with Danny van Dyk and Keri Vos
arXiv:2308.04347

New determination of $|V_{ub}/V_{cb}|$ from $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$

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12th International Workshop on the CKM Unitarity Triangle
September 20, 2023, Santiago de Compostela

Motivation

Puzzling CKM elements

$$V_{CKM} \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- Elements need to be measured as inputs for predictions
- Exclusive and inclusive determinations *should* be the same

Exclusive → form factors

Inclusive → no form factors, OPE calculation

$$\left|V_{ub}\right|_{\text{excl}} = (3.67 \pm 0.15) \cdot 10^{-3} \quad 1.5\sigma \quad \left|V_{ub}\right|_{\text{incl}} = (4.13 \pm 0.26) \cdot 10^{-3}$$

$$\left|V_{cb}\right|_{\text{excl}} = (39.4 \pm 0.8) \cdot 10^{-3} \quad 2.5\sigma \quad \left|V_{cb}\right|_{\text{incl}} = (42.2 \pm 0.8) \cdot 10^{-3}$$

Particle Data Group, 2022

- Ratios ⇒ additional information to clarify the puzzle

see Delaney's talk on Tuesday!

$B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$ ratio

LHCb measurement

- First observation of $B_s^0 \to K^-\mu^+\nu_\mu$ decay and determination of branching ratio
- Normalised to $B_s^0 \to D_s^-\mu^+\nu_\mu$: reduce experimental systematic uncertainty

$$\frac{\mathcal{B}\left(B_s^0 \to K^-\mu^+\nu_\mu\right)}{\mathcal{B}\left(B_s^0 \to D_s^-\mu^+\nu_\mu\right)} = \frac{\left|V_{ub}\right|^2}{\left|V_{cb}\right|^2} \frac{\mathrm{FF}_K}{\mathrm{FF}_{D_s}} \Rightarrow \mathrm{FF}_Y \equiv |V_{xb}|^{-2} \int \frac{\mathrm{d}\Gamma(B_s^0 \to Y\mu^+\nu_\mu)}{\mathrm{d}q^2}\mathrm{d}q^2$$

Form factors needed as theory input!

see Delaney's talk on Tuesday!

$B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$ ratio

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$$\frac{\mathcal{B}\left(B_s^0 \to K^-\mu^+\nu_\mu\right)}{\mathcal{B}\left(B_s^0 \to D_s^-\mu^+\nu_\mu\right)} = \frac{|V_{ub}|^2}{|V_{cb}|^2} \frac{\mathrm{FF}_K}{\mathrm{FF}_{D_s}} \Rightarrow \mathrm{FF}_Y \equiv |V_{xb}|^{-2} \int \frac{\mathrm{d}\Gamma(B_s^0 \to Y\mu^+\nu_\mu)}{\mathrm{d}q^2} \mathrm{d}q^2$$

Form factors needed as theory input!

HPQCD, Phys. Rev. D 101 (2020) 074513

- $\mathrm{FF}_{D_s} \Rightarrow$ known from lattice
- $\mathrm{FF}_K \Rightarrow$ different theoretical approaches apply to the two q^2 ranges!

$$q^2 = (p_\mu + p_\nu)^2 \begin{cases} < 7\ \mathrm{GeV}^2 \\ > 7\ \mathrm{GeV}^2 \end{cases}$$

$\bar{B}_s \to K$ form factors for $B_s^0 \to K^-\mu^+\nu_\mu$

$$\langle K^+(k)\,|\,\bar{u}\gamma^\mu b\,|\,\bar{B}_s(p)\rangle = f_+(q^2)\left[(p+k)^\mu - \frac{m_{B_s}^2 - m_K^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_{B_s}^2 - m_K^2}{q^2}q^\mu$$

Light-Cone Sum Rules: low q^2

- Duplančić, Melić 2008 arXiv:0805.4170

⇩ update

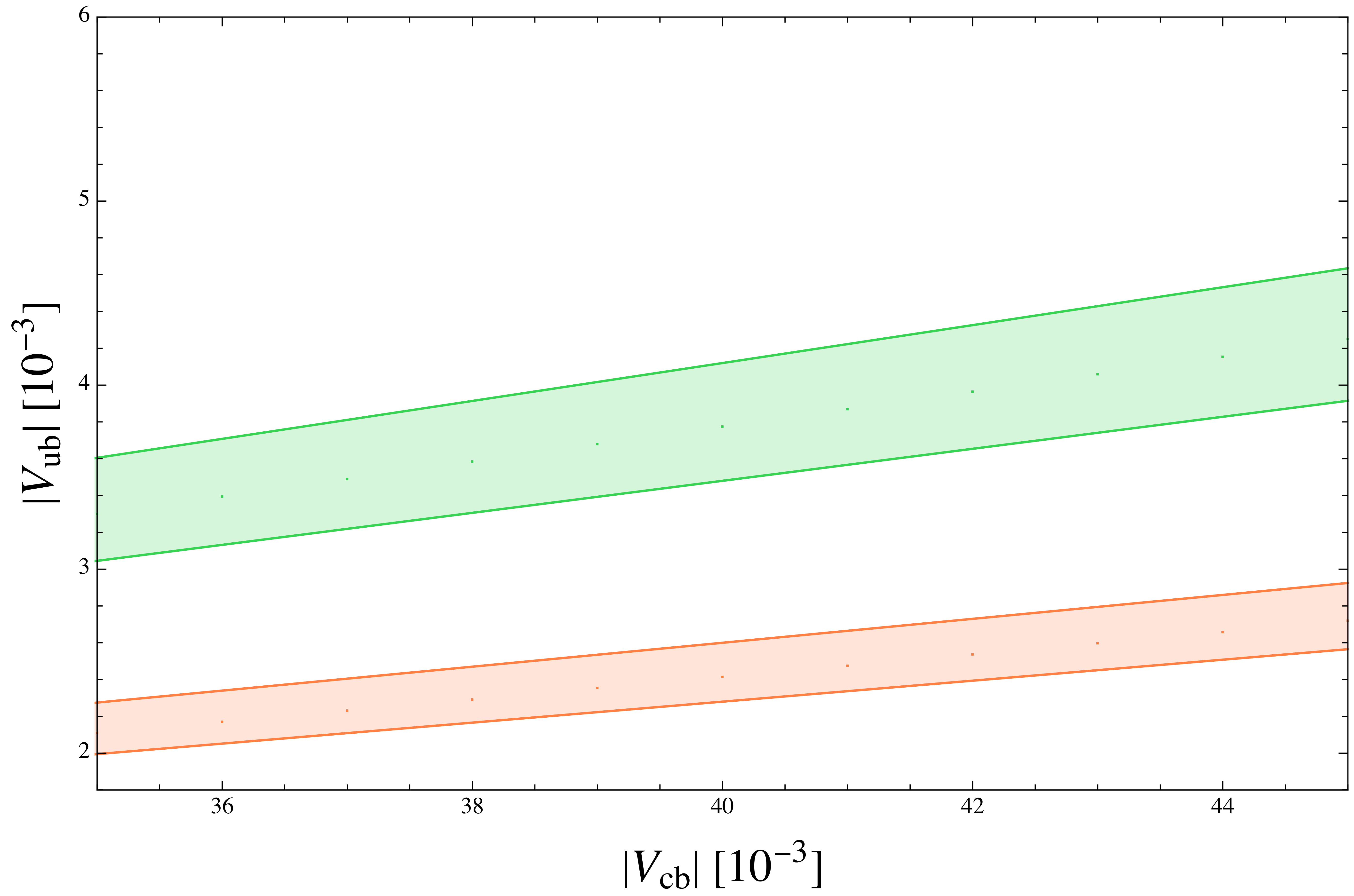
★ - Khodjamirian, Rusov 2017 arXiv:1703.04765

Lattice QCD: high q^2

- HPQCD 2014 arXiv:1406.2279
- RBC/UKQCD 2015 arXiv:1501.05373
- RBC/UKQCD 2023 arXiv:2303.11280 ⇩
- ★ - FNAL/MILC 2019 arXiv:1901.02561

★ : used by LHCb , 2021

Puzzling ratio of CKM elements



LHCb high q^2 ratio: FF_K determined with LQCD

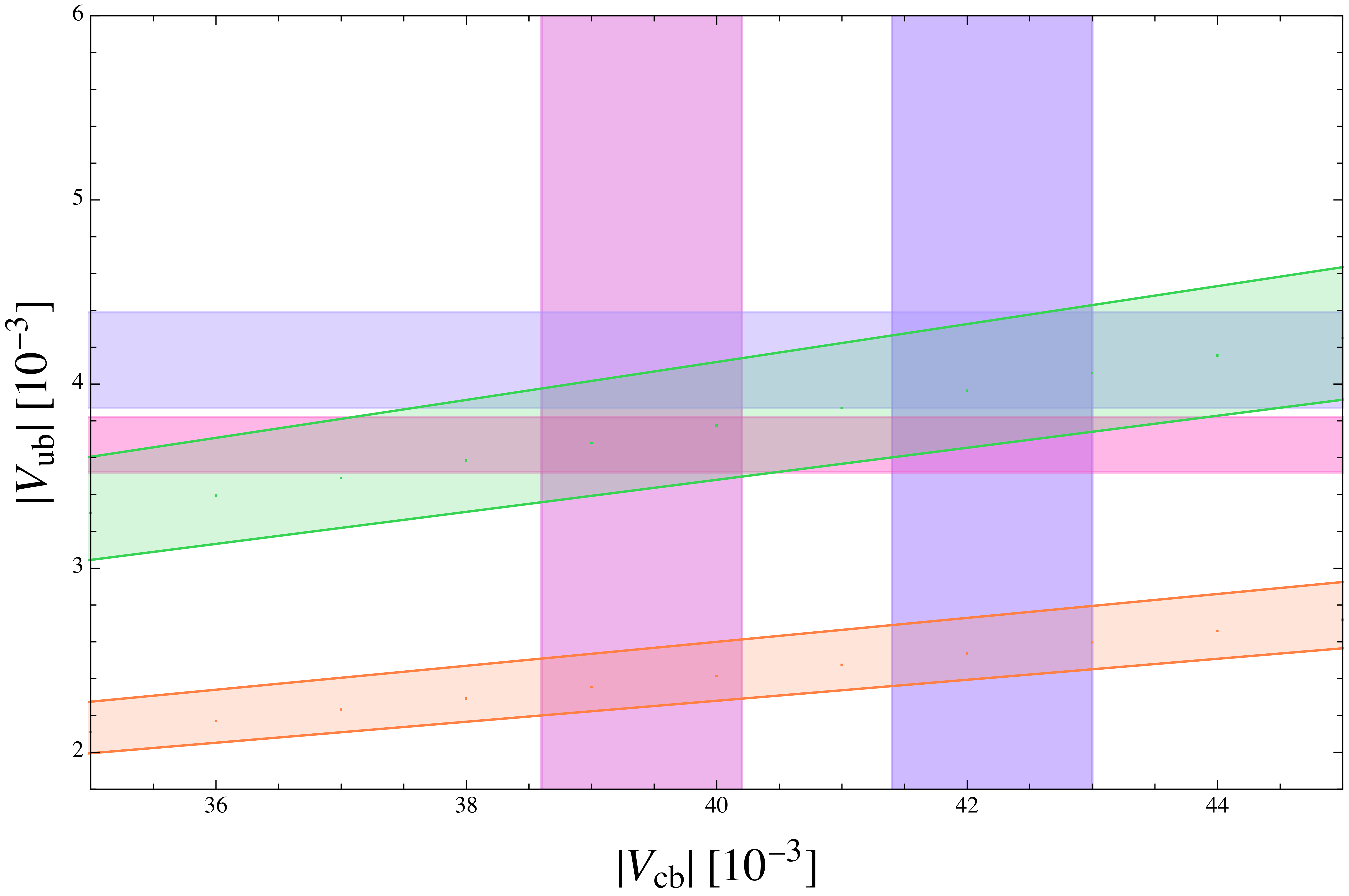
LHCb low q^2 ratio: FF_K determined with LCSR

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{\text{low } q^2} = 0.061 \pm 0.004$$

$$\left|\frac{V_{ub}}{V_{cb}}\right|_{\text{high } q^2} = 0.095 \pm 0.008$$

3.8σ

Puzzling ratio of CKM elements



Exclusive PDG averages

Inclusive averages

LHCb high q^2 ratio: FF_K determined with LQCD

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New determination of $|V_{ub}/V_{cb}|$

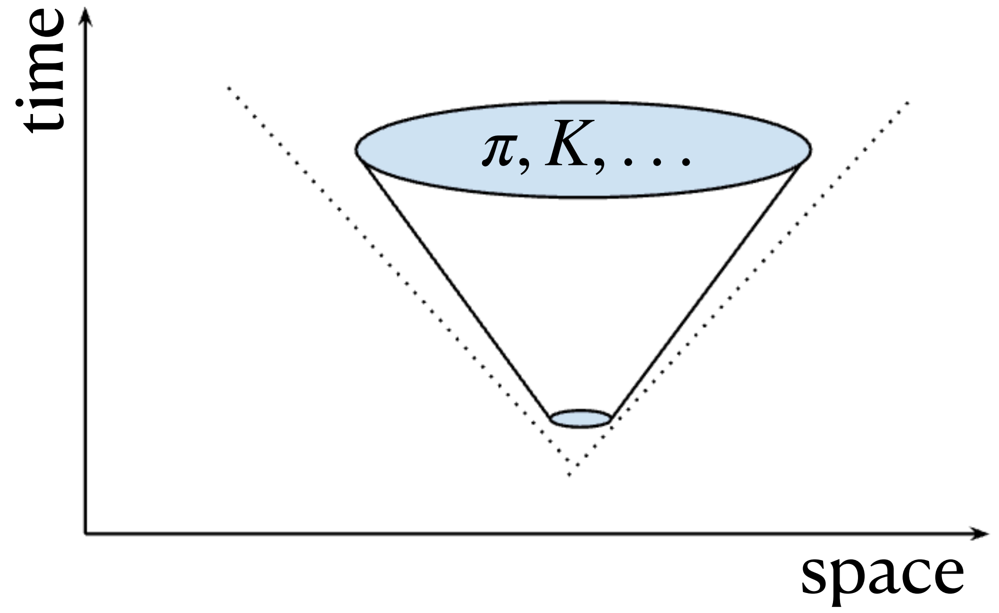
- Infer full set of $\bar{B}_s \to K$ form factors over the full q^2 range

- Steps:
  - ★ Update LCSR form factor results with study of duality threshold parameters
  - ★ Add LQCD results to constrain the parametrisation at high q^2
  - ★ Fit to both theory inputs using a unitarity-bounded parametrisation
  - ★ Extract $|V_{ub}/V_{cb}|$ from the $B_s^0 \to K^- \mu^+ \nu_\mu$ LHCb measurement

- Analysis done with EOS flavour physics software

arXiv:2111.15428

Light-Cone Sum Rules



- Vacuum to kaon correlation function ⇒ weak current and B_s current
- Expand near light-cone ⇒ LCOPE

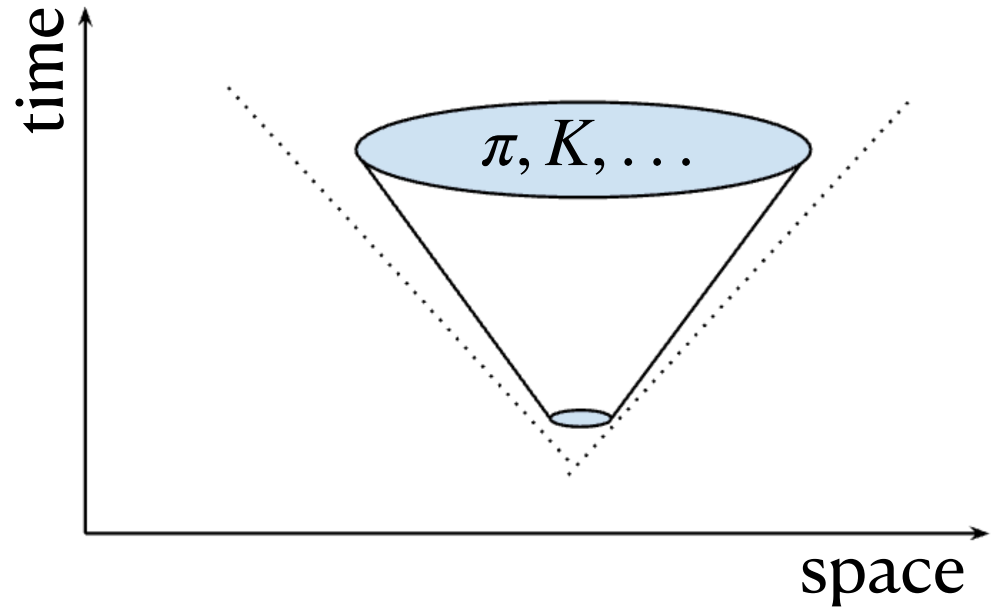
Perturbative: hard scattering kernels

$$\int d^4x\, e^{iqx}\, T\{J_{B_s}(x), [\bar{u}\gamma^\mu b](0)\} \propto T(q^2, \vec{u}) \otimes \phi(\vec{u}) + \text{higher corrections}$$

Non-perturbative: universal LCDAs

- Dispersion relations + quark-hadron duality
- Duality threshold parameter s_0^f ⇒ extract $\bar{B}_s \to K$ matrix elements

Light-Cone Sum Rules



- Vacuum to kaon correlation function ⇒ weak current and B_s current
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Perturbative: hard scattering kernels

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Non-perturbative: universal LCDAs

- Dispersion relations + quark-hadron duality
- Duality threshold parameter s_0^f ⇒ extract $\bar{B}_s \to K$ matrix elements

Main novelties of our work:
- determination of s_0^f from daughter sum rule
- explicit $m_s \pm m_q$ terms in the RGE

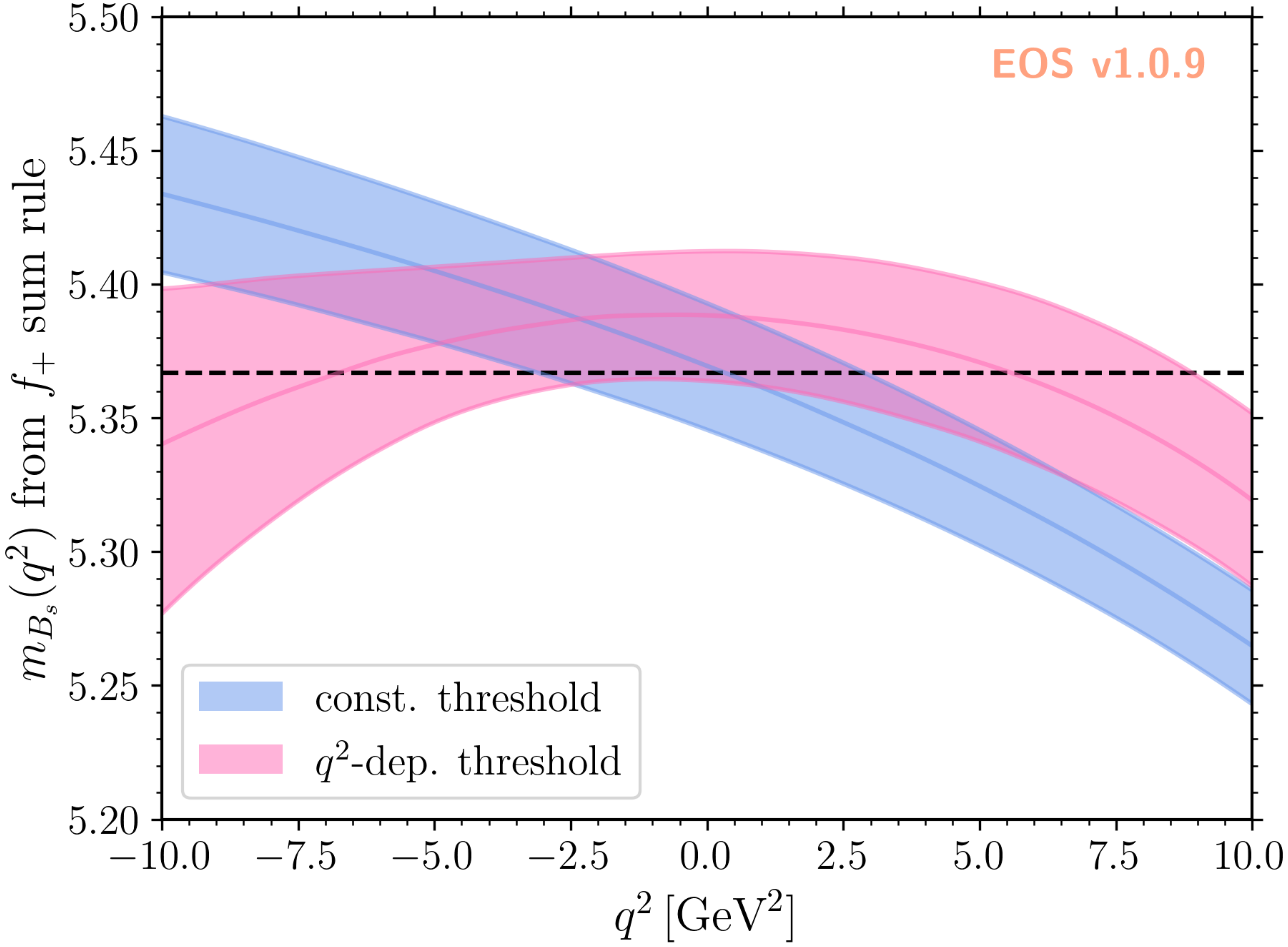
Light-Cone Sum Rules

Duality threshold parameters

- Mass estimator

$$\left[m^2_{B_s}(q^2; f_i)\right]_{\text{LCSR}} = \frac{\int_0^{s_0} \mathrm{d}s\, s\, \rho^{f_i}(s, q^2)\, e^{-s/M^2}}{\int_0^{s_0} \mathrm{d}s\, \rho^{f_i}(s, q^2)\, e^{-s/M^2}}$$

- Compare ansatz for s_0^f



Light-Cone Sum Rules

Duality threshold parameters

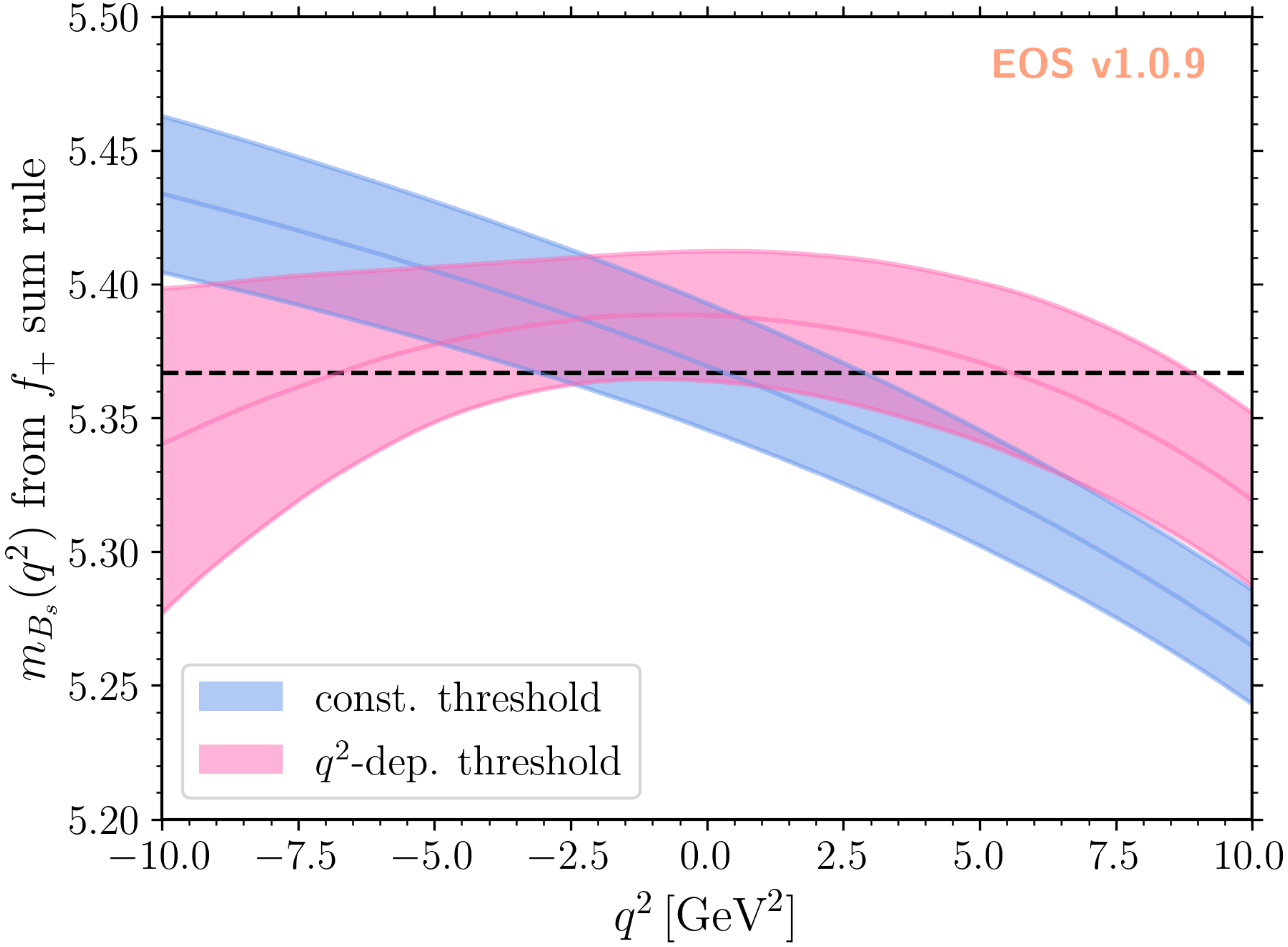
- Mass estimator

$$\left[m^2_{B_s}(q^2; f_i)\right]_{\text{LCSR}} = \frac{\int_0^{s_0} \mathrm{d}s\, s\, \rho^{f_i}(s, q^2)\, e^{-s/M^2}}{\int_0^{s_0} \mathrm{d}s\, \rho^{f_i}(s, q^2)\, e^{-s/M^2}}$$

- Compare ansatz for s_0^f

- LCSR points determined where mass estimator is consistent with m_{B_s}

  no LCSR points at $q^2 \geq 10\ \text{GeV}^2$



Form factors in the full q^2 range

Form factor data

$$f_+(q^2=0) = f_0(q^2=0)$$

LCSR + LQCD:

LCSR:

4 f_+ points

3 f_0 points

4 f_T points

LQCD:

2 f_+ points

3 f_0 points

RBC/UKQCD23

3 f_+ points

3 f_0 points

HPQCD14

Form factors in the full q^2 range

$z\left(q^2 = t_0\right) = 0$

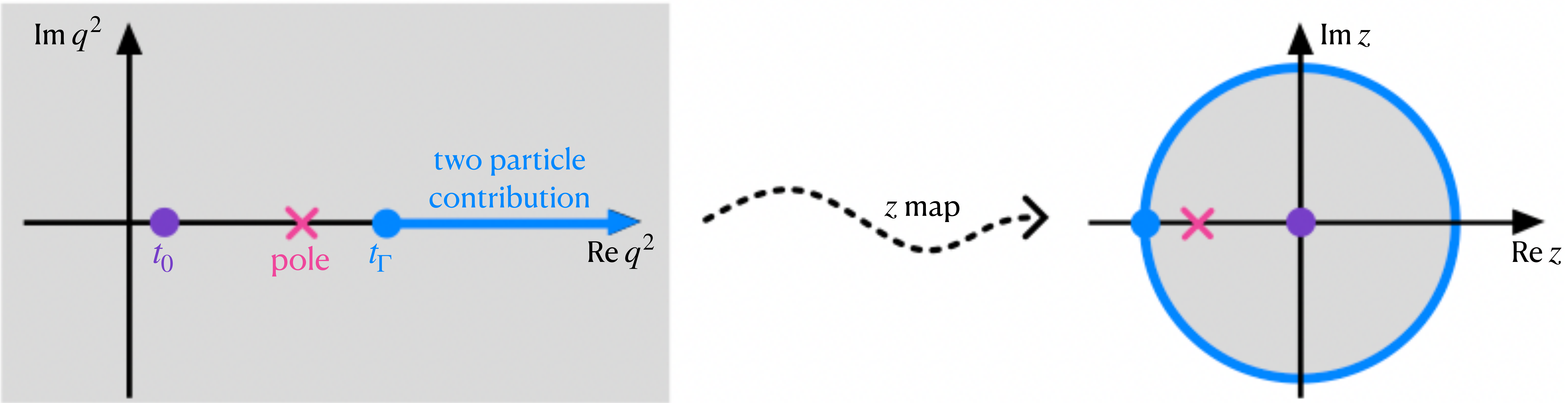
Parametrisation

- Modified BGL: analyticity + unitarity

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma - t_0}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma - t_0}}$$

$$f(q^2) = \frac{1}{\sqrt{\chi}\,\phi(q^2)\,B(q^2)} \sum_k^K a_k\, p_k\left(z(q^2)\right)$$

$$f_+(q^2 = 0) = f_0(q^2 = 0)$$



Used $K = 4$

Form factors in the full q^2 range

$z\left(q^2 = t_0\right) = 0$
$t_\Gamma = (m_B + m_\pi)^2$
$t_+ = (m_{B_s} + m_K)^2$

Parametrisation

- Modified BGL: analyticity + unitarity + (pair production ≠ first branch point)

$$q^2 \mapsto z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma - t_0}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma - t_0}}$$

$$f(q^2) = \frac{1}{\sqrt{\chi}\,\phi(q^2)\,B(q^2)} \sum_k^K a_k\, p_k\left(z(q^2)\right)$$

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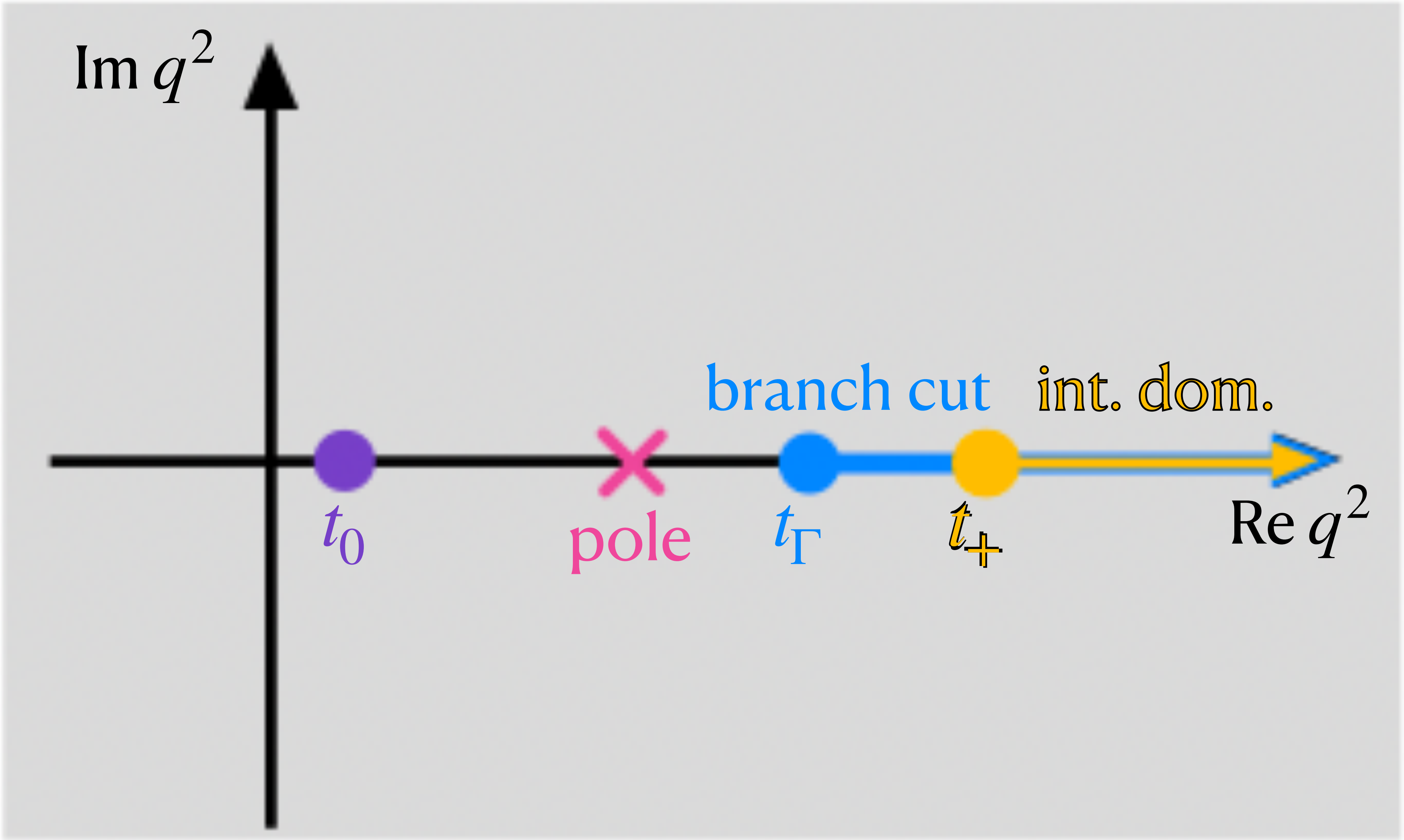
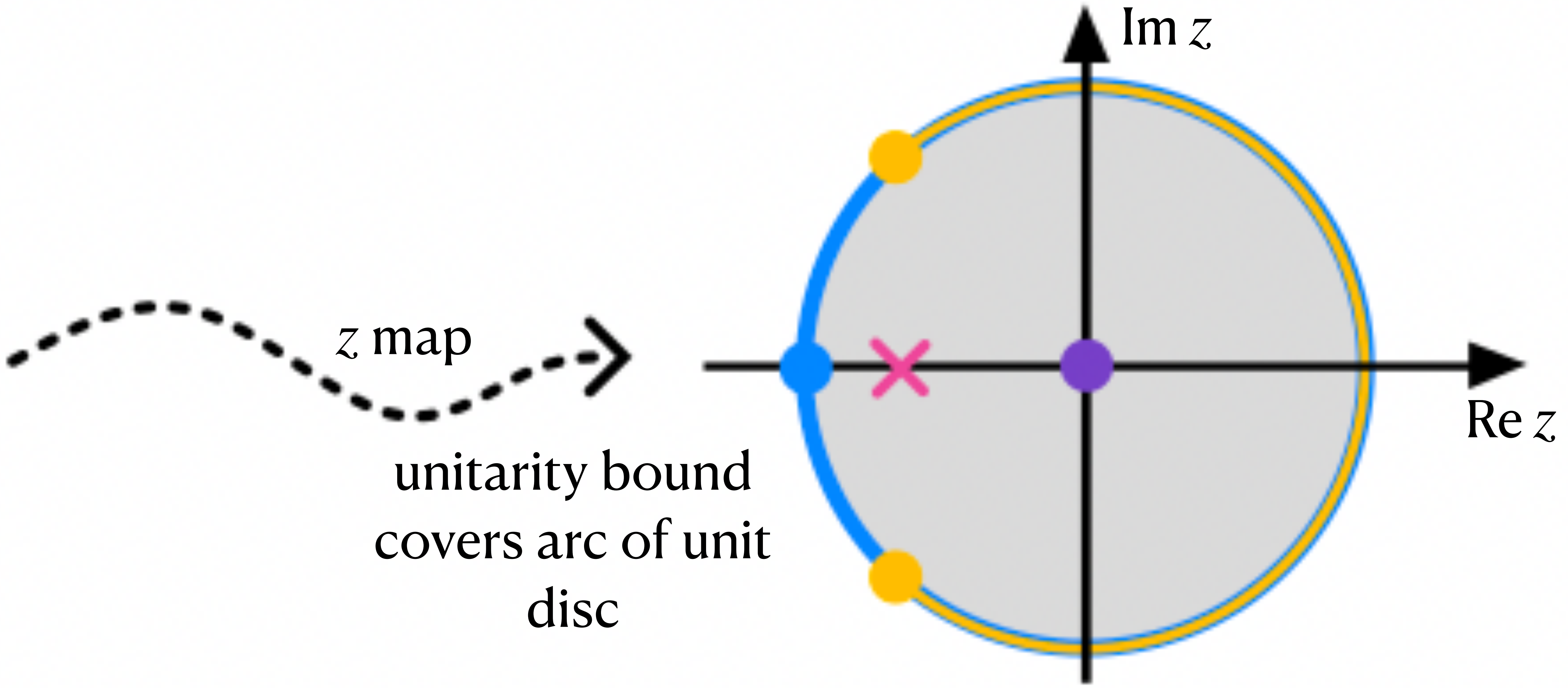


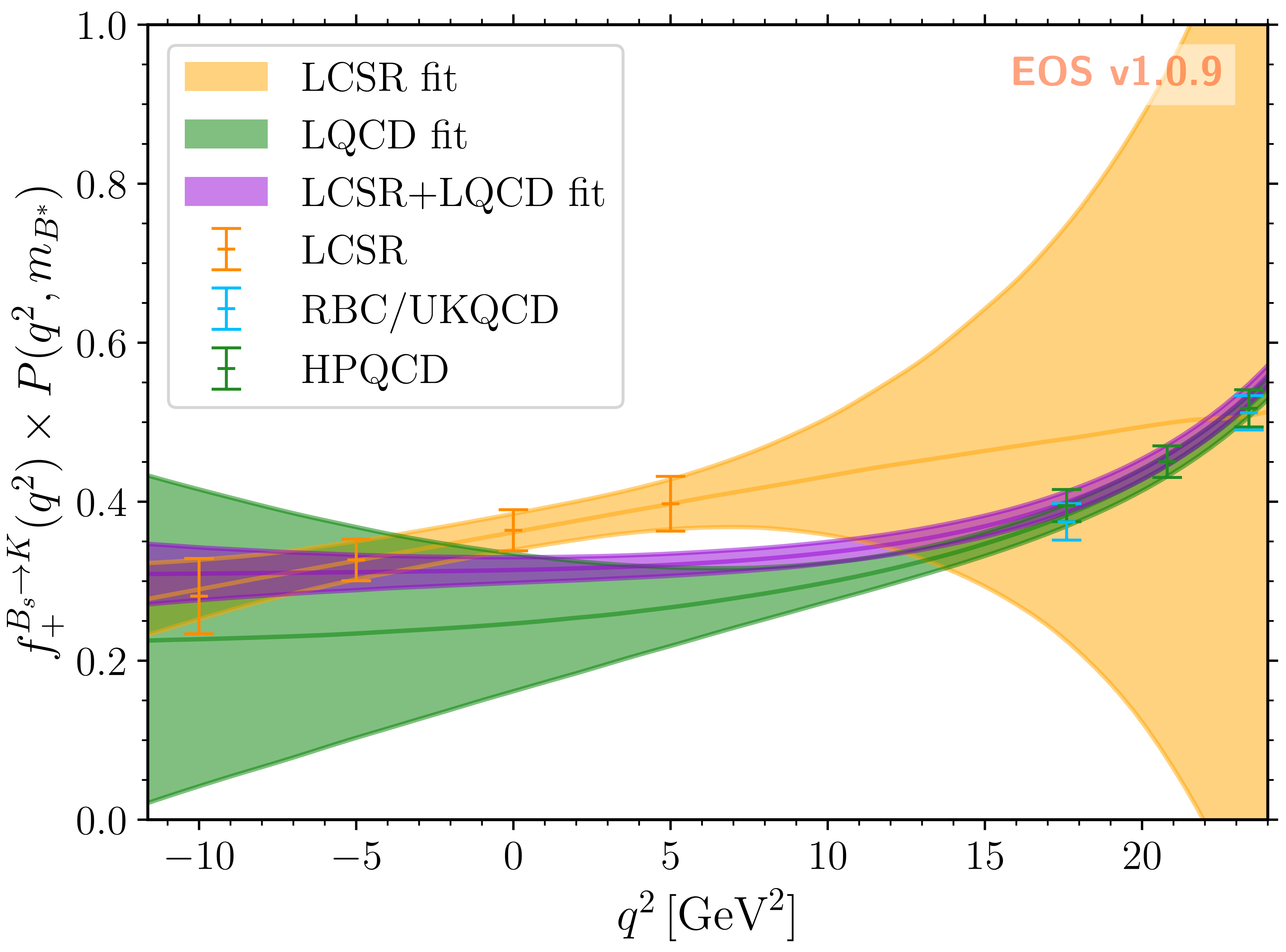


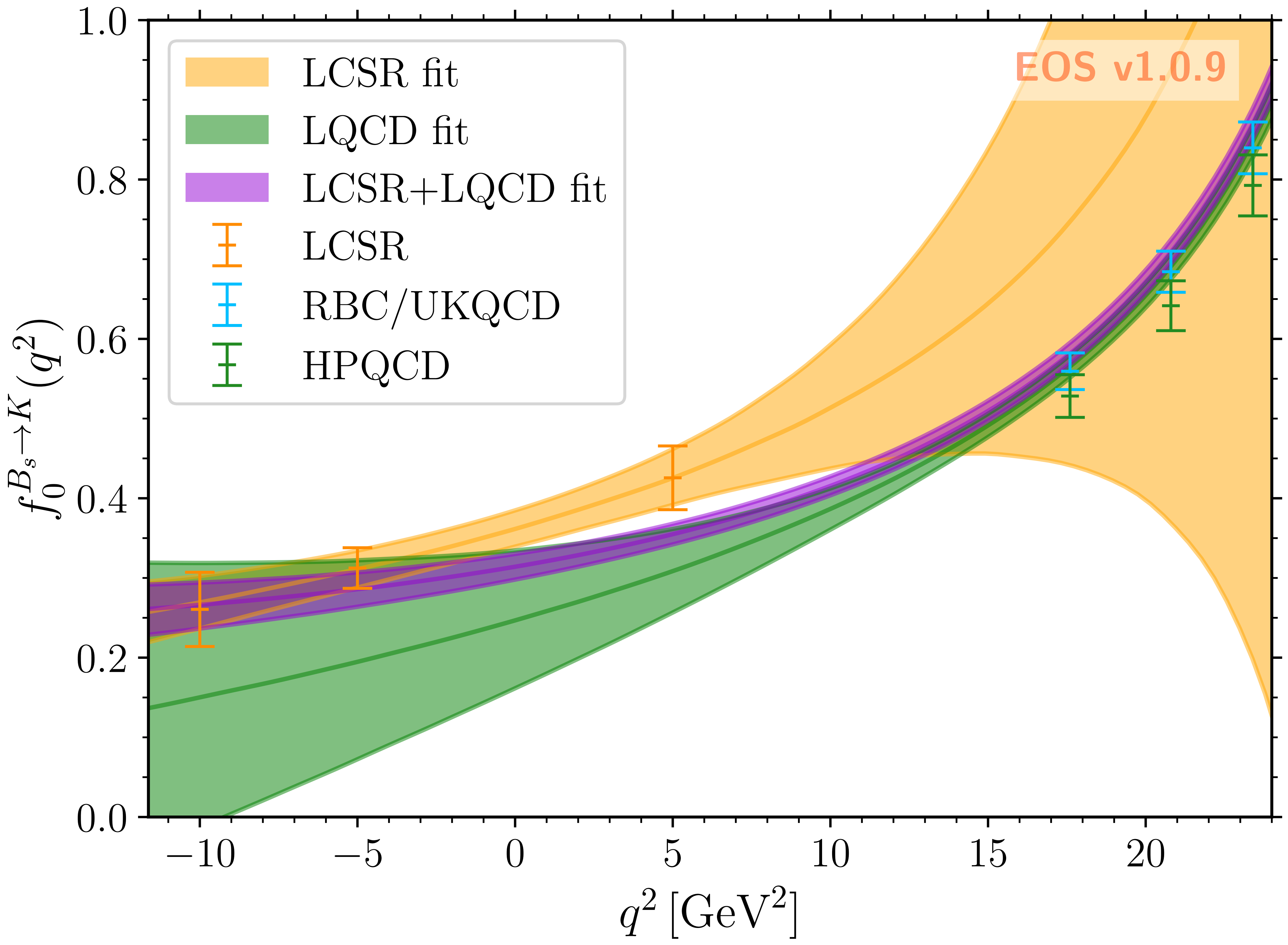
Used $K = 4$

Further discussion on form factor approach: Gubernari, (Reboud), van Dyk, Virto 2021 & 2022; Blake et al. 2022; Flynn et al. 2023

Form factors over the full q^2 range

Results





Nominal result: LCSR+LQCD fit

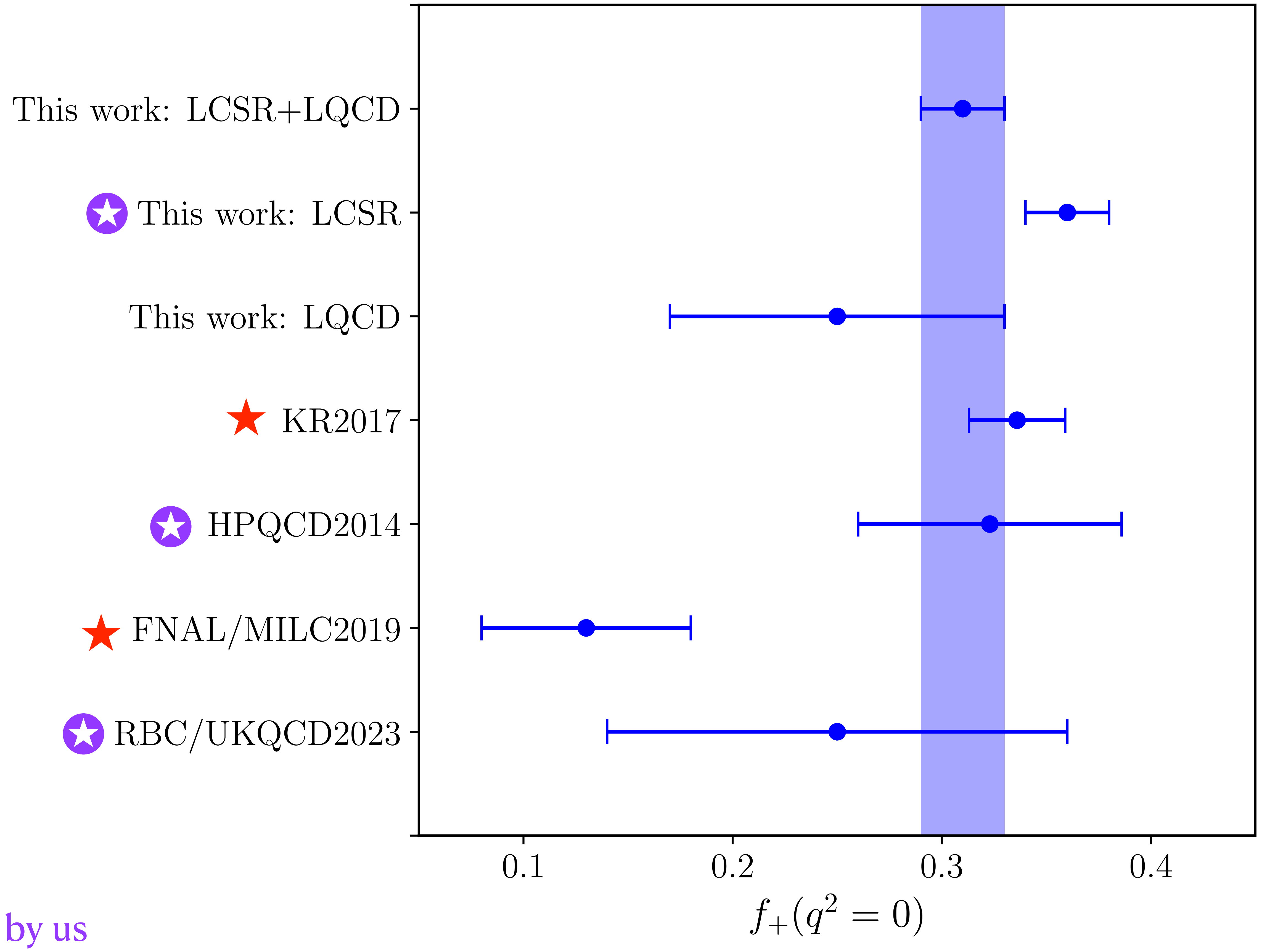
p-value=6% : acceptable fit over full range!

LCSR and LQCD show different slopes

Strongly correlated LCSR points

Form factors over the full q^2 range

Comparison to previous results at $q^2 = 0$

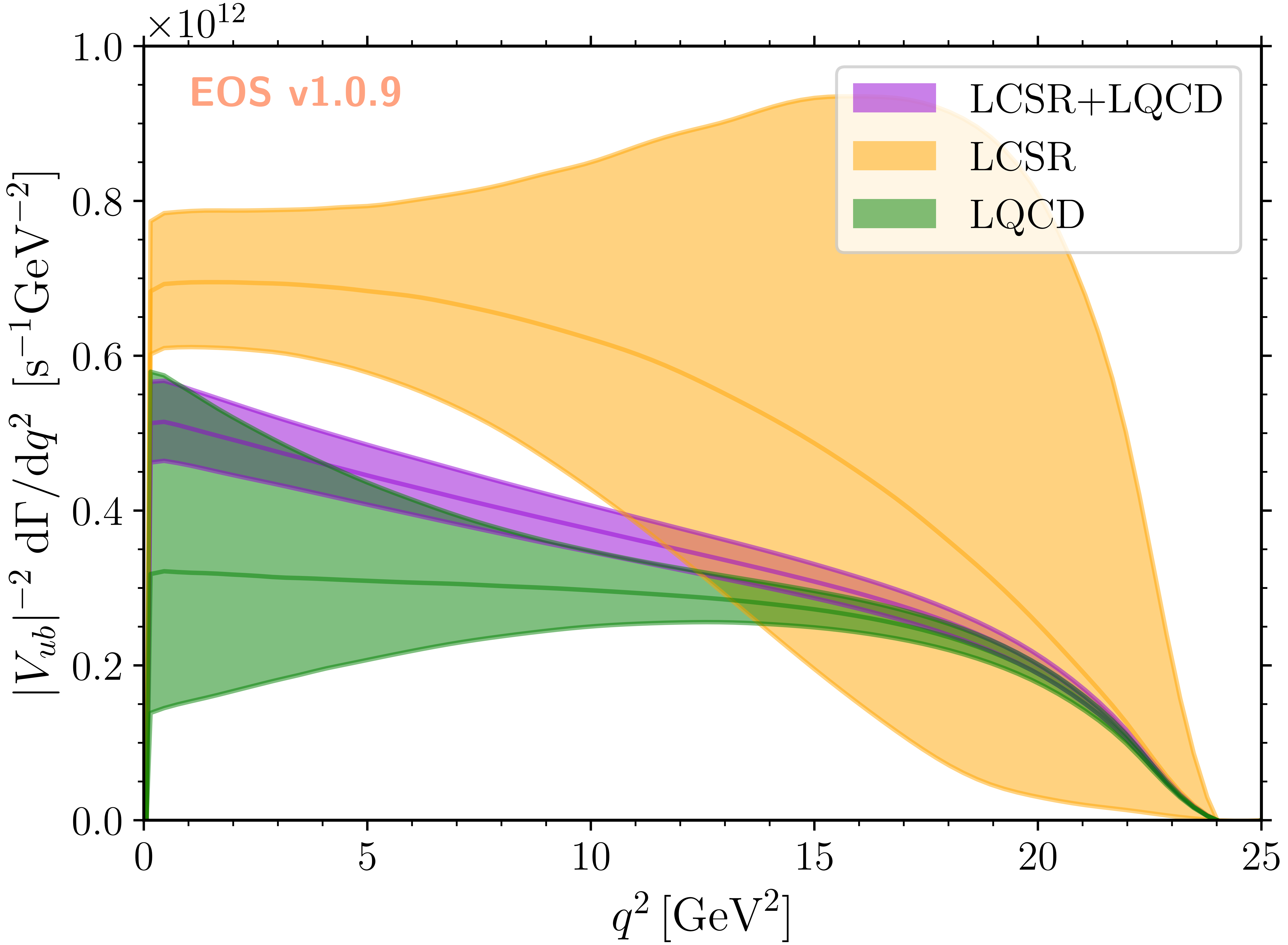


- Compatible with previous determinations
- Discussion on LQCD determinations:
  RBC/UKQCD23: Phys. Rev. D 107 (2023) 114512

☆: used by us
★: used by LHCb

The $B_s^0 \to K^- \mu^+ \nu_\mu$ decay

Decay rate



- Consistent relative uncertainties over full range for nominal fit
  - Smaller than LCSR at high q^2
  - Smaller than LQCD at low q^2

Determination of $|V_{ub}/V_{cb}|$

LHCb Collaboration, Phys.Rev.Lett. 126 (2021) 8

➡ From LHCb $B_s^0 \to K^- \mu^+ \nu_\mu$ determination

$$q^2 < 7\ \text{GeV}^2 \Rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| = 0.0681 \pm 0.0040$$

1.9σ

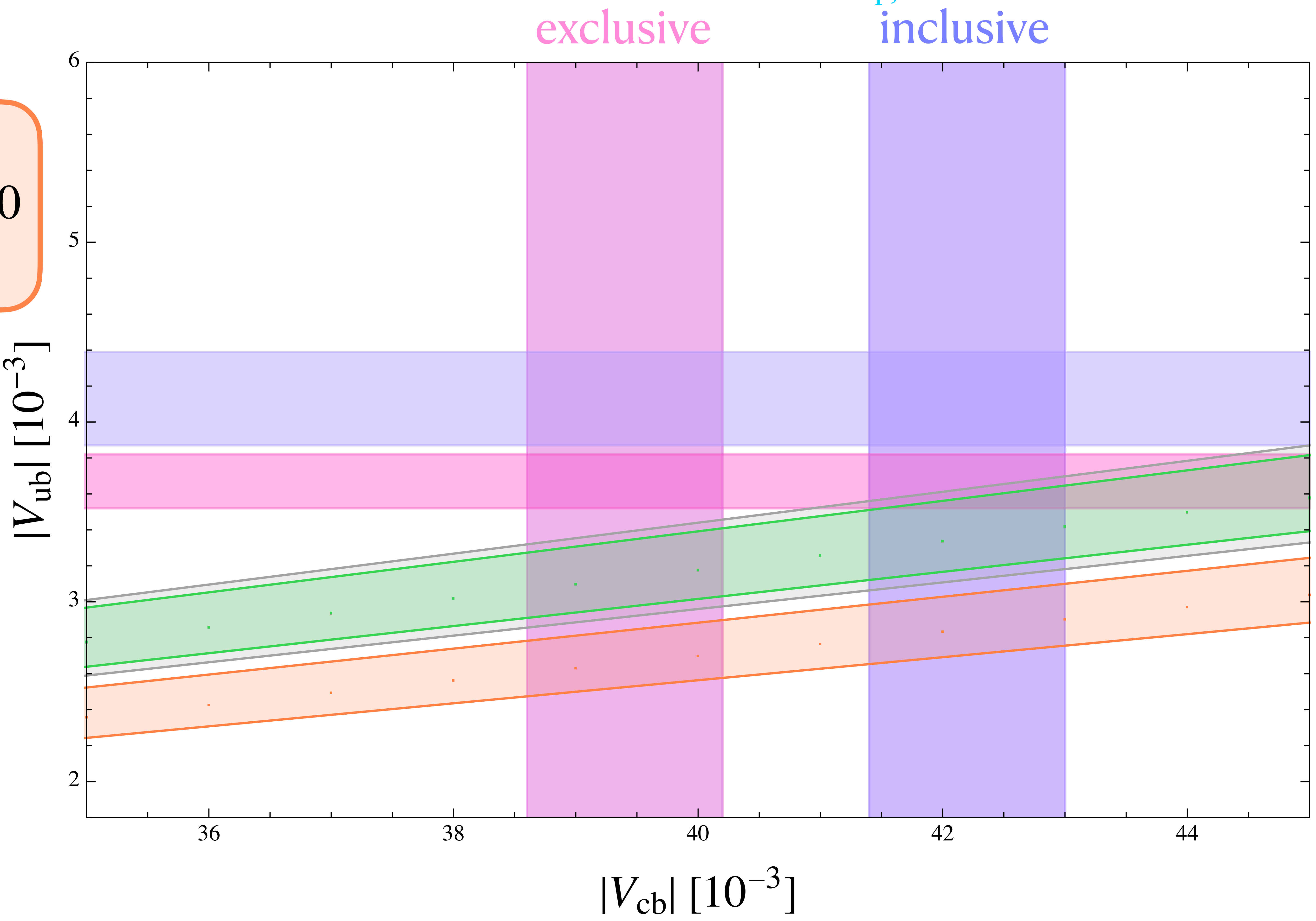
$$q^2 > 7\ \text{GeV}^2 \Rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| = 0.0801 \pm 0.0047$$

➡ Compare with LHCb baryon determination

$$\left|\frac{V_{ub}}{V_{cb}}\right|^{\Lambda_b \to \{p, \Lambda_c\}\mu^- \bar{\nu}}_{q^2 > 15} = 0.080 \pm 0.006$$

LHCb Collaboration, Nature Phys. 11 (2015) 743-747

Particle Data Group, 2022



$q^2 < 7\ \mathrm{GeV}^2 \Rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| = 0.0681 \pm 0.0040$

$q^2 > 7\ \mathrm{GeV}^2 \Rightarrow \left|\frac{V_{ub}}{V_{cb}}\right| = 0.0801 \pm 0.0047$

Conclusions

arXiv:2308.04347

- Determination of $\bar{B}_s \to K$ form factors:
  - ★ Update of LCSR results → evaluation of s_0^f **new**
  - ★ Combination with more precise LCQD results **new**

    → consistent description over full q^2 range respecting unitarity

- Not discussed here: investigation of BSM reach for $B_s^0 \to K^-\mu^+\nu_\mu$ **new**

- Improved compatibility between $|V_{ub}/V_{cb}|$ determinations

- Desired for the future...

  → update of experimental $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu_\mu$ with shape of q^2 distribution

Obrigada!

C. Bolognani, D. van Dyk, K. Vos
arXiv:2308.04347

BACK-UP

New determination of $|V_{ub}/V_{cb}|$ from $B_s^0 \to \{K^-, D_s^-\}\mu^+\nu$

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Determination of $|V_{ub}/V_{cb}|$

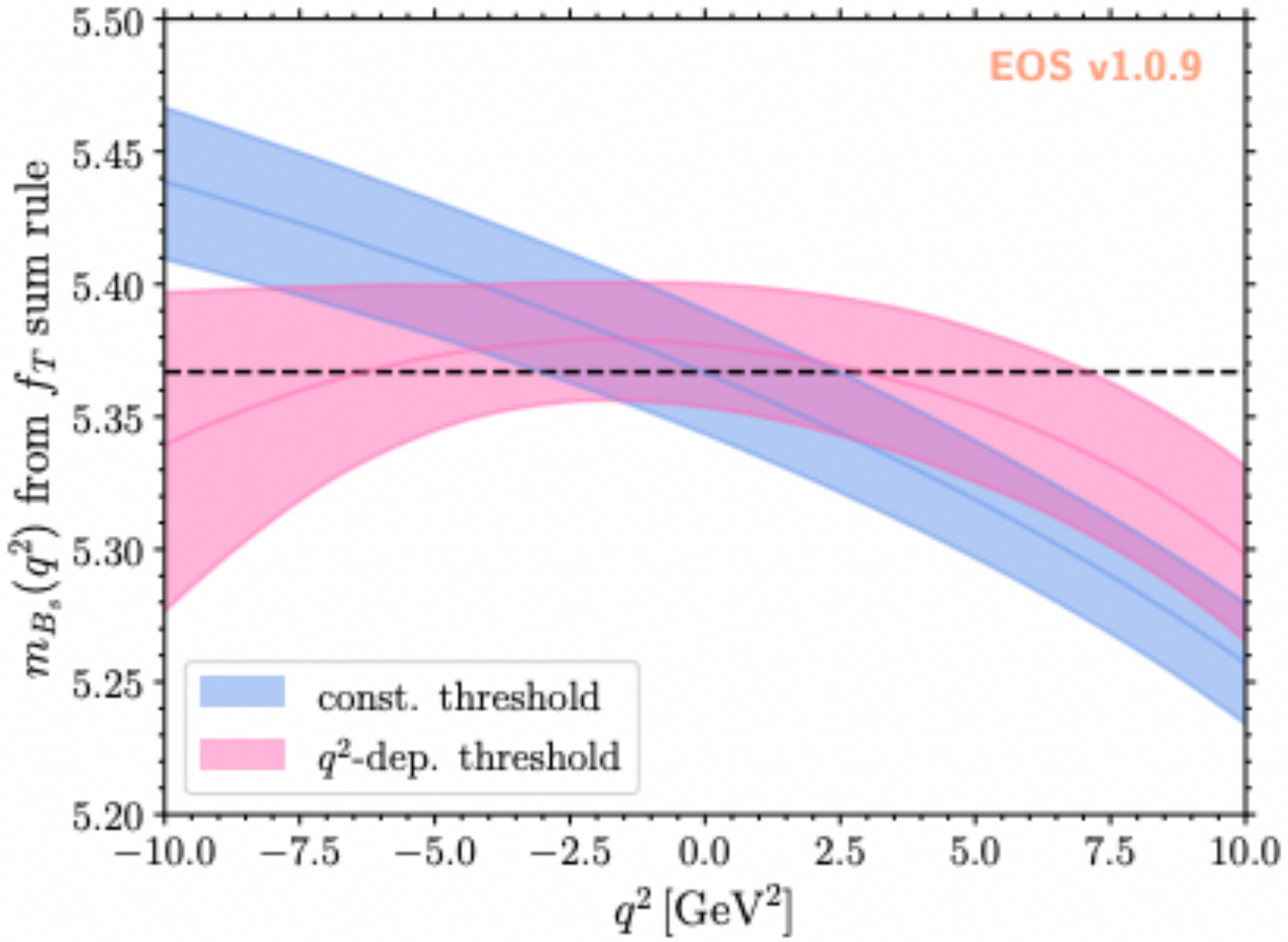
- LHCb measurement:
  - ★ $B_s^0 \to D_s^- \mu^+ \nu_\mu$ in the full q^2 range, FF_{D_s} from LQCD arXiv:1906.00701

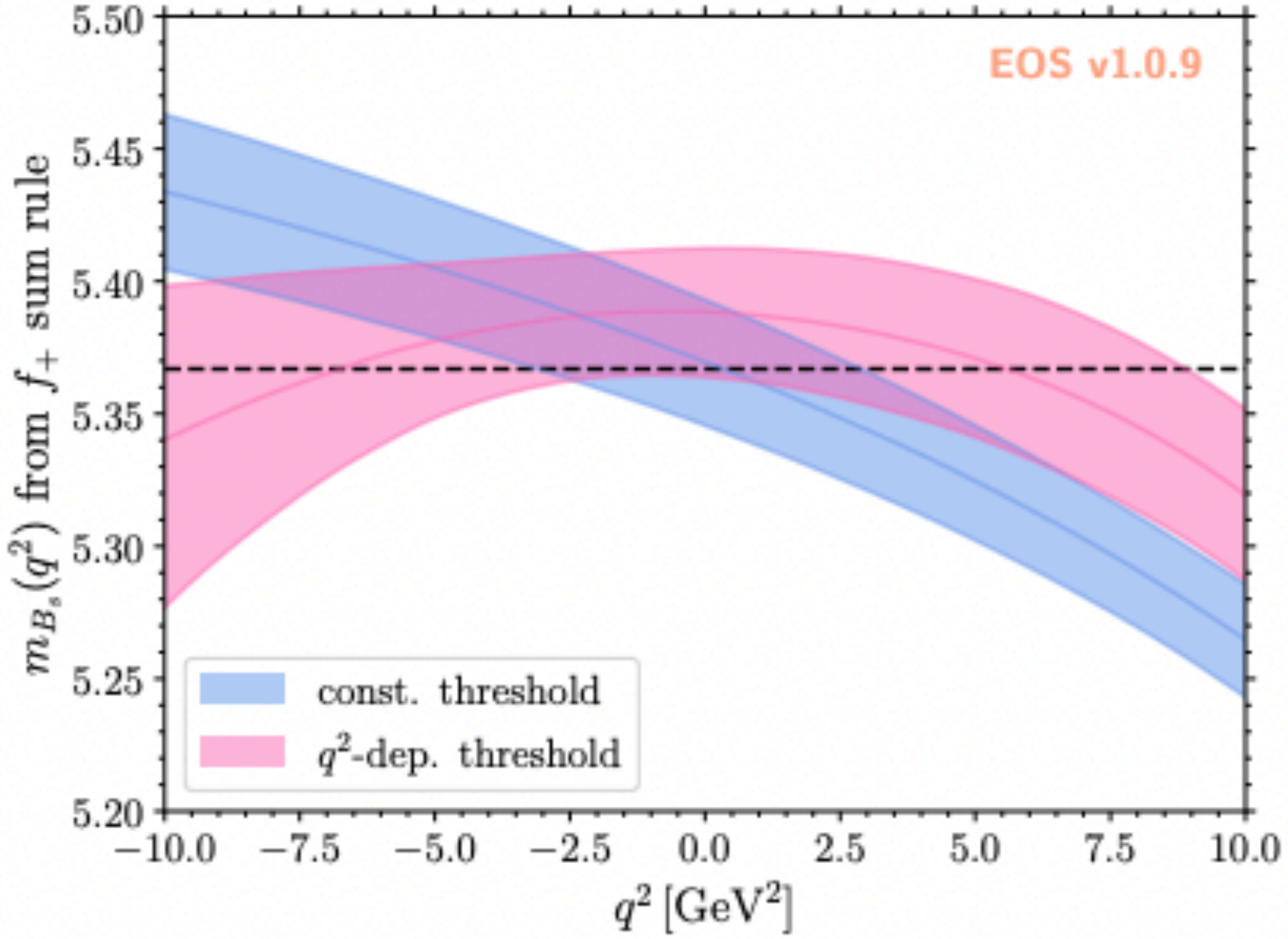
$$R_{\mathrm{BF}} = \frac{\mathcal{B}(B_s^0 \to K^- \mu^+ \nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^- \mu^+ \nu_\mu)} \quad \rightarrow \quad \begin{aligned} R_{\mathrm{BF}}^{q^2<7} &= (1.65 \pm 0.11) \cdot 10^{-3} \\ R_{\mathrm{BF}}^{q^2>7} &= (3.24 \pm 0.28) \cdot 10^{-3} \end{aligned}$$

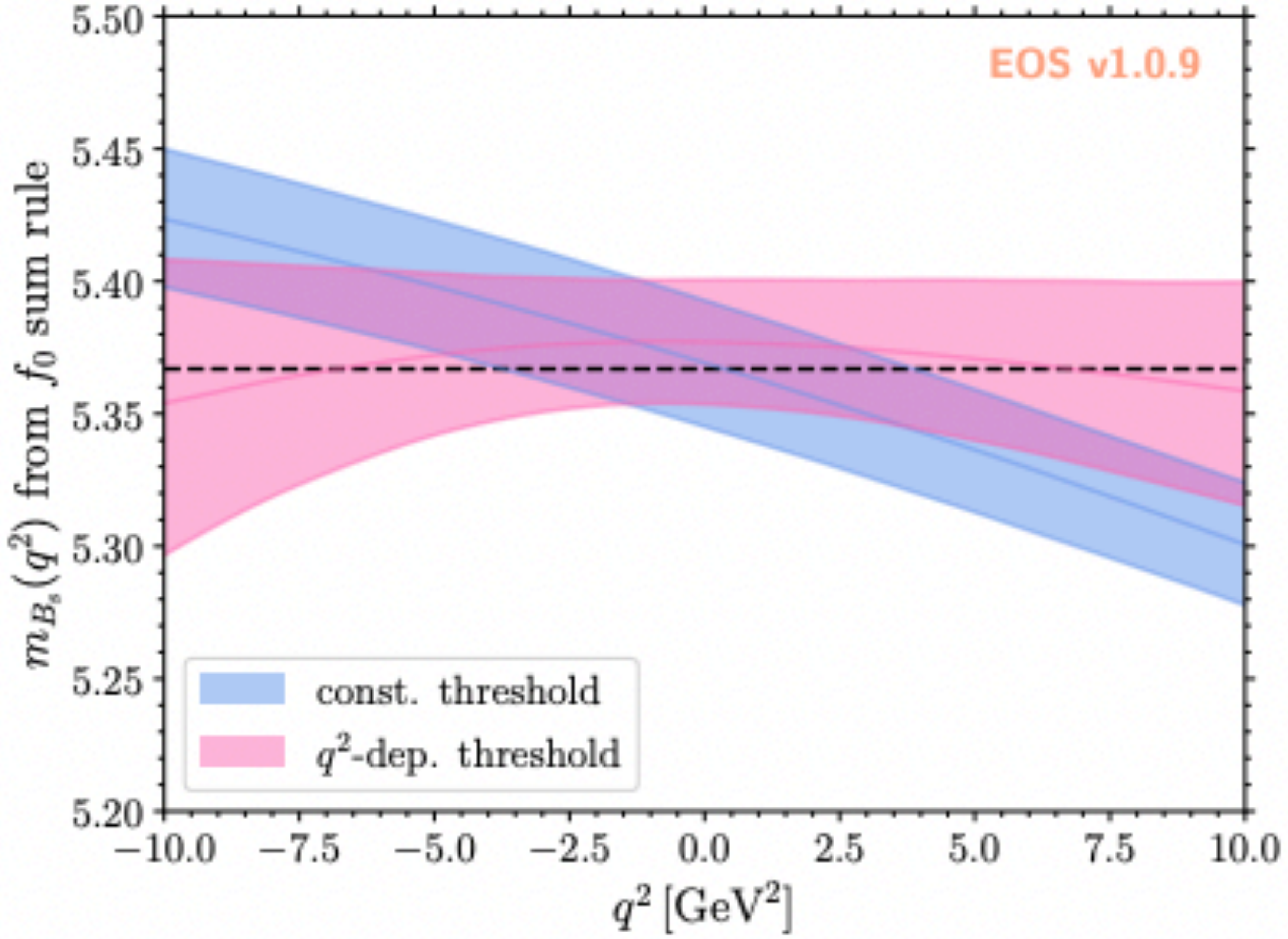
- Our theoretical determination

$$\sqrt{R_{\mathrm{FF}}} = \sqrt{\frac{\mathrm{FF}_{D_s}}{\mathrm{FF}_K}} \quad \rightarrow \quad \left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{R_{\mathrm{BF}}} \times \sqrt{R_{\mathrm{FF}}}$$

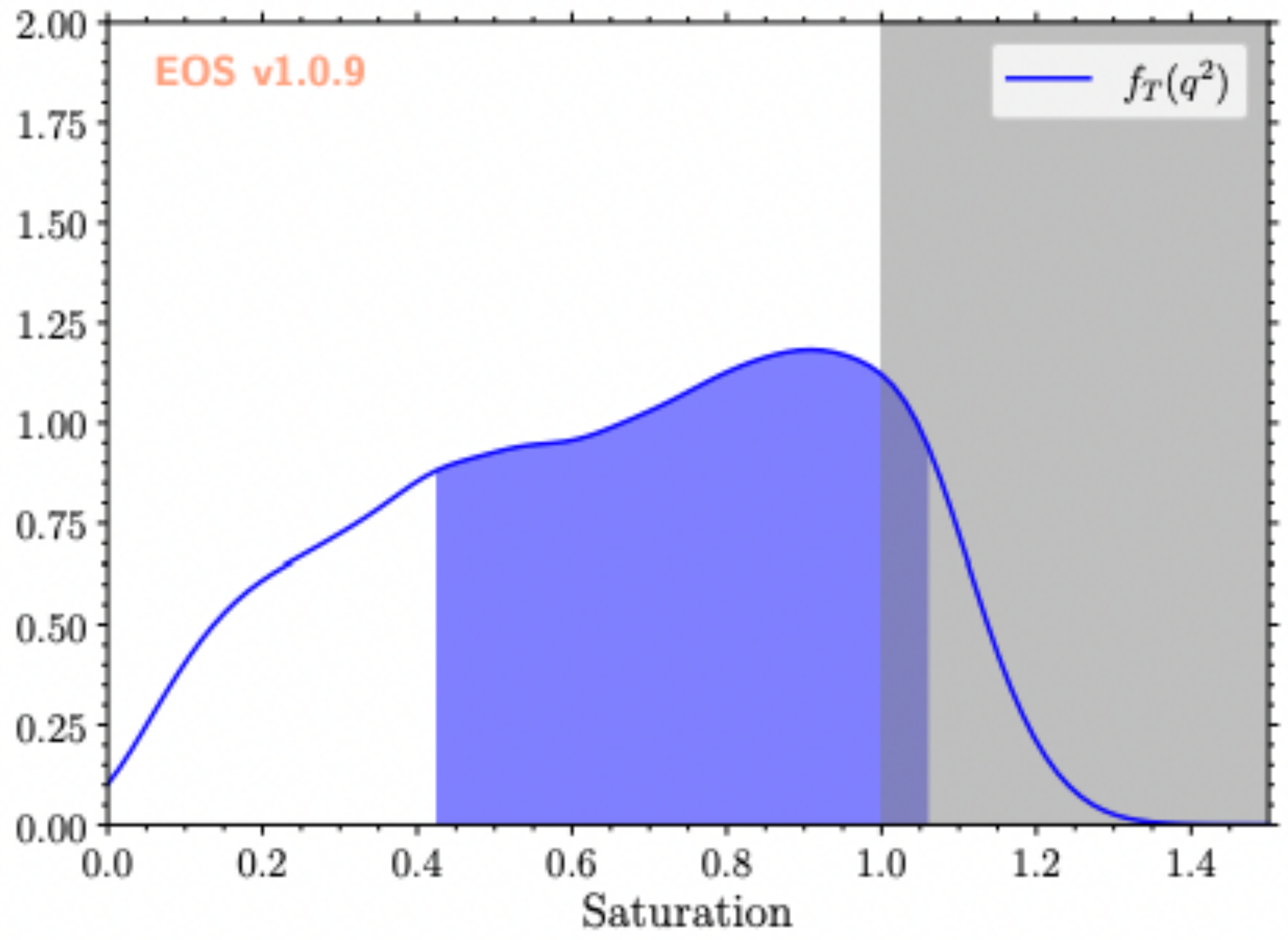
Mass predictor and duality threshold

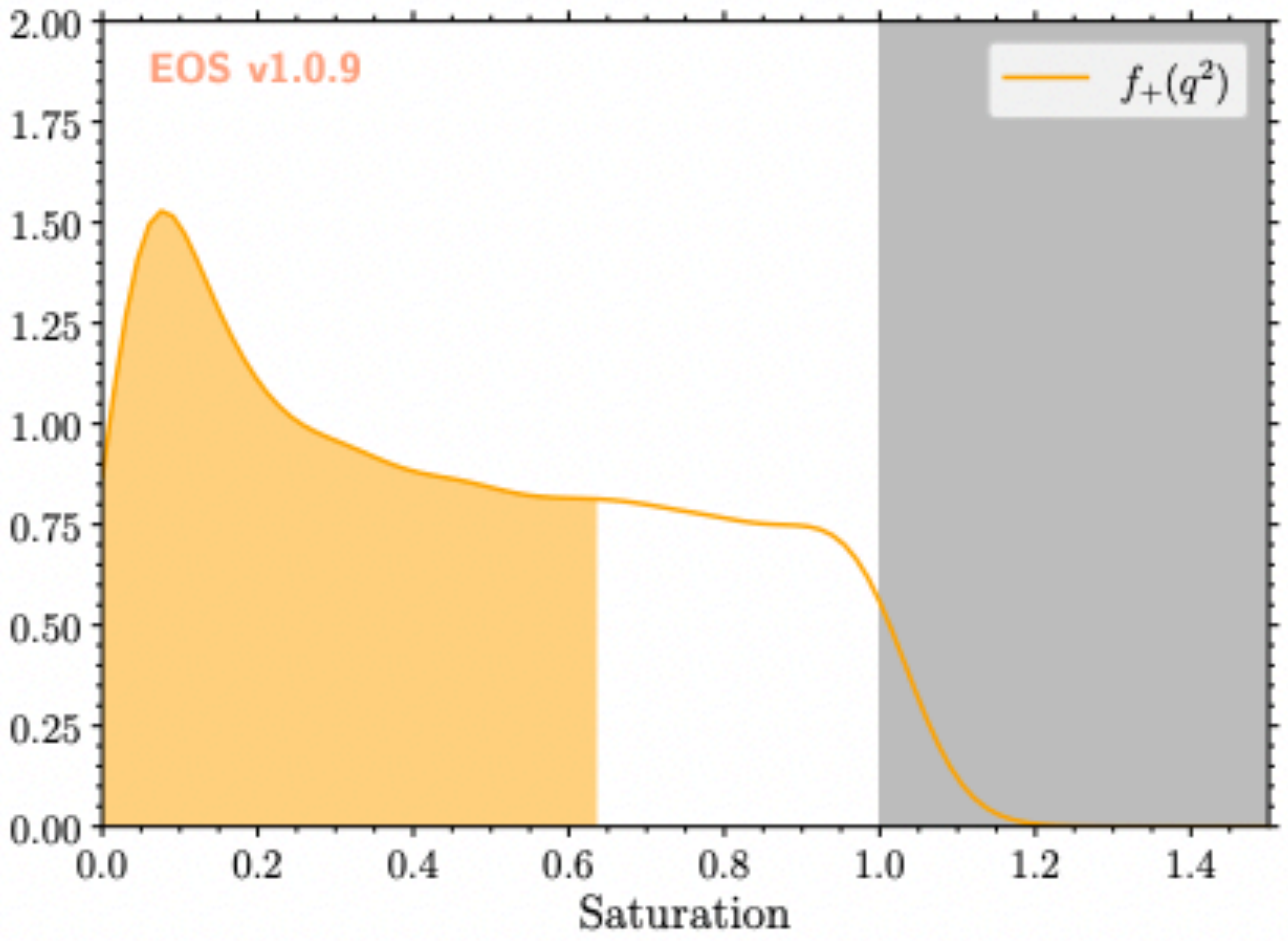


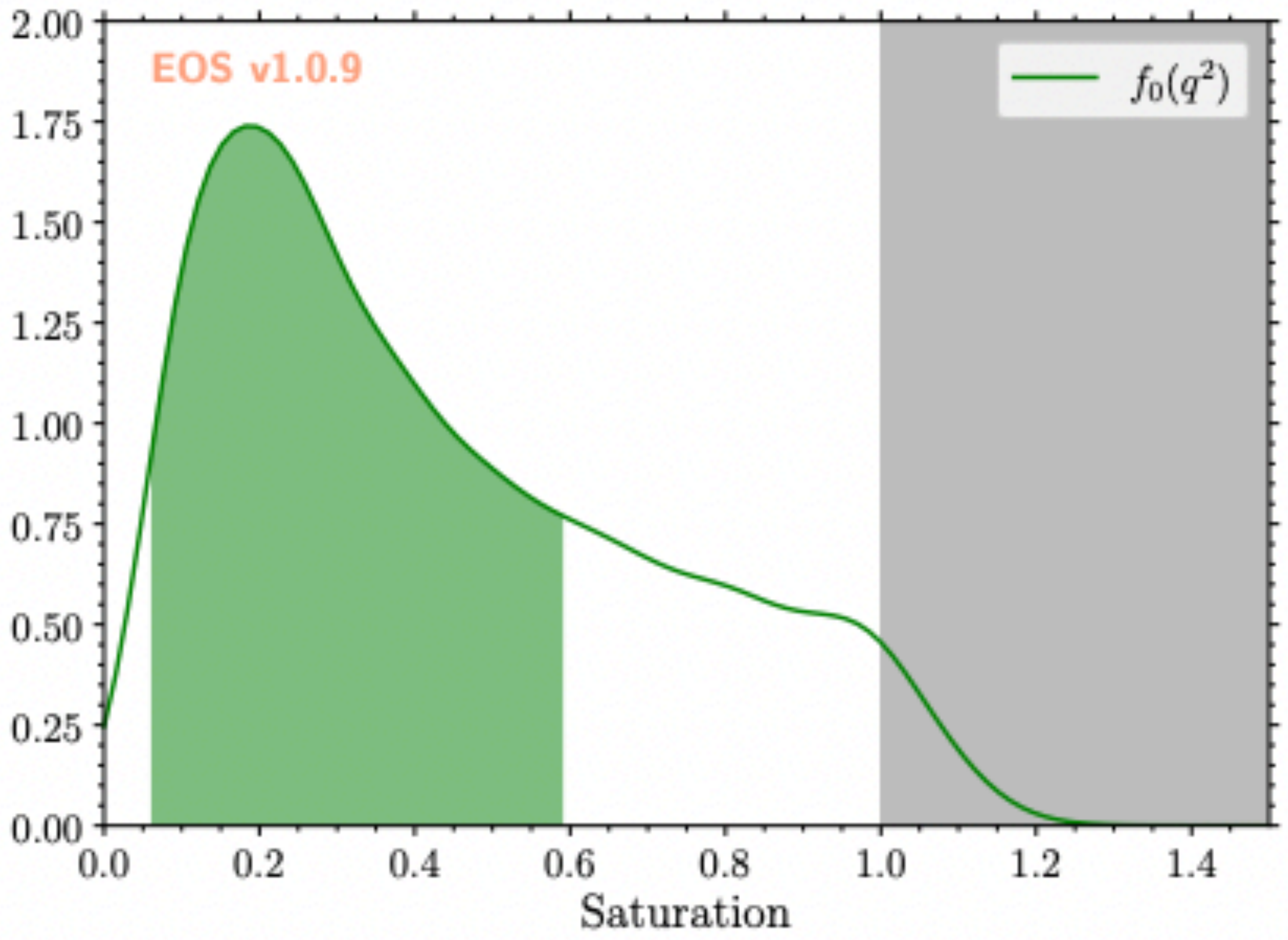




Saturation of unitarity bound

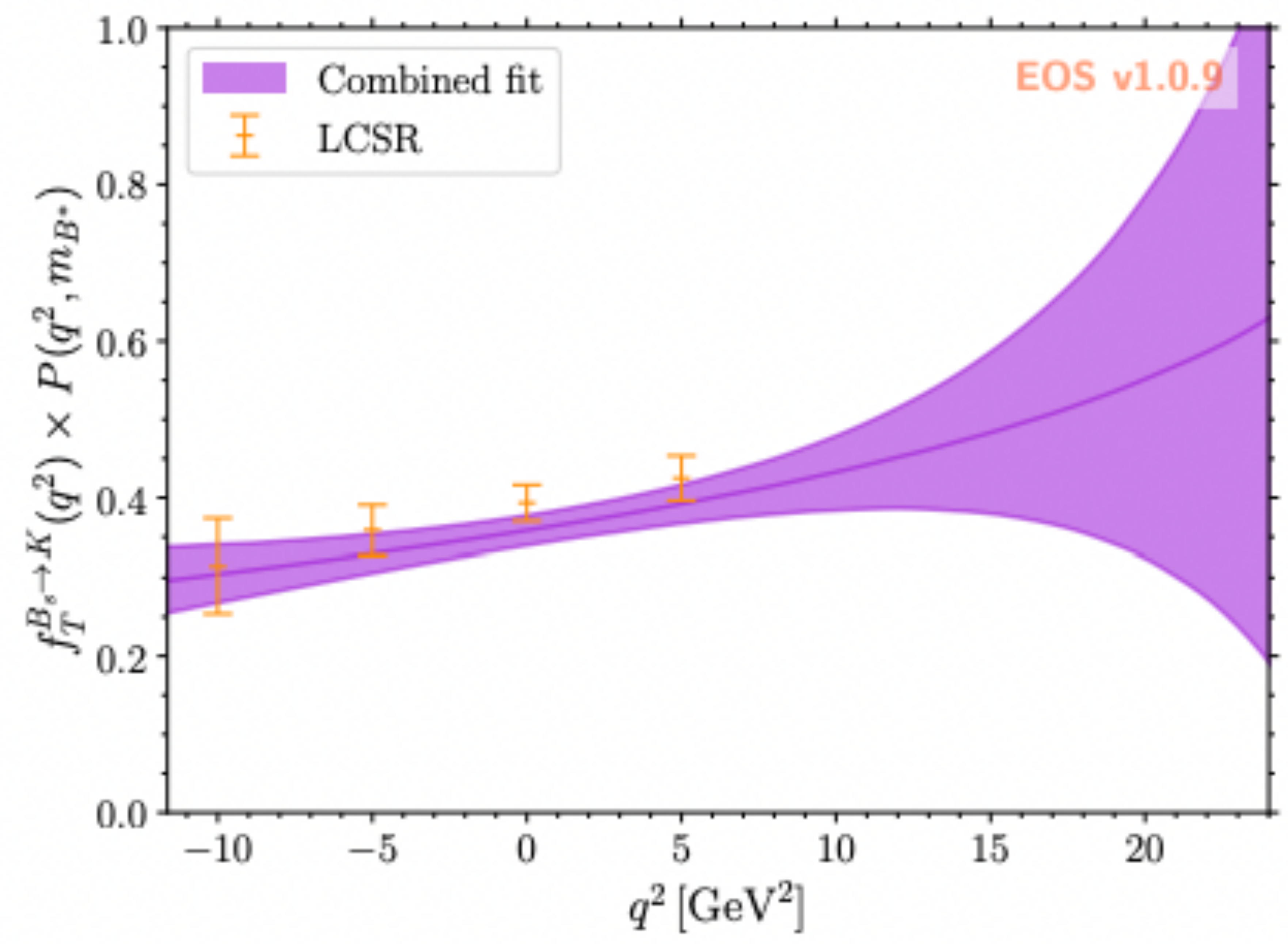


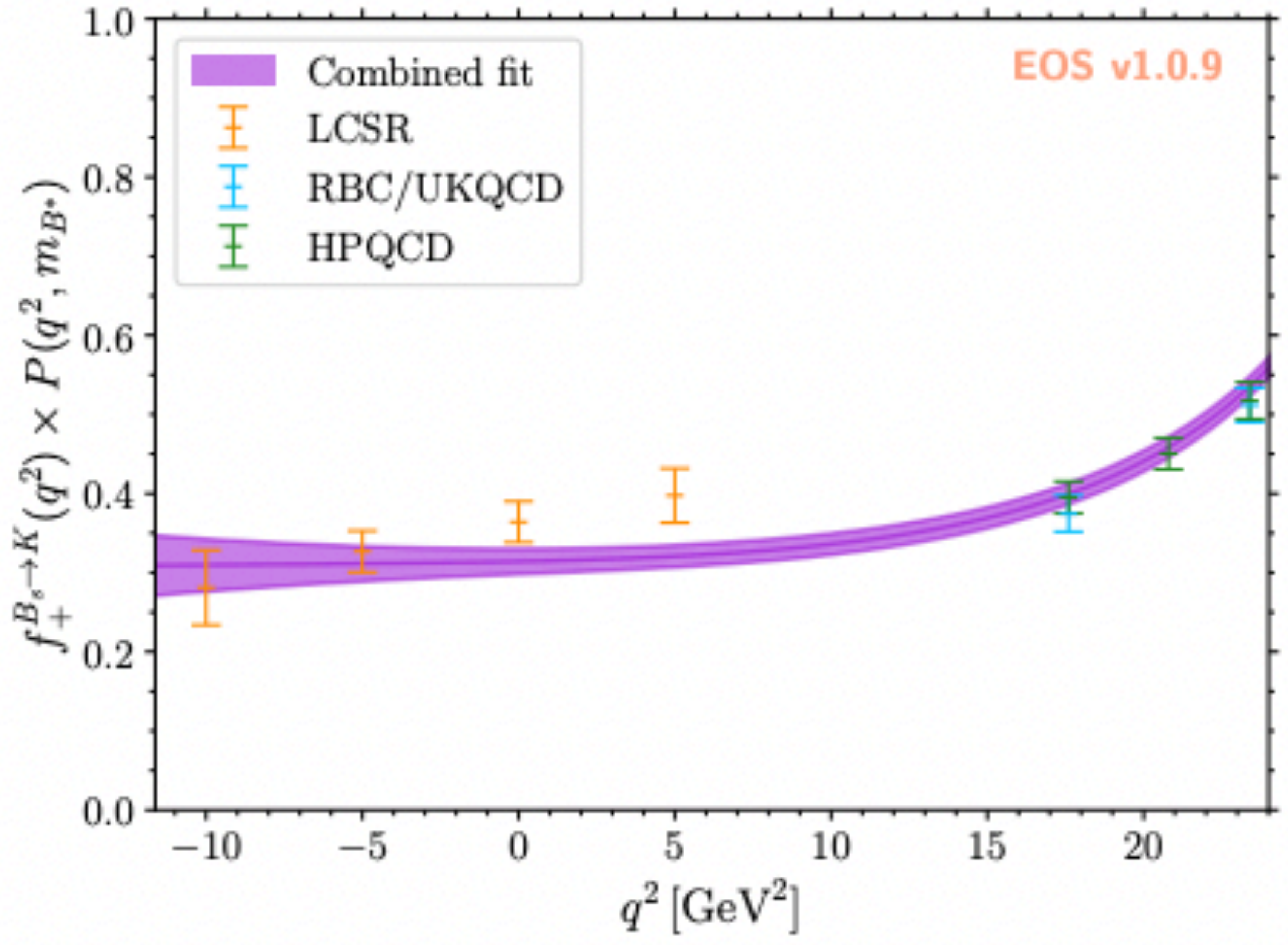


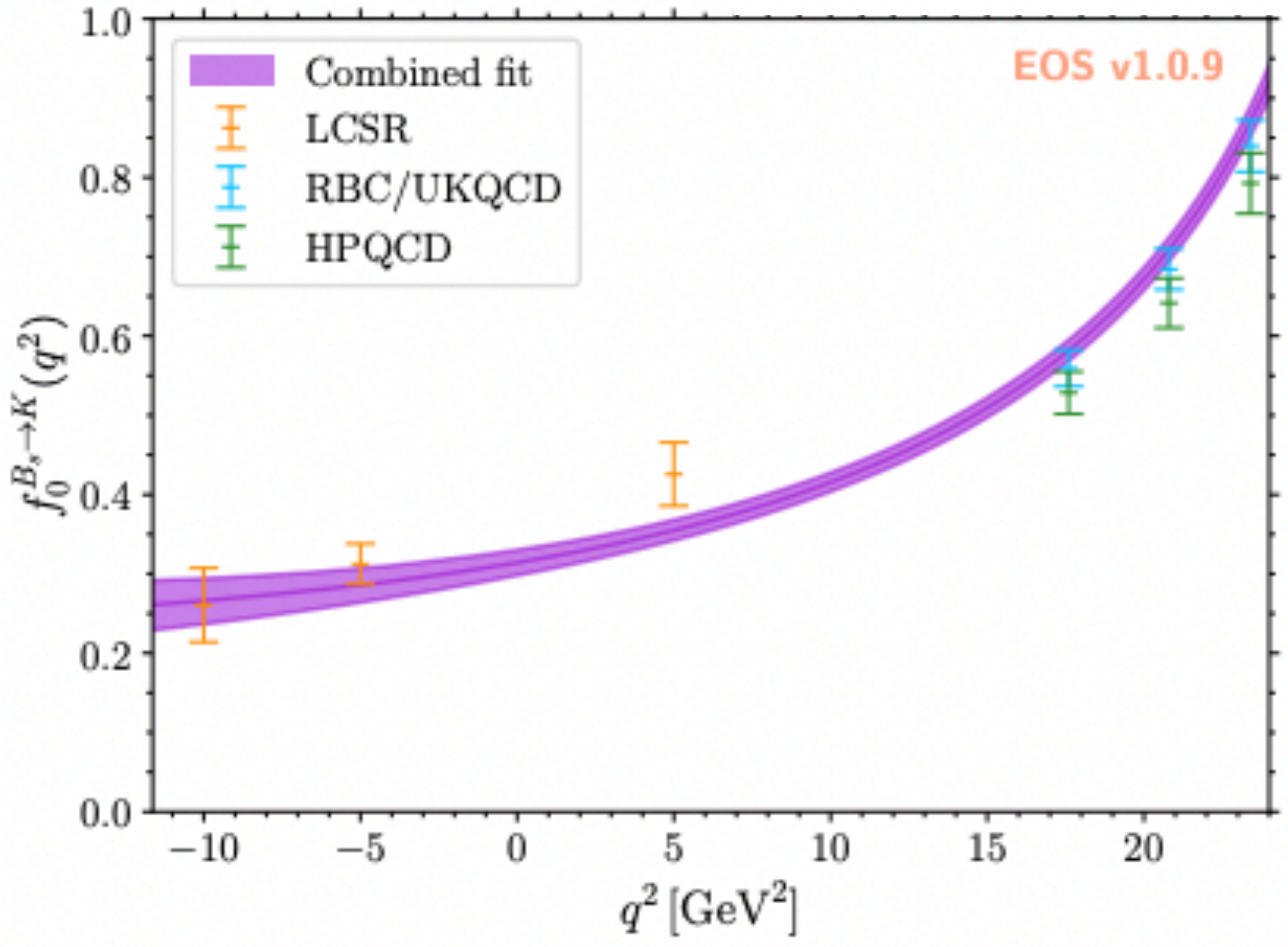


Fit results

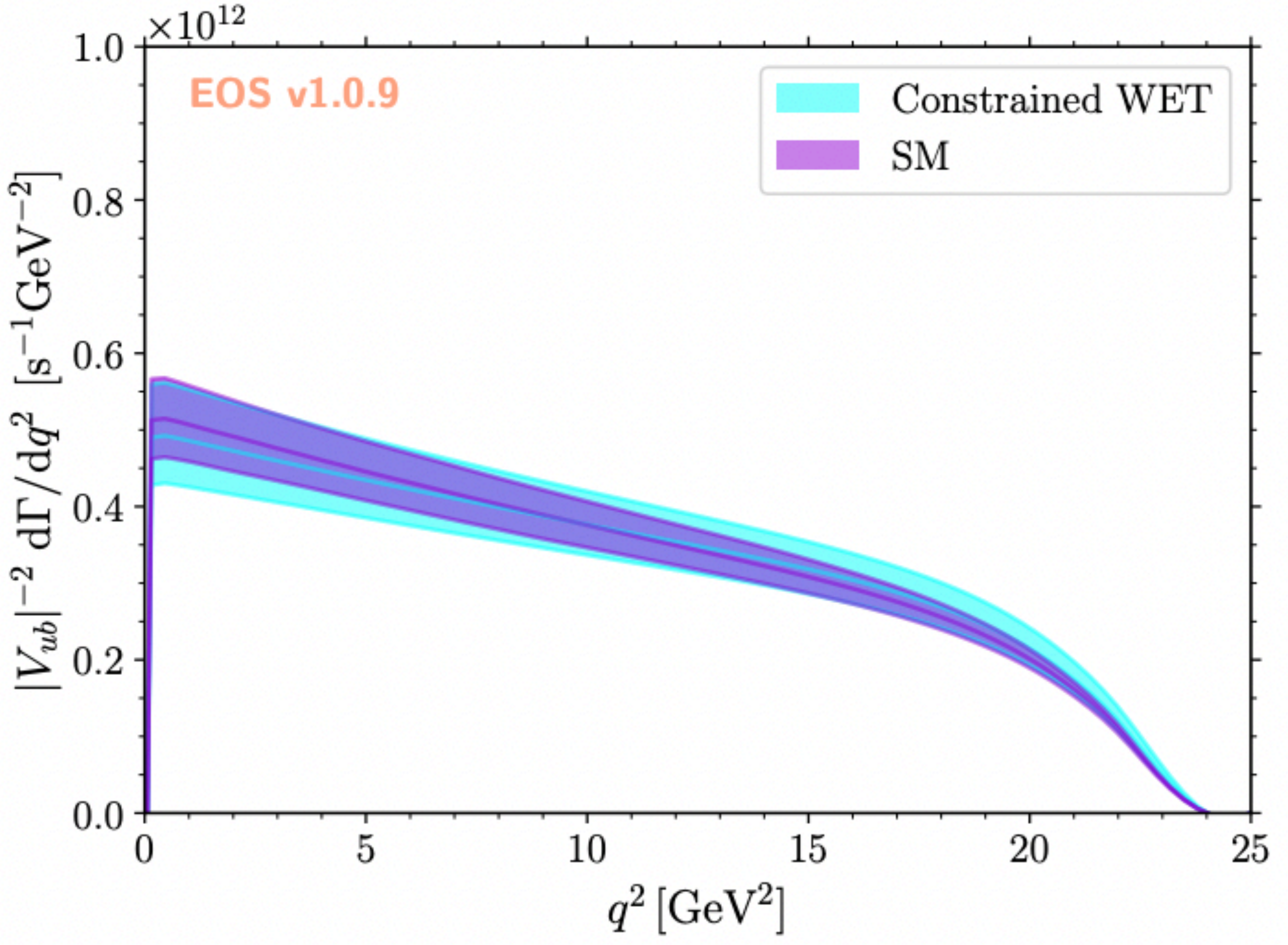
posterior	goodness of fit χ^2	d.o.f.	p-value	BFP saturation sat$_+$	sat$_0$	sat$_T$	extrapolation $f_+(q^2=0)$	$f_T(q^2=0)$
LCSR	0.0	-3	—	0.93	1.00	1.00	0.36 ± 0.02	0.39 ± 0.02
LQCD	5.7	-3	—	0.45	0.52	—	0.25 ± 0.08	—
LCSR+LQCD	15.0	8	6.0%	1.01	0.34	1.00	0.31 ± 0.02	0.36 ± 0.02







BSM available space for $b \to u\ell\bar{\nu}$



$|V_{ub}/V_{cb}|$

$$\left|\frac{V_{ub}}{V_{cb}}\right|^{\mathrm{LCSR}}_{q^2<7\ \mathrm{GeV}^2} = 0.057 \pm 0.005 \qquad \left|\frac{V_{ub}}{V_{cb}}\right|^{\mathrm{LCSR}}_{q^2>7\ \mathrm{GeV}^2} = 0.068 \pm 0.021$$

$$\left|\frac{V_{ub}}{V_{cb}}\right|^{\mathrm{LQCD}}_{q^2<7\ \mathrm{GeV}^2} = 0.087 \pm 0.020 \qquad \left|\frac{V_{ub}}{V_{cb}}\right|^{\mathrm{LQCD}}_{q^2>7\ \mathrm{GeV}^2} = 0.087 \pm 0.006$$

Statistical treatment in EOS

Bayesian inference

- (Dynamic) nested sampling:
  Access probabilities of the predicted samples

LCSR

Comparison to previous determination

Khodjamirian, Rusov arXiv:1703.04765

Same: kaon LCDAs

Ball, Braun, Lenz arXiv:0603063

Update: input parameters

Main differences:

- explicit $m_s \pm m_q$ terms in the RGE (before expanded in m_q/m_s)
- determination of duality threshold parameters