

Exclusive semileptonic *B* decays at Belle and Belle II

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Semileptonic *B* decays **Determination of the CKM elements** $|V_{cb}|$ and $|V_{\mu b}|$

- SL *B* decays are studied to determine the CKM elements $|V_{cb}|$ and $|V_{\mu b}|$
 - $|V_{xb}|$ are limiting the global constraining power of UT fits
 - Important inputs in predictions of SM rates for ultrarare decays such as $B_s \rightarrow \mu\mu$ and $K \rightarrow \pi\nu\nu$
- The determinations can be
 - *Exclusive* from a single final state
 - Inclusive sensitive to all SL final states \rightarrow talk by M. Prim Tue 12:30 pm (WG1&2)



	Experiment	Theory
Exclusive V _{cb}	$B \rightarrow Dlv, D^*lv$ (low backgrounds)	Lattice QC light cone s rules
Inclusive V _{cb}	B → Xlv (higher background)	Operator pro expansio

 V_{qb}



Experimental status $|V_{cb}|$ and $|V_{ub}|$



- Determinations of both $|V_{ch}|$ and $|V_{ub}|$ exhibit a discrepancy at the level of ~3 σ between exclusive and inclusive
- The current experimental focus is on understanding the origin of this discrepancy, as this inconsistency limits the power of precision flavour physics

Outline **Results covered in this presentation**

• Belle II

• $B^0 \to D^{*-} \ell^+ \nu$ and determination of $|V_{ch}|$ [189/fb, to be submitted to Phys. Rev. D]

• Belle

- $B \to D^* \ell \nu$ differential decay distributions and $|V_{ch}|$ [711/fb, Phys.Rev.D 108 (2023) 1, 012002, <u>arXiv:2301.07529 [hep-ex]]</u>
- Angular coefficients in $B \to D^* \ell \nu$ and $|V_{ch}|$ [711/fb, EPS-HEP23 preliminary]
- Measurement of $B \to \bar{D}^{(*)} \pi \ell^+ \nu, D^{(*)} \pi^+ \pi^- \ell^+ \nu$ [711/fb, Phys.Rev.D 107 (2023) 9, 092003, <u>arXiv:2211.09833 [hep-ex]]</u>

1999 – 2010: B factory at KEK (Japan)

KEKB double ring e+e- collider

$e^+e^- \to \Upsilon(4S) \to B\bar{B}$

Belle detector



Comparison of B factories (1999-2010)



 $> 1 ab^{-1}$ **On resonance:** $Y(5S): 121 \text{ fb}^{-1}$ $Y(4S): 711 \text{ fb}^{-1}$ $Y(3S): 3 \text{ fb}^{-1}$ $Y(2S): 24 \text{ fb}^{-1}$ $Y(1S): 6 \text{ fb}^{-1}$ **Off reson./scan:** $\sim 100 {\rm ~fb}^{-1}$

~ 550 fb⁻¹ **On resonance:** $Y(4S): 433 \text{ fb}^{-1}$ $Y(3S): 30 \text{ fb}^{-1}$ $Y(2S): 14 \text{ fb}^{-1}$ **Off resonance:** $\sim 54 \text{ fb}^{-1}$

The Belle II detector







Untagged vs. Tagged

Untagged:

only $B_{\rm sig}$ is reconstructed

high signal yield (+) high backgrounds (-) poor neutrino reconstruction (-)





Tagged:

 $B_{\rm sig}$ and $B_{\rm tag}$ are reconstructed to take advantage of $\Upsilon(4S)$ kinematics

signal yield O(10³) lower (-) low backgrounds (+) good neutrino reconstruction (+) tag calibration (-)

n

 π^+

 $\bar{\mathbf{D}}_0$

 e^+





$B^0 \rightarrow D^{*-} \ell^+ \nu$ untagged (189/fb) remains to be submitted to Phys. Rev. D

Updated numbers with respect to winter 2022/23 presentations

Parameterisation of $B \rightarrow D^* \ell \nu$

Three form-factors as function of $w = v_B \cdot v_{D^*}$ parameterise the non-perturbative physics

 $d^4\Gamma$ $\frac{1}{dwd\cos\theta_{\ell}d\cos\theta_{V}d\chi} \propto |V_{cb}|^{2}A(w,\cos\theta_{\ell},\cos\theta_{V},\chi)$

- Form factor parameterisations
 - Boyd, Grinstein, Lebed (BGL) lacksquare[Phys. Rev. D56, 6895 (1997)].

Caprini, Lellouch, Neubert (CLN) [Nucl. Phys. B530, 153 (1998)]

$$g(z) = rac{1}{P_g(z)}$$
 $f(z) = rac{1}{P_f(z)}$
 $\mathcal{F}_1(z) = rac{1}{P_{\mathcal{F}_1}(z)}$

$$h_{A_1}(z) = h_{A_1}(w = 1) \left(1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3 \right)$$

$$R_1(w) = R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2$$

$$R_2(w) = R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2$$





Measurement

- $D^{*+} \rightarrow D^0(\rightarrow K^-\pi^+)\pi^+$ is reconstructed and combined with an appropriately charged lepton (*e* or μ)
- The neutrino direction is reconstructed inclusively using the known angle $\cos \theta_{BY}$ between the B and the $Y = D^* + \ell$ direction

$$\cos \theta_{BY} = \frac{2E_B^{\rm CM} E_Y^{\rm CM} - m_B^2 c^4 - m_Y^2 c^4}{2|\vec{p}_B^{\rm CM}||\vec{p}_Y^{\rm CM}|c^2}$$

- The yield in 10 (8) bins of w, $\cos \theta_{\ell}$, $\cos \theta_{V}$ and χ is extracted by fitting $\cos \theta_{BY}$ and $\Delta M = M(K\pi\pi) M(K\pi)$
- Bin-to-bin migration is corrected with SVD unfolding [arXiv:hep-ph/9509307]
- Main challenges: accurate background model, slow pion tracking and statistical correlations between bins



BGL fit result

BGL truncation order determined by Nested Hypothesis Test [Phys. Rev. D100, 013005]

Values	Correlations	s χ^2/nd	Preliminary
$\tilde{a}_0 imes 10^3$ 0.88 ± 0.05 1.00	0.26 - 0.23	8 0.19	
${ ilde b}_0^\circ imes 10^3 ext{ } 0.54 \pm 0.01 ext{ } 0.26$	5 1.00 -0.3	7 -0.43	Belle I
${ ilde b}_1^{"} imes 10^3 \; -0.31\pm 0.30 \; -0.28$	3 - 0.37 1.00	$\begin{array}{c} & 39/31 \\ 0 & 0.57 \end{array}$	$60 = \overline{B}^0 \to D^{*+}$
$ ilde{c}_1 imes 10^3 - 0.04 \pm 0.03 - 0.19$	0 - 0.43 0.5'	7 1.00	- 25
Relative uncertainty (%)	Preliminary		¥ 30
	$ ilde{a}_0 ilde{b}_0$	$ ilde{b}_1 ilde{c}_1$	dL/c
Statistical	3.7 0.8	65.1 50.8	10
Background subtraction	2.1 0.4	31.3 21.8	Q <u>[</u> 1.0 1
Finite MC samples	1.5 0.3	26.4 20.5	D
Lepton ID efficiency	1.6 0.3	3.4 2.8	
Tracking of K, π, ℓ	0.4 0.4	0.5 0.4	$ [\overline{B}^0 \to D^{*+}] $
Slow pion efficiency	1.6 1.5	23.8 24.7	Ge Ge
$N_{P\overline{P}}$	0.8 0.8	0.8 0.8	ម្នុ ¹⁵
$f_{\pm 0}$	1.3 1.3	1.3 1.2	
$\mathcal{B}(D^{*+} \to D^0 \pi^+)$	0.4 0.4	0.4 0.4	
$\mathcal{B}(D^0 \to K^- \pi^+)$	0.4 0.4	0.4 0.4) COS
B^{0} lifetime	0.1 0.1	0.1 0.1	
Signal modelling	2.3 0.5	52.1 35.0	
Total	5.8 2.5	96.0 73.0	-1.00 -0.75

LQCD used only for normalisation at zero recoil (w = 1)







Fit with non-zero recoil LQCD

LQCD constraints at w = 1.03, 1.10, 1.17[Eur. Phys. J. C 82, 1141 (2022)]

	Constraints on	Constraints on
	$h_{A_1}(w)$	$h_{A_1}(w), R_1(w), R_2(w)$
$a_0 imes 10^3$	$21.7 \hspace{0.2cm} \pm 1.3 \hspace{0.2cm}$	$25.6 \pm 0.8 $
$b_0 imes 10^3$	13.19 ± 0.24	13.61 ± 0.23
$b_1 imes 10^3$	-6 ± 6	2 ± 6
$c_1 imes 10^3$	$-0.9 ext{ }\pm 0.7 ext{ }$	$0.0 \ \pm 0.7$
$ V_{cb} \times 10^3$	$40.3 \hspace{0.2cm} \pm 1.2 \hspace{0.2cm}$	$38.3 \hspace{0.2in} \pm 1.1 \hspace{0.2in}$
χ^2/ndf	39/33	75/39
p-value	21%	0.04%

Preliminary

Inclusion of $h_{A_1}(w)$ Inclusion of $h_{A_1}(w)$ $R_1(w)$ and $R_2(w)$ FNAL/MILC



Summary of the measurement

Branching fraction

$$\mathcal{B}(\overline{B}^0 \to D^{*+} \ell^- \bar{\nu}_\ell) = (4.922)$$

• Value of $|V_{ch}|$

$$|V_{cb}|_{
m BGL} = (40.57 \pm 0.31 \pm 0.9)$$

 $|V_{cb}|_{
m CLN} = (40.13 \pm 0.27 \pm 0.9)$

Lepton flavour universality tests

 $R_{e/\mu} = 0.998 \pm 0.009 \pm 0.020$ $\Delta A_{\rm FB} = (-17 \pm 16 \pm 16) \times 10^{-3}$ $\Delta F_L = 0.006 \pm 0.007 \pm 0.005$

 $\pm 0.023 \pm 0.220)\%$

Preliminary





$|V_{cb}|$ and $|V_{ub}|$ from Belle II

• Recent, preliminary results from exclusive decays

	$ V_{cb} \times 10^3$			Reference	
Belle II $B^0 \to D^{*-} \ell^+ \nu$ untagged	40.57 ± 1.16 (BGL)	Prelim	inary	To be submitted to PRD	
Belle II $B^0 \to D^{*-} \ell^+ \nu$ tagged	37.9 ± 2.7 (CLN)	Prelim	inary	[arXiv:2301.04716]	
Belle II $B \to D\ell\nu$ untagged	38.28 ± 1.16 (BGL)	Prelim	inary	[arXiv:2210.13143]	
	$ V_{ub} \times 10^3$			Reference	
Belle II $B \to \pi e \nu$ tagged	3.88 ± 0.45	Prelin	ninary	[arXiv:2206.08102]	
Belle II $B \to \pi \ell \nu$ untagged	3.55 ± 0.25	Prelin	ninary	[arXiv:2210.04224]	

WA values [HFLAV 2021] $|V_{cb}|_{excl} = (39.10 \pm 0.50) \times 10^{-3}$ $|V_{ub}|_{excl} = (4.19 \pm 0.17) \times 10^{-3}$





$B \rightarrow D^* \ell^+ \nu$ differential decay distributions (711/fb) Phys.Rev.D 108 (2023) 1, 012002, arXiv:2301.07529 [hep-ex]]

Reconstruction

- One *B* meson is fully reconstructed in a hadronic mode (tag side)
- $D^{*+} \rightarrow D^0 \pi^+, D^+ \pi^0, D^{*0} \rightarrow D^0 \pi^0$ are searched on signal side
 - D^{*0} channel has higher signal efficiency than D^{*+} near zero recoil
- And combined with an appropriately charged $\ell'=e,\mu$ to identify a $B \to D^*\ell' \nu$ decay



Extraction Method: Missing Mass Squared $0 = m_{\nu}^2 = M_{miss}^2 = (p_{e^+e^-} - p_B - p_{D^*} - p_{\ell})^2$



Results

- Normalisation is not measured, \mathscr{B} taken from HFLAV
- Fit to marginal distributions of $w, \cos \theta_{\ell}, \cos \theta_{V}, \chi$ (40 bin



$$|V_{cb}|^{BGL} = (40.6 \pm 0.9) \times 10^{-3}$$

 $|V_{cb}|^{CLN} = (40.1 \pm 0.9) \times 10^{-3}$

using
$${\cal F}(1)=0.906\pm 0.013$$

LFV observables

$$\Delta A_{FB} = A_{FB}^{\mu} - A_{FB}^{e} = 0.022 \pm 0.027$$

$$\Delta F_{L} = F_{L}^{\mu} - F_{L}^{e} = 0.034 \pm 0.024$$

$$R_{e\mu} = \frac{\mathcal{B}(B \to D^{*} e \bar{\nu}_{e})}{\mathcal{B}(B \to D^{*} \mu \bar{\nu}_{\mu})} = 0.990 \pm 0.031$$

BGL332 is the LQCD prediction Eur.Phys.J.C 82 (2022) 12, 1141, Eur.Phys.J.C 83 (2023) 1, 21 (erratum) arXiv:2105.14019 [hep-lat]



Angular coefficients of $B \rightarrow D^* \ell^+ \nu$ (711/fb) EPS-HEP 2023 preliminary



Preliminary

Reconstruction

- Same dataset, fit 12 angular coefficients $J_i = J_i(w)$ in 4 bins of w
- In total 144 $M_{\rm miss}^2$ fits per mode, 576 in total



$$\frac{V_{cb}^{2}m_{B}^{4}m_{D^{*}}}{2\pi^{4}} \times \left(J_{1s}\sin^{2}\theta_{V} + J_{1c}\cos^{2}\theta_{V}\right)$$

$$in^{2}\theta_{V} + J_{2c}\cos^{2}\theta_{V})\cos 2\theta_{\ell} + J_{3}\sin^{2}\theta_{V}\sin^{2}\theta_{\ell}\cos 2\chi$$

$$2\theta_{V}\sin 2\theta_{\ell}\cos \chi + J_{5}\sin 2\theta_{V}\sin \theta_{\ell}\cos \chi + (J_{6s}\sin^{2}\theta_{V} + J_{6c}\cos^{2}\theta_{V})c$$

$$2\theta_{V}\sin \theta_{\ell}\sin \chi + J_{8}\sin 2\theta_{V}\sin 2\theta_{\ell}\sin \chi + J_{9}\sin^{2}\theta_{V}\sin^{2}\theta_{\ell}\sin 2\chi\right).$$



Fitted angular coefficients



Preliminary

Results

Belle

1.0 ·

0.9

0.8

0.6

0.5

0.4

1.0

1.1

(M)¹ 0.7

$$|V_{cb}|^{BGL} = (41.0 \pm 0.7) \times 10^{-3}$$

 $|V_{cb}|^{CLN} = (40.9 \pm 0.7) \times 10^{-3}$

1.3

w [1]

CLN^{Fit} BGL₃₃₂

1.2

Preliminary

1.4

1.6

1.4 ·

1.2 μ¹(Μ)

1.0

0.8 -

1.5

1.0

1.1



• Combined fit with non-zero recoil lattice data (FNAL/MILC, HPQCD, JLQCD) • p = 0.75 for BGL₃₃₂ • Average $\mathcal{F}(1)$ of new LQCD results: $\mathcal{F}(1) = 0.895 \pm 0.007$





Measurement of $B \rightarrow \bar{D}^{(*)}\pi \ell^+ \nu$ and $B \rightarrow \bar{D}^{(*)}\pi^+\pi^- \ell^+ \nu$ Phys.Rev.D 107 (2023) 9, 092003, arXiv:2211.09833 [hep-ex]]

Reconstruction

- 711/fb of hadronically tagged events
- 16 final states are searched for on the signal side: $\bar{D}^0\pi^-, D^-\pi^+, \bar{D}^{*0}\pi^-, D^{*-}\pi^+, D^-\pi^+\pi^-, \bar{D}^0\pi^+\pi^-, D^{*-}\pi^+, D^{*-}$ with $\ell = e, \mu$
- Normalisation modes: $B^0 \rightarrow D^{*-} \ell^+ \nu$ and $B^+ \rightarrow \bar{D}^{*0} \ell^+ \nu$
- Signal extracted from $U = E_{\text{miss}} p_{\text{miss}}$

$$\pi^{+}\pi^{-}, D^{*0}\pi^{+}\pi^{-}\ell^{+}\nu^{-}$$



Branching fraction results (π) $D^0\pi$ \vdash 0.50.30.350.450.550.4





 $D\pi$





D composition**

Candidates / (0.0075 GeV/c²) Candidates / (0.0075 GeV/c²) 80 120 $B^0 \to \overline{D}{}^0 \pi^- \ell^+ \nu_\ell$ $B^+ \rightarrow D^- \pi^+ \ell^+ \nu_\ell$ Belle Belle 70 È 100 — Total — Total 60 \overline{D}_2^{*0} D_2^* 50 80 $- \overline{D}_{0}^{*0}$ other $\overline{D}{}^0\pi^-$ 40 E 60 - other $D^-\pi^+$ 30 20 40 20 Pull Pull 0 հինը-շրենն-2.1 2.2 2.3 2.4 2.7 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.5 2.6 2.8 2.8 m(**D**⁰π[−]) (GeV/c²) $m(D^{-}\pi^{+})$ (GeV/c²)

	yield	branching fraction [%]	PDG [%]
$B^0 \! ightarrow D_0^{*-} \ell^+ u_\ell$ with $D_0^{*-} \! ightarrow \overline{D}{}^0 \pi^-$	-	$<\!0.044$ at 90% CL	0.30 ± 0.12
$B^0 \! ightarrow D_2^{st -} \ell^+ u_\ell$ with $D_2^{st -} \! ightarrow \overline{D}{}^0 \pi^-$	$457{\pm}45$	$0.157\pm0.015(ext{stat})\pm0.005(ext{syst})$	0.121 ± 0.033
other $B^0 ightarrow \overline{D}{}^0 \pi^- \ell^+ u_\ell$	547 ± 45	-	
$B^+ \! ightarrow \overline{D}_0^{*0} \ell^+ u_\ell$ with $\overline{D}_0^{*0} \! ightarrow D^- \pi^+$	$180{\pm}72$	$0.054\pm0.022(ext{stat})\pm0.005(ext{syst})$	$0.25 \hspace{0.2cm} \pm \hspace{0.2cm} 0.05 \hspace{0.2cm}$
$B^+ ightarrow \overline{D}_2^{*0} \ell^+ u_\ell$ with $\overline{D}_2^{*0} ightarrow D^- \pi^+$	$590{\pm}39$	$0.163 \pm 0.011({ m stat}) \pm 0.008({ m syst})$	0.153 ± 0.016
other $B^+ \rightarrow D^- \pi^+ \ell^+ \nu_\ell$	520 ± 70	-	

 $M(D\pi)$





 $353\pm~93$

 382 ± 74

 $0.138 \pm 0.036 \,({\rm stat}) \pm 0.009 \,({\rm syst})$

 $0.137 \pm 0.026 \,(\text{stat}) \pm 0.009 \,(\text{syst})$

 $B^+ \to \overline{D}_2^{*0} \ell^+ \nu_\ell$ with $\overline{D}_2^{*0} \to D^{*-} \pi^+$

 $0.27 \hspace{0.2cm} \pm \hspace{0.2cm} 0.06 \hspace{0.2cm}$

 0.101 ± 0.024

 $M(D\pi^+\pi^-)$



First observation of $B \to \overline{D}_1 (\to \overline{D}\pi^+\pi^-) \ell^+ \nu$



 $\mathcal{B}(B^0 \to D_1^- \ell^+ \nu_\ell) \times \mathcal{B}(D_1^- \to D^- \pi^+ \pi^-) = (0.102 \pm 0.013 \, (\text{stat}) \pm 0.009 \, (\text{syst}))\%$ $\mathcal{B}(B^+ \to \overline{D}_1^0 \ell^+ \nu_\ell) \times \mathcal{B}(\overline{D}_1^0 \to \overline{D}^0 \pi^+ \pi^-) = (0.105 \pm 0.011 \,(\text{stat}) \pm 0.009 \,(\text{syst}))\%$

Summary and conclusions

- Two new results for $|V_{cb}|$ from $B \to D^* \ell \nu$: Belle II $B^0 \to D^{*-}\ell^+\nu$ untagged (189/fb), Belle $B \to D^*\ell\nu$ tagged (711/fb)
 - Obtain values closer to the inclusive result/expection from CKM unitary
 - No significant form factor dependence
 - Is experiment compatible with non zero recoil lattice data?
- Improved Belle measurement of $B \rightarrow D^{**} \ell \nu$ modes
 - First observation of $B \to \overline{D}_1 (\to \overline{D}\pi^+\pi^-) \ell^+ \nu$
- Many more Belle II analyses on exclusive semileptonic B decays are in the making stay tuned!



Backup

$B^0 \rightarrow D^{*-} \ell^+ \nu$ untagged (189/fb) CLN fit

Fitted values without LQCD predictions				
	Values	Correlations	χ^2/ndf	
$- ho^2$	1.22 ± 0.05	1.00 0.36 -0.81 0.29		
$R_1(1)$	1.14 ± 0.07	0.36 1.00 -0.60 -0.10	20/21	
$R_{2}(1)$	0.89 ± 0.03	-0.81 -0.60 1.00 -0.08	39/31	
$ V_{cb} imes 10^3$	$40.1 \hspace{0.2cm} \pm \hspace{0.2cm} 1.1 \hspace{0.2cm}$	0.29 -0.10 -0.08 1.00		

Relative uncertainties (in %)

Preliminary

	$ ho^2$	$R_{1}(1)$	$R_{2}(1)$	$ V_{cb} $
Statistical	3.0	4.1	2.8	0.7
Background subtraction	1.4	2.2	1.2	0.3
Finite MC samples	1.2	1.7	1.1	0.3
Lepton ID efficiency	0.2	1.6	0.1	0.3
Slow pion efficiency	1.0	0.9	0.8	1.5
Tracking of K, π, ℓ				0.4
$N_{B\overline{B}}$				0.8
$f_{\pm 0}$				1.3
$\mathcal{B}(D^{*+} o D^0 \pi^+)$				0.4
${\cal B}(D^0\to K^-\pi^+)$				0.4
B^0 lifetime				0.1
Signal modelling	2.6	2.6	2.0	0.5
Total	4.5	5.9	3.9	2.4

Preliminary

$|V_{cb}|_{ m CLN} = (40.13 \pm 0.27 \pm 0.93 \pm 0.58) \times 10^{-3}$

using ${\cal F}(1)=0.906\pm 0.013$

Fitted values with LQCD constraints
at $w = 1.03, 1.10, 1.17$
[Eur. Phys. J. C 82, 1141 (2022)]

Preliminary

	Constraints on	Constraints on
	$h_{A_1}(w)$	$h_{A_1}(w), R_1(w), R_2(w)$
$h_{A_1}(1)$	0.91 ± 0.02	0.94 ± 0.02
$ ho^2$	1.22 ± 0.05	1.21 ± 0.04
$R_1(1)$	1.14 ± 0.07	1.26 ± 0.04
$R_{2}(1)$	0.88 ± 0.03	0.88 ± 0.03
$ V_{cb} \times 10^3$	$40.3 \hspace{0.2cm} \pm 1.2 \hspace{0.2cm}$	38.7 ± 1.1
χ^2/ndf	39/33	70/39
p-value	23%	0.2%
$\chi^2/\mathrm{ndf} \ p ext{-value}$	$39/33 \\ 23\%$	$70/39 \\ 0.2\%$







Nested hypothesis test Phys.Rev.D 100 (2019) 1, 013005, arXiv:1902.09553 [hep-ph]

Belle II $B^0 \to D^{*-} \ell^+ \nu$ untagged

(n_a, n_b, n_c)	$ V_{cb} \times 10^3$	$ ho_{ m max}$	χ^2	Ndf	<i>p</i> -value
(1,1,2)	40.2 ± 1.1	0.43	40	32	16%
(2,1,2)	40.1 ± 1.1	0.97	38.6	31	16%
(1, 2, 2)	$40.6{\pm}1.2$	0.57	38.9	31	16%
(1,1,3)	40.1 ± 1.1	0.96	39.5	31	14%
(2,2,2)	40.3 ± 1.3	0.99	38.6	30	13%
(1,3,2)	40.0 ± 1.3	0.98	38	30	15%
(1,2,3)	40.5 ± 1.2	0.96	38.8	30	13%

Belle $B \to D^* \ell^+ \nu$ tagged

	$ V_{ m cb} $	χ^2	dof	Ν	$ ho_{ m max}$
BGL_{111}	40.3 ± 0.8	45.7	32	3	0.71
BGL_{112}	40.8 ± 0.8	42.6	31	4	0.97
\mathbf{BGL}_{121}	$\textbf{40.6} \pm \textbf{0.9}$	45.3	31	4	0.62
BGL_{122}	41.4 ± 1.0	41.5	30	5	0.97
BGL_{131}	39.9 ± 0.9	42.4	30	5	0.61
BGL_{132}	40.7 ± 1.0	39.3	29	6	0.98
BGL_{211}	39.8 ± 0.9	42.1	31	4	0.99
BGL_{212}	40.4 ± 0.9	37.5	30	5	0.99
BGL_{221}	40.9 ± 1.0	45.1	30	5	0.93
BGL_{222}	39.2 ± 1.0	36.5	29	6	0.96
BGL_{231}	40.3 ± 1.0	41.8	29	6	0.94
BGL_{232}	41.0 ± 1.0	39.0	28	7	0.97
BGL_{311}	39.8 ± 0.9	42.1	30	5	0.99
BGL_{312}	40.4 ± 0.9	37.4	29	6	0.99
BGL_{321}	38.5 ± 0.9	39.4	29	6	0.65
BGL_{322}	39.2 ± 1.0	36.4	28	7	0.96
BGL_{331}	38.3 ± 0.9	38.1	28	7	0.86
BGL_{332}	38.7 ± 1.5	36.0	27	8	0.99