Updated predictions for $R(D^{(*)})$ using the residual chiral expansion

- BLPRXP Form Factors

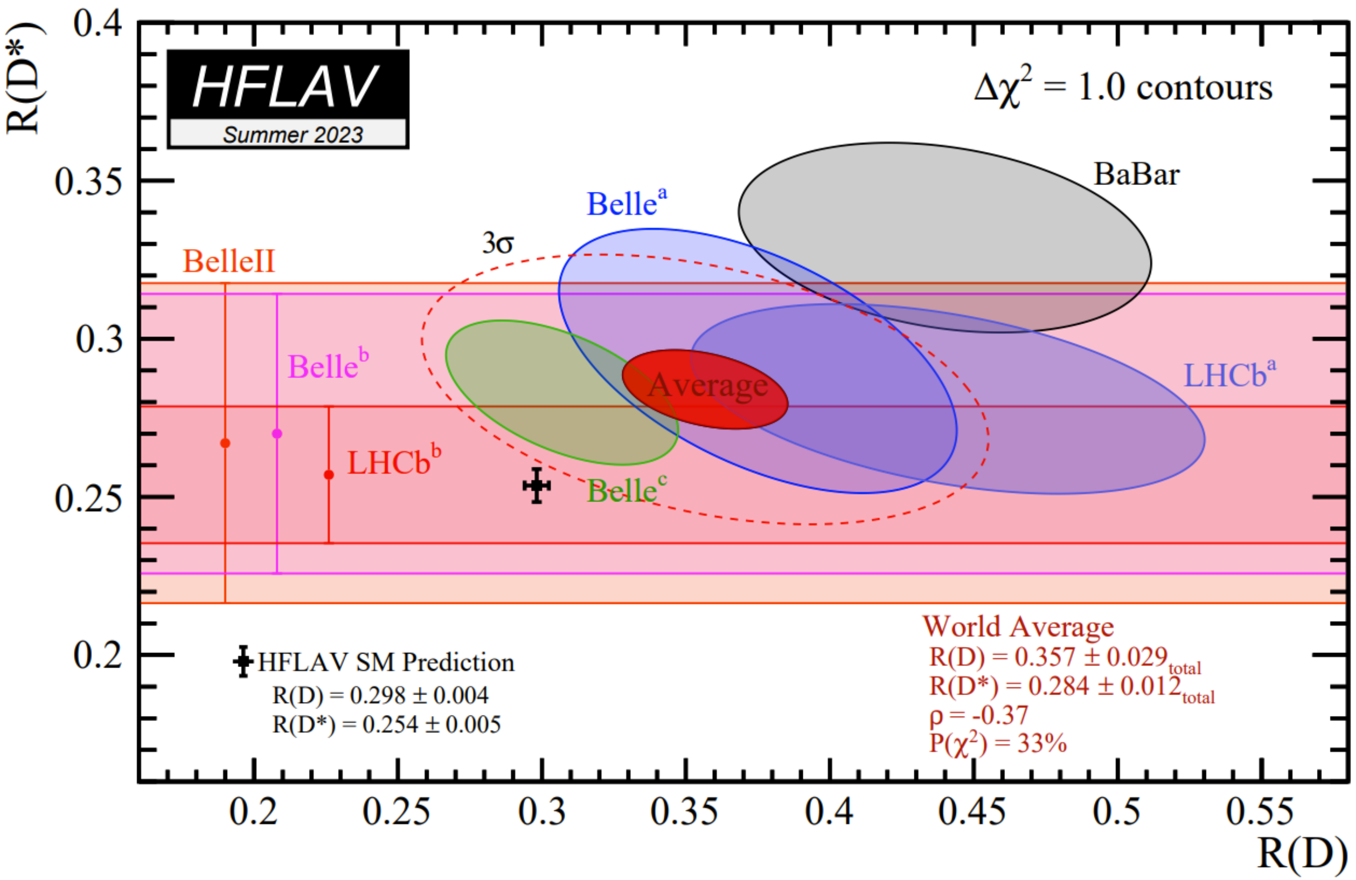
Dean Robinson, Florian Bernlochner, Markus Prim,
Michele Papucci, Zoltan Ligeti

markus.prim@uni-bonn.de

Bundesministerium
für Bildung
und Forschung

UNIVERSITÄT BONN

Experimental Status of $R\left(D^{(*)}\right)$

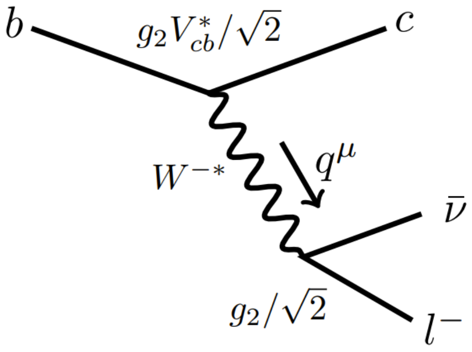


The difference with the SM prediction corresponds to 3.3σ (HFLAV '23)

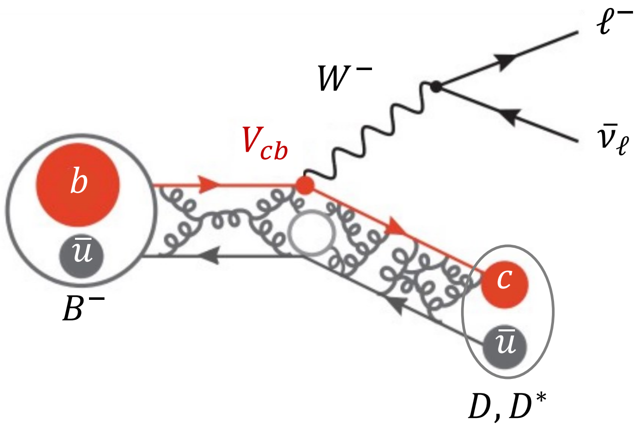
$R(D)$ exceeds SM by 2.0σ

$R(D^*)$ exceeds SM by 2.2σ

The $b \to c\ell\bar{\nu}_\ell$ Laboratory



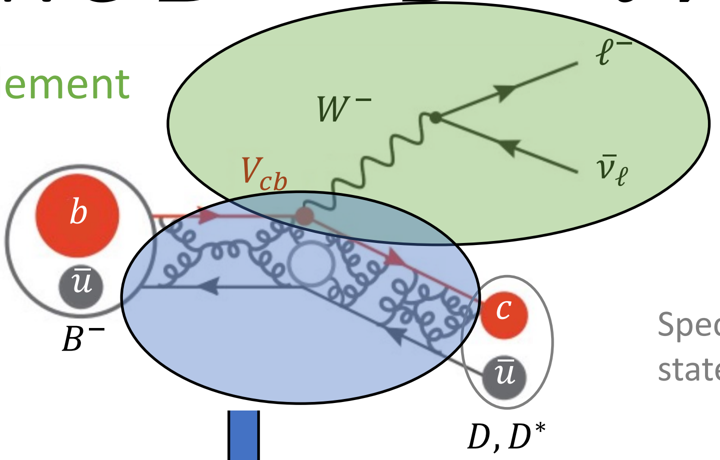
Main Theory Challenge



- Tree level process: large branching fraction (10%), theoretically relatively clean
- Universal lepton gauge coupling in the SM
- Experimentally only access to bound states $B \to D^{(*)}, D^{**}, \dots, \Lambda_b \to \Lambda_c^*, \dots$
- Precision description of the hadronic matrix element required for precision measurements

Exclusive $B \to D^{(*)}\ell\bar{\nu}_\ell$

Leptonic Matrix Element



$$\Gamma(B \to D\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{G}(1) \qquad \mathcal{G}(1) = h_+(1)$$

$$\Gamma(B \to D^*\ell\bar{\nu}_\ell) \propto |V_{cb}|^2 \mathcal{F}(1) \qquad \mathcal{F}(1) = h_{A_1}(1)$$

Specific final
state meson

Hadronic Matrix Elements can not be calculated from first principles
→ Can be parameterized with form factors $h_X = h_X(w)$ and extracted from data
→ Lattice QCD must provide (at least) inputs on their normalization

$$\frac{\langle D(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_D}} = h_+(v+v')^{\mu} + h_-(v-v')^\mu$$

$$\frac{\langle D^*(p')|\bar{c}\gamma^\mu b|B(p)\rangle}{\sqrt{m_B m_{D^*}}} = h_V \epsilon^{\mu\nu\alpha\beta}\epsilon^*_\nu v'_\alpha v_\beta$$

$$\frac{\left\langle D^*(p')\middle|\bar{c}\gamma^\mu\gamma^5 b\middle|B(p)\right\rangle}{\sqrt{m_B m_{D^*}}} = h_{A_1}(w+1)\epsilon^{*\mu} - h_{A_2}(\epsilon^* \cdot v)v^{\mu} - h_{A_3}(\epsilon^* \cdot v)v'^{\mu}$$

Heavy Quark Symmetry Basis

One Slide Intro to HQET

- Mass-subtracted field redefinition of quarks $Q^v_\pm(x) = \Pi_\pm e^{im_Q v\cdot x} Q(x)$
- Rewrite QCD as

light massless

double heavy

$$\mathcal{L}_{QCD} = \bar{Q}^v_+ iv\cdot D Q^v_+ + \bar{Q}^v_+ i\not{D}_\perp Q^v_- + \bar{Q}^v_- i\not{D}_\perp Q^v_+ - \bar{Q}^v_- (iv\cdot D + 2m_Q) Q^v_-$$

NB: $D^\mu_\perp = D^\mu - v\cdot D v^\mu$

- Integrate out the double heavy fields to generate HQET
  Power expansion in $\sim iv\cdot\frac{D}{2m_Q} \sim \frac{\Lambda_{QCD}}{2m_Q}$
  - Lagrangian corrections: $\mathcal{L}_{HQET} = \bar{Q}^v_+ iv\cdot D Q^v_+ + \sum_{n=1}\mathcal{L}_n/\left(2m_Q\right)^n$
  - Current corrections: $\mathcal{J}_{HQET} = 1 + \sum_{n=1}\mathcal{J}_n/\left(2m_Q\right)^n$
  - Perturbative $\mathcal{O}(\alpha_s)$ radiative corrections are fully calculable
- Obtain EFT of 'light muck' in definite s^P state around a HQ static color source
  - Hadrons embed into HQ supermultiplets, e.g., $s^P = \frac{1}{2}^- \Rightarrow J^P = \frac{1}{2}\times\frac{1}{2}^- = 0^- \oplus 1^-$: The D and D^*
  - HQET relates $B \to D$ and $B \to D^*$ form factors
- Match QCD matrix element onto HQET matrix elements, each represented by Isgur-Wise functions

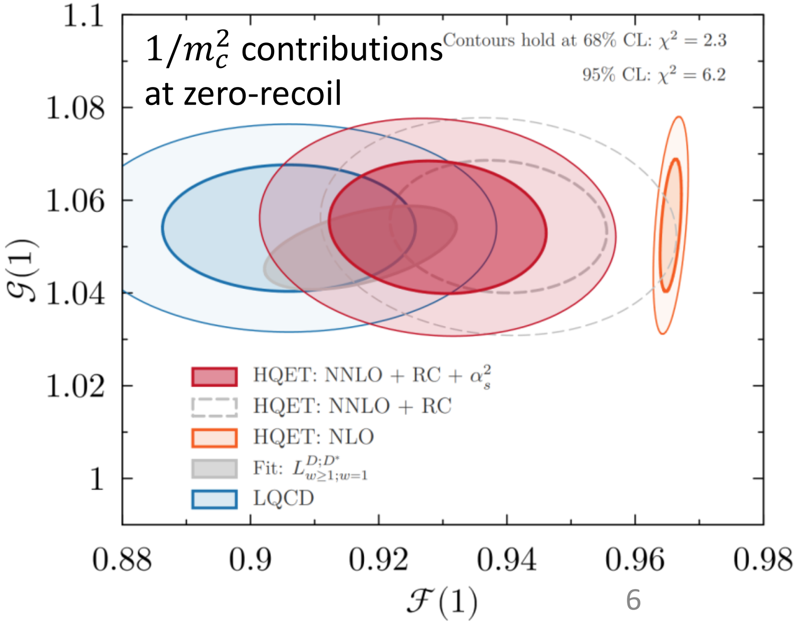
BLPRXP Form Factors for $B \to D^{(*)}\ell\bar{\nu}_\ell$

Expansion to order $\mathcal{O}\left(1/m_{b,c}^{(2)}\right), \mathcal{O}(1/(m_b m_c))$

$$\frac{\langle H_c|\bar{c}\Gamma b|H_b\rangle}{\sqrt{m_{H_c}m_{H_b}}} \propto 1 + \underbrace{\frac{1}{2m_c} + \frac{1}{2m_b}}_{+3} + \overbrace{\underbrace{\frac{1}{4m_c^2}}_{+20} + \frac{1}{2m_b^2} + \frac{1}{4m_c m_b}}^{+32}$$

Proliferation of non-perturbative parameters

- At NLO HQET requires 4 IW functions → Predictive HQE constraints
- At NNLO there are 32 additional IW functions → Loss of predictivity
  - Largest NNLO correction: $\sim \frac{\Lambda_{QCD}^2}{4m_c^2}$ is larger than current exp. precision
  - Also, $\frac{\alpha_s}{\pi} \times \frac{\Lambda_{QCD}}{2m_c} \sim 2\%$ and $\frac{\alpha_s}{\pi} \times \frac{\Lambda_{QCD}^2}{4m_c m_b} \sim 0.8\%$ needed in the future



$1/m_c^2$ contributions at zero-recoil

Chiral Structure and Residual Chiral Expansion

Back to mass-subtracted QCD

$$\mathcal{L}_{QCD} = \underset{\text{light massless}}{\bar{Q}_+^v iv \cdot D Q_+^v} + \boxed{\bar{Q}_+^v i\not{D}_\perp Q_-^v + \bar{Q}_-^v i\not{D}_\perp Q_+^v} - \underset{\text{double heavy}}{\bar{Q}_-^v (iv \cdot D + 2m_Q) Q_-^v}$$

- Kinetic terms have accidental $U(1) \times U(1)$ chiral symmetry broken to $U(1)$ by $\not{D}_\perp$ terms

  They also break HQ spin symmetry
- HQET corrections
  - Each Lagrangian correction $\mathcal{L}_n$ generated by $\bar{Q}_+^v i\not{D}_\perp Q_-^v + \bar{Q}_-^v i\not{D}_\perp Q_+^v$: two $\not{D}_\perp$ insertions
  - Each current correction $\mathcal{J}_n$ generated by one insertion of $\not{D}_\perp$ insertion

Key Idea: Counting $\not{D}_\perp$ insertions provides and additional classification of terms vs $1/m_Q$ power expansion.
Deform QCD by including a $\not{D}_\perp$ power-counting parameter θ

RCE conjecture: matrix elements involving (many) $\not{D}_\perp$ OP insertions are typically small

Truncate at $\mathcal{O}(\theta^2)$

⇒ Captures all NLO + NNLO with zero OP insertions

BLPRXP Form Factors for $B \to D^{(*)}\ell\bar{\nu}_\ell$

Expansion to order $\mathcal{O}\left(1/m_{b,c}^{(2)}\right)$, $\mathcal{O}(1/(m_b m_c))$

$\varphi_1(w), \beta_2(w), \beta_3(w)$

$$\frac{\langle H_c|\bar{c}\Gamma b|H_b\rangle}{\sqrt{m_{H_c} m_{H_b}}} \propto 1 + \underbrace{\frac{1}{2m_c} + \frac{1}{2m_b}}_{\substack{+3 \\ \eta(w),\chi_2(w),\chi_3(w)}} + \overbrace{\underbrace{\frac{1}{4m_c^2}}_{\substack{+20 \to +1 \\ \varphi_1(w)}} + \frac{1}{2m_b^2} + \frac{1}{4m_c m_b}}^{+32 \to +3}$$

Number of non-perturbative parameters under control

Supplemental power counting in the transverse residual momentum $\not{p}_\perp$
→ Drastic reduction of the non-perturbative parameters

Parameterization of the IW functions

- Parameterization required to fit the experimental/lattice data
- Leading order IW function expressed wrt. to the conformal map $w \to z_*$

$$\frac{\xi(w)}{\xi(w_0)} = 1 - 8a^2\rho_*^2 z_* + 16(2c_* a^4 - \rho_*^2 a^2)z_*^2 + \cdots$$

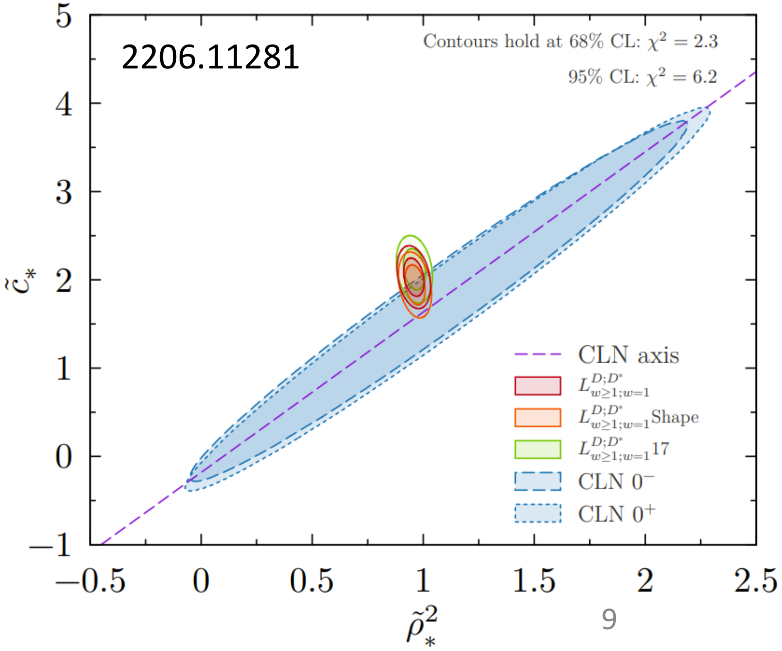
Slope at w_0 Curvature at w_0 Conformal parameter $z_*(w), z_*(w_0) = 0$

No CLN-type major-axis approximation

- (Sub-)Subleading IW functions

$$X(w) = X(1) + X'(1)(w-1) + \cdots$$

- This yields to many free parameters ⇒ overfitting/biases
- How to truncate? ⇒ Nested hypothesis test (NHT)



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Nested Hypothesis Test

- The free parameters in our model are
  - entering at zero-recoil: $|V_{cb}|, m_b^{1S}, \delta m_{bc}, \rho_1, \lambda_2, \rho_*^2, c_*, \hat{\eta}(1)$
  - and beyond: $\hat{\eta}'(1)$, $\hat{\chi}_2(1)$, $\hat{\chi}_2'(1)$, $\hat{\chi}_3'(1)$, $\hat{\varphi}_1'(1)$, $\hat{\beta}_2(1)$, $\hat{\beta}_3'(1)$
- Nested Hypothesis Test to determine optimal set of fit parameters
  - Starting point are the parameters contribution at zero-recoil
  - Subsequently add parameters to the model in all combinations
  - Test alternative fit hypothesis with cut-off $\Delta\chi^2 = \chi_N^2 - \chi_{N+1}^2 < 1$
  - Reject combinations with highly correlated parameters

$B \to D^{(*)}\ell\bar{\nu}_{\ell}$ - Updated Results

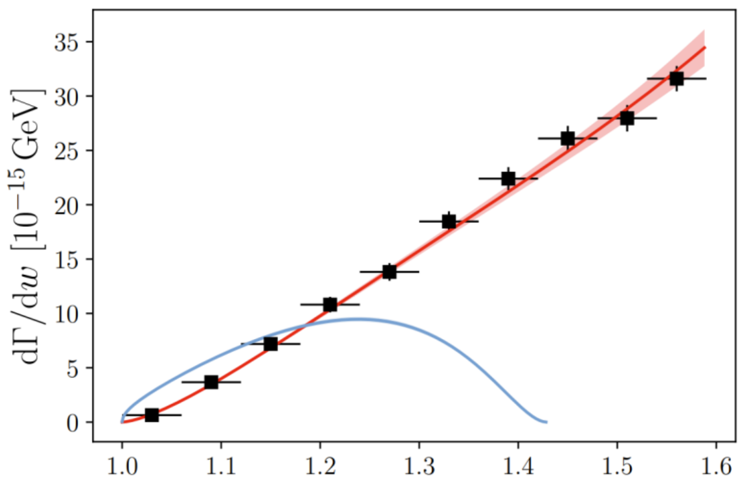
With new experimental and lattice QCD inputs

Experimental Inputs

- A) Belle $B \to D\ell\bar{\nu}_\ell$ tagged ’15 → Only use shape and BR world average
- B) Belle $B \to D^*\ell\bar{\nu}_\ell$ untagged ’19
- C) Belle $B \to D^*\ell\bar{\nu}_\ell$ tagged ’23 → Updated measurement wrt ‘17
- Today: Only use measured hadronic recoil spectra

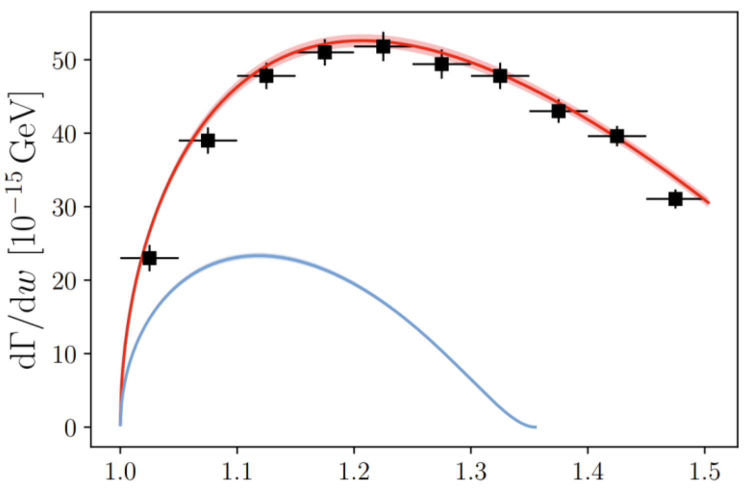
BLPRXP ’23 Fit $B \to D^{(*)}\ell\bar{\nu}_\ell$
BLPRXP ’23 $B \to D^{(*)}\tau\bar{\nu}_\tau$

A) $B \to D\ell\bar{\nu}_\ell$



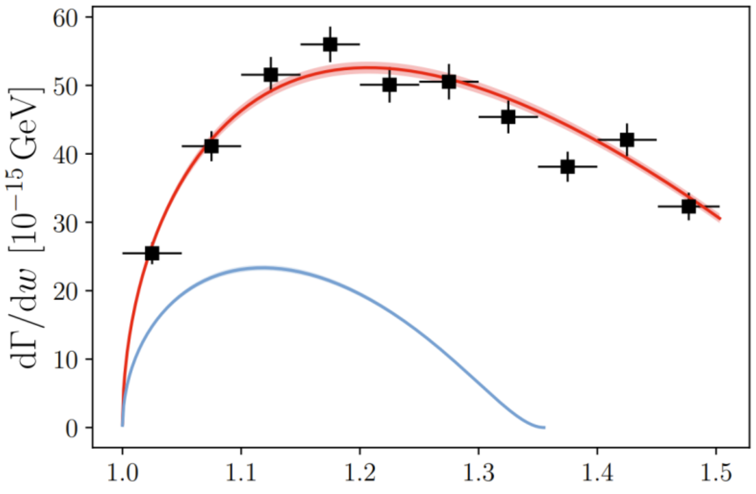
$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

B) $B \to D^*\ell\bar{\nu}_\ell$



$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

C) $B \to D^*\ell\bar{\nu}_\ell$



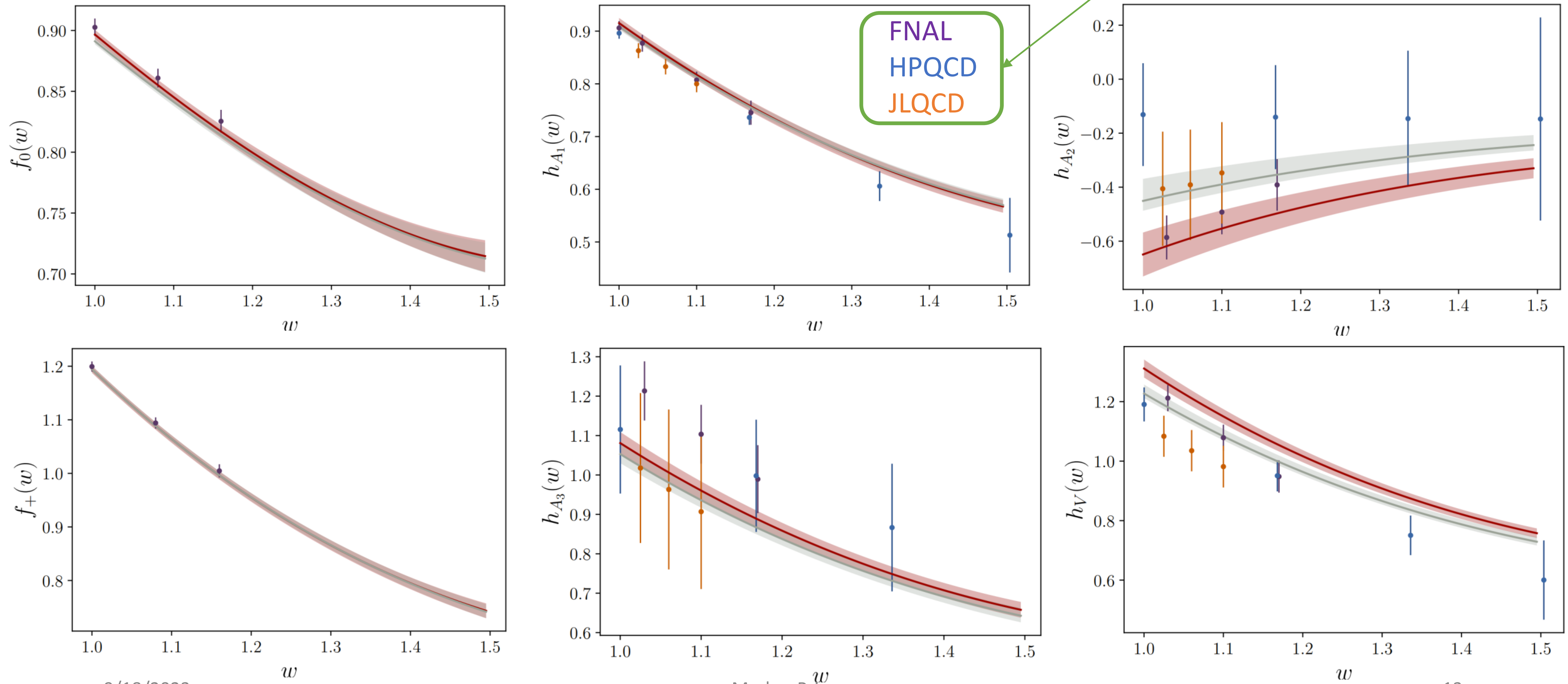
$$w = v_B \cdot v_{D^{(*)}} = \frac{m_B^2 + m_{D^{(*)}}^2 - q^2}{2m_B m_{D^{(*)}}}$$

Lattice Inputs

BLPRXP '23 $f_{+/0}, h_{A_1}(1), B \to D^{(*)}\ell\bar{\nu}_\ell$

BLPRXP '23 $f_{+/0}\ h_X(w), B \to D^{(*)}\ell\bar{\nu}_\ell$

New nonzero recoil lattice QCD



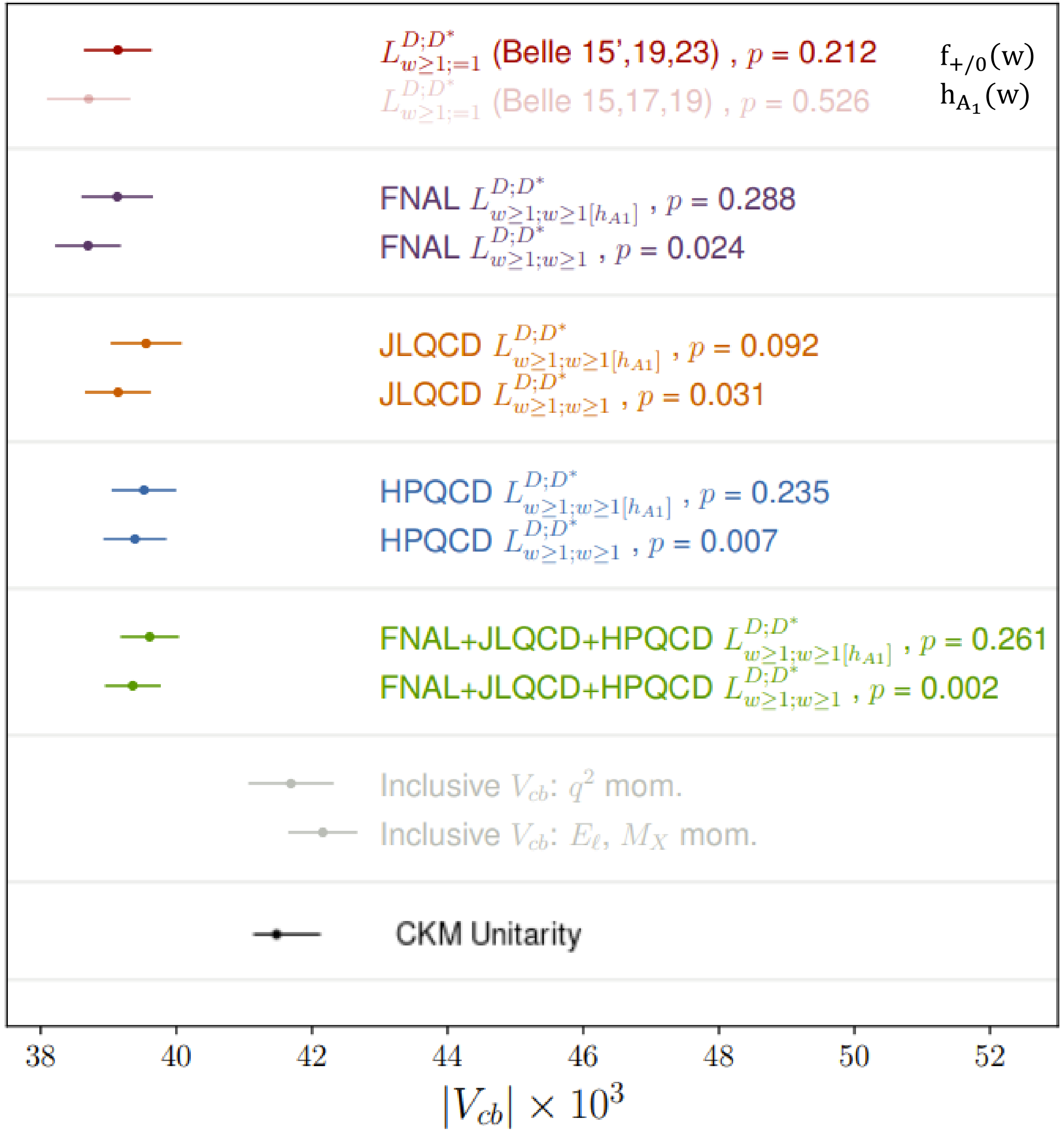
$|V_{cb}|$ from BLPRXP

non-zero recoil lattice inputs:

- $h_{A_1}(w)$ only has good p-values
- full set $h_X(w)$ results in worse p-values

$|V_{cb}| = (39.1 \pm 0.5) \times 10^{-3}$
using $\mathrm{f}_{+/0}(\mathrm{w}), \mathrm{h}_{\mathrm{A}_1}(\mathrm{w})$
$|V_{cb}| = (38.7 \pm 0.6) \times 10^{-3}$

$|V_{cb}| = (39.6 \pm 0.4) \times 10^{-3}$ $[h_{A_1}(w)]$
$|V_{cb}| = (39.4 \pm 0.4) \times 10^{-3}$ $[h_X(w)]$
with the same NHT hypothesis

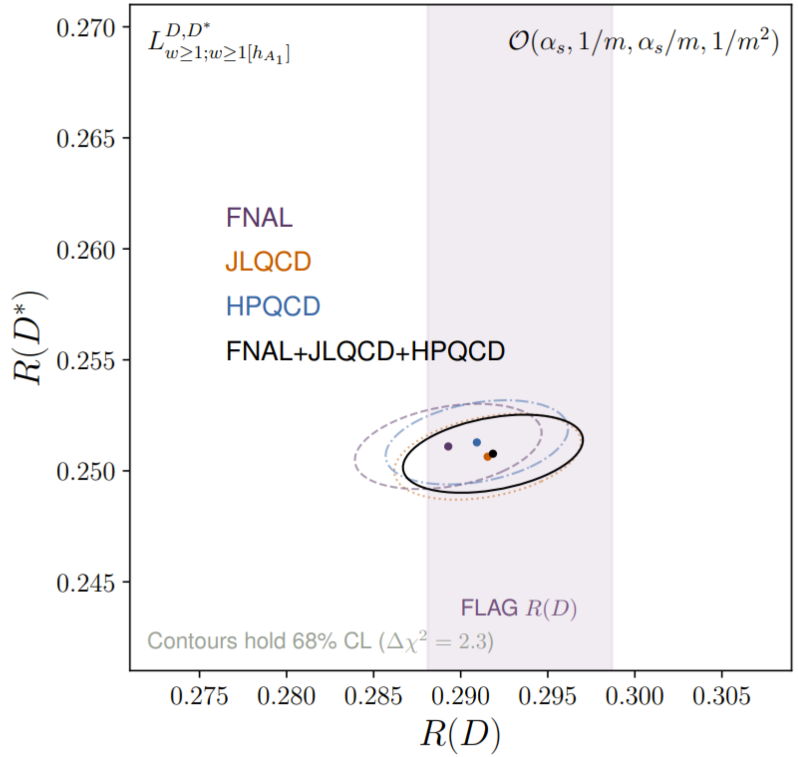


$R(D^{(*)})$ Predictions – Lattice Inputs

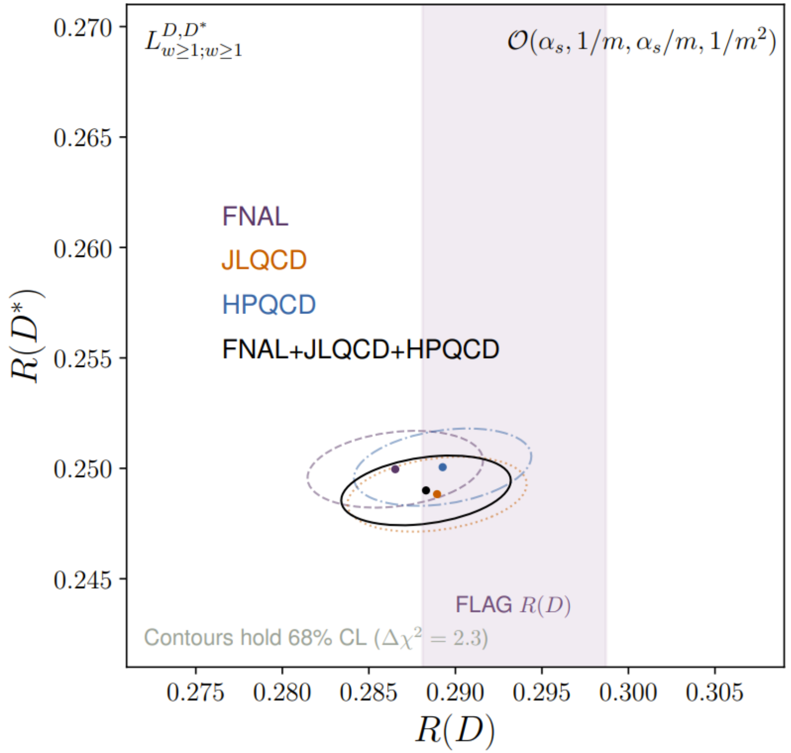
How big is the impact of the LQCD inputs?

Prediction depends on the lattice input, but is compatible within uncertainties

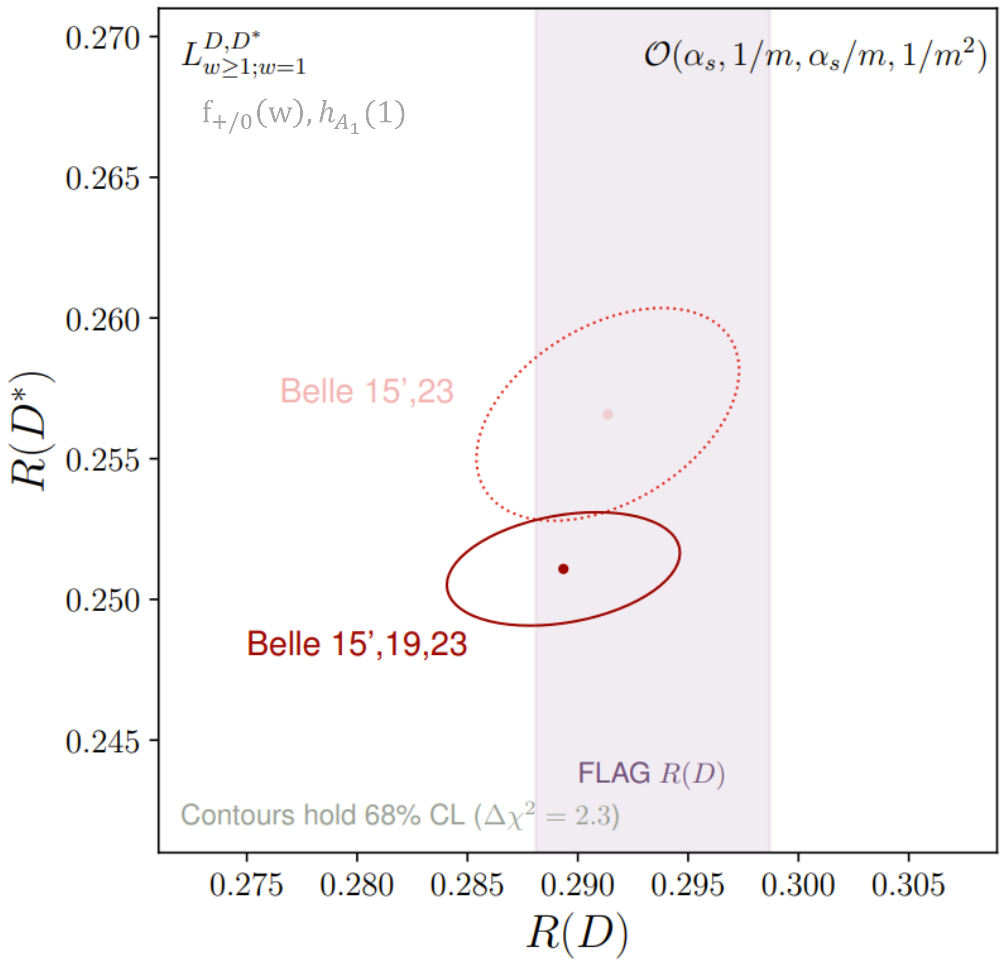
$f_{+/0}(w), h_{A_1}(w)$



$f_{+/0}(w), h_X(w)$



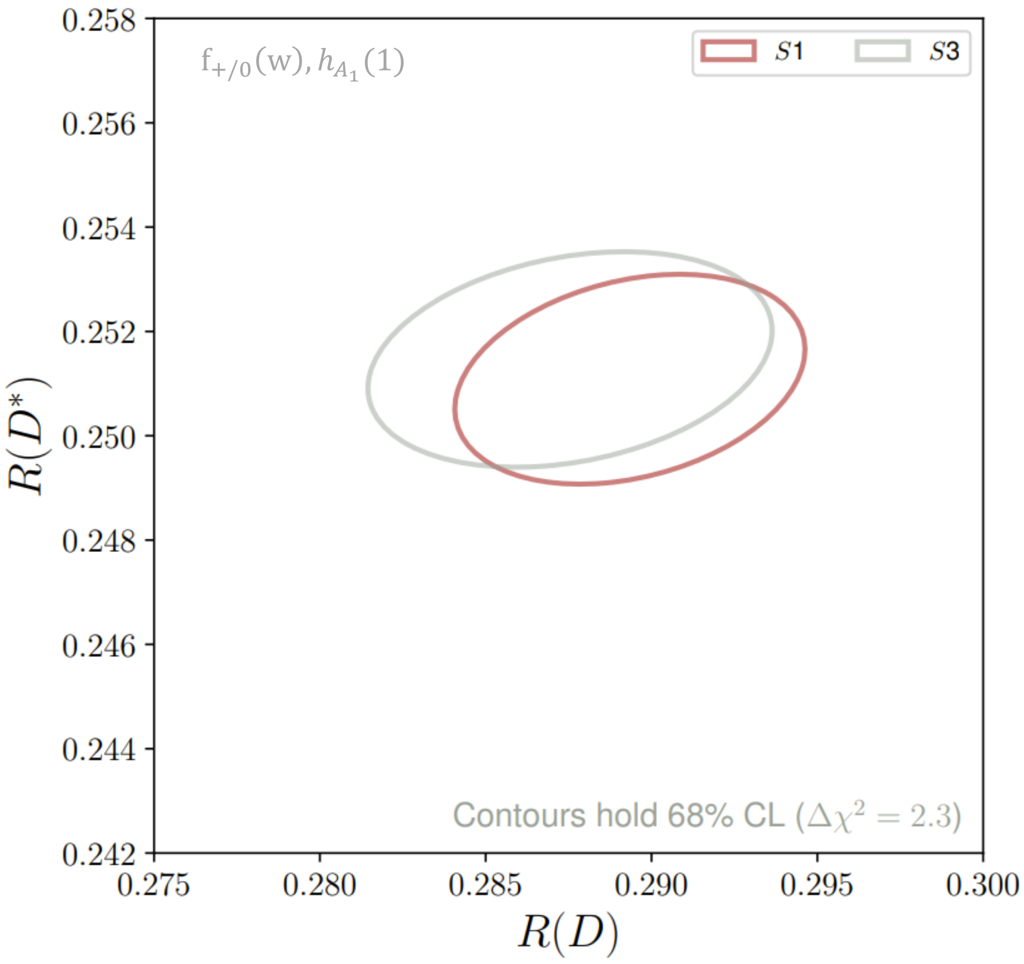
$R(D^{(*)})$ Predictions – Experimental Inputs



Are the experimental results compatible with each other?

Small tension in the $R(D^*)$ prediction using different inputs.
→ PDG scale factor of 2 applied in our final prediction

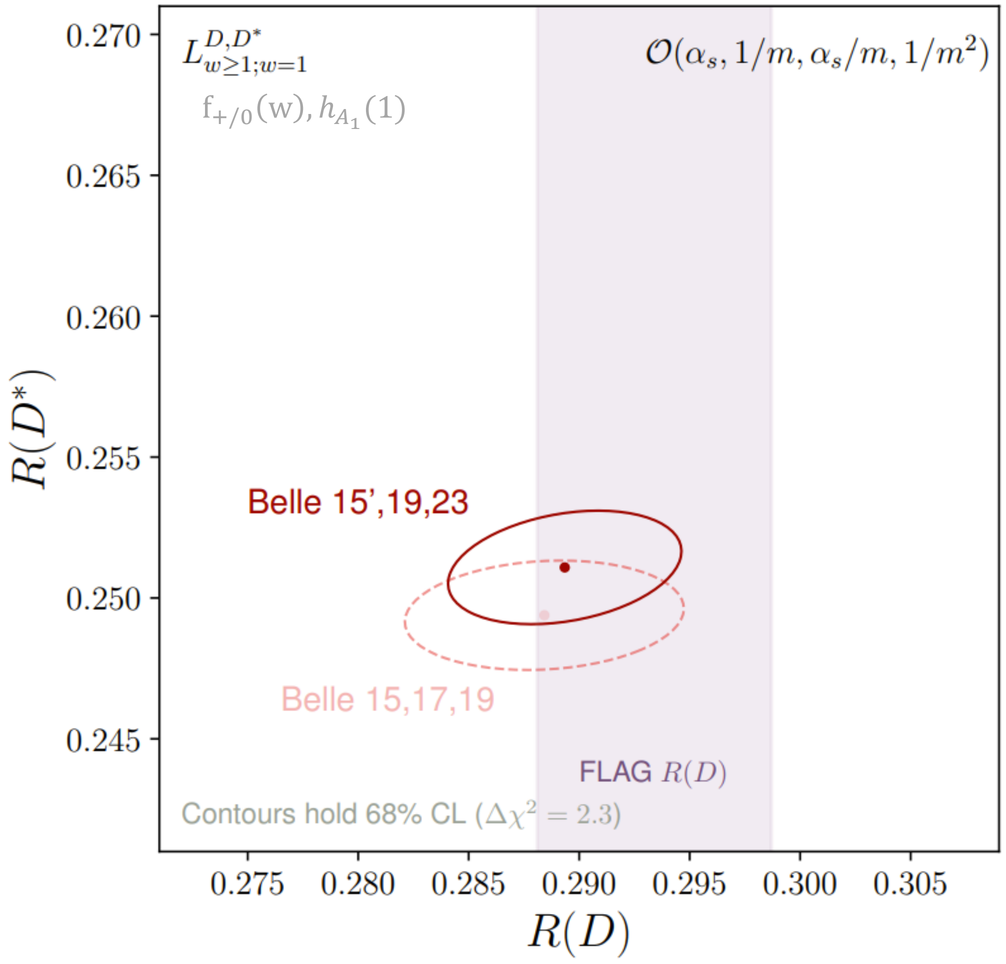
$R(D^{(*)})$ Predictions – Model Dependence



How strongly does the result depend on the choice of the NHT?

$R(D^{(*)})$ dependence on selected hypothesis from the NHT is small and compatible within the uncertainty.

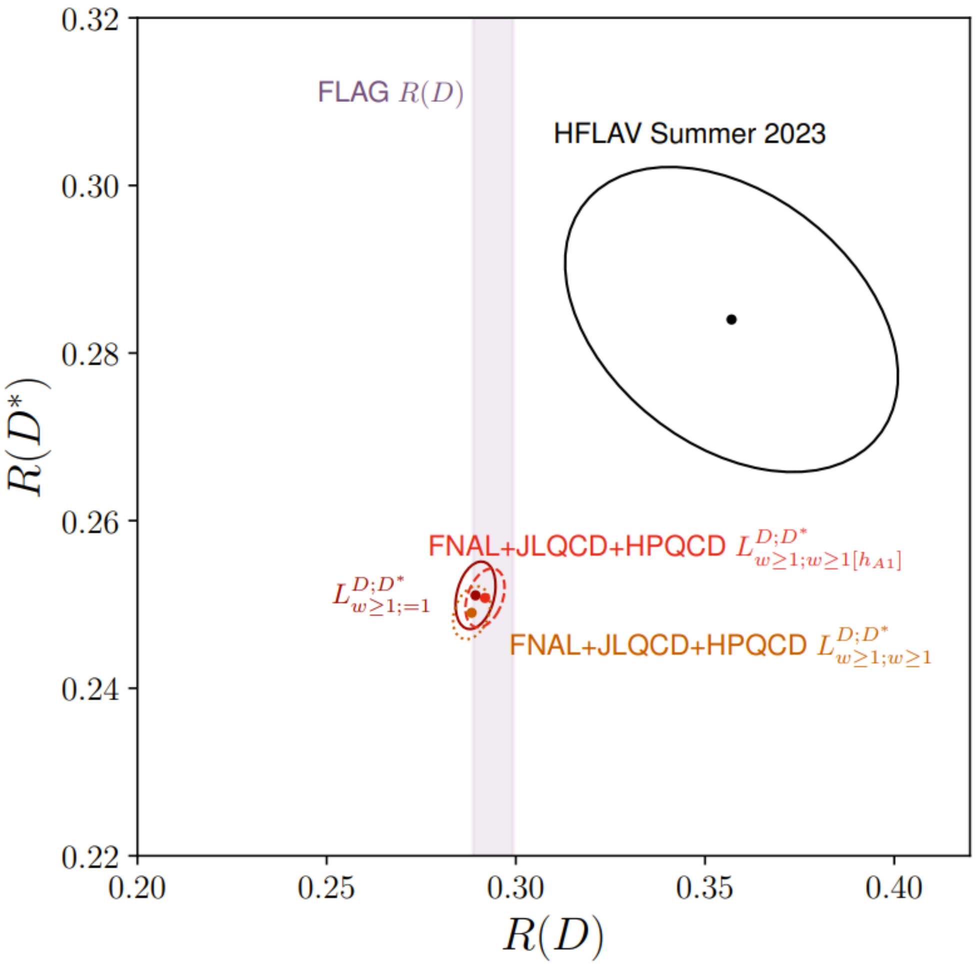
$R(D^{(*)})$ Predictions – Impact of Updates



How big is the impact of the new experimental data?

1σ shift in $R(D^*)$

$R(D^{(*)})$ Predictions – vs. Experiment



What is the current picture of $R(D^{(*)})$?

The picture with respect to the experimental measurements did not change, still a strong tension!

$$R(D) = 0.289 \pm\ 0.003$$
$$R(D^*) = 0.251 \pm 0.003$$
$\rho = 0.286$ using $\mathrm{f}_{+/0}(\mathrm{w}), h_{A_1}(1)$

$\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}_\ell$

An alternative process to test the RC

$\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$

- Only 2 subleading IW at $\mathcal{O}\left(\frac{1}{m_c^2}\right)$ for $\Lambda_b \to \Lambda_c \ell \bar{\nu}_\ell$ (no $\mathcal{O}\left(\frac{1}{m_b m_c}\right)$ included)
- Fit without RC yields only 1 significant parameter contributing to subleading IW

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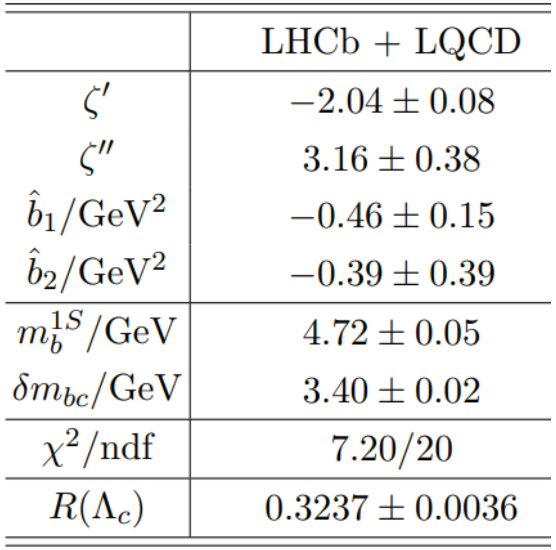
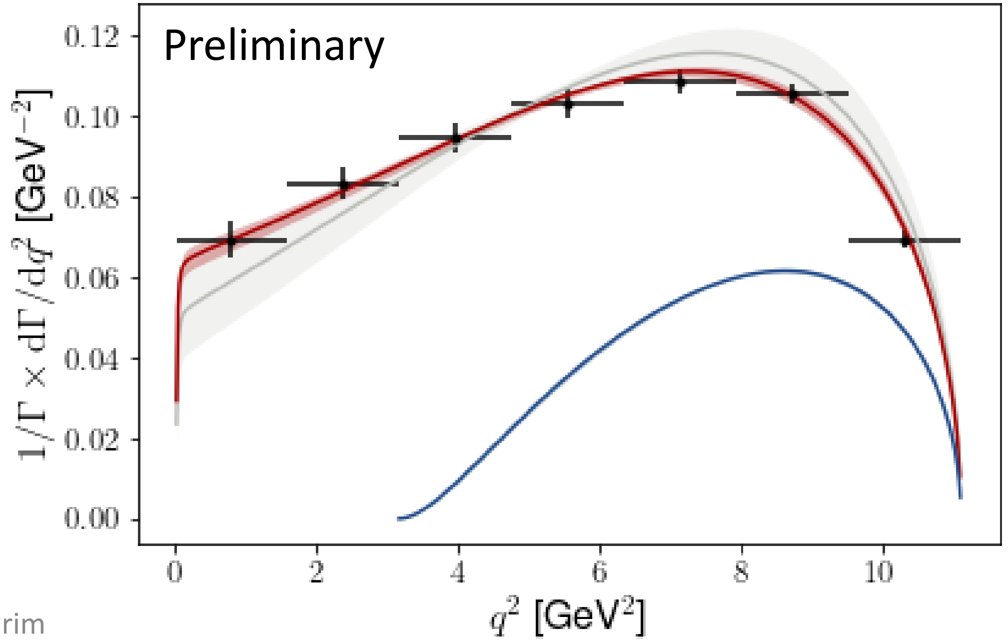


	LHCb + LQCD
ζ'	-2.04 ± 0.08
ζ''	3.16 ± 0.38
$\hat{b}_1/\mathrm{GeV}^2$	-0.46 ± 0.15
$\hat{b}_2/\mathrm{GeV}^2$	-0.39 ± 0.39
m_b^{1S}/GeV	4.72 ± 0.05
$\delta m_{bc}/\mathrm{GeV}$	3.40 ± 0.02
χ^2/ndf	$7.20/20$
$R(\Lambda_c)$	0.3237 ± 0.0036

- After application of the RC: ($\mathcal{O}\left(\frac{1}{m_b m_c}\right)$ included) only 1 free parameter remaining to describe the subleading IW functions: φ_1

→Ideal process to test if RC yields compatible results

RC $R(\Lambda_c) = 0.321 \pm 0.004$



Summary & Conclusion

- Residual Chiral Expansion allows us to fit NNLO HQET in exclusive $b \to c\ell\bar{\nu}_\ell$ decays
  - $1/m^2$ corrections are important to match HQET result to LQCD result
- Updated $R(D^{(*)})$ with new experimental and lattice data for $B \to D^{(*)}\ell\bar{\nu}_\ell$
  - Results stable with new lattice / experimental data
  - No significant changes with respect to previous result
- Updated $R(\Lambda_c)$ with the RC

Preliminary Results

$R(D) = 0.289 \pm 0.003$
$R(D^*) = 0.251 \pm 0.003$
$\rho = 0.286$ using
$\mathrm{f}_{+/0}(\mathrm{w}), h_{A_1}(1)$

$R(\Lambda_c) = 0.321 \pm 0.004$