Extraction of the ratio $|V_{ub}|/|V_{cb}|$ from a combined study of the exclusive decays.

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The CKM matrix

$$V_{CKM} = V_{uL} V_{dL}^{\dagger}$$

- Parameterized by 4 parameters: three mixing angles and the CP-violating phase.

$$\sum_{k=1\ldots3} V_{ik}^{*} V_{kj\neq i} = 0,$$

- Can be represented as triangles in the complex plane.

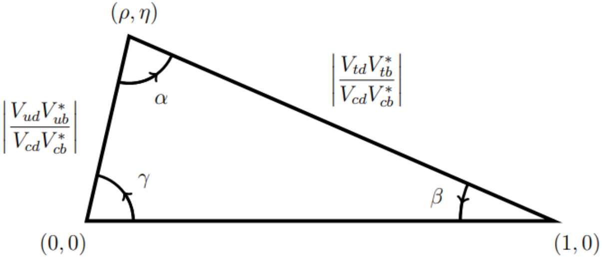
$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- Upto $\mathcal{O}(\lambda^3)$, complex numbers only in 1-3 and 3-1 matrix elements.

The Unitarity Triangle

- The one obtained for i = 1 and j = 3 involves the sum of three terms all of the same order in λ,

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

The CKM matrix

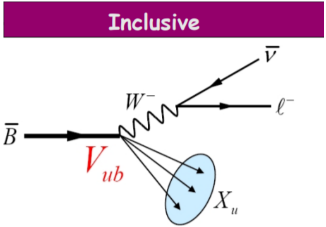
- Precision determinations of the CKM elements necessary to probe the quark mixing mechanism of the Standard Model.
- Important ingredients in the theoretical predictions of several observables in the flavor sector.
- V_{ub} $\rightarrow$ Source of CP violation within the SM
  $\rightarrow$ Less precisely known.

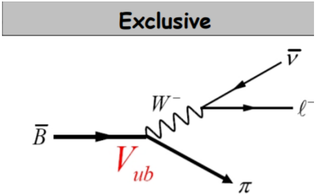
$$V_{CKM} = \begin{pmatrix} 0.97435 \pm 0.00016 & 0.22500 \pm 0.00067 & 0.00369 \pm 0.00011 \\ 0.22486 \pm 0.00067 & 0.97349 \pm 0.00016 & 0.04182^{+0.00085}_{-0.00074} \\ 0.00857^{+0.00020}_{-0.00018} & 0.04110^{+0.00083}_{-0.00072} & 0.999118^{+0.000031}_{-0.000036} \end{pmatrix}$$

[PDG, 2022]

Measurements of $|V_{xb}|$

- The transition $b \to u(c) l \bar{\nu}$ provides two avenues for determining $|V_{xb}|$ -





- Experimental and theoretical techniques for these two approaches different and largely independent → Important cross checks of our understanding.

- Mutual disagreement between exclusive and inclusive measurements.

$$|V_{ub}|^{exc} = (3.70 \pm 0.16) \times 10^{-3}, \quad |V_{ub}|^{inc} = (4.13 \pm 0.12^{+0.13}_{-0.14}) \times 10^{-3},$$
$$|V_{cb}|^{exc} = (39.4 \pm 0.8) \times 10^{-3}, \quad |V_{cb}|^{inc} = (42.2 \pm 0.8) \times 10^{-3},$$

[PDG, 2022]

$|V_{xb}|$ from inclusive decays

- The theoretical description of inclusive $\bar{B} \to X_{u(c)} l\bar{\nu}$ decays based on the Heavy Quark Expansion (an expansion in Λ_{QCD}/m_b).
- Total decay rate hard to measure due to the large background from $\bar{B} \to X_c l\bar{\nu}$ transitions $\to$ experimental cuts are necessary.
- In regions of phase space where $\bar{B} \to X_c l\bar{\nu}$ decays are suppressed, can't use HQE $\to$ introduce non-perturbative distribution functions(SF).
- Different approaches to model the shape function $\to$ extracted values of $|V_{ub}|$ model dependent.
- Recent analysis of the inclusive spectra with hadronic-tagging by Belle [arXiv:2102.00020] -

$$|V_{ub}|^{inc} = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3} \,.$$

[Talks by M.Prim, K.Vos]

$|V_{xb}|$ from exclusive decays

- Exclusive determinations require knowledge of the form factors.
- Pseudoscalar meson in final state -

$$\langle M(p_M)|V_\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_M^\mu - \frac{m_B^2 - m_M^2}{q^2} q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_M^2}{q^2} q^\mu$$

- $f_+(q^2 = 0) = f_0(q^2 = 0) \rightarrow$ cancel the divergence at $q^2 = 0$.

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- Vector meson in final state -

$$\langle V(k)|\bar{f}\gamma_\mu(1-\gamma_5)b|B(p)\rangle = -i\epsilon_\mu^*(m_B + m_V)A_1(q^2) + i(2p-q)_\mu(\epsilon^* \cdot q)\,\frac{A_2(q^2)}{m_B + m_V}$$
$$+ iq_\mu(\epsilon^* \cdot q)\,\frac{2m_V}{q^2}\,\left[A_3(q^2) - A_0(q^2)\right] + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p^\rho k^\sigma\,\frac{2V(q^2)}{m_B + m_V},$$
$$\text{with } A_3(q^2) = \frac{m_B + m_V}{2m_V}\,A_1(q^2) - \frac{m_B - m_V}{2m_V}\,A_2(q^2) \text{ and } A_0(0) = A_3(0)$$

Form factors

- Model-independent parametrization based on general properties of analyticity, unitarity, and crossing symmetry.
- Change of variables from q^2 to z maps the semileptonic region within a disc of radius $|z| < 1$ in the complex z plane.

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

where $t_\pm \equiv (m_B \pm m_f)^2$ and $t_0 \equiv t_+(1 - \sqrt{1 - t_-/t_+})$

- Can be expanded as a simple power series in z:

$$P_i(q^2)\phi_i(q^2, t_0)f_i(q^2) = \sum_{k=0}^{\infty} a_i^{(k)}(t_0)z(q^2, t_0)^k$$

- $P_i(q^2)$ chosen to vanish at any subthreshold poles to ensure analyticity of $f_i(q^2)$.

Form factors

- For $\bar{B} \to M l \bar{\nu}_l$ decays, $m_l^2 \leq q^2 \leq (m_B - m_M)^2$.
- BSZ parametrization (Bharucha et al. 1503.05534) -

$$f_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2} \sum_{k=0}^{N} a_k^i \left[z(q^2) - z(0)\right]^k$$

- Kinematic constraint $\to a_0^0 = a_0^+$
- BGL parametrization (Boyd et al. hep-ph/9412324) -

$$\mathcal{F}_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{j=0}^{N} a_j^i z^j,$$

- $q^2 = m_B^2 + m_F^2 - 2 m_B m_F w.$

Form factors

- Unitarity constraints -

$$\sum_{j=0}^{N}(a_j^{f_+})^2 < 1, \quad \sum_{j=0}^{N}(a_j^{f_0})^2 < 1, \quad \sum_{j=0}^{N}(a_j^{g})^2 < 1$$

$$\sum_{j=0}^{N}(a_j^{f})^2 + (a_j^{\mathcal{F}_1})^2 < 1, \quad \sum_{j=0}^{N}(a_j^{\mathcal{F}_2})^2 < 1.$$

- The kinematical constraints on the form factors, at zero and maximum recoil are given as

$$\begin{aligned} \mathcal{F}_1(1) &= m_B(1-r)f(1), \\ \mathcal{F}_2(w_{max}) &= \frac{1+r}{m_B^2(1+w_{max})(1-r)r}\mathcal{F}_1(w_{max}). \end{aligned}$$

References for $|V_{ub}|$ and $|V_{cb}|$ determinations

Mode	References
$B \to \pi l\nu$	RBC/UKQCD(1501.05373), Fermilab/MILC(1507.01618), JLQCD(2203.04938), LCSR(1811.00983, 2102.07233), Belle(1012.0090, 1306.2781), BaBar(1005.3288, 1208.1253)
$B \to \rho l\nu$	LCSR(1811.00983, 1503.05534, 1907.11092), Belle(1306.2781), BaBar(1005.3288)
$B \to \omega l\nu$	LCSR(1503.05534, 1907.11092), Belle(1306.2781), BaBar(1205.6245, 1308.2589)
$B_s \to K l\nu$	RBC/UKQCD(1501.05373), Fermilab/MILC(1901.02561), HPQCD(1406.2279), LCSR(1703.04765)
$B \to D l\nu$	HPQCD(1505.03925), Fermilab/MILC(1503.07237), LCSR(1811.00983), Belle(1510.03657)
$B \to D^* l\nu$	Fermilab/MILC(2105.14019), LCSR(1811.00983), Belle(1809.03290,2301.07529)
$B_s \to D_s l\nu$	HPQCD(1906.00701), Fermilab/MILC(1202.6346), LCSR(1912.09335), LHCb(2001.03225)
$B_s \to D_s^* l\nu$	HPQCD(2105.11433), LCSR(1912.09335), LHCb(2001.03225)

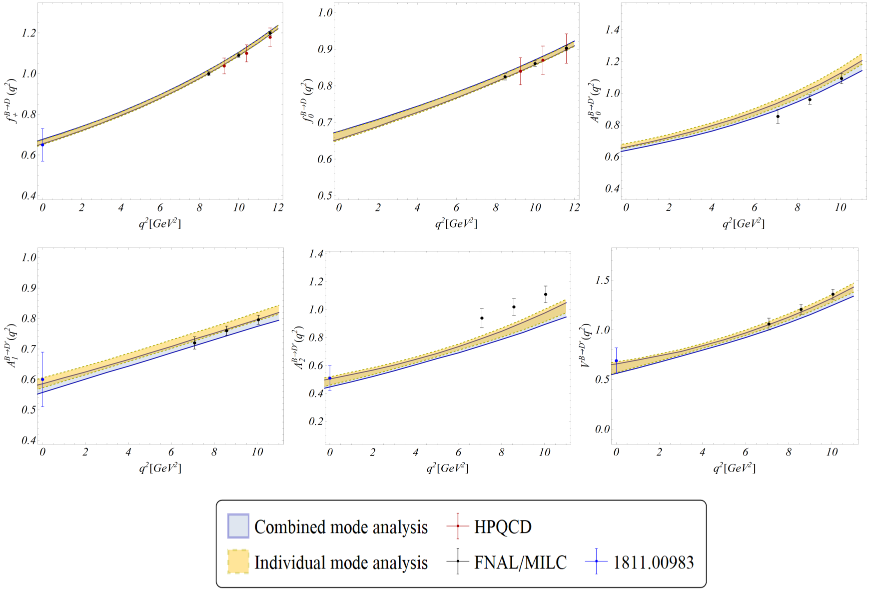
$R_{BF} = \mathcal{BR}(B_s \to K\mu\nu)/\mathcal{BR}(B_s \to D_s\mu\nu)$ from LHCb [2012.05143] in two bins of the $B_s \to K^-$ momentum transfer, $q^2 < 7$ GeV^2 and $q^2 > 7$ GeV^2.

Comparative study for extraction of $|V_{cb}|^{exc}$

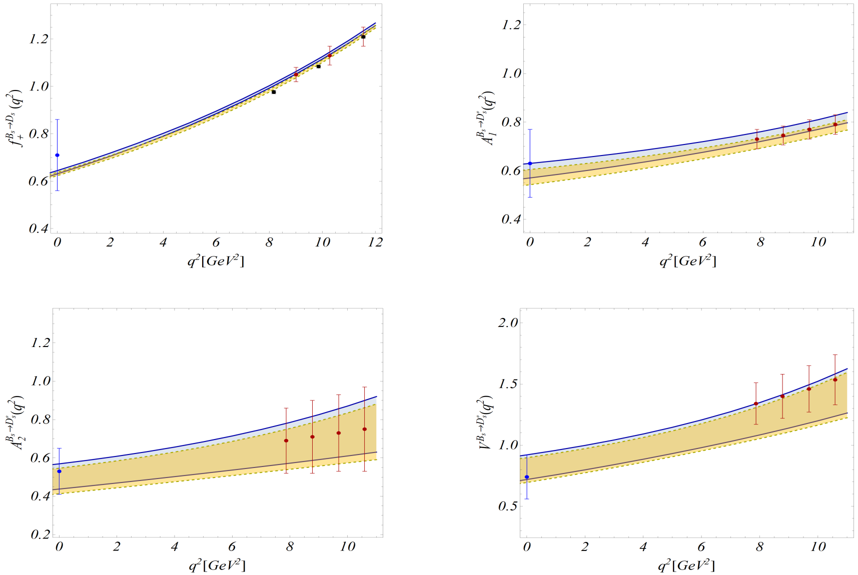
Mode	Inputs	χ^2_{min}/DOF	p-value (%)	$\lvert V_{cb}\rvert \times 10^3$
$\bar{B} \to D l^- \bar{\nu}$	Experiment+Fermilab/MILC [6] + HPQCD [7] + LCSR [25]	33.0/47	93.9	41.4 ± 1.1
$\bar{B} \to D^* l^- \bar{\nu}$	Experiment+Fermilab-MILC [10] + LCSR [25]	120.5/81	0.3	39.2 ± 0.8
$\bar{B}_s \to D_s^{(*)} l^- \bar{\nu}$	Experiment+ HPQCD [23, 24] + LCSR [26]	2.98/17	99.9	42.2 ± 1.3
	Experiment+ Fermilab/MILC [22] + HPQCD [23, 24] +LCSR [26]	29.3/20	8.2	44.0 ± 1.3
All modes	All Experiments + All lattice + LCSR	273.9/156	1.7×10^{-6}	$\mathbf{40.5 \pm 0.6}$
	All Experiments + All lattice (except $f_{+,0}^{B_s \to D_s}$ Fermilab/MILC [22]) +LCSR	163.2/150	21.8	40.3 ± 0.6

- JLQCD ($\bar{B} \to D^{(*)} l\nu$) $\to (40.5 \pm 0.8) \times 10^{-3}$
- Deviation from the inclusive determination of $(41.69 \pm 0.63) \times 10^{-3}$ obtained from the analyses of the q^2 moments and the differential rates (Bernlochner et al. 2205.10274) $\sim 1.4\ \sigma$.
- Deviation from the inclusive determination of $(42.16 \pm 0.51) \times 10^{-3}$ obtained using lepton energy and hadronic invariant mass moments distributions (Bordone et al. 2107.00604) $\sim 2.1\ \sigma$.
- $|V_{cb}| = (41.2 \pm 0.8) \times 10^{-3}$ [Martinelli et al. 2204.05925].

Plots for $B \to D^{(*)}$



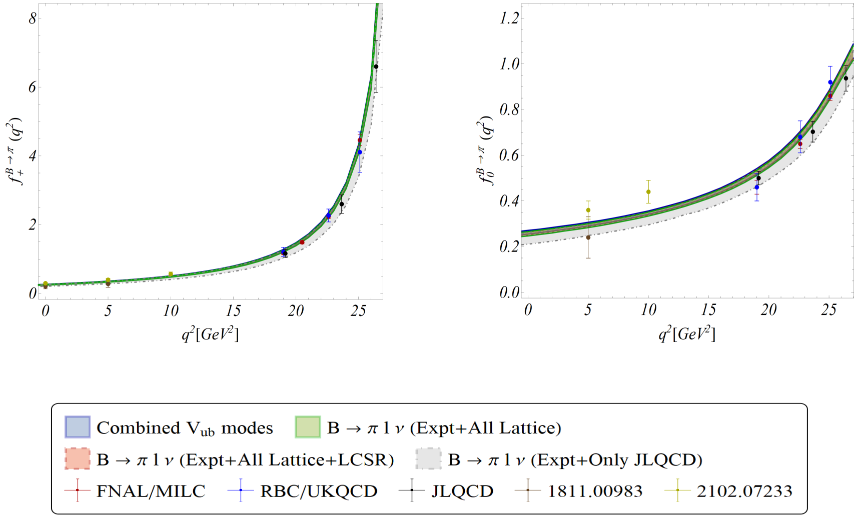
Plots for $B_s \to D_s^{(*)}$



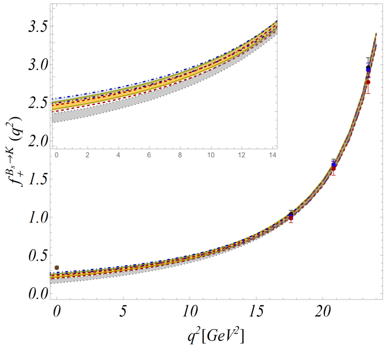
Comparative study for extraction of $|V_{ub}|^{exc}$

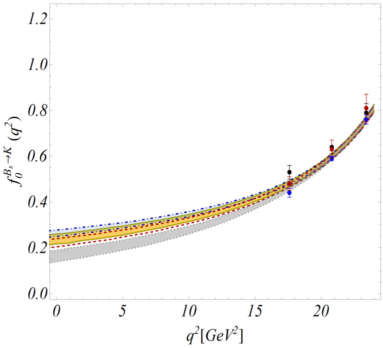
Mode	Fits with the Inputs	χ^2_{min}/DOF	p-value(%)	$\lvert V_{ub}\rvert \times 10^3$	$\lvert V_{ub}\rvert/\lvert V_{cb}\rvert$
$\bar{B} \to \pi l^- \bar{\nu}$	(I) All Experiment+JLQCD	10.8/11	45.7	3.98 ± 0.42	0.098 ± 0.010
	(II) Fit-I + LCSR [25, 38]	16.3/29	97.2	3.73 ± 0.24	0.092 ± 0.006
	(III) All Experiment+Fermilab/MILC	11.7/12	46.6	3.78 ± 0.16	0.093 ± 0.004
	(IV) Fit-III + LCSR [25, 38]	24.2/30	76.2	3.84 ± 0.15	0.095 ± 0.004
	(V) All Experiment+RBC/UKQCD	11.0/11	44.3	3.75 ± 0.34	0.093 ± 0.009
	(VI) Fit-V + LCSR [25, 38]	17.5/29	95.3	3.57 ± 0.23	0.088 ± 0.006
	(VII) All Experiment+ RBC/UKQCD+JLQCD	27.2/17	5.5	3.69 ± 0.24	0.091 ± 0.006
	(VIII) All Experiment+ All Lattice	50.8/24	0.1	3.69 ± 0.13	0.091 ± 0.003
	(IX) Fit-VIII + LCSR [25, 38]	55.7/42	7.7	3.69 ± 0.12	0.091 ± 0.003
$\bar{B} \to \pi l^- \bar{\nu}$ and $\bar{B}_s \to K l^- \bar{\nu}$	(X) All Experiment + All Lattice	74.9/38	0.03	3.60 ± 0.11	0.089 ± 0.003
	(XI) Fit-(X) + LCSR [25, 38, 41]	95.5/57	0.1	3.50 ± 0.10	0.086 ± 0.003
	(XII) All Experiment+ All Lattice except inputs from HPQCD on $B_s \to K$ [40]	58.8/32	0.3	3.74 ± 0.12	0.092 ± 0.003
	(XIII) Fit-(XII) + LCSR [25, 38, 41]	88.4/51	0.1	3.52 ± 0.10	0.087 ± 0.003
$\bar{B} \to \rho l^- \bar{\nu}$	Experiment + LCSR [25, 36, 37]	31.2/41	86.6	3.22 ± 0.26	0.080 ± 0.007
$\bar{B} \to \omega l^- \bar{\nu}$	Experiment+LCSR [36, 37]	8.6/15	89.7	3.09 ± 0.33	0.076 ± 0.008
$\bar{B} \to \pi(\rho,\omega) l^- \bar{\nu}$	All inputs combined	99.6/100	49.4	3.58 ± 0.11	0.088 ± 0.003
$\bar{B} \to \pi(\rho,\omega) l^- \bar{\nu}$ and $\bar{B}_s \to K l^- \bar{\nu}$	$B \to \pi$ and $B_s \to K$: Experiment + Lattice; $B \to (\rho,\omega)$: Experiment + LCSR	117.8/96	6.5	3.52 ± 0.10	0.087 ± 0.003
	For all modes: Experiment + lattice + LCSR	137.2/115	7.7	3.45 ± 0.09	0.085 ± 0.003

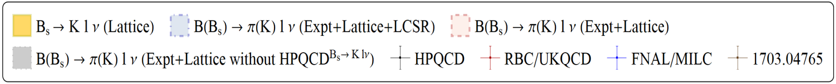
Plots for $B \to \pi$



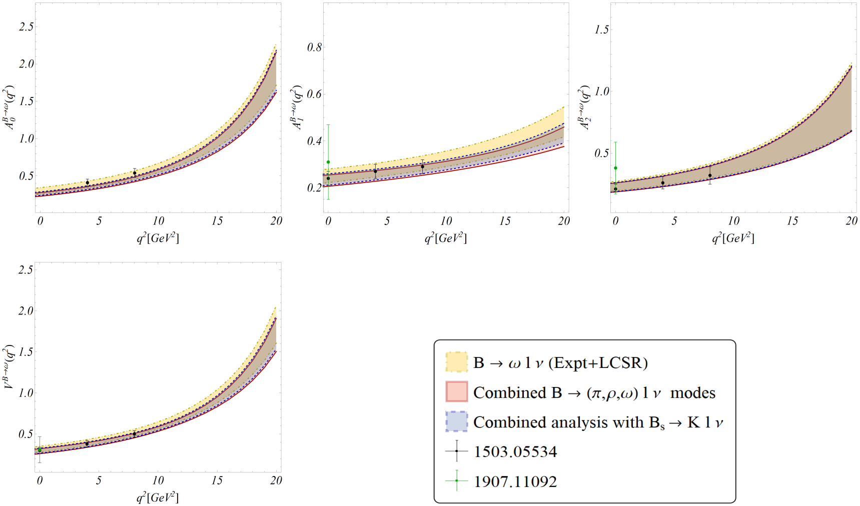
Plots for $B_s \to K$



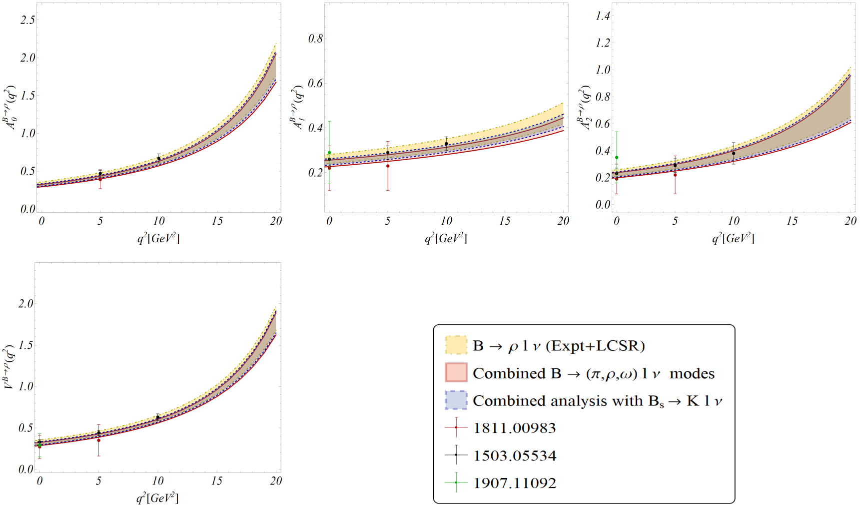




Plots for $B \to \omega$



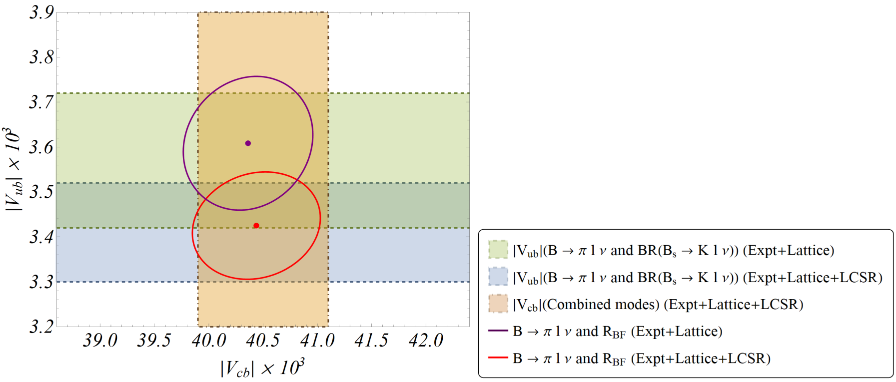
Plots for $B \to \rho$



Comparative study for extraction of $|V_{ub}|/|V_{cb}|$

- $\frac{|V_{ub}|}{|V_{cb}|} = 0.079 \pm 0.006$ (from $\frac{\Gamma(\Lambda_b \to p\mu\nu)}{\Gamma(\Lambda_b \to \Lambda_c\mu\nu)}$ [LHCb 1504.01568])

Mode(s)	Inputs	$\chi^2_{min}/\mathrm{DOF}$	$\lvert V_{ub}\rvert \times 10^3$	$\lvert V_{cb}\rvert \times 10^3$	$\frac{\lvert V_{ub}\rvert}{\lvert V_{cb}\rvert}$	Correlation (%) $[\lvert V_{ub}\rvert, \lvert V_{cb}\rvert]$
$\bar{B} \to \pi l^- \bar{\nu}$ and $b \to c l^- \bar{\nu}$	Experiment+ All Lattice	171.1/156	3.61 ± 0.15	40.4 ± 0.6	0.089 ± 0.004	12.8
	Experiment+All Lattice+LCSR	191.7/167	3.43 ± 0.12	40.4 ± 0.6	0.085 ± 0.003	14.1
	Experiment+ All Lattice +LCSR (without $f_+^{B_s \to K}$)	176.1/166	3.57 ± 0.13	40.3 ± 0.6	0.089 ± 0.003	11.5
$b \to c l^- \bar{\nu}$, $b \to u l^- \bar{\nu}$	Experiment+Lattice+LCSR	231.8/225	3.34 ± 0.10	40.4 ± 0.6	0.083 ± 0.003	12.4
	Experiment+ All Lattice +LCSR (without $f_+^{B_s \to K}$)	219.5/224	3.51 ± 0.11	40.2 ± 0.6	0.087 ± 0.003	10.4



3.9
3.8
3.7
3.6
3.5
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$|V_{ub}| \times 10^3$
39.0
39.5
40.0
40.5
41.0
41.5
42.0
$|V_{cb}| \times 10^3$
$|V_{ub}|(B \to \pi\, l\, \nu$ and $BR(B_s \to K\, l\, \nu))$ (Expt+Lattice)
$|V_{ub}|(B \to \pi\, l\, \nu$ and $BR(B_s \to K\, l\, \nu))$ (Expt+Lattice+LCSR)
$|V_{cb}|$(Combined modes) (Expt+Lattice+LCSR)
$B \to \pi\, l\, \nu$ and R_{BF} (Expt+Lattice)
$B \to \pi\, l\, \nu$ and R_{BF} (Expt+Lattice+LCSR)

Estimated values of R_{BF}

Experimental : R_{BF} (Low) $= (1.66 \pm 0.12) \times 10^{-3}$, R_{BF} (High) $= (3.25 \pm 0.28) \times 10^{-3}$			
Mode	Inputs	Predictions of R_{BF} (10^{-3})	
		Low ($q^2 < 7$ GeV^2)	High ($q^2 > 7$ GeV^2)
$\bar{B} \to \pi l^- \bar{\nu}$ and $b \to c l^- \bar{\nu}$	Experiment+All Lattice	1.67 ± 0.11	3.32 ± 0.20
	Experiment+All Lattice+LCSR	1.79 ± 0.10	3.17 ± 0.18
	Experiment+All Lattice+LCSR (without $f_+^{B_s \to K}$)	1.67 ± 0.11	3.30 ± 0.19
$b \to c l^- \bar{\nu}$, $b \to u l^- \bar{\nu}$	Experiment+All Lattice+LCSR	1.80 ± 0.10	3.08 ± 0.17
	Experiment+All Lattice+LCSR (without $f_+^{B_s \to K}$)	1.63 ± 0.11	3.30 ± 0.19

Conclusions

- We have extracted exclusive $|V_{ub}|$ from $\bar{B} \to (\pi, \rho, \omega)\ell^- \bar{\nu}_\ell$ and $\bar{B}_s \to K\mu^- \bar{\nu}_\mu$ decays separately as well as from a combined analysis, studying the impact of the various form factor inputs. From the combined analysis we obtain $|V_{ub}| = (3.52 \pm 0.10) \times 10^{-3}$.
- For the various $b \to cl\nu$ transitions also, we have studied $|V_{cb}|^{exc}$ determination from $\bar{B} \to D^{(*)}\ell^- \bar{\nu}_\ell$ and $\bar{B}_s \to D_s^{(*)}\ell^- \bar{\nu}_\ell$ decays. From the combined analysis we obtain $|V_{cb}| = (40.5 \pm 0.6) \times 10^{-3}$.
- We have also determined the ratio $|V_{ub}|/|V_{cb}|$ from the $b \to u(c)l\nu$ modes independently and also after introducing the inputs on the BRs $\mathrm{BR}(B_s \to K\mu\nu)/\mathrm{BR}(B_s \to D_s\mu\nu)$ in the 2 bins and compared the results. We have found a good agreement between the measured value of $\frac{|V_{ub}|}{|V_{cb}|}$ obtained by LHCb from the measurement of $\frac{\Gamma(\Lambda_b \to p\mu^- \bar{\nu}_\mu)}{\Gamma(\Lambda_b \to \Lambda_c\mu^- \bar{\nu}_\mu)}$ and our estimated values from the semileptonic mesonic decays.

Thank you!