CKM metrology with semileptonic B decays at LHCb

CKM 2023 @ Santiago de Compostela, Spain

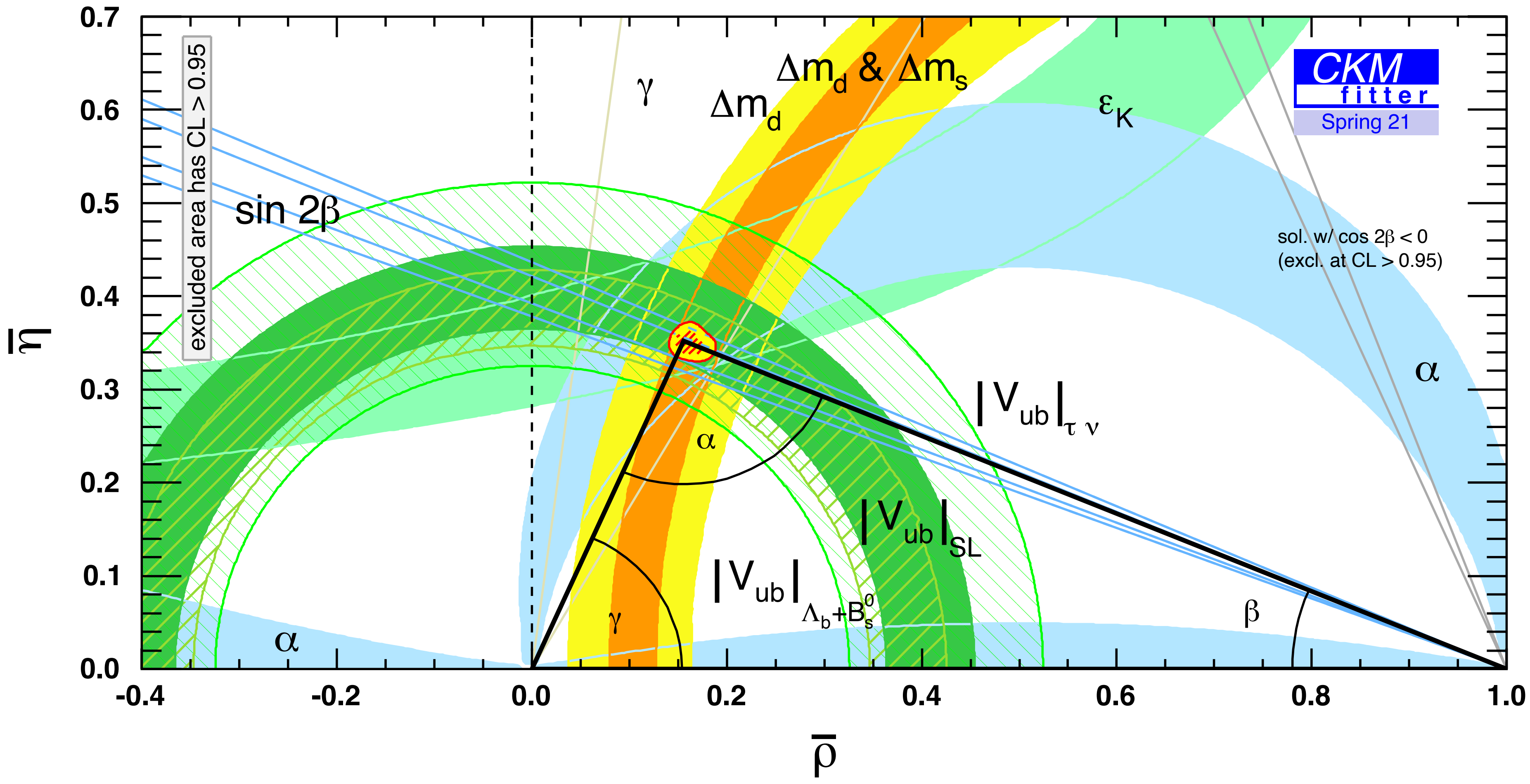
September 19th, 2023

Blaise Delaney, on behalf of the LHCb Collaboration

Introduction

Probing the CKM picture using semileptonic decays

- CKM matrix elements are **fundamental** SM parameters
- **Closure** of the Unitarity Triangle a null test of the SM
- Semileptonic decays of heavy hadrons involve **one hadronic current**
  → *clean* laboratory to perform CKM metrology



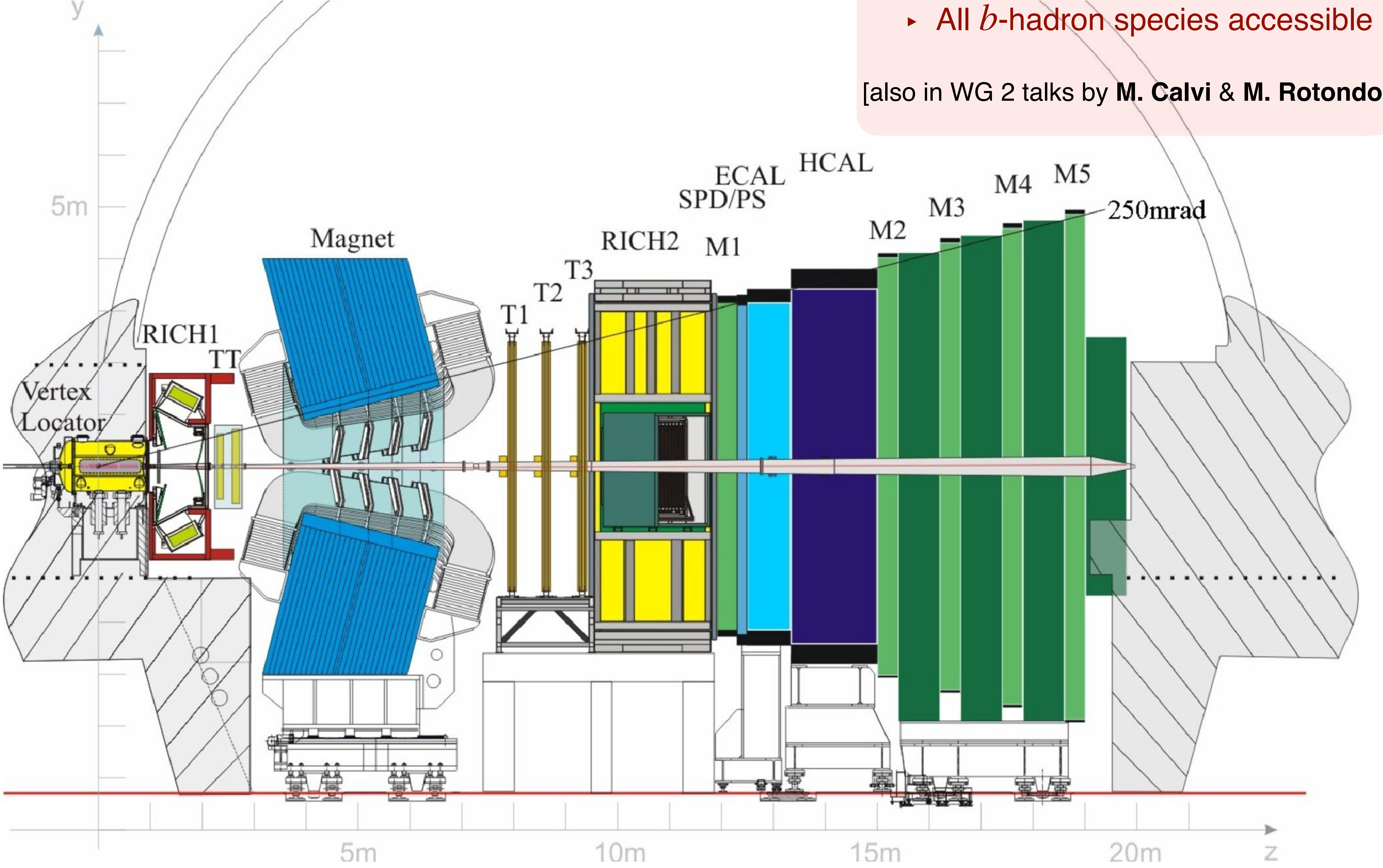
The LHCb detector

Int. J. Mod. Phys. A 30 (2015) 1530022

Single-arm forward spectrometer

- ~25 kHz $b\bar{b}$, ~500 kHz
- Large samples of semileptonic decays
- All b-hadron species accessible

[also in WG 2 talks by **M. Calvi** & **M. Rotondo**]



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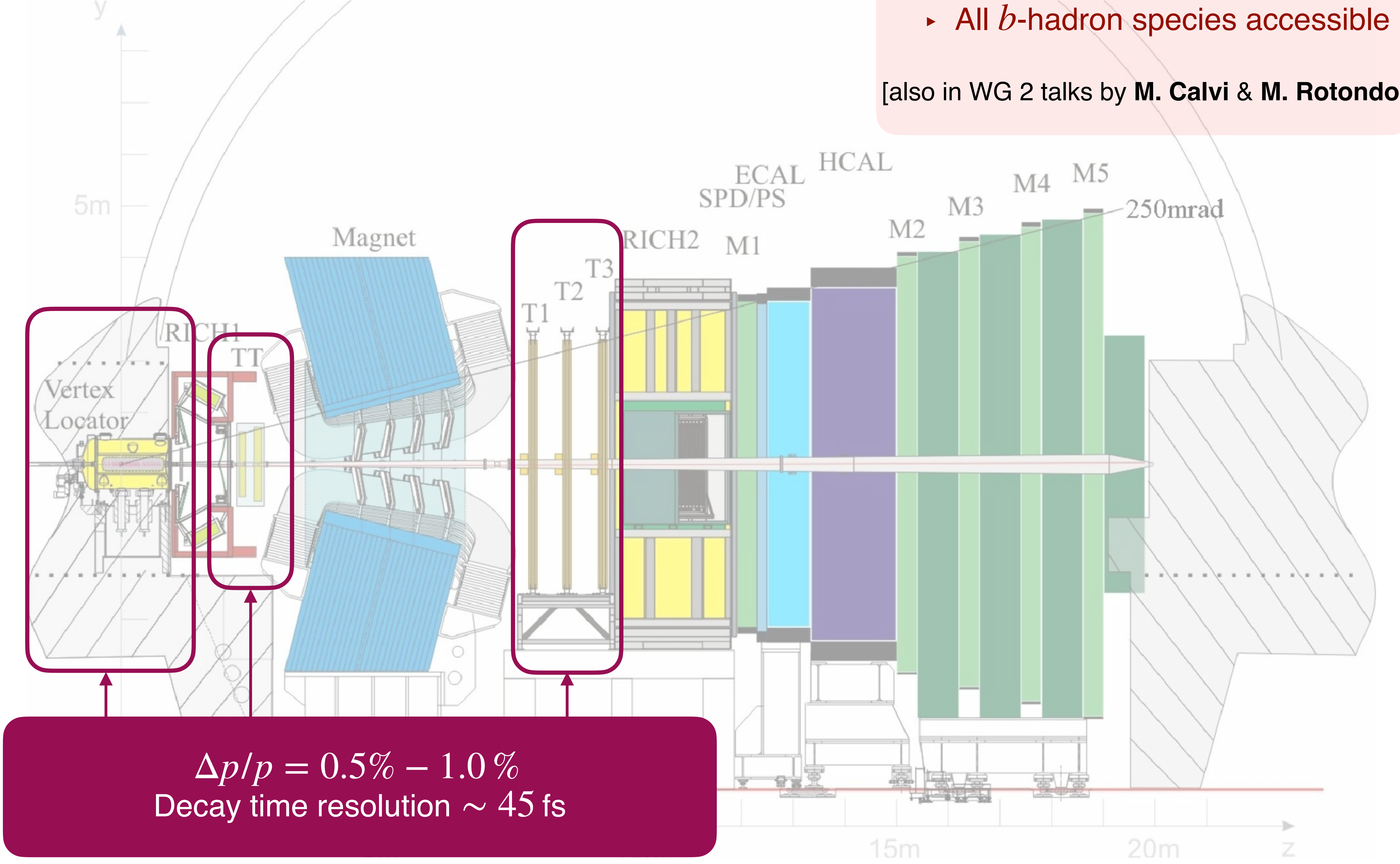

$\Delta p/p = 0.5\% - 1.0\,\%$

Decay time resolution ~ 45 fs

The LHCb detector

Int. J. Mod. Phys. A 30 (2015) 1530022

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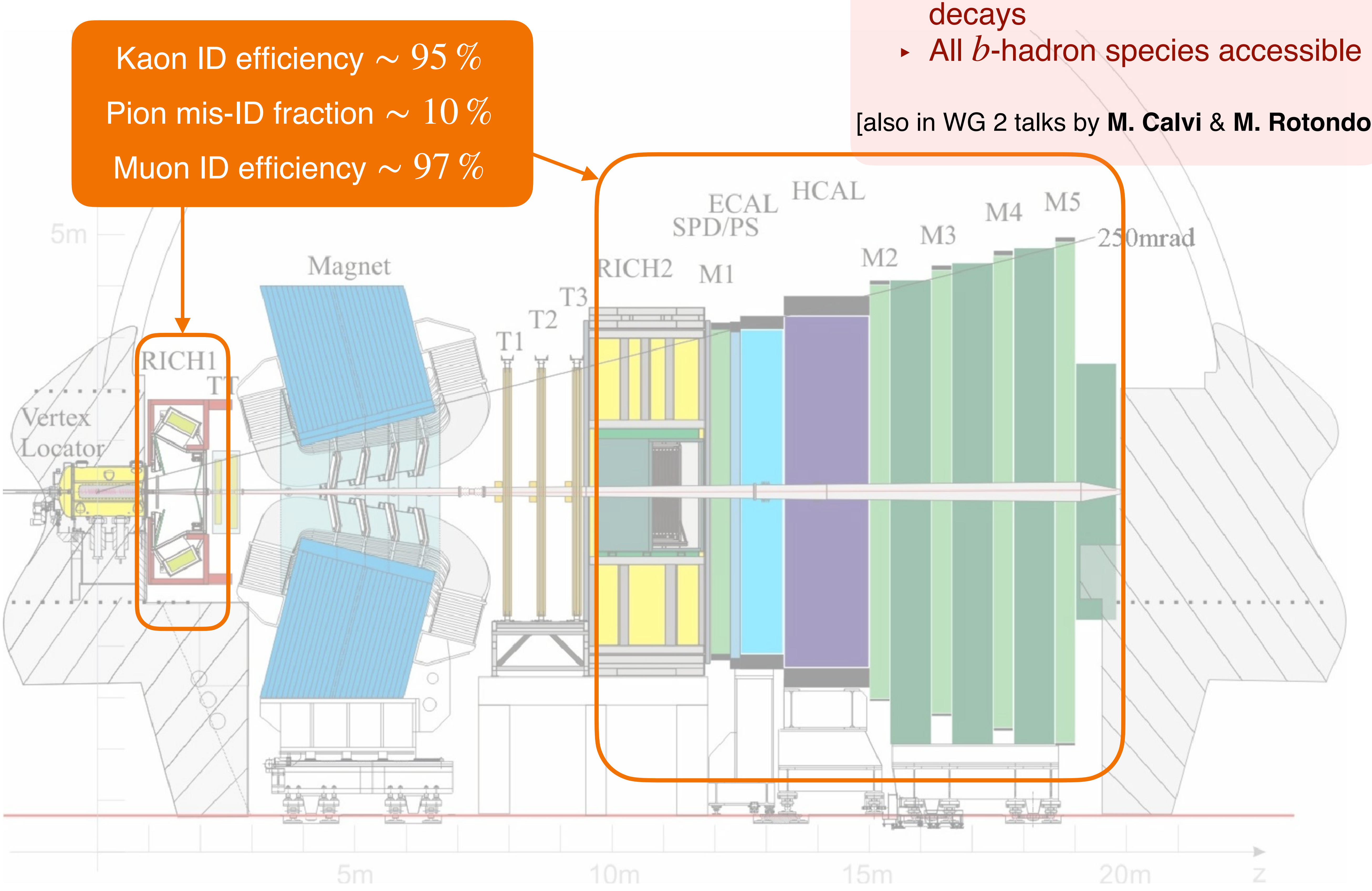


Kaon ID efficiency $\sim 95\,\%$

Pion mis-ID fraction $\sim 10\,\%$

Muon ID efficiency $\sim 97\,\%$

Introduction

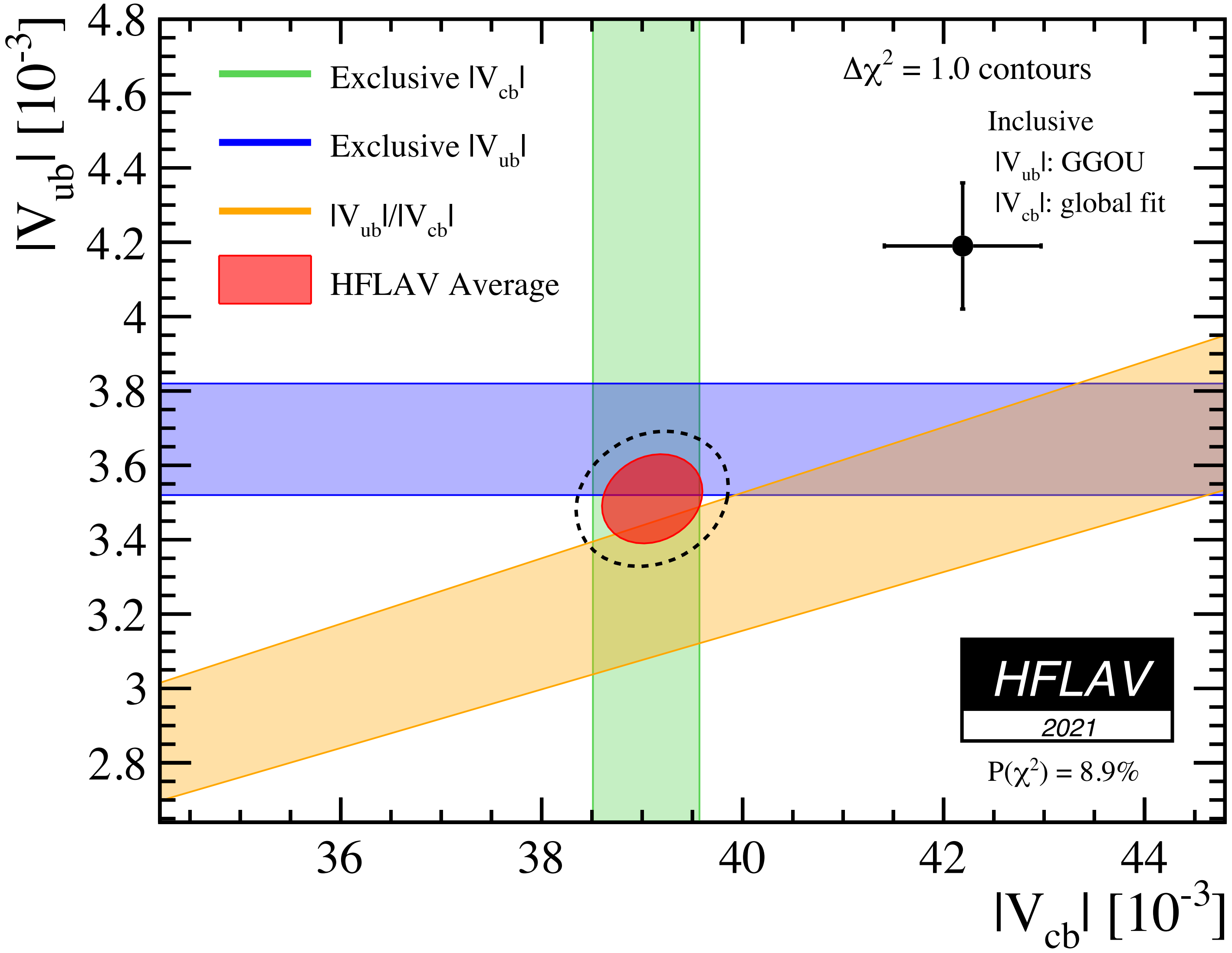
$|V_{cb}|$ and $|V_{ub}|/|V_{cb}|$ @ LHCb

Long-standing tension ($\sim 3\sigma$) between $|V_{\{c,u\}b}|$ **inclusive** and **exclusive** determinations.

LHCb:

- $|V_{ub}|/|V_{cb}|$ via Λ_b^0 decays [Nature Physics 11 (2015)]

- B_s^0 **system**:
  a) Theoretically advantageous : $m_s \gg m_u, m_d$
  b) Experimentally appealing:
    - $\sim 10^{10}$ B_s^0 per fb^{-1} produced
    - Reduced *part-reco* pollution than $B^{0/+}$



Introduction

$|V_{cb}|$ and $|V_{ub}|/|V_{cb}|$ @ LHCb

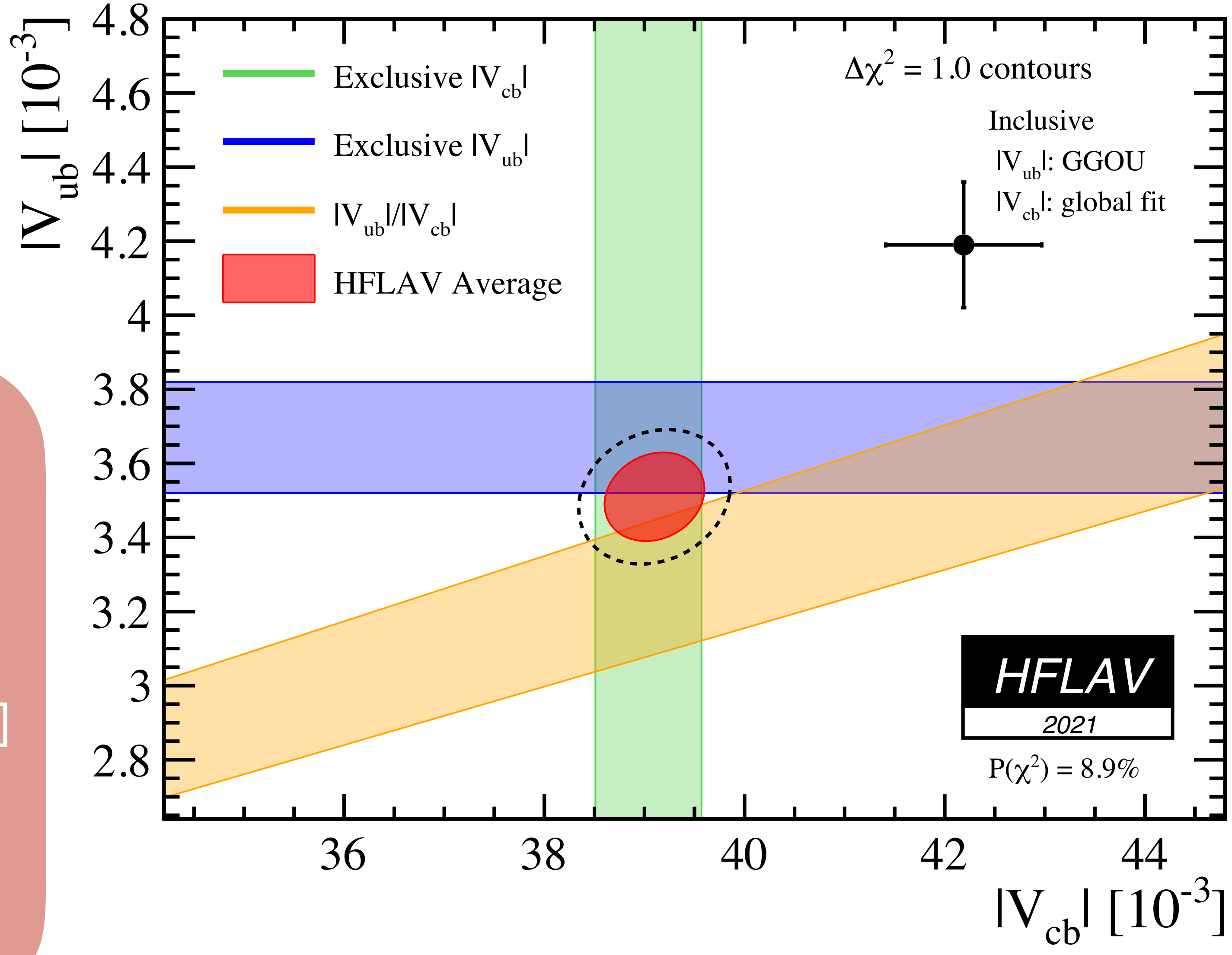
Long-standing tension (~3σ) between $|V_{\{c,u\}b}|$ **inclusive** and **exclusive** determinations.

LHCb:

- $|V_{ub}|/|V_{cb}|$ via Λ_b^0 decays [Nature Physics 11 (2015)]
- B_s^0 **system**:

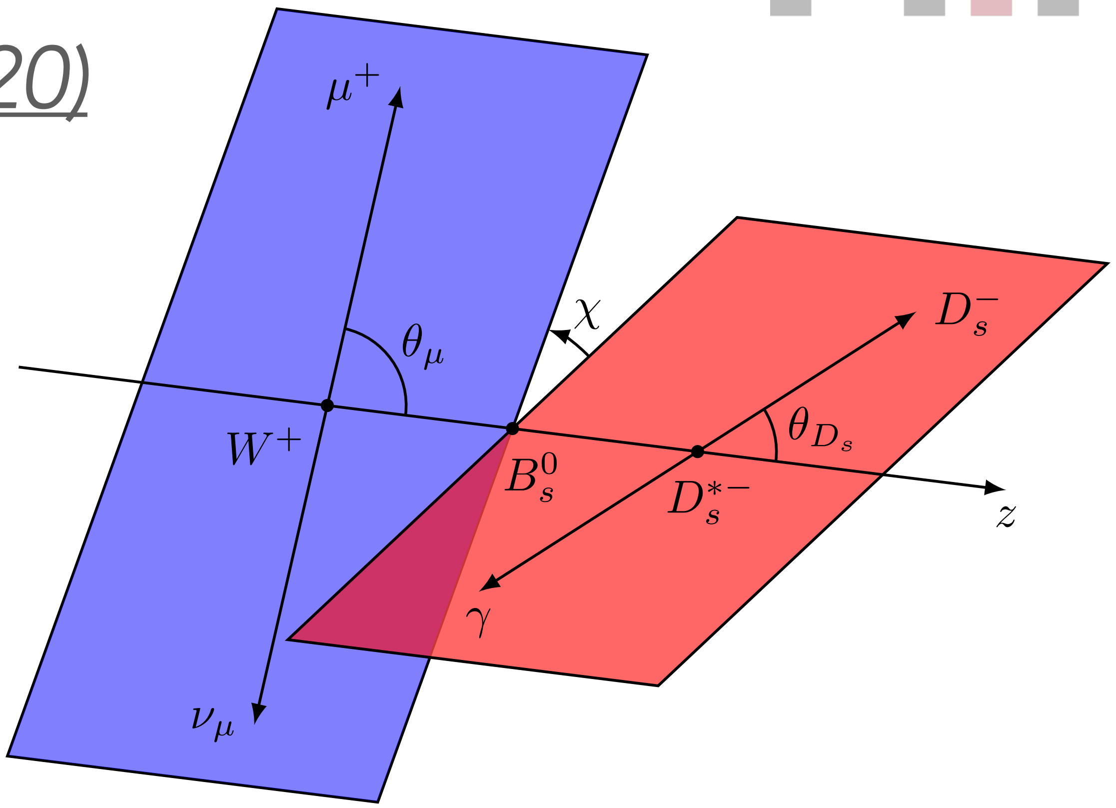
Today:

1. Extraction of $|V_{cb}|$ via $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ [PRD 101, 072004]
2. The differential decay rate of $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ [JHEP 2020, 144 (2020)]
3. Extraction of $|V_{ub}|/|V_{cb}|$ and observation of $B_s^0 \to K^-\mu^+\nu_\mu$ [PRL 126, 081804]



$B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ decay rate formalism

PRD 101, 072004, JHEP 2020, 144 (2020)



$$\frac{\mathrm{d}\Gamma(B_s^0 \to D_s^- \mu^+ \nu_\mu)}{\mathrm{d}w} = \frac{G_F^2 m_{D_s}^3}{48\pi^3}(m_{B_s} + m_{D_s})^2 \eta_{\mathrm{EW}}^2 \times$$
$$|V_{cb}|^2 (w^2-1)^{3/2} \underbrace{|\mathcal{G}(w)|^2}_{1\ \mathrm{FF}}$$

$$\frac{\mathrm{d}\Gamma(B_s^0 \to D_s^{*-} \mu^+ \nu_\mu)}{\mathrm{d}w\,\mathrm{d}\cos\theta_\mu\,\mathrm{d}\cos\theta_{D_s}\,\mathrm{d}\chi} = \frac{3 m_{B_s}^3 m_{D_s^*}^2 G_F^2}{16(\pi)^4}\eta_{\mathrm{EW}}^2 \times$$
$$|V_{cb}|^2 \underbrace{|\mathcal{A}(w, \theta_\mu, \theta_{D_s}, \chi)|^2}_{3\ \mathrm{FF}}$$

with
$$w = v_B \cdot v_{D_s^{(*)}}$$
$$= (m_{B_s}^2 + m_{D_s^{(*)}} - q^2)/(2 m_B m_{D_s^{(*)}})$$
$$q^2 = (p_{B_s} - p_{D_s^{(*)}})^2$$

Parameterisations to model the FF adopted in exclusive $|V_{cb}|$:

a) Caprini, Lellouch, Neubert (**CLN**) [Nucl. Phys. B530 (1998) 153]

b) Boyd, Grinstein, Lebed (**BGL**) [Phys. Rev. Lett. 74 (1995) 4603]

Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ decays

Phys. Rev. D 101 2020, 072004

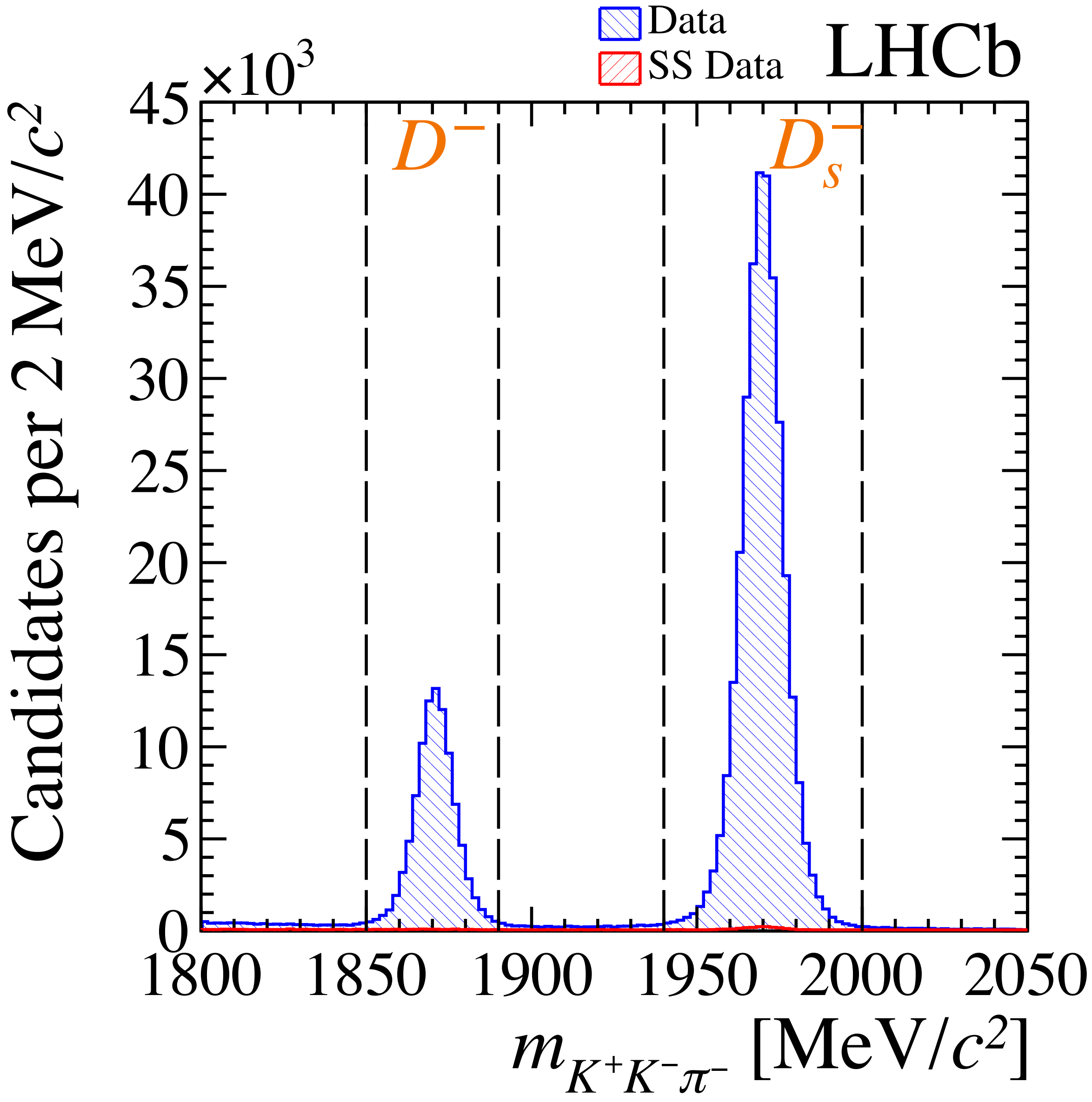
Analysis strategy

Phys. Rev. D 101 2020, 072004

Dataset: Full Run 1 dataset,
1 fb^{-1}@ 7 TeV + 2 fb^{-1}@ 8 TeV

Signal: $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$

Normalisation: $B^0 \to D^{(*)-}\mu^+\nu_\mu$



PRD D 101, 072004 supplementary material

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1 fb^{-1}@ 7 TeV + 2 fb^{-1}@ 8 TeV

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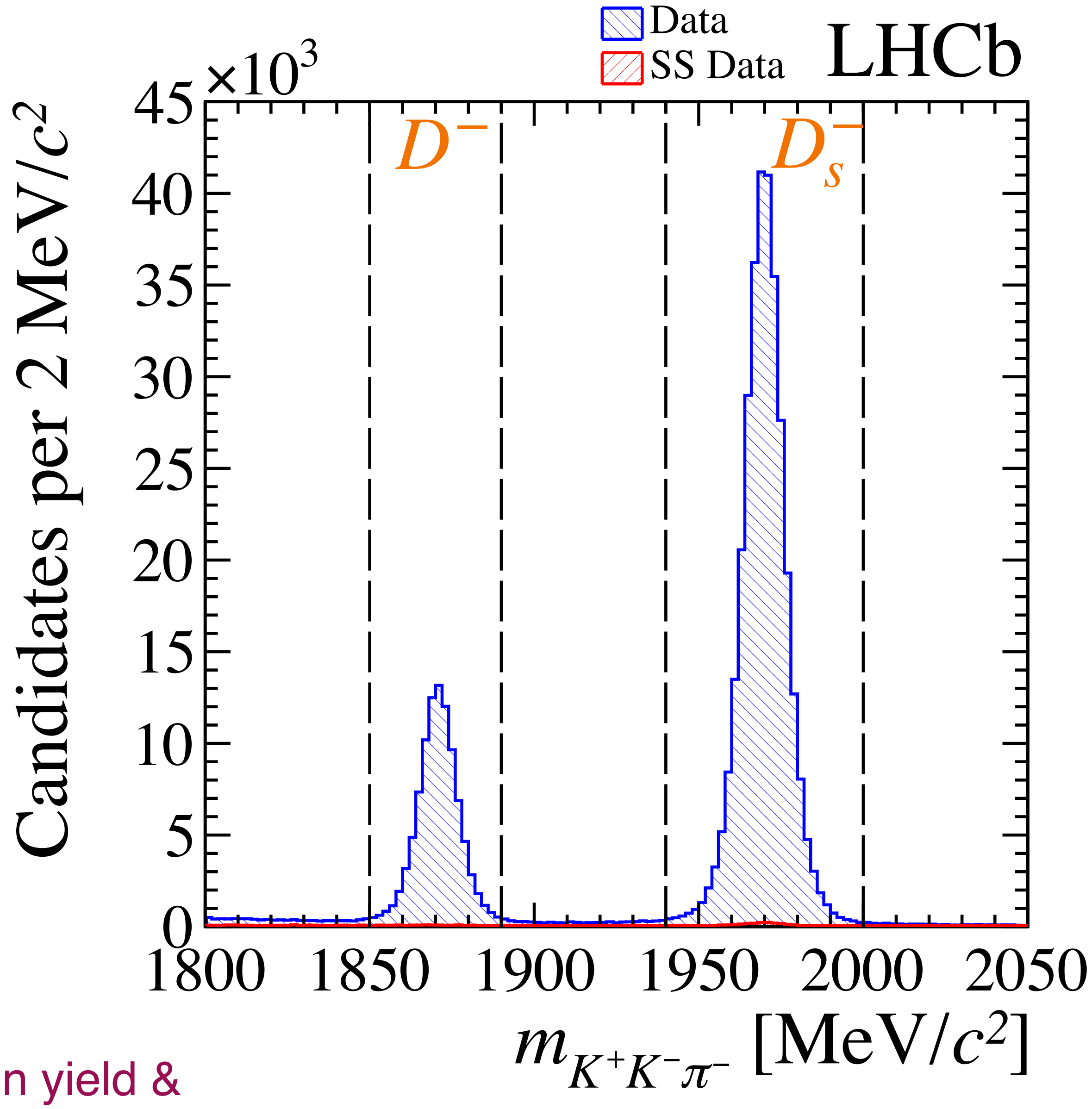
Normalisation: $B^0 \to D^{(*)-}\mu^+\nu_\mu$

Strategy:

- Both channels reconstructed in the $[K^-K^+]_\phi\pi^+$final state → **reduced syst.**
- Fit data to **simultaneously** determine $|V_{cb}|$ and FF parameters

→ Templates of the form



PRD D 101, 072004 supplementary material

Normalisation yield & external inputs

$$\frac{\mathrm{d}N(B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu)}{\mathrm{d}\xi} = \kappa\,\frac{\mathrm{d}\Gamma(|V_{cb}|, \{\mathrm{FF\ pars}\})}{\mathrm{d}\xi}\,\varepsilon(\xi)$$

Set of fit variables

Efficiency

A novel fit method

Phys. Rev. D 101 2020, 072004

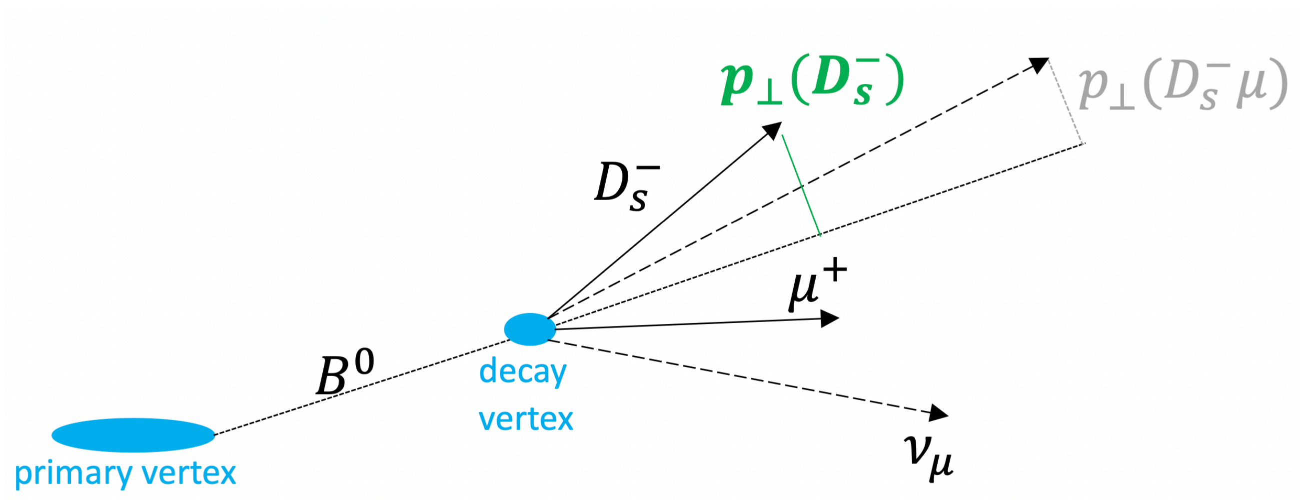
Challenge @ LHCb: reconstruct B_s^0 peak with *unreconstructed neutrino*

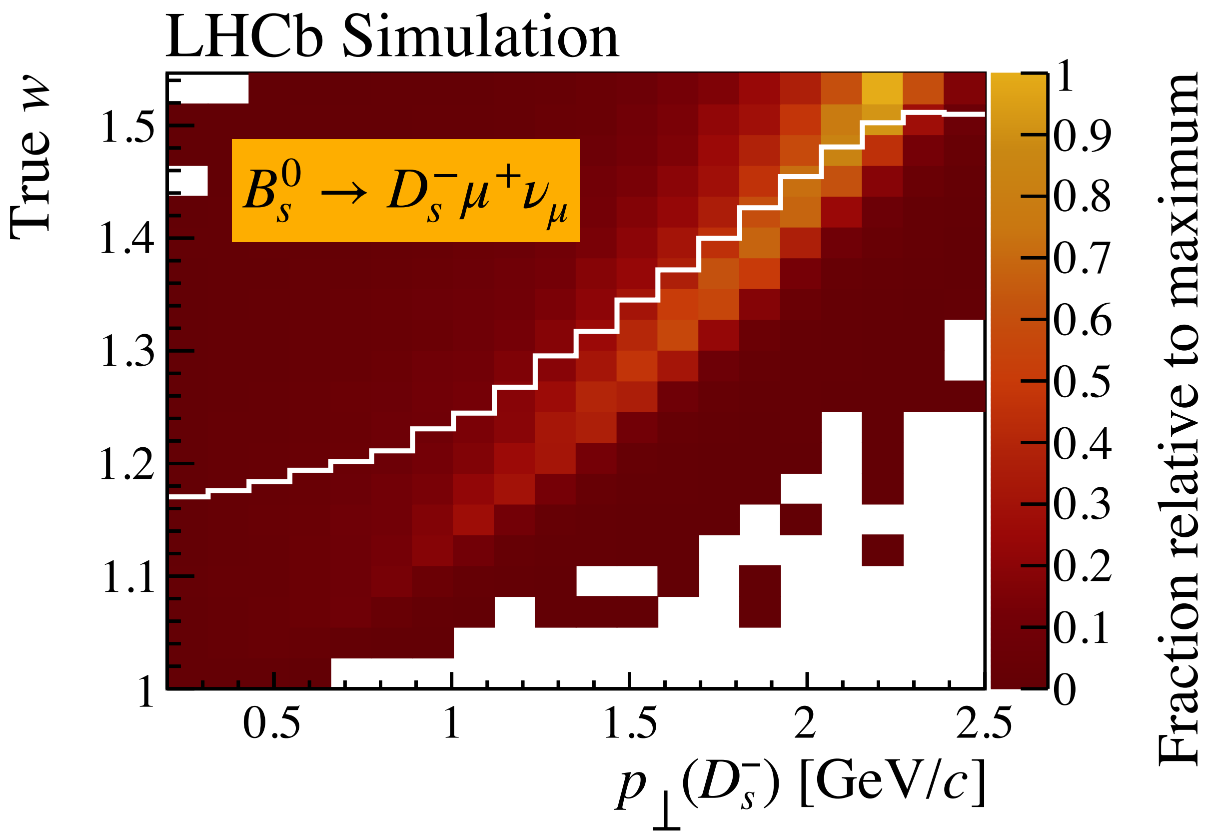
Solution: 2D fit to the *plane* in

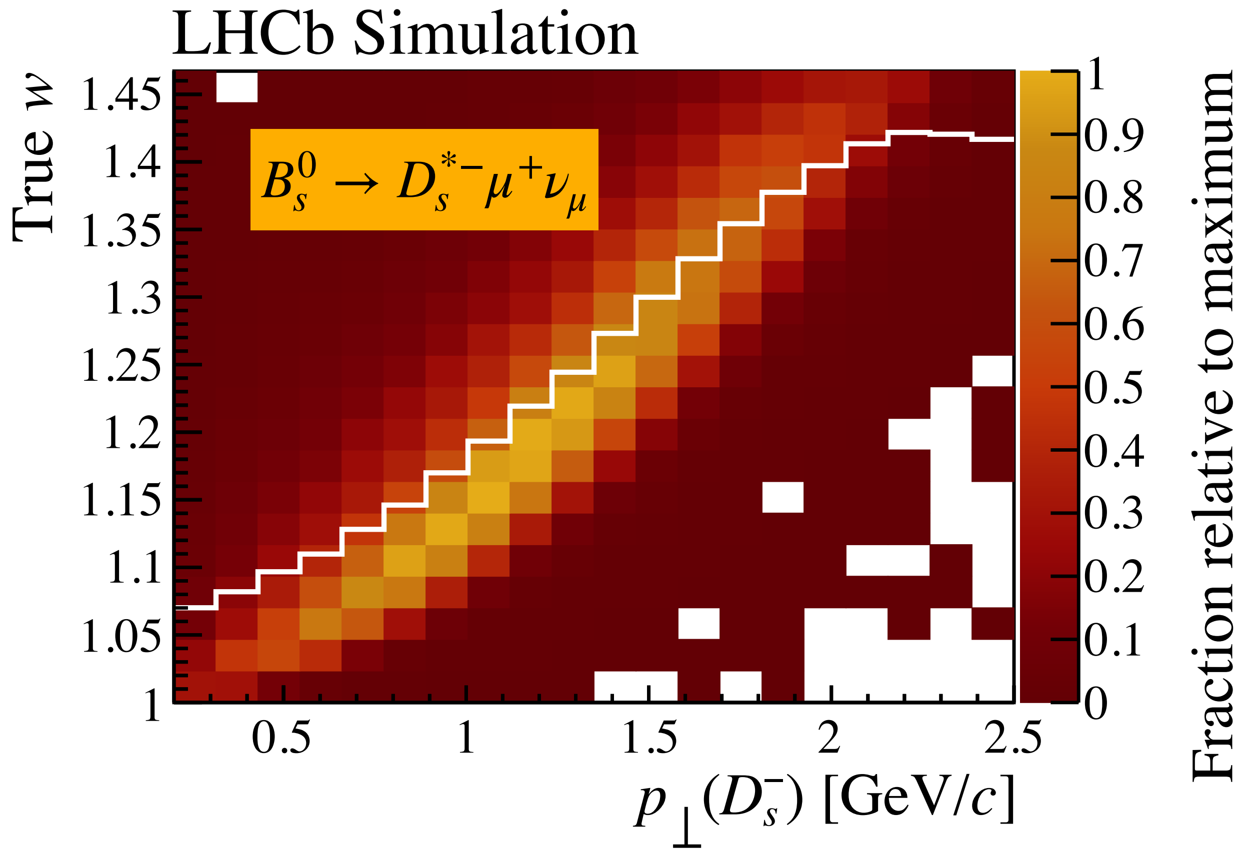
- Corrected mass

$$\boxed{m_{\text{corr}} \equiv \sqrt{m^2(D_s^-\mu^+) + p_\perp(D_s^-\mu^+)} + p_\perp(D_s^-\mu^+)}$$

- $p_\perp(D_s^-)$
  - a) Fully reconstructed observable
  - b) Correlated with **hadron recoil**, w



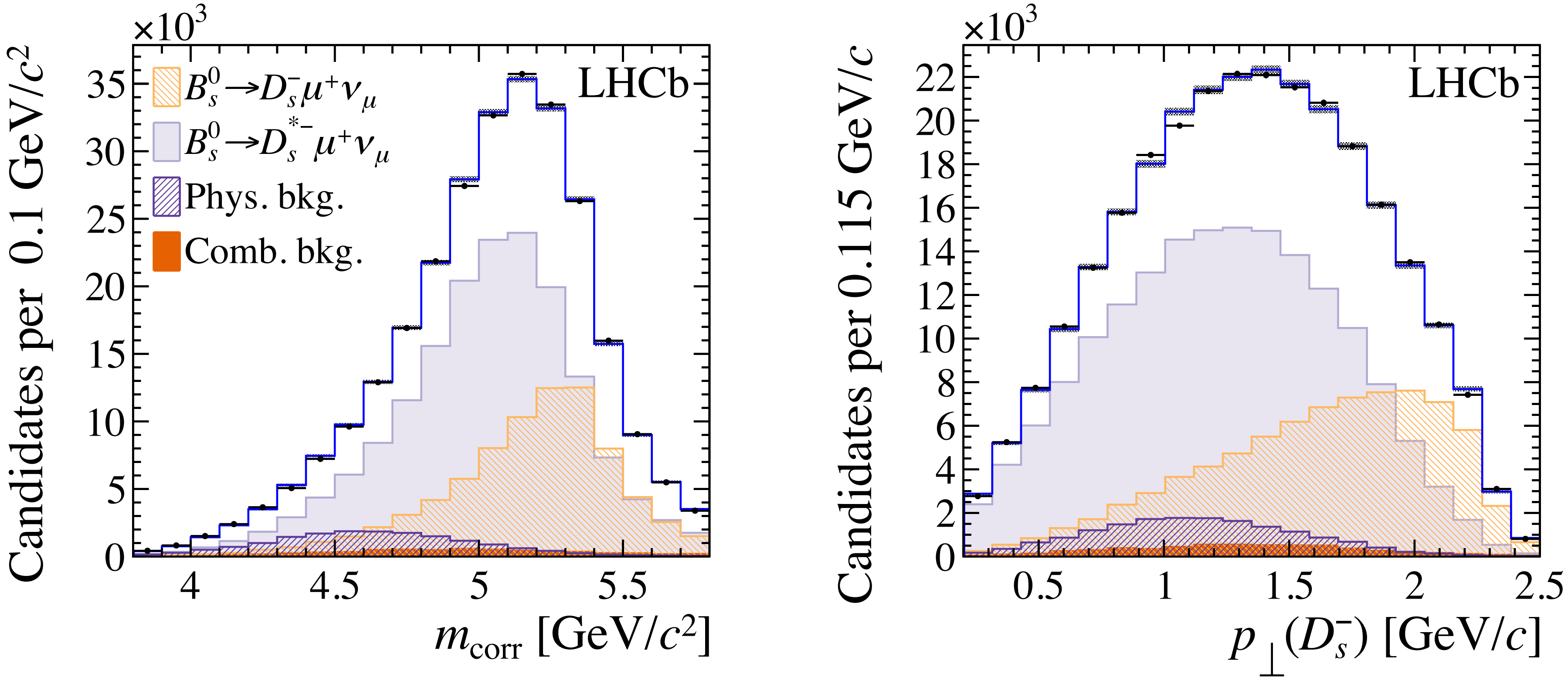




Fit results

Phys. Rev. D 101 2020, 072004

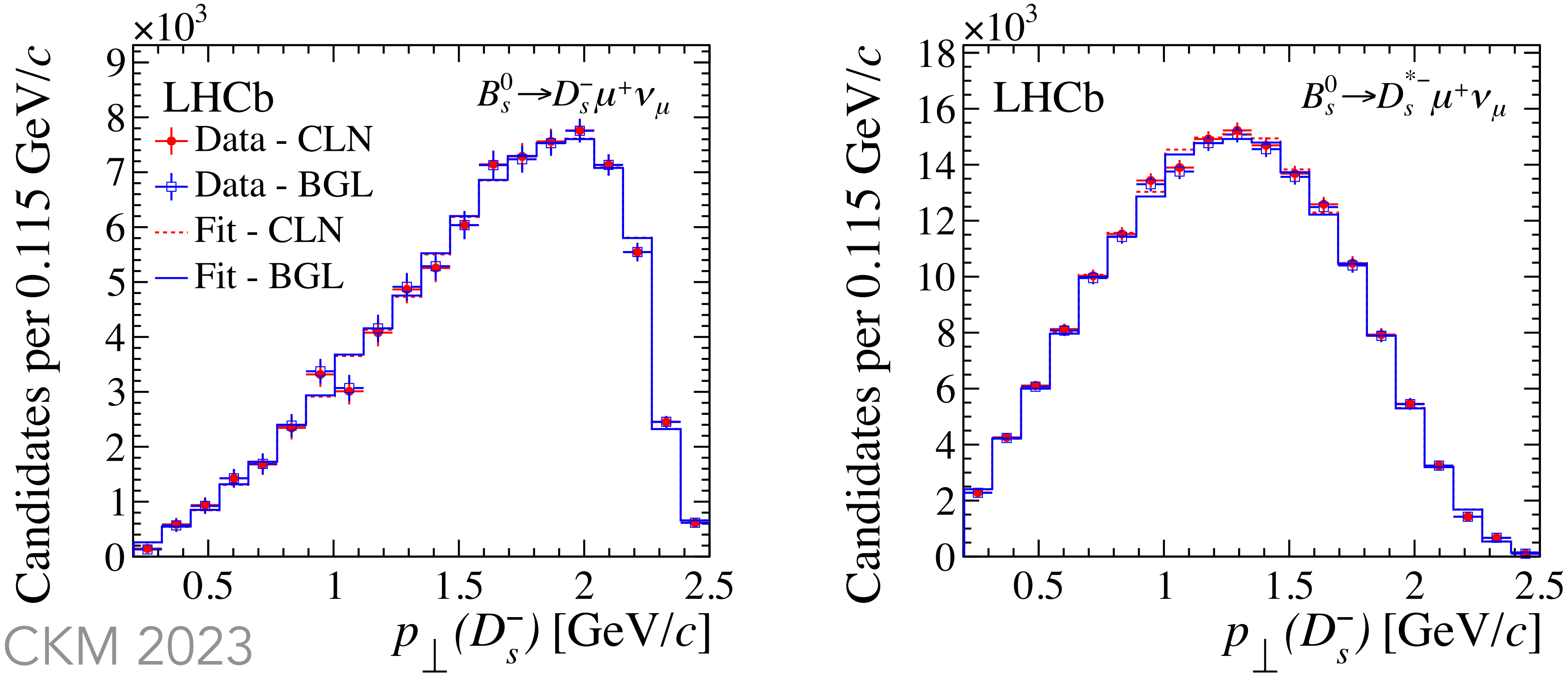
Signal fit using the **CLN** parameterisation:



$\chi^2/\mathrm{ndf} = 279/285$

p-value $= 58\%$

Bkg-subtracted distributions:



Extraction of $|V_{cb}|$

Phys. Rev. D 101 2020, 072004

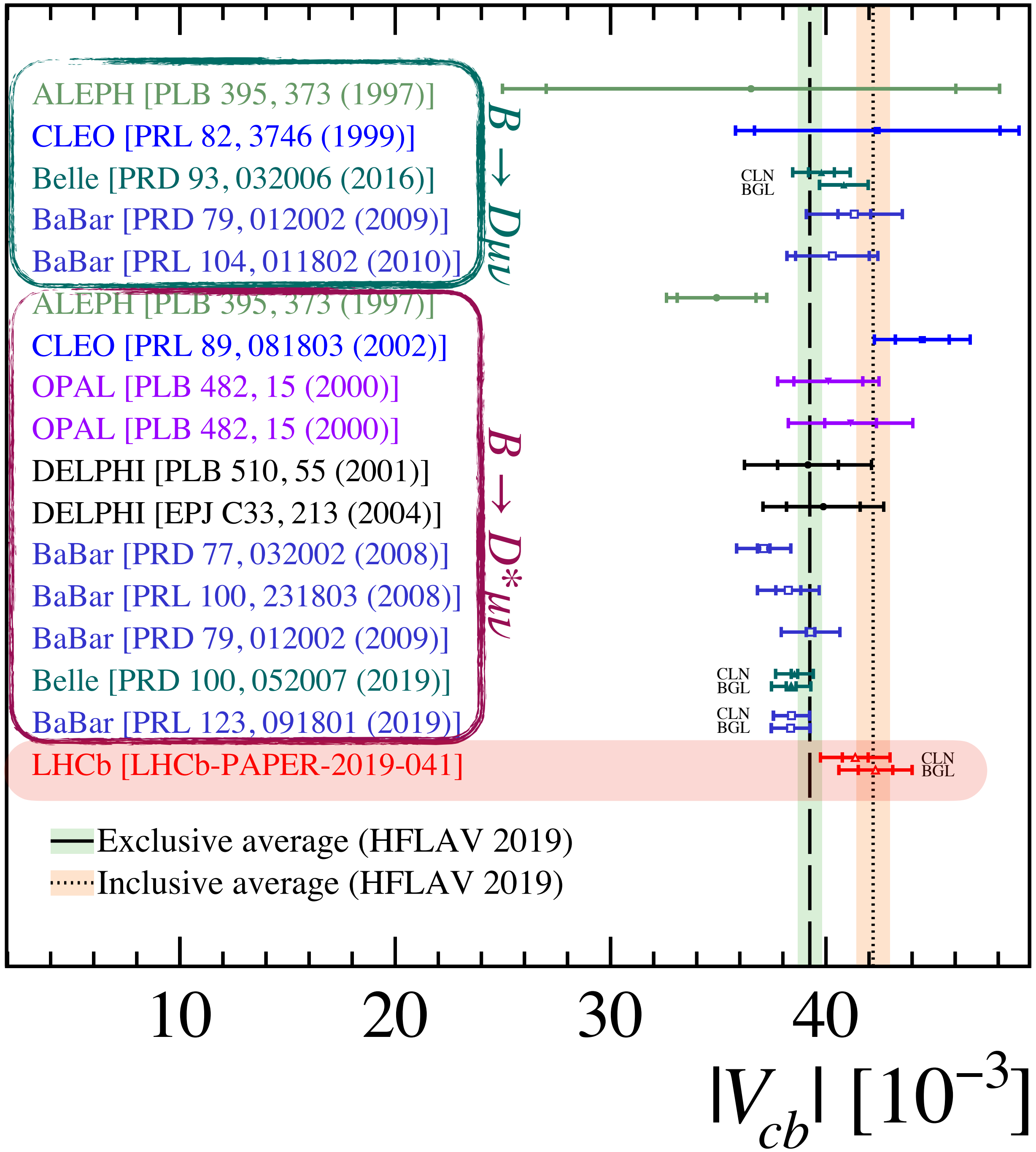
First exclusive $|V_{cb}|$ extraction at a **hadron collider** *and* first determination using B_s^0 **decays**

$$|V_{cb}|_{\text{CLN}} = (41.6 \pm 0.6(\text{stat}) \pm 0.9(\text{syst}) \pm 1.2(\text{ext})) \times 10^{-3}$$

$$|V_{cb}|_{\text{BGL}} = (42.3 \pm 0.8(\text{stat}) \pm 0.9(\text{syst}) \pm 1.2(\text{ext})) \times 10^{-3}$$

- Both extractions are compatible with each other
- Agreement with **exclusive** via $B^{0/+}$ and **inclusive** $|V_{cb}|$ determinations.

[PRD **101**, 072004 supplementary material]



Measurement of the shape of the $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ differential decay rate

J. High Energ. Phys. 2020, 144 (2020)

Analysis strategy

J. High Energ. Phys. **2020**, 144 (2020)

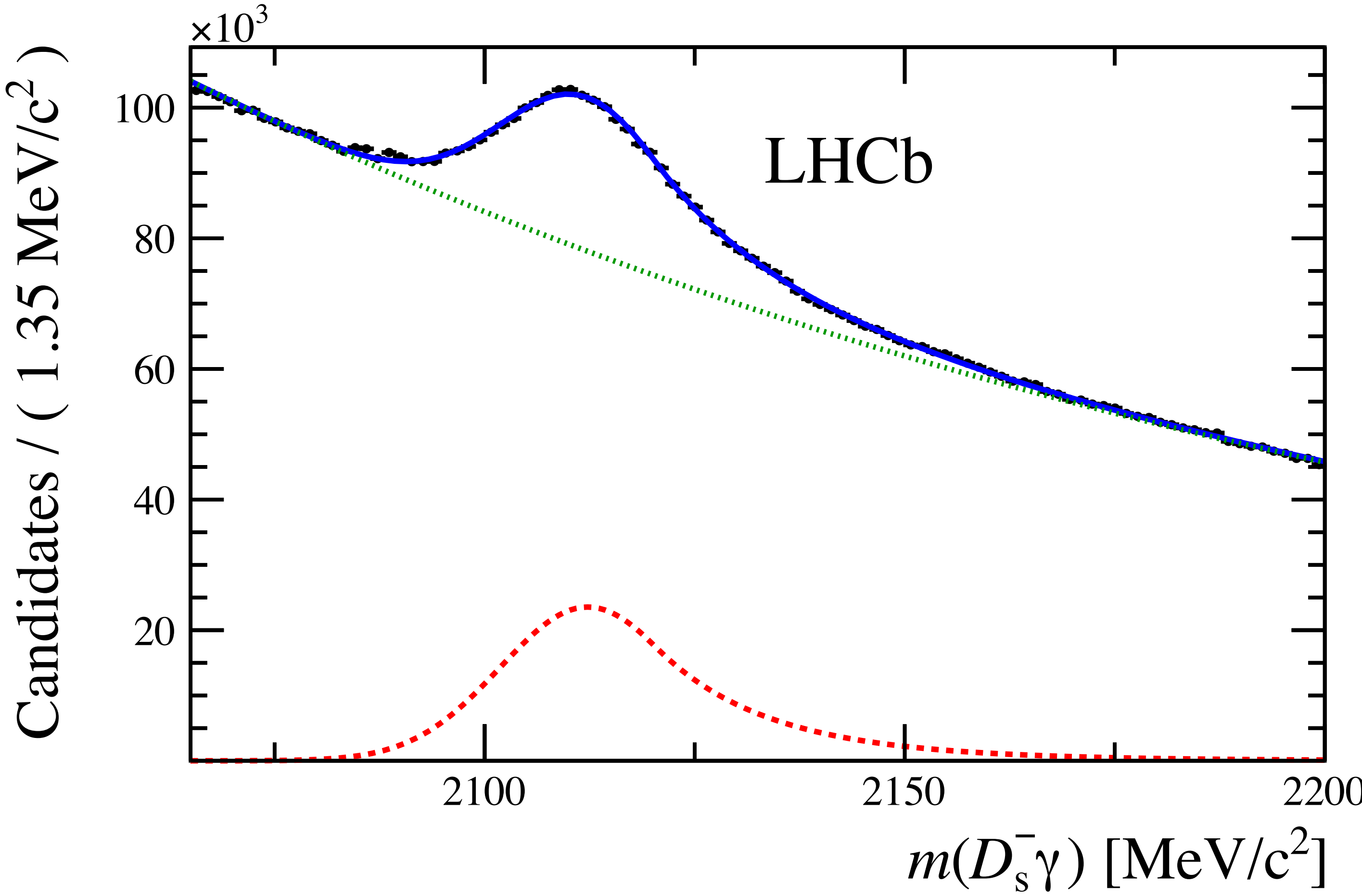
Dataset: 2016 data, 1.7 fb^{-1}@ 13 TeV

Selection:

- $B_s^0 \to D_s^{*-}(\to D_s^- \gamma)\mu^+\nu_\mu$

with $D_s^+ \to [K^-K^+]_\phi \pi^-, [K^+\pi^-]_{K^{*0}}K^-$

- Reconstruct soft γ in a cone around the D_s^+
- Bkg subtraction via fit to $m(D_s^+\gamma)$



Analysis target:

Unfold the $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ **spectrum in** w

a) accounting for detector resolution on w

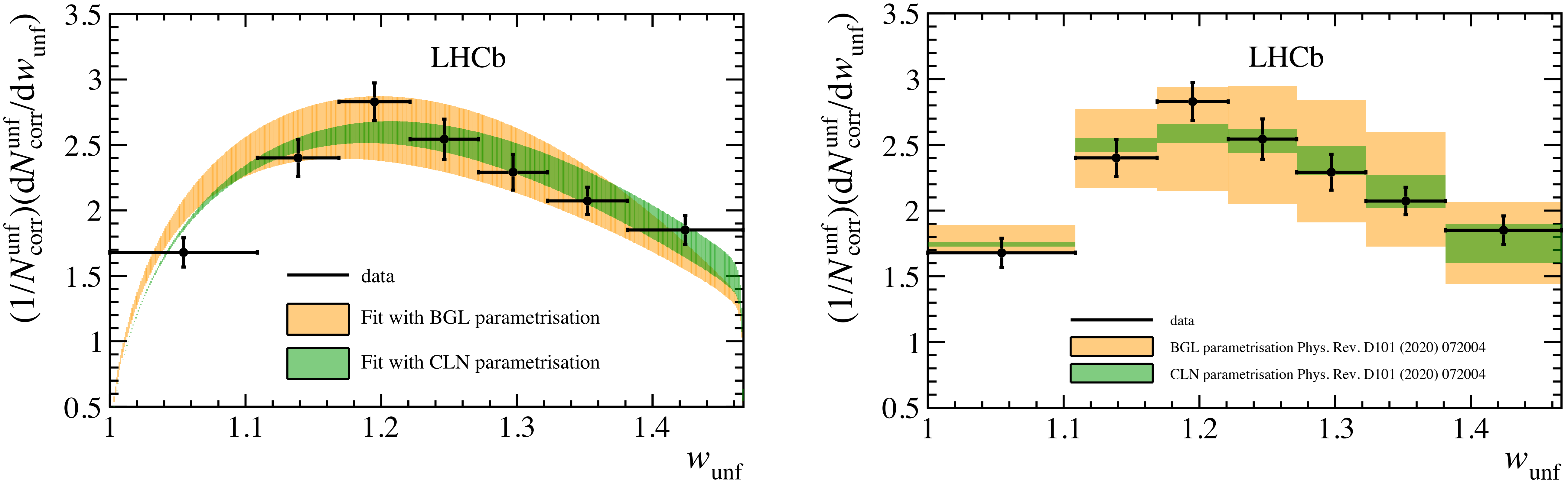
b) corrected for the reconstruction and selection efficiency

Form factor fits

J. High Energ. Phys. **2020**, 144 (2020)

The measured $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ spectrum unfolded accounting for efficiency and experimental resolution

→ unfolded normalised differential decay rate fit with **CLN** and **BGL**



First unfolded $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ as a function of w demonstrating **compatibility of BGL and CLN parameterisations**

First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ and measurement of $|V_{ub}|/|V_{cb}|$

Phys. Rev. Lett. ***126***, *081804*

Analysis strategy

Phys. Rev. Lett. **126** (2021), 081804

Dataset: 2012 data, 2 fb^{-1}@ 8TeV

Signal: $B_s^0 \to K^-\mu^+\nu_\mu$

Normalisation: $B_s^0 \to D_s^-\mu^+\nu_\mu$

CKM extraction:

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} \times \overbrace{\frac{\mathrm{FF}_K}{\mathrm{FF}_{D_s}}}^{\text{Theory input}} = \overbrace{\frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}}^{\text{Experiment}}$$

$$= \underbrace{\frac{N_K}{N_{D_s}}}_{\text{Fit}} \underbrace{\frac{\varepsilon_{D_s}}{\varepsilon_K}}_{\text{Simulation}} \times \underbrace{\mathcal{B}(D_s^+ \to K^+K^-\pi^+)}$$

PTEP 2020 (2020) 8, 083C01

with $$\mathrm{FF}_Y = |V_{xb}|^{-2} \int [\mathrm{d}\Gamma(B_s^0 \to Y\mu^+\nu_\mu)/\mathrm{d}q^2]\mathrm{d}q^2,$$

$$Y \in \{K^-, D_s^-\}; x \in \{u, c\}$$

Analysis strategy

*Phys. Rev. Lett. **126** (2021), 081804*

$$\underbrace{\frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)}}_{\text{Experiment}} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \times \underbrace{\frac{\mathrm{FF}_K}{\mathrm{FF}_{D_s}}}_{\text{Theory input}}$$

with $\mathrm{FF}_Y = |V_{xb}|^{-2} \int [\mathrm{d}\Gamma(B_s^0 \to Y \mu^+ \nu_\mu)/\mathrm{d}q^2]\mathrm{d}q^2,$

$$Y \in \{K^-, D_s^-\}; x \in \{u, c\}$$

$|V_{ub}|/|V_{cb}|$ extraction in **two regions of q^2 using FF$_K$ calculations** from

a) LCSR @ $q^2 < 7$ GeV^2/c^4 [JHEP 2017, 112 (2017)]
b) Lattice QCD @ $q^2 > 7$ GeV^2/c^4 [Phys. Rev. D 100, 034501]

→ Maximise precision of theoretical inputs
→ Similar signal-fit yields

with **full-q^2 FF$_{D_s}$ from lattice QCD** [Phys. Rev. D 101, 074513]

Fits to data

Phys. Rev. Lett. **126** (2021), 08180

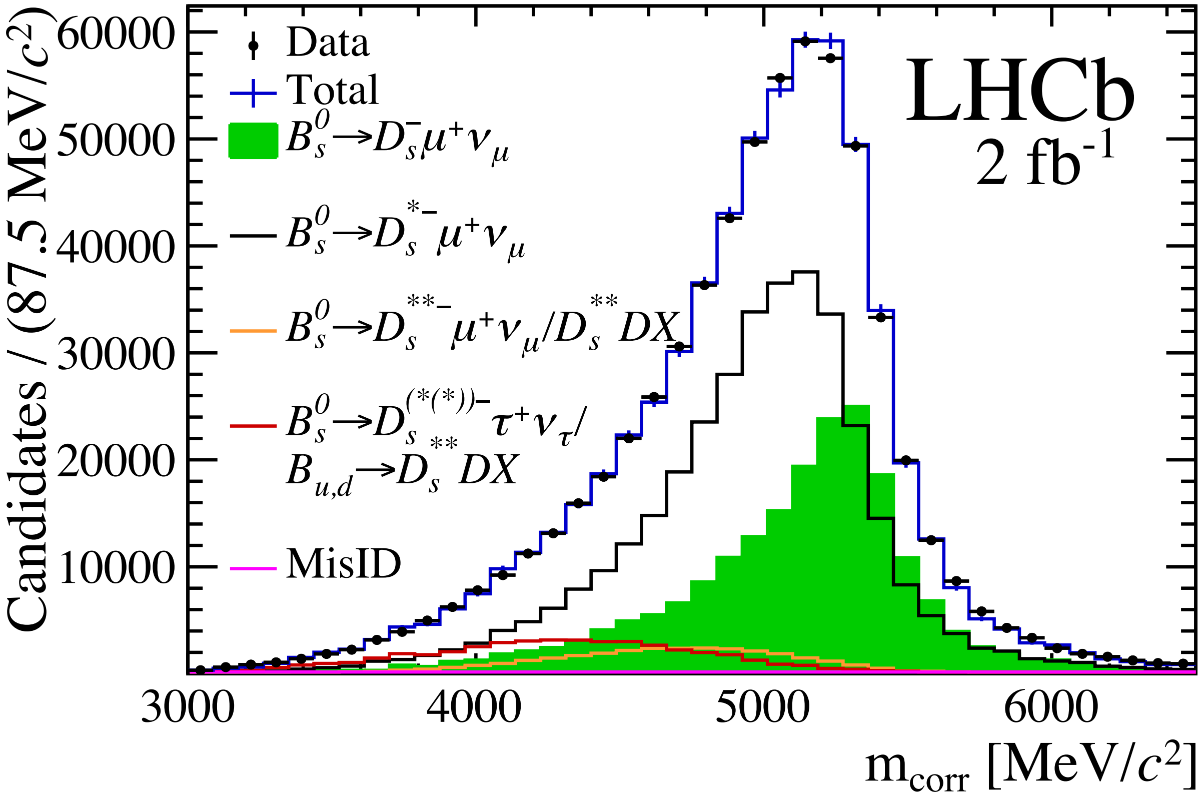
Yields:

$$N(B_s^0 \to D_s^- \mu^+ \nu_\mu) = 201450 \pm 5200$$

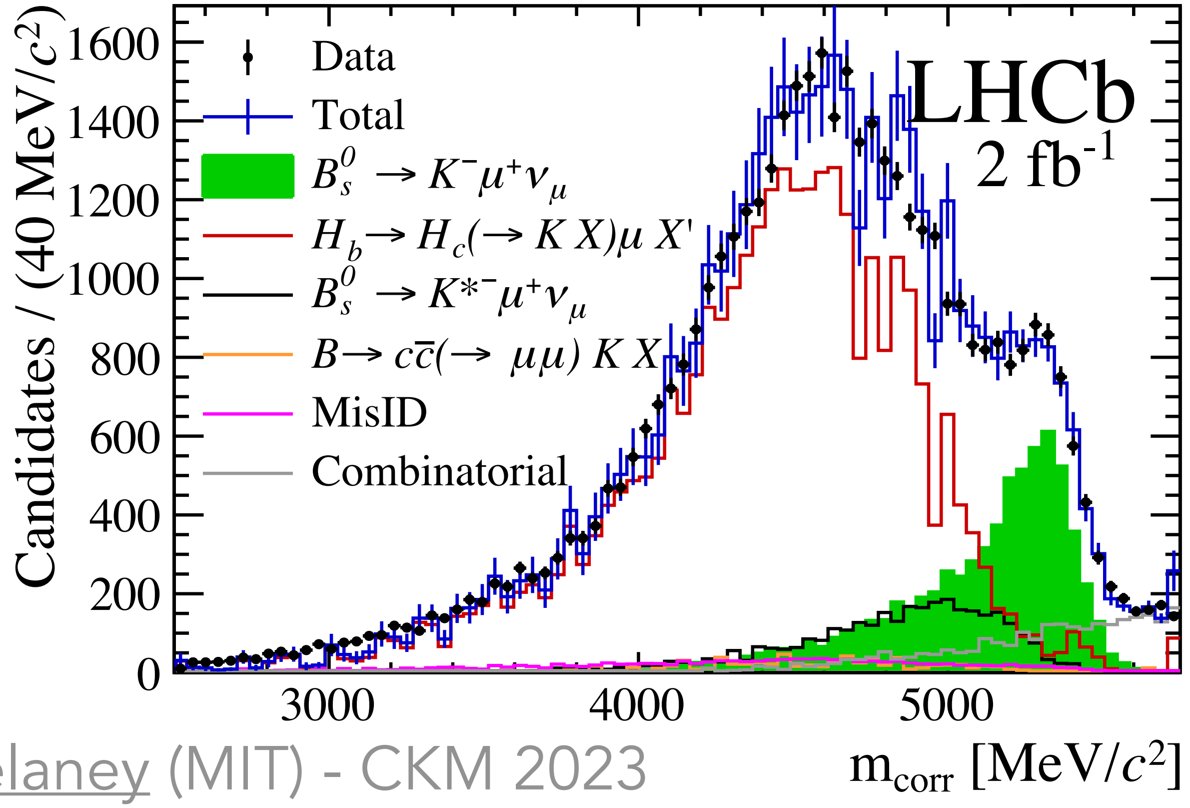
$$N(B_s^0 \to K^- \mu^+ \nu_\mu)_{q^2<7} = 6922 \pm 285$$

$$N(B_s^0 \to K^- \mu^+ \nu_\mu)_{q^2>7} = 6399 \pm 370$$

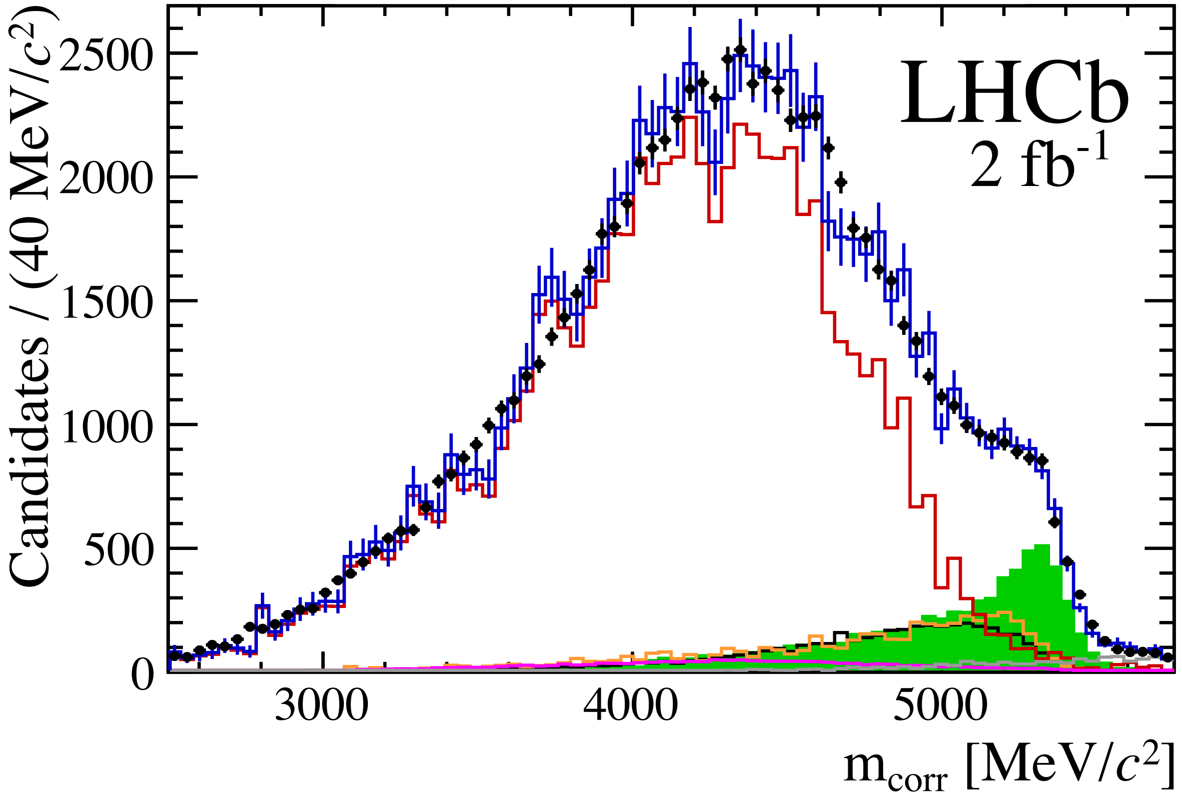
$B_s^0 \to D_s^- \mu^+ \nu_\mu$ **full** q^2



$B_s^0 \to K^- \mu^+ \nu_\mu$ **low** q^2



$B_s^0 \to K^- \mu^+ \nu_\mu$ **high** q^2



Extraction of $|V_{ub}|/|V_{cb}|$

Phys. Rev. Lett. ***126*** *(2021), 081804*

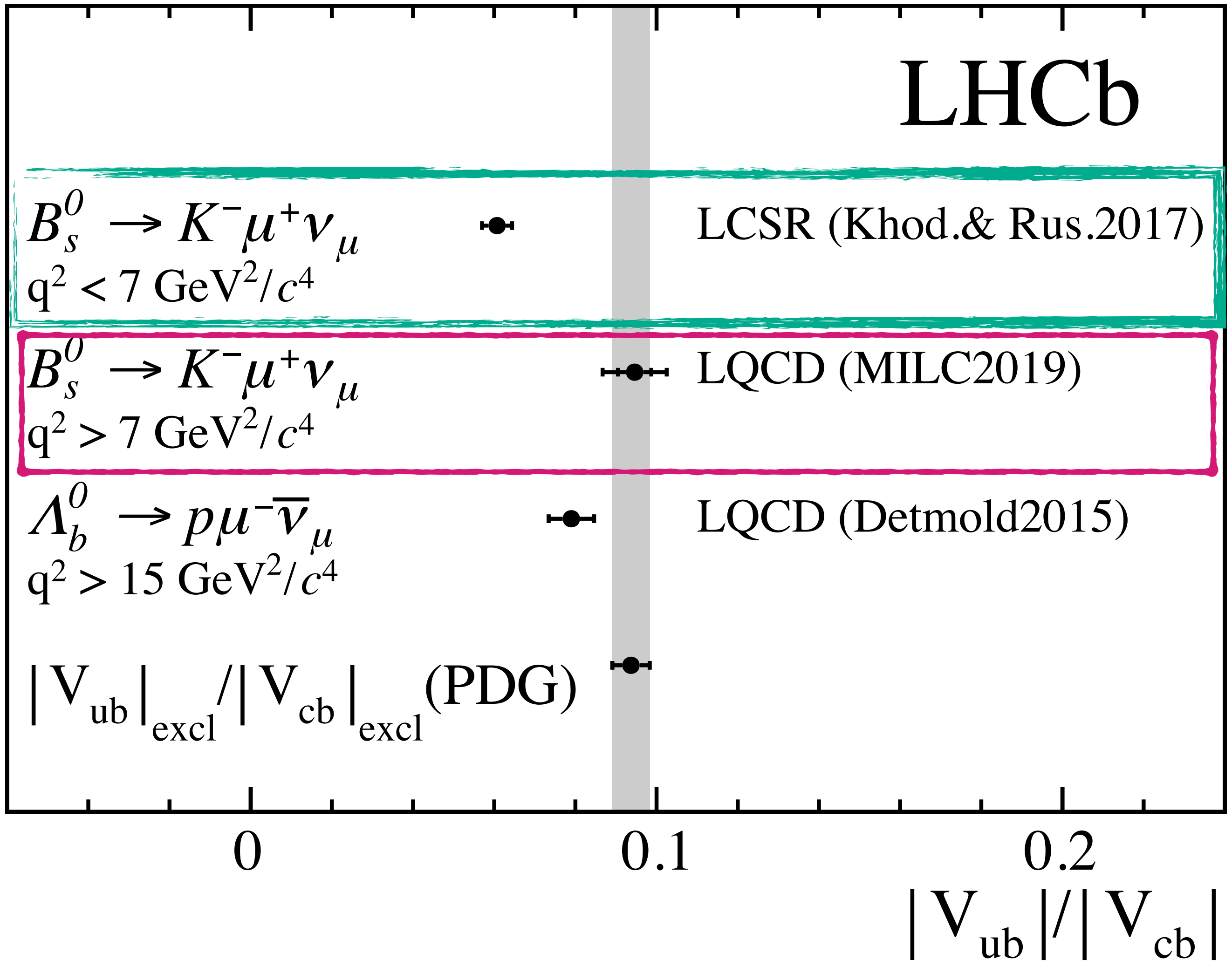
Low q^2: $\dfrac{\mathcal{B}(B^0_s \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B^0_s \to D^-_s\mu^+\nu_\mu)} = 1.66 \pm 0.08(\text{stat}) \pm 0.07(\text{syst}) \pm 0.05(D_s) \times 10^{-3}$

High q^2: $\dfrac{\mathcal{B}(B^0_s \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B^0_s \to D^-_s\mu^+\nu_\mu)} = 3.25 \pm 0.21(\text{stat})^{+0.16}_{-0.17}(\text{syst}) \pm 0.09(D_s) \times 10^{-3}$

→ with FF predictions from **LCSR** [JHEP 112 (2017)] and **LQCD** [PRD 100, 034501]:

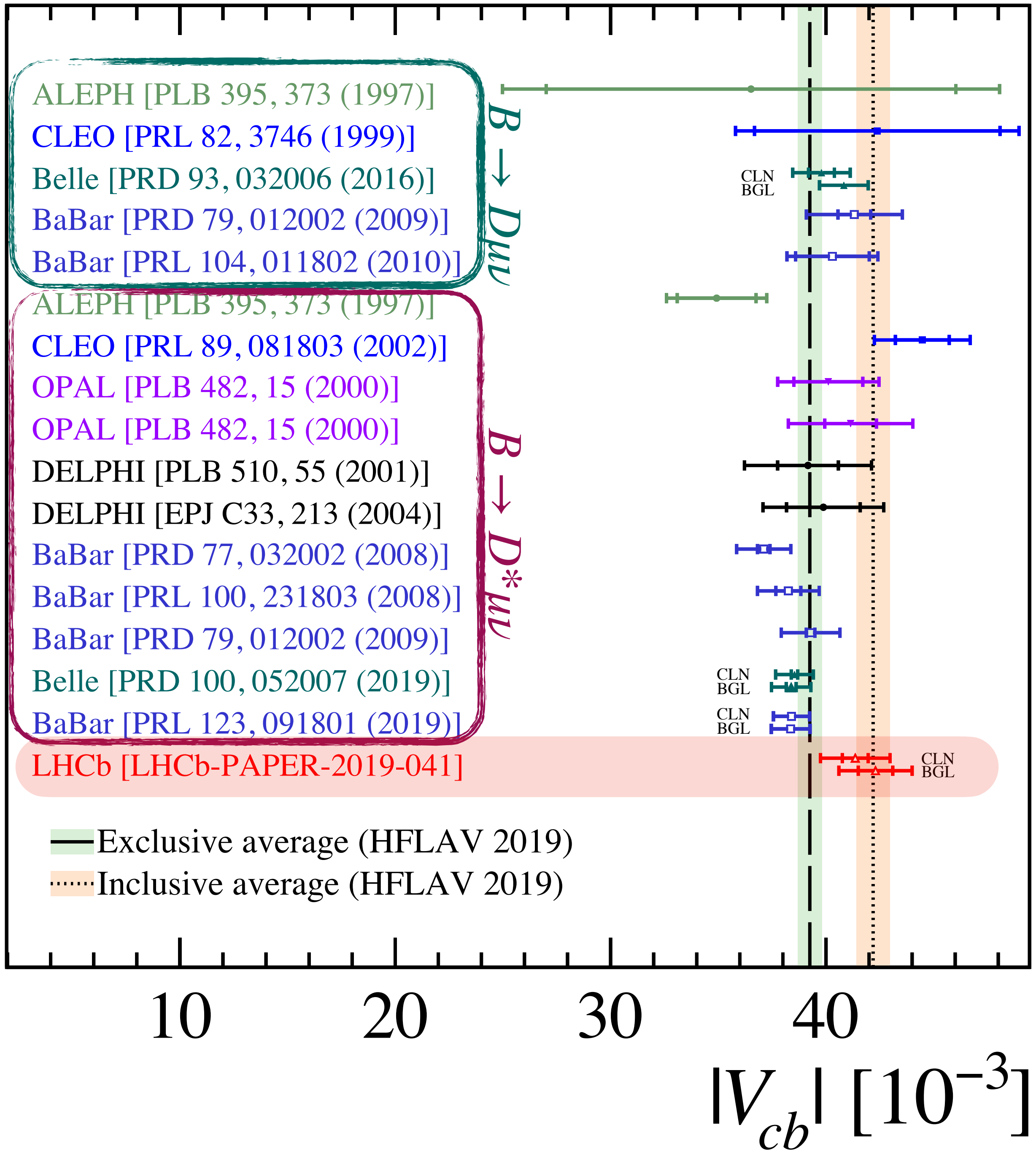
$$\begin{aligned}|V_{ub}|/|V_{cb}|_{\text{low}} &= 0.0607 \pm 0.0015(\text{stat})\\ &\pm 0.0013(\text{syst}) \pm 0.0008(D_s)\\ &\pm 0.0030(\text{FF})\end{aligned}$$

$$\begin{aligned}|V_{ub}|/|V_{cb}|_{\text{high}} &= 0.0946 \pm 0.0030(\text{stat})\\ &{}^{+0.0024}_{-0.0025}(\text{syst}) \pm 0.0013(D_s)\\ &\pm 0.0068(\text{FF})\end{aligned}$$

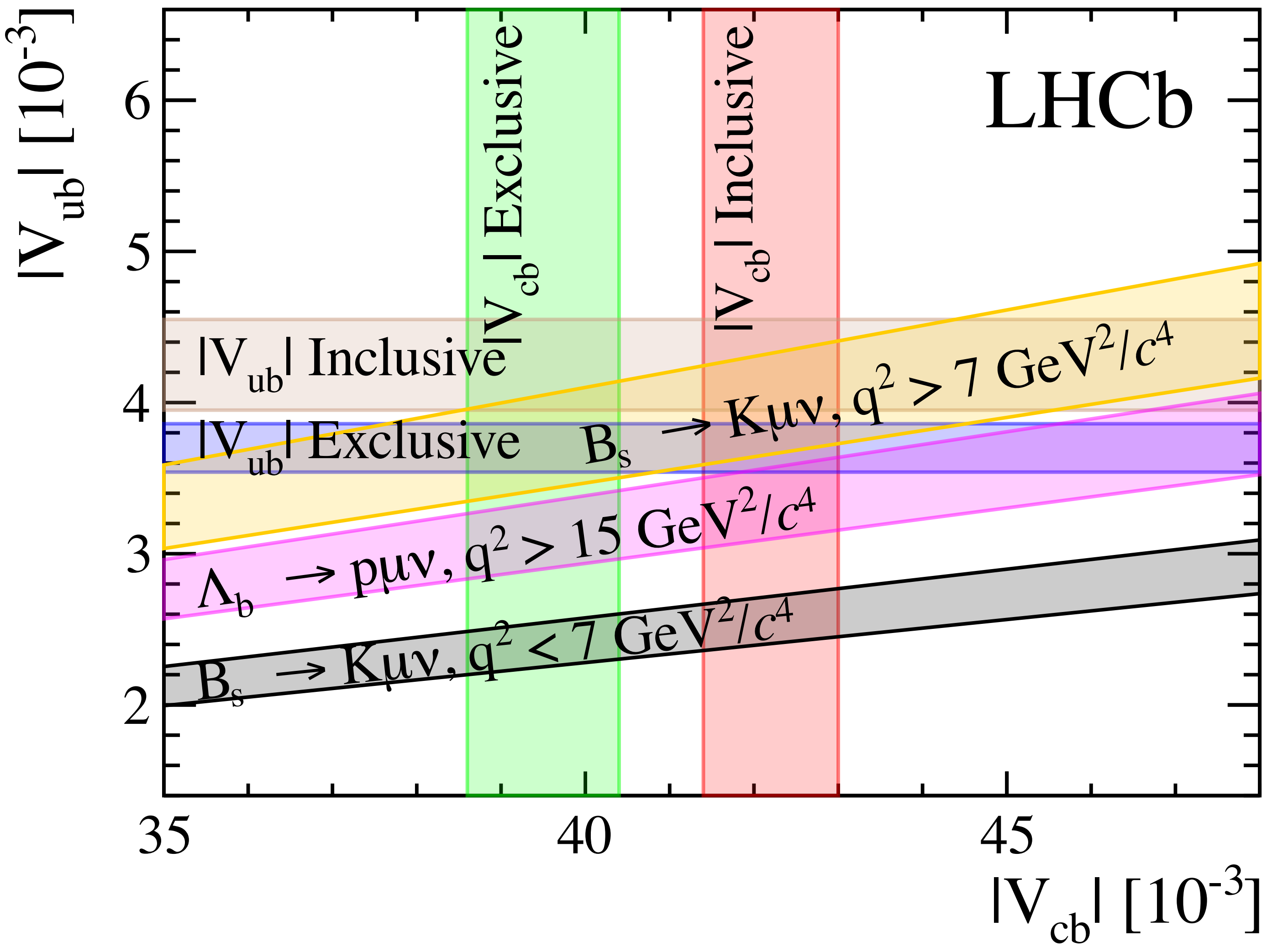


Summary

[PRD **101**, 072004 supplementary material]



[PRL **126**, 081804 supplementary material]



In the pipeline…

[LHCb Implications Workshop 2022]

- $B_s^0 \to K^-\mu^+\nu_\mu$:
  - Planned **Run 2 update** executed in nominally **8 bins** in $q^2 \to$ aim for **sensitivity to the shape** of the differential decay rate
  - Investigating $B^+ \to J/\psi K^+$ data as **normalisation** → simultaneously fit $|V_{ub}|$ and coeffs of FF parameterisation
  - Profit from **updated** $B_s^0 \to K^-$ **FF calculation** with reduced uncertainty at **low** q^2
- Additional channels reaching advanced state:

a) $B_c^+ \to D^{(*)0}\mu^+\nu_\mu$ **:** extract $|V_{ub}|/|V_{cb}|$ by normalising to $B_c^+ \to J/\psi\mu^+\nu_\mu$ with full 9 fb^{-1} dataset → profit from **LQCD** $B_c^+ \to D^{*0}$ **FF across full** q^2

b) $B^+ \to \rho\mu^+\nu_\mu$: access $|V_{ub}|$ and diff. decay rate

c) $\Lambda_b^0 \to \Lambda_c^{(*)+}\mu^-\overline{\nu}_\mu$: access $|V_{cb}|$ and diff. decay rate [Phys. Rev. D 99, 055008]

Thank you for your attention

blaise.delaney at cern.ch

Appendix

Measurement of $|V_{cb}|$ with $B_s^0 \to D_s^{(*)-}\mu^+\nu_\mu$ decays

Phys. Rev. D 101, 072004

External inputs

Phys. Rev. D 101, 072004

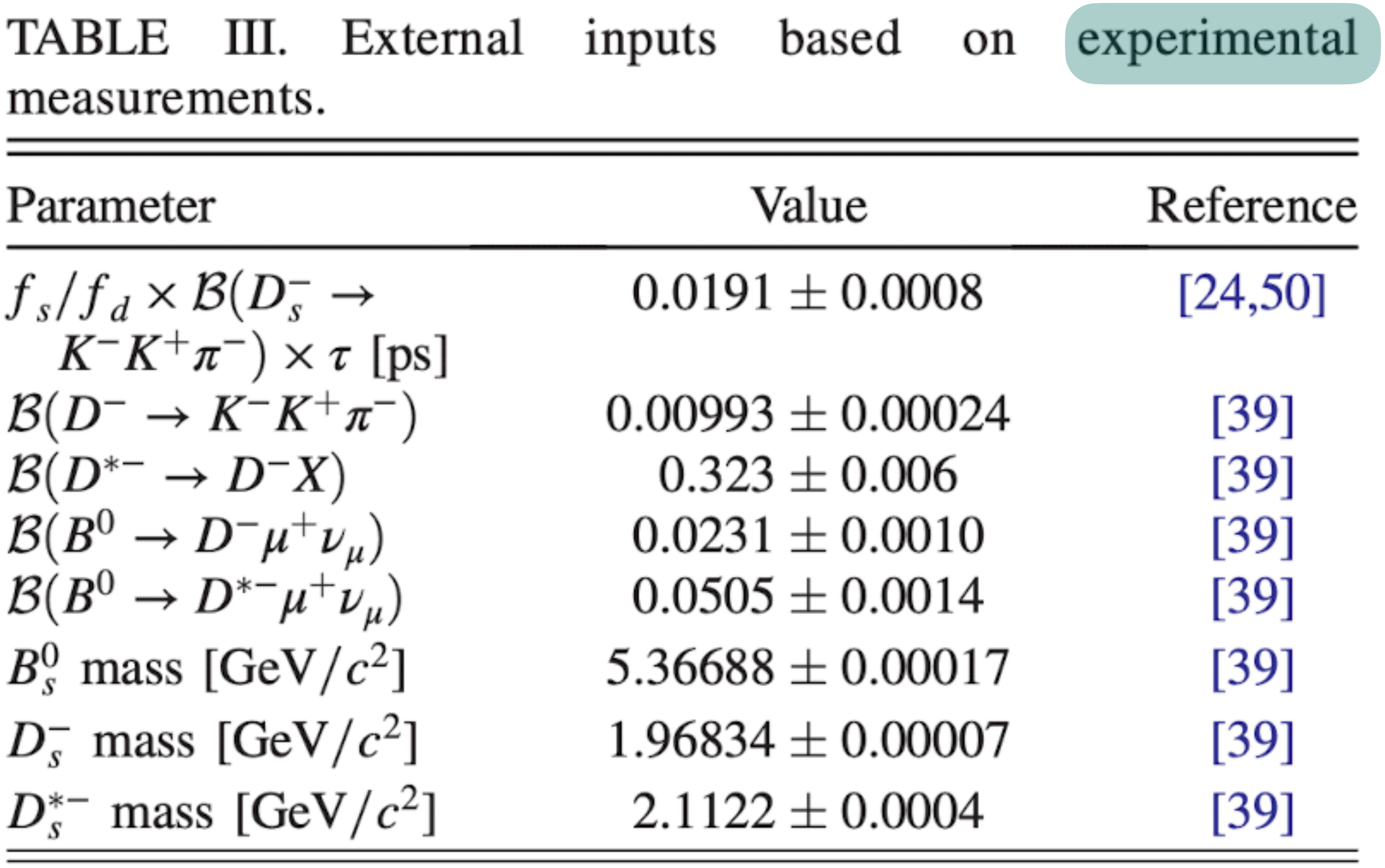


TABLE III. External inputs based on experimental measurements.

Parameter	Value	Reference
$f_s/f_d \times \mathcal{B}(D_s^- \to K^-K^+\pi^-) \times \tau$ [ps]	0.0191 ± 0.0008	[24,50]
$\mathcal{B}(D^- \to K^-K^+\pi^-)$	0.00993 ± 0.00024	[39]
$\mathcal{B}(D^{*-} \to D^-X)$	0.323 ± 0.006	[39]
$\mathcal{B}(B^0 \to D^-\mu^+\nu_\mu)$	0.0231 ± 0.0010	[39]
$\mathcal{B}(B^0 \to D^{*-}\mu^+\nu_\mu)$	0.0505 ± 0.0014	[39]
B_s^0 mass [GeV/c^2]	5.36688 ± 0.00017	[39]
D_s^- mass [GeV/c^2]	1.96834 ± 0.00007	[39]
D_s^{*-} mass [GeV/c^2]	2.1122 ± 0.0004	[39]

External inputs

Phys. Rev. D 101, 072004

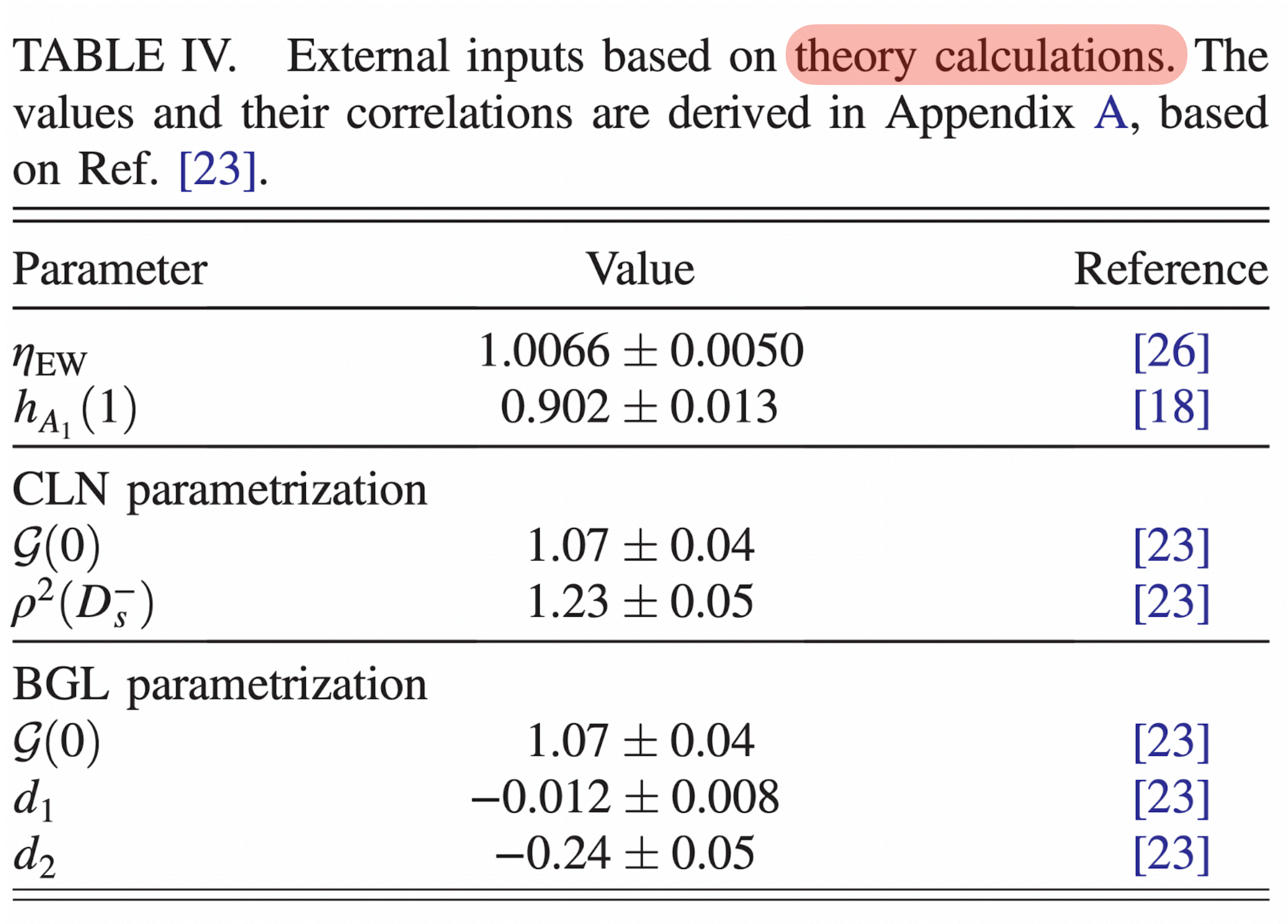


TABLE IV. External inputs based on theory calculations. The values and their correlations are derived in Appendix A, based on Ref. [23].

Parameter	Value	Reference
η_{EW}	1.0066 ± 0.0050	[26]
$h_{A_1}(1)$	0.902 ± 0.013	[18]
CLN parametrization		
$\mathcal{G}(0)$	1.07 ± 0.04	[23]
$\rho^2(D_s^-)$	1.23 ± 0.05	[23]
BGL parametrization		
$\mathcal{G}(0)$	1.07 ± 0.04	[23]
d_1	-0.012 ± 0.008	[23]
d_2	-0.24 ± 0.05	[23]

Systematic uncertainties

Phys. Rev. D 101, 072004

Source	Uncertainty															
	CLN parametrization						BGL parametrization									
	$\lvert V_{cb}\rvert$ $[10^{-3}]$	$\rho^2(D_s^-)$ $[10^{-1}]$	$\mathcal{G}(0)$ $[10^{-2}]$	$\rho^2(D_s^{*-})$ $[10^{-1}]$	$R_1(1)$ $[10^{-1}]$	$R_2(1)$ $[10^{-1}]$	$\lvert V_{cb}\rvert$ $[10^{-3}]$	d_1 $[10^{-2}]$	d_2 $[10^{-1}]$	$\mathcal{G}(0)$ $[10^{-2}]$	b_1 $[10^{-1}]$	c_1 $[10^{-3}]$	a_0 $[10^{-2}]$	a_1 $[10^{-1}]$	$\mathcal{R}$ $[10^{-1}]$	$\mathcal{R}^*$ $[10^{-1}]$
$f_s/f_d \times \mathcal{B}(D_s^- \to K^+K^-\pi^-)(\times\tau)$	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.4	0.4
$\mathcal{B}(D^- \to K^-K^+\pi^-)$	0.5	0.0	0.0	0.0	0.0	0.0	0.5	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.3
$\mathcal{B}(D^{*-} \to D^-X)$	0.2	0.0	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.1	0.0	0.2	0.0	0.3	–	0.2
$\mathcal{B}(\mathrm{B}^{0} \to D^-\mu^+\nu_\mu)$	0.4	0.0	0.3	0.1	0.2	0.1	0.5	0.1	0.0	0.1	0.1	0.4	0.1	0.7	–	–
$\mathcal{B}(\mathrm{B}^{0} \to D^{*-}\mu^+\nu_\mu)$	0.3	0.0	0.2	0.1	0.1	0.1	0.2	0.0	0.0	0.1	0.1	0.3	0.1	0.4	–	–
$m(B_s^0)$, $m(D^{(*)-})$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
η_{EW}	0.2	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.1	–	–
$h_{A_1}(1)$	0.3	0.0	0.2	0.1	0.1	0.1	0.3	0.0	0.0	0.1	0.1	0.3	0.1	0.5	–	–
External inputs (ext)	1.2	0.0	0.4	0.1	0.2	0.1	1.2	0.1	0.0	0.1	0.1	0.6	0.1	0.8	0.5	0.5
$D_{(s)}^- \to K^+K^-\pi^-$ model	0.8	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.5	0.4
Background	0.4	0.3	2.2	0.5	0.9	0.7	0.1	0.5	0.2	2.3	0.7	2.0	0.5	2.0	0.4	0.6
Fit bias	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.0	0.0	0.0	0.2	0.4	0.2	0.4	0.0	0.0
Corrections to simulation	0.0	0.0	0.5	0.0	0.1	0.0	0.0	0.1	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0
Form-factor parametrization	–	–	–	–	–	–	–	–	–	–	–	–	–	–	0.0	0.1
Experimental (syst)	0.9	0.3	2.2	0.5	0.9	0.7	0.9	0.5	0.2	2.3	0.7	2.1	0.5	2.0	0.6	0.7
Statistical (stat)	0.6	0.5	3.4	1.7	2.5	1.6	0.8	0.7	0.5	3.4	0.7	2.2	0.9	2.6	0.5	0.5

Fit results

Phys. Rev. D 101, 072004

Parameter	Value
$\lvert V_{cb}\rvert$ $[10^{-3}]$	41.4 ± 0.6 (stat) ± 1.2 (ext)
$\mathcal{G}(0)$	1.102 ± 0.034 (stat) ± 0.004 (ext)
$\rho^2(D_s^-)$	1.27 ± 0.05 (stat) ± 0.00 (ext)
$\rho^2(D_s^{*-})$	1.23 ± 0.17 (stat) ± 0.01 (ext)
$R_1(1)$	1.34 ± 0.25 (stat) ± 0.02 (ext)
$R_2(1)$	0.83 ± 0.16 (stat) ± 0.01 (ext)

CLN

Parameter	Value
$\lvert V_{cb}\rvert$ $[10^{-3}]$	42.3 ± 0.8 (stat) ± 1.2 (ext)
$\mathcal{G}(0)$	1.097 ± 0.034 (stat) ± 0.001 (ext)
d_1	-0.017 ± 0.007 (stat) ± 0.001 (ext)
d_2	-0.26 ± 0.05 (stat) ± 0.00 (ext)
b_1	-0.06 ± 0.07 (stat) ± 0.01 (ext)
a_0	0.037 ± 0.009 (stat) ± 0.001 (ext)
a_1	0.28 ± 0.26 (stat) ± 0.08 (ext)
c_1	0.0031 ± 0.0022 (stat) ± 0.0006 (ext)

BGL

Measurement of the shape of the $B_s^0 \to D_s^{*-}\mu^+\nu_\mu$ differential decay rate

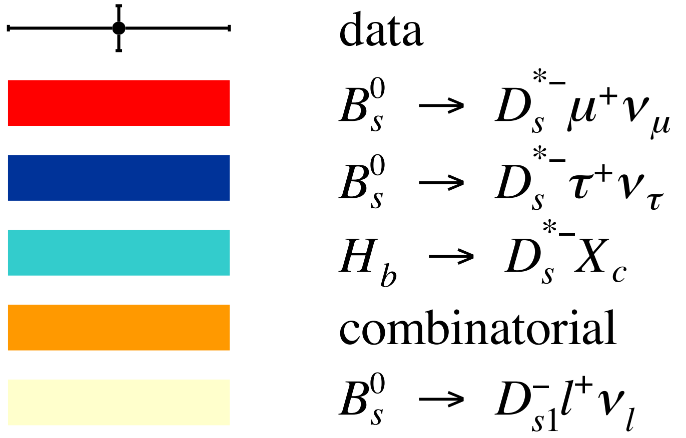
J. High Energ. Phys. 2020, 144 (2020)

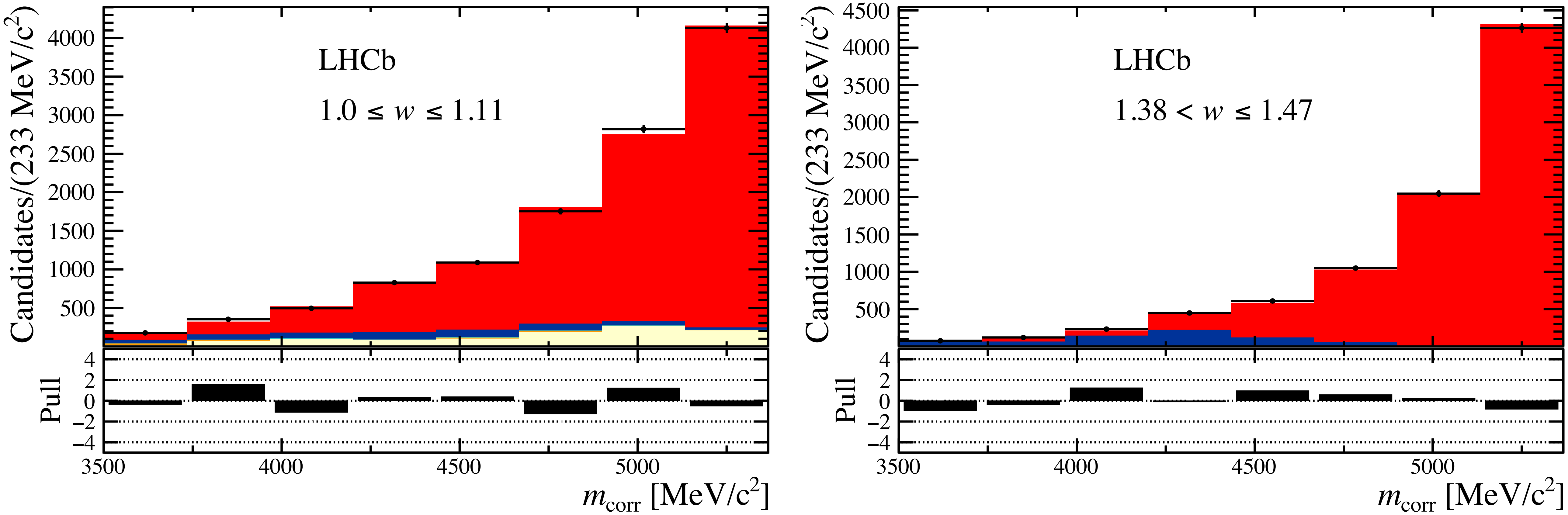
Signal Fits

J. High Energ. Phys. **2020**, 144 (2020)

Binned maximum-likelihood fit to $m_{\text{corr}}(D_s^{*-}\mu^+)$ in **7 bins of *reconstructed*** w [1] → extract *raw* yields

Binning optimised to ensure **comparable signal yield** in each bin





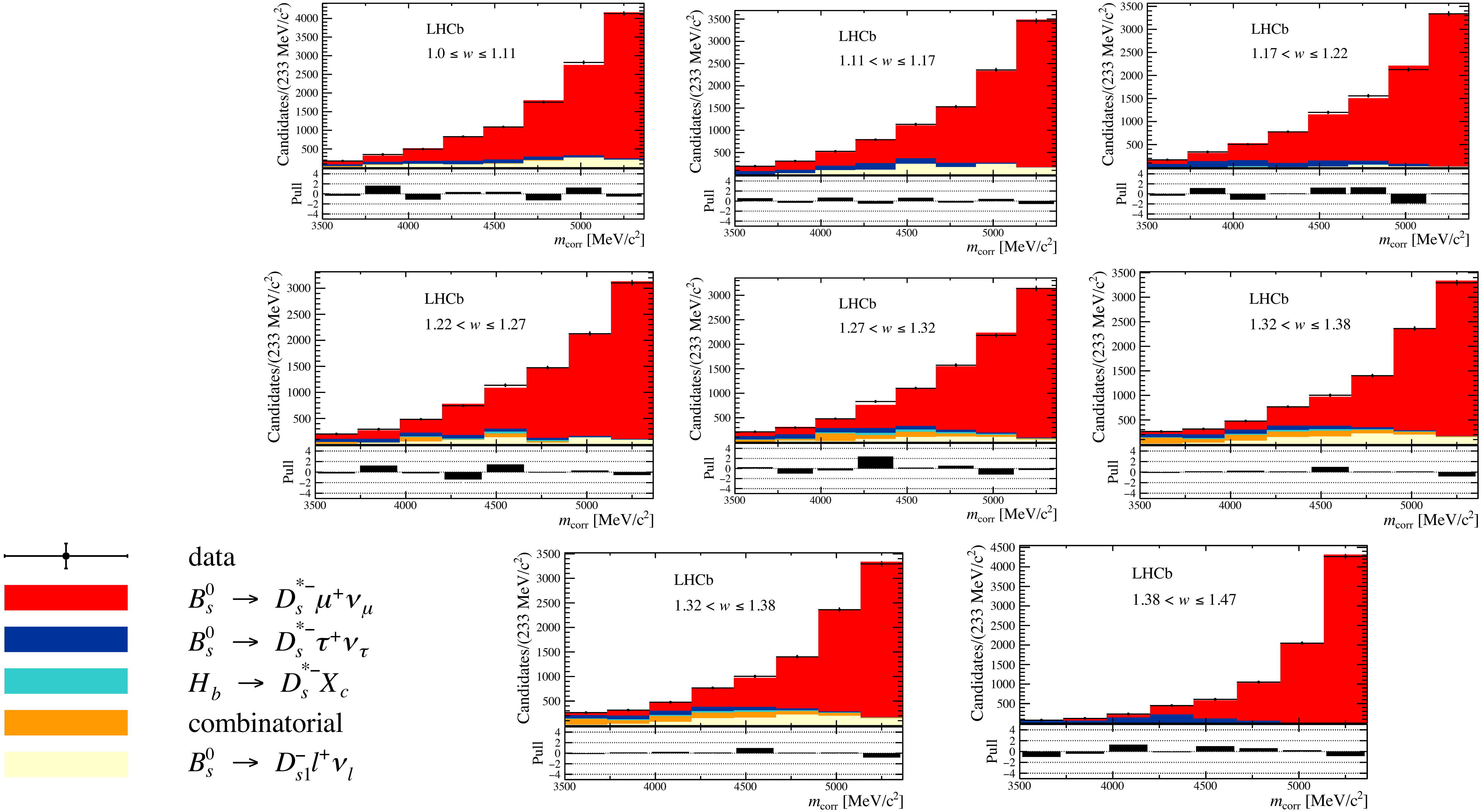
[1] JHEP 02 (2017) 021

Signal Fits

J. High Energ. Phys. **2020**, 144 (2020)

Binning scheme in reconstructed w

bin	1	2	3	4	5	6	7
w	1.1087	1.1688	1.2212	1.2717	1.3226	1.3814	1.4667



Systematic uncertainties

J. High Energ. Phys. **2020**, 144 (2020)

Unfolded yields in each bin of w &
breakdown of stat. and syst. uncertainties

	w bin						
	1	2	3	4	5	6	7
Fraction of $N^{\mathrm{unf}}_{\mathrm{corr},i}$	0.183	0.144	0.148	0.128	0.117	0.122	0.158
Uncertainties (%)							
Simulation sample size	3.5	3.0	2.8	3.1	3.4	3.0	3.7
Sample sizes for effs and corrections	3.6	3.2	3.0	2.8	2.8	2.7	2.8
SVD unfolding regularisation	0.5	0.5	0.1	0.7	1.2	0.0	0.5
Radiative corrections	0.1	0.2	0.1	0.3	0.4	0.2	0.2
Simulation FF parametrisation	0.3	0.1	0.1	0.1	0.2	0.4	0.2
Kinematic corrections	2.4	1.0	1.1	0.1	0.2	0.1	0.9
Hardware-trigger efficiency	0.3	0.3	0.0	0.2	0.2	0.3	0.1
Software-trigger efficiency	0.0	0.1	0.0	0.0	0.1	0.0	0.0
D_s^- selection efficiency	0.5	0.2	0.3	0.3	0.2	0.1	0.3
Photon background subtraction	0.0	2.3	0.8	2.9	2.0	0.9	0.4
Total systematic uncertainty	5.6	5.1	4.4	5.2	5.0	4.2	4.8
Statistical uncertainty	3.4	2.9	2.7	3.1	3.2	2.9	3.4

Systematic uncertainties

J. High Energ. Phys. **2020**, 144 (2020)

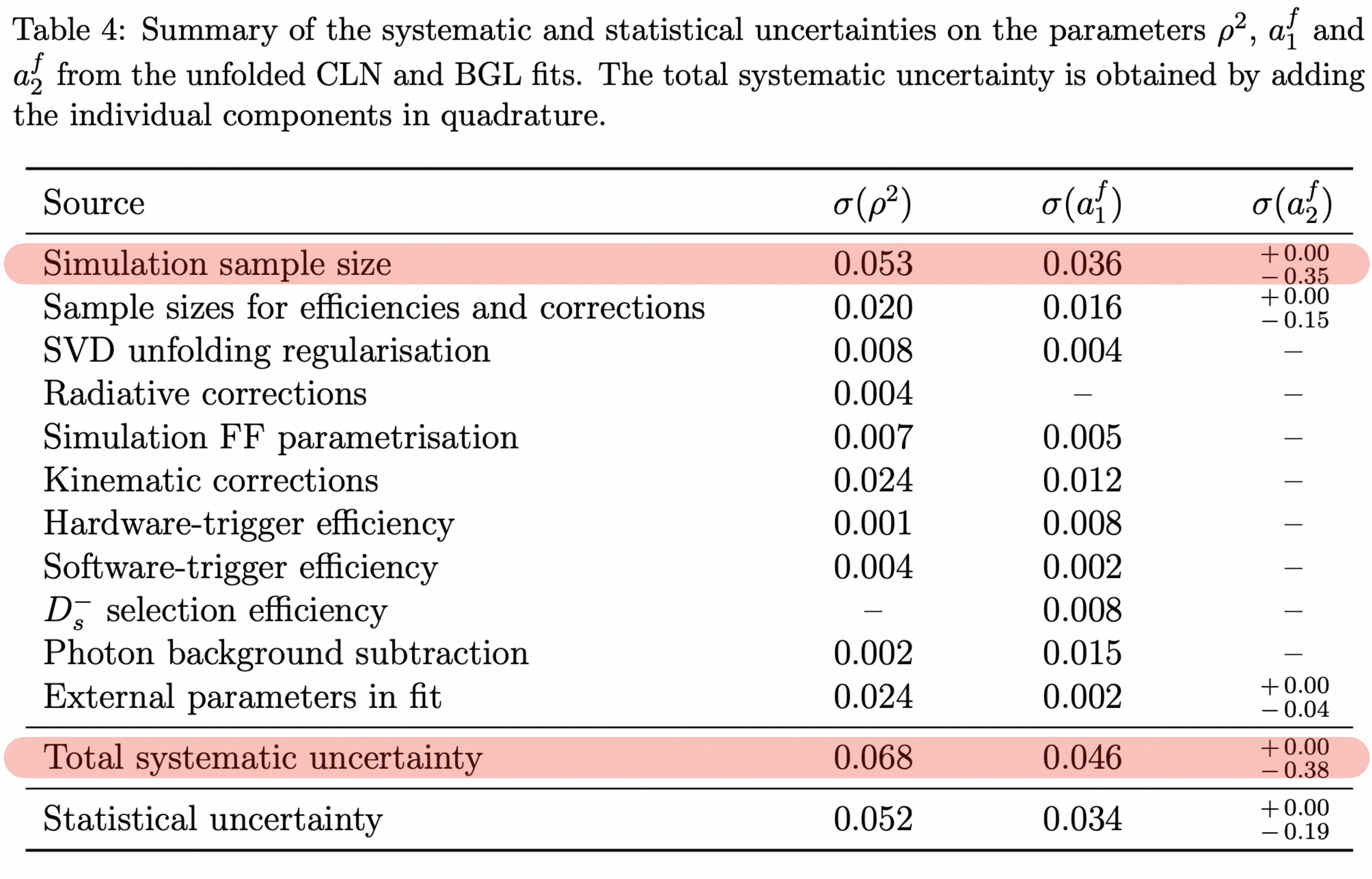


Table 4: Summary of the systematic and statistical uncertainties on the parameters ρ^2, a_1^f and a_2^f from the unfolded CLN and BGL fits. The total systematic uncertainty is obtained by adding the individual components in quadrature.

Source	$\sigma(\rho^2)$	$\sigma(a_1^f)$	$\sigma(a_2^f)$
Simulation sample size	0.053	0.036	$^{+0.00}_{-0.35}$
Sample sizes for efficiencies and corrections	0.020	0.016	$^{+0.00}_{-0.15}$
SVD unfolding regularisation	0.008	0.004	–
Radiative corrections	0.004	–	–
Simulation FF parametrisation	0.007	0.005	–
Kinematic corrections	0.024	0.012	–
Hardware-trigger efficiency	0.001	0.008	–
Software-trigger efficiency	0.004	0.002	–
D_s^- selection efficiency	–	0.008	–
Photon background subtraction	0.002	0.015	–
External parameters in fit	0.024	0.002	$^{+0.00}_{-0.04}$
Total systematic uncertainty	0.068	0.046	$^{+0.00}_{-0.38}$
Statistical uncertainty	0.052	0.034	$^{+0.00}_{-0.19}$

Inputs to BGL fit

J. High Energ. Phys. **2020**, 144 (2020)

Theory inputs used for the BGL fit

[Phys.Lett.B 795 (2019) 386-390] [JHEP 11 (2017) 061]

BGL parameter	Value
a_0^f	0.01221 ± 0.00016
$a_1^{\mathcal{F}_1}$	0.0042 ± 0.0022
$a_2^{\mathcal{F}_1}$	$-0.069^{+0.041}_{-0.037}$
a_0^g	$0.024^{+0.021}_{-0.009}$
a_1^g	$0.05^{+0.39}_{-0.72}$
a_2^g	$1.0^{+0.0}_{-2.0}$
$a_0^{\mathcal{F}_2}$	0.0595 ± 0.0093
$a_1^{\mathcal{F}_2}$	-0.318 ± 0.170

Fit results

J. High Energ. Phys. **2020**, 144 (2020)

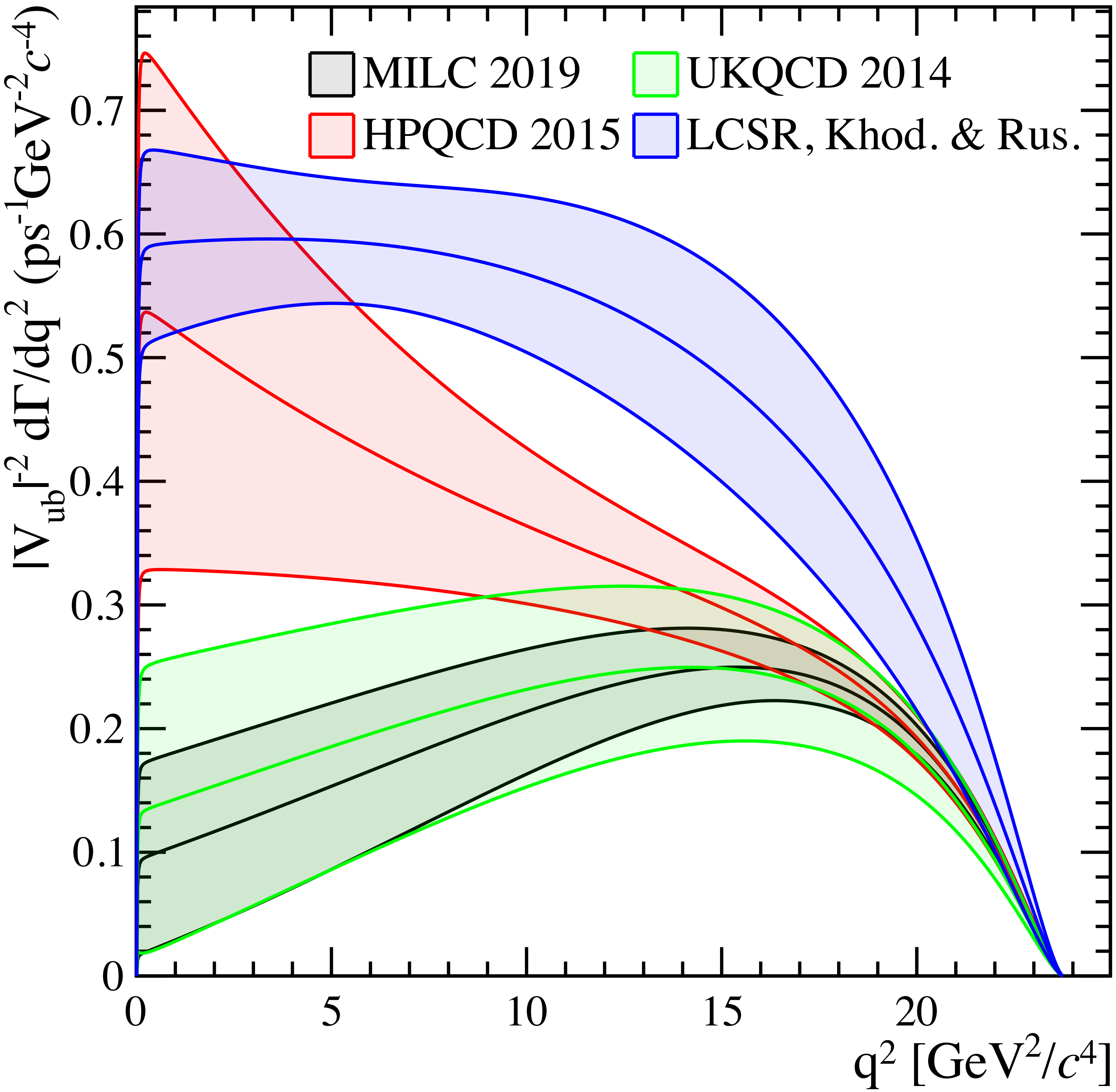
CLN fit	**Stat** **Syst**
Unfolded fit	$\rho^2 = 1.16 \pm 0.05 \pm 0.07$
Unfolded fit with massless leptons	$\rho^2 = 1.17 \pm 0.05 \pm 0.07$
Folded fit	$\rho^2 = 1.14 \pm 0.04 \pm 0.07$
BGL fit	
Unfolded fit	$a_1^f = -0.005 \pm 0.034 \pm 0.046$
	$a_2^f = 1.00\,^{+0.00}_{-0.19}\,^{+0.00}_{-0.38}$
Folded fit	$a_1^f = 0.039 \pm 0.029 \pm 0.046$
	$a_2^f = 1.00\,^{+0.00}_{-0.13}\,^{+0.00}_{-0.34}$

First observation of the decay $B_s^0 \to K^-\mu^+\nu_\mu$ and measurement of $|V_{ub}|/|V_{cb}|$

Phys. Rev. Lett. ***126****, 081804*

FF_K calculations

Phys. Rev. Lett. 126, 081804 [supplementary material]



BF result and systematic uncertainties

Phys. Rev. Lett. 126, 081804

$$\frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)} = [4.89 \pm 0.21(\text{stat})^{+0.20}_{-0.21}(\text{syst}) \pm 0.14(D_s)] \times 10^{-3}$$ full q^2

Relative systematic uncertainties on $\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)/\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)$

Uncertainty	All q^2	low q^2	high q^2
Tracking	2.0	2.0	2.0
Trigger	1.4	1.2	1.6
Particle identification	1.0	1.0	1.0
$\sigma(m_{\text{corr}})$	0.5	0.5	0.5
Isolation	0.2	0.2	0.2
Charged BDT	0.6	0.6	0.6
Neutral BDT	1.1	1.1	1.1
q^2 migration	–	2.0	2.0
Efficiency	1.2	1.6	1.6
Fit template	$^{+2.3}_{-2.9}$	$^{+1.8}_{-2.4}$	$^{+3.0}_{-3.4}$
Total	$^{+4.0}_{-4.3}$	$^{+4.3}_{-4.5}$	$^{+5.0}_{-5.3}$