Unitarity constraints and the dispersive matrix

Work in collaboration with G. Martinelli and S. Simula

[PRD ’21 (2105.02497), PRD ’21 (2105.07851), PRD ’22 (2105.08674), EPJC ‘22 (2109.15248), …]

Ludovico Vittorio (LAPTh & CNRS, Annecy, France)

12th International Workshop on the CKM Unitarity Triangle - September 20th, 2023

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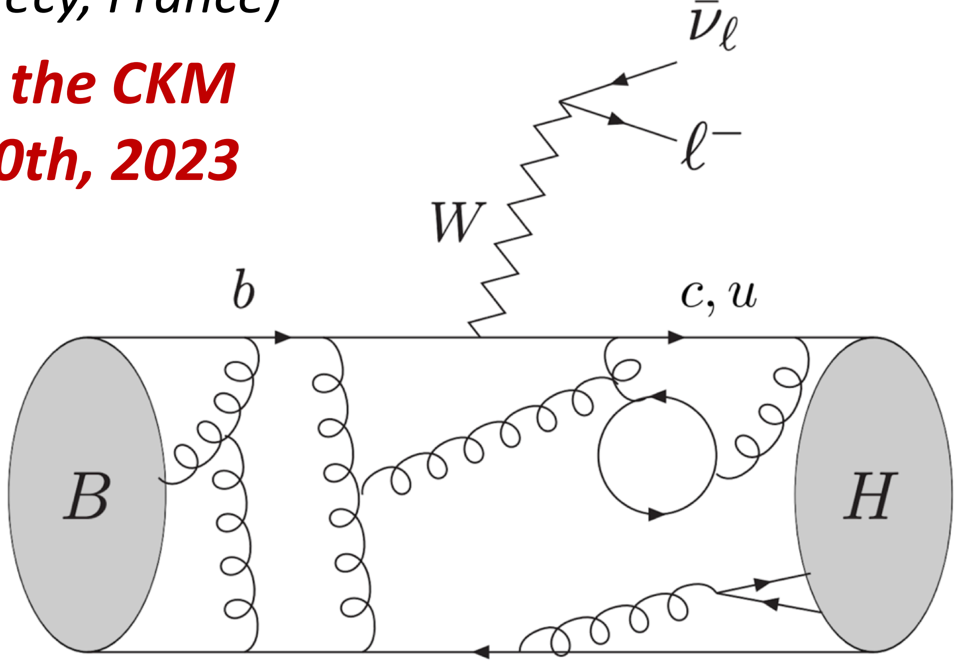

(from J.Phys.G 46 (2019) 2, 023001)

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1. CKM matrix elements puzzles

A non-negligible tension exists between the inclusive and the exclusive determinations of |Vcb| and |Vub|, for instance in the latter case:

$$|V_{cb}| \times 10^3 = 39.36(68)$$ *EXCLUSIVE*

VS

$$|V_{cb}| \times 10^3 = 42.00(65)$$ *INCLUSIVE*

$$|V_{cb}|_{\rm incl} \times 10^3 = 42.16 \pm 0.50$$

Bordone et al., Phys.Lett.B '21 [2107.00604]

$$|V_{cb}|_{\rm incl} \times 10^3 = 41.69 \pm 0.63$$

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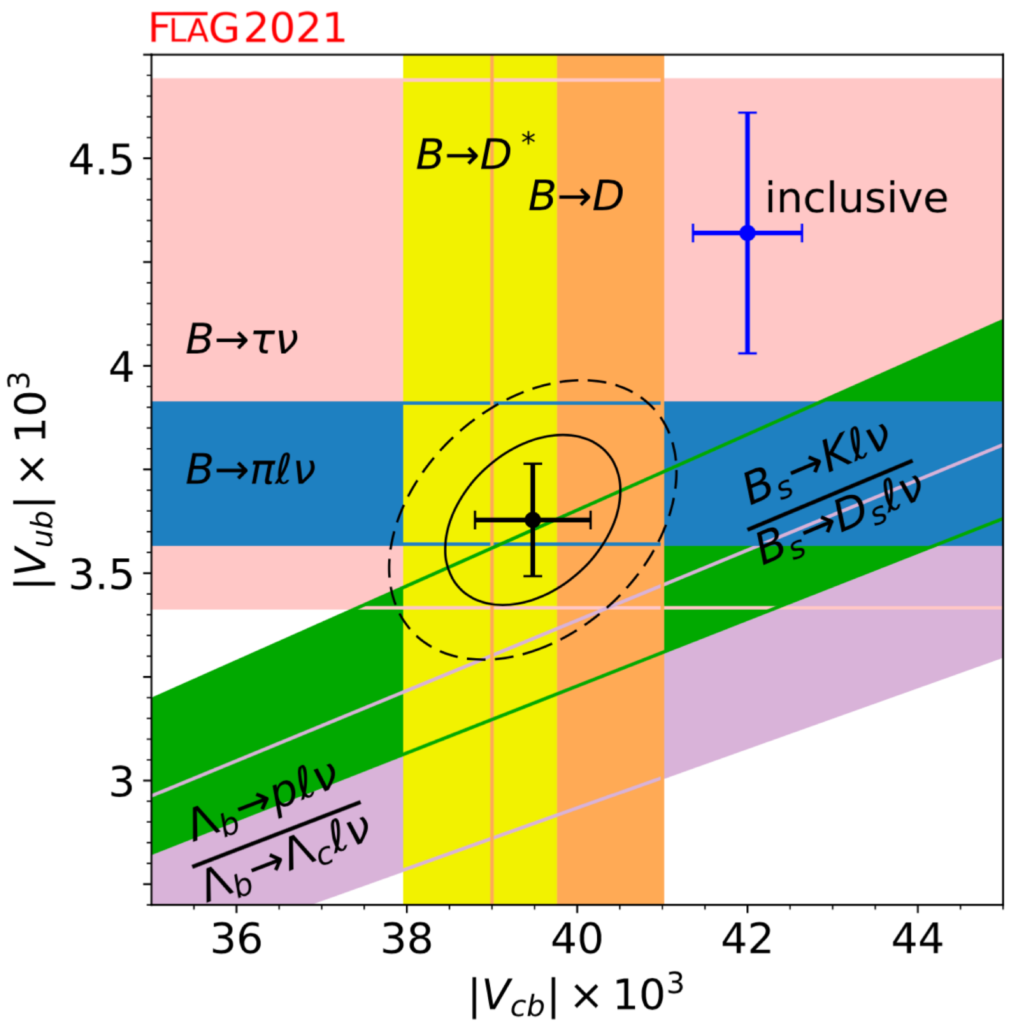

FLAG Review 2021 [EPJC '22 (2111.09849)]

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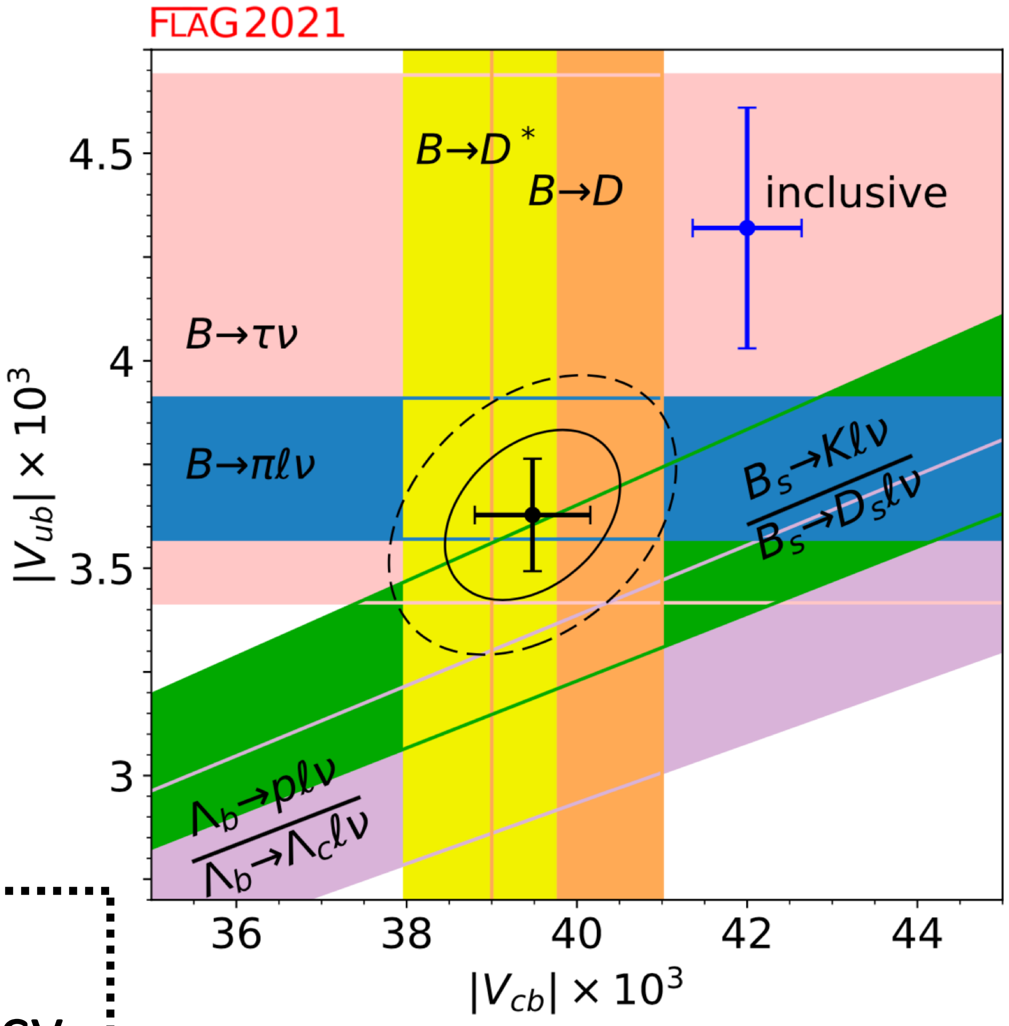

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2. Lepton Flavour Universality (Violation)

Lepton Flavour Universality (LFU) is one of the pillars of the SM. According to this principle, all the three types of charged lepton particles (namely the electrons, the muons and the taus) interact in the same way with the gauge bosons, independently of their generation. In other words, in the SM the gauge interactions are LFU.

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Lepton Flavour Universality Violation in charged currents

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)},$$

$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)}$$

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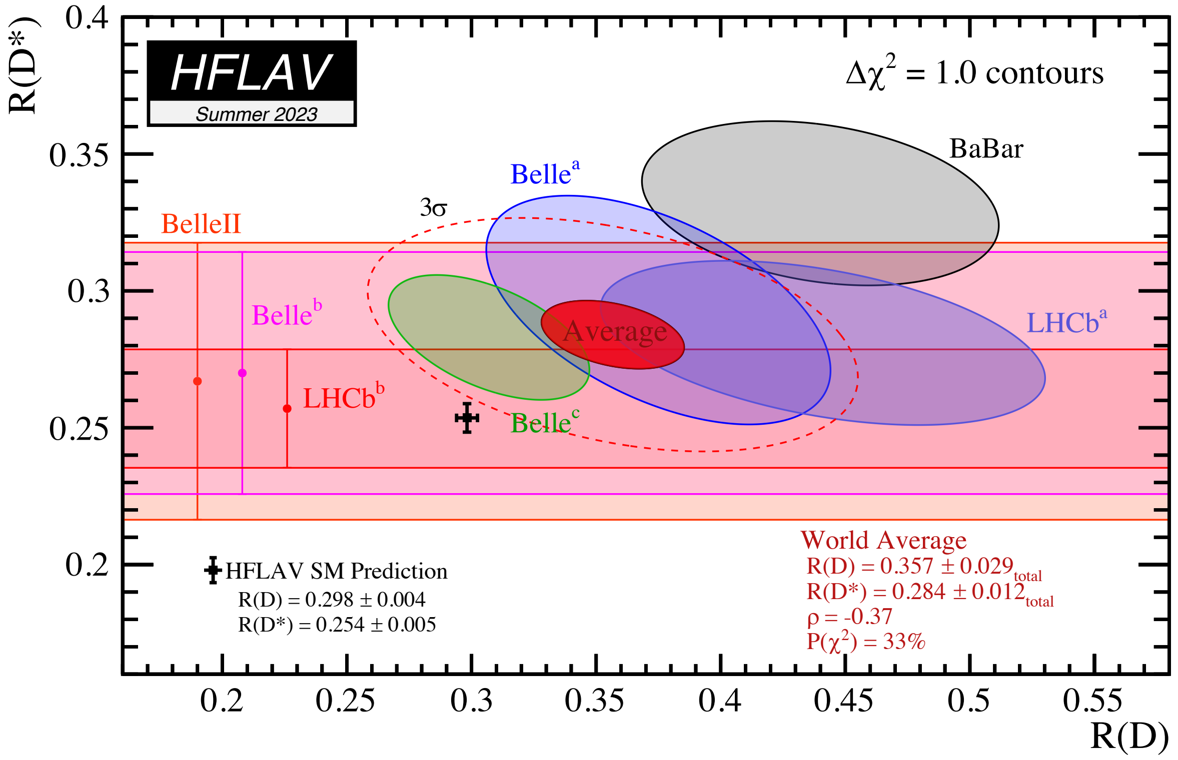

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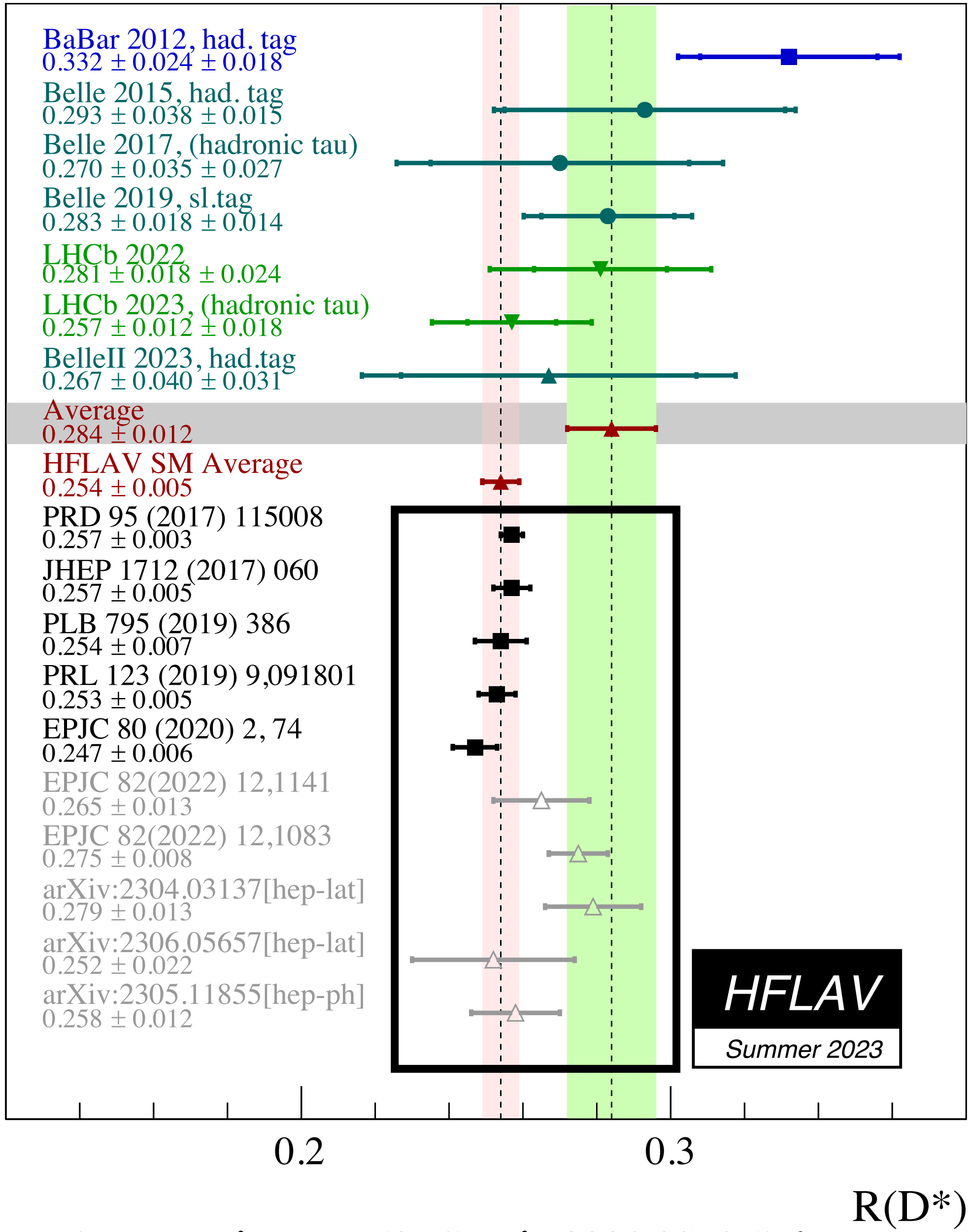

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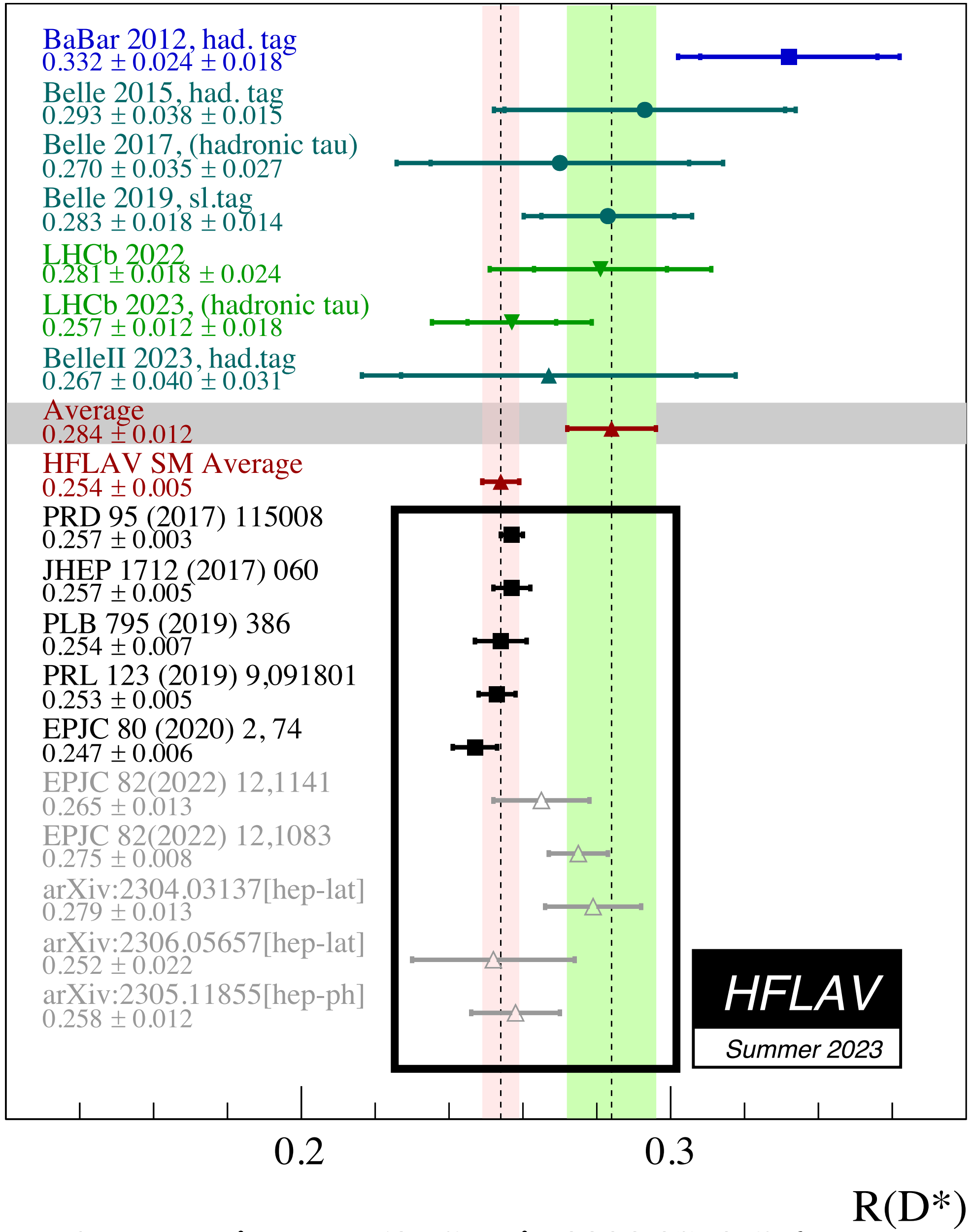
The final SM result on R(D*) depends on the lattice input chosen and/or on the analysis strategy adopted!



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The Dispersive Matrix (DM) method

Our goal is to describe the FFs using a novel, non-perturbative and model independent approach: starting from the available LQCD computations of the FFs in the high-q^2 (or low-w) regime, we extract the FFs behaviour in the low-q^2 (or high-w) region!

- Pioneering works from S. Okubo [PRD, 3 (1971); PRD, 4 (1971)], C. Bourrely et al [NPB, 189 (1981)] and L. Lellouch [NPB, 479 (1996)]
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The resulting description of the FFs

- **is entirely based on first principles (LQCD evaluation of 2- and 3-point Euclidean correlators)**
- **is independent of any assumption on the functional dependence of the FFs on the momentum transfer**
- **can be applied to theoretical calculations of the FFs, but also to experimental data**
- **keep theoretical calculations and experimental data separated**
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How does it work?

The DM method

Let us focus on a generic FF *f*: **we will determine f(t) with $f(t_i)$ known at positions t_i (i=1, …, N)**

$$z(t) = \frac{\sqrt{\frac{t_+ - t}{t_+ - t_-}} - 1}{\sqrt{\frac{t_+ - t}{t_+ - t_-}} + 1}$$

$$t_\pm \equiv (m_B \pm m_D)^2$$

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- inner product $$\langle h_1 | h_2 \rangle = \int_{|z|=1} \frac{dz}{2\pi i z} \bar{h}_1(z) h_2(z)$$
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We build up the matrix M of the scalar products of ϕf, g_t, g_{t1}, …, g_{tN} :

$$\mathbf{M} = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle \end{pmatrix}$$

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L. Lellouch, NPB, 479 (1996), p. 353-391
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CENTRAL ISSUE: since **M** contains only inner products, by construction its determinant is semipositive definite

$$\det \mathbf{M} \geq 0 \quad \Rightarrow \quad f_{\rm lo}(z) \leq f(z) \leq f_{\rm up}(z)$$

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DISPERSION RELATIONS:

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→ **Simple implementation!**

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The DM results for the FFs entering semileptonic B \to D(*) decays have been also twice applied to global New Physics (NP) fit:

- **Global NP study of semilept. B \to D(*) decays**:

  Fedele, Blanke, Crivellin, Iguro, Nierste, Simula, LV: **arxiv:2305.15457 [hep-ph]**

  → *Implementation of DM FFs in HEPfit! {EPJC '20 [arXiv:1910.14012]}*

- **Interplay between b \to s data and R(D(*))**:

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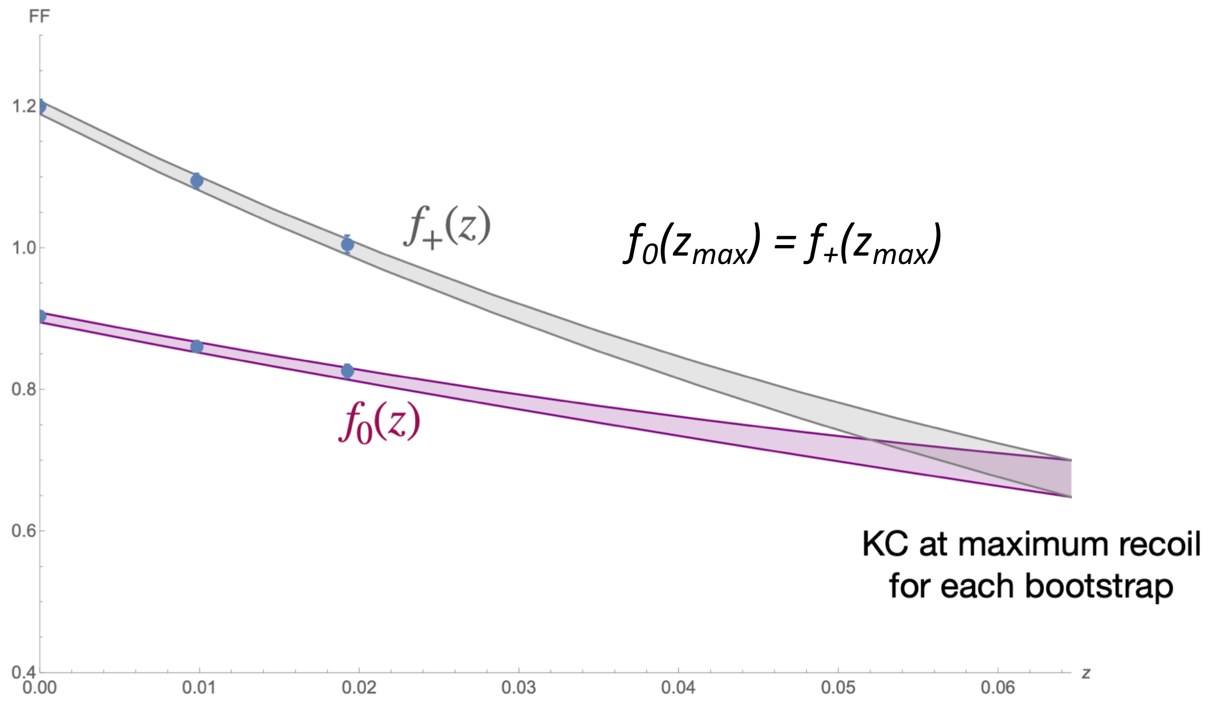
The DM results can be directly taken for further use!

The simplest example: semileptonic **B → D** decays

In **PRD '21 (arXiv:2105.08674)**, our DM method has been applied to B → D decays:

- **3 FNAL/MILC data for each FF**: final results contained in **PRD '15 (arXiv:1503.07237)**



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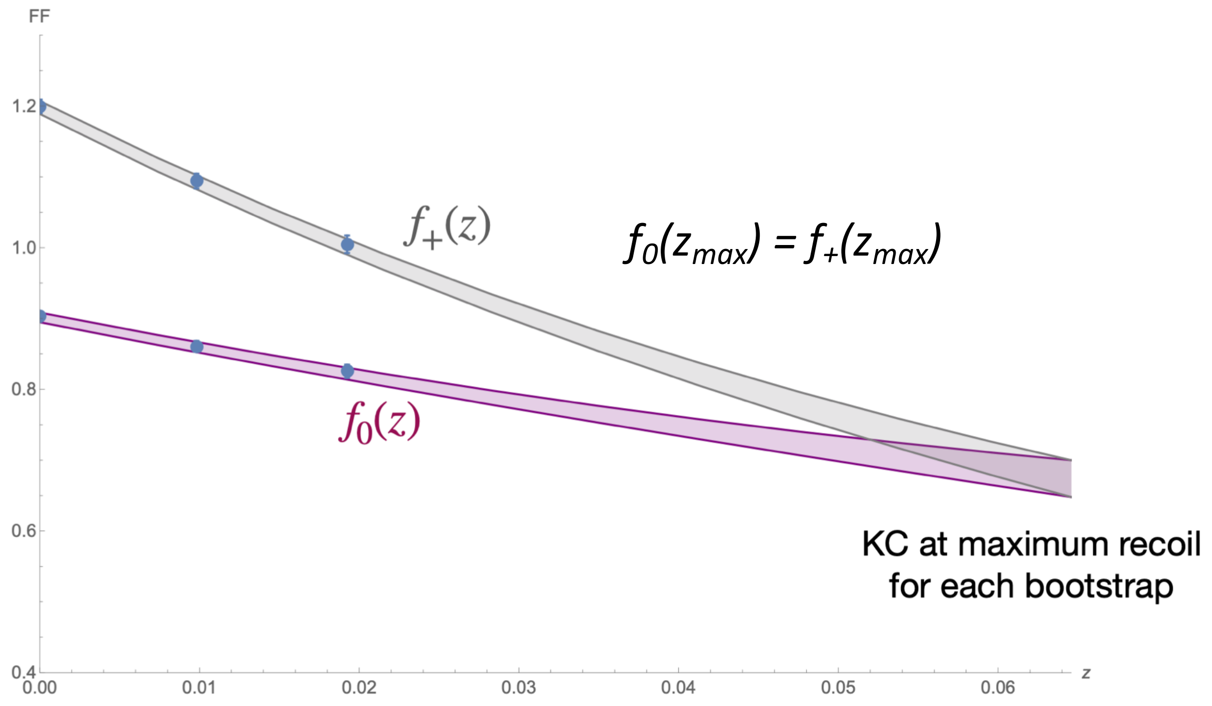

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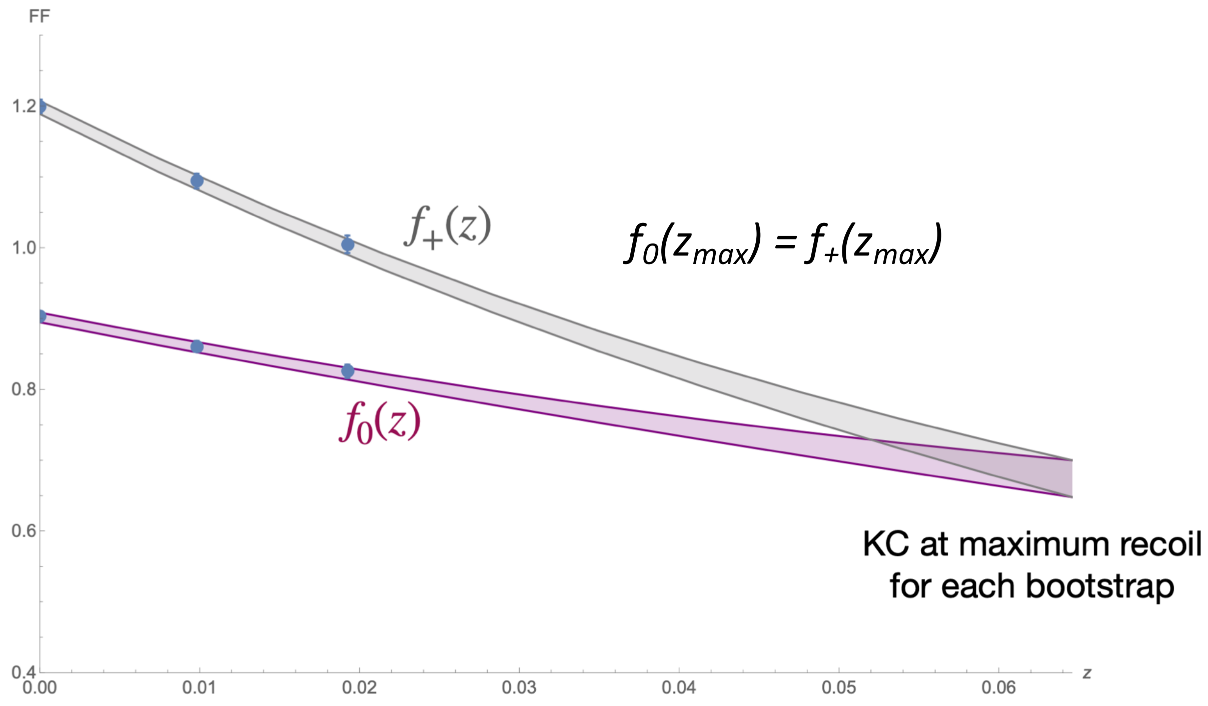
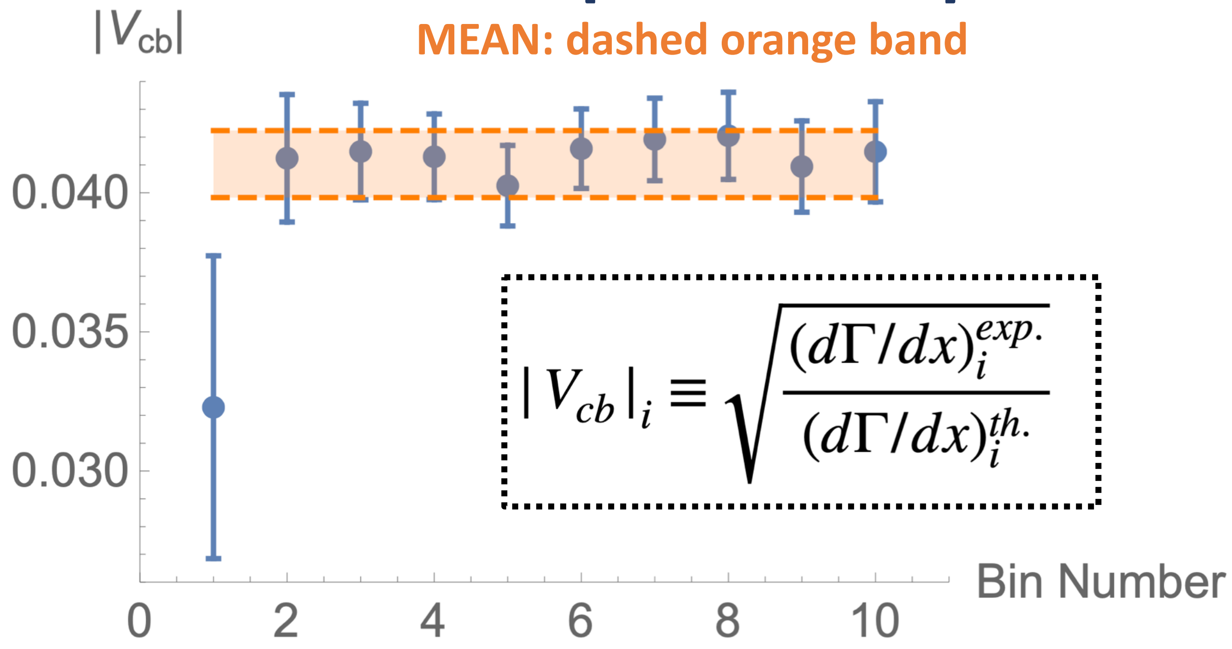


Blue points from Belle data:
PRD '16 [arXiv:1510.03657]
MEAN: dashed orange band



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$$|V_{cb}| \times 10^3 = 41.0 \pm 1.2$$

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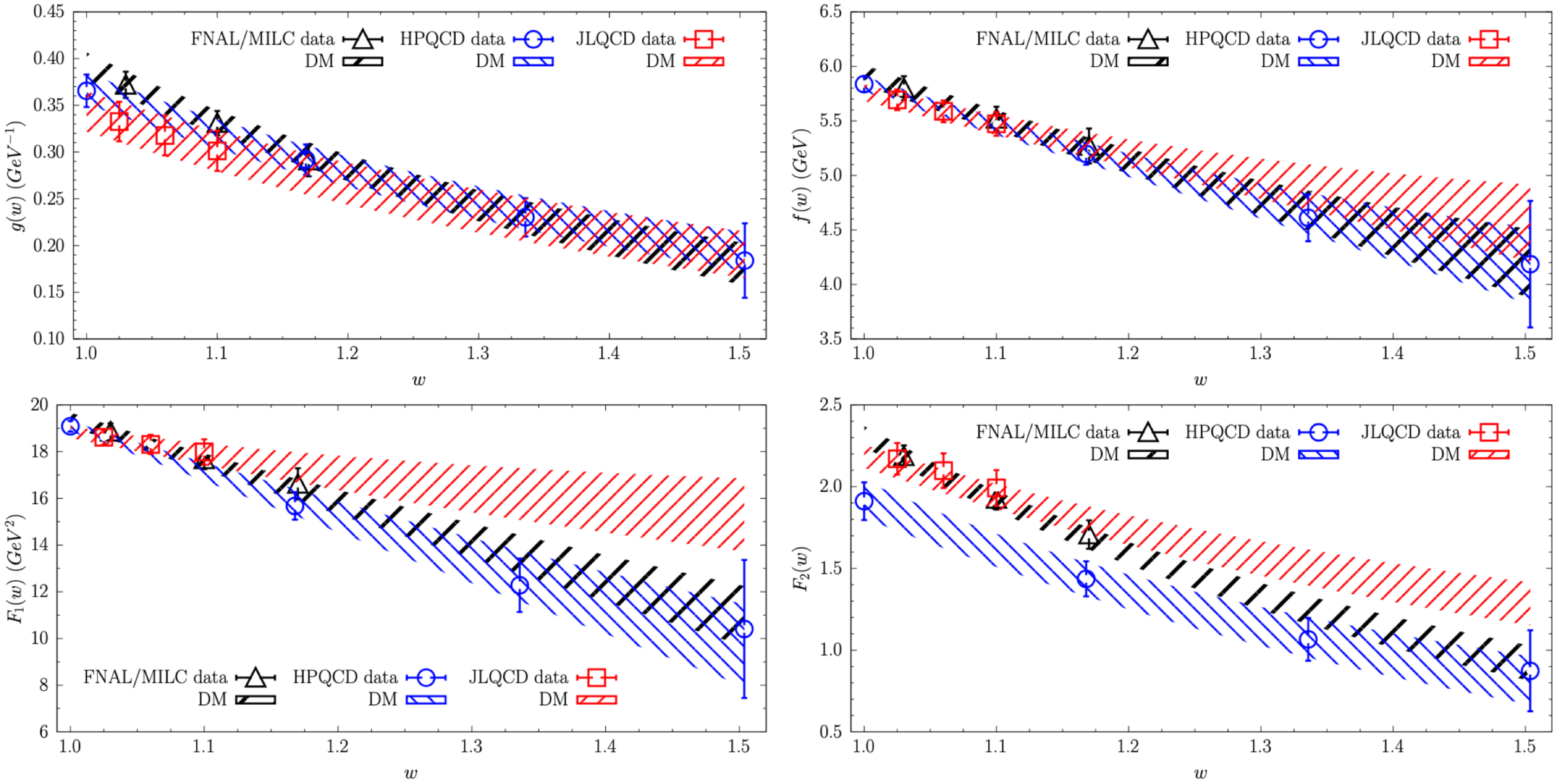
Updates on the “problematic” semileptonic **B → D*** channel

Some updates are now available:



FNAL/MILC:
EPJC ‘22
(arXiv:2105.14019)

HPQCD:
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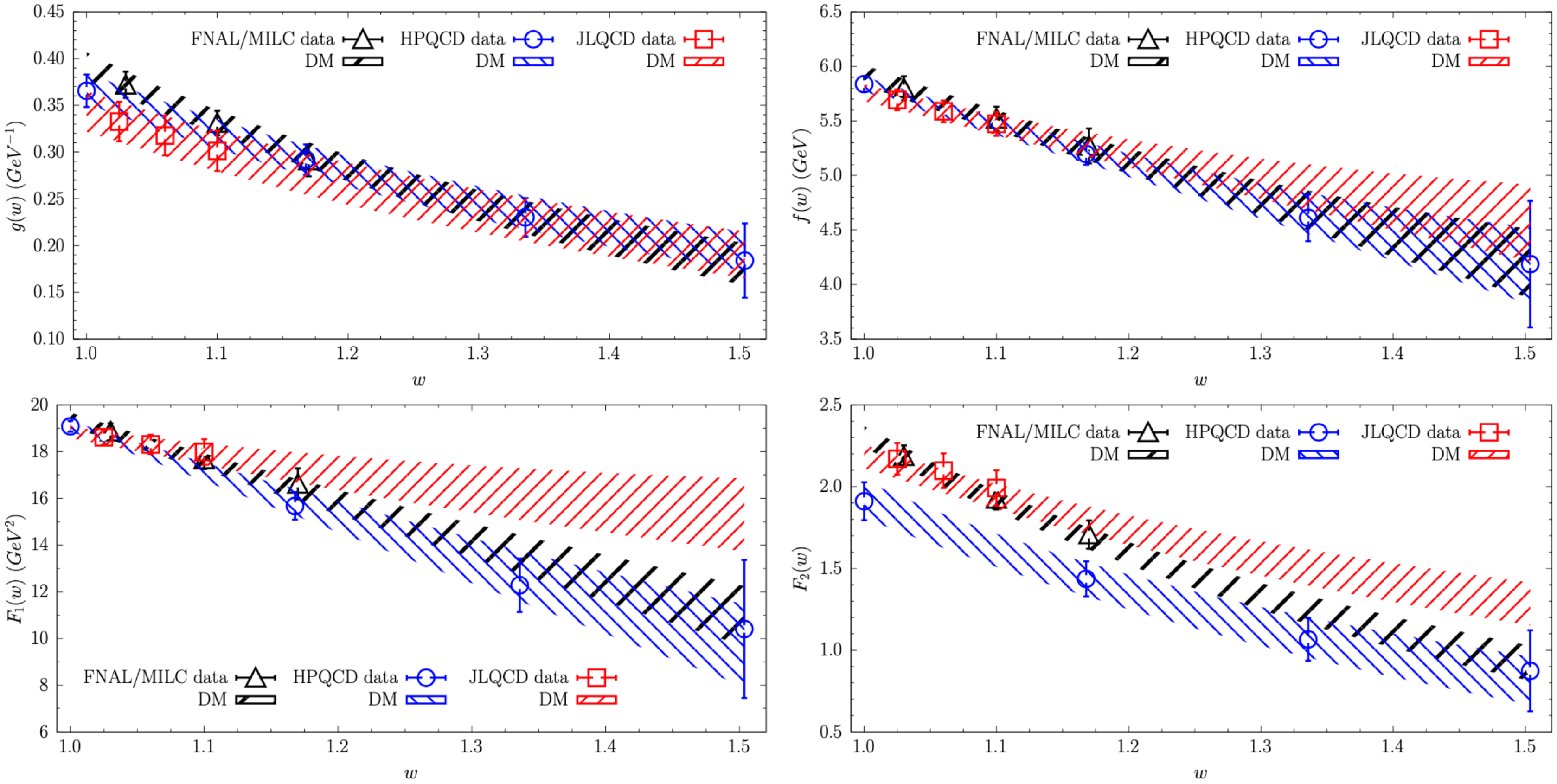

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i) There is a **strong tension between the values of F2(w) from HPQCD and those of the other two collaborations**;
ii) Although at small *w* **the values of F2(w) from FNAL/MILC and JLQCD** are close, the extrapolated values **are different**;
iii) The results for g(w), f(w) and F1(w) are in good agreement where all the collaborations have computed the FFs (**at w ≤ 1.2**);
iv) The allowed band of the **extrapolated values of F1(w) from JLQCD**, however, is **very different from the bands obtained for this quantity using the** values by FNAL/MILC and HPQCD (see the different slope of F1(w) at the smaller w values).

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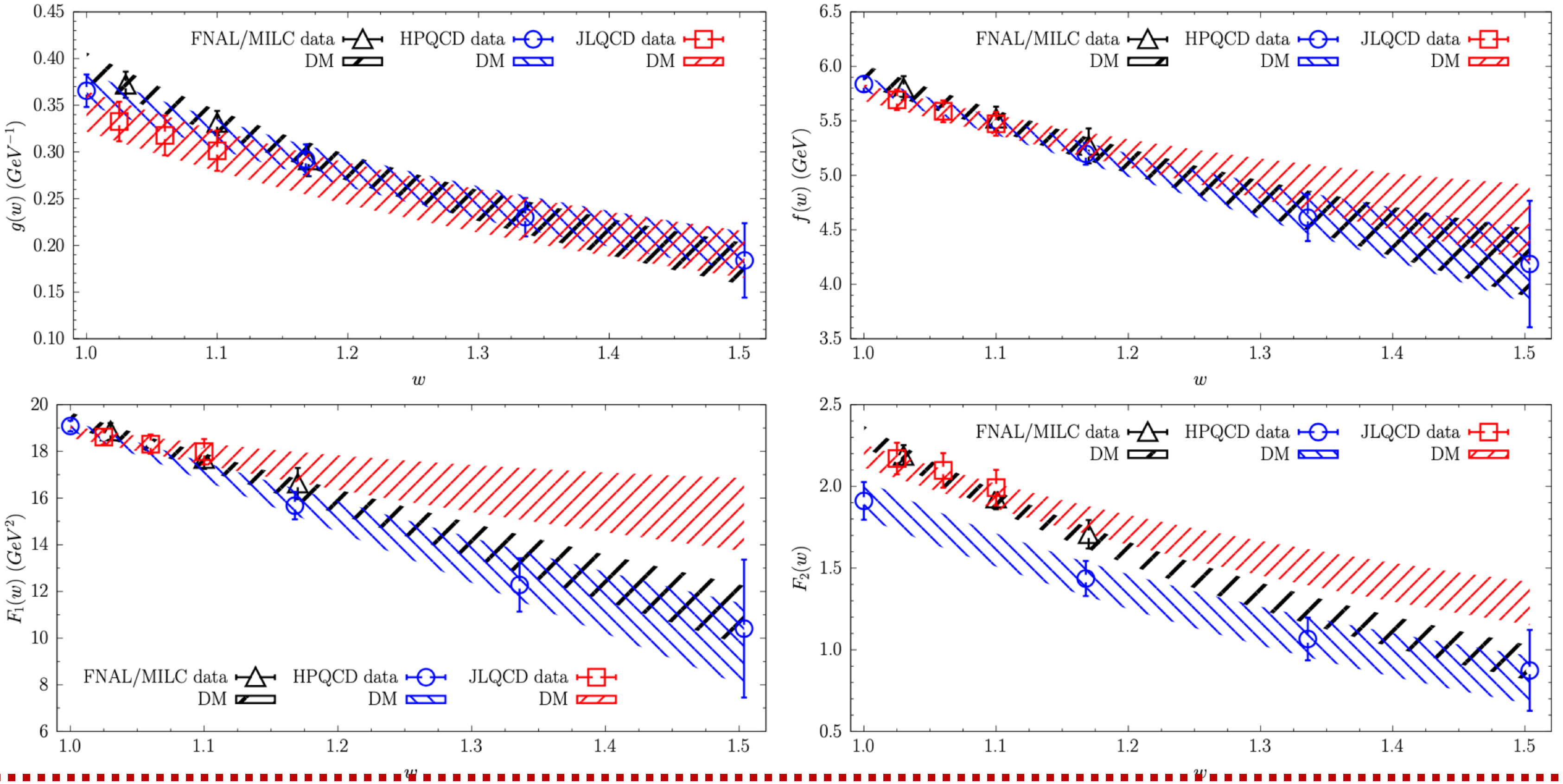

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IMPORTANT: the DM results always correspond to a vanishing value of the χ2-variable in a frequentist language, while having unitarity built-in (and the KCs exactly implemented)!!

Updates on |Vcb| extraction

Two sets of data by Belle Collaboration to be used:

Belle Collaboration: PRD '19 [arXiv:1809.03290]

- Belle 2018: $d\Gamma/dx, \;\; x = w, \cos\theta_l, \cos\theta_v, \chi$

- Belle 2023: $(d\Gamma/dx)/\Gamma, \;\; x = w, \cos\theta_l, \cos\theta_v, \chi$

Belle Collaboration: PRD '23 [arXiv:2301.07529]

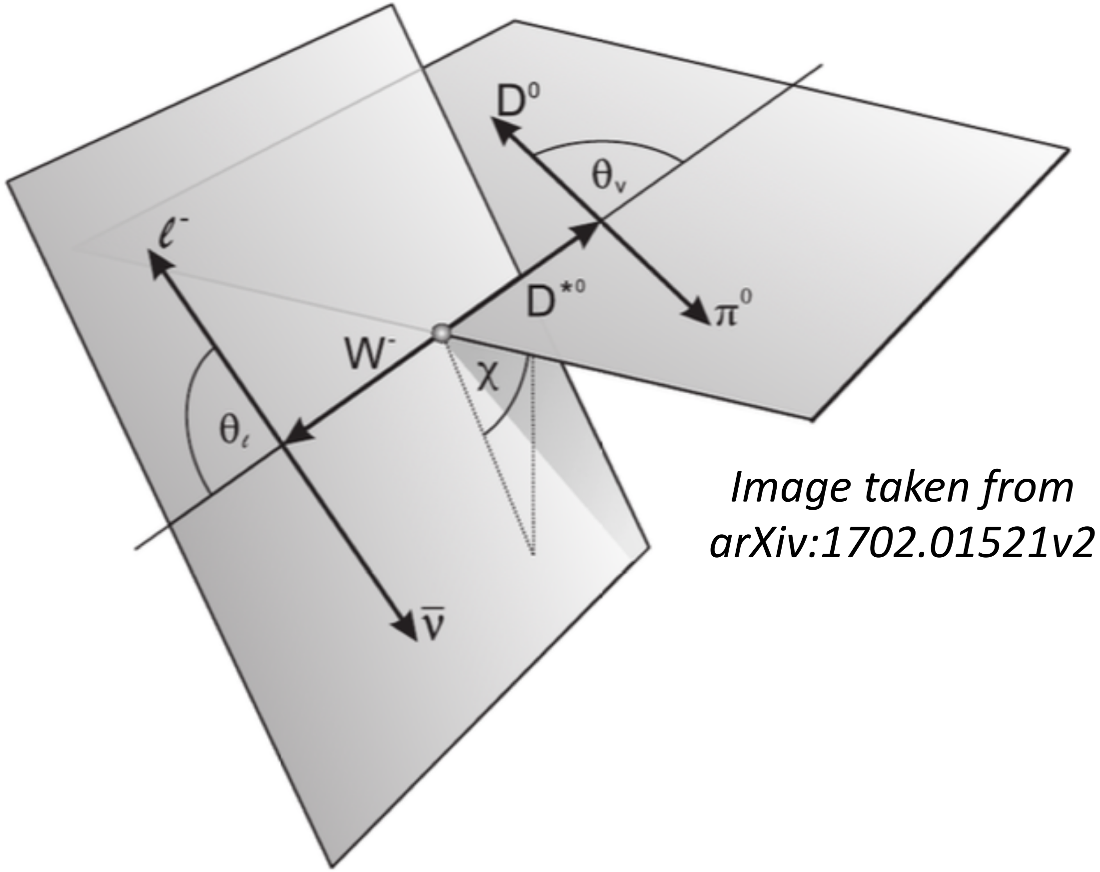

Image taken from arXiv:1702.01521v2

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Belle Collaboration: PRD '23 [arXiv:2301.07529]

For **Belle 2018** data:

- we use a modified covariance matrix to take into account the correct numbero of zero eigenvalues (see **PRD '21 (arXiv:2105.08674)**)
- we can compute |Vcb| from the experimental total decay rate (see **LV's PhD Thesis "The D(M)M perspective on Flavour Physics"** and **arxiv:2305.15457 [hep-ph]**)

For **Belle 2023** data:

- the covariance matrix is already in the correct form
- we can <u>NOT</u> compute |Vcb| from the experimental total decay rate
- we have to use an external number for the total decay rate, *i.e.* $\Gamma(B \to D^* \ell \nu) = 2.20(9) \cdot 10^{-14}$ GeV

Updates on |Vcb| extraction

Then, we determine |Vcb| bin-per-bin through the formula

$$|V_{cb}|_i \equiv \sqrt{\frac{(d\Gamma/dx)_i^{exp.}}{(d\Gamma/dx)_i^{th.}}}$$

For **Belle 2018** data:

- we use a modified covariance matrix to take into account the correct numbero of zero eigenvalues (see **PRD ’21 (arXiv:2105.08674)**)
- we can compute |Vcb| from the experimental total decay rate (see **LV’s PhD Thesis “The D(M)M perspective on Flavour Physics”** and **arxiv:2305.15457 [hep-ph]**)

For **Belle 2023** data:

- the covariance matrix is already in the correct form
- we can NOT compute |Vcb| from the experimental total decay rate
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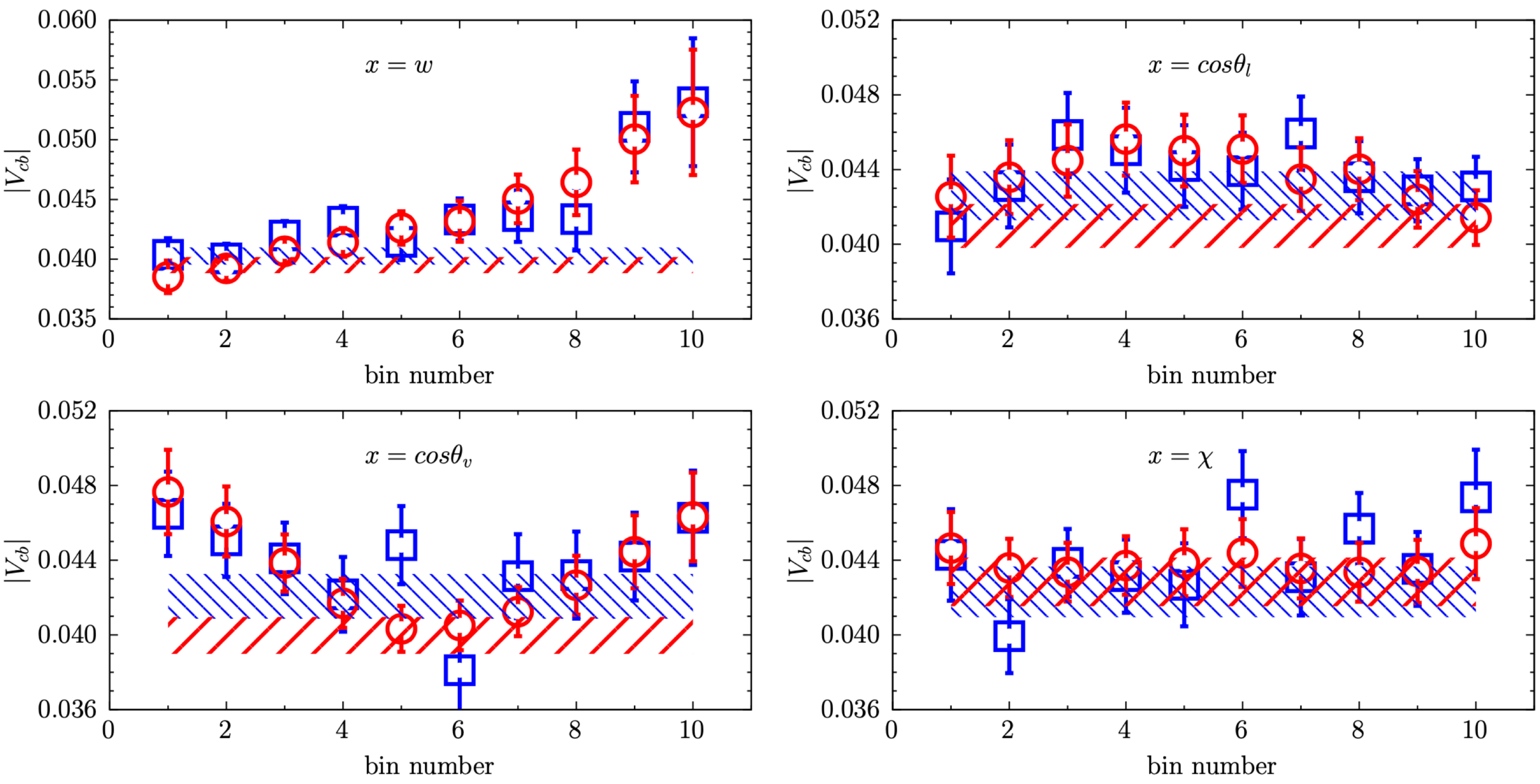
computed by using the BRs in HFLAV, PRD ’23 [arXiv:2206.07501]

Our proposal: *bin-per-bin exclusive Vcb* determination through unitarity



FNAL/MILC: EPJC '22 (arXiv:2105.14019)

~~**HPQCD: arXiv:2304.03137**~~

~~**JLQCD: arXiv:2306.05657**~~

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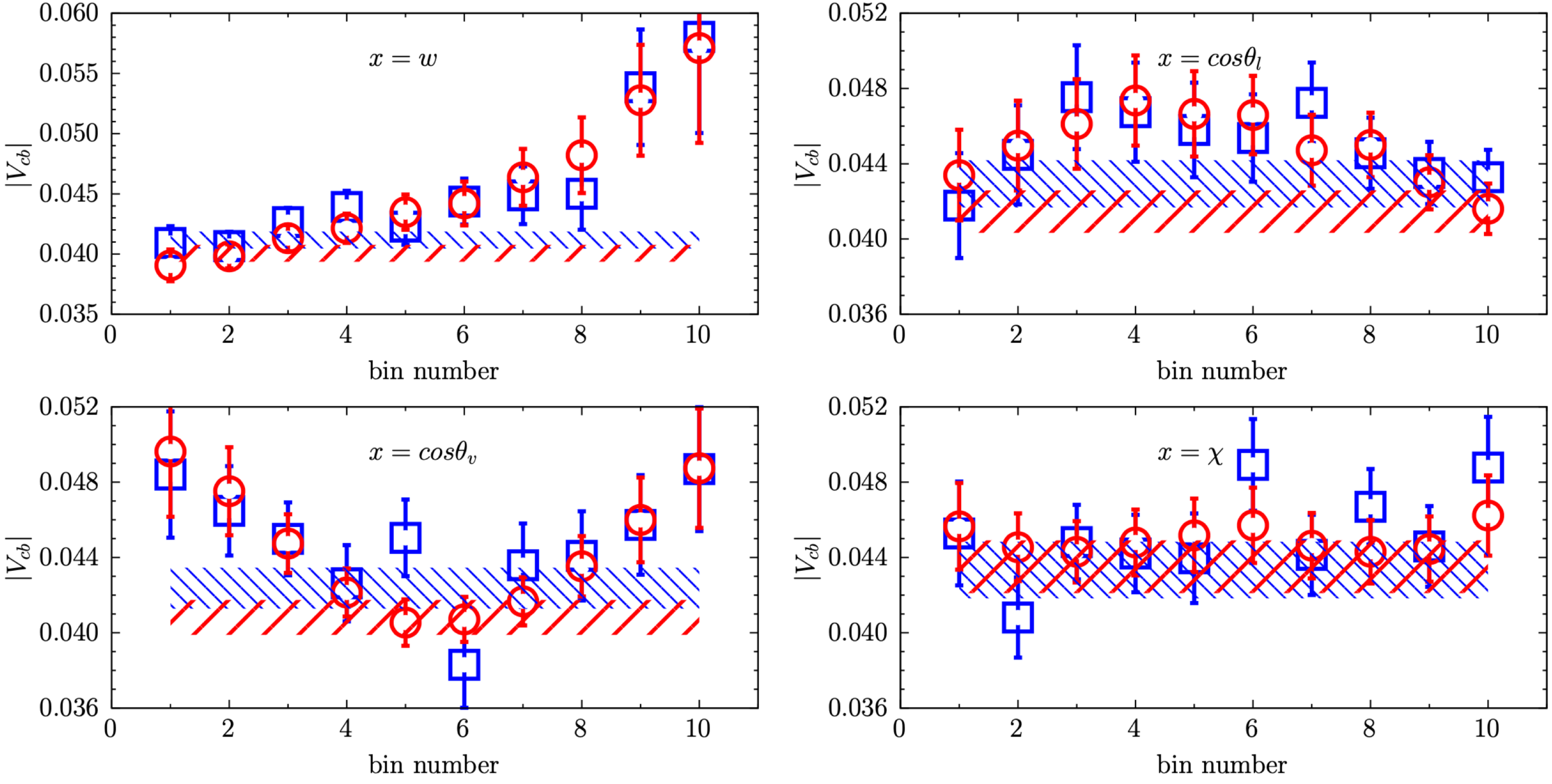

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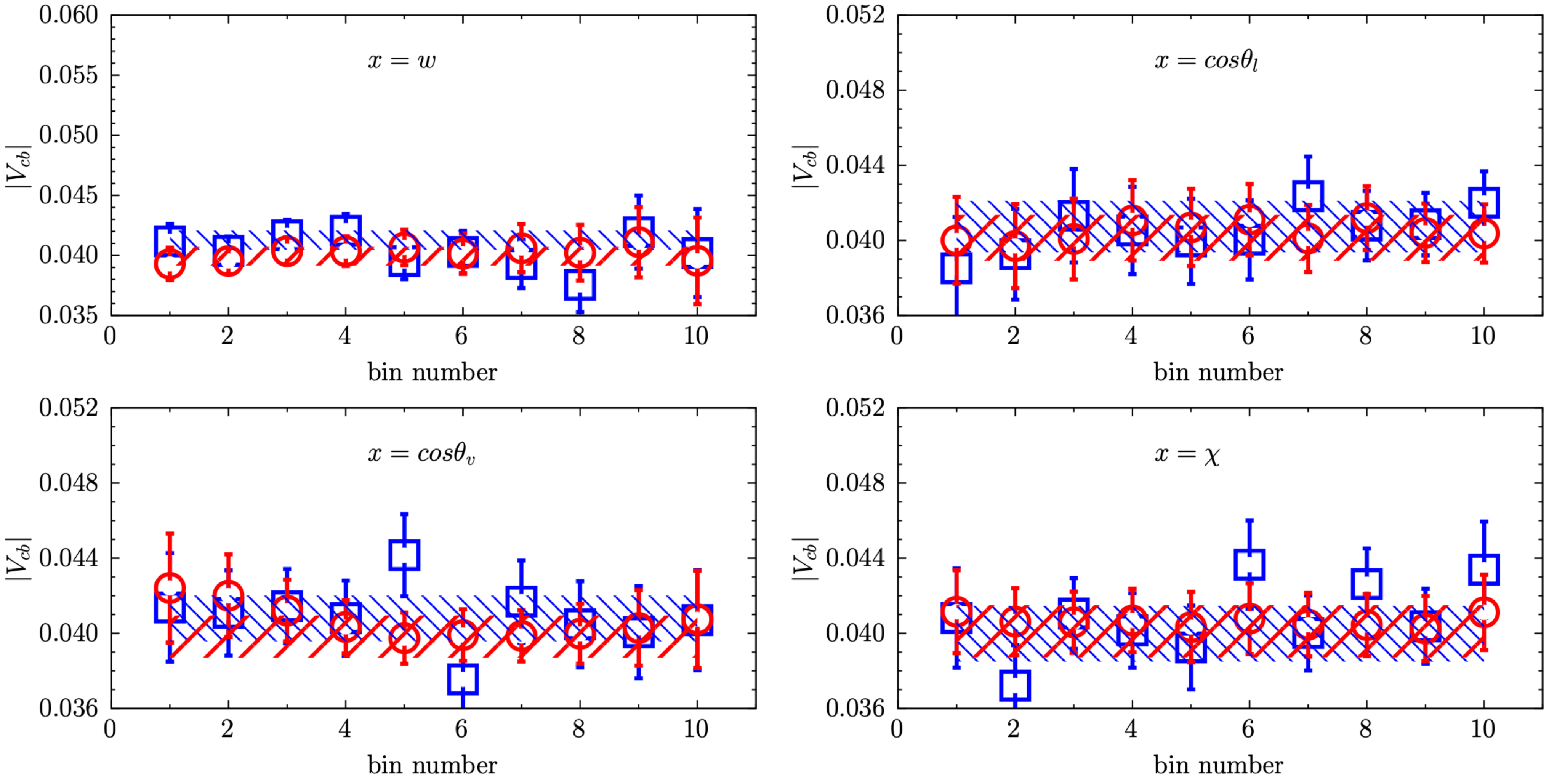

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Exclusive Vcb determination through unitarity

The averages of |Vcb| for each of the kinematic distributions are:

FNAL/MILC				
experiment	$\lvert V_{cb}\rvert_{x=w} \times 10^3$	$\lvert V_{cb}\rvert_{x=\cos\theta_l} \times 10^3$	$\lvert V_{cb}\rvert_{x=\cos\theta_v} \times 10^3$	$\lvert V_{cb}\rvert_{x=\chi} \times 10^3$
Belle 2018	39.50 (68)	40.9 (12)	39.98 (99)	42.8 (13)
$\chi^2/(\text{d.o.f.})$	1.21	1.36	1.99	0.38
Belle 2023	40.26 (72)	42.6 (13)	42.1 (12)	42.3 (14)
$\chi^2/(\text{d.o.f.})$	1.94	0.85	1.23	1.87
HPCQD				
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Belle 2018	40.06 (70)	41.5 (11)	40.82 (93)	43.5 (14)
$\chi^2/(\text{d.o.f.})$	1.33	1.15	1.37	0.40
Belle 2023	41.16 (71)	42.9 (13)	42.4 (11)	43.3 (15)
$\chi^2/(\text{d.o.f.})$	1.64	0.95	1.09	1.98
JLQCD				
experiment	$\lvert V_{cb}\rvert_{x=w} \times 10^3$	$\lvert V_{cb}\rvert_{x=\cos\theta_l} \times 10^3$	$\lvert V_{cb}\rvert_{x=\cos\theta_v} \times 10^3$	$\lvert V_{cb}\rvert_{x=\chi} \times 10^3$
Belle 2018	39.94 (77)	40.1 (12)	39.8 (11)	40.1 (14)
$\chi^2/(\text{d.o.f.})$	0.25	0.16	0.53	0.11
Belle 2023	41.28 (80)	40.7 (14)	40.8 (12)	40.0 (15)
$\chi^2/(\text{d.o.f.})$	1.87	0.52	0.65	1.72

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FROM TOTAL DECAY RATE:

$$|V_{cb}| = (43.3 \pm 1.6) \cdot 10^{-3}$$

$$|V_{cb}| = (44.6 \pm 1.7) \cdot 10^{-3}$$

$$|V_{cb}| = (40.2 \pm 1.6) \cdot 10^{-3}$$

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----

consistent with arXiv:2304.03137

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Exclusive Vcb determination through unitarity

To compute the final average of these Vcb estimates:

- **CORRELATED AVERAGE among the four values of |Vcb| at fixed lattice inputs and at fixed experiment:**

$\|V_{cb}\| \times 10^3$			
experiment	FNAL/MILC	HPCQD	JLQCD
Belle 2018	39.72 (64)	40.02 (63)	39.89 (76)
Belle 2023	40.41 (71)	41.22 (69)	41.24 (79)

$$|V_{cb}| = (40.55 \pm 0.54) \cdot 10^{-3}$$

(scaling factor à la PDG of 1.58)

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- **OUR AVERAGE [see EPJC '22 (2109.15248)] among all the values of |Vcb| of the Table of previous slide:**

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N}\sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k-\mu_x)^2$$

$$|V_{cb}| = (40.79 \pm 1.46) \cdot 10^{-3}$$

Exclusive Vcb determination through unitarity

Very similar mean values!!

$\lvert V_{cb}\rvert \times 10^3$			
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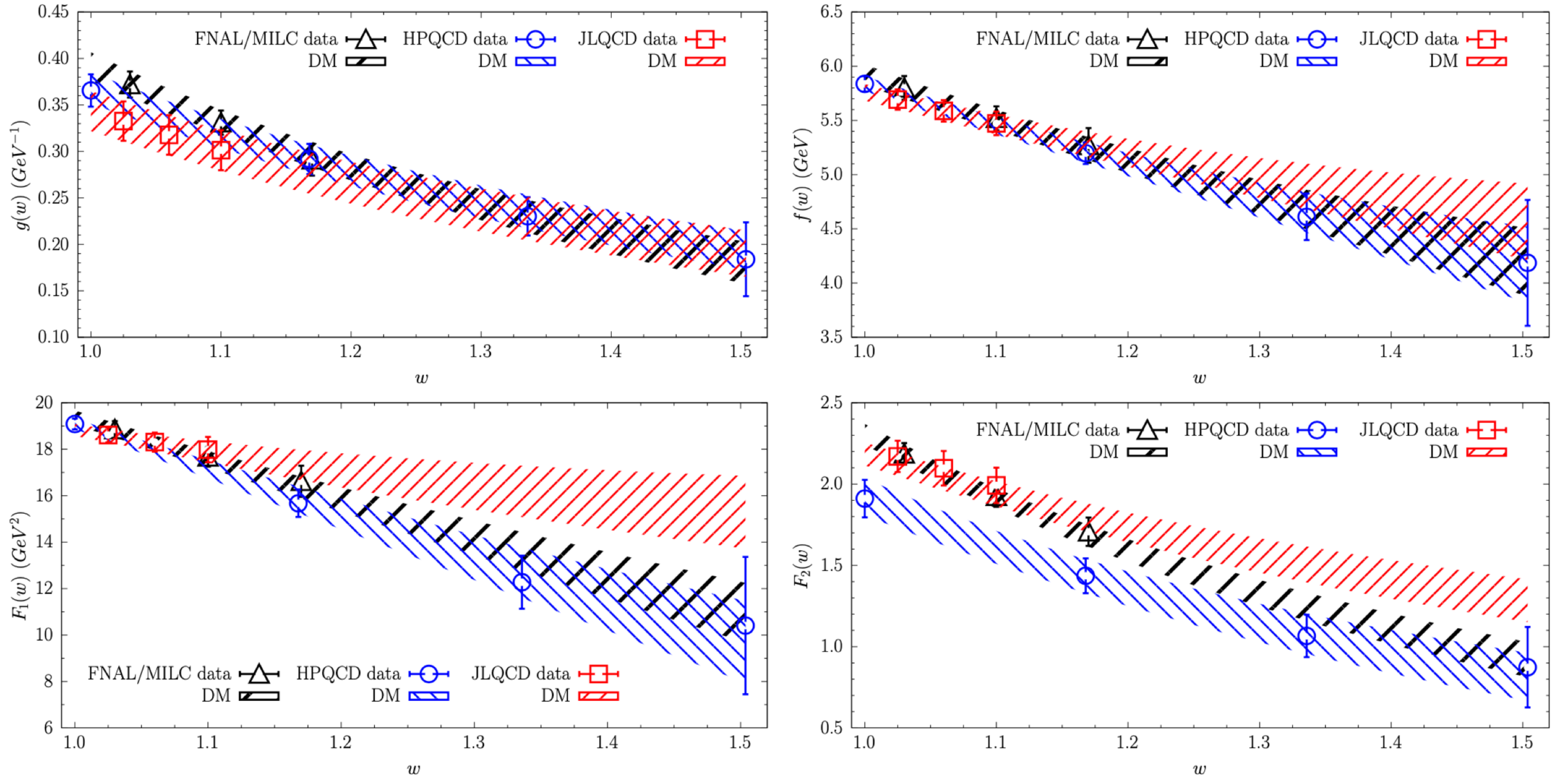
(scaling factor à la PDG of 1.58)

This |Vcb| result is in striking agreement with |Vcb| = (40.3 ± 0.5) × 10^{-3} recently obtained by I. Ray and S. Nandi, see ***arXiv:2305.11855 [hep-ph]***

Combined study of all the lattice data?

What about a combined study of FNAL/MILC + HPQCD + JLQCD lattice data? ***(The R(D*) values are the DM results!)***



FNAL/MILC:
EPJC '22
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$$R(D^*) = 0.275(8)$$

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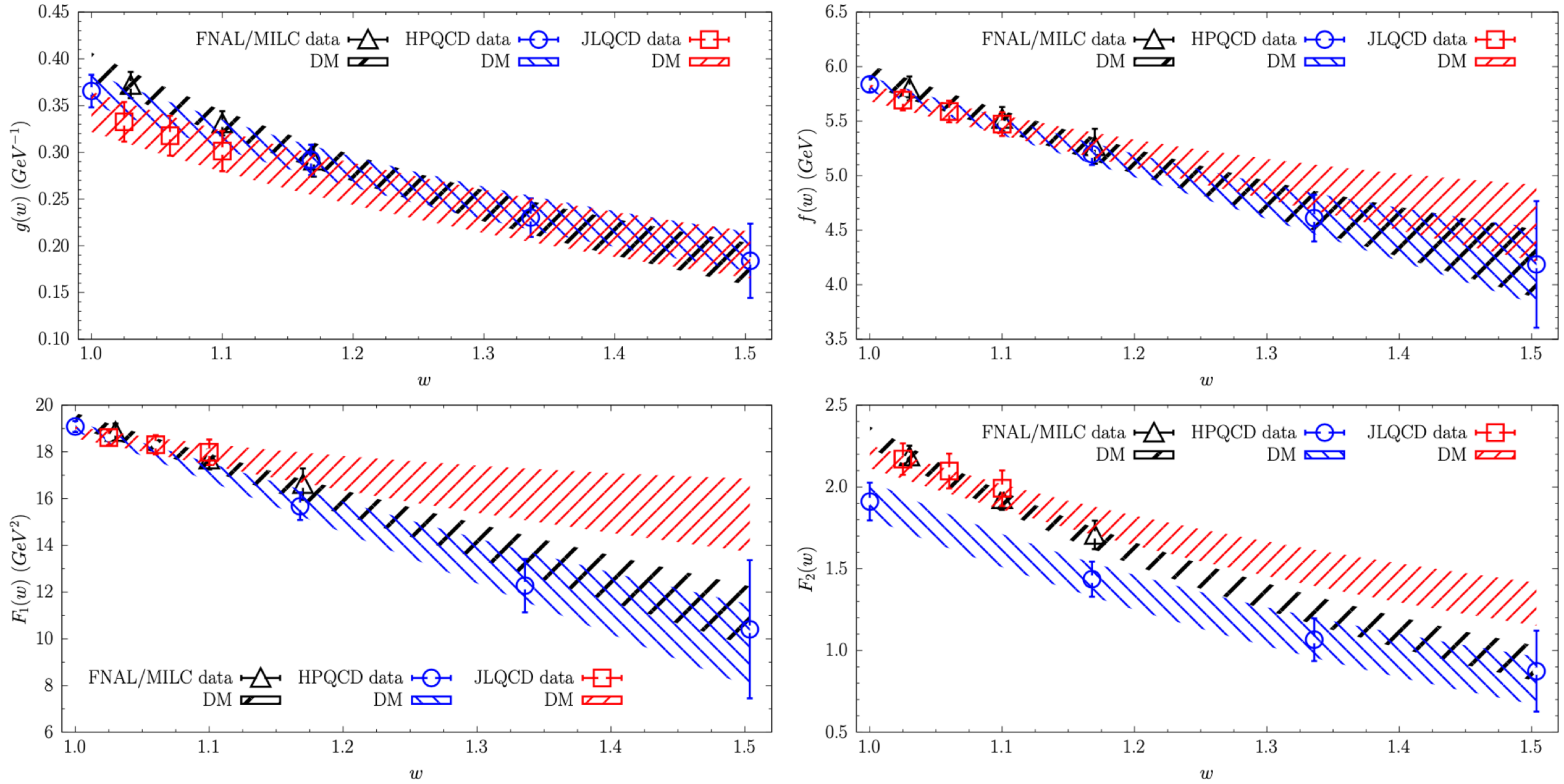

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Important differences on R(D*) !

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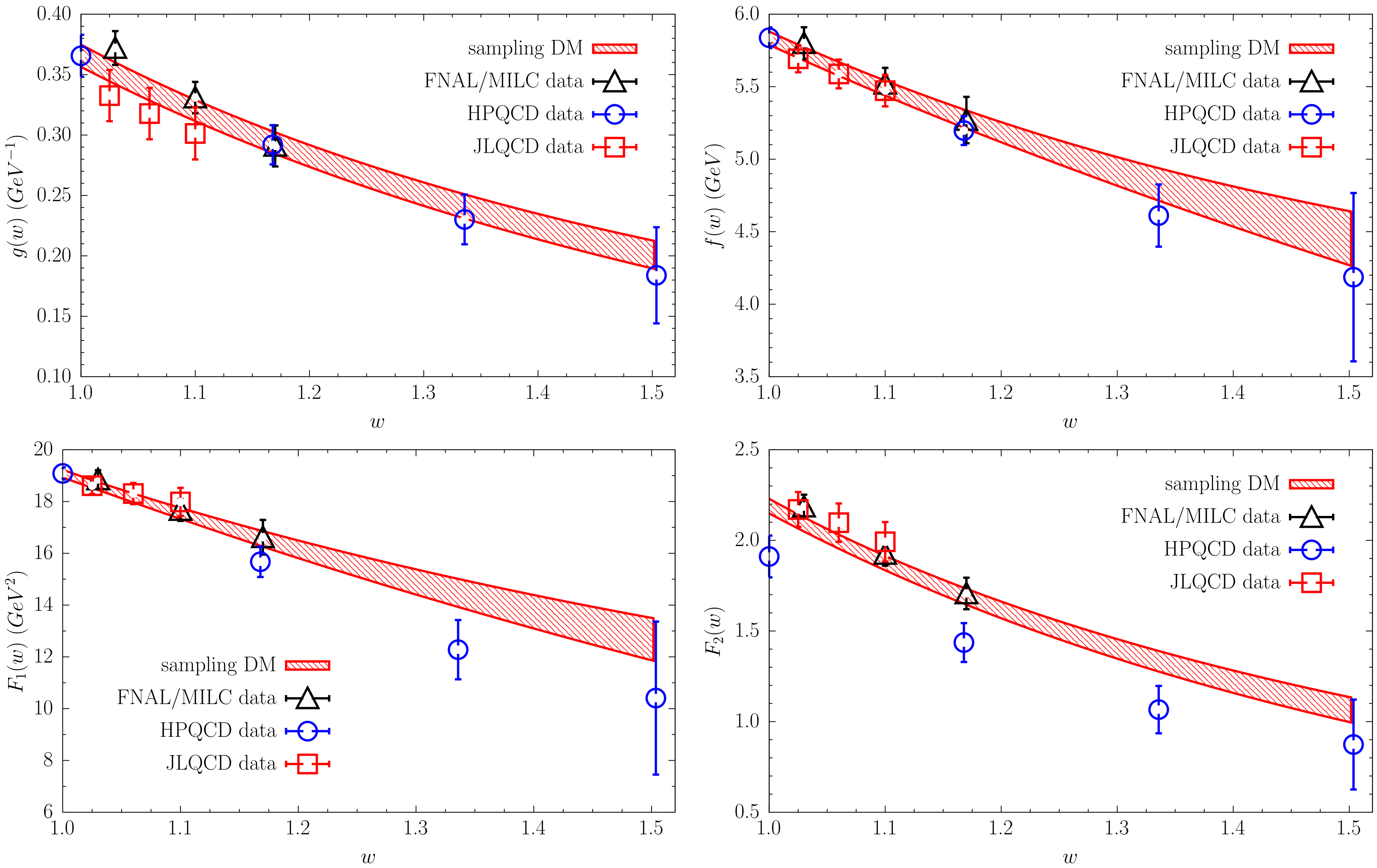

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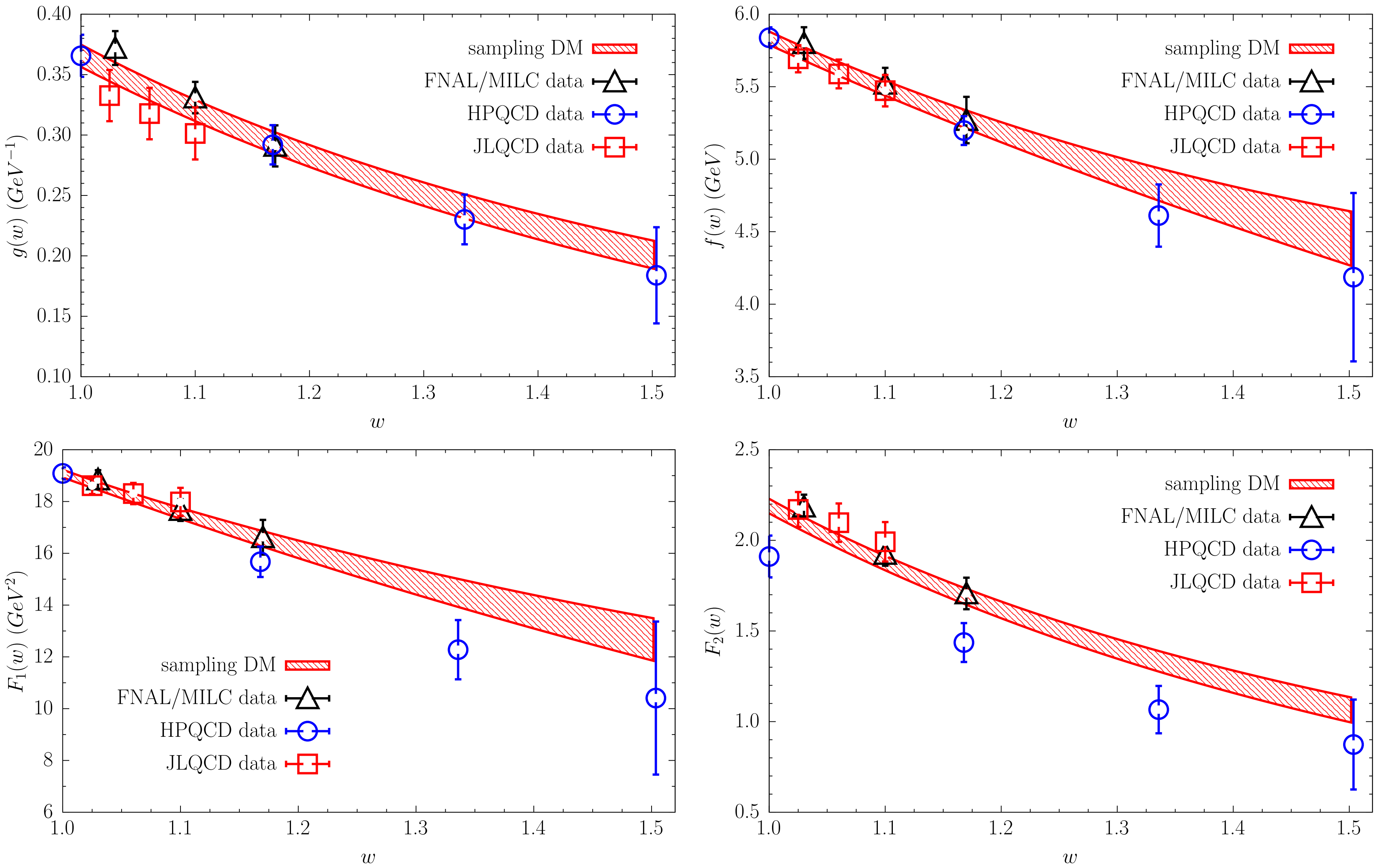

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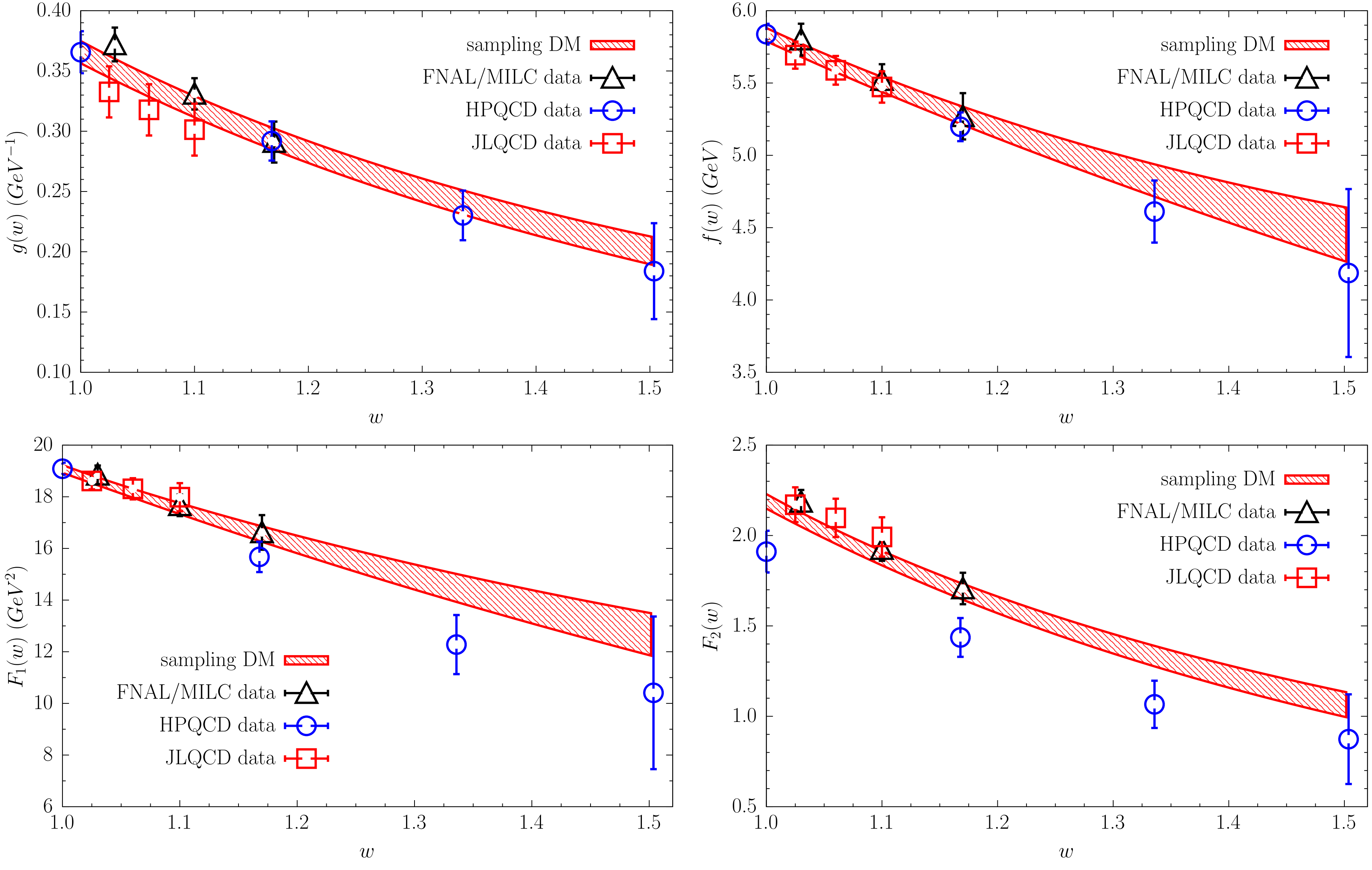

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Final number for R(D*):

$$R(D^*)_{IS} = 0.262(5)$$

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iv) Different ways to study all the available lattice data to have a «combined» answer for |Vcb| and R(D*)

Conclusions

Global fits of the Unitarity Triangle within the Standard Model. Updates from the UTfit collaboration.

Marcella Bona[1] Marco Ciuchini[2] Denis Derkach[3] Fabio Ferrari[4,5] Vittorio Lubicz[2,7] Guido Martinelli[6,8] Davide Morgante[9,10] Maurizio Pierini[11] Luca Silvestrini[6] Silvano Simula[2] Achille Stocchi[12] Cecilia Tarantino[2,7] Vincenzo Vagnoni[4] Mauro Valli[6] and Ludovico Vittorio[14]

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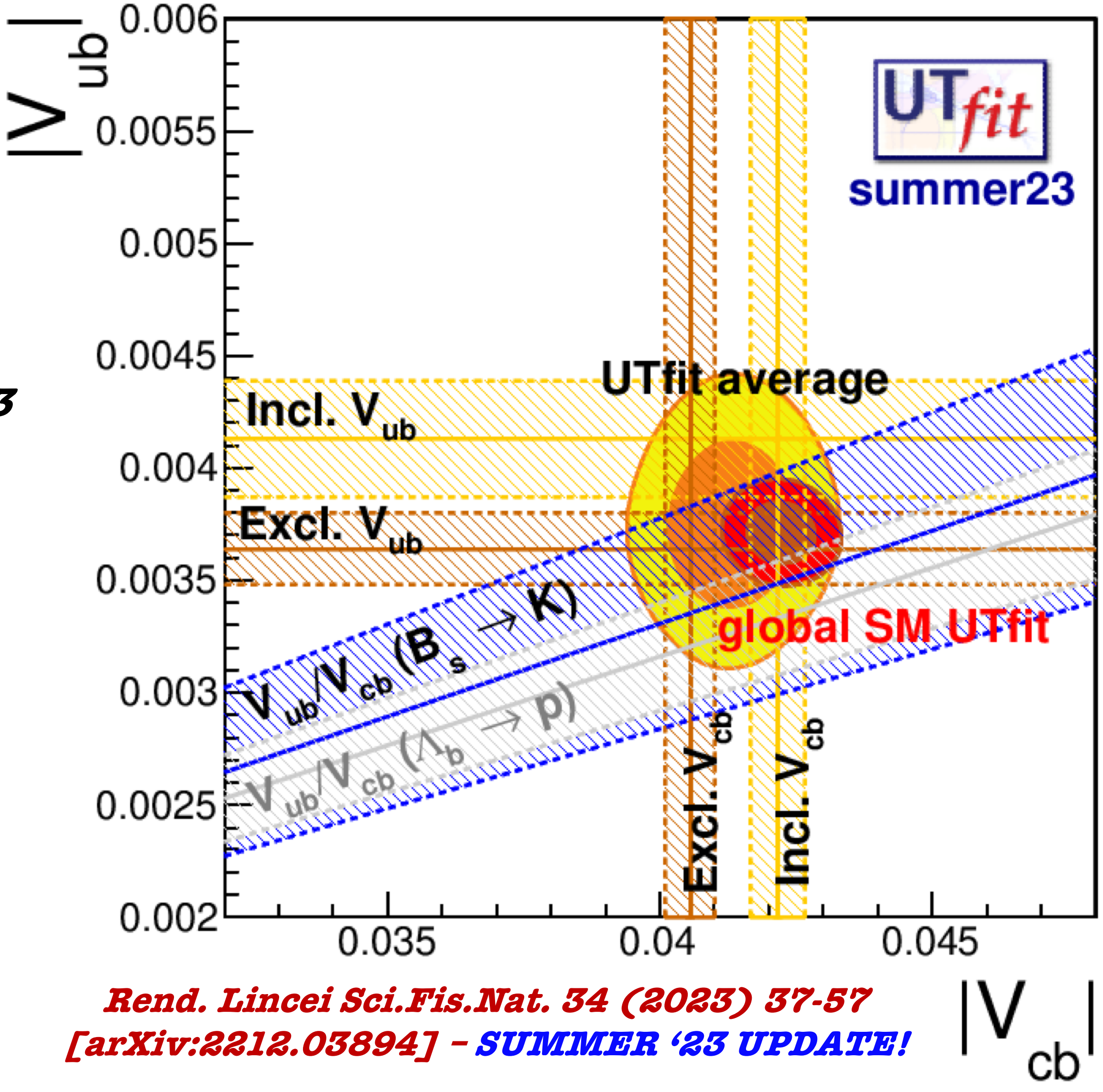

Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894] – SUMMER '23 UPDATE!

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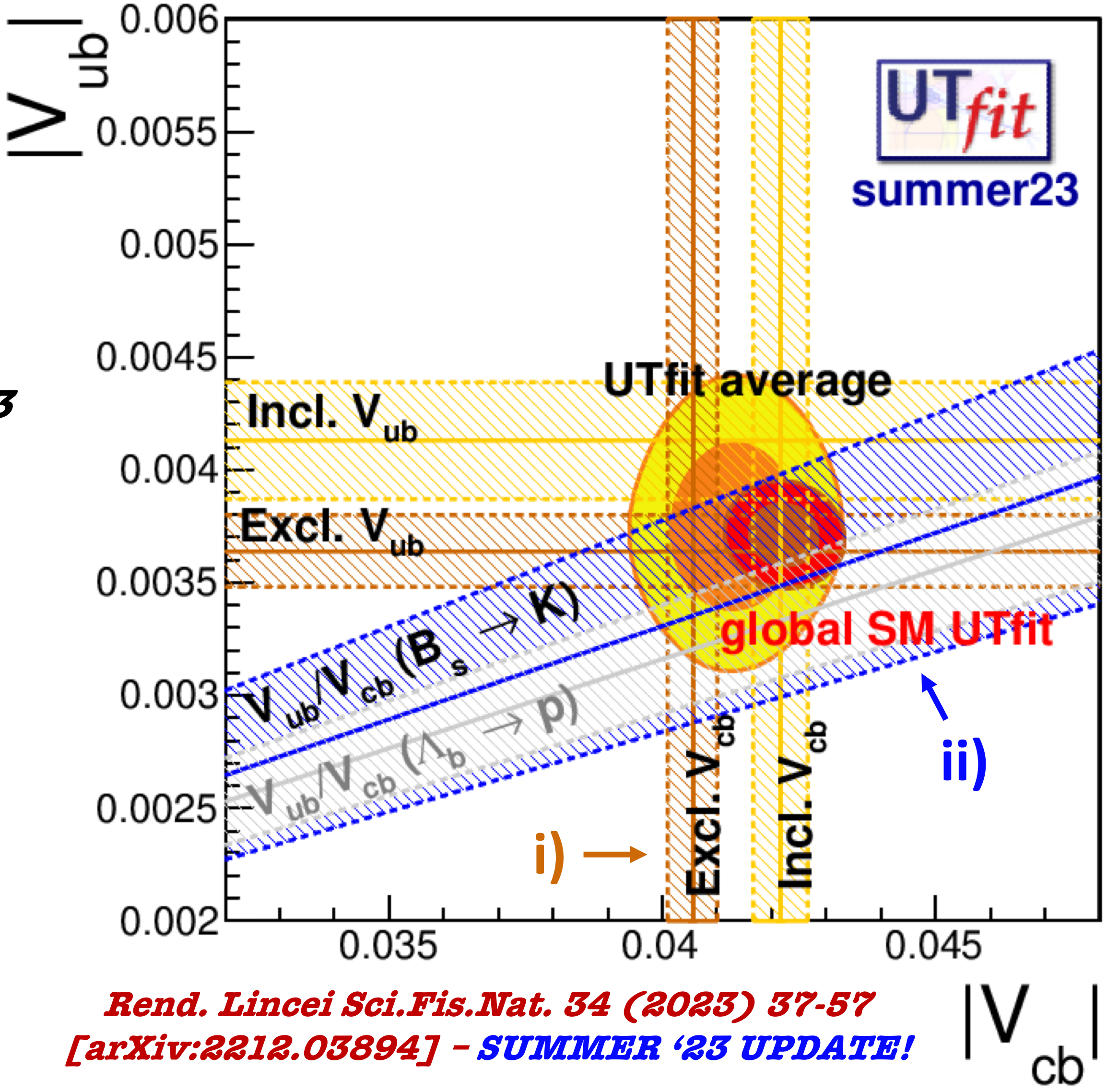

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«the unitarity triangle analysis, namely the analysis without including the experimental measurements from semileptonic decays, favours a large value of ***|Vcb|, close to the inclusive determination****, and a smaller value of* ***|Vub|, close to the exclusive determination»***

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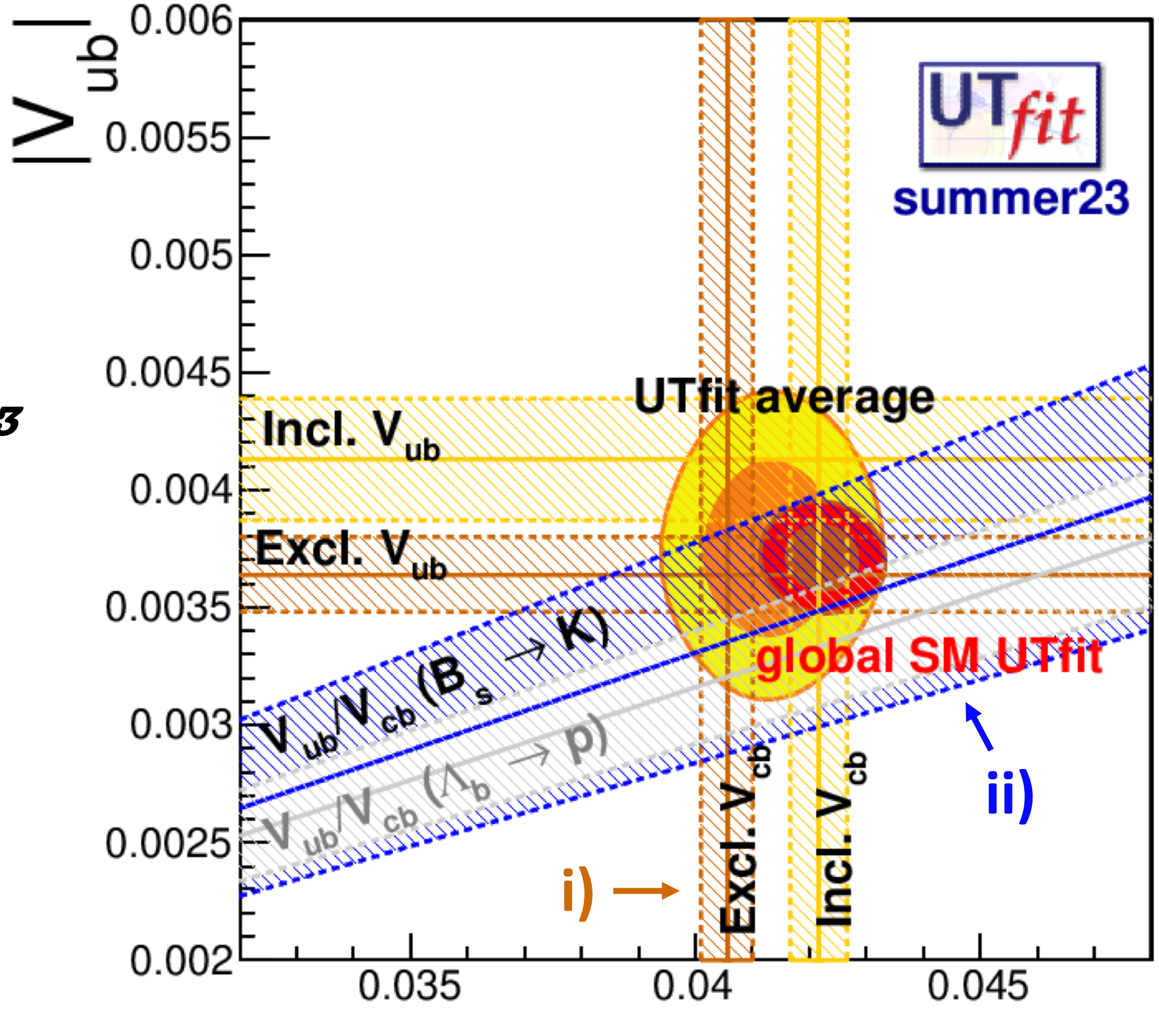

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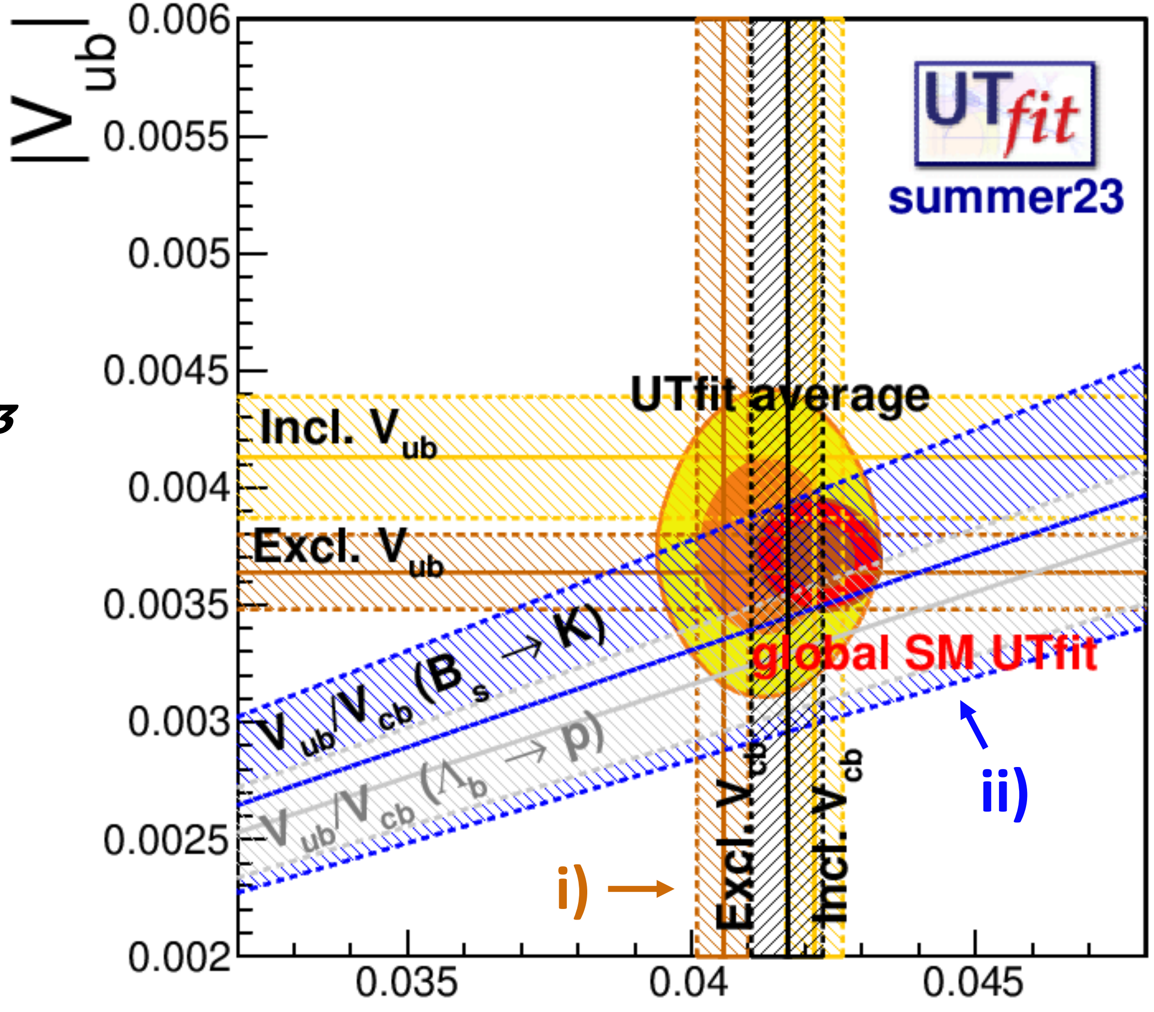


$|V_{cb}|$

Rend. Lincei Sci.Fis.Nat. 34 (2023) 37-57 [arXiv:2212.03894] – SUMMER '23 UPDATE!

THANKS FOR
YOUR ATTENTION!

BACK-UP SLIDES

Statistical and systematic uncertainties

How can we finally combine all the N_U lower and upper bounds of both the FFs??

One bootstrap event case:

after a single extraction, we have one value of the lower bound f_L and one value of the upper one f_U for each FF. Assuming that the true value of each FF can be **everywhere inside the range (f_U - f_L) with equal probability**, we associate to the FFs a *flat* distribution

$$P(f_{0(+)}) = \frac{1}{f_{U,0(+)} - f_{L,0(+)}} \Theta(f_{0(+)} - f_{L,0(+)}) \Theta(f_{U,0(+)} - f_{0(+)})$$

Many bootstrap events case:

how to mediate over the whole set of bootstrap events? Since the lower and the upper bounds of a generic FF are deeply correlated, we will assume a multivariate Gaussian distribution:

$$P(f_L, f_U) = \frac{\sqrt{\det \rho}}{2\pi} \exp\left[-\frac{\rho_{up,up}(f_U - \langle f_U \rangle)^2 + \rho_{lo,lo}(f_L - \langle f_L \rangle)^2 + 2\rho_{lo,up}(f_U - \langle f_U \rangle)(f_L - \langle f_L \rangle)}{2}\right]$$

In conclusion, we can combine the bounds of each FF in a final mean value and a final standard deviation, defined as

$$\langle f \rangle = \frac{\langle f_L \rangle + \langle f_U \rangle}{2},$$

$$\sigma_f = \frac{1}{12}(\langle f_U \rangle - \langle f_L \rangle)^2 + \frac{1}{3}(\sigma^2_{f_{lo}} + \sigma^2_{f_{up}} + \rho_{lo,up}\sigma_{f_{lo}}\sigma_{f_{up}})$$

NO PARAMETRIZATION ADOPTED!!!

Kinematical Constraints (KCs)

REMINDER: after the unitarity filter we were left with $N_U < N$ *survived events!!!*

Let us focus on the pseudoscalar case. Since by construction the following *kinematical constraint* holds

$$f_0(0) = f_+(0)$$

we will filter only the $N_{KC} < N_U$ events for which the two bands of the FFs intersect each other @ $t = 0$.
Namely, for each of these events we also define

$$\phi_{lo} = \max[F_{+,lo}(t=0), F_{0,lo}(t=0)]$$

$$\phi_{up} = \min[F_{+,up}(t=0), F_{0,up}(t=0)]$$

From WE theorem

$$\langle D(p_D)|\, V^\mu \,|B(p_B)\rangle \;=\; f_+(p_B+p_D)^\mu + f_-(p_B-p_D)^\mu$$

One then defines

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2-m_D^2} f_-(q^2)$$

$$\langle D(p_D)\,|\,V^\mu\,|\,B(p_B)\rangle = f^+(q^2)\left(p_B^\mu + p_D^\mu - \frac{m_B^2-m_D^2}{q^2}q^\mu\right) + f^0(q^2)\frac{m_B^2-m_D^2}{q^2}q^\mu$$

Kinematical Constraints (KCs)

We then consider a **modified matrix**

$$\mathbf{M_C} = \begin{pmatrix} \langle\phi f|\phi f\rangle & \langle\phi f|g_t\rangle & \langle\phi f|g_{t_1}\rangle & \cdots & \langle\phi f|g_{t_n}\rangle & \langle\phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix}$$

with $t_{n+1} = 0$. Hence, we compute the new lower and upper bounds of the FFs in this way. For each of the N_{KC} events, we extract $N_{KC,2}$ values of $f_0(0) = f_+(0) \equiv f(0)$ with uniform distribution defined in the range $[\phi_{lo}, \phi_{up}]$. Thus, for both the FFs and for each of the N_{KC} events we define

$$\begin{aligned} F_{lo}(t) &= \min[F_{lo}^1(t), F_{lo}^2(t), \cdots, F_{lo}^{N_{KC,2}}(t)], \\ F_{up}(t) &= \max[F_{up}^1(t), F_{up}^2(t), \cdots, F_{up}^{N_{KC,2}}(t)] \end{aligned}$$

Non-perturbative computation of the susceptibilities

In **PRD '21 [arXiv:2105.07851]**, we have presented the results of the first computation on the lattice of the susceptibilities for the $b \to c$ quark transition, using the N_f=2+1+1 gauge ensembles generated by ETM Collaboration.

How are they defined? The starting point is the HVP tensor:

$$\Pi^V_{\mu\nu}(Q) = \int d^4x \; e^{-iQ\cdot x} \langle 0 | T \left[\bar{b}(x) \gamma^E_\mu c(x) \; \bar{c}(0) \gamma^E_\nu b(0) \right] | 0 \rangle$$
$$= -Q_\mu Q_\nu \Pi_{0^+}(Q^2) + (\delta_{\mu\nu} Q^2 - Q_\mu Q_\nu) \Pi_{1^-}(Q^2)$$

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2} \left[Q^2 \Pi_{0^+}(Q^2) \right] = \int_0^\infty dt \; t^2 j_0(Qt) \; C_{0^+}(t) \;, \quad \xrightarrow{W.\,I.} \quad \frac{1}{4} \int_0^\infty dt' \; t'^4 \; \frac{j_1(Qt')}{Qt'} \left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t') \right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} \left[Q^2 \Pi_{1^-}(Q^2) \right] = \frac{1}{4} \int_0^\infty dt \; t^4 \frac{j_1(Qt)}{Qt} \; C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2} \left[Q^2 \Pi_{0^-}(Q^2) \right] = \int_0^\infty dt \; t^2 j_0(Qt) \; C_{0^-}(t) \;, \quad \xrightarrow{W.\,I.} \quad \frac{1}{4} \int_0^\infty dt' \; t'^4 \; \frac{j_1(Qt')}{Qt'} \left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t') \right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2} \frac{\partial^2}{\partial^2 Q^2} \left[Q^2 \Pi_{1^+}(Q^2) \right] = \frac{1}{4} \int_0^\infty dt \; t^4 \frac{j_1(Qt)}{Qt} \; C_{1^+}(t)$$

Non-perturbative computation of the susceptibilities

Let us choose for the moment zero Q^2:

$$\chi_{0^+}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^+}(t)\ ,$$

$$\chi_{1^-}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^-}(t)\ ,$$

$$\chi_{0^-}(Q^2=0) = \int_0^\infty dt\ t^2\ C_{0^-}(t)\ ,$$

$$\chi_{1^+}(Q^2=0) = \frac{1}{12}\int_0^\infty dt\ t^4\ C_{1^+}(t)\ .$$

$$\chi_{0^+}(Q^2=0) = \frac{1}{12}(m_b - m_c)^2\int_0^\infty dt\ t^4\ C_S(t)$$

$$\chi_{0^-}(Q^2=0) = \frac{1}{12}(m_b + m_c)^2\int_0^\infty dt\ t^4\ C_P(t)$$

$$C_{0^+}(t) = \boxed{\widetilde{Z}_V^2} \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0 q_2(x)\ \bar{q}_2(0)\gamma_0 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^-}(t) = \boxed{\widetilde{Z}_V^2}\, \frac{1}{3}\sum_{j=1}^{3} \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j q_2(x)\ \bar{q}_2(0)\gamma_j q_1(0)\right]|0\rangle\ ,$$

$$C_{0^-}(t) = \boxed{\widetilde{Z}_A^2} \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_0\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_0\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_{1^+}(t) = \boxed{\widetilde{Z}_A^2}\, \frac{1}{3}\sum_{j=1}^{3} \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_j\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_j\gamma_5 q_1(0)\right]|0\rangle\ ,$$

$$C_S(t) = \boxed{\widetilde{Z}_S^2} \int d^3x \langle 0|T\left[\bar{q}_1(x) q_2(x)\ \bar{q}_2(0) q_1(0)\right]|0\rangle\ ,$$

$$C_P(t) = \boxed{\widetilde{Z}_P^2} \int d^3x \langle 0|T\left[\bar{q}_1(x)\gamma_5 q_2(x)\ \bar{q}_2(0)\gamma_5 q_1(0)\right]|0\rangle\ ,$$

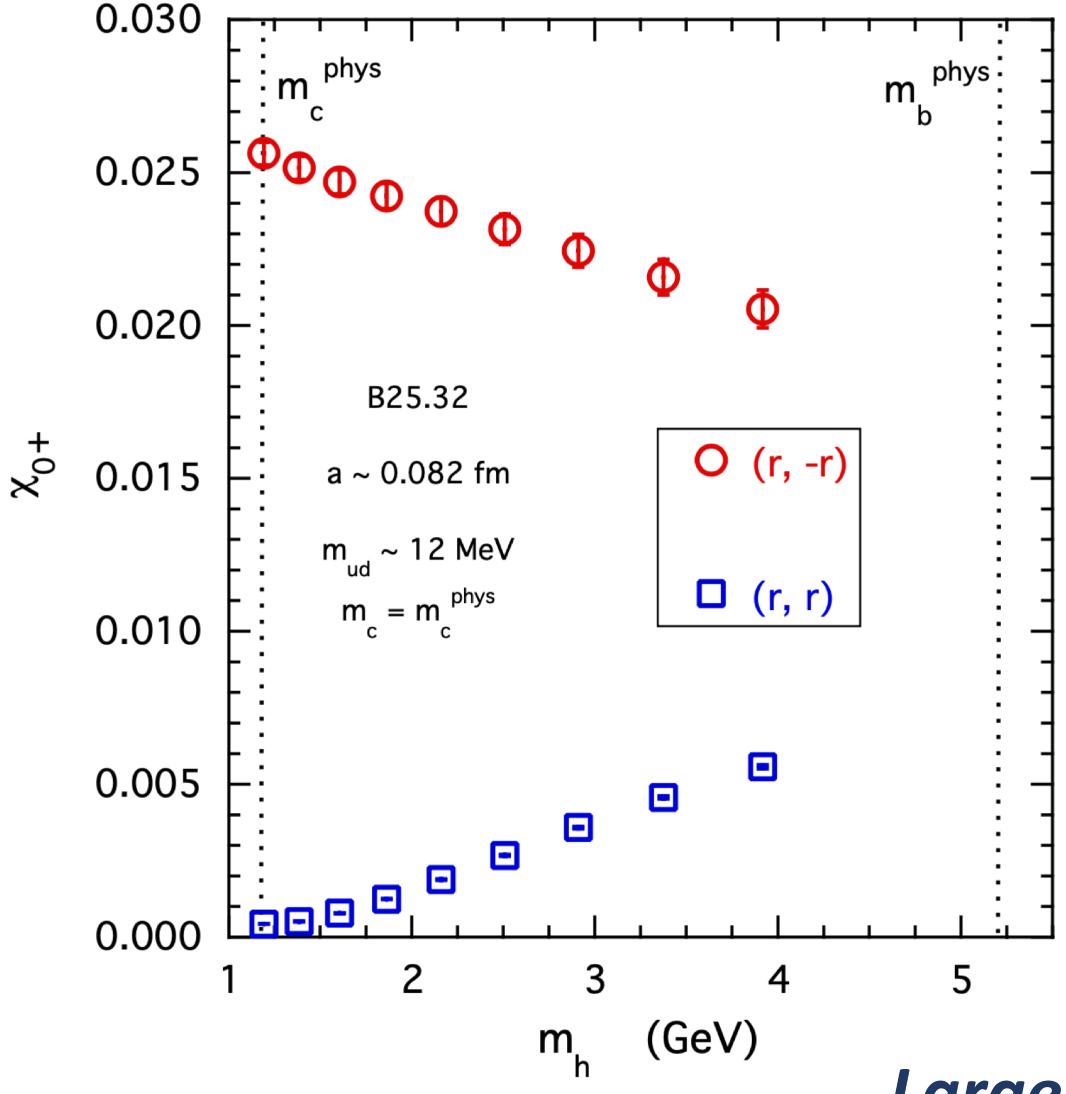
We are working in twisted mass LQCD: the Wilson parameter *r* can be equal or opposite for the two quarks in the currents

➡ Two possible **independent** combinations of *(r_1,r_2)!*

Z: appropriate renormalization constants

N. Carrasco et al. [ETM Coll.], NPB 887 (2014) [arXiv:1403.4504]

Non-perturbative computation of the susceptibilities



Following set of masses:

$$m_h(n) = \lambda^{n-1}\, m_c^{phys} \qquad \text{for } n = 1, 2, \ldots$$

$$m_h = a\mu_h/(Z_P a)$$

$$\lambda \equiv [m_b^{phys}/m_c^{phys}]^{1/10} = [5.198/1.176]^{1/10} \simeq 1.1602$$

Nine masses values!

$$m_h(1) = m_c^{phys}$$

$$m_h(9) \simeq 3.9 \text{ GeV} \simeq 0.75\, m_b^{phys}$$

***r*: Wilson parameter**

Large discretisation effects and contact terms

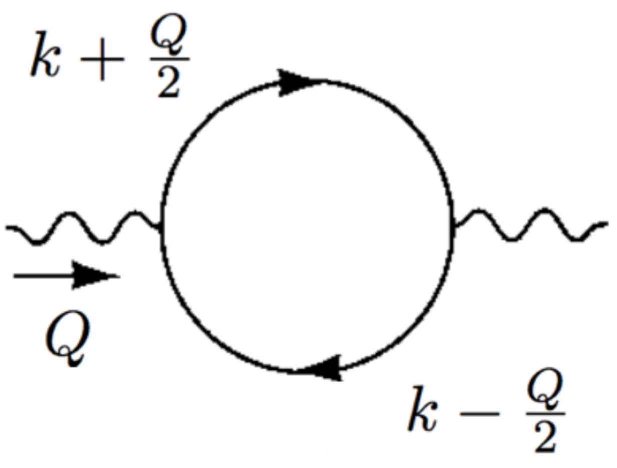
Contact terms & perturbative subtraction

In **twisted mass LQCD:**

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \,\mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i r_i \mu_{q,i}\gamma_5}{\mathring{p}_\mu^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}$$

$$\mathring{p}_\mu \equiv \frac{1}{a}\sin(ap_\mu), \qquad \mathcal{M}_i(p) \equiv m_i + \frac{r_i}{2}a\hat{p}_\mu^2, \qquad \hat{p} \equiv \frac{2}{a}\sin\left(\frac{ap_\mu}{2}\right).$$



$$\begin{aligned}\Pi_V^{\alpha\beta} &= a^{-2}(Z_1^I + (r_1^2 - r_2^2)Z_2^I + (r_1^2-r_2^2)(r_1^2+r_2^2)Z_3^I)g^{\alpha\beta} \\ &+ (\mu_1^2 Z^{\mu_1^2} + \mu_2^2 Z^{\mu_2^2} + \mu_1\mu_2 Z^{\mu_1\mu_2})g^{\alpha\beta} + (Z_1^{Q^2} + (r_1^2-r_2^2)Z_2^{Q^2})Q\cdot Q g^{\alpha\beta} \\ &+ (Z_1^{Q^\alpha Q^\beta} + (r_1^2-r_2^2)Z_2^{Q^\alpha Q^\beta})Q^\alpha Q^\beta + r_1 r_2(a^{-2}Z_1^{r_1r_2}g^{\alpha\beta} + (Z_2^{r_1r_2} + (r_1^2+r_2^2)Z_3^{r_1r_2} \\ &+ (r_1^4+r_2^4)Z_4^{r_1r_2})Q\cdot Q g^{\alpha\beta} + (\mu_1^2 Z_5^{r_1r_2} + \mu_2^2 Z_6^{r_1r_2})g^{\alpha\beta}) + O(a^2),\end{aligned}$$

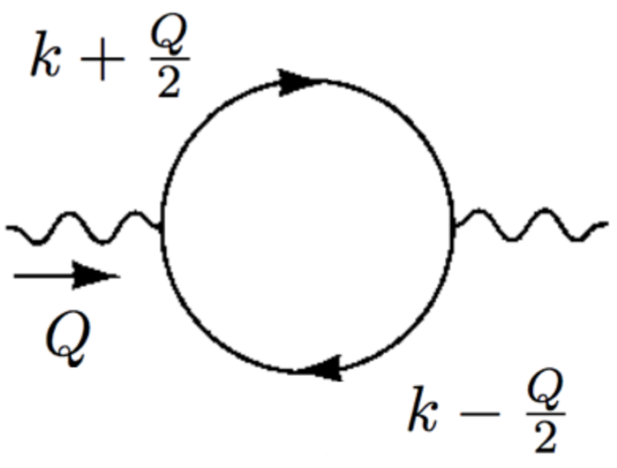
CONTACT TERMS!!!

F. Burger et al., ETM Coll., JHEP '15 [arXiv:1412.0546]

Contact terms & perturbative subtraction

In **twisted mass LQCD**:

$$\Pi_V^{\alpha\beta} = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4} \, \mathrm{Tr}\left[\gamma^\alpha G_1(k+\frac{Q}{2})\gamma^\beta G_2(k-\frac{Q}{2})\right],$$



Thus, by separating the *longitudinal* and the *transverse* contributions, we can compute the susceptibilities for all the spin-parity quantum numbers **in the free theory on the lattice**, *i.e.* at order $\mathcal{O}(\alpha_s^0)$ using twisted-mass fermions!

$$\chi_j^{free} = \chi_j^{LO} + \chi_j^{discr}$$

LO term of PT @ $\mathcal{O}(\alpha_s^0)$ contact terms and discretization effects @ $\mathcal{O}(\alpha_s^0 a^m)$ with $m \geq 0$

Perturbative subtraction:

$$\chi_j \to \chi_j - \left[\chi_j^{free} - \chi_j^{LO}\right]$$

ETMC ratio method & final results

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \boxed{\frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}}$$

to ensure that $\lim_{n\to\infty} R_j(n) = 1$

$$\boxed{\begin{aligned} \rho_{0^+}(m_h) &= \rho_{0^-}(m_h) = 1 \ , \\ \rho_{1^-}(m_h) &= \rho_{1^+}(m_h) = (m_h^{pole})^2 \end{aligned}}$$

All the details are deeply discussed in **PRD ’21 [2105.07851]**. In this way, we have obtained the first lattice QCD determination of susceptibilities of heavy-to-heavy (and heavy-to-light, see **JHEP ‘22 [2202.10285]**) transition current densities:

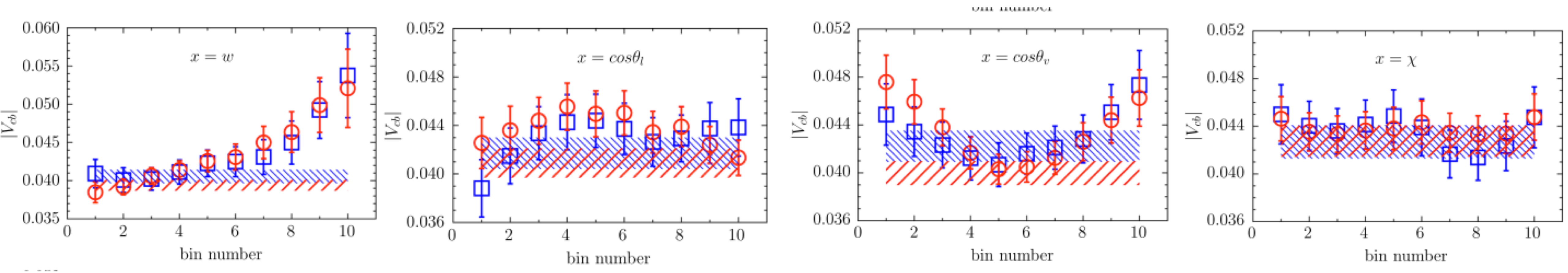
b → c

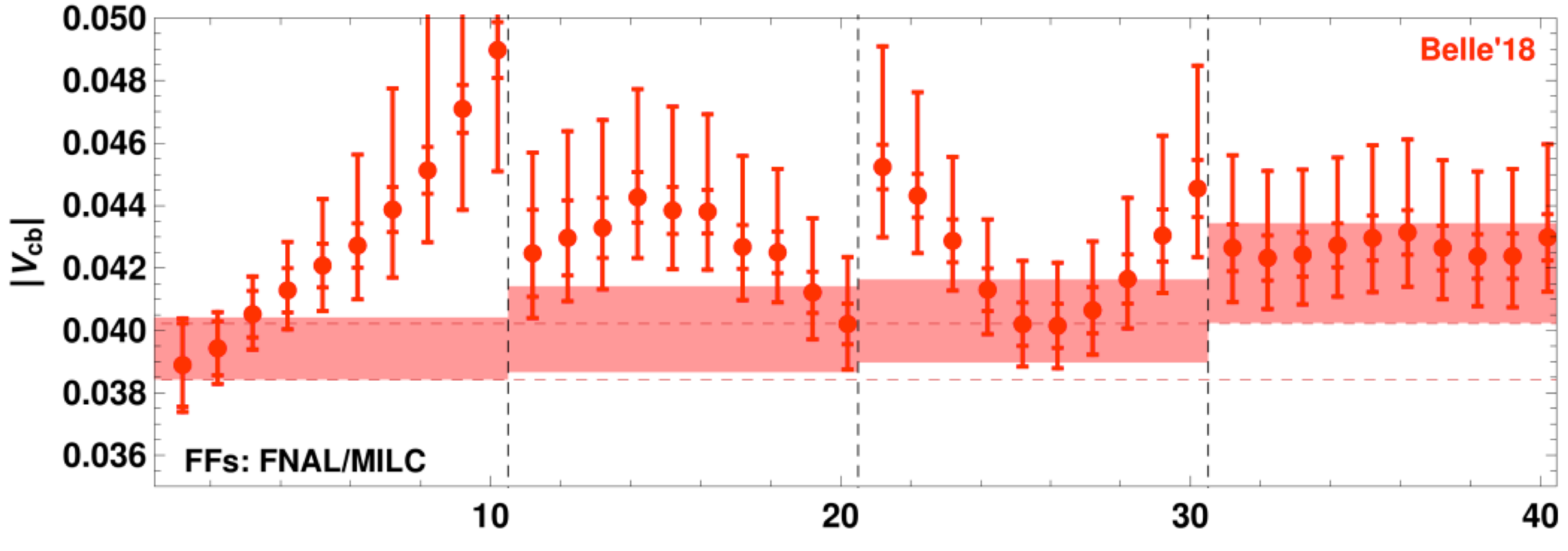
	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}\,[10^{-3}]$	6.204(81)	—	7.58(59)	—
$\chi_{A_L}\,[10^{-3}]$	24.1	19.4	25.8(1.7)	21.9(1.9)
$\chi_{V_T}\,[10^{-4}\text{ GeV}^{-2}]$	6.486(48)	5.131(48)	6.72(41)	5.88(44)
$\chi_{A_T}\,[10^{-4}\text{ GeV}^{-2}]$	3.894	—	4.69(30)	—

Differences with PT? **~4% for 1^-, ~7% for 0^-, ~20 % for 0^+ and 1^+**

Critical understanding of the results obtained so far

1. Does the DM method modify the mean values/the correlations of the FFs?





Jung's talk @ LHCb Implications Workshop 2022 (CERN)

Critical understanding of the results obtained so far

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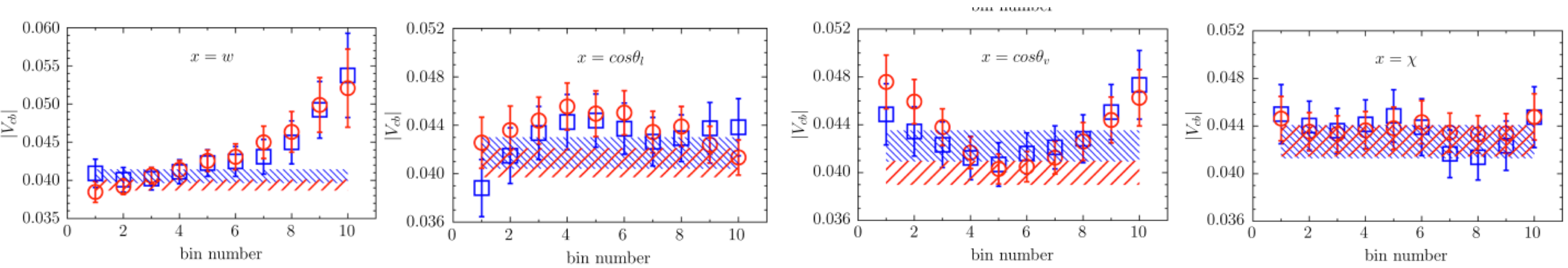
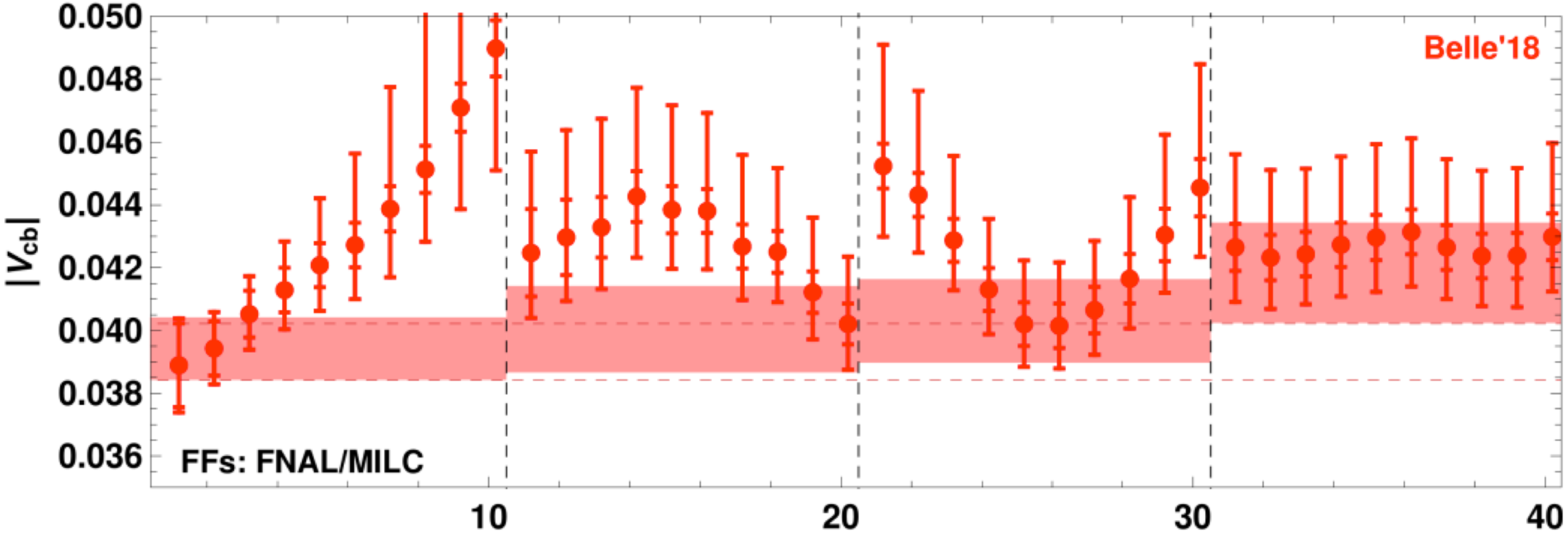


i) Same bin-per-bin values of Vcb

ii) Same mean values for each kinematical variable!



Jung's talk @ LHCb Implications Workshop 2022 (CERN)

Basics of IS DM

The basic idea is a substitution of the usual probability density function (PDF) adopted in our analyses:

$$PDF(f_i) \propto e^{-\frac{1}{2}\sum_{i,j=0}^{N}(f_i - F_i)C_{ij}^{-1}(f_j - F_j)}$$

All the details are contained in **arXiv: 2309.02135**

$$PDF_{DM}(f_i) \propto PDF(f_i) \cdot e^{-\frac{s}{\chi_T(\overline{Q}_0^2)}\chi_{DM}(\overline{Q}_0^2)}$$

<u>In short</u>: a new set of input data $\{\widetilde{F}_i, \widetilde{C}_{ij}\}$ is introduced in order to increase the likelihood of small values of χDM !

$$\beta - \sqrt{\gamma} \leq f(z) \leq \beta + \sqrt{\gamma}$$

χDM

$$\beta = \frac{1}{d(z)\phi(z)}\sum_{j=1}^{N} f_j \phi_j d_j \frac{1 - z_j^2}{z - z_f} \qquad \gamma = \frac{1}{d^2(z)\phi^2(z)}\frac{1}{1 - z^2}\left[\chi - \sum_{i,j=1}^{N} f_i f_j \phi_i \phi_j d_i d_j \frac{(1 - z_i^2)(1 - z_j^2)}{1 - z_i z_j}\right]$$

Relevant quantities for monitoring the results of IS DM

Recall that the **DM** remains a **fitting procedure with a vanishing value of the χ2-variable in a frequentist language**!
Then, we have to monitorate the deviation of the new input data from the initial ones thorugh the quantities

$$\Delta \equiv \left\{ \frac{1}{N+1} \sum_{i,j=0}^{N} (\widetilde{F}_i - F_i) C_{ij}^{-1} (\widetilde{F}_j - F_j) \right\}^{1/2}$$

Δ < 1 means that on average the new data deviate from the original ones by less than one standard deviation

$$\eta \equiv \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{F}_i^2}{F_i^2} \right\}^{1/2}$$

The value of η can be less or larger than unity depending on whether the new data are (on average) less or larger than original ones

$$\epsilon \equiv \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{C}_{ii}}{C_{ii}} \right\}^{1/2} = \left\{ \frac{1}{N+1} \sum_{i=0}^{N} \frac{\widetilde{\sigma}_i^2}{\sigma_i^2} \right\}^{1/2}$$

Same physical meaning of η, but now referred to the uncertaintities of the new data in comparison to the original ones

A counter-check of the IS DM results