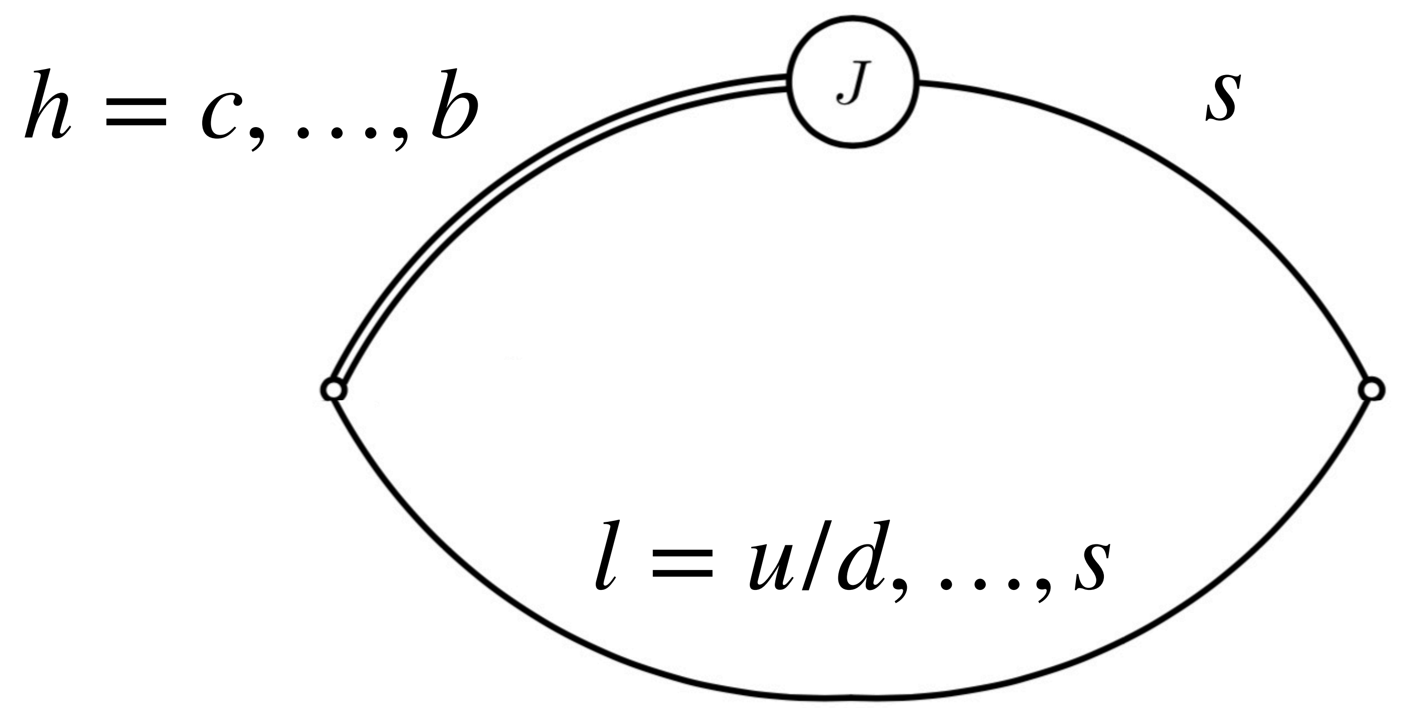


- FNAL/MILC heavy HISQ $B \to K$ calculation underway with more statistics on finer lattices
  - should further improve upon Statistics, HQET, and Discretization

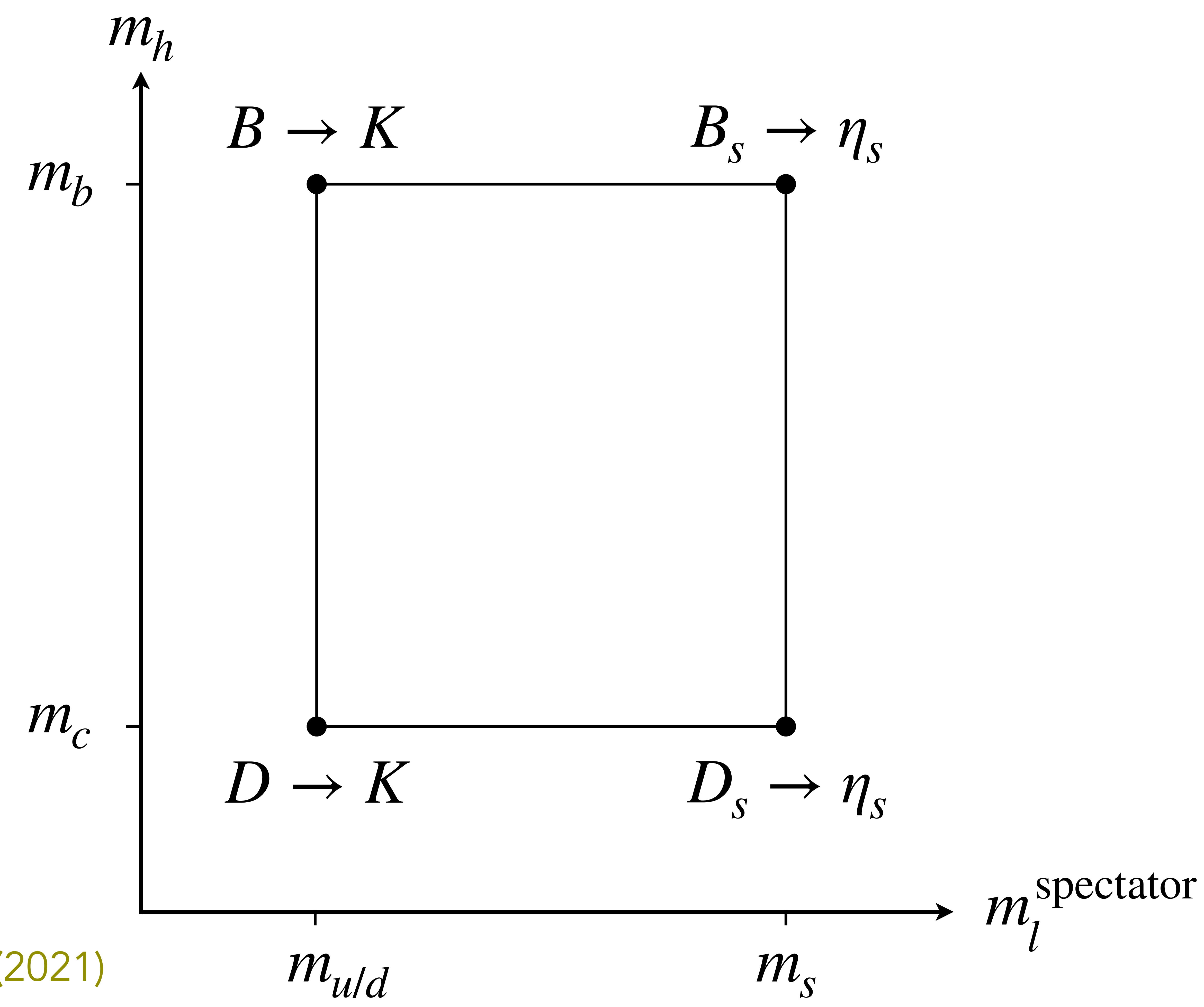
- heavy HISQ $B \to \pi$ will see similar improvement

Outlook

- variable initial state m_h and spectator m_l

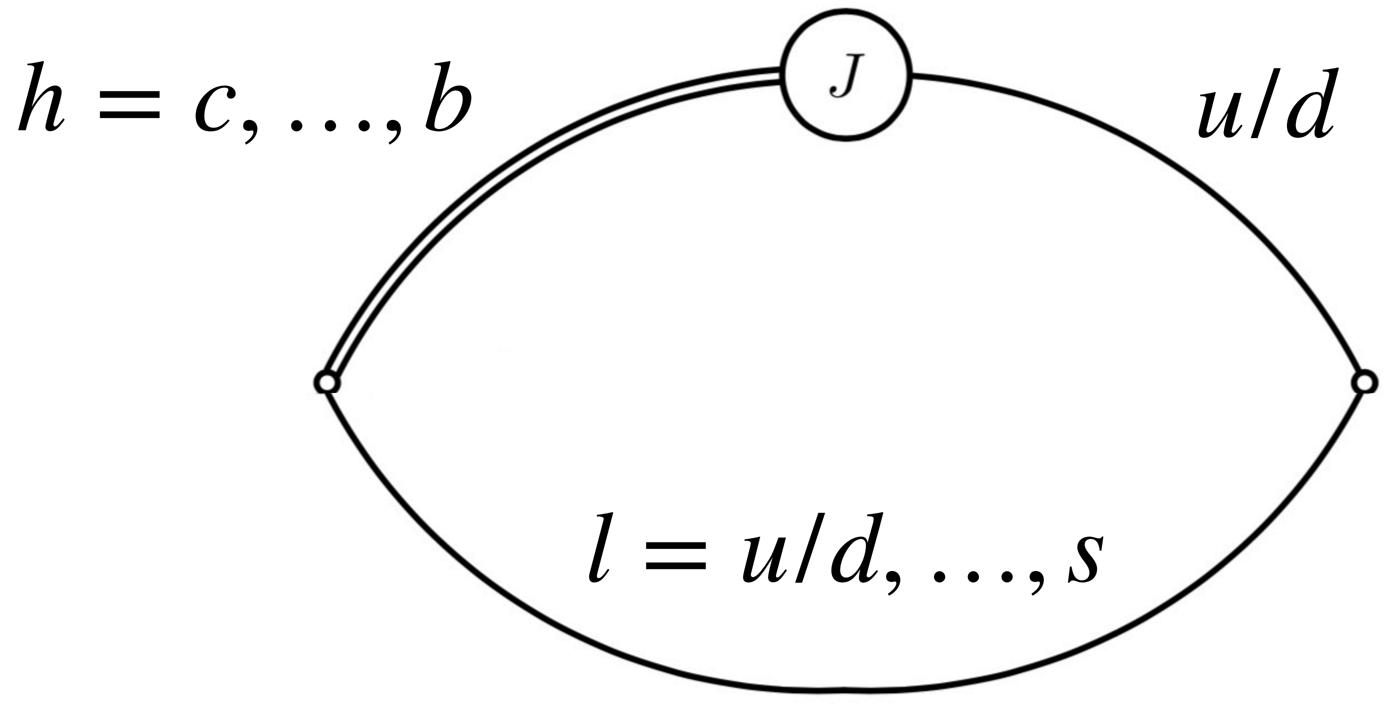


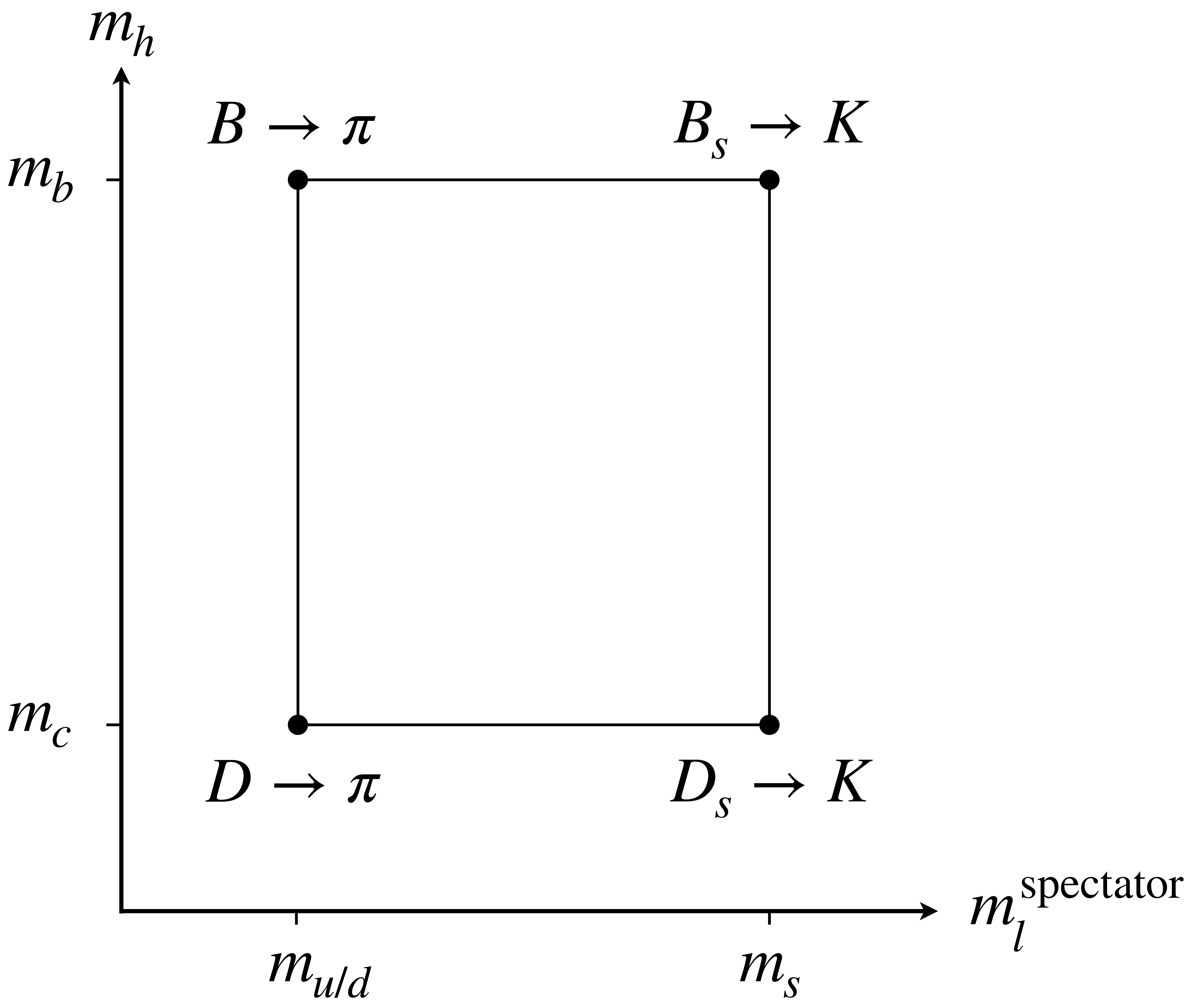
- one calculation gives multiple form factors
- we attacked in piecemeal fashion
  - $H_s \to \eta_s$ Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)
  - $D \to K$ Chakraborty, Parrott, Bouchard, Davies, Koponen, and Lepage, PRD 104 (2021) 034505
  - $B \to K$ Parrott, Bouchard, and Davies, 2207.12468 and 2207.13371



Outlook

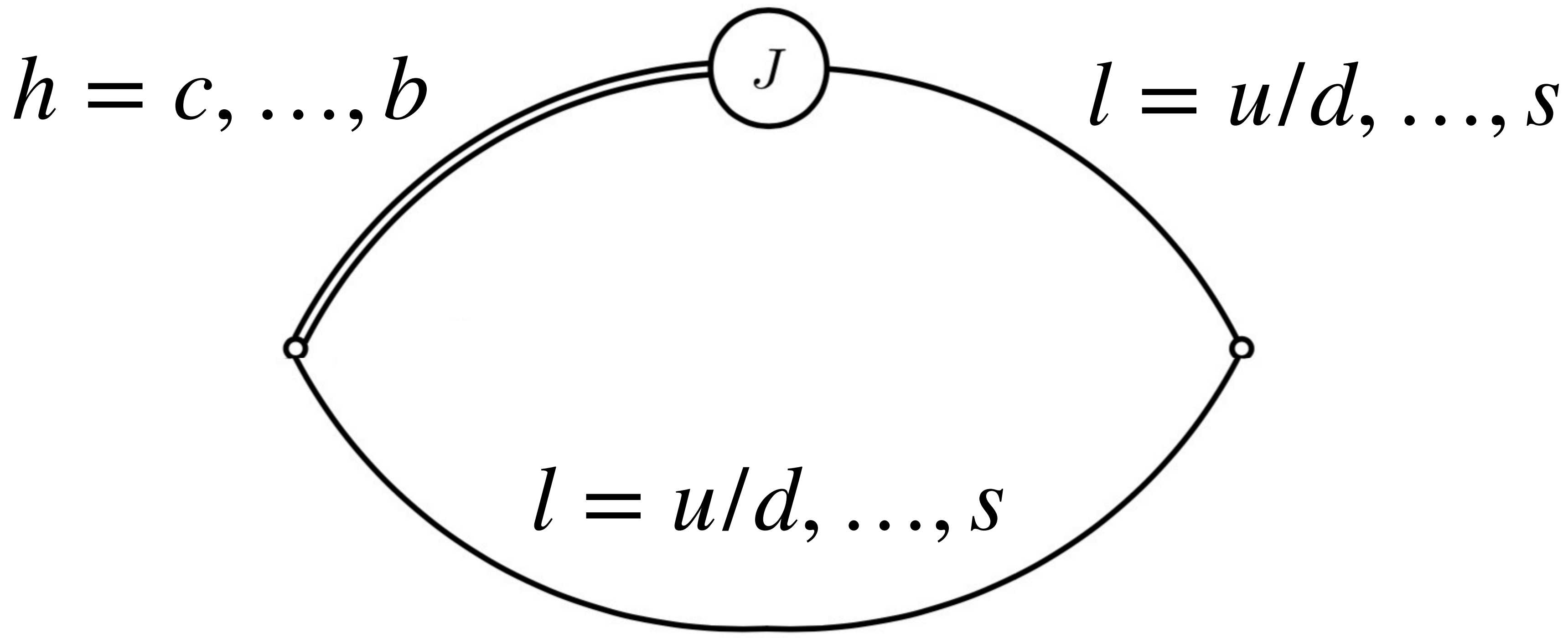
- changing final state quark to u/d



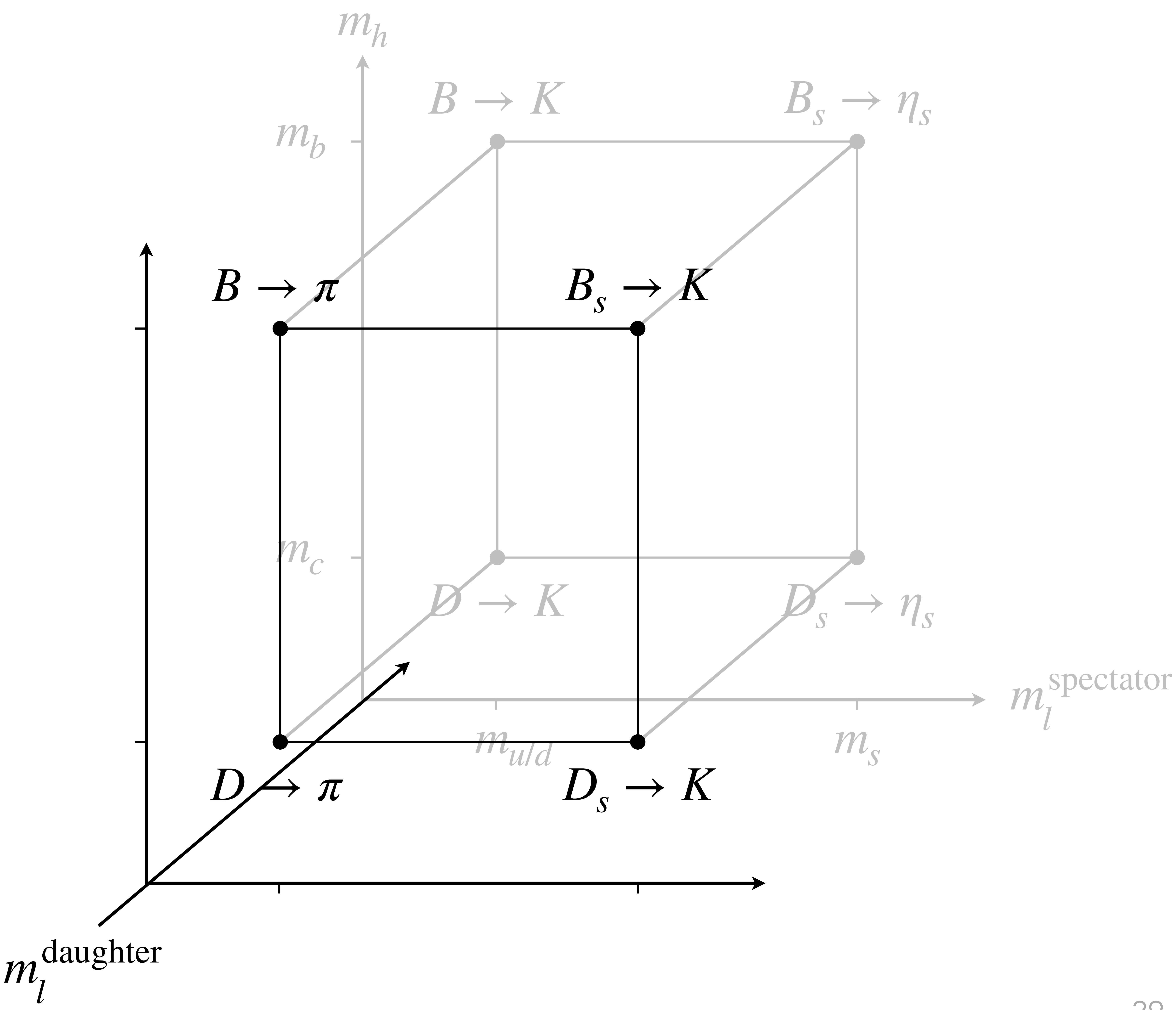


Outlook

- a lever arm for daughter u/d



- form factor calculations can inform one another, and permit correlated, combined phenomenology



Thank you

and thanks to collaborators:

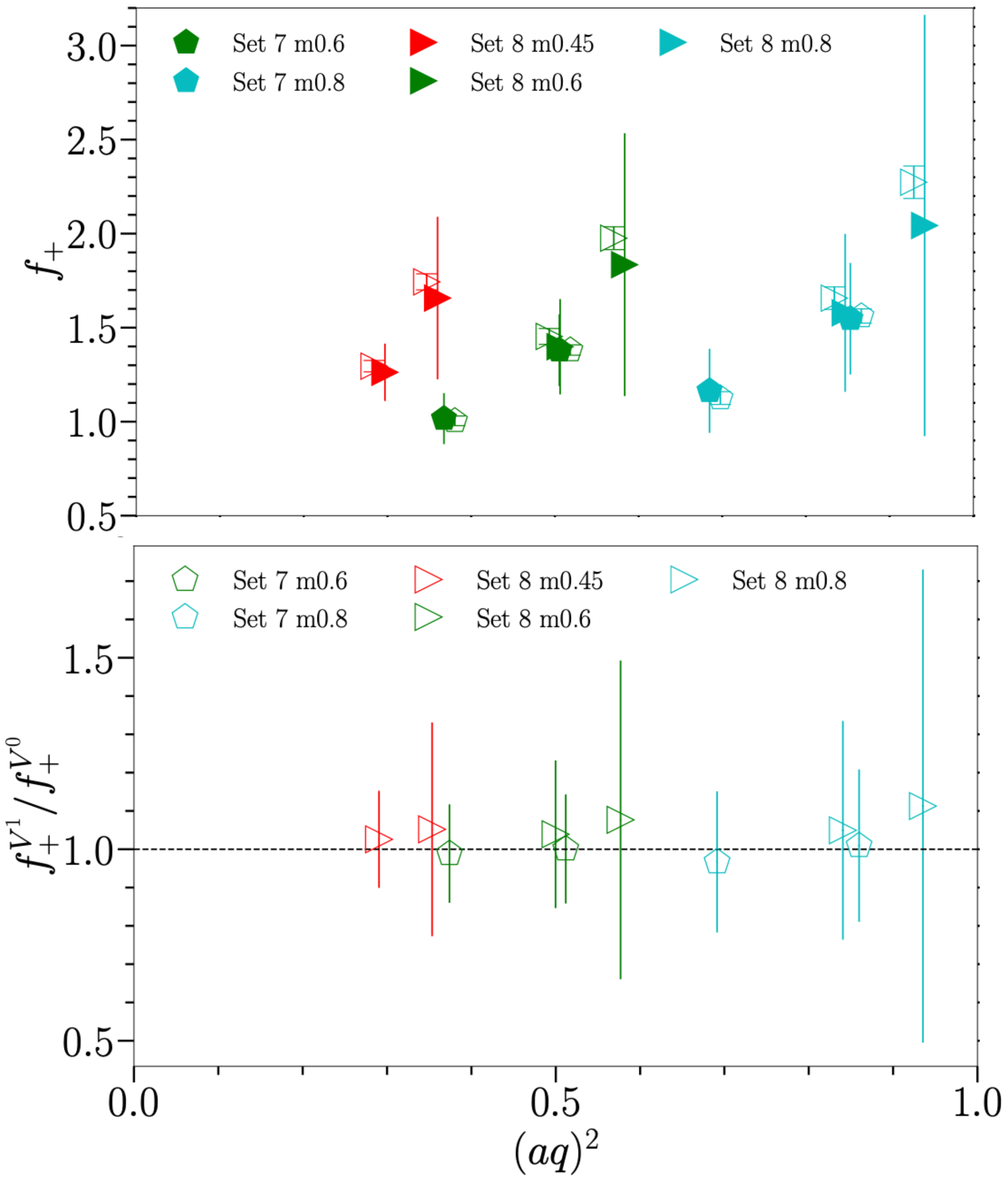
- Bipasha Chakraborty
- Christine Davies
- Dan Hatton
- Jonna Kopponen
- Peter Lepage
- Will Parrott

Form Factor calculation: ensembles

MILC $n_f = 2 + 1 + 1$ HISQ ensembles Bazavov et al., PRD 82, 074501 (2010); Bazavov et al., PRD 87, 054505 (2012)

$\approx a/\mathrm{fm}$	$N_s^3 \times N_t$	N_{cfg}	N_{src}	$am_l^{\mathrm{val,\,sea}}$	am_h
0.15	32^3 x 48	998	16	$0.00235 \approx am_l^{\mathrm{phys}}$	0.8605
0.15	16^3 x 48	1020	16	$0.013 \approx am_s/5$	0.888
0.12	48^3 x 64	985	16	$0.00184 \approx am_l^{\mathrm{phys}}$	0.643
0.12	24^3 x 64	1053	16	$0.0102 \approx am_s/5$	0.664, 0.8, 0.9
0.09	64^3 x 96	620	8	$0.0012 \approx am_l^{\mathrm{phys}}$	0.433, 0.683, 0.8
0.09	32^3 x 96	499	16	$0.0074 \approx am_s/5$	0.449, 0.566, 0.683, 0.8
0.06	48^3 x 144	413	8	$0.0048 \approx am_s/5$	0.274, 0.45, 0.6, 0.8
0.045	64^3 x 192	375	4	$0.00316 \approx am_s/5$	0.194, 0.45, 0.6, 0.8

Form Factor calculation: V^0 vs V^k



- filled symbols: V^0
- open symbols: V^k
- $f_+(q^2_{\max})$ from V^0 relies on a delicate cancelation

$$f_+(q^2) = \frac{Z_V\langle K|V^\mu|H\rangle - f_0 B^\mu}{p_H^\mu + p_K^\mu - B^\mu}, \quad B^\mu = \frac{M_H^2 - M_K^2}{q^2} q^\mu$$

- no evidence, within errors, of discretisation effects
- accommodate possibility in fit via

$$f_+^{V^1}(q^2) = \left(1 + \mathscr{C}^{a,m_h}(aq)^2\right) f_+^{V^0}(q^2)$$

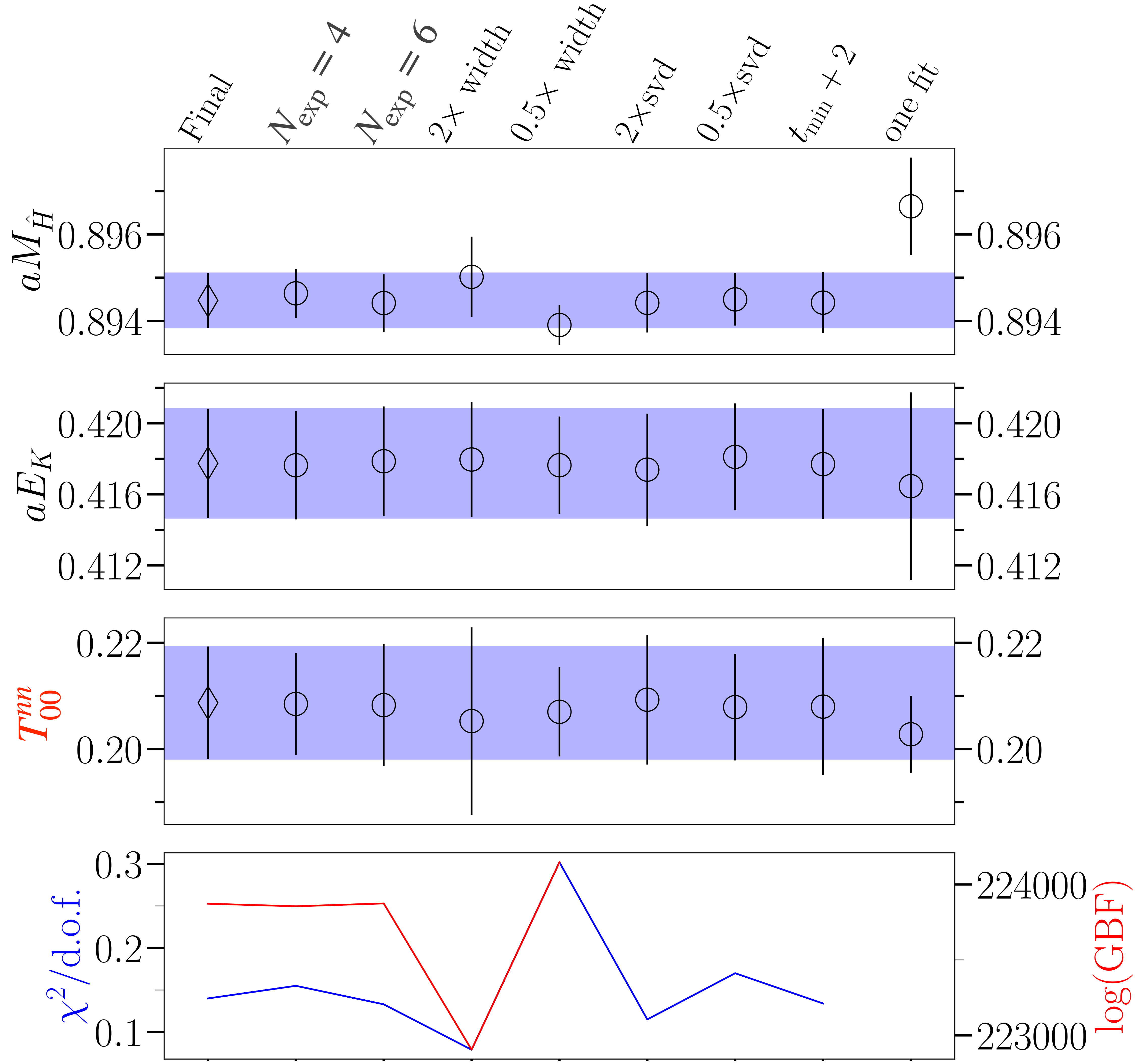
$$\text{prior}[\mathscr{C}] = 0.0(1)$$

Form Factors: matrix elements from correlators

- matrix element extracted from amplitudes of 3pt correlators (built with MILC code)
- simultaneous fit 2pt and 3pt correlators, e.g.
  Parrott, Bouchard, Davies, Hatton, PRD 103, 094506 (2021)
- fits use Lepage's gvar, lsqfit and corrfitter

$$C_2(t) = \sum_{i=0}^{N_{\rm exp}} \left[|d_i^n|^2 (e^{-E_i^n t} + e^{-E_i^n (N_t - t)}) - (-1)^t |d_i^o|^2 (e^{-E_i^o t} + e^{-E_i^o (N_t - t)}) \right]$$

$$C_3^J(t,T) = \sum_{i,j=0}^{N_{\rm exp}} \Big[d_i^{H,n} J_{ij}^{nn} d_j^{K,n} e^{-E_i^{H,n} t} e^{-E_j^{K,n}(T-t)}$$
$$+ (-1)^{(T-t)} d_i^{H,n} J_{ij}^{no} d_j^{K,o} e^{-E_i^{H,n} t} e^{-E_j^{K,o}(T-t)}$$
$$+ (-1)^t d_i^{H,o} J_{ij}^{on} d_j^{K,n} e^{-E_i^{H,o} t} e^{-E_j^{K,n}(T-t)}$$
$$+ (-1)^T d_i^{H,o} J_{ij}^{oo} d_j^{K,o} e^{-E_i^{H,o} t} e^{-E_j^{K,o}(T-t)} \Big]$$



representative fit stability, from $a \approx 0.045$ fm

Form Factors: modified z-expansion

- form factors at simulated a, $m_{\rm quarks}$, V and q^2
- extrapolate to $a \to 0$, $m_{\rm quarks} \to m_{\rm quarks}^{\rm phys}$ and $V \to \infty$ using modified z-expansion

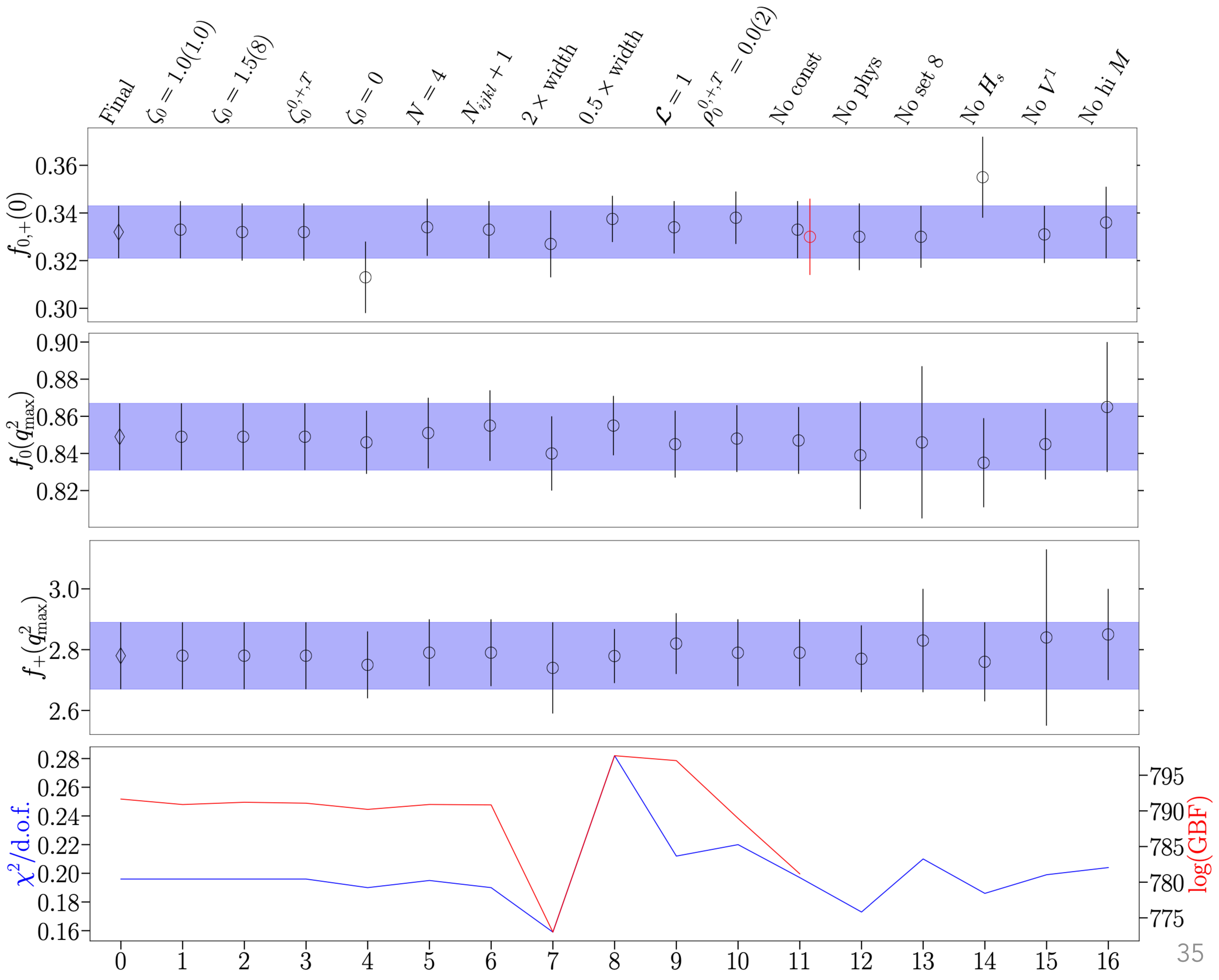
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}} \;;\quad t_+ = (M_H + M_K)^2 \;,\quad \text{we choose} \quad t_0 = 0$$

$$f_{+,T}(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^*}^2} \sum_{n=0}^{N-1} a_n^{+,T}\left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right) \;,\quad f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M_{H_s^0}^2} \sum_{n=0}^{N-1} a_n^0 z^n$$

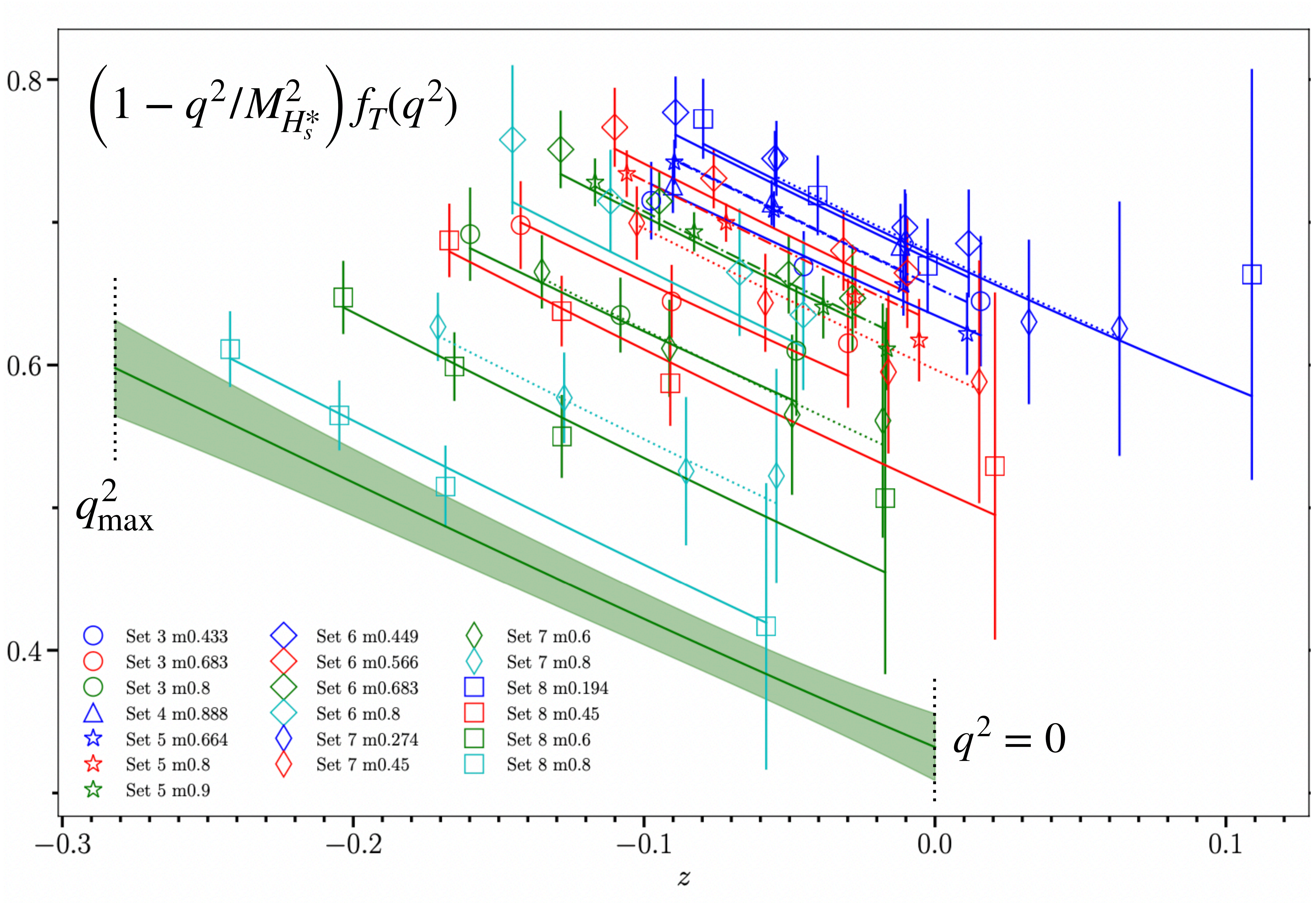
- $\mathscr{L}(V)$ are hard pion ChPT logs including (small) FV corrections Bijnens, Jemos, NPB 846, 145-166 (2011)
- a_n contains mistuning, heavy quark expansion, discretization, and analytic chiral terms

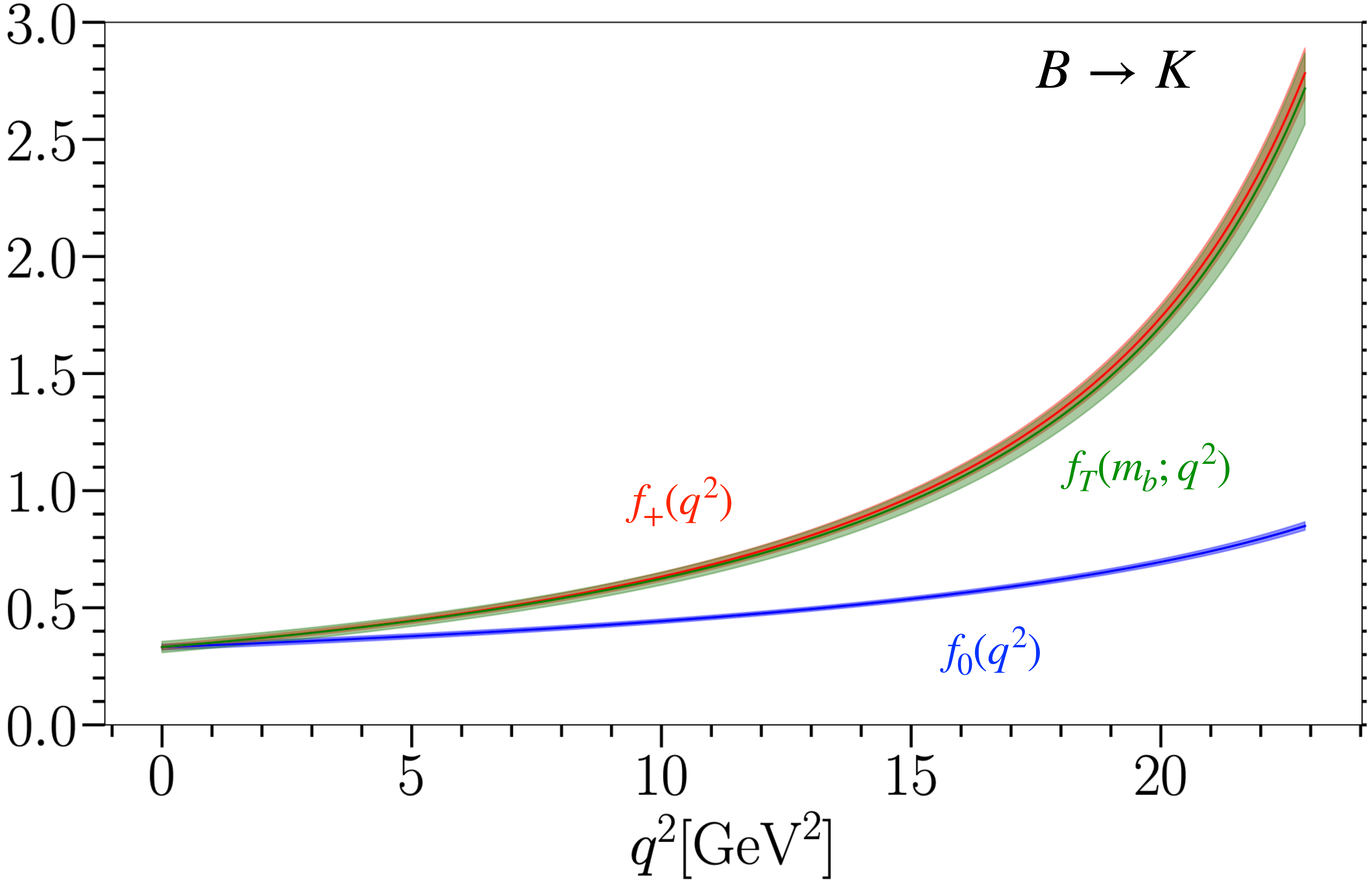
$$a_n^f = \left(1 + \mathscr{N}_n^f\right)\left(\frac{M_D}{M_H}\right)^{\zeta_n}\left(1 + \rho_n^f \log\left(\frac{M_H}{M_D}\right)\right) \sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln}^f \left(\frac{\Lambda}{M_H}\right)^i \left(\frac{am_h}{\pi}\right)^{2j} \left(\frac{a\Lambda}{\pi}\right)^{2k} \left(\frac{m_\pi^2 - (m_\pi^{\rm phys})^2}{(4\pi f_\pi)^2}\right)^l$$

Form Factors: modified z-expansion stability for $B \to K\ell\bar{\ell}$

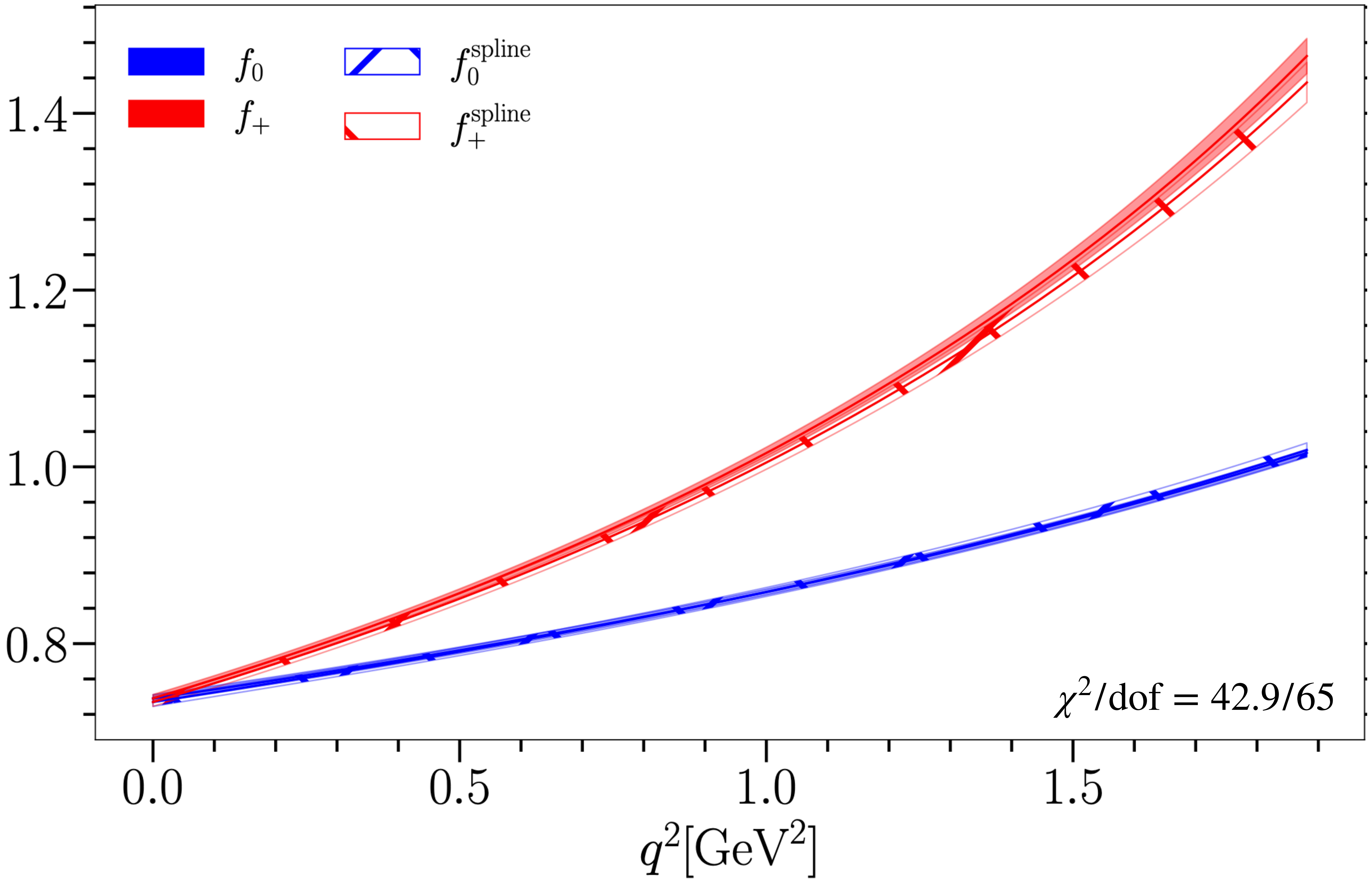


Form Factors: $B \to K\ell\bar{\ell}$ extrapolation results





Form Factors: $B \to K\ell\bar{\ell}$ test of modified z-expansion



- for $D \to K$, try cubic spline instead of modified z-expansion

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \sum_{n=0}^{N-1} a_n^0 z^n$$

$$\downarrow$$

$$f_0(q^2) = \frac{\mathscr{L}(V)}{1 - q^2/M^2_{H_s^0}} \left[\sum_{j=0}^{N} g_j(q^2) \left(\frac{am_c}{\pi} \right)^{2j} + \mathscr{N} \right]$$

- $g_j(q^2)$ are Steffen spline functions
- 4 knots {-3.25, -1.5, 0.25, 2.0} GeV^2

Form Factors: $B \to K\ell\bar{\ell}$ variation with m_h

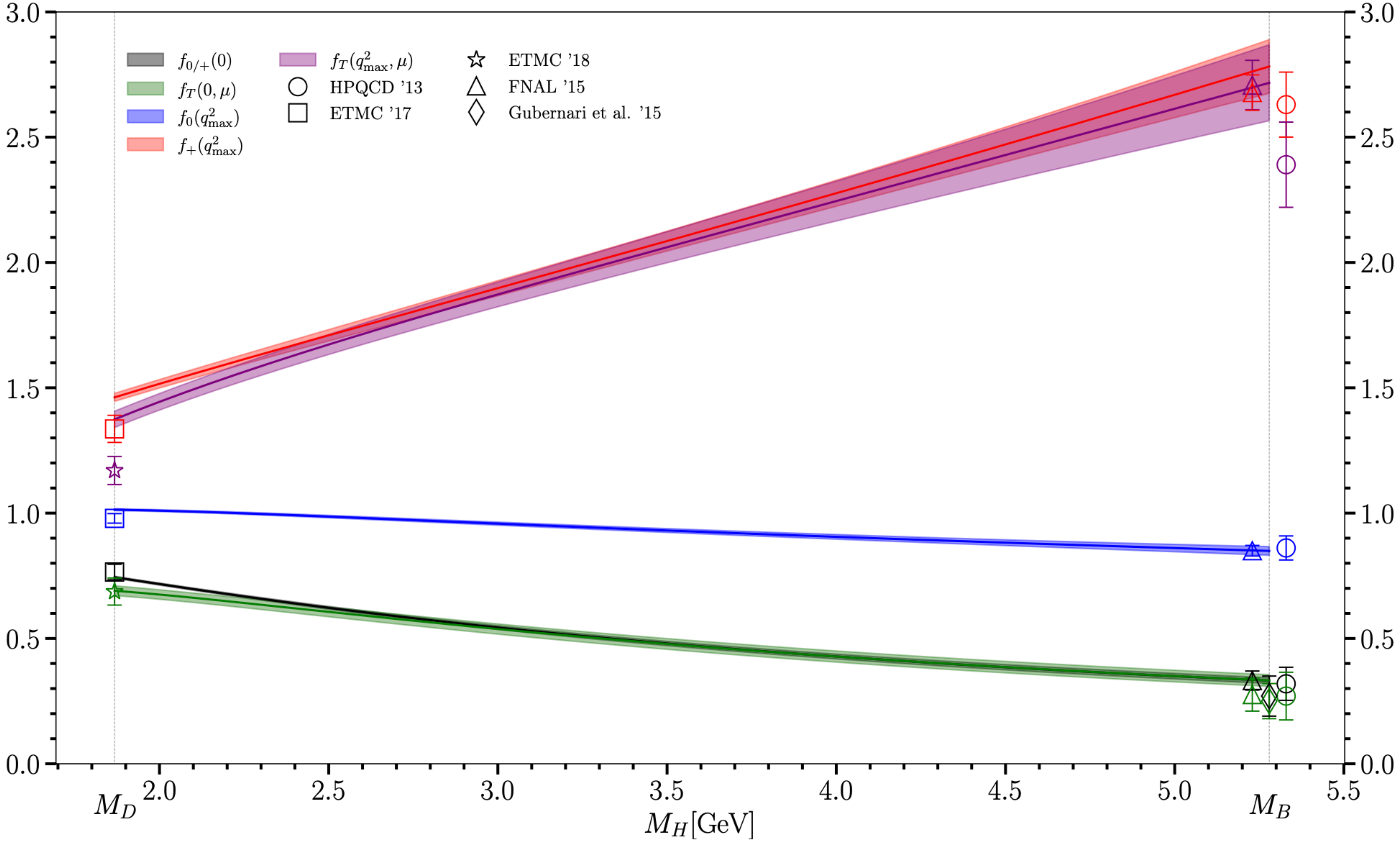
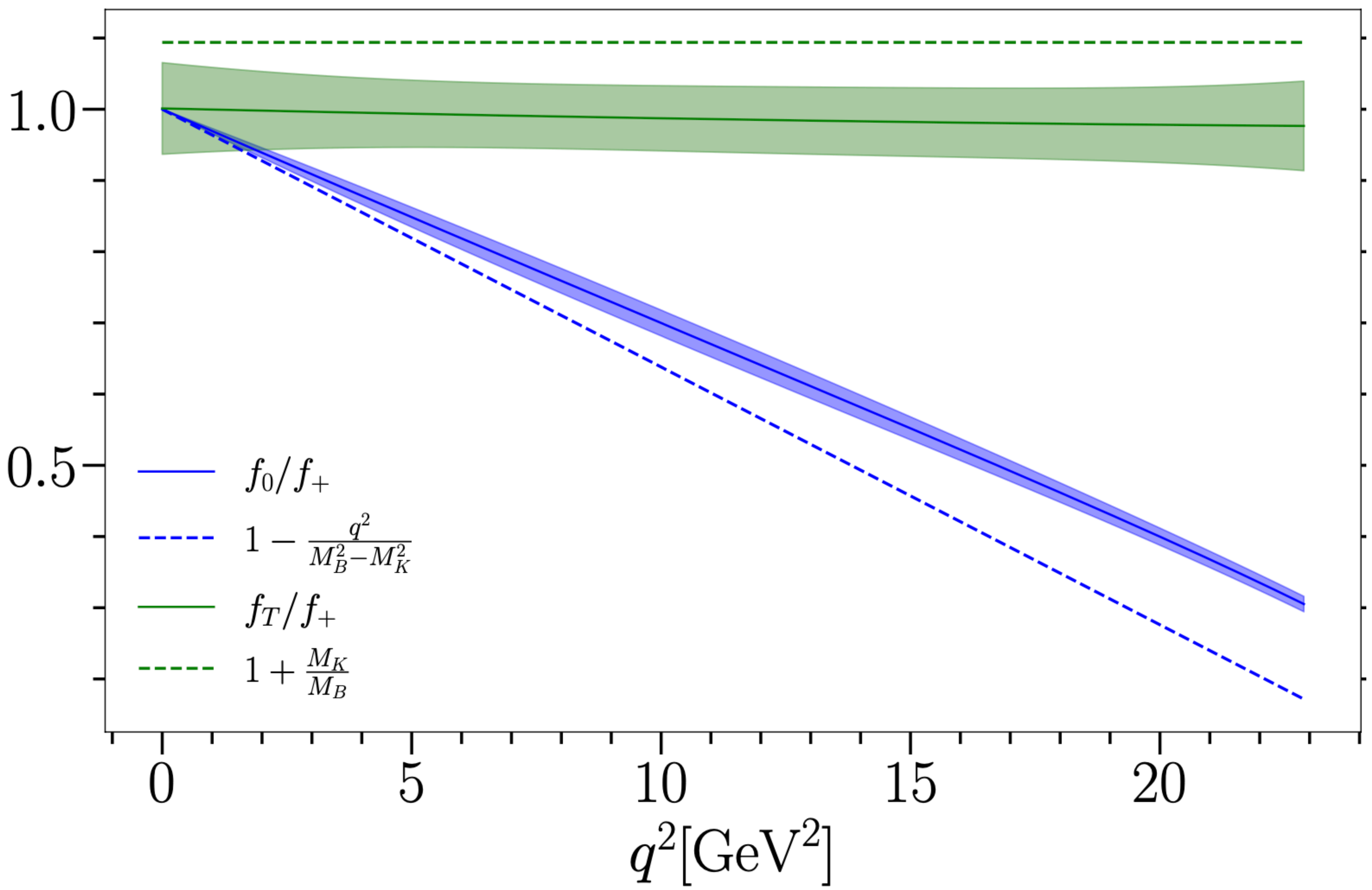


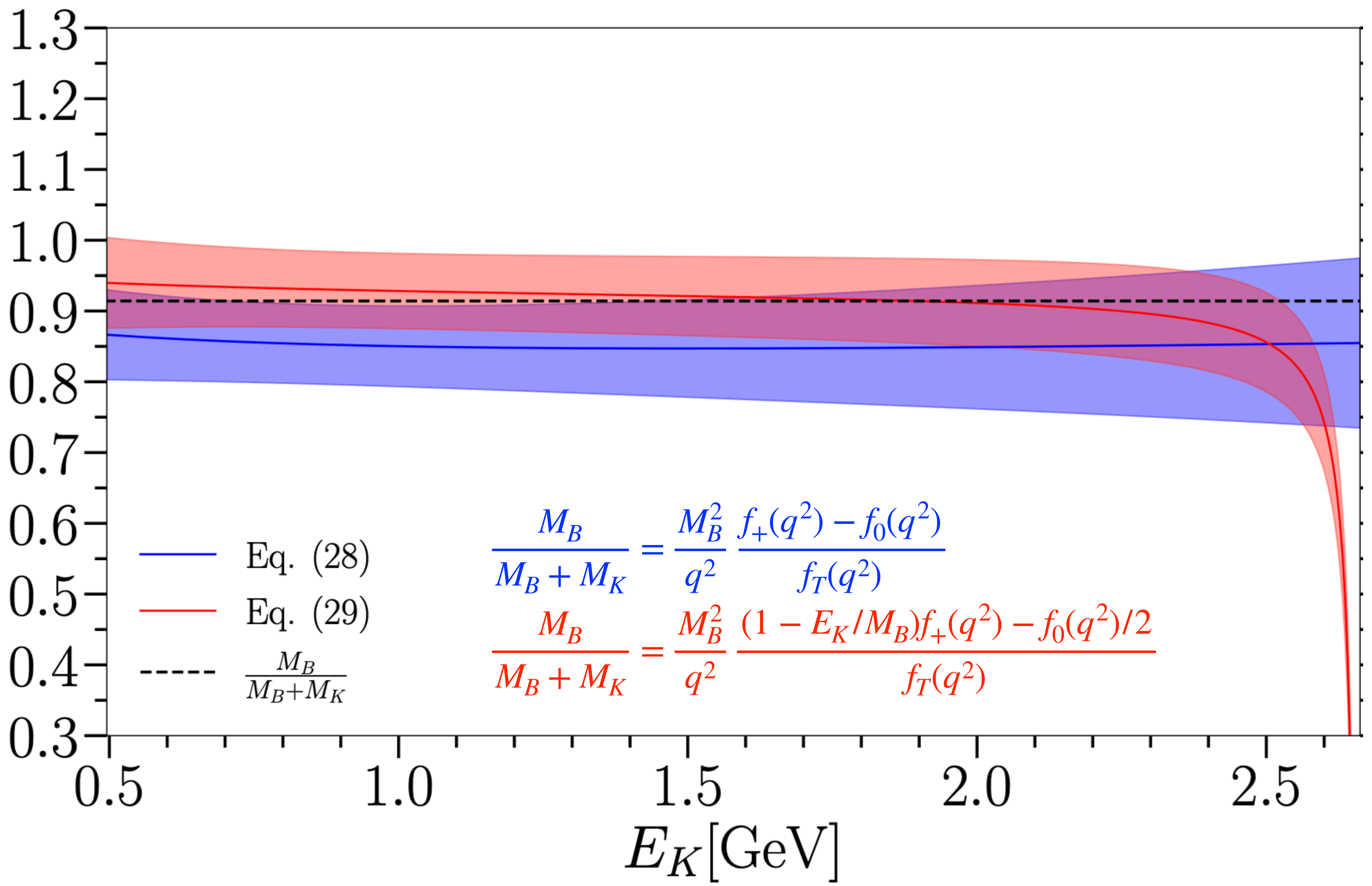
FIG. 10. The form factors at q^2_{max} and $q^2 = 0$ evaluated across the range of physical heavy masses from the D to the B. Other lattice studies [25, 28, 68, 69] of both $D \to K$ and $B \to K$ are shown for comparison. We also include some $B \to K$ results at $q^2 = 0$ from Gubernari et al. [70], a calculation using light cone sum rules. We do not include HPQCD's $D \to K$ results that share data with our calculation here [36]; see text for a discussion of that comparison. At the B end, data points are offset from M_B for clarity. Note that we have run Z_T to scale μ in this plot, where μ is defined linearly between 2 GeV and $m_b = 4.8$ GeV, according to Equation (26). The full running to 2 GeV from m_b results in a factor of 1.0773(17), applied to $f_T^{D\to K}$.

Form Factors: $B \to K\ell\bar{\ell}$ testing EFT expectation



Large Energy Effective Theory expectations

Charles, Le Yaouanc, Oliver, Pene, Raynal, PRD 60, 014001 (1999)



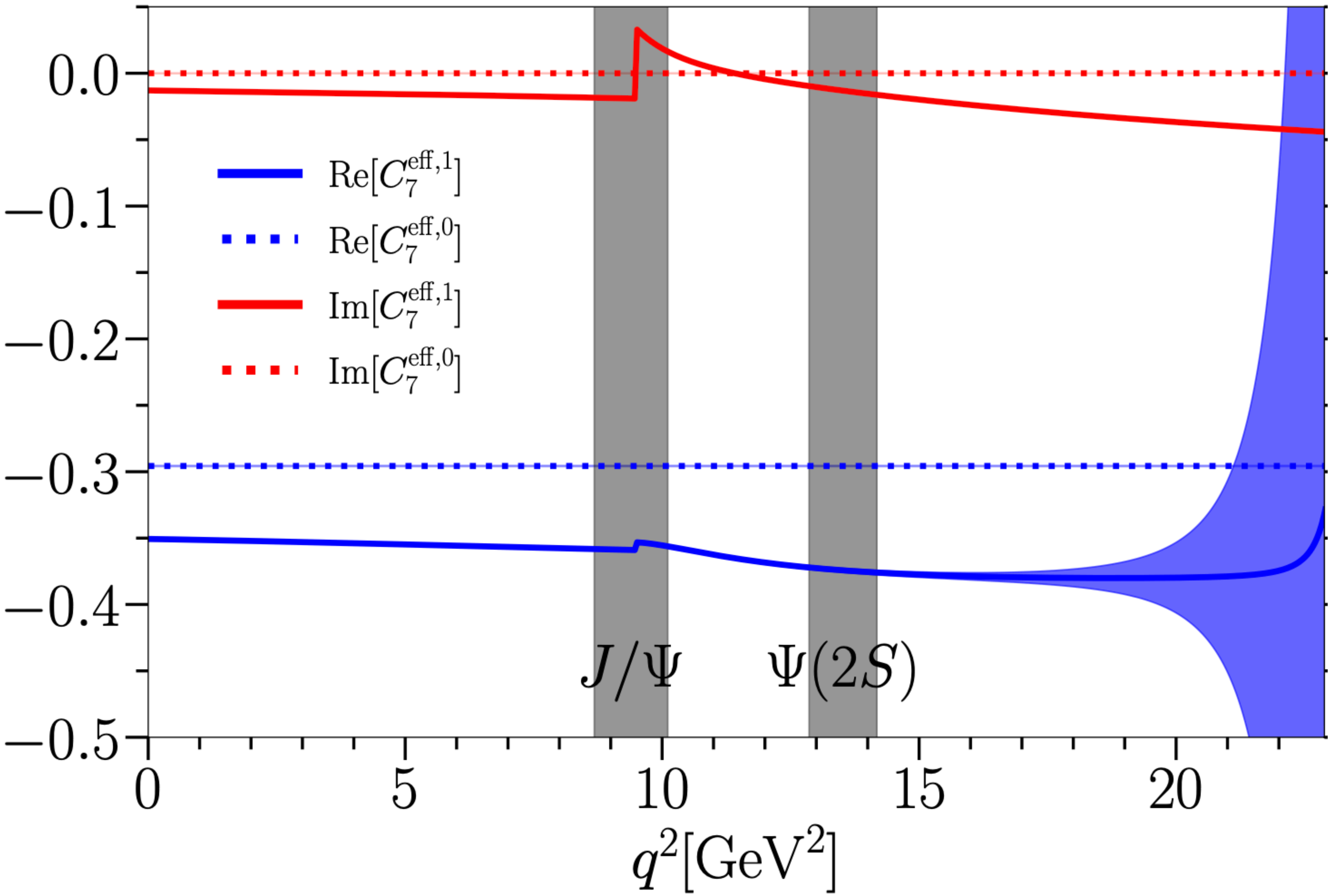
HQET expectations

Hill, PRD 73, 014012 (2006)

Phenomenology: $B \to K\ell\bar{\ell}$ inputs

Parameter	Value	Reference
G_F	$1.1663787(6) \times 10^{-5}$ GeV^{-2}	[43]
$m_c^{\overline{\mathrm{MS}}}(m_c^{\overline{\mathrm{MS}}})$	1.2719(78) GeV	See caption
$m_b^{\overline{\mathrm{MS}}}(\mu_b)$	4.209(21) GeV	[48]
m_c	1.68(20) GeV	-
m_b	4.87(20) GeV	-
f_{K+}	0.1557(3) GeV	[49–52]
f_{B+}	0.1894(14) GeV	[53]
τ_{B^0}	1.519(4) ps	[54]
$\tau_{B^\pm}$	1.638(4) ps	[54]
$1/\alpha_{\mathrm{EW}}(\mu_b)$	132.32(5)	-
$\lvert V_{tb}V_{ts}^*\rvert$	0.04185(93)	[55]
$C_1(\mu_b)$	-0.294(9)	[56]
$C_2(\mu_b)$	1.017(1)	[56]
$C_3(\mu_b)$	-0.0059(2)	[56]
$C_4(\mu_b)$	-0.087(1)	[56]
$C_5(\mu_b)$	0.0004	[56]
$C_6(\mu_b)$	0.0011(1)	[56]
$C_7^{\mathrm{eff},0}(\mu_b)$	-0.2957(5)	[56]
$C_8^{\mathrm{eff}}(\mu_b)$	-0.1630(6)	[56]
$C_9(\mu_b)$	4.114(14)	[56]
$C_9^{\mathrm{eff},0}(\mu_b)$	$C_9(\mu_b) + Y(q^2)$	-
$C_{10}(\mu_b)$	-4.193(33)	[56]

Phenomenology: $B \to K\ell\bar{\ell}$ corrections

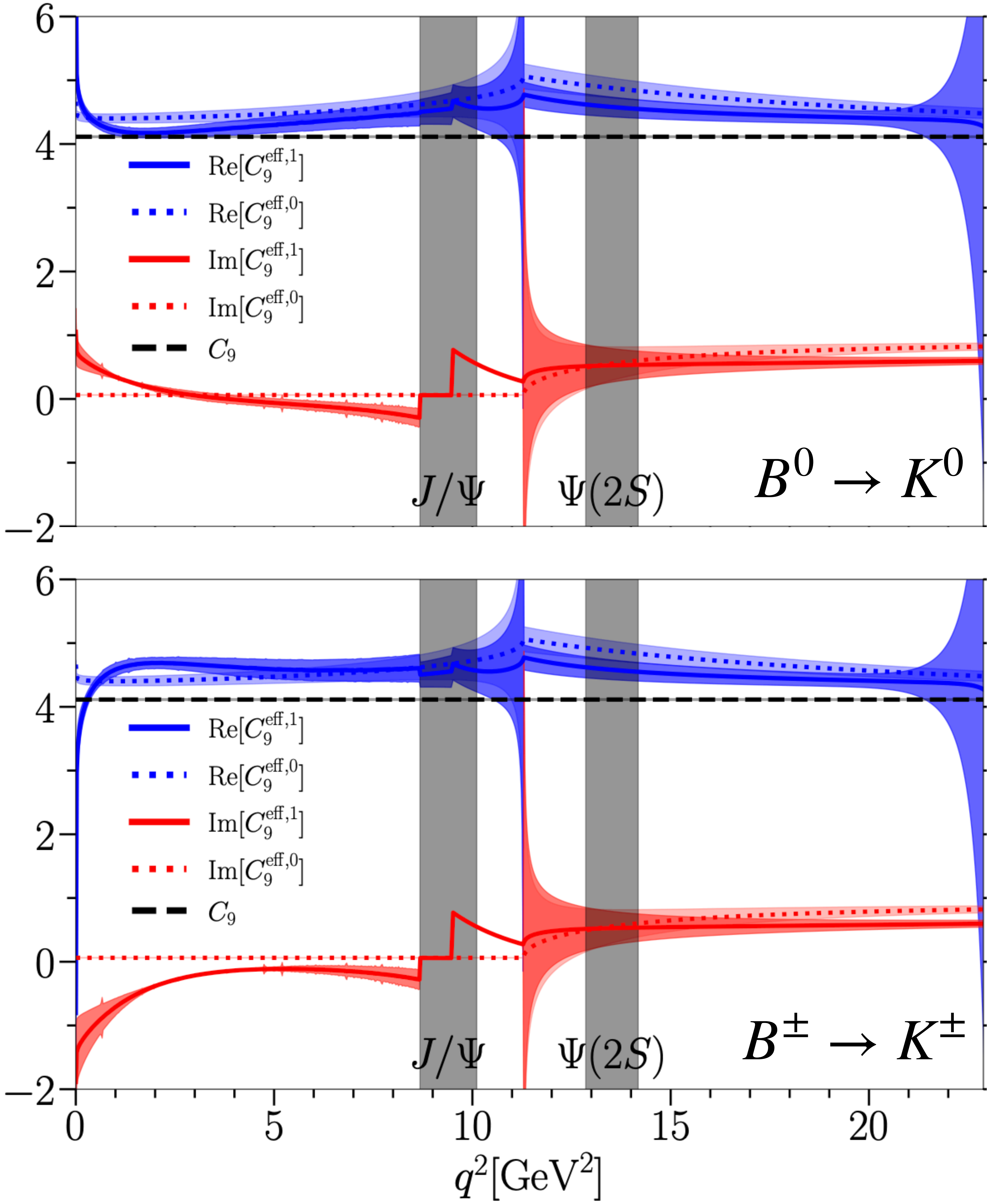


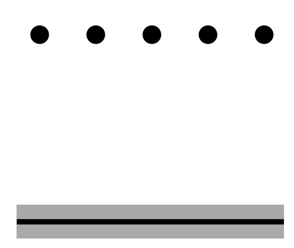
uncorrected C_7^{eff}

corrected C_7^{eff}

Corrections to C_7^{eff} are $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2

Phenomenology: $B \to K\ell\bar{\ell}$ corrections





uncorrected C_9^{eff}

corrected C_9^{eff}

corrections to C_9^{eff} include:

- $\mathcal{O}(\alpha_s)$ perturbative QCD effects for all q^2
- non-factorizable corrections at low q^2

Beneke, Feldmann, Seidel, NPB 612, 25-58 (2001)

- would be interesting to compare non-factorizable corrections to results of data driven determination

talk by Andrea Mauri (LHCb), Wed@11:30