



A model independent description of $B \rightarrow D\pi\ell\nu$ decays

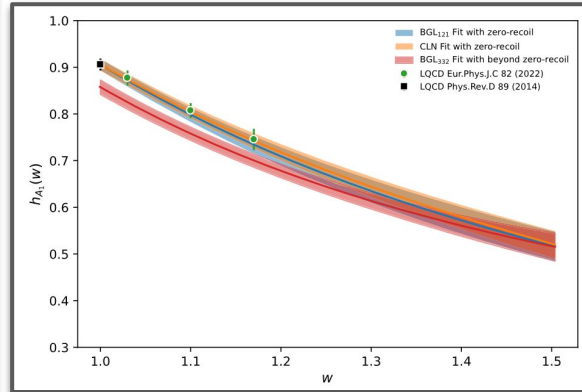
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In collaboration with Erik Gustafson, Ruth Van de Water, Raynette van Tonder & Mike Wagman

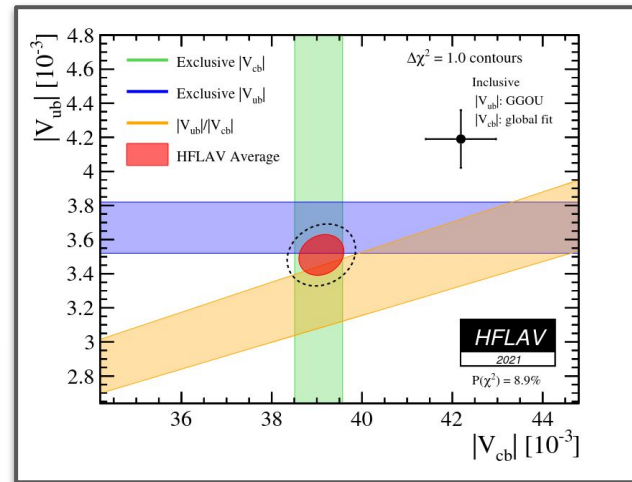
Motivation

Semileptonic B decays

- Several puzzling observations: inclusive vs. exclusive puzzles, the gap, ... (see Talks by Luiz Vale Silva [yesterday](#), Keri Vos [today](#))
- Challenges in modelling of inclusive decay width through exclusive states
→ Many analyses systematically limited (e.g. R(X), see Talks by Bob Kowalewskis [yesterday](#) & Markus Prim [today](#))
- Tensions in $B \rightarrow D^*$ form factors from Lattice & Belle



Taken from: [arXiv:2301.07529](https://arxiv.org/abs/2301.07529)



Decay	$\mathcal{B}(B^+)$	$\mathcal{B}(B^0)$
$B \rightarrow D \ell^+ \nu_\ell$	$(2.4 \pm 0.1) \times 10^{-2}$	$(2.2 \pm 0.1) \times 10^{-2}$
$B \rightarrow D^* \ell^+ \nu_\ell$	$(5.5 \pm 0.1) \times 10^{-2}$	$(5.1 \pm 0.1) \times 10^{-2}$
$B \rightarrow D_1 \ell^+ \nu_\ell$	$(6.6 \pm 0.1) \times 10^{-3}$	$(6.2 \pm 0.1) \times 10^{-3}$
$B \rightarrow D_2^* \ell^+ \nu_\ell$	$(2.9 \pm 0.3) \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$
$B \rightarrow D_0^* \ell^+ \nu_\ell$	$(4.2 \pm 0.8) \times 10^{-3}$	$(3.9 \pm 0.7) \times 10^{-3}$
$B \rightarrow D_1' \ell^+ \nu_\ell$	$(4.2 \pm 0.9) \times 10^{-3}$	$(3.9 \pm 0.8) \times 10^{-3}$
$B \rightarrow D \pi \pi \ell^+ \nu_\ell$	$(0.6 \pm 0.9) \times 10^{-3}$	$(0.6 \pm 0.9) \times 10^{-3}$
$B \rightarrow D^* \pi \pi \ell^+ \nu_\ell$	$(2.2 \pm 1.0) \times 10^{-3}$	$(2.0 \pm 1.0) \times 10^{-3}$
$B \rightarrow X_c \ell \nu_\ell$	$(10.8 \pm 0.4) \times 10^{-2}$	$(10.1 \pm 0.4) \times 10^{-2}$

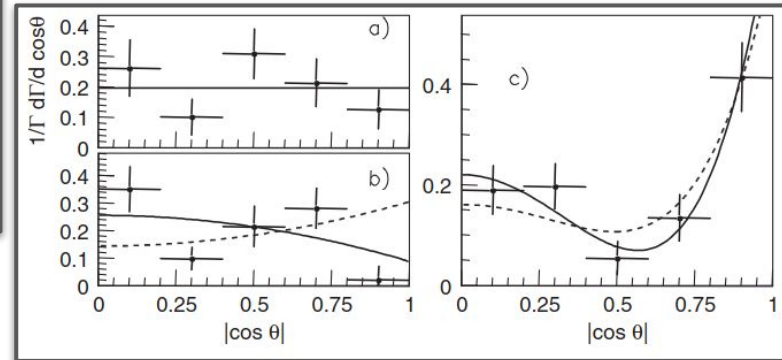
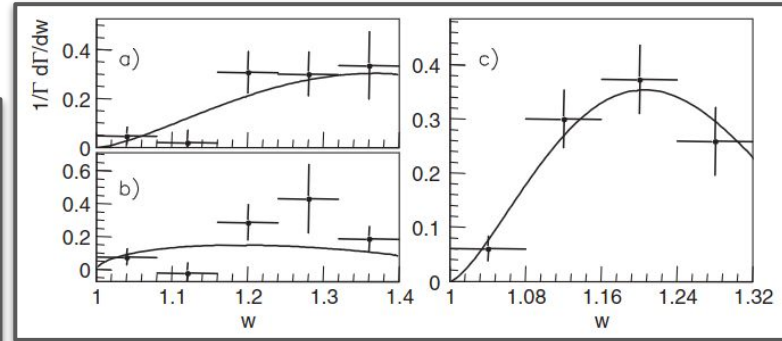
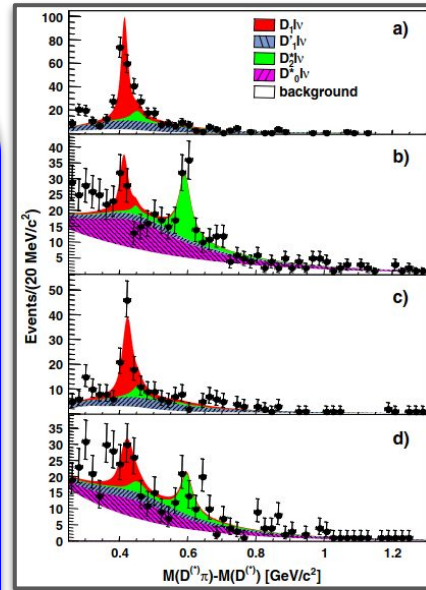


Taken from Raynette van Tonder's talk at [Challenges in Semileptonic B decays](#)

Motivation

$B \rightarrow D\pi\ell\nu$ decays

- Proceed through $B \rightarrow D^*, D_2^*, D_0^*$
- w-spectra from Belle & mass-spectra from Belle + BaBar
[Liventsev et al. (Belle) [PRD 77 \(2008\) 091503](#),
Aubert et al. (BaBar) [PRL 101 \(2008\) 261802](#)]
- In most Belle (II) analyses the fit from [Bernlochner, Ligeti [PRD 95 \(2017\) 1, 014022](#), Bernlochner, Ligeti, Robinson [PRD 97 \(2018\) 7, 075011](#)] (BLR) to the LLSW parametrization is used to model these decays



Motivation

Problems

- Recent LLSW fit points out that virtual $B \rightarrow D^*$ component should be included

[Le Yaouanc, Leroy, Roudeau [PRD 99 \(2019\) 7, 073010](#); [PRD 105 \(2022\) 1, 013004](#)]

(Orsay)

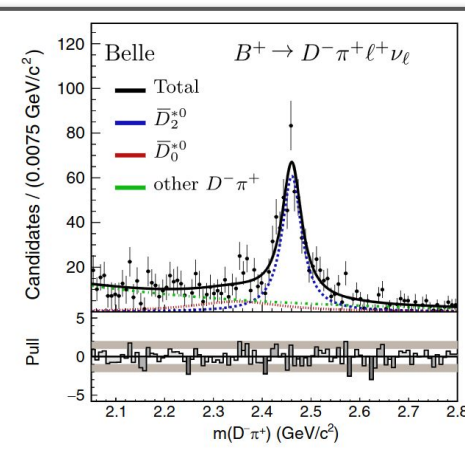
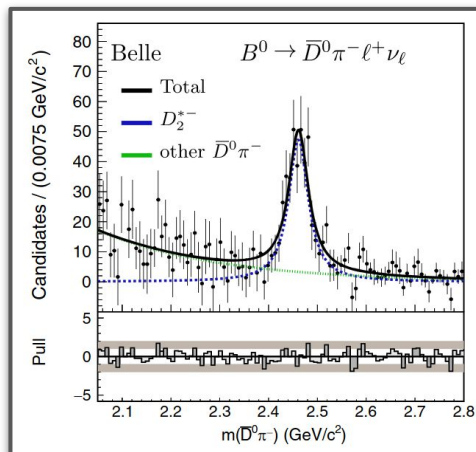
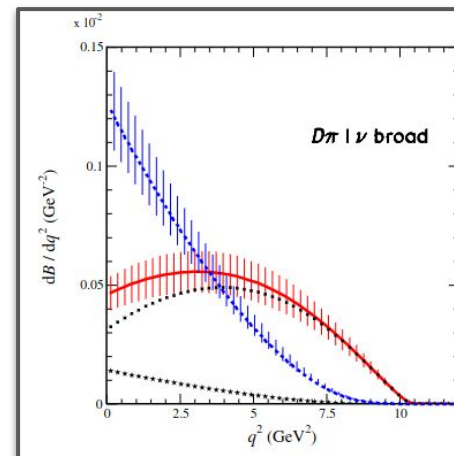
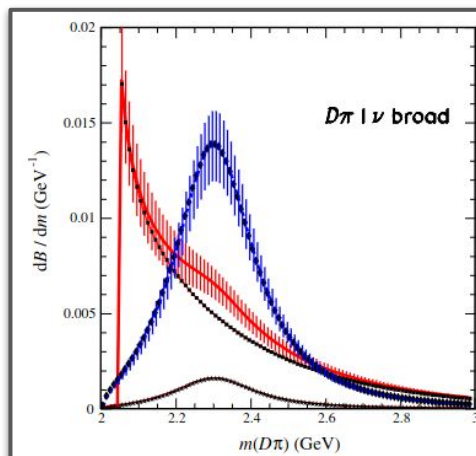
- New Belle measurement finds significantly lower $B \rightarrow D_0^*$ BF but a large falling component

[Meier et al. (Belle) [PRD 107 \(2023\) 9, 092003](#)]

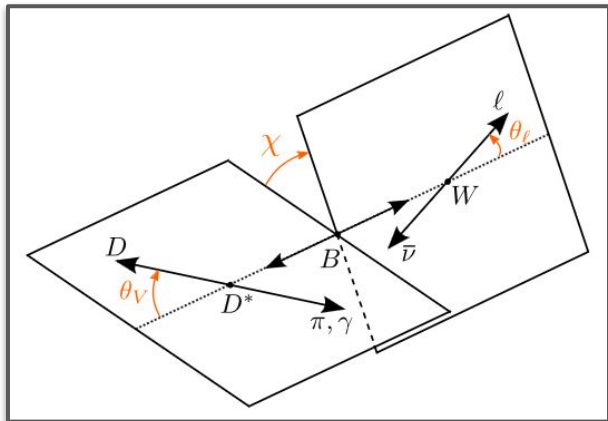
(see Christoph Schwandas Talk [earlier](#))

- D_0^* found to be a two-pole structure with masses of 2.1 & 2.45 GeV instead of 2.3 GeV, compatible with LHCb data

[Du, Kubis, Hanhart, Meißner [PRL 126 \(2021\) 19, 192001](#)]



Formalism



Taken from: [arXiv:2301.07529](https://arxiv.org/abs/2301.07529)

Form Factor decomposition

- We perform a partial wave decomposition in the D- π system
- 2 FFs for $l = 0, 4$ for every higher partial wave
- Setting the D- π invariant mass to a resonance mass, picking a specific partial wave and replacing L by the polarization tensors yields the standard expressions for the D^* and D_2^*
- In general, the FFs are complex

$$\langle D(p_D)\pi(p_\pi)|V_\mu|B(p_B)\rangle = \frac{2i}{M_B + M_{D\pi}} \epsilon_{\mu\nu\rho\sigma} p_{D\pi}^\rho p_B^\sigma \sum_l L^{(l),\nu} V^{(l)}(q^2, M_{D\pi}^2),$$

$$\langle D(p_D)\pi(p_\pi)|A_\mu|B(p_B)\rangle = 2M_{D\pi} \frac{q^\mu}{q^2} \sum_l L^{(l),\nu} q_\nu A_0^{(l)}(q^2, M_{D\pi}^2)$$

$$+ (M_B + M_{D\pi}) \sum_l \left(L^{(l),\mu} - \frac{L^{(l),\nu} q_\nu}{q^2} q^\mu \right) A_1^{(l)}(q^2, M_{D\pi}^2)$$

$$+ \left[\frac{(p_B + p_{D\pi})^\mu}{M_B + M_{D\pi}} - \frac{M_B - M_{D\pi}}{q^2} q^\mu \right] \sum_l L^{(l),\nu} q_\nu A_2^{(l)}(q^2, M_{D\pi}^2)$$

$$L_\mu^{(l)} p_{D\pi}^\mu = 0,$$

$$L_\mu^{(l)} q^\mu = M_B \left(\frac{|\vec{q}| |p_{D\pi}|}{M_B M_{D\pi}} \right)^l P_l(\cos \theta)$$

Differential decay rate

- Fully general expression including all PWs
- Allows for interference terms
- Five-fold differential
- Basis change to simplify expressions

$$V^{(l)} = \frac{M_B + M_{D\pi}}{2} g_l ,$$

$$A_0^{(l)} = \frac{1}{2} \mathcal{F}_{2,l} ,$$

$$A_1^{(l)} = \frac{1}{2(M_B + M_{D\pi})} f_l ,$$

$$A_2^{(l)} = \frac{M_B + M_{D\pi}}{\lambda(M_B^2, M_{D\pi}^2, q^2)} \left[M_{D\pi} \mathcal{F}_{1,l} - \frac{(M_B^2 - q^2 - M_{D\pi}^2)}{2} f_l \right]$$

$$\begin{aligned} |\mathcal{M}_{ab}|^2 &= \langle B | (V - A)^\mu | (D\pi)_a \rangle \langle (D\pi)_b | (V - A)^\nu | B \rangle L_{\mu\nu} \\ &= M_B^2 M_{D\pi}^2 (q^2 - m_l^2) W^{a+b} \left\{ \left[\frac{\mathcal{F}_{1,a} \mathcal{F}_{1,b}^*}{\lambda(M_B^2, M_{D\pi}^2, q^2) q^2} + \frac{m_l^2}{q^4} \mathcal{F}_{2,a} \mathcal{F}_{2,b}^* - \frac{f_a f_b^*}{\lambda(M_B^2, M_{D\pi}^2, q^2)} - g_a g_b^* \right] P_a^0 P_b^0 \right. \\ &\quad - \left(1 - \frac{m_l^2}{q^2} \right) \frac{\mathcal{F}_{1,a} \mathcal{F}_{1,b}^*}{\lambda(M_B^2, M_{D\pi}^2, q^2) q^2} \cos^2 \theta_l P_a^0 P_b^0 \\ &\quad + \left(\frac{f_a f_b^*}{\lambda(M_B^2, M_{D\pi}^2, q^2)} + g_a g_b^* \right) \left[P_{a-1}^0 P_{b-1}^0 + \frac{P_{a-1}^1 P_{b-1}^1}{ab} \right] - \left(1 - \frac{m_l^2}{q^2} \right) g_a g_b^* \frac{P_a^1 P_b^1}{ab} (1 - \cos^2 \theta_l) \\ &\quad - \left(1 - \frac{m_l^2}{q^2} \right) \left(\frac{f_a f_b^*}{\lambda(M_B^2, M_{D\pi}^2, q^2)} - g_a g_b^* \right) \frac{P_a^1 P_b^1}{ab} \cos^2 \chi (1 - \cos^2 \theta_l) \\ &\quad - \frac{f_a g_b^* + g_a f_b^*}{\sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \left[P_{a-1}^0 P_{b-1}^0 + \frac{P_{a-1}^1 P_{b-1}^1}{ab} \right] \cos \theta_l + \left(\frac{f_a g_b^* + g_a f_b^*}{\sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} + \frac{m_l^2}{q^4} \frac{\mathcal{F}_{1,a} \mathcal{F}_{2,b}^* + \mathcal{F}_{2,a} \mathcal{F}_{1,b}^*}{\sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \right) \cos \theta_l P_a^0 P_b^0 \\ &\quad + i \left(1 - \frac{m_l^2}{q^2} \right) \frac{g_a f_b^* - f_a g_b^*}{\sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \frac{P_a^1 P_b^1}{ab} (1 - \cos^2 \theta_l) \cos \chi \sin \chi \\ &\quad + \left(1 - \frac{m_l^2}{q^2} \right) \left[\frac{\mathcal{F}_{1,a} f_b^*}{\sqrt{q^2} \lambda(M_B^2, M_{D\pi}^2, q^2)} \frac{P_a^0 P_b^1}{b} + \frac{f_a \mathcal{F}_{1,b}^*}{\sqrt{q^2} \lambda(M_B^2, M_{D\pi}^2, q^2)} \frac{P_a^1 P_b^0}{a} \right] \cos \theta_l \sin \theta_l \cos \chi \\ &\quad + i \left(1 - \frac{m_l^2}{q^2} \right) \left[\frac{\mathcal{F}_{1,a} g_b^*}{\sqrt{q^2} \sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \frac{P_a^0 P_b^1}{b} - \frac{g_a \mathcal{F}_{1,b}^*}{\sqrt{q^2} \sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \frac{P_a^1 P_b^0}{a} \right] \cos \theta_l \sin \theta_l \sin \chi \\ &\quad - \left[\left(\mathcal{F}_{1,a} g_b^* + \frac{m_l^2}{q^2} \mathcal{F}_{2,a} f_b^* \right) \frac{P_a^0 P_b^1}{b} + \left(g_a \mathcal{F}_{1,b}^* + \frac{m_l^2}{q^2} f_a \mathcal{F}_{2,b}^* \right) \frac{P_a^1 P_b^0}{a} \right] \frac{\cos \chi \sin \theta_l}{\sqrt{q^2} \sqrt{\lambda(M_B^2, M_{D\pi}^2, q^2)}} \\ &\quad - i \left[\left(\mathcal{F}_{1,a} f_b^* + \frac{m_l^2}{2q^2} \mathcal{F}_{2,a} g_b^* \right) \frac{P_a^0 P_b^1}{b} - \left(f_a \mathcal{F}_{1,b}^* + \frac{m_l^2}{2q^2} g_a \mathcal{F}_{2,b}^* \right) \frac{P_a^1 P_b^0}{a} \right] \frac{\sin \chi \sin \theta_l}{\sqrt{q^2} \lambda(M_B^2, M_{D\pi}^2, q^2)} \left. \right\} \end{aligned}$$

Unitarity bounds

- Derivation of BGL ([Boyd, Grinstein, Lebed [PRL 74 \(1995\) 4603-4606](#); [PRD 56 \(1997\) 6895-6911](#);...]) can be generalized to multi-hadron final states
- However, a z-expansion is not straightforward, due to the dependence of the FFs on 2 variables
- Weak interaction and final state interactions can be factorized ([Watson [PR 88 \(1952\) 1163-1171](#)])
- Approximate weak interaction dependence on invariant mass
- Corrections can be systematically incorporated

$$\chi_{(J)}^L(Q^2) \equiv \left. \frac{\partial \Pi_{(J)}^L}{\partial q^2} \right|_{q^2=Q^2} = \frac{1}{\pi} \int_0^\infty dq^2 \frac{\text{Im} \Pi_{(J)}^L(q^2)}{(q^2 - Q^2)^2},$$

$$\chi_{(J)}^T(Q^2) \equiv \left. \frac{1}{2} \frac{\partial^2 \Pi_{(J)}^T}{\partial (q^2)^2} \right|_{q^2=Q^2} = \frac{1}{\pi} \int_0^\infty dq^2 \frac{\text{Im} \Pi_{(J)}^T(q^2)}{(q^2 - Q^2)^3}$$

$$\text{Im} \Pi_A^L \supset \frac{1}{32\pi^3} \frac{M_B^4}{q^4} \sum_{l=0} \int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} dM_{D\pi}^2 M_{D\pi}^2 W^{2l+1} \frac{1}{2l+1} |\mathcal{F}_2^{(l)}(q^2, M_{D\pi}^2)|^2,$$

$$\text{Im} \Pi_A^T \supset \frac{1}{96\pi^3} \frac{M_B^4}{q^4} \sum_{l=0} \int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} dM_{D\pi}^2 M_{D\pi}^2 \frac{W^{2l+1}}{\lambda(M_B^2, M_{D\pi}^2, q^2)} \frac{1}{2l+1} |\mathcal{F}_1^{(l)}(q^2, M_{D\pi}^2)|^2$$

$$+ \frac{1}{96\pi^3} \frac{M_B^4}{q^2} \sum_{l=1} \int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} dM_{D\pi}^2 M_{D\pi}^2 \frac{W^{2l+1}}{\lambda(M_B^2, M_{D\pi}^2, q^2)} \frac{l+1}{l(2l+1)} |f^{(l)}(q^2, M_{D\pi}^2)|^2$$

$$\text{Im} \Pi_V^T \supset \frac{1}{96\pi^3} \frac{M_B^4}{q^2} \sum_{l=1} \int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} dM_{D\pi}^2 M_{D\pi}^2 W^{2l+1} \frac{l+1}{l(2l+1)} |g^{(l)}(q^2, M_{D\pi}^2)|^2.$$

$$f(q^2, M_{D\pi}^2) = \hat{f}(q^2, M_{D\pi}^2) g(M_{D\pi}^2) \approx (\tilde{f}(q^2) + \mathcal{O}((M_R^2 - M_{D\pi}^2)/M_B^2)) g(M_{D\pi}^2)$$

Unitarity bounds

- For the q^2 -dependent remainder, a standard z -expansion can be derived
- Invariant mass dependent terms can be treated in the same way
- Outer functions more complicated
- Recent developments concerning states above the first branch cut can be straightforwardly included
[\[Blake, Meinel, Rahimi, van Dyk arXiv:2205.06041; Flynn, Jüttner, Tsang arXiv:2303.11285\]](#)
- Standard formulae recovered in the narrow-width limit

$$\mathcal{I}^{(l)}(q^2) = \int_{(M_D+m_\pi)^2}^{(\sqrt{q^2}-M_B)^2} dM_{D\pi}^2 M_{D\pi}^2 W^{2l+1} g^{(l)}(M_{D\pi}^2)$$

$$\Phi_{\mathcal{I}} = e^{i\phi} \exp\left(\frac{1}{2\pi} \int_{-\pi}^{\pi} dt \frac{e^{it} + z}{e^{it} - z} \log |\mathcal{I}^{(l)}(e^{it})|\right)$$

$$\tilde{f}(q^2) = \frac{1}{P(q^2)\Phi(q^2)} \sum_{i=0}^{\infty} a_i z^i$$

$$\sum_i |a_i|^2 \leq 1$$

$$|f(q^2, M_{D\pi})|^2 = |\tilde{f}(q^2)|^2 \frac{g^2}{(M_{D\pi}^2 - M_R^2)^2 + M_R^2 \Gamma_R^2} \approx |\tilde{f}(q^2)|^2 \frac{\pi g^2}{M_R \Gamma_R} \delta(M_{D\pi}^2 - M_R^2)$$

S-Wave

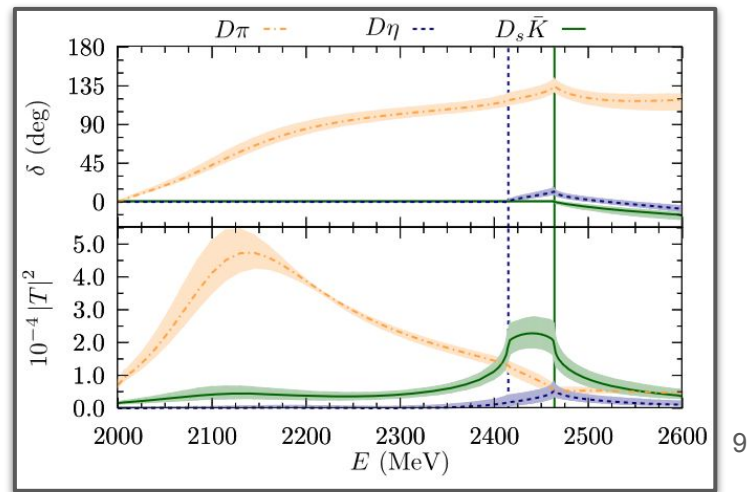
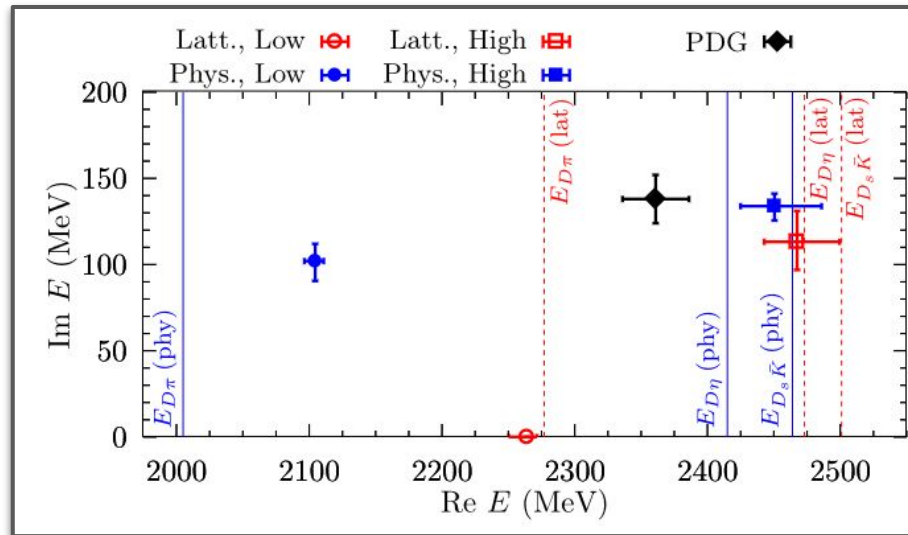
Coupled-channel treatment

- The T-Matrix for coupled-channel $D\pi$, $D\eta$ & $D_s\bar{K}$ scattering has been obtained using NLO χ^{PT} up to 2.6 GeV
[\[Albaladejo, Fernandez-Soler, Guo, Nieves PLB 767 \(2017\) 465-469, Moir et al. \(Hadron Spectrum\) JHEP 10 \(2016\) 011\]](#)
- We obtain a dispersive representation for the S-Wave FFs and treat the 3 final states on common grounds

$$\text{Im } \vec{f}(q^2, M_{D\pi}^2 + i\epsilon) = T^*(M_{D\pi}^2 + i\epsilon) \Sigma(M_{D\pi}^2) \vec{f}(q^2, M_{D\pi}^2 + i\epsilon)$$

$$\vec{f}(q^2, M_{D\pi}^2) = \Omega(M_{D\pi}^2) \vec{P}(q^2, M_{D\pi}^2) \approx \Omega(M_{D\pi}^2) \vec{P}(q^2)$$

$$\text{Im } \Omega(s + i\epsilon) = \frac{1}{\pi} \int_{s_{\text{thr}}}^{\infty} \frac{T^*(s') \Sigma(s') \Omega(s')}{s' - s - i\epsilon} ds'$$



S-Wave

$$\mathcal{M}_{B^+ \rightarrow D_s^- K^+ l^+ \nu} \xrightarrow{p_K \rightarrow 0} 0$$

$$\mathcal{M}_{B^+ \rightarrow \bar{D}^0 \eta^+ l^+ \nu} \xrightarrow{p_\eta \rightarrow 0} \frac{1}{\sqrt{6}f} \mathcal{M}_{B^+ \rightarrow \bar{D}^0 l^+ \nu}$$

$$\mathcal{M}_{B^+ \rightarrow (D\pi)^0 l^+ \nu} = \sqrt{\frac{2}{3}} \mathcal{M}_{B^+ \rightarrow D^- \pi^+ l^+ \nu} + \sqrt{\frac{1}{3}} \mathcal{M}_{B^+ \rightarrow \bar{D}^0 \pi^0 l^+ \nu} \xrightarrow{p_\pi \rightarrow 0} \frac{1}{\sqrt{6}f} \mathcal{M}_{B^+ \rightarrow \bar{D}^0 l^+ \nu}$$

Coupled-channel treatment

- In the soft Goldstone boson limit the 3 channels can be related to known form factors
- $SU(3)_F$ breaking is partially accounted for by using the respective leptonic decay constants for π and η

$$\Omega(M_D^2) = \mathbb{1}$$

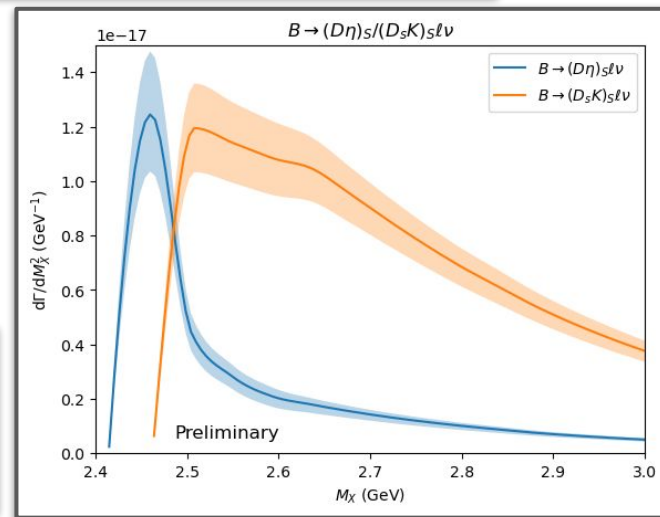
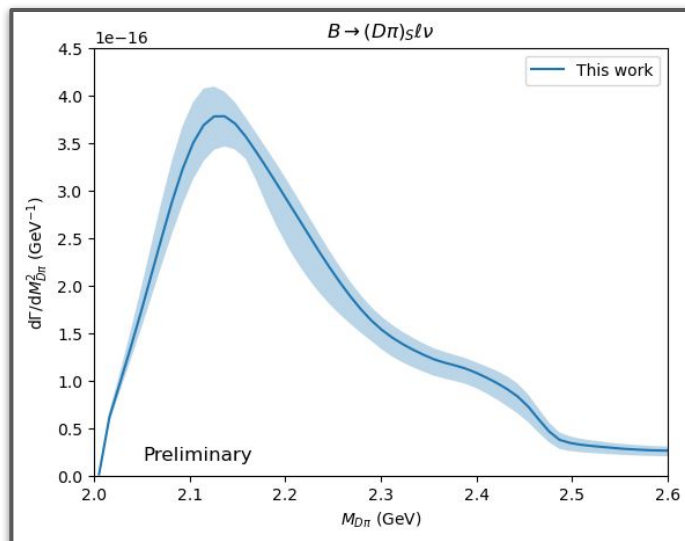
$$\vec{f}(q^2, M_D^2) = \frac{1}{\sqrt{6}} f_D(q^2) \begin{pmatrix} \frac{1}{f_\pi} \\ \frac{1}{f_\eta} \\ 0 \end{pmatrix}$$

$$\vec{f}(q^2, M_{D\pi}^2) = \frac{1}{\sqrt{6}} f_D(q^2) \Omega(M_{D\pi}^2) \begin{pmatrix} \frac{1}{f_\pi} \\ \frac{1}{f_\eta} \\ 0 \end{pmatrix}$$

S-Wave

Results

- Peak near 2.14 GeV in the $D\pi$ spectrum
- $B \rightarrow D\pi \ell \nu$ S-Wave contribution significantly smaller than previously determined
- $B \rightarrow D\eta \ell \nu$ S-Wave contribution negligibly small
- $B \rightarrow D_s K \ell \nu$ S-Wave contribution does not saturate Belle measurement
[\[Stypula et al. \(Belle\) PRD 86 \(2012\) 072007\]](#)
 → either D_2^* or $D_1^*(2600)$ likely contribute remainder



Channel	Prediction (Preliminary)	Measured
$\text{Br}(B \rightarrow (D\pi)_{S}\ell\nu)$	$1.22^{+0.07}_{-0.11} \times 10^{-3}$	$(4.20 \pm 0.75) \times 10^{-3}$
$\text{Br}(B \rightarrow (D_sK)_{S}\ell\nu)$	$(0.75 \pm 0.08) \times 10^{-4}$	$3.0^{+1.4}_{-1.2} \times 10^{-4}$
$\text{Br}(B \rightarrow (D\eta)_{S}\ell\nu)$	$(2.1 \pm 0.3) \times 10^{-5}$	—

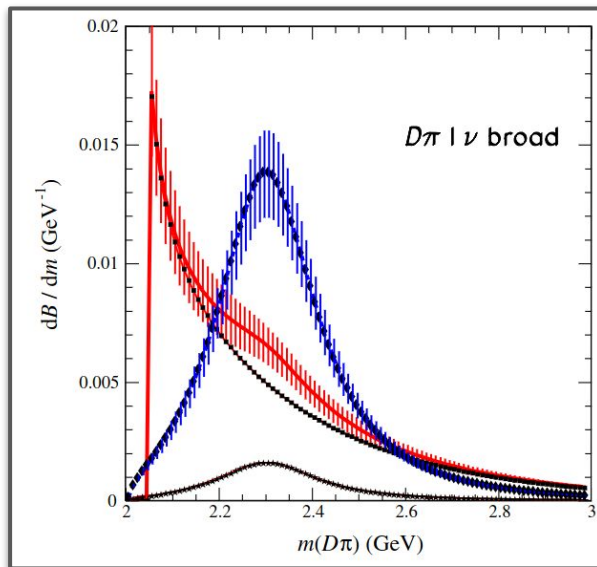
P & D-Wave

$$\text{Br}(D^* \rightarrow D\pi) \frac{d^4\Gamma}{dq^2 d\cos\theta d\cos\theta_l d\chi} \Big|_{\text{FNAL/MILC}} = \int dM_{D\pi}^2 \frac{d^5\Gamma}{dM_{D\pi}^2 dq^2 d\cos\theta d\cos\theta_l d\chi} \Big|_{\text{BFF}}$$

$$f^{(l)}(q^2, M_{D\pi}) = \tilde{f}^{(l)}(q^2) \frac{g}{(M_{D\pi}^2 - M_R^2) + iM_R\Gamma_R(M_{D\pi}^2)} X^{(l)}(|\vec{p}_D| r_{\text{BW}}, |\vec{p}_{D,0}| r_{\text{BW}})$$

Treatment

- Recently, it was pointed out that virtual D^* contributions should be taken into account in semileptonic decays
[\[Le Yaouanc, Leroy, Roudeau PRD 99 \(2019\) 7, 073010; PRD 105 \(2022\) 1, 013004\]](#)
- We introduce Blatt-Weisskopf damping factors and include r_{BW} as fit parameter
- For the D^* we use FNAL/MILC FFs and fit after integrating over the $D\pi$ invariant mass

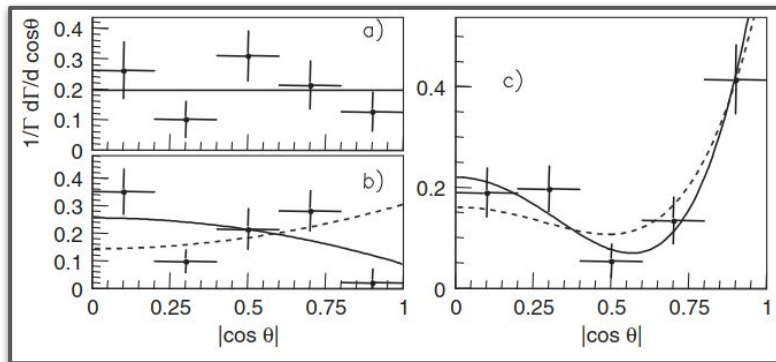
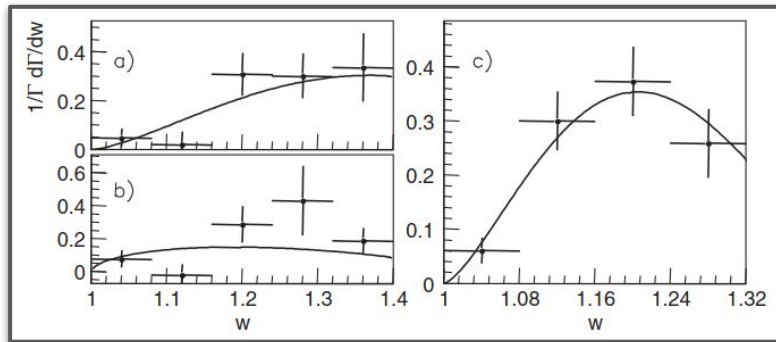


$$X^{(1)}(z, z_0) = \sqrt{\frac{1 + z_0^2}{1 + z^2}}$$

$$X^{(2)}(z, z_0) = \sqrt{\frac{9 + 3z_0^2 + z_0^4}{9 + 3z^2 + z^4}}$$

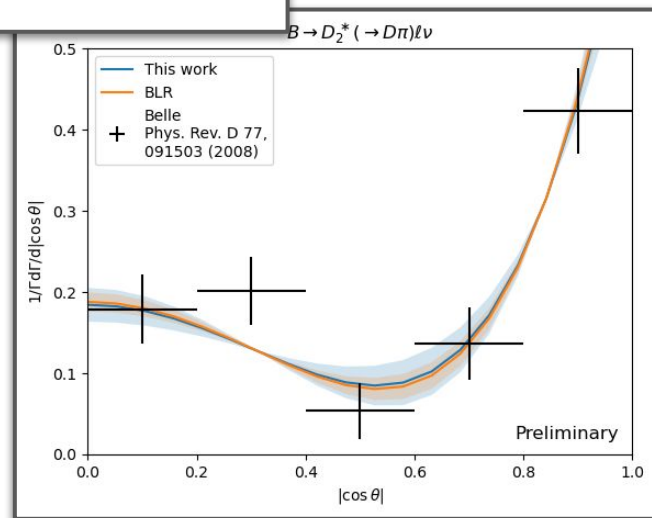
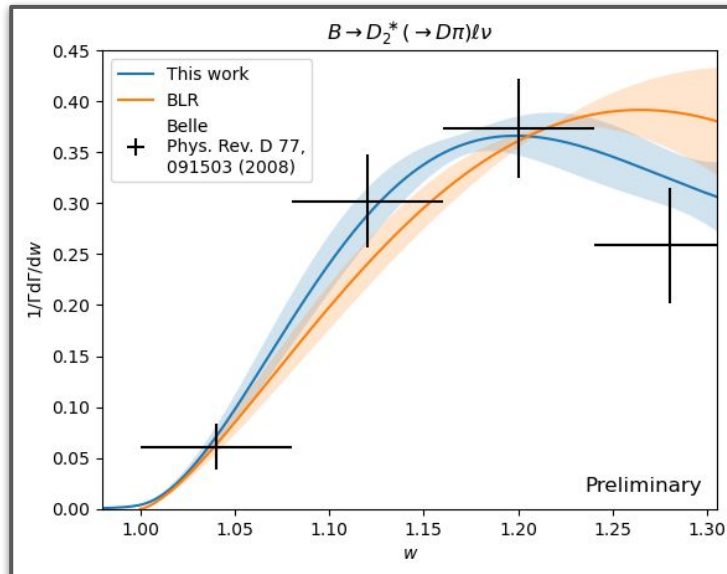
Treatment

- The D_2^* FFs are fitted to the spectra measured by Belle, with a loose constraint on the $B \rightarrow D_2^*(\rightarrow D\pi)\ell\nu$ decay rate, as well as to the $B^0 \rightarrow D_2^*(\rightarrow D\pi)\pi/K$ BFs
[\[Liventsev et al. \(Belle\) PRD 77 \(2008\) 091503\]](#)



Results for the D-Wave

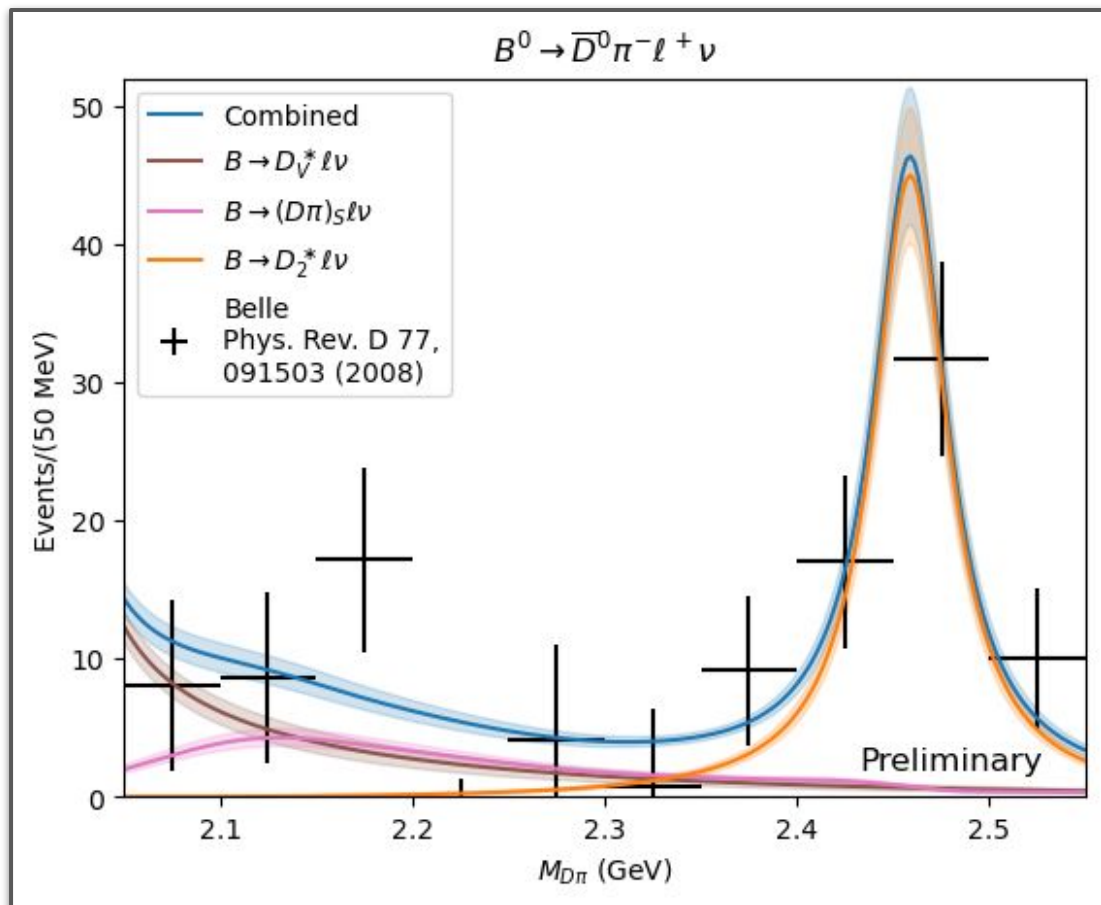
- The q^2 -spectrum we obtain is harder than the one obtained by BLR
- Possible reason: model independent Ansatz in our approach
- We obtain $R(D_2^*) = 0.11 \pm 0.06$, compared to $R(D_2^*) = 0.07 \pm 0.01$ (BLR)
- Uncertainties could be decreased by implementing the HQET constraints present in the LLSW parametrization on our more general FFs



Putting everything together

Fit to Belle (2007)

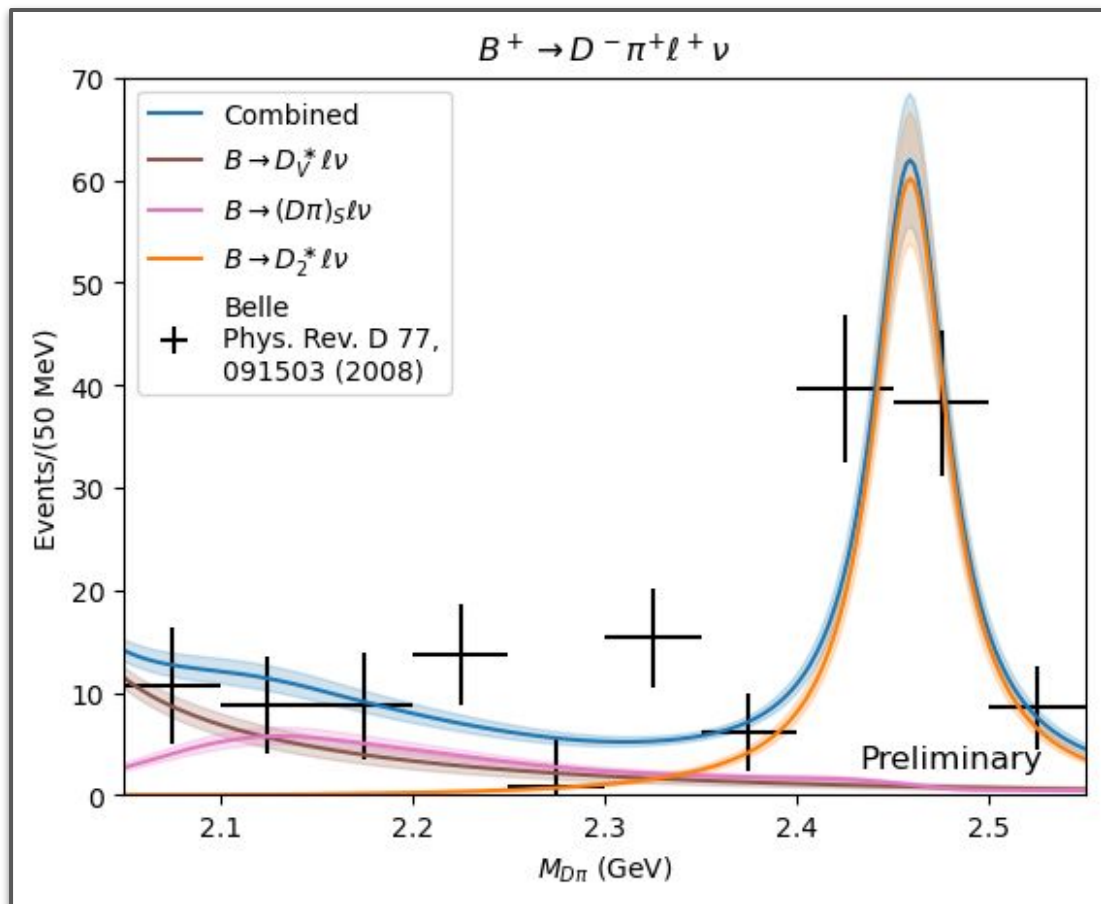
- Combined fit to both charge modes
- Do not include data above 2.55 GeV
- PDG averages for D_2^* mass and width
- Overall good agreement with data:
 $\chi^2/\text{dof} = 1.2$ (20 dof)
- Largest tension on left flank of D_2^* , can be resolved by increasing uncertainty in mass by 10-20 MeV



Putting everything together

Fit to Belle (2007)

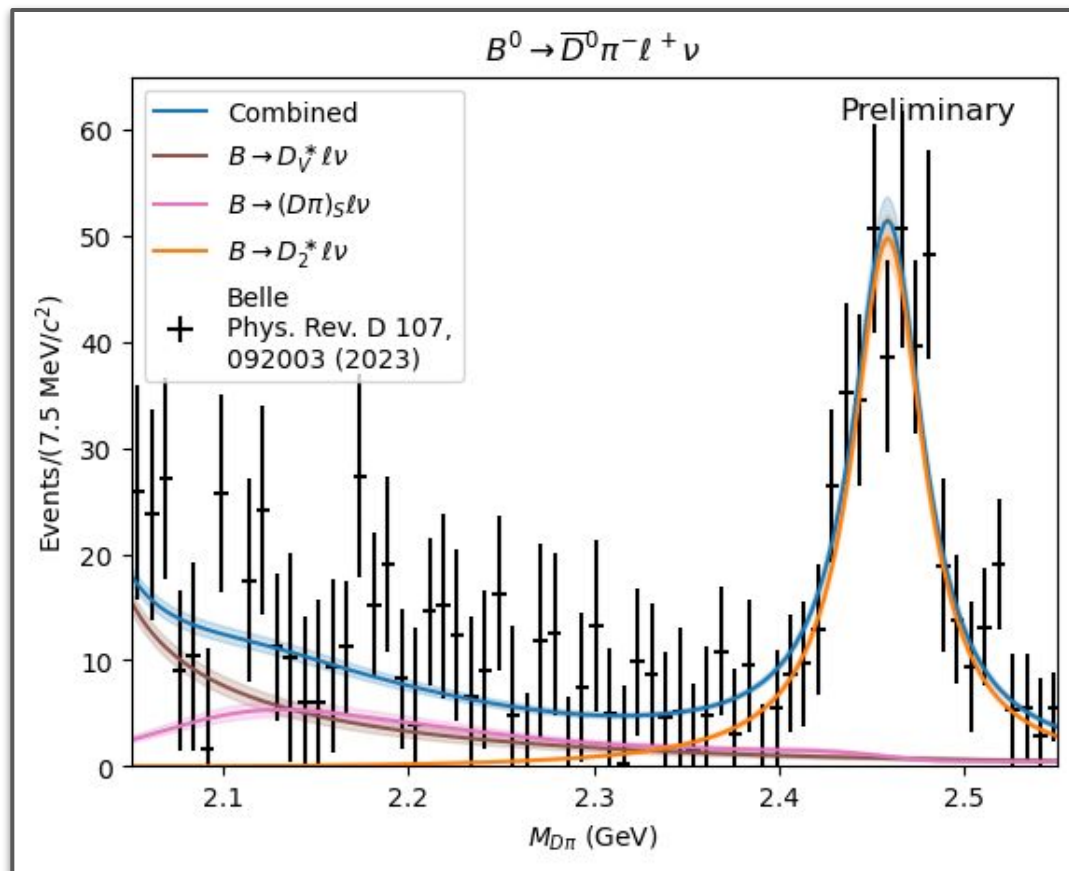
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Putting everything together

Fit to Belle (2022)

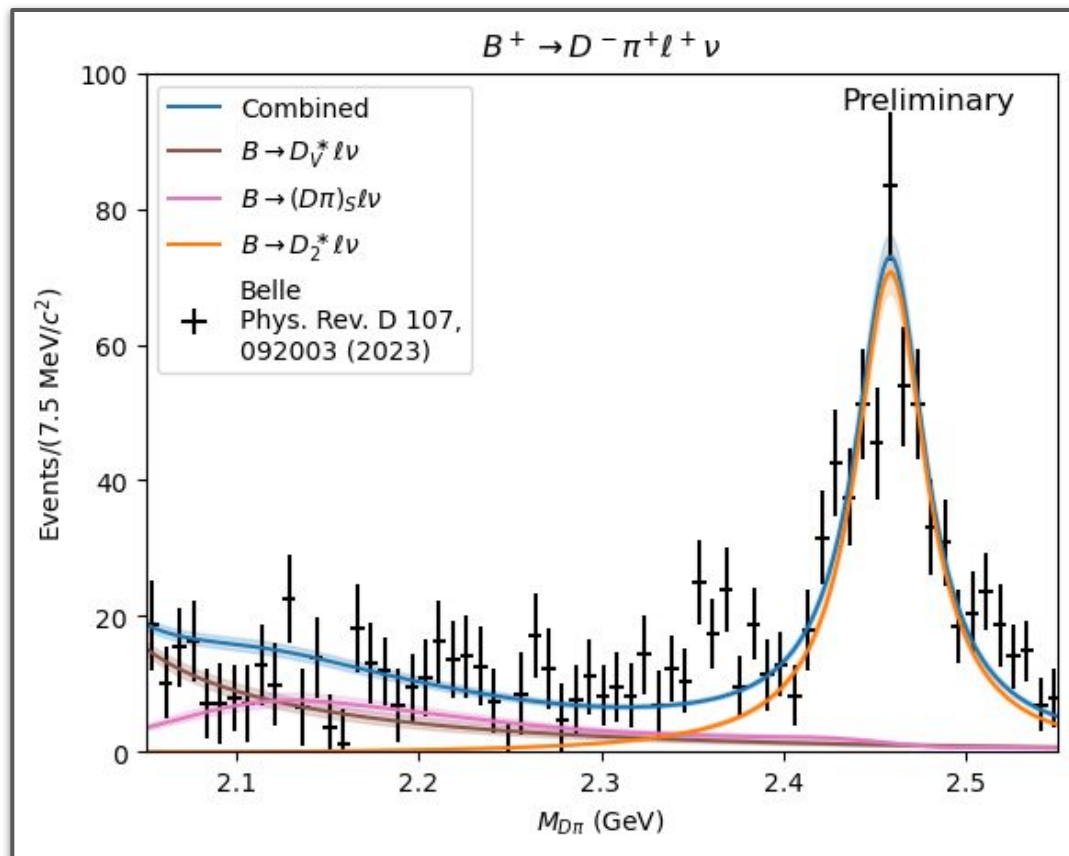
- Combined fit to both charge modes
- Do not include data above 2.55 GeV
- PDG averages for D_2^* mass and width
- Excellent agreement with data:
 $\chi^2/\text{dof} = 1.0$ (134 dof)
- Compared to Belle, our D^*+S -Wave contribution drops off faster than the falling exponential in the analysis
- Larger D_2^* yield than PDG and Belle



Putting everything together

Fit to Belle (2022)

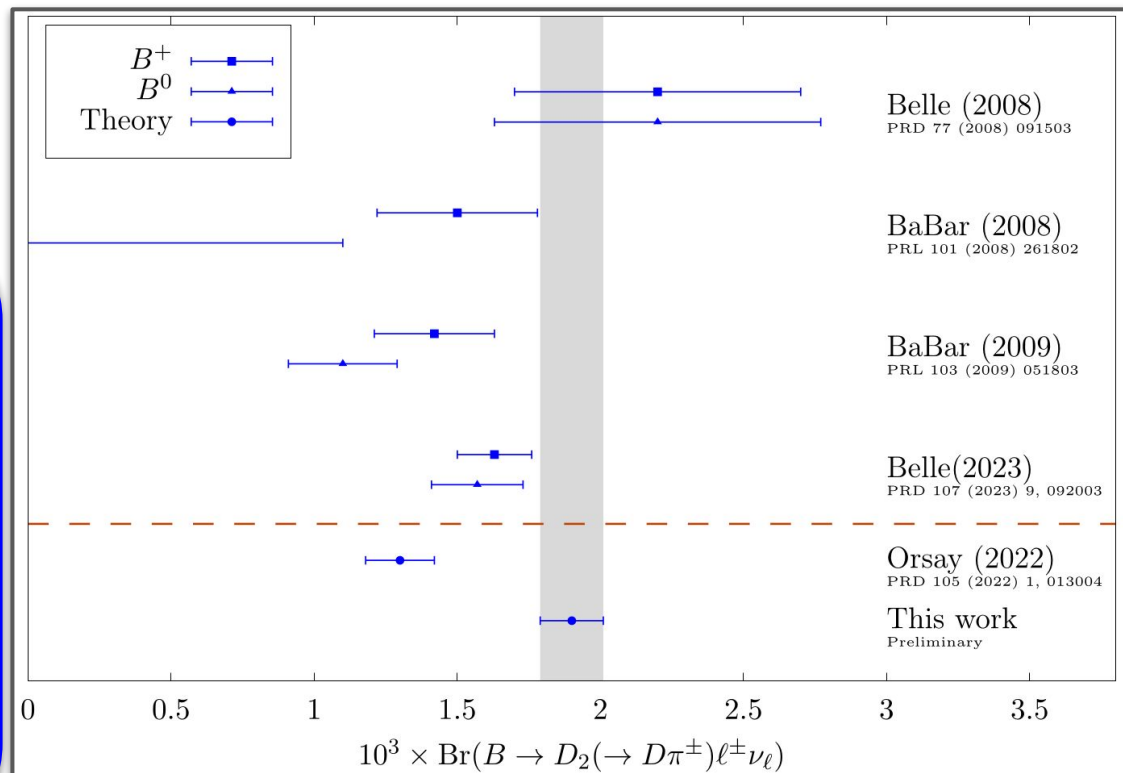
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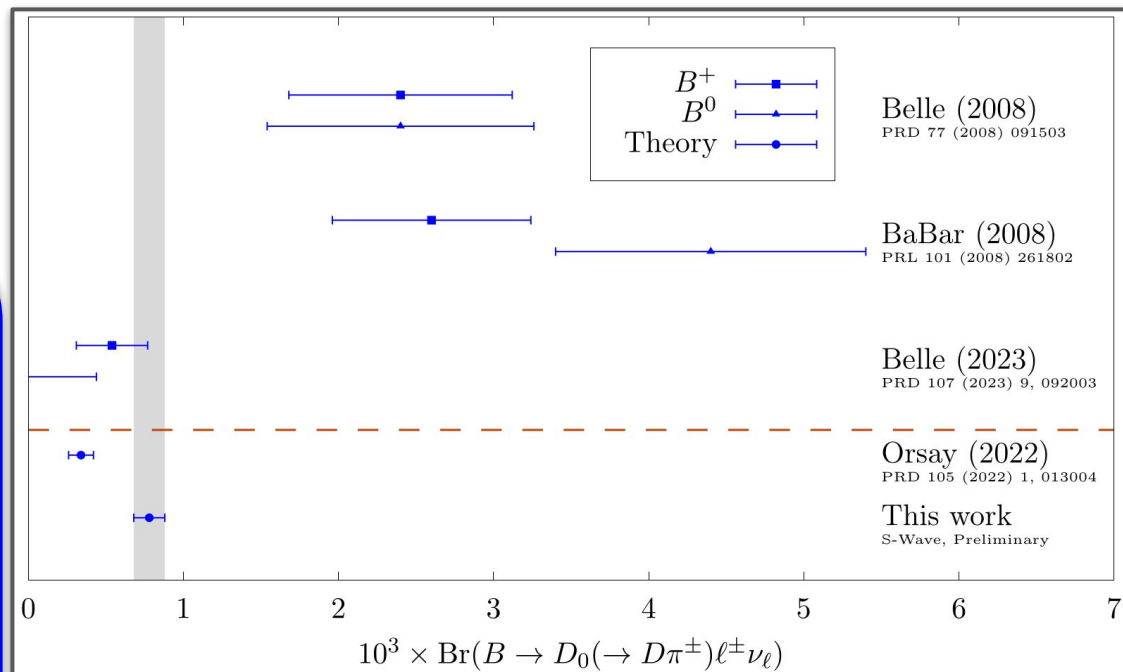
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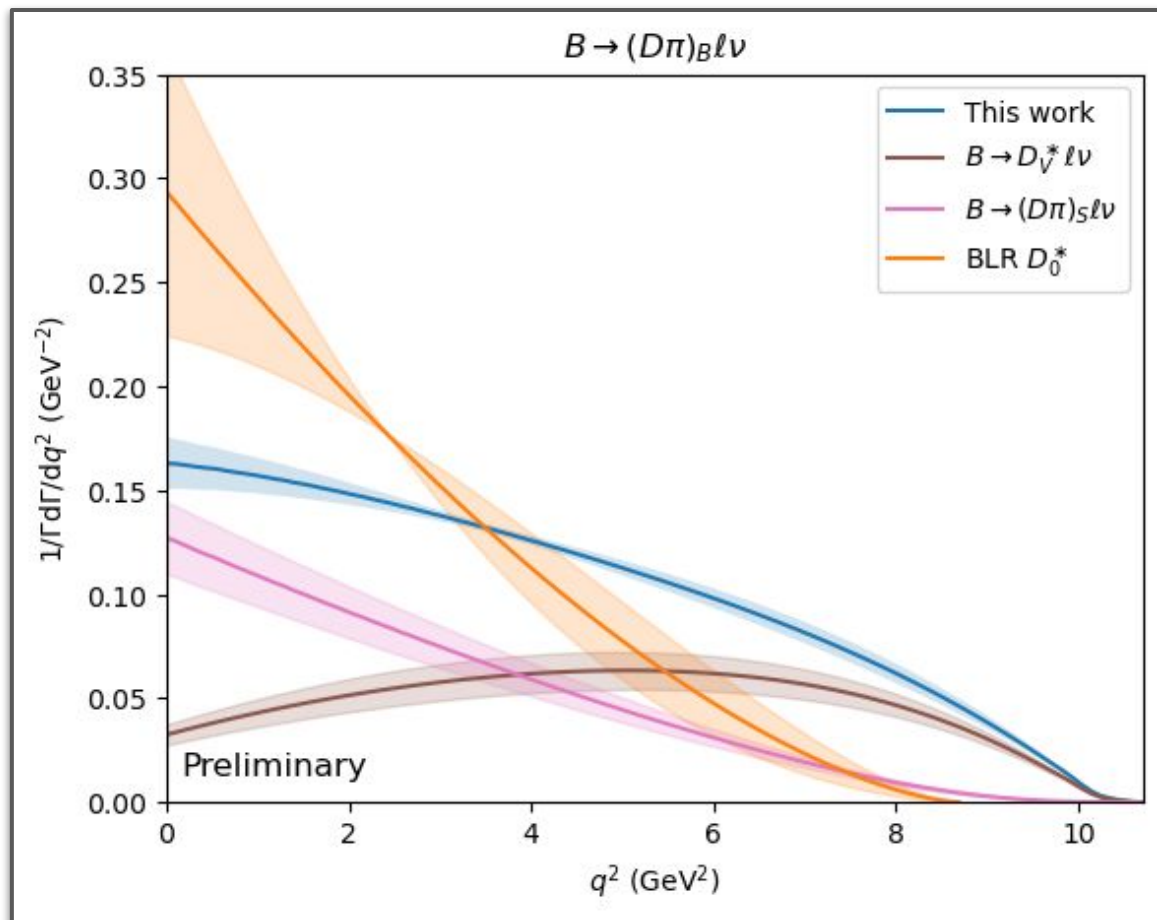
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Putting everything together

Possible impact on inclusive analyses

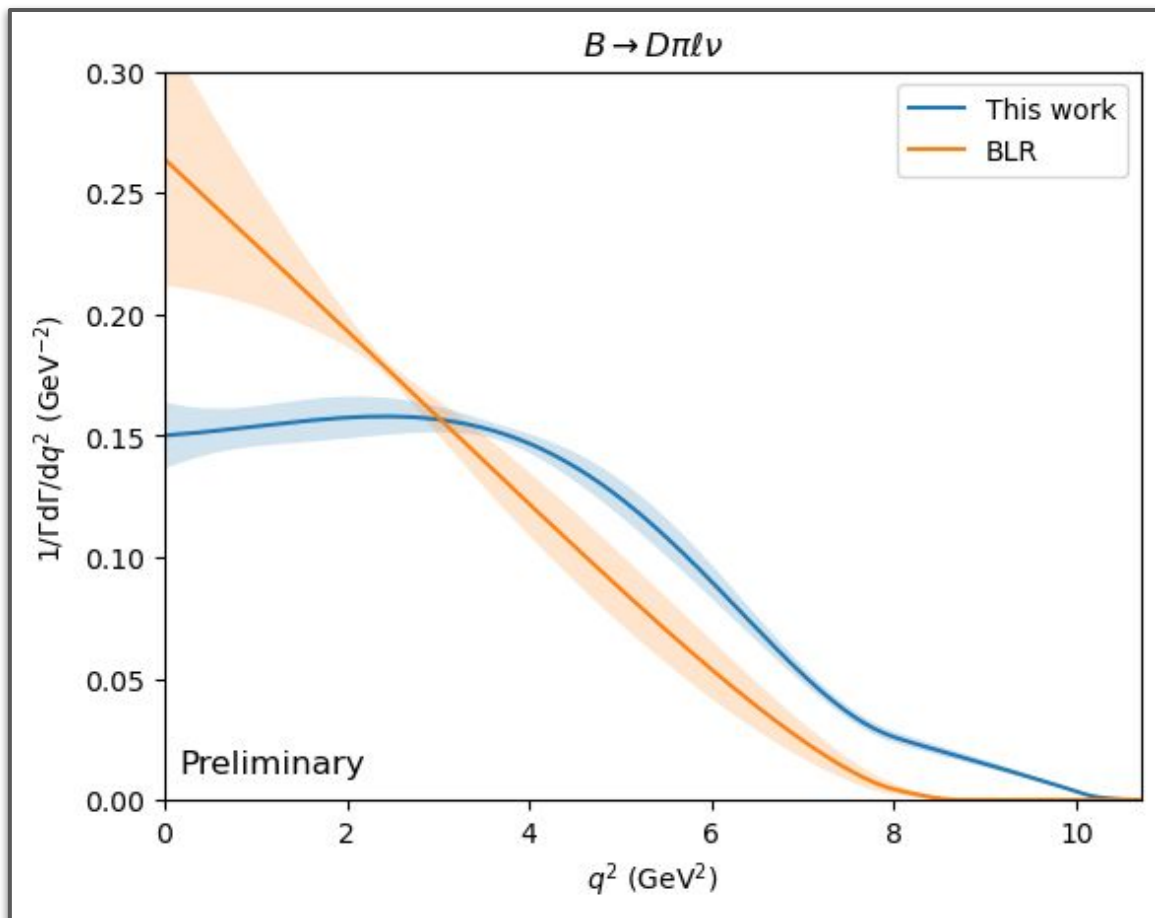
- Harder q^2 -spectrum in the narrow & broad components
- Possibly resolves the small tension seen in the inclusive q^2 -spectrum
- S-Wave $B \rightarrow D\eta\ell\nu$ decays can not account for the gap $\rightarrow B \rightarrow D^*\eta\ell\nu$ decays will also be subdominant
- N.B.: Endpoint of BLR will be washed out by MC generator



Putting everything together

Possible impact on inclusive analyses

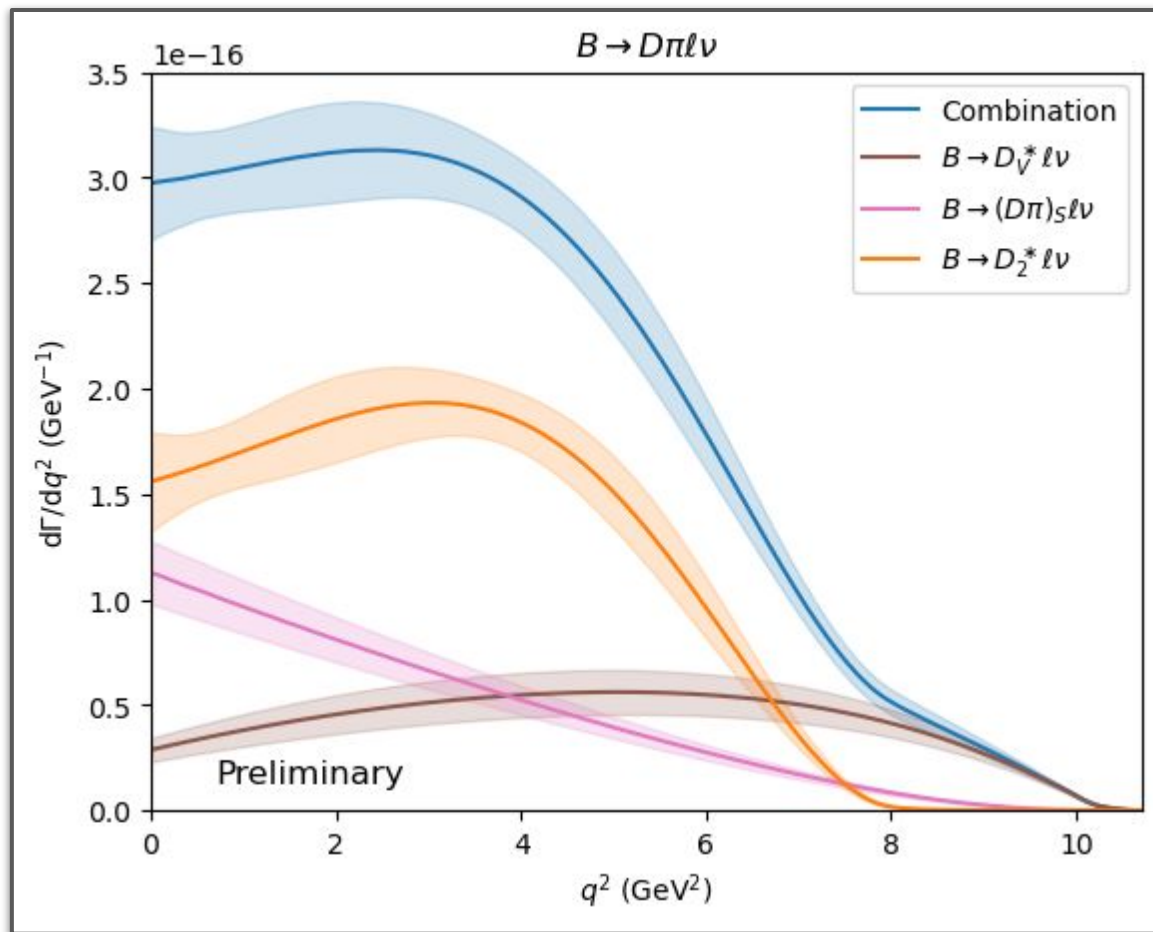
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Putting everything together

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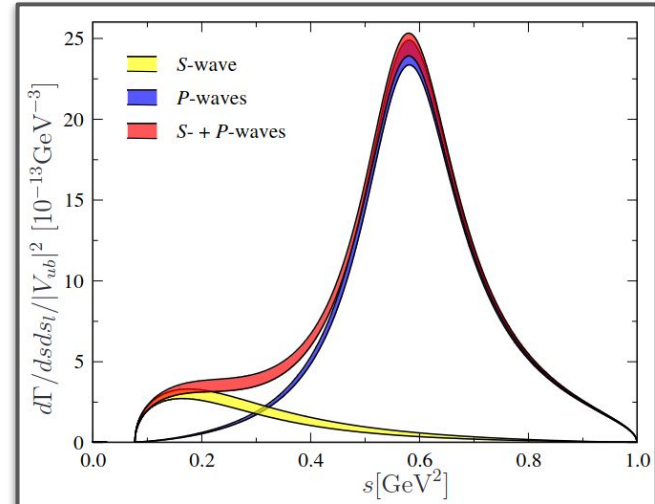
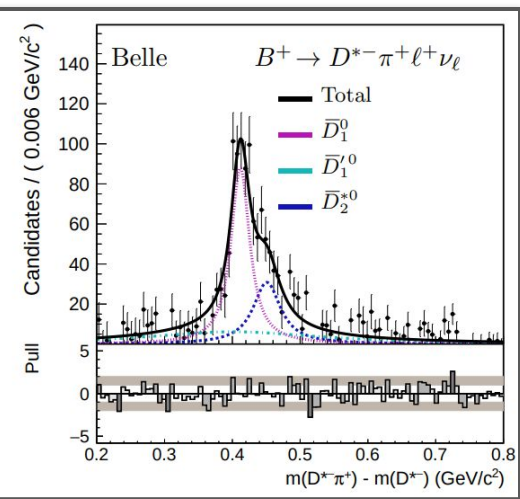
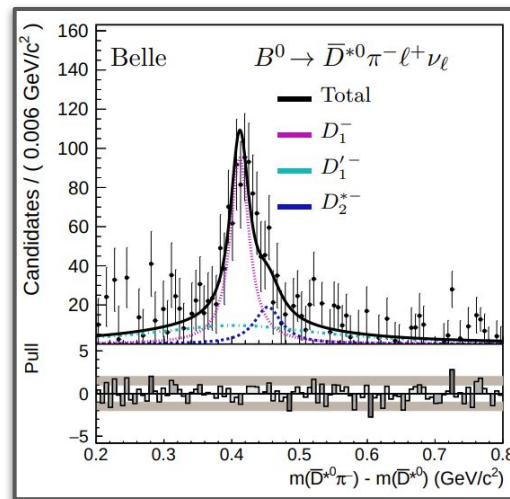


Outlook

Future work

- $B \rightarrow D^* \pi \ell \nu$ & $B \rightarrow D \gamma \ell \nu$ obvious next targets, but richer angular structure and less data
- $B \rightarrow D^* \pi \ell \nu$ 1^+ S-Wave expected to show similar features to $B \rightarrow D \pi \ell \nu$ S-Wave, but complicated by the presence of mixing with narrow D-Wave state \rightarrow More input from Hadron spectroscopy Lattice calculations would be helpful
- Inclusion of $B \rightarrow B^*(\rightarrow D \ell \nu) \pi$ and corresponding interference effects
- Apply to $B \rightarrow \pi \pi \ell \nu$ S-Wave

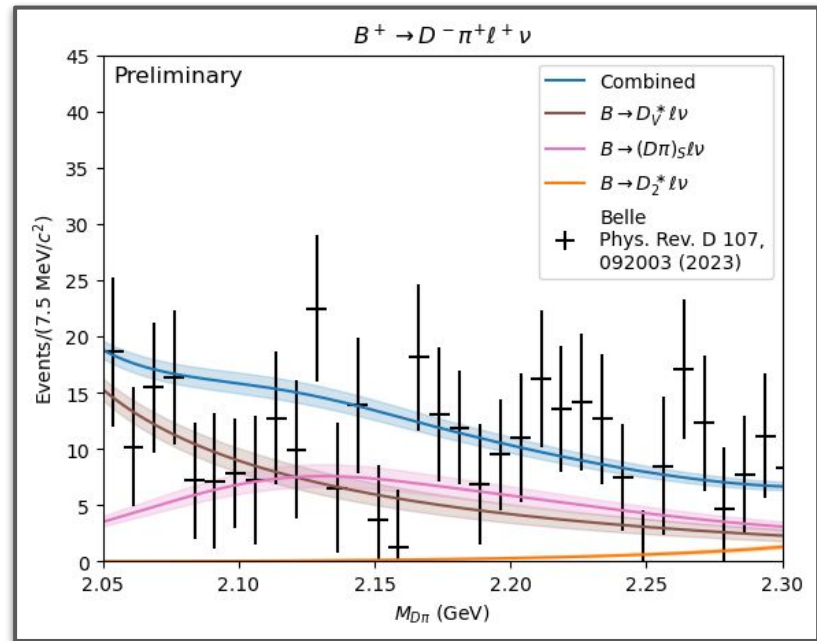
[Kang, Kubis, Meißner [PRD 89 \(2014\) 053015](#)]



Outlook

How can the experiments improve the situation?

- Releasing data including correlations...
- Partial-wave analysis of $B \rightarrow D^{(*)} \pi \ell \nu$ decays, large invariant mass bins sufficient to distinguish between BLR, Orsay & our work
- Measurements of $B \rightarrow D^{(*)} \pi \ell \nu$ q^2 - and E_1 -spectra in bins of $D^{(*)} \pi$ invariant mass, especially around the D_2^* and the narrow D_1^*
- Measurements of $B^0 \rightarrow D^{(*)} \pi \pi$, $B^0 \rightarrow D^{(*)} \pi K$ & $B^0 \rightarrow D^{(*)} \pi D_s$ decays



$$\frac{d \langle P_i \rangle}{dM_{D\pi}^2} = \int_{-1}^1 d \cos \theta \frac{d^2 \Gamma}{dM_{D\pi}^2 d \cos \theta} P_i(\cos \theta)$$

$$10 \left(\left(\frac{d \langle P_0 \rangle}{dM_{D\pi}^2} \right)^{-1} \frac{d \langle P_2 \rangle}{dM_{D\pi}^2} \right)_{[2.05^2, 2.30^2]} \approx \begin{cases} 0 & \text{pure S-Wave} \\ 1/2 & \text{50/50, this work} \\ 1 & \text{pure P-Wave} \end{cases}$$

Conclusion

- We developed a model-independent description of $B \rightarrow D\pi\ell\nu$
- By combining meson-meson scattering phase-shifts with $B \rightarrow D\ell\nu$ in the soft-Goldstone limit we obtained predictions for the S-Wave $B \rightarrow D\pi\ell\nu$, $B \rightarrow D\eta\ell\nu$ & $B \rightarrow D_s K\ell\nu$ decays
- We re-analyzed $B \rightarrow D_2^* \ell\nu$ decays and found discrepancies with the literature
- The framework developed is extendable to other final states, as well as Cabibbo-suppressed decays

