

12th International Workshop on the CKM Unitarity Triangle

Impact of $\Lambda_b \rightarrow \Lambda_c \tau \nu$ on New Physics in $b \rightarrow c \tau \nu$ transitions

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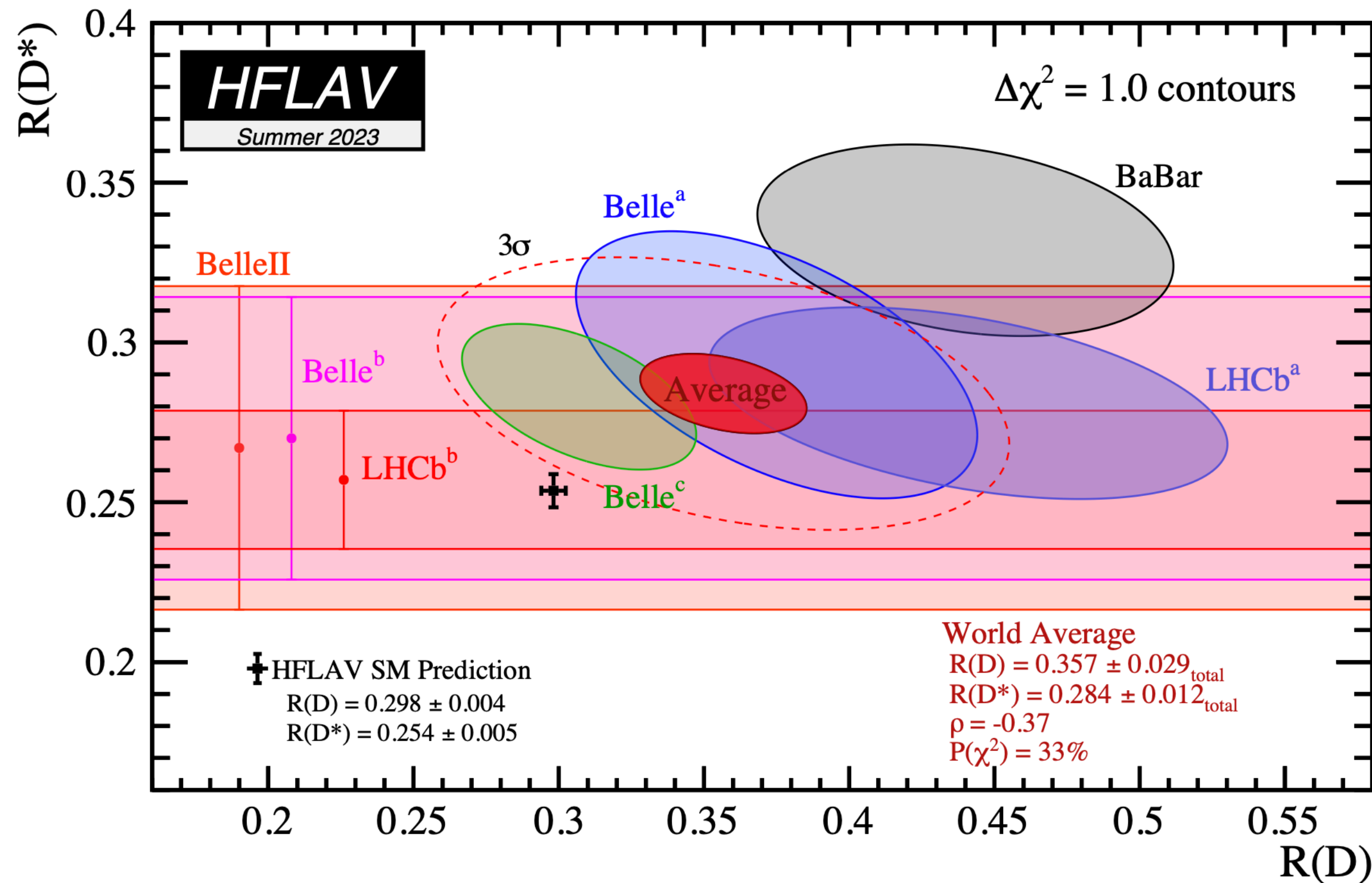
based on [arXiv:2211.14172](#) and [arXiv:2305.15457](#) in collaboration with:

M. Blanke, A. Crivellin, S. Iguro, T. Kitahara, U. Nierste, S. Simula, L. Vittorio & R. Watanabe

Introduction to $b \rightarrow c$ anomalies

- Tree level, theoretically clean processes with large Br (\sim few %)
- Sensitive to NP via LFUV tests

$$R(D^{(*)}) = \frac{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu}_\ell)} \quad \begin{array}{l} l = e, \mu, \tau \\ \ell = e, \mu \end{array}$$



Experimental average (HFLAV):

$$R(D) = 0.357 \pm 0.029$$

$$R(D^*) = 0.284 \pm 0.012$$

SM predictions:

$$R(D) = 0.298 \pm 0.004$$

$$R(D^*) = 0.254 \pm 0.005$$

Comb. discrepancy at $\sim 3.3\sigma$ level hinting at τ over-abundance

The partner LFUV ratio observable

Interesting x-check coming from $R(\Lambda_c) = \frac{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \tau \nu)}{\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \ell \nu)}$, mediated by the same $b \rightarrow c \ell \nu$ transition

⇒ Analogous τ over-abundance predicted in this sector!

$$\frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\text{SM}}(\Lambda_c)} \simeq 0.262 \frac{\mathcal{R}(D)}{\mathcal{R}_{\text{SM}}(D)} + 0.738 \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\text{SM}}(D^*)}$$

[Blanke, Crivellin, De Boer, Kitahara, Moscati, Nieste, Nišandzić \(1811.09603, 1905.08253\)](#)

However this is not what was found when LHCb measured $R(\Lambda_c)$

$$R(\Lambda_c)_{\text{exp}} = 0.242 \pm 0.076$$

[LHCb \(2201.03497\)](#)

$$R(\Lambda_c)_{\text{SM}} = 0.324 \pm 0.004$$

[Bernlochner, Ligeti, Robinson, Sutcliffe \(1808.09464\)](#)

No strong tension w.r.t SM, actually tiny hint to τ under-abundance...?

A matter of normalization?

LHCb actually measures $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \tau \nu) / \mathcal{B}(\Lambda_b \rightarrow \Lambda_c 3\pi)$ to extract $\mathcal{B}(\Lambda_b \rightarrow \Lambda_c \tau \nu)$, which is normalized to the pdg value of $\Gamma(\Lambda_b \rightarrow \Lambda_c \mu \nu)$ to obtain $R(\Lambda_c)$
 \Rightarrow potential underestimation of systematics?



$$R(\Lambda_c)_{\text{SM}} = 0.324 \pm 0.004$$

$$R(\Lambda_c)_{\text{exp}'} = (0.285 \pm 0.073) \left| \frac{0.04}{V_{cb}} \right|^2$$

[Bernlochner, Ligeti, Papucci, Robinson \(2206.11282\)](#)

Better agreement with SM, but tension with $R(D^{(*)})$ still present!

Open questions

For $R(D^{(*)})$ we have multiple experiments giving a coherent pattern of deviations, but a new element of the puzzle actually points to the opposite direction...

- Can new data be accommodated by a violation of the sum rule, i.e. by assuming NP coupling not only to τ , but also to μ and e ? (✓)
- Or, is this pointing to data incompatibility, requiring further scrutiny? ((exp?) ✗)

NP analysis

To study NP effects in $b \rightarrow cl\nu$ we employ the effective Hamiltonian

$$C_i^{l(SM)} = 0$$

$$\mathcal{H}_{\text{eff}} = 2\sqrt{2}G_F V_{cb} \left[(1 + C_{V_L}^l) O_{V_L}^l + C_{S_R}^l O_{S_R}^l + C_{S_L}^l O_{S_L}^l + C_T^l O_T^l \right]$$

$$O_{V_L}^l = (\bar{c}\gamma^\mu P_L b) (\bar{l}\gamma_\mu P_L \nu_l)$$

$$O_{S_L}^l = (\bar{c}P_L b) (\bar{l}P_L \nu_l)$$

$$O_{S_R}^l = (\bar{c}P_R b) (\bar{l}P_L \nu_l)$$

$$O_T^l = (\bar{c}\sigma^{\mu\nu} P_L b) (\bar{l}\sigma_{\mu\nu} P_L \nu_l)$$

We include RGE effects when going from the matching scale $\Lambda = 2\text{TeV}$

$$C_{V_L}^l(\mu_b) = 1.12 C_{V_L}^l(2\text{TeV})$$

$$C_{S_R}^l(\mu_b) = 2.00 C_{S_R}^l(2\text{TeV})$$

$$\begin{pmatrix} C_{S_L}^l(\mu_b) \\ C_T^l(\mu_b) \end{pmatrix} = \begin{pmatrix} 1.91 & -0.38 \\ 0. & 0.89 \end{pmatrix} \begin{pmatrix} C_{S_L}^l(2\text{TeV}) \\ C_T^l(2\text{TeV}) \end{pmatrix}$$

Update of the sum rule

As a first step, we updated the sum rules due to update in $B \rightarrow D^*$ FF

$$\frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\text{SM}}(\Lambda_c)} = 0.280 \frac{\mathcal{R}(D)}{\mathcal{R}_{\text{SM}}(D)} + 0.720 \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\text{SM}}(D^*)} + \delta_{\Lambda_c} \quad \text{with}$$

$$\delta_{\Lambda_c} = \text{Re} \left[\left(1 + C_{V_L}^\tau \right) \left(0.314 C_T^{\tau*} - 0.003 C_{S_R}^{\tau*} \right) \right] + 0.014 \left(|C_{S_L}^\tau|^2 + |C_{S_R}^\tau|^2 \right) + 0.004 \text{Re} \left(C_{S_L}^\tau C_{S_R}^{\tau*} \right) - 1.30 |C_T^\tau|^2$$

Coefficients slightly changed, overall stability of the sum rule

$$\mathcal{R}(\Lambda_c) \simeq \mathcal{R}_{\text{SM}}(\Lambda_c) \left(0.280 \frac{\mathcal{R}(D)}{\mathcal{R}_{\text{SM}}(D)} + 0.720 \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\text{SM}}(D^*)} \right)$$

$$= \mathcal{R}_{\text{SM}}(\Lambda_c) (1.172 \pm 0.038)$$

$$= 0.380 \pm 0.012 \pm 0.005$$

to be compared with

$$R(\Lambda_c)_{\text{exp}} = 0.242 \pm 0.076$$

$$R(\Lambda_c)_{\text{exp}'} = (0.285 \pm 0.073) \left| \frac{0.04}{V_{cb}} \right|^2$$

Interlude: How to obtain the sum rule

$$\frac{R_D}{R_D^{\text{SM}}} = |1 + C_{V_L} + C_{V_R}|^2 + 1.01|C_{S_L} + C_{S_R}|^2 + 0.84|C_T|^2 + 1.49\text{Re}[(1 + C_{V_L} + C_{V_R})(C_{S_L}^* + C_{S_R}^*)] + 1.08\text{Re}[(1 + C_{V_L} + C_{V_R})C_T^*]$$

$$\frac{R_{D^*}}{R_{D^*}^{\text{SM}}} = |1 + C_{V_L}|^2 + |C_{V_R}|^2 + 0.04|C_{S_L} - C_{S_R}|^2 + 16.0|C_T|^2 - 1.83\text{Re}[(1 + C_{V_L})C_{V_R}^*] - 0.11\text{Re}[(1 + C_{V_L} - C_{V_R})(C_{S_L}^* - C_{S_R}^*)] - 5.17\text{Re}[(1 + C_{V_L})C_T^*] + 6.60\text{Re}[C_{V_R}C_T^*],$$

$$\frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\text{SM}}(\Lambda_c)} = |1 + C_{V_L}^\tau|^2 + 0.50\text{Re}[(1 + C_{V_L}^\tau)C_{S_R}^{\tau*}] + 0.33\text{Re}[(1 + C_{V_L}^\tau)C_{S_L}^{\tau*}] + 0.52\text{Re}(C_{S_L}^\tau C_{S_R}^{\tau*}) + 0.32(|C_{S_L}^\tau|^2 + |C_{S_R}^\tau|^2) - 3.11\text{Re}[(1 + C_{V_L}^\tau)C_T^{\tau*}] + 10.4|C_T^\tau|^2,$$

$$a \frac{\mathcal{R}(D)}{\mathcal{R}_{\text{SM}}(D)} + b \frac{\mathcal{R}(D^*)}{\mathcal{R}_{\text{SM}}(D^*)} = \frac{\mathcal{R}(\Lambda_c)}{\mathcal{R}_{\text{SM}}(\Lambda_c)} \Rightarrow$$

$$a + b = 1$$

$$1.49a + 0.11b = 0.5$$

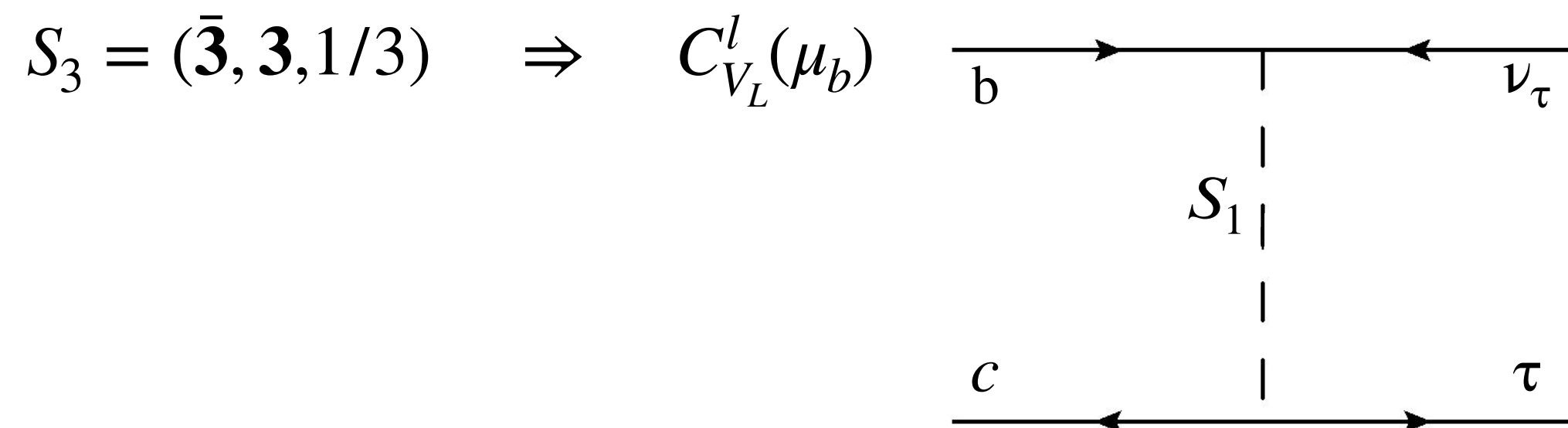
What is not canceled ends in δ_{Λ_c} !

Usual NP scenarios not complying with data!

Scalar Leptoquarks

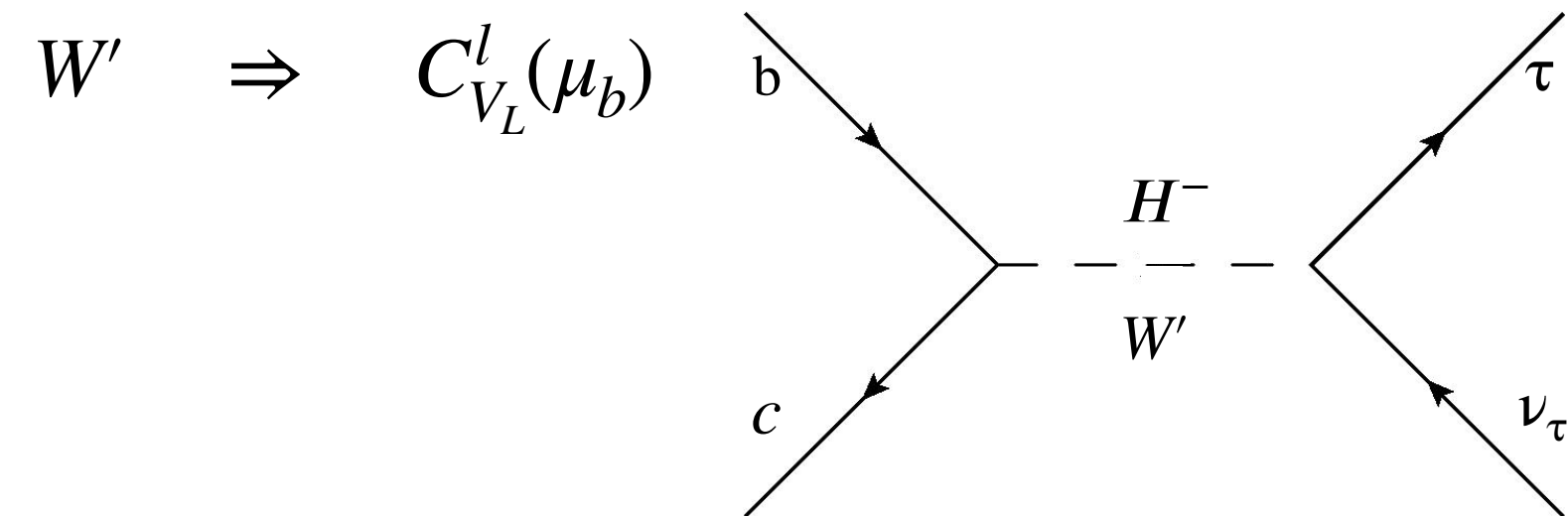
$$S_1 = (\bar{\mathbf{3}}, \mathbf{1}, 1/3) \Rightarrow C_{S_L}^l(\mu_b) \simeq -8.9 C_T^l(\mu_b)$$

$$R_2 = (\mathbf{3}, \mathbf{2}, 7/6) \Rightarrow C_{S_L}^l(\mu_b) \simeq 8.4 C_T^l(\mu_b)$$



Charged Bosons

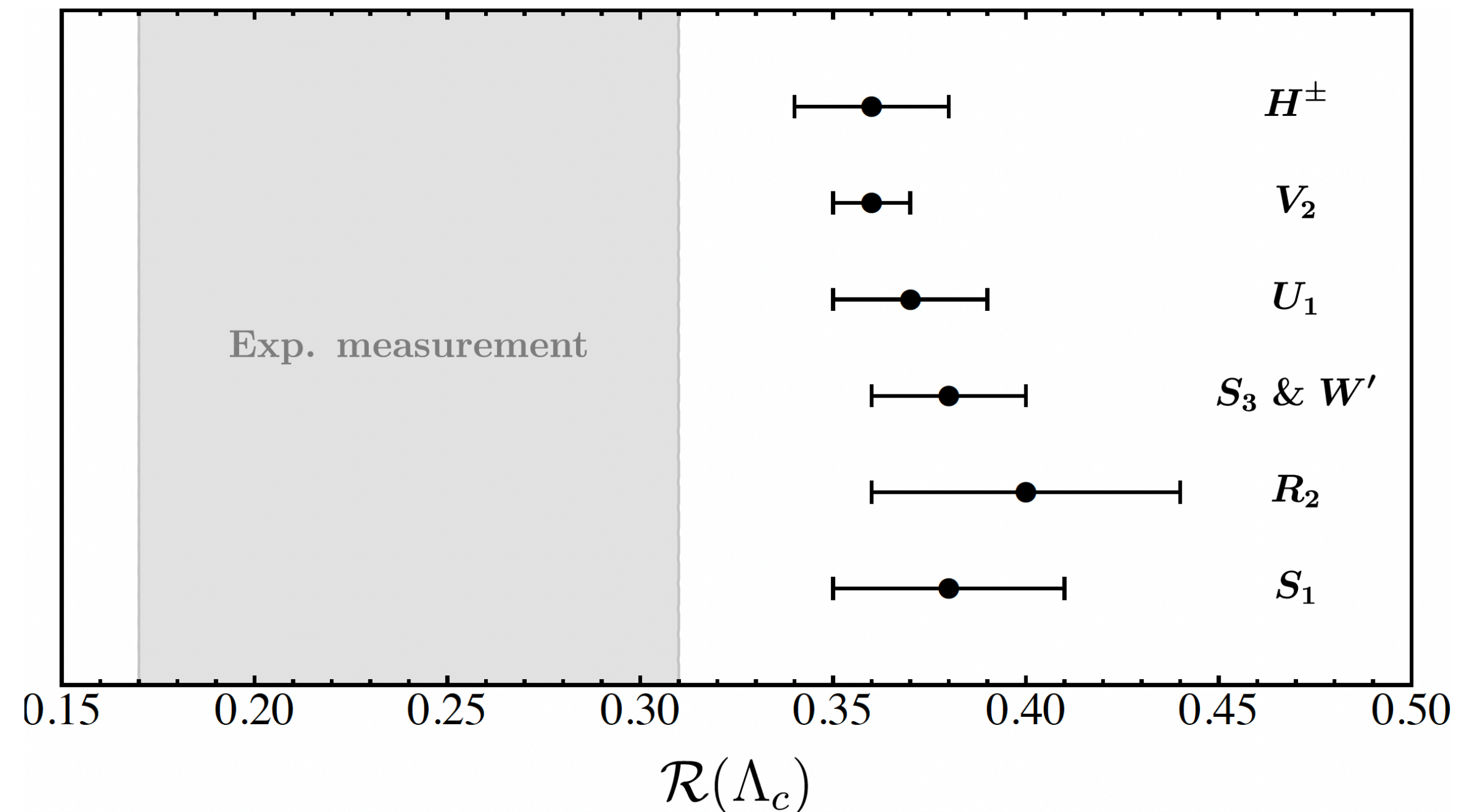
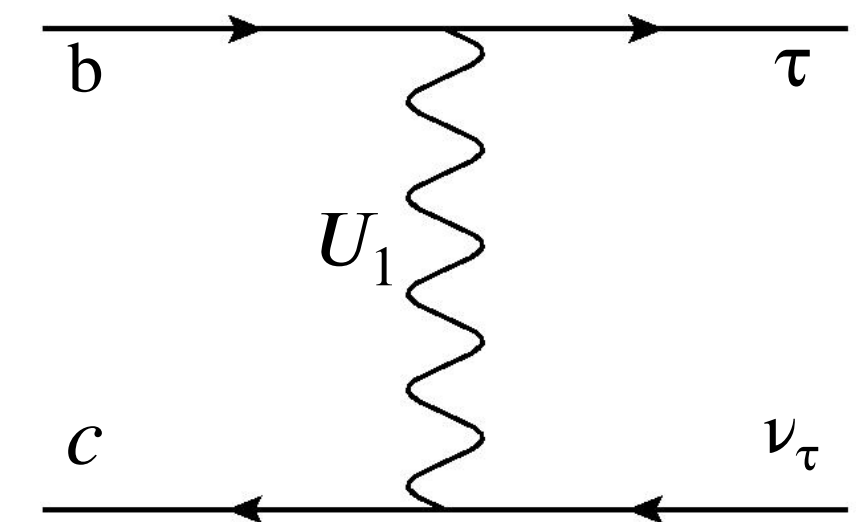
$$H^\pm \Rightarrow C_{S_L}^l(\mu_b), C_{S_R}^l(\mu_b)$$



Vector Leptoquarks

$$U_1 = (\mathbf{3}, \mathbf{1}, 2/3) \Rightarrow C_{V_L}^l(\mu_b), C_{S_R}^l(\mu_b)$$

$$V_2 = (\bar{\mathbf{3}}, \mathbf{2}, 5/6) \Rightarrow C_{S_R}^l(\mu_b)$$



Predictions consistent with sum rule, not with data... 9

Could NP in light leptons rescue data?

Sum rule violated by NP in ℓ : we studied 36 2D scenarios, 1st NP field coupled to τ , 2nd to $\mu = e$

\Rightarrow Only 2 scenarios capable to reproduce all LFUV found

$$S_1^\ell \text{ \& \ } R_2^\tau$$

$$C_{S_L}^\ell = -8.9 C_T^\ell \simeq \pm 1$$

$$S_1^\ell \text{ \& \ } H^{\pm\tau}$$

BUT: in both cases S_1^ℓ requires

$$C_{V_L}^\ell \simeq -1$$

This is however strongly incompatible with bounds from: high-PT searches, $B \rightarrow K^* \ell \nu$, angular distribution and D^{*-} polarization data in $B \rightarrow D^{*-} \ell \nu$, $|V_{cb}|$ fits

$$\Rightarrow \begin{cases} |C_{V_L}^\ell| < 0.03 \\ |C_T^\ell| < 0.05 \end{cases}$$

Could NP in light leptons rescue data?

As a final test, we inspected the general 8dim NP fit

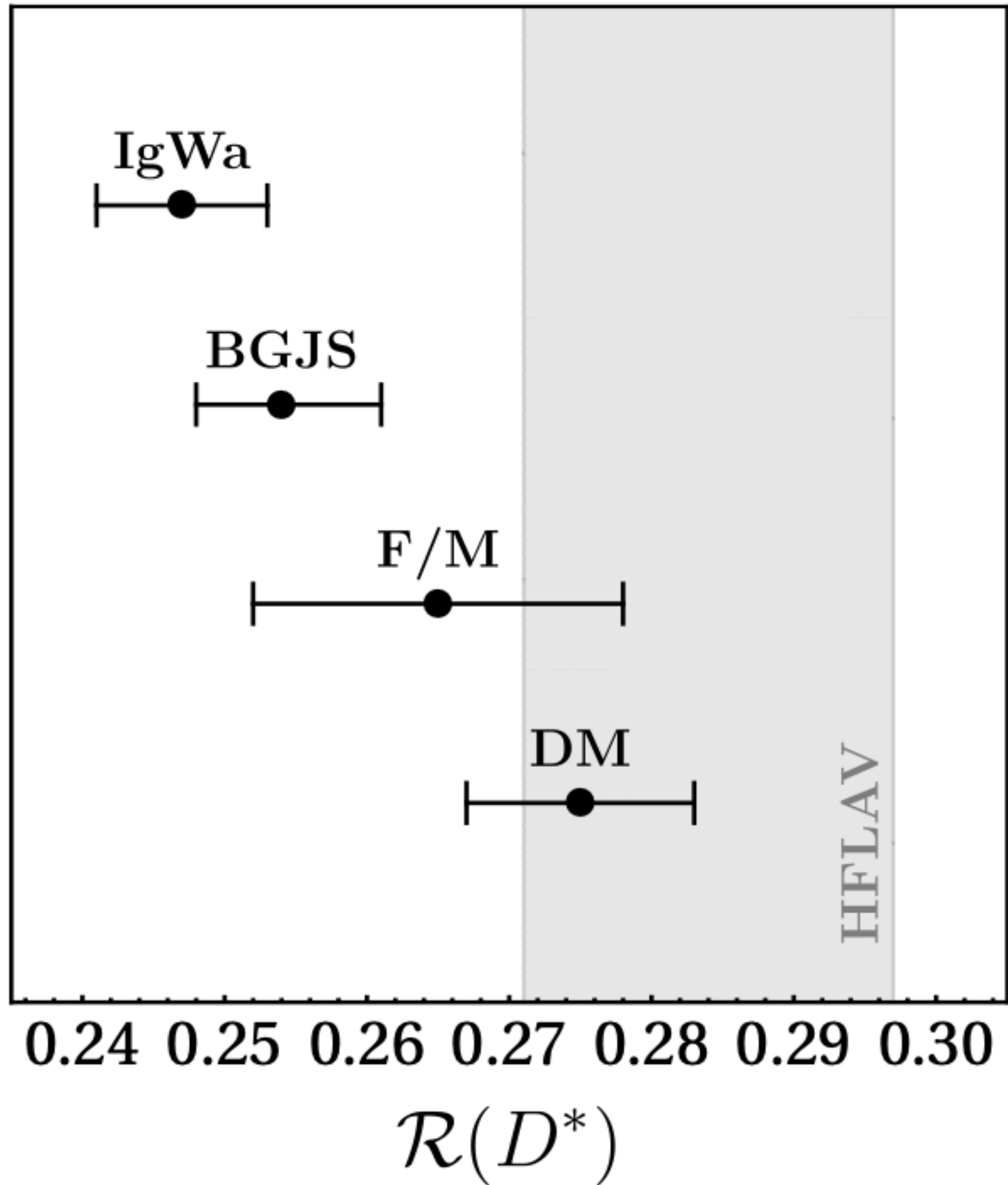
$$\Rightarrow C_{V_L}^\tau, C_{S_L}^\tau, C_{S_R}^\tau, C_T^\tau, C_{V_L}^\ell, C_{S_L}^\ell, C_{S_R}^\ell, C_T^\ell$$

but, analogously to the 2D case, we found a viable fit only for

$$C_T^\ell \simeq \pm 0.1 \qquad C_{V_L}^\ell \simeq -1$$

Again strongly incompatible with bounds from: high-PT searches, $B \rightarrow K^* \nu \nu$, angular distribution and D^{*-} polarization data in $B \rightarrow D^{*-} \ell \nu$, $|V_{cb}|$ fits $\Rightarrow \begin{cases} |C_{V_L}^\ell| < 0.03 \\ |C_T^\ell| < 0.05 \end{cases}$

What if it's a FF issue?



The SM prediction for $\mathcal{R}(D^*)$ might not be as stable as originally thought!

Different Form Factors approaches have different predictions, with noticeable increase on the prediction for the latest determinations

Could the discrepancy actually arise from issues on the FF estimates?

The Dispersive Matrix approach

Exploits the dispersion relation valid for each FF

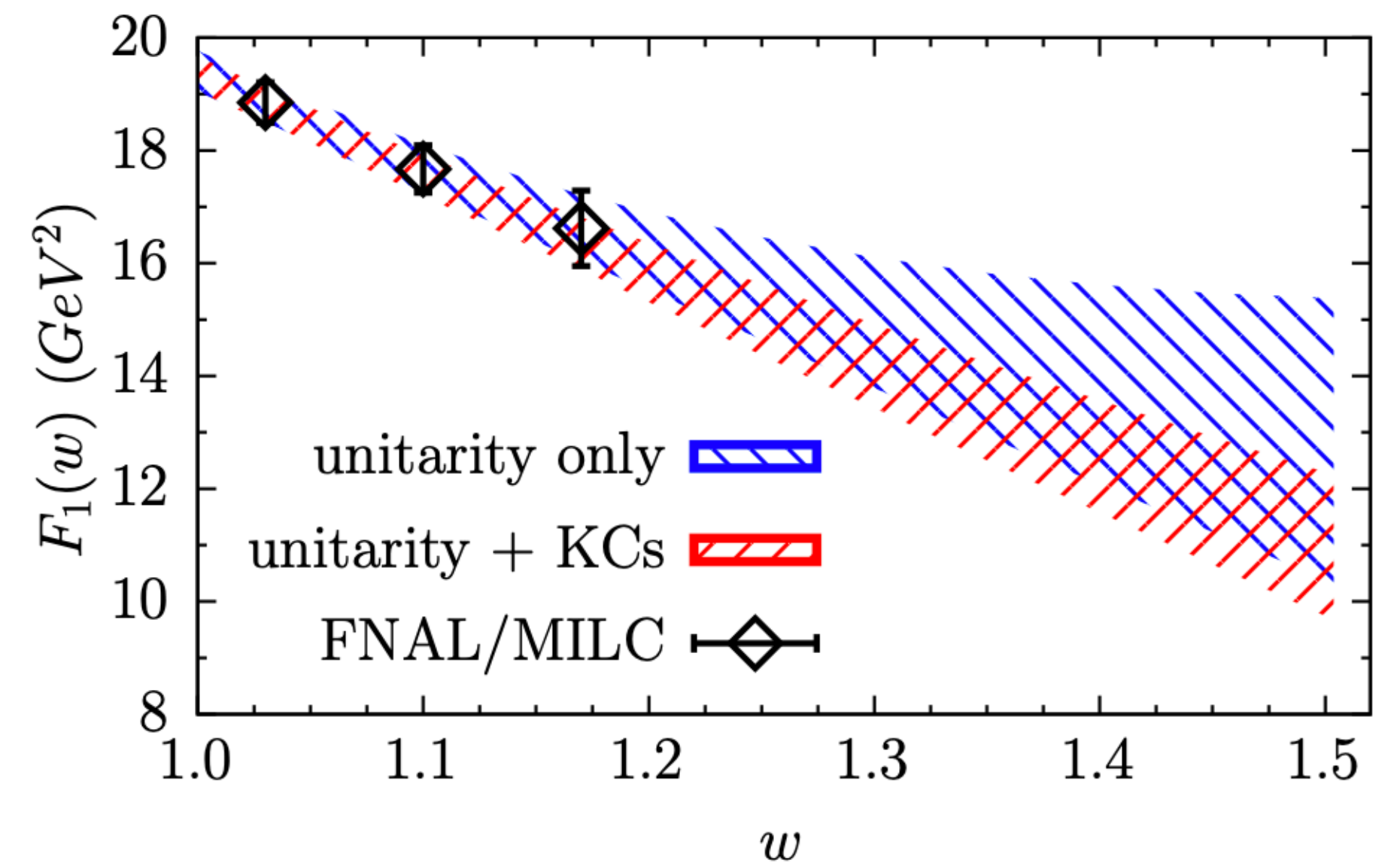
$$\frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z} |\phi(z)f(z)|^2 \leq \chi \xrightarrow{\langle g|h \rangle = \frac{1}{2\pi i} \oint_{|z|=1} \frac{dz}{z} \bar{g}(z)h(z)} 0 \leq \langle \phi f | \phi f \rangle \leq \chi$$

After defining $g_t(z) \equiv \frac{1}{1 - \bar{z}(t)z}$, it is possible to define the matrix with positive semidefinite determinant

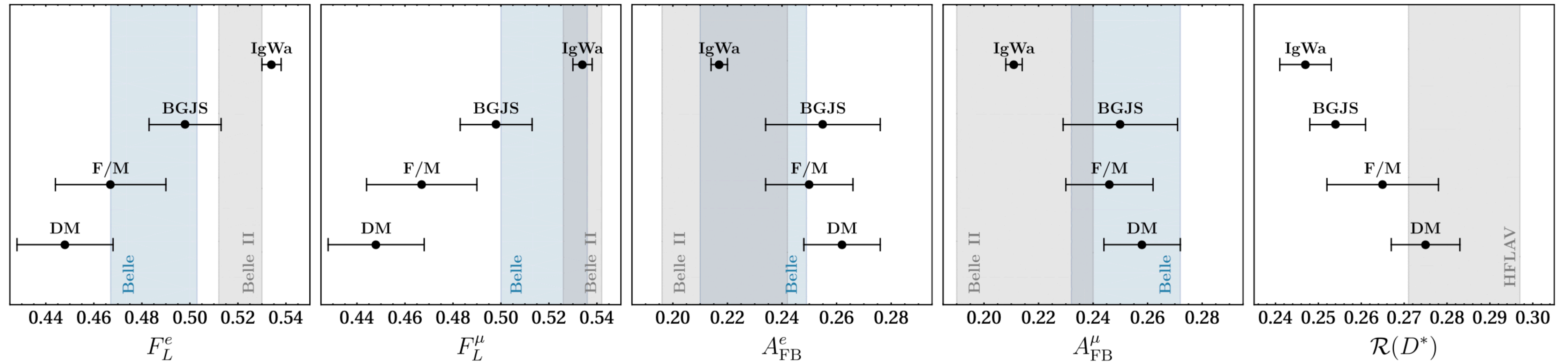
$$\mathbf{M} \equiv \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_N} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_N} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_N} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_N} | \phi f \rangle & \langle g_{t_N} | g_t \rangle & \langle g_{t_N} | g_{t_1} \rangle & \cdots & \langle g_{t_N} | g_{t_N} \rangle \end{pmatrix} \xrightarrow{\begin{aligned} \langle g_t | \phi f \rangle &= \phi(z(t)) f(z(t)) \\ \langle g_{t_m} | g_{t_l} \rangle &= \frac{1}{1 - \bar{z}(t_l)z(t_m)} \end{aligned}} \mathbf{M}_\chi = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \cdots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-zz_1} & \cdots & \frac{1}{1-zz_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \cdots & \frac{1}{1-z_1 z_N} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \cdots & \frac{1}{1-z_N^2} \end{pmatrix}$$

t_i corresponds to values of q^2 where FF is known, e.g. on Lattice

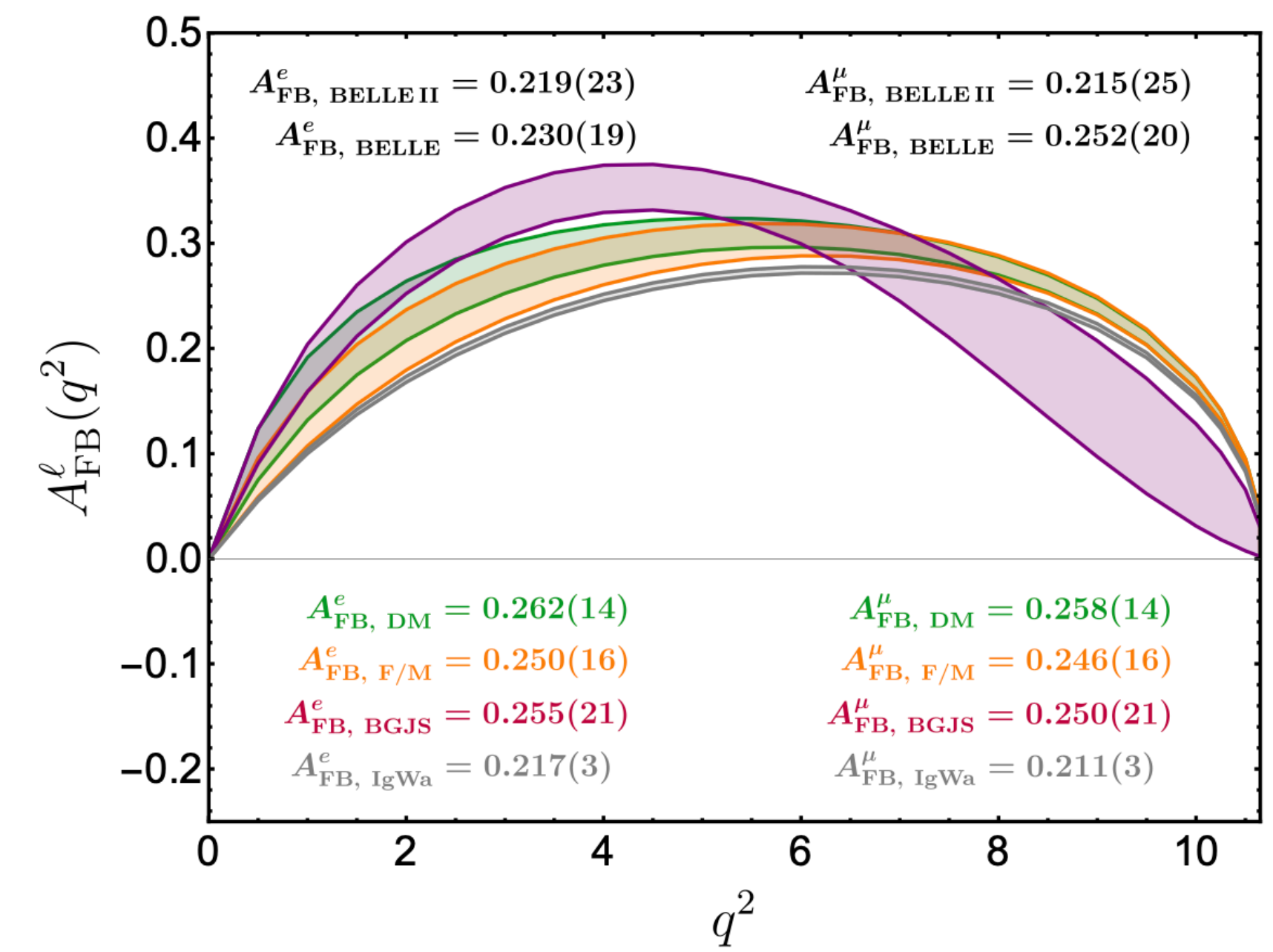
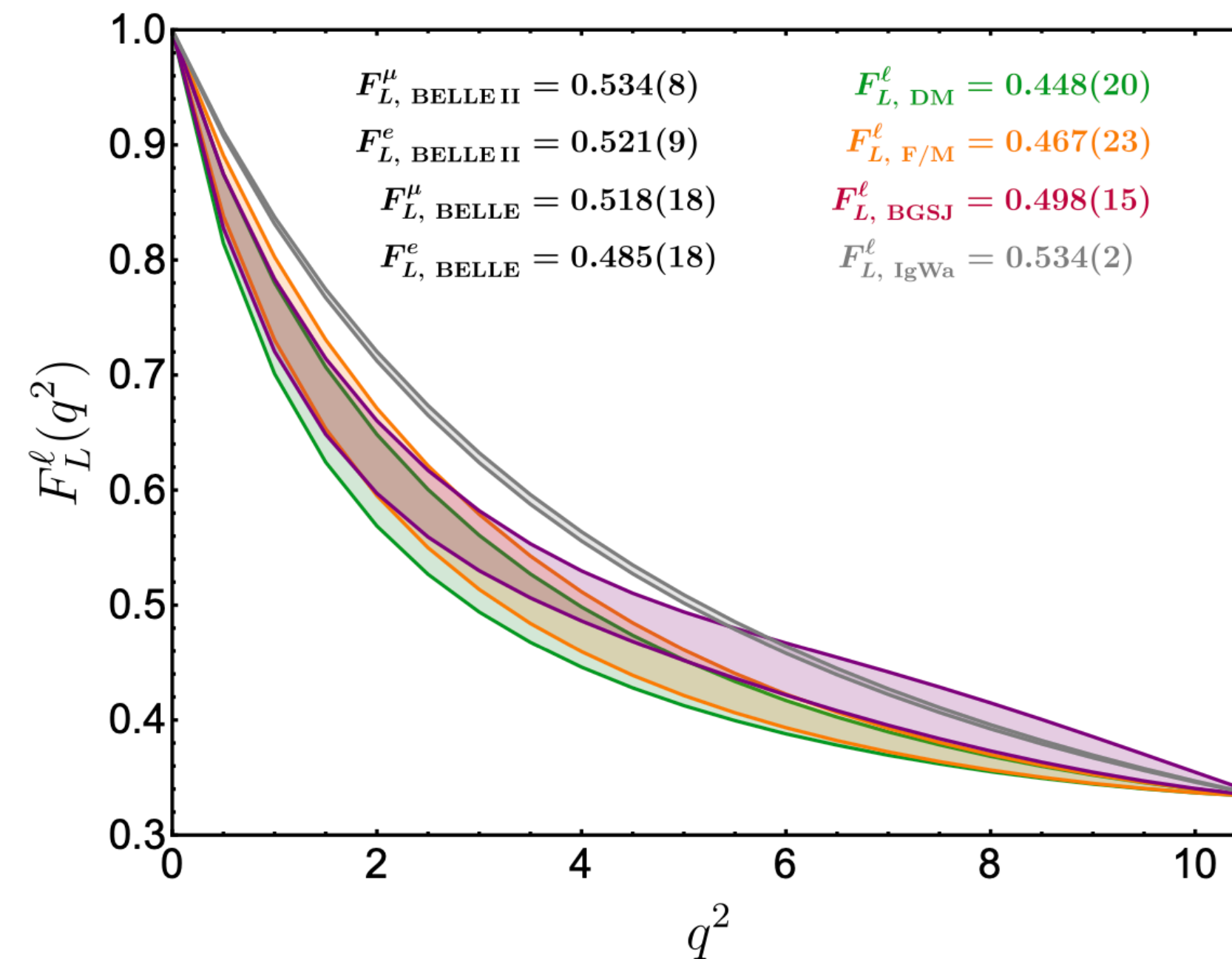
Requiring the positiveness of the determinant allows to obtain a band for the FF, representing the envelope of the results of all possible (non) truncated z -expansions, like BGL ones



Not all that glitters is gold...



The DM FF approach is capable to address tension in $\mathcal{R}(D^*)$ (and $|V_{cb}|$ incl. vs excl. discrepancy), but however in tension with new F_L^ℓ and A_{FB}^ℓ data!



DM FF cannot fix all at once

Indeed, if one tries to perform a SM fit to data obtains

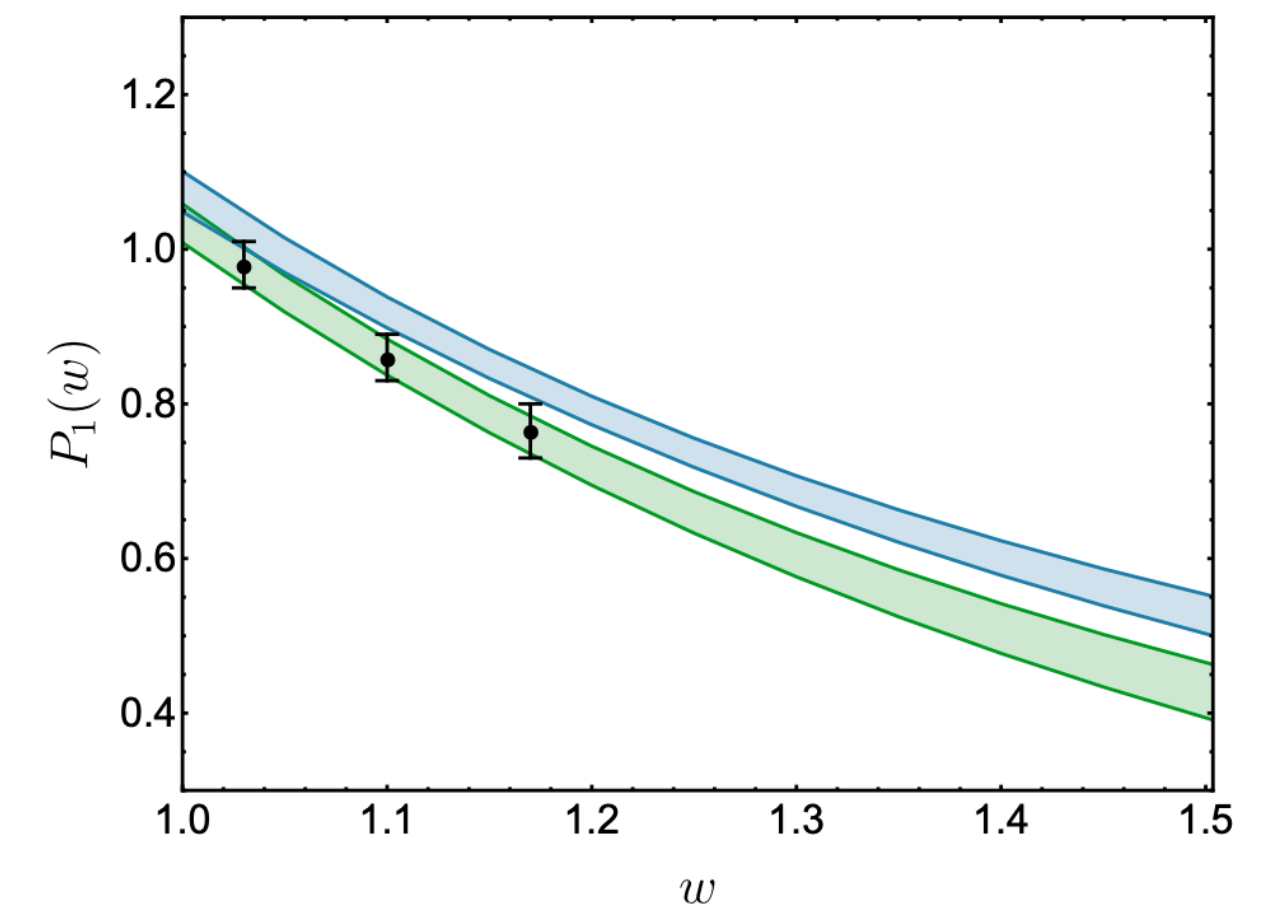
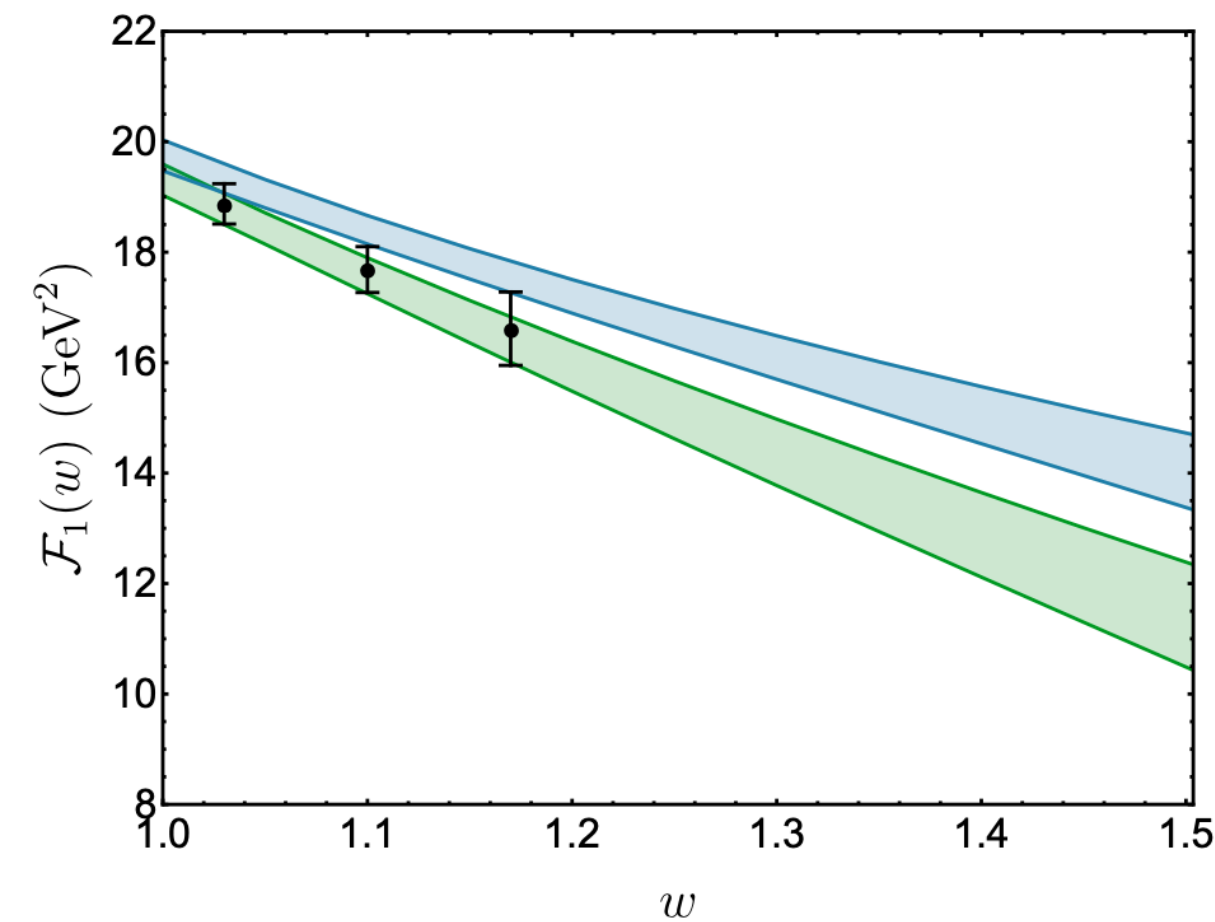
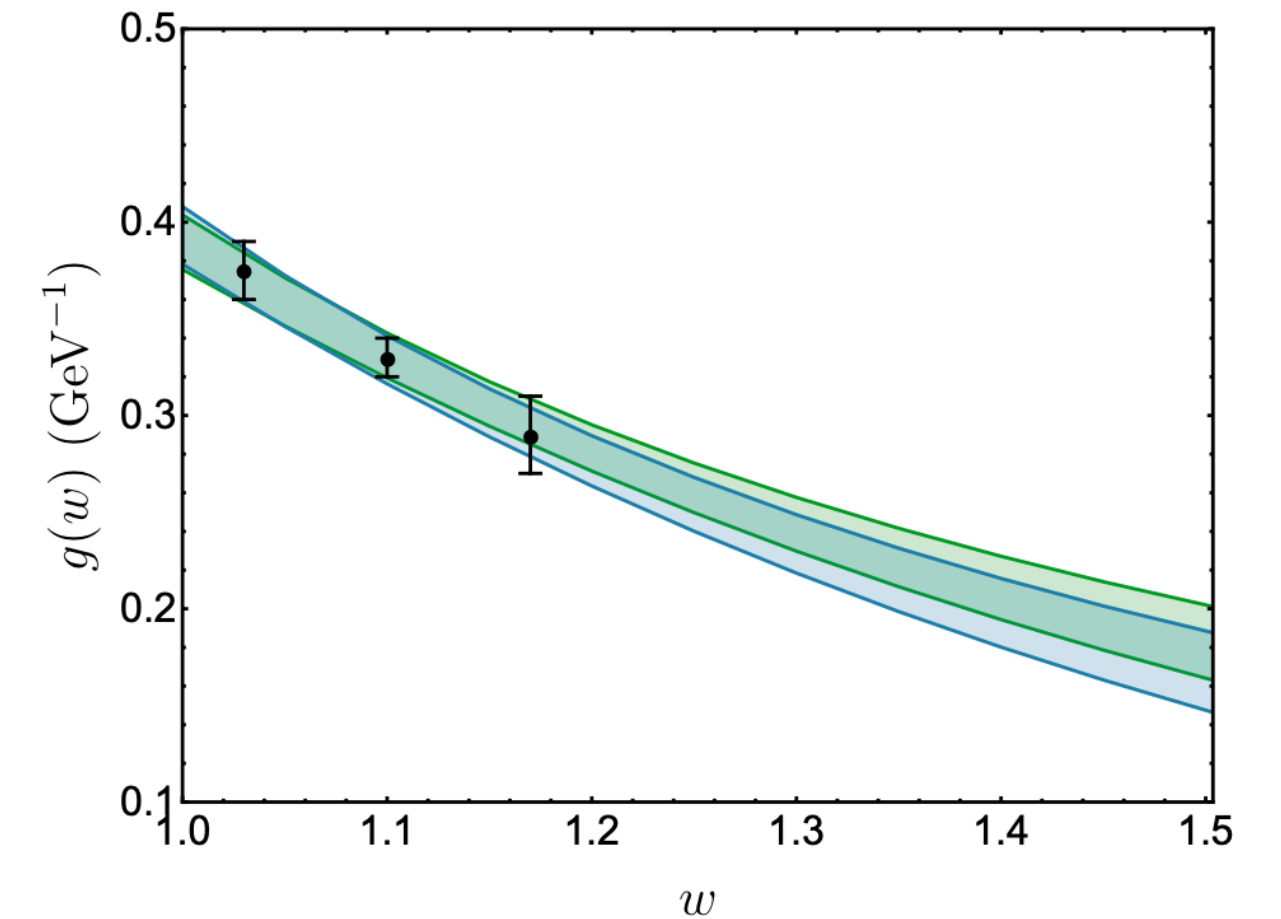
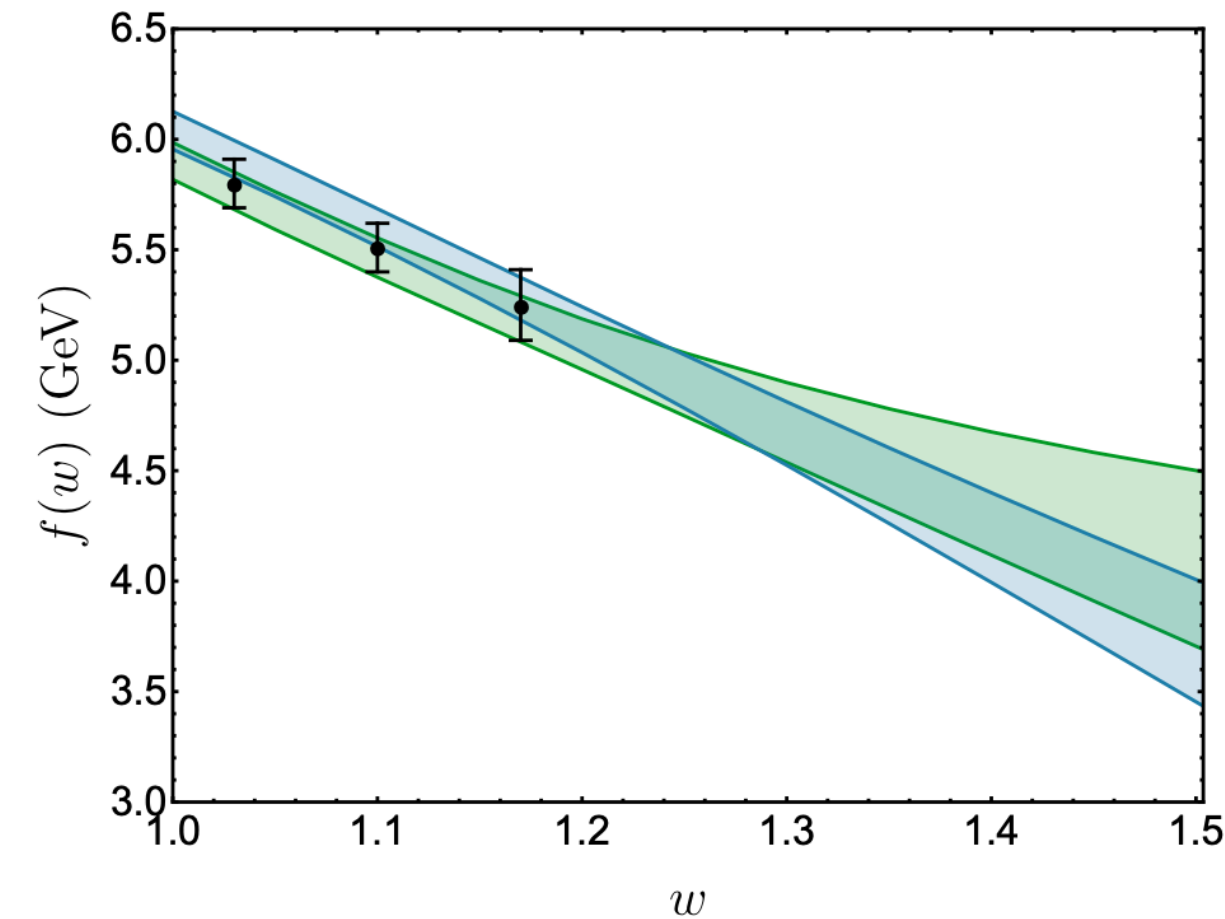
$$\mathcal{R}(D^*)_{\text{fit}} = 0.265 \pm 0.005$$

$$F_L^\ell_{\text{fit}} = 0.515 \pm 0.005$$

$$A_{\text{FB}}^e_{\text{fit}} = 0.227 \pm 0.007$$

$$A_{\text{FB}}^\mu_{\text{fit}} = 0.222 \pm 0.007$$

Re-emergence of $R(D^*)$ anomaly,
disappearance of F_L^ℓ and A_{FB}^ℓ ones,
increase of $F_1(w)$ values



This is however in contrast to original lattice data the method is based on!

What about (again) NP in light leptons?

The DM FF offer the unique possibility to employ NP in light leptons to address anomalies (forbidden in other scenarios due to CKM limits)

Could this fix the issue?

$$g_{V_R} \in [-0.04, 0.01]$$

$$g_{S_L} \in [-0.07, 0.02]$$

$$g_{S_R} \in [-0.05, 0.03]$$

$$g_T \in [-0.01, 0.02]$$

Only evidence found for $g_{V_L} = -0.054 \pm 0.015$

however F_L^ℓ and A_{FB}^ℓ are insensitive to it, so not helpful

No, it can't! Anomalies in F_L^ℓ and A_{FB}^ℓ cannot be addressed using DM FFs!

Conclusions

- The measurement of $R(\Lambda_c)$ is not following the pattern of $R(D^{(*)})$, which would predict an higher value in the presence of NP coupled to τ . Adding NP to e/μ does not help
- However, several concerns might point to a problem on the exp. side, NP is still viable
- Recent developments in the determinations of $B \rightarrow D^*$ FF suggested the possibility of addressing $R(D^*)$ in the SM. However, incompatibility with recent data on A_{FB}^ℓ and F_L^ℓ invalidate the possibility

Back-up

Where is this coming from?

In order to understand the origin of the FF behaviours, it's instrumental to take a look at the helicity amp.

$$H_0(w) = \frac{\mathcal{F}_1(w)}{\sqrt{m_B^2 + m_{D^*}^2 - 2m_B m_{D^*} w}}$$

$$H_{\pm}(w) = f(w) \mp m_B m_{D^*} \sqrt{w^2 - 1} g(w)$$

which are used to build

$$\frac{d\Gamma}{dw} \propto |H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2$$

$$F_L^\ell(w) = \frac{|H_0(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

$$A_{\text{FB}}^\ell(w) = \frac{|H_-(w)|^2 - |H_+(w)|^2}{|H_0(w)|^2 + |H_+(w)|^2 + |H_-(w)|^2}$$

this implies for light leptons a peculiar behaviour for $\mathcal{F}_1(w)$

$\mathcal{F}_1(w)$	$d\Gamma^\ell/dw$	$\mathcal{R}(D^*)$	A_{FB}^ℓ	F_L^ℓ

The IgWa approach

Expand the FF $h_X(w) = \xi(w)\hat{h}_X(w)$, with $\xi(w)$ the leading Isgur-Wise function, in α_s and $1/m_{b,c}$

$$\hat{h}_X = \hat{h}_{X,0} + \frac{\alpha_s}{\pi} \delta\hat{h}_{X,\alpha_s} + \frac{\bar{\Lambda}}{2m_b} \delta\hat{h}_{X,m_b} + \frac{\bar{\Lambda}}{2m_c} \delta\hat{h}_{X,m_c} + \left(\frac{\bar{\Lambda}}{2m_c}\right)^2 \delta\hat{h}_{X,m_c^2}$$

$\propto m_i$ \propto sub-lead. I-W functs. $\xi_3(w), \chi_{2,3}(w)$ \propto sub-lead. I-W functs. $\ell_{1-6}(w)$

Expand each of the 10 I-W functs. as a power of z , and fit to theory and experiment data up to a different order for each of the functions, selected by goodness-of-fit

$$f(w) = f^{(0)} + 8f^{(1)}z + 16(f^{(1)} + 2f^{(2)})z^2 + \frac{8}{3}(9f^{(1)} + 48f^{(2)} + 32f^{(3)})z^3 + \mathcal{O}(z^4)$$

The BGJS approach

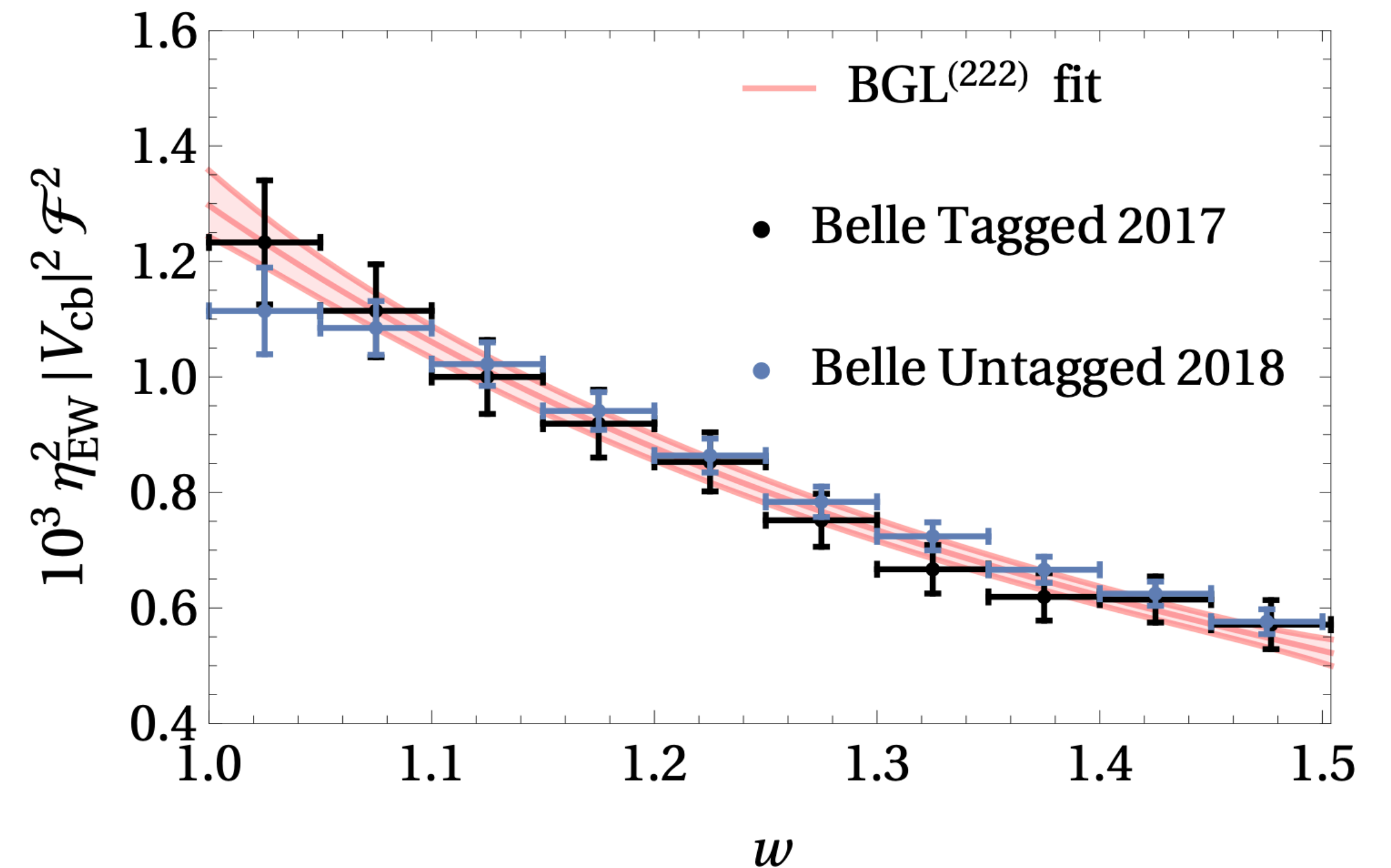
Expand the FF as a series in $z = (\sqrt{w+1} - \sqrt{2})/(\sqrt{w+1} + \sqrt{2})$, where $w = (m_B^2 + m_{D^*}^2 - q^2)/(2m_B m_{D^*})$

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

Different expansion order for each FF
(selected by goodness-of-fit)

Weak unitarity constraints imposed on series coefficients to ensure a rapid convergence of the series in the physical region, $0 < z < 0.056$

$$\sum_{k=0}^{n_g} (a_k^g)^2 < 1, \quad \sum_{i=0}^{n_f} (a_k^f)^2 + \sum_{k=0}^{n_{F_1}} (a_k^{F_1})^2 < 1$$

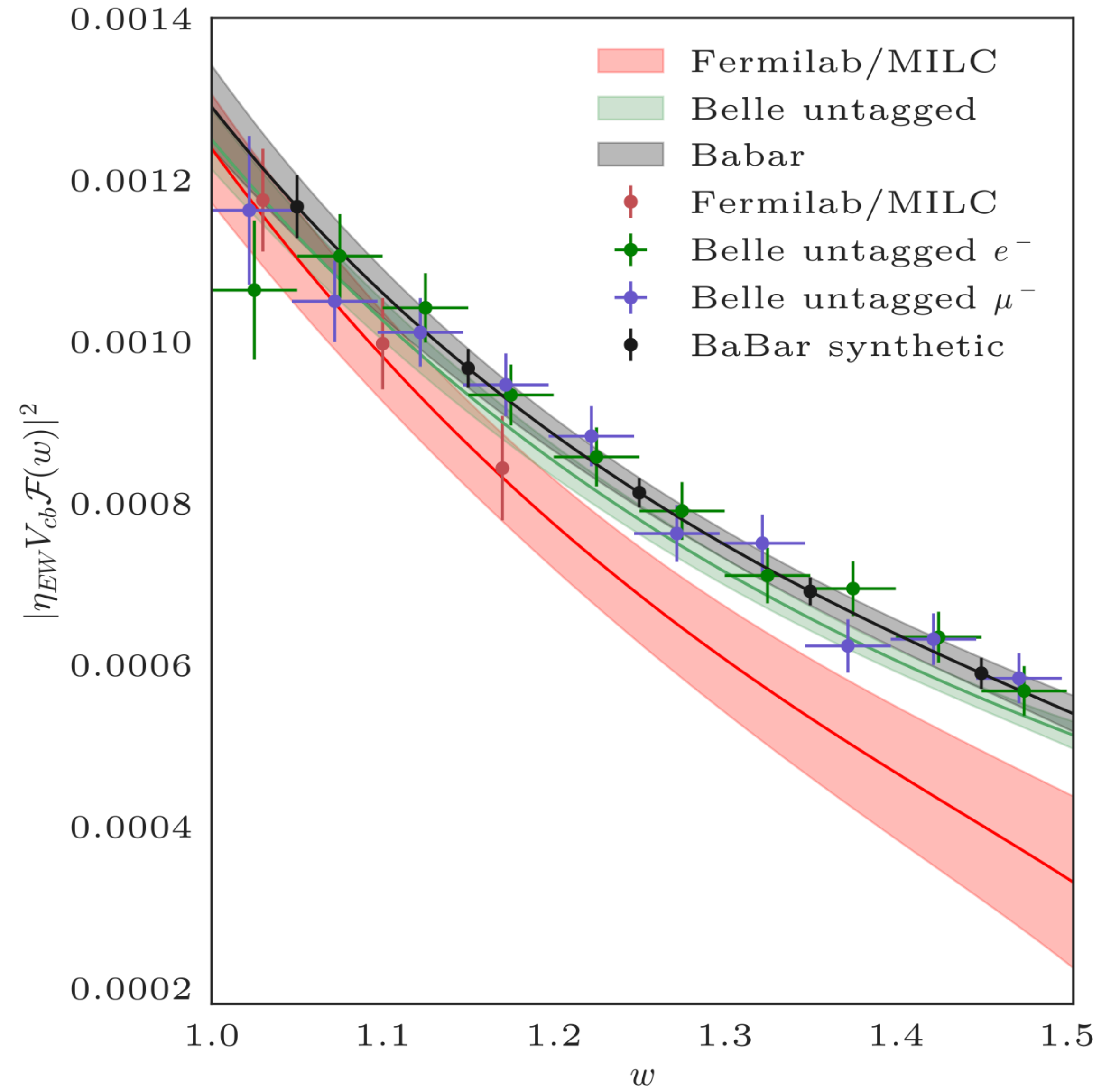


The Lattice approach

Employs the same parameterization as the BGL approach, first results beyond non-zero recoil have been recently obtained

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

Result is however not fully compatible with exp.
Problem with the slope?



The Dispersive Matrix approach

$$\beta(z) - \sqrt{\gamma(z)} \leq f(z) \leq \beta(z) + \sqrt{\gamma(z)}$$

$$\beta(z) \equiv \frac{1}{\phi(z)d(z)} \sum_{j=1}^N \phi_j f_j d_j \frac{1 - z_j^2}{z - z_j},$$

$$\gamma(z) \equiv \frac{1}{1 - z^2} \frac{1}{\phi^2(z)d^2(z)} (\chi - \chi_{\text{DM}}),$$

$$\chi_{\text{DM}} \equiv \sum_{i,j=1}^N \phi_i f_i \phi_j f_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}, \quad \Rightarrow$$

$$d(z) \equiv \prod_{m=1}^N \frac{1 - z z_m}{z - z_m},$$

$$d_j \equiv \prod_{m \neq j=1}^N \frac{1 - z_j z_m}{z_j - z_m}.$$

FF is given by convolution of $\beta(z)$ and $\gamma(z)$ with the distribution of input lattice data with $\chi > \chi_{\text{DM}}$, corresponding to the unitarity requirement: it represents the envelope of the results of all possible (non) truncated z-expansions

NP global fits w/out $R(\Lambda_c)$

- Model independent

	Pull [χ^2_{best}]	Fitted C_X
SM	- [22.4]	-
C_{V_L}	4.4 [2.8]	+0.08(2)
C_{V_R}	1.9 [18.8]	-0.05(3)
C_{S_L}	3.0 [13.3]	+0.17(5)
C_{S_R}	3.8 [7.9]	+0.20(5)
C_T	3.4 [10.6]	-0.03(1)

$$C_{V_R} \simeq 0.02 \pm i 0.43 \quad \text{Pull} = 4.1$$

$$C_{S_L} \simeq -0.58 \pm i 0.88 \quad \text{Pull} = 4.2$$

Re-emergence of scalar solutions due to latest measurement, which require smaller NP in $R(D^*)$ compared to $R(D)$

- Some model dependent

$$U_1 \text{ LQ: } C_{V_L} = 0.07, C_{S_R} = 0.06, \quad \text{Pull} = 3.8$$

$$S_1 \text{ LQ } (C_{V_L} = 0): C_{S_L} = -8.9 C_T = 0.19, \quad \text{Pull} = 3.9$$

$$R_2 \text{ LQ } (C_{V_R} = 0): C_{S_L} = 8.4 C_T = -0.07 \pm i 0.58, \quad \text{Pull} = 4.0$$