

New physics contributions to moments of inclusive $b \rightarrow c$ decays

Matteo Fael (CERN)

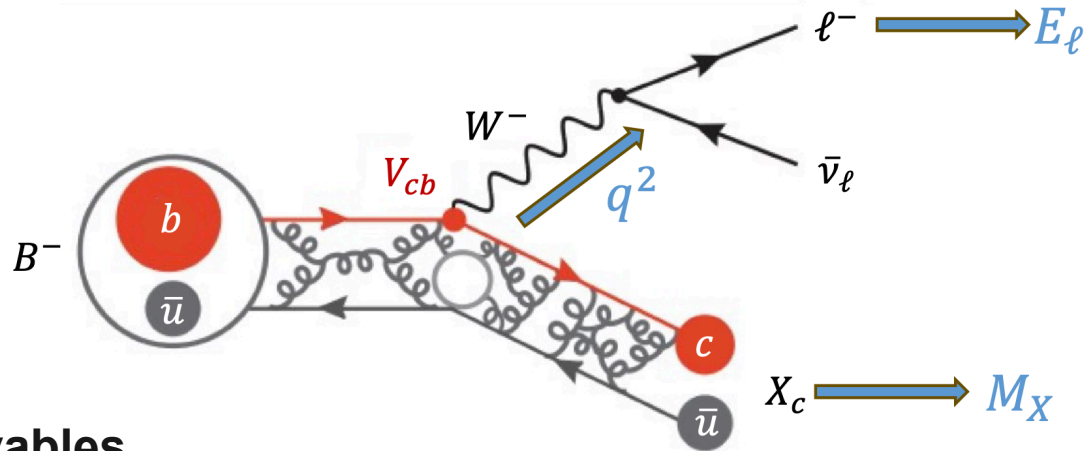
CKM 2023 - Santiago de Compostela - 20 Sept. 2023

in collaboration with M. Rahimi, K. Vos



Funded by
the European Union

Extraction of V_{cb} from inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$ decays



Observables

- Total rate $\Gamma_{sl} = \Gamma(B \rightarrow X_c \ell \bar{\nu}_\ell)$
- Moments of the differential distribution of an observables O

$$\langle (O)^n \rangle_{\text{cut}} = \frac{\int_{\text{cut}} (O)^n \frac{d\Gamma}{dO} dO}{\int_{\text{cut}} \frac{d\Gamma}{dO} dO}$$

- $O = E_\ell$: energy of the charged lepton in the B rest frame
- $O = M_X^2$: hadronic invariant mass
- $O = q^2$: leptonic invariant mass

see talks by M. Prim & K. Vos

Heavy Quark Expansion

Double series expansion in the **strong coupling constant** α_s and **power suppressed terms** Λ_{QCD}/m_b

- Total rate

$$\Gamma_{\text{sl}} = \frac{G_F^2 m_b^5 A_{\text{ew}} |V_{cb}|^2}{192\pi^3} \left[\left(1 - \frac{\mu_\pi^2}{2m_b^2} \right) \left(X_0(\rho) + \frac{\alpha_s}{\pi} X_1(\rho) + \left(\frac{\alpha_s}{\pi} \right)^2 X_2(\rho) + \left(\frac{\alpha_s}{\pi} \right)^3 X_3(\rho) + \dots \right) \right. \\ \left. + \left(\frac{\mu_G^2}{m_b^2} - \frac{\rho_D^3}{m_b^3} \right) \left(g_0(\rho) + \frac{\alpha_s}{\pi} g_1(\rho) + \dots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0(\rho) + \frac{\alpha_s}{\pi} d_1(\rho) + \dots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

- Moments of differential distribution

$$\langle O^n \rangle_{\text{cut}} = (m_b)^{mn} \left[X_0^{(O,n)} + \frac{\alpha_s}{\pi} X_1^{(O,n)} + \left(\frac{\alpha_s}{\pi} \right)^2 X_2^{(O,n)} + \frac{\mu_\pi^2}{m_b^2} \left(p_0^{(O,n)} + \frac{\alpha_s}{\pi} p_1^{(O,n)} + \dots \right) \right. \\ \left. + \frac{\mu_G^2}{m_b^2} \left(g_0^{(O,n)} + \frac{\alpha_s}{\pi} g_1^{(O,n)} + \dots \right) + \frac{\rho_D^3}{m_b^3} \left(d_0^{(O,n)} + \frac{\alpha_s}{\pi} d_1^{(O,n)} + \dots \right) + \frac{\rho_{LS}}{m_b^2} \left(l_0^{(O,n)} + \frac{\alpha_s}{\pi} l_1^{(O,n)} + \dots \right) + O\left(\frac{1}{m_b^4}\right) \right]$$

Global fits

$$\text{Br}(E_{\text{cut}}) \quad \langle E^n \rangle_{E_{\text{cut}}} \quad \langle (M_X^2)^n \rangle_{E_{\text{cut}}} \quad \langle (q^2)^n \rangle_{q_{\text{cut}}^2}$$

$$\mu_\pi, \mu_G, \rho_D, \rho_{LS}, m_b, (m_c)$$

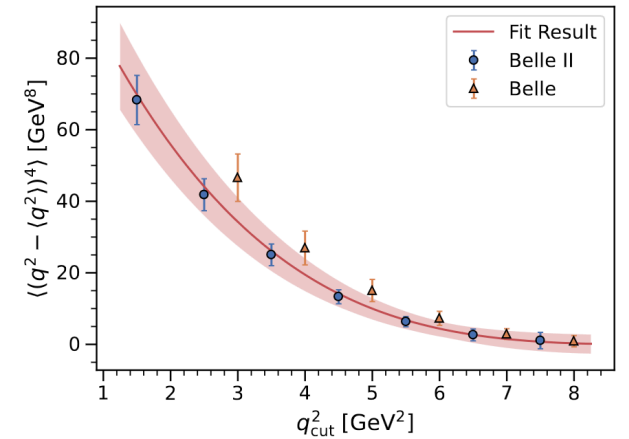
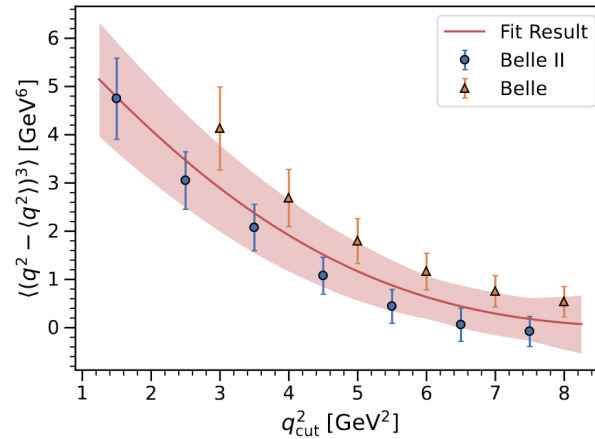
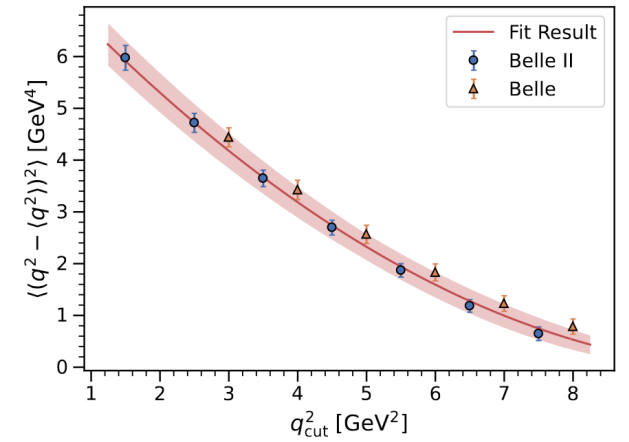
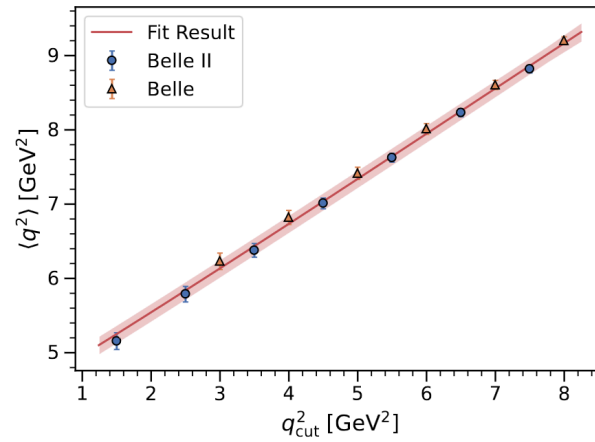
$$\text{Br}(\bar{B} \rightarrow X_c l \bar{\nu}) \propto \frac{|V_{cb}|^2}{\tau_B} \left[\Gamma_0 + \Gamma_{\mu_\pi} \frac{\mu_\pi^2}{m_b^2} + \Gamma_{\mu_G} \frac{\mu_G^2}{m_b^2} + \Gamma_{\rho_D} \frac{\rho_D^3}{m_b^3} \right]$$

$$|V_{cb}| = (42.16 \pm 0.51) \times 10^{-3}$$

Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

$$|V_{cb}|^{q^2} = (41.69 \pm 0.63) \times 10^{-3}$$

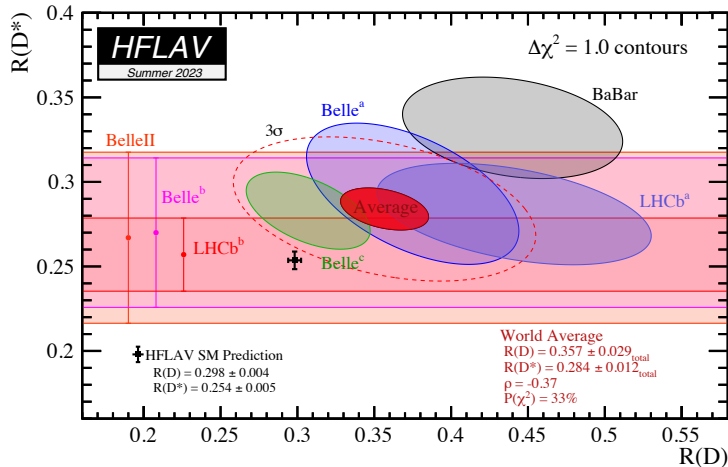
Bernlochner, MF, Olschewsky, Persson
van Tonder, Vos, Welsch JHEP 10 (2022) 068



Bernlochner, et al, JHEP 10 (2022) 068

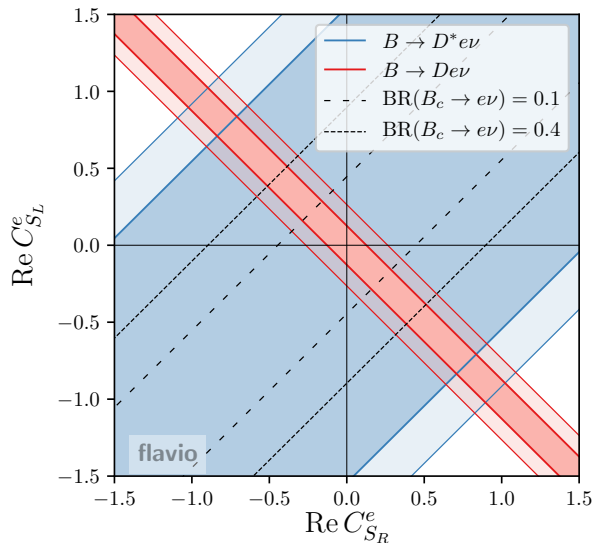
- HQE parameters extracted from **global fits**
- Experimental data from CLEO, CDF, DELPHI, BaBar, Belle, Belle II
- **Extraction is perform within the SM**

Motivation



- Tree-level processes in principle affected by NP
- Long-standing **tensions between excl. and incl.**
- Significant deviations from SM in $b \rightarrow c\tau\bar{\nu}_\tau$
- Fit of exclusive $b \rightarrow c\ell\bar{\nu}_\ell$ decays.

Jung, Straub, *JHEP* 01 (2019) 009



Jung, Straub, *JHEP* 01 (2019) 009

- Precision measurements of the moments by B -factories
- Ongoing studies inclusive $b \rightarrow c\ell\bar{\nu}_\ell$ on lattice

Barone, Hashimoto, Jüttner, Kanedo, Kellermann, *JHEP* (2023) 145
 Hashimoto, Gambino, *Phys.Rev.Lett.* 125 (2020) 032001 and hep-lat/2203.11762
 Gambino, Melis, Simula, *Phys.Rev.D* 96 (2017) 014511
 Hansen, Meyer, Robaina, *Phys.Rev.D* 96 (2017) 9, 094513

see talk by R. Kellermann

GOAL: comprehensive model-independent analysis of all possible types of NP effects in $B \rightarrow X_c \ell \bar{\nu}_\ell$

NP contributions

Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left[\left(1 + C_{V_L}\right) O_{V_L} + \sum_{i=V_R, S_L, S_R, T} C_i O_i \right]$$

Dimension-six operators

$$O_{V_{L(R)}} = \left(\bar{c} \gamma_\mu P_{L(R)} b \right) \left(\bar{\ell} \gamma^\mu P_L \nu_\ell \right)$$

$$O_{S_{L(R)}} = \left(\bar{c} P_{L(R)} b \right) \left(\bar{\ell} P_L \nu_\ell \right)$$

$$O_T = \left(\bar{c} \sigma_{\mu\nu} P_L b \right) \left(\bar{\ell} \sigma^{\mu\nu} P_L \nu_\ell \right)$$

- In the SM all $C_i = 0$
- In the **Weak Effective Theory** the expansion parameter is $1/v^2$, i.e. Wilson coefficients are $O(1)$
- NP effects from SMEFT are suppressed by $1/\Lambda_{\text{NP}}^2$. The matching to WET lead to a suppression $(v/\Lambda_{\text{NP}})^2$

Aebischer, Crivellin, MF, Greub, JHEP 05 (2016) 037

- In the following we assume $|C_i| \ll 1$

Heavy Quark Expansion with NP effects

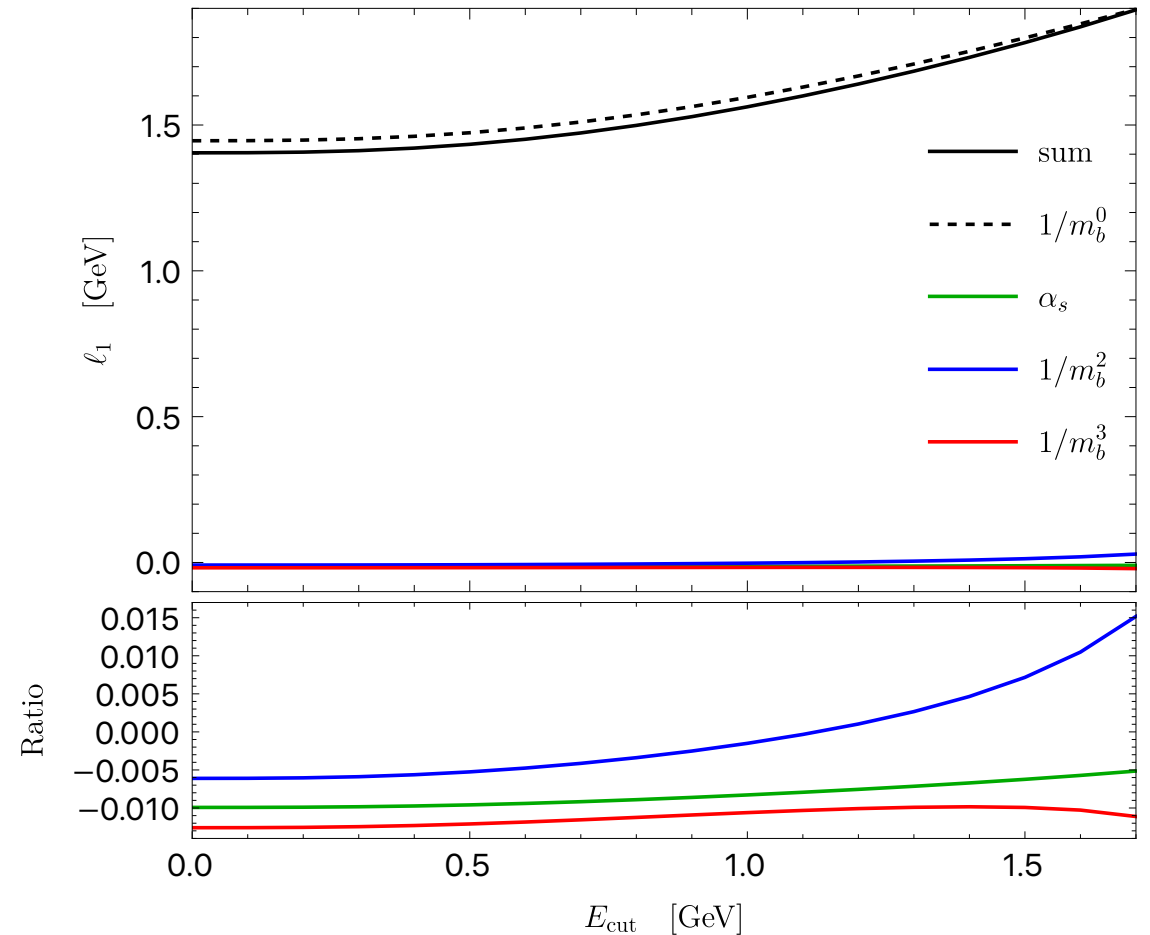
$$\ell_1 = \langle E_e \rangle E_{\text{cut}}$$

Series expansion in three parameters:

- Λ_{QCD}/m_b
- α_s
- $(v/\Lambda_{\text{NP}})^2$

To properly catch the leading effects:

- $(v/\Lambda_{\text{NP}})^2 \times \alpha_s^0 \times (1/m_b)^0$: NP at tree level in the free-quark approximation.



values of HQE param. from
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

Heavy Quark Expansion with NP effects

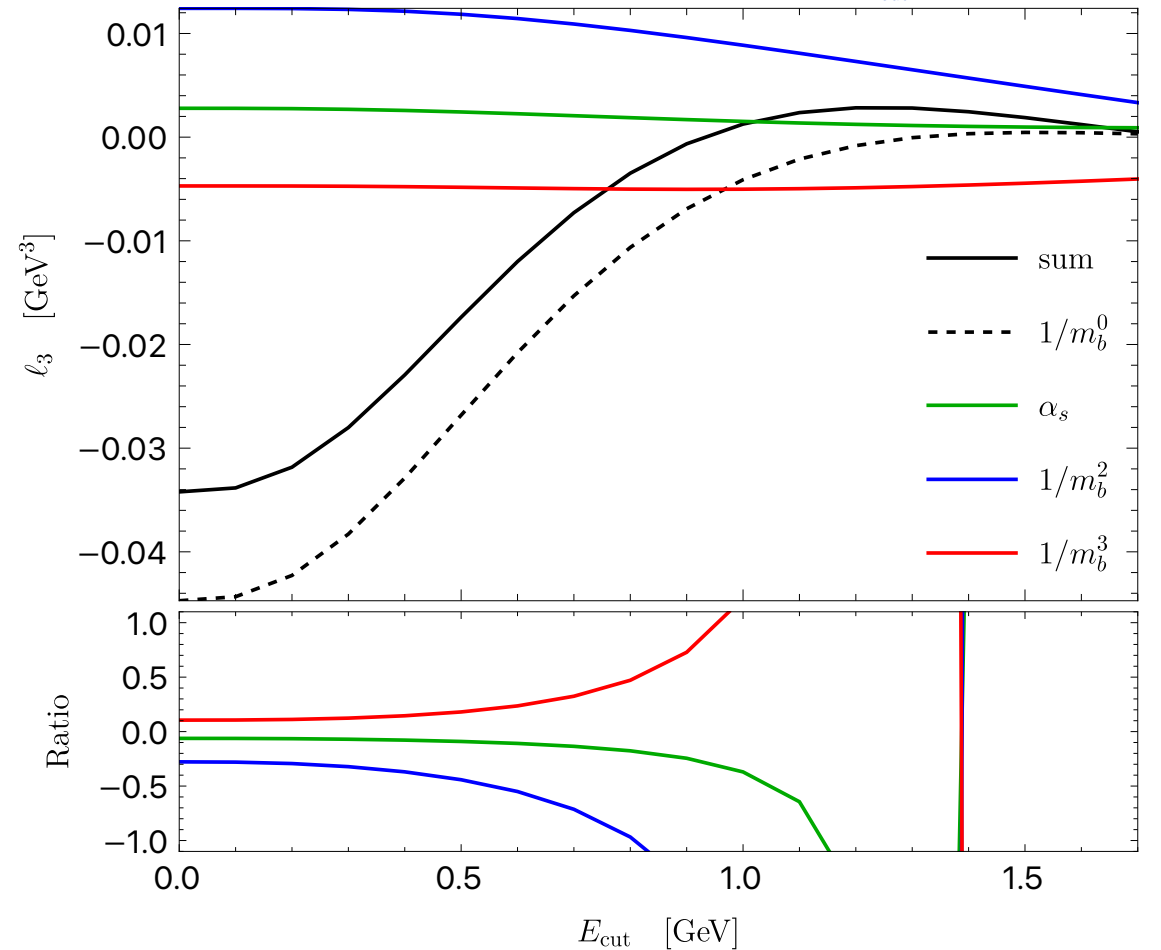
$$\ell_e = \left\langle \left(E_e - \langle E_e \rangle \right)^3 \right\rangle_{E_{\text{cut}}}$$

Series expansion in three parameters:

- Λ_{QCD}/m_b
- α_s
- $(v/\Lambda_{\text{NP}})^2$

To properly catch the leading effects:

- $(v/\Lambda_{\text{NP}})^2 \times \alpha_s^0 \times (1/m_b)^0$: NP at tree level in the free-quark approximation.
- $(v/\Lambda_{\text{NP}})^2 \times \alpha_s^0 \times (1/m_b)^{2,3}$: power-suppressed terms for NP effects



values of HQE param. from
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

Heavy Quark Expansion with NP effects

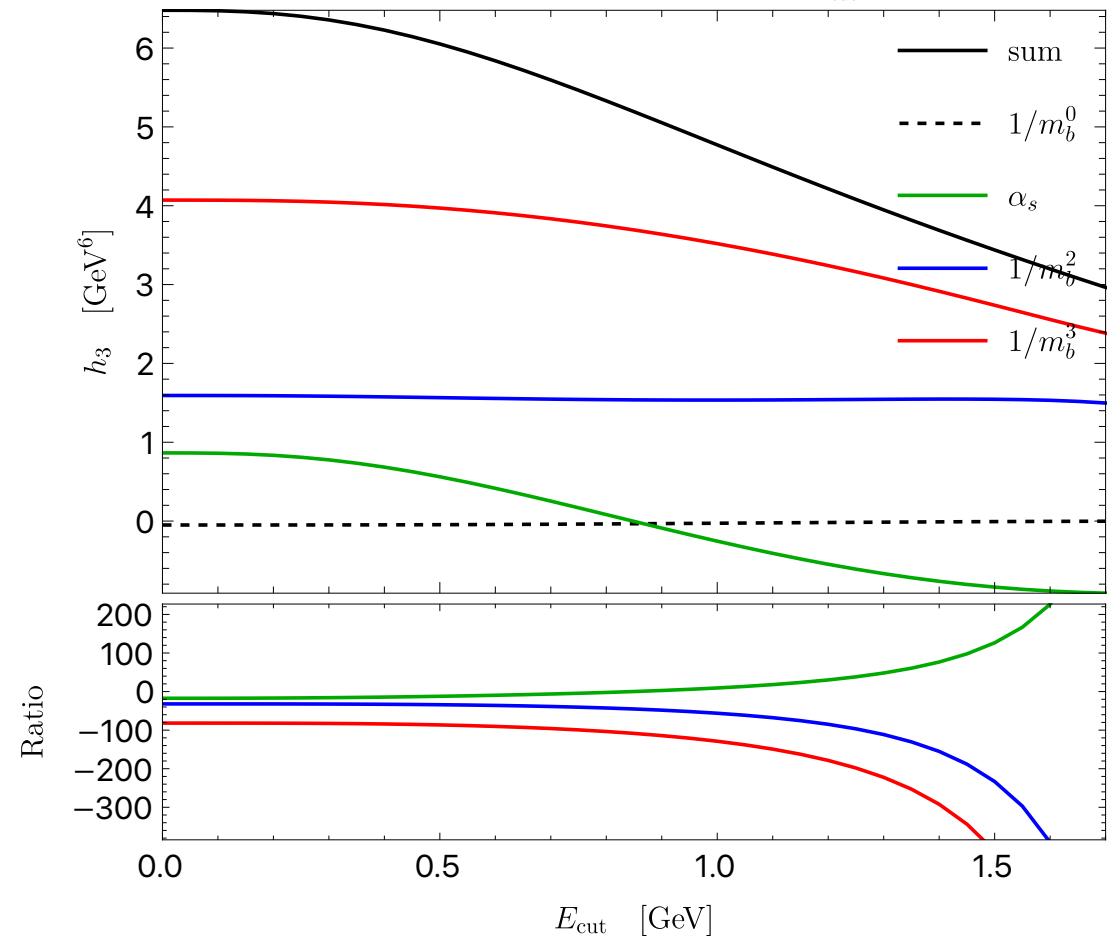
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To properly catch the leading effects:

- $(v/\Lambda_{NP})^2 \times \alpha_s^0 \times (1/m_b)^0$: NP at tree level in the free-quark approximation.
- $(v/\Lambda_{NP})^2 \times \alpha_s^0 \times (1/m_b)^{2,3}$: power-suppressed terms for NP effects
- $(v/\Lambda_{NP})^2 \times \alpha_s^1 \times (1/m_b)^0$: QCD NLO corrections to NP effects

$$h_3 = \left\langle \left(M_X^2 - \langle M_X^2 \rangle \right)^3 \right\rangle_{E_{\text{cut}}}$$



values of HQE param. from
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

The calculation

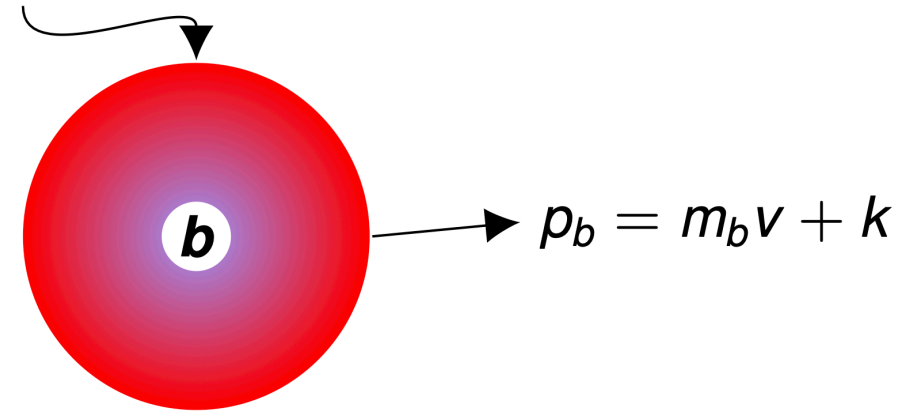
We calculate triple differential rate

$$\begin{aligned} \frac{d\Gamma_{\text{SM+NP}}}{dE_\ell dq^2 dE_\nu} &= \frac{d\Gamma_{\text{SM+NP}}^{\text{LO}}}{dE_\ell dq^2 dE_\nu} + \frac{d\Gamma_{\text{SM+NP}}^{\text{Pow}}}{dE_\ell dq^2 dE_\nu} + \left(\frac{\alpha_s}{\pi}\right) \frac{d\Gamma_{\text{SM+NP}}^{\text{NLO}}}{dE_\ell dq^2 dE_\nu} \\ &= \frac{G_F^2 |V_{cb}|^2}{16\pi^3} \tilde{W} \otimes \tilde{L} \end{aligned}$$

- leptonic tensor $L = \sum_{\text{lepton spin}} \langle 0 | J_L^\dagger | \ell \bar{\nu}_\ell \rangle \langle \ell \bar{\nu}_\ell | J_L | 0 \rangle$
- hadronic tensor

$$\begin{aligned} W &= \sum_{X_c} \frac{1}{2m_B} (2\pi)^3 \langle \bar{B} | J_H^\dagger | X_c \rangle \langle X_c | J_H | \bar{B} \rangle \delta^{(4)}(p_B - q - p_{X_c}) = \frac{1}{m_B} \text{Im} \int dx e^{im_b v \cdot x} \langle \bar{B} | T\{J_H^\dagger(x) J_H(0)\} | \bar{B} \rangle \\ &= \sum_{n=0}^{\infty} \frac{C^{(n)}(v, q)}{m_b^n} \langle B | \mathcal{O}_n | B \rangle \end{aligned}$$

quark-gluon cloud



The calculation

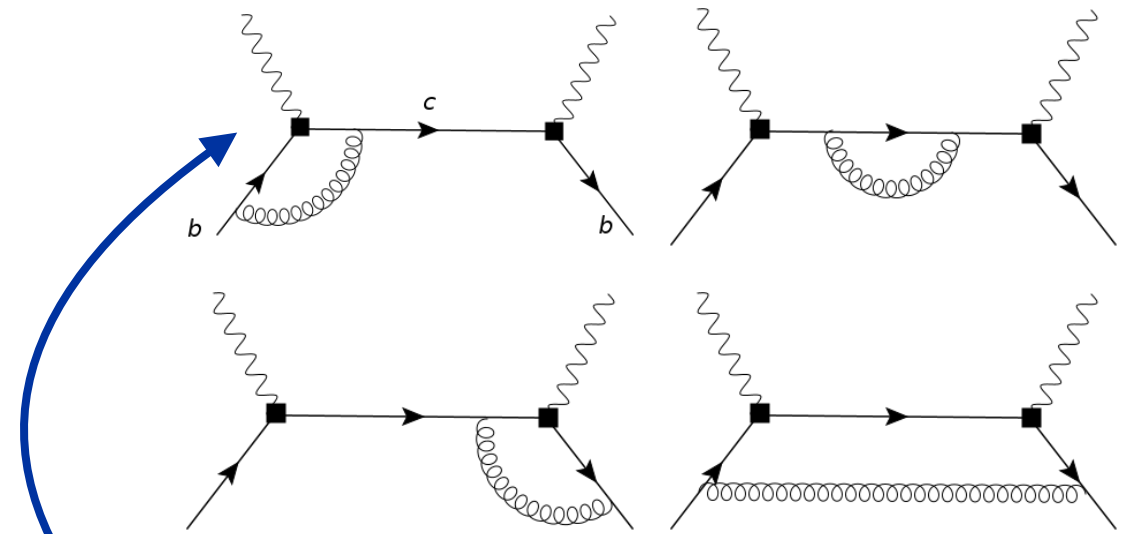
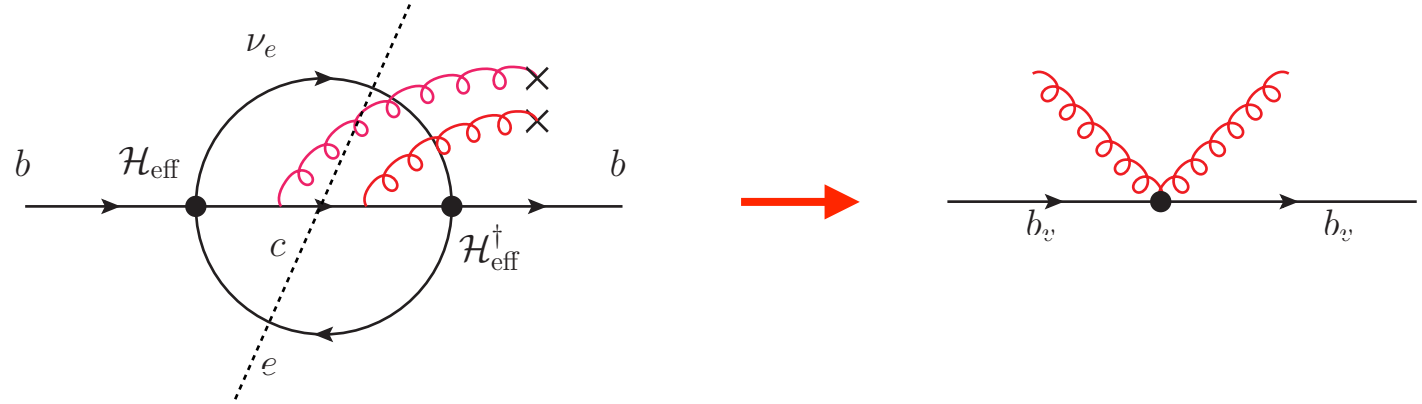
- Power corrections

- Background field method
- Trace formulas up to $1/m_b^3$
- Analytic integration

Manohar, Wise, Heavy quark physics;
 Mannel, *Phys.Rev.D* 50 (1994) 428;
 Gramm, Kapustin, *Phys.Rev.D* 55 (1997) 6924.

- NLO corrections

- Dimensional regularisation for both UV and IR divergences
- Larin scheme for γ_5 Larin, *Phys.Lett.B* 303 (1993) 113.
- Numerical phase space integration
 see also: Alberti et al, *Nucl.Part.Phys.Proc.* 273(2016) 1325



Insertion of any NP effective operator

Moments of differential distributions

We obtain results for:

- Charged-lepton energy moments, lower cut on E_ℓ

$$L_n = \frac{1}{\Gamma_0} \int_{E_\ell > E_{\text{cut}}} \left(\frac{E_\ell}{m_b} \right)^n \frac{d\Gamma}{dq^2 dE_\ell dE_\nu} dq^2 dE_\ell dE_\nu.$$

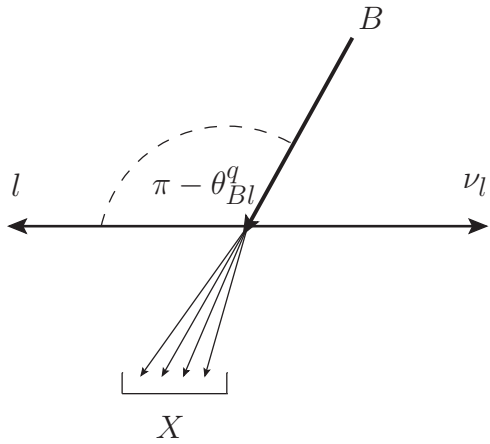
- Leptonic invariant mass moments, lower cut on q^2

$$Q_n = \frac{1}{\Gamma_0} \int_{q^2 > q_{\text{cut}}^2} \left(\frac{q^2}{m_b^2} \right)^n \frac{d\Gamma}{dq^2 dE_\ell dE_\nu} dq^2 dE_\ell dE_\nu.$$

- Hadronic invariant mass moments, lower cut on E_ℓ

$$M_n = \frac{1}{\Gamma_0} \int_{E_\ell > E_{\text{cut}}} \left(\frac{M_B^2 + q^2 - 2M_B q_0}{m_b^2} \right)^n \frac{d\Gamma}{dq^2 dE_\ell dE_\nu} dq^2 dE_\ell dE_\nu.$$

Forward-backward asymmetry



$$A_{FB} \equiv \frac{\int_{-1}^0 dz \frac{d\Gamma}{dz} - \int_0^1 dz \frac{d\Gamma}{dz}}{\int_{-1}^1 dz \frac{d\Gamma}{dz}}$$

where $z \equiv \cos \theta = \frac{v \cdot p_{\bar{\nu}_\ell} - v \cdot p_\ell}{\sqrt{(v \cdot q)^2 - q^2}}$

- Study of A_{FB} with a lower cut on q^2
- Good sensitivity to the power corrections

Turczyk, JHEP 04 (2016) 131
Herren, SciPost Phys. 14 (2023) 020

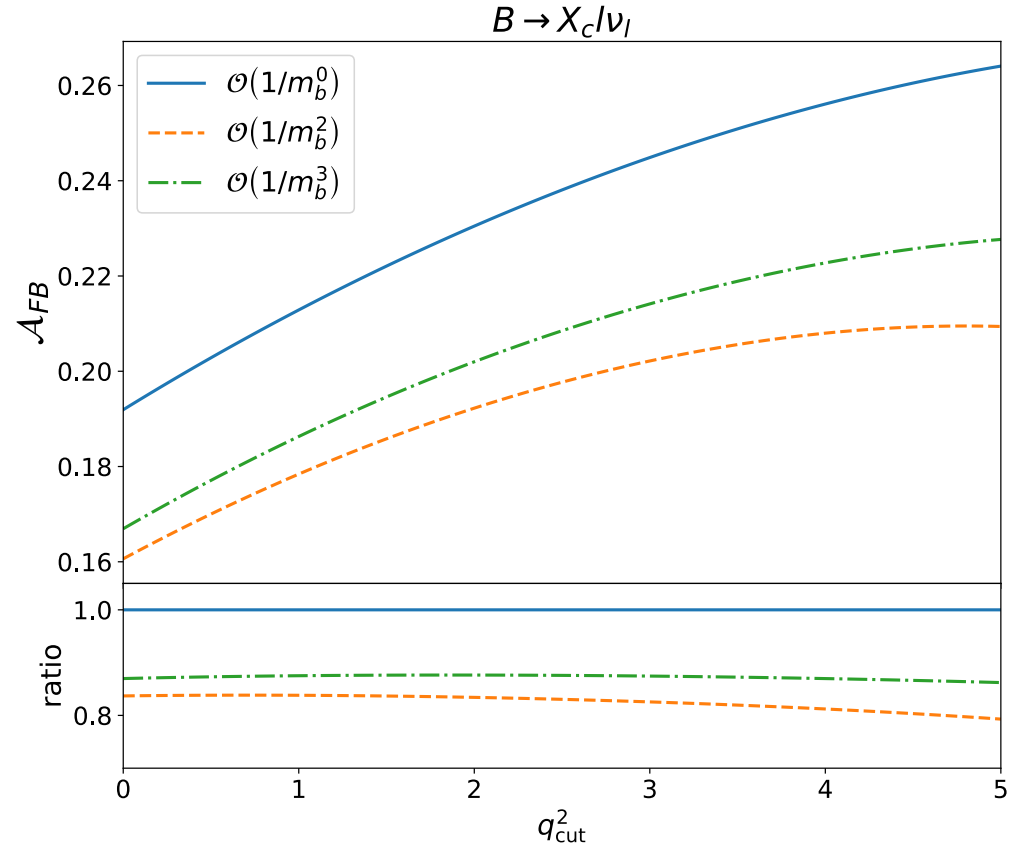


fig: Herren, SciPost Phys. 14 (2023) 020

Git repository

<https://gitlab.com/vcb-inclusive/npinb2xclv>

README.md

New physics contributions to moments of inclusive $b \rightarrow c$ semileptonic decays

by M. Fael, M. Rahimi, K. K. Vos, hep-ph/2208.04282

Content

This repository contains the analytic expressions for the evaluation of the various kinematic moments in the presence of NP operators. All expressions are given in the on-shell scheme. We include power corrections up to $1/m_b^3$ and NLO corrections at LO in $1/m_b$. Content of the directories:

- `Eemoments_Eecut` : the directory contains expressions for the electron energy moments up to the third moment, with a lower cut on the electron energy.
- `MXmoments_Eecut` : the directory contains expressions for the hadronic invariant mass moments up to the third moment, with a lower cut on the electron energy.
- `Q2moments_q2cut` : the directory contains expressions for the dilepton invariant mass (q^2) moments up to the third moment, with a lower cut on q^2 .
- `AFB` : the directory contains expressions for the total rate, with emission in the forward and backward direction (in the rest frame of the two leptons). They corresponds to the two terms at numerator in Eq. (13).
- `alphas` : expressions of the NLO corrections. The NLO corrections can be evaluated with the Mathematica package `EvaluateAlphaSNP.m`.

QCD NLO Corrections

The files contained in the directories `AFB`, `Eemoments_Eecut`, `MXmoments_Eecut` and `Q2moments_q2cut` give analytic expressions for the moments at LO. However the coefficients of order α_s is represented in this files by the functions:

```
X1AFBForward[cNP, q2cutthat, m2, mu2hat]
X1AFBBackward[cNP, q2cutthat, m2, mu2hat]
X1mix[nq2, nq0, cNP, Ycut, m2, mu2hat]
X1E1[n, cNP, Ycut, m2, mu2hat]
X1Q2[n, cNP, q2cutthat, m2, mu2hat]
```

The numerical evaluation of the NLO corrections requires non-trivial numerical integration over the phase-space. The package `EvaluateAlphaSNP.m` provides the necessary subroutines for the integration. One needs to load the package in Mathematica:

```
In[ ] := << "alphas/EvaluateAlphaSNP.m"
```

Afterwards, calling these functions with numerical arguments will execute the numerical integration. For example, the NLO correction the the first q^2 moment, with a cut $q_{\text{cut}}^2 = 4 \text{ GeV}^2$ is evaluated in the following way:

```
In[ ] := mb = 4.6; mc = 1.15; mus = mb; q2cut = 1; q2hatcut = q2cut/mb^2;
In[ ] := X1Q2[1, SM^2, q2hatcut, (mc/mb)^2, (mus/mb)^2]
Out[ ] := -0.215785
```

Illustration for specific NP scenarios

Centralized moments:

$$\langle (O)^n \rangle_{\text{cut}} = \int_{\text{cut}} (O)^n \frac{d\Gamma}{dO} dO \Big/ \int_{\text{cut}} \frac{d\Gamma}{dO} dO$$

$$\langle (O - \langle O \rangle)^n \rangle = \sum_{i=0}^n \binom{n}{i} \langle O^i \rangle (\langle O \rangle)^{n-i}$$

- Expansion in α_s and $1/m_b$
- Kinetic scheme for m_b and $\overline{\text{MS}}$ for m_c
- Numerical inputs from

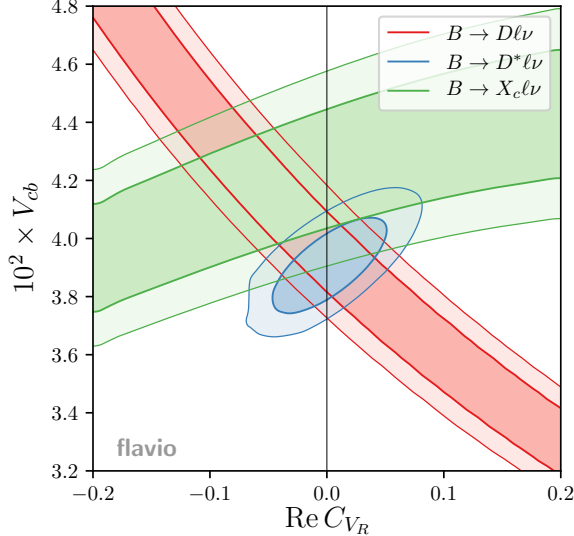
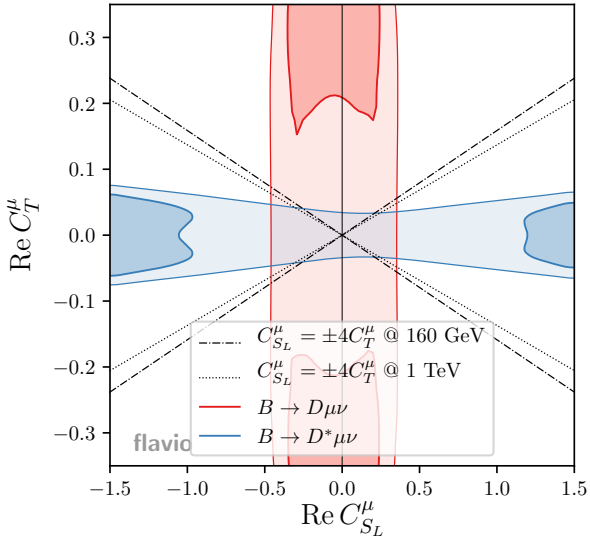
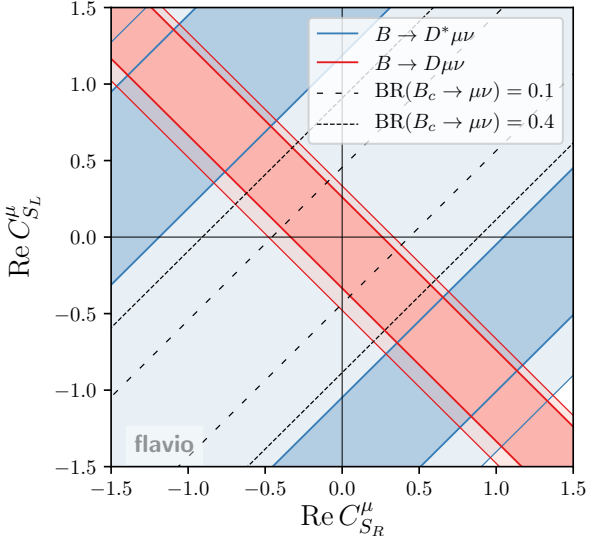
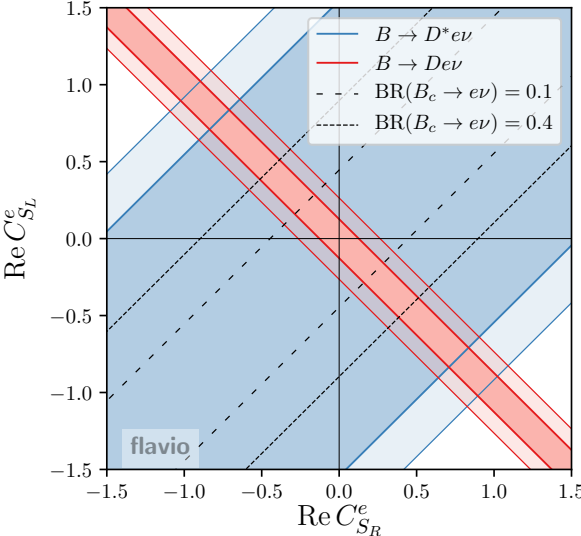
Bordone, Gambino, Capdevila, PLB 822 (2021) 136679

NP Scenarios	C_{V_L}	C_{V_R}	C_{S_R}	C_{S_L}	C_T
I	0	0	1	1	0
II	0	0	0	-1	0.5
III	-1	0.5	0	0	0

- The 3 scenarios are for illustration!
- Scen. II excluded by exclusive $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$
- In this case, we do not reexpand in v/Λ_{NP}
- SM uncertainty:
 - Renormalization scale $m_b/2 \leq \mu_s \leq 2m_b$
 - Parametric uncertainty on HQE parameters

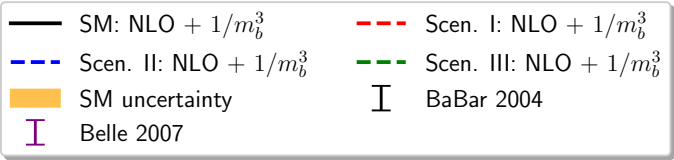
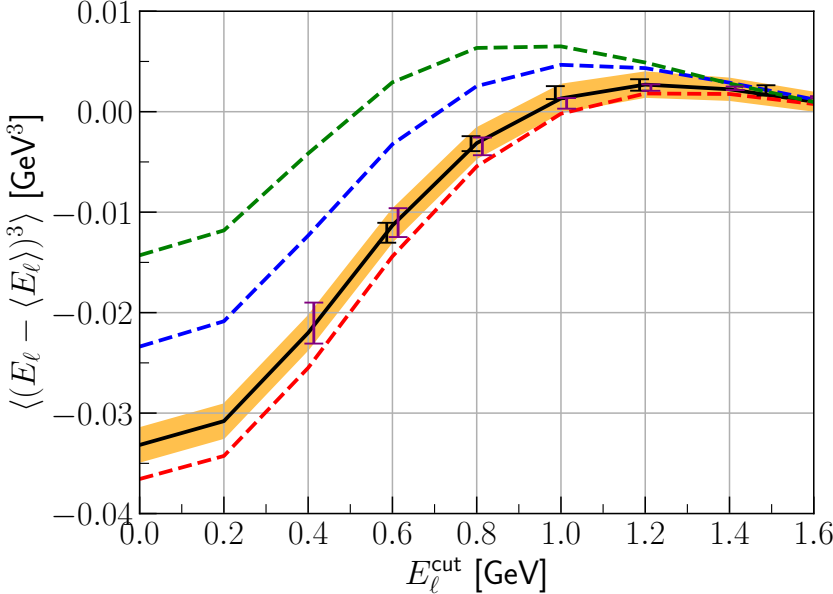
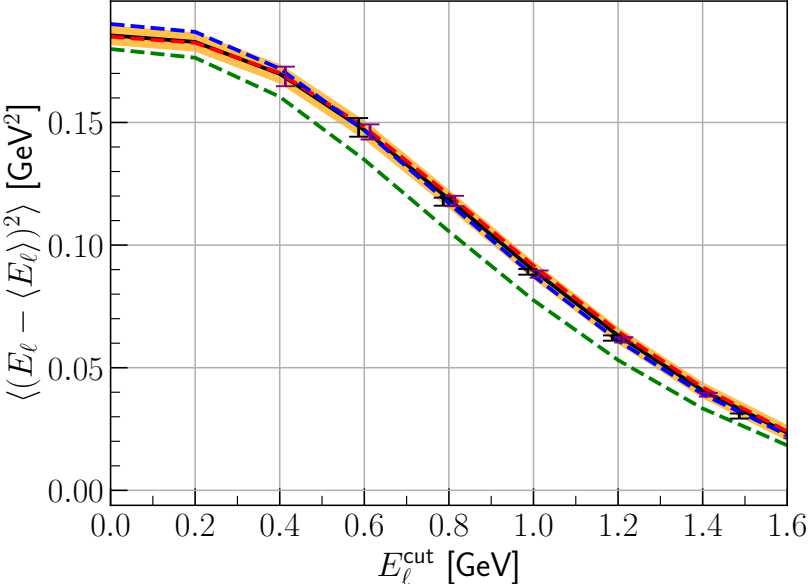
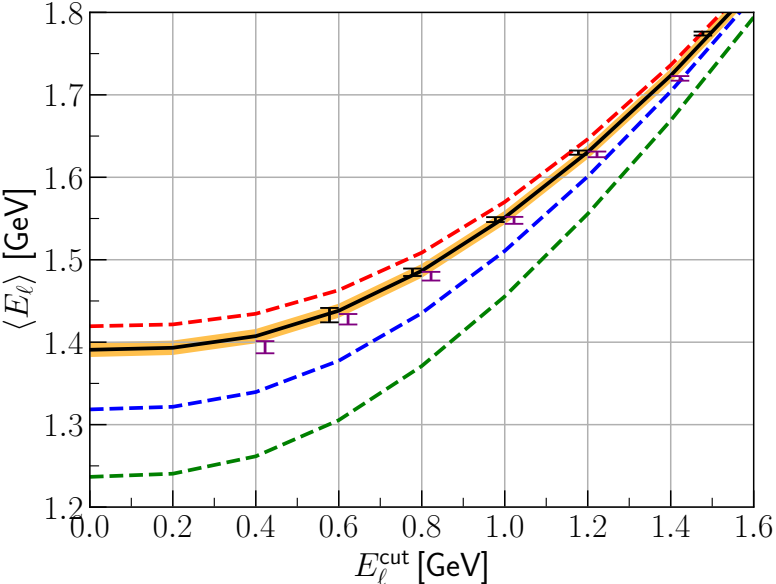
Illustration for specific NP scenarios

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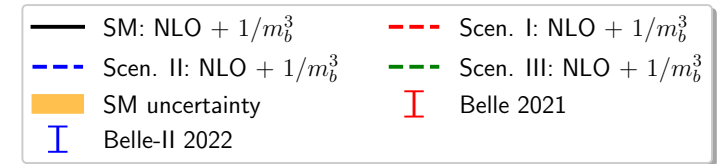
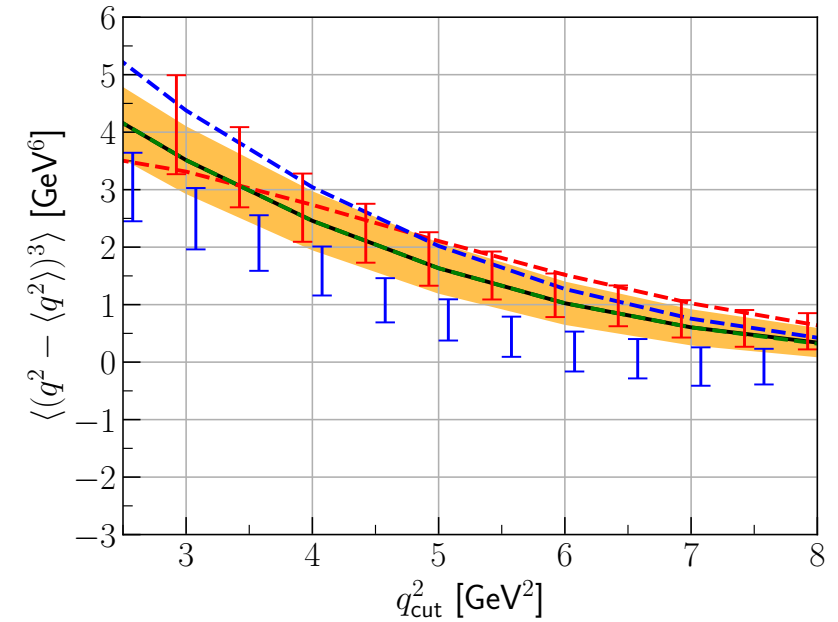
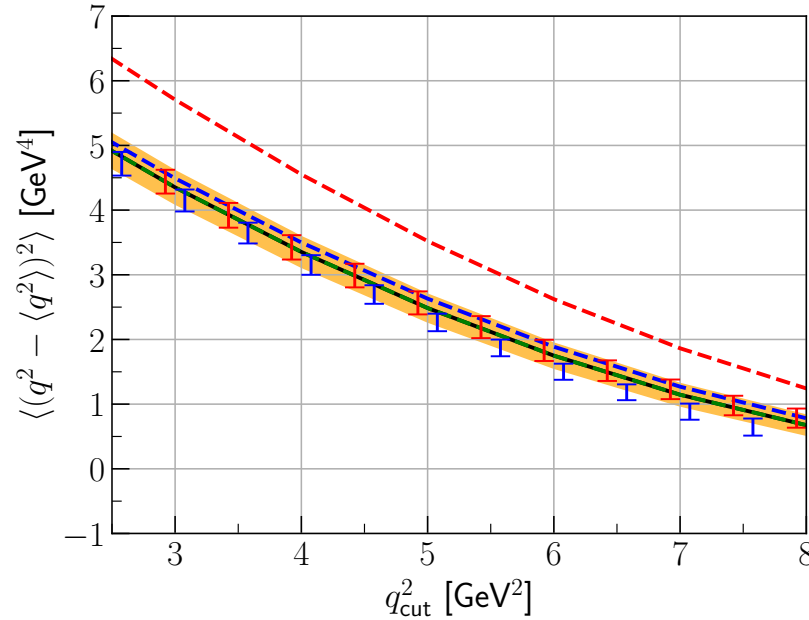
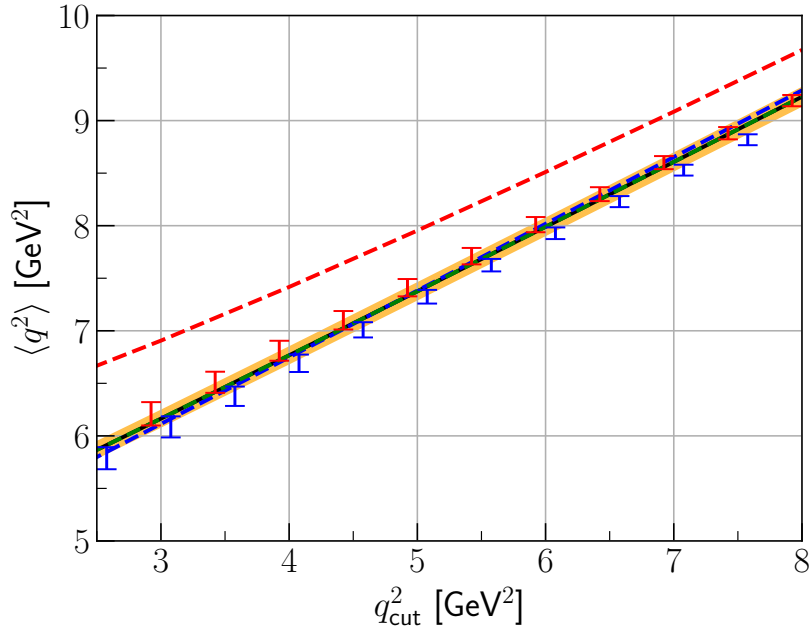
plots from Jung, Straub, *JHEP* 01 (2019) 009

Electron energy moments



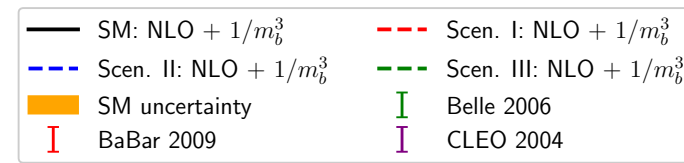
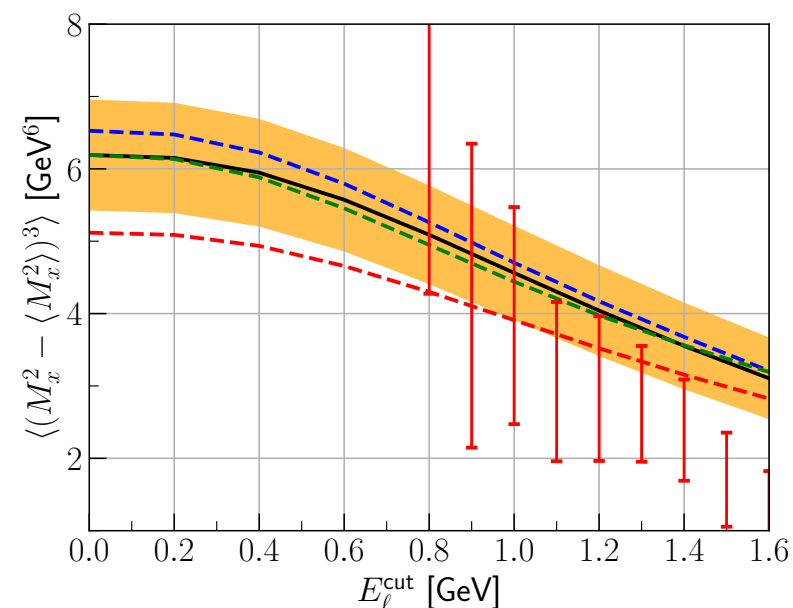
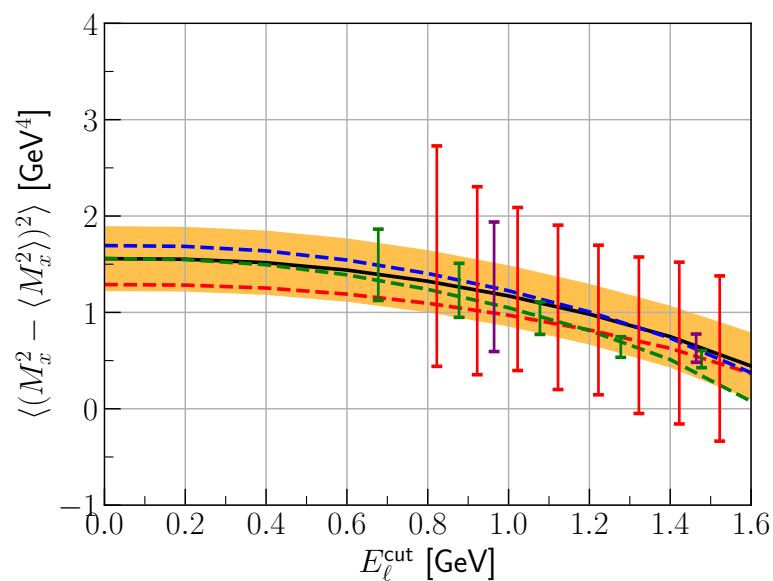
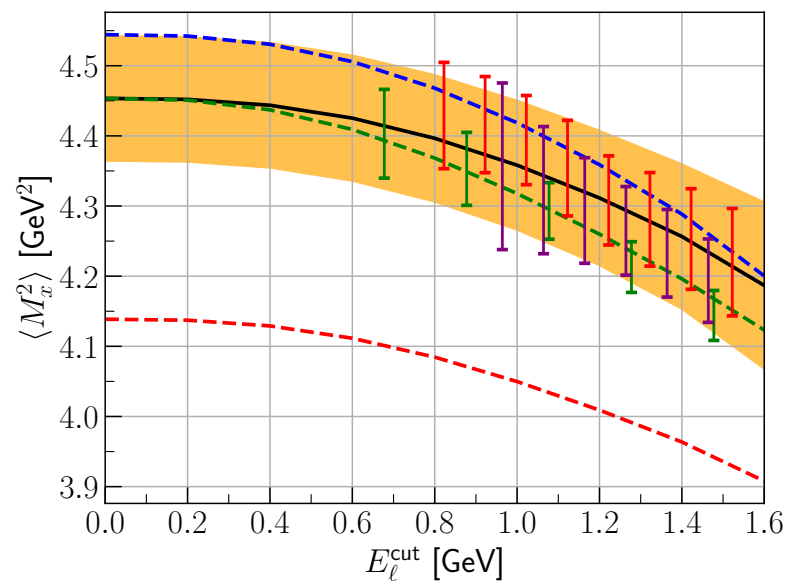
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q^2 moments



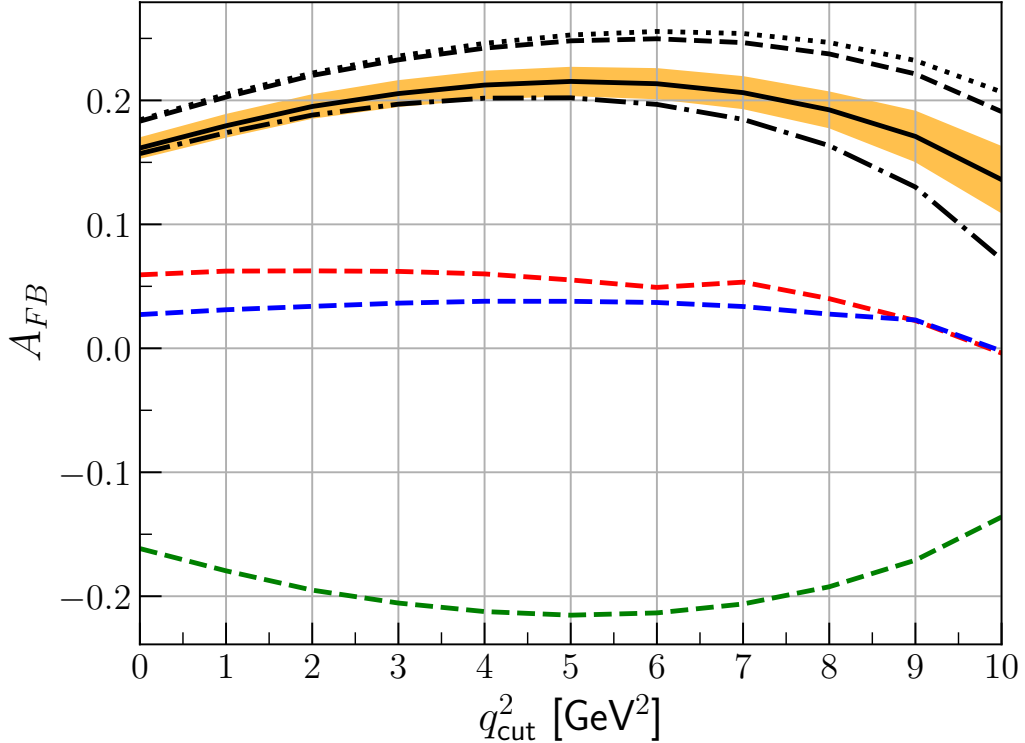
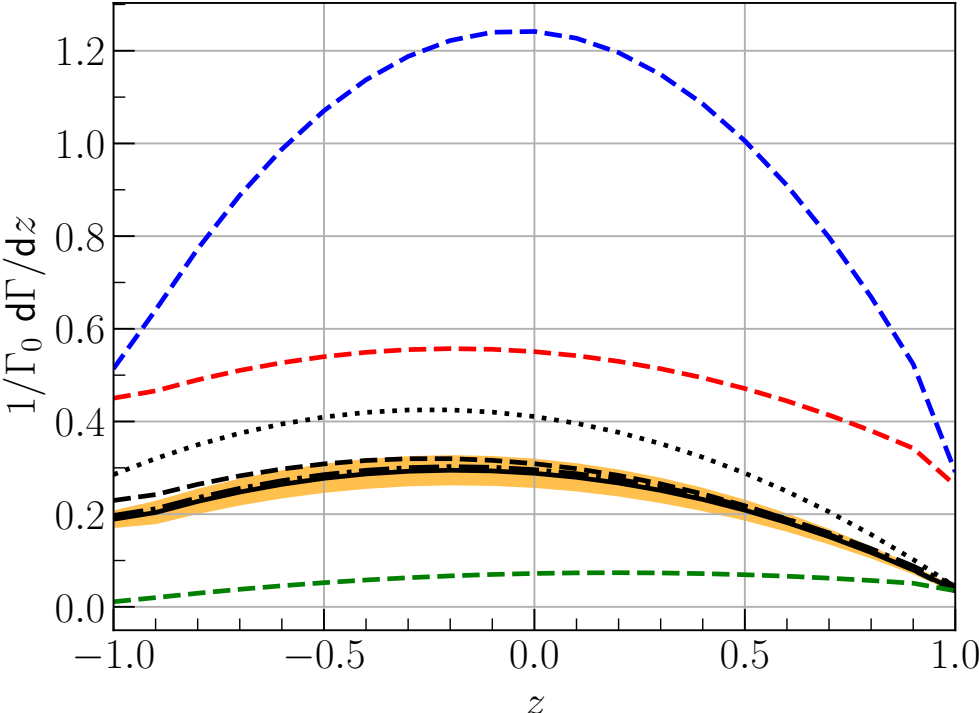
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M_x moments

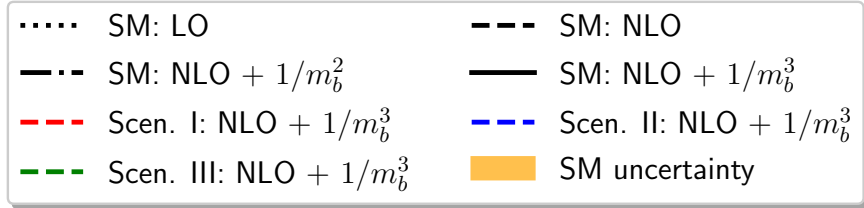


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Forward-backward asymmetry



NP Scenarios	C_{V_L}	C_{V_R}	C_{S_R}	C_{S_L}	C_T
I	0	0	1	1	0
II	0	0	0	-1	0.5
III	-1	0.5	0	0	0



Towards a global fit with NP effects

with F. Bernlochner, M. Prim, K.K. Vos

<https://gitlab.com/vcb-inclusive/kolya>

- **Open-source** python package
- Python interface to **CRunDec** for automatic α_s , m_b^{kin} and \bar{m}_c RGE evolution

Chetyrkin., Kühn, Steinhauser, *Comput. Phys. Commun.* 133 (2000) 43

Herren, Steinhauser, *Comput.Phys.Commun.* 224 (2018) 333

see also: <https://github.com/DavidMStraub/rundec-python>

- We included SM and NP effects
- Kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017

Czarnecki, Melnikov, Uraltsev, *Phys.Rev.Lett.* 80 (1998) 3189

MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003

- Observables
 - $\Gamma_{\text{sl}}, \Delta\text{Br}(E_{\text{cut}})$
 - Centralised moments $\langle E_\ell \rangle_{E_{\text{cut}}}, \langle M_X^2 \rangle_{E_{\text{cut}}}$
 - Centralised moments $\langle q^2 \rangle_{q_{\text{cut}}^2}$



Nikolai Uraltsev 1957 - 2013

- **Complete:** NLO corrections at order $1/m_b^0$
- **Completing:**
 - NNLO QCD corrections at $1/m_b^0$ in the SM
 - NLO QCD corrections for power-suppressed terms in the SM

Included in Kolya

- Total rate

NLO: Nir, *Phys.Lett.B* 221 (1989) 184

NNLO: Czarniecki, Pak, *Phys.Rev.Lett.* 100 (2008) 241807, *Phys.Rev.D* 78 (2008) 114015

N3LO: MF, Schönwald, Steinhauser, *Phys.Rev.D* 104 (2021) 016003, *JHEP* 08 (2022) 039

NLO at $1/m_b^2$: Mannel, Pivovarov, Rosental, *Phys.Rev.D* 92 (2015) 5, 054025

- E_ℓ and M_X^2 moments

NLO differential rate: Aquila, Gambino, Ridolfi, Uraltsev, *Nucl.Phys.B* 719 (2005) 77;
MF, Rahimi, Vos, *JHEP* 02 (2023) 086.

NNLO: Biswas, Melnikov, *JHEP* 02 (2010) 089; Gambino, *JHEP* 09 (2011) 055.

NLO at $1/m_b^2$: Alberti, Gambino, Nandi, *Nucl.Phys.B* 870 (2013) 16, *JHEP* 01 (2014) 147

- q^2

NLO: Jezebel, Kühn, *Nucl.Phys.B* 320 (1989) 20

NLO up to $1/m_b^2$: Mannel, Moreno, Pivovarov, *JHEP* 08 (2020) 089

see talk by D. Moreno

- Kinetic scheme

Bigi, Shifman, Uraltsev, Vainshtein, *Phys.Rev.D* 56 (1997) 4017

Czarniecki, Melnikov, Uraltsev, *Phys.Rev.Lett.* 80 (1998) 3189

MF, Schönwald, Steinhauser, *Phys.Rev.Lett.* 125 (2020) 052003, *Phys.Rev.D* 103 (2021) 1, 014005

```
[1]: import kolya
import numpy as np
```

Physical parameters

They are declared in the class `parameters.physical_parameters`. Initialization set default values

```
[5]: par = kolya.parameters.physical_parameters()
```

Bottom mass in the kinetic scheme $m_b^{\text{kin}}(\mu_{WC})$ in GeV

```
[6]: par.mbkin
```

```
[6]: 4.573
```


HQE parameters

They are declared in the class `parameters.HQE_parameters`. They are initialized to zero. We can set their values in the following way

```
[8]: hqe = kolya.parameters.HQE_parameters(  
      muG = 0.306,  
      rhoD = 0.185,  
      rhoLS = -0.13,  
      mupi = 0.477,  
      )
```

Wilson coefficients

They are declared in the class `parameters.WCoefficients`. They are initialized to zero and can be set in the following way

```
[9]: wc = kolya.parameters.WCoefficients(  
      VL = 0,  
      VR = 0,  
      SL = 0.1,  
      SR = 0,  
      T = 0,  
      )
```

Total Rate

The branching ratio is given by the function `BranchingRatio_KIN_MS(Vcb, par, hqe, wc)`

```
[10]: Vcb = 42.2e-2  
kolya.TotalRate.BranchingRatio_KIN_MS(Vcb, par, hqe, wc)
```

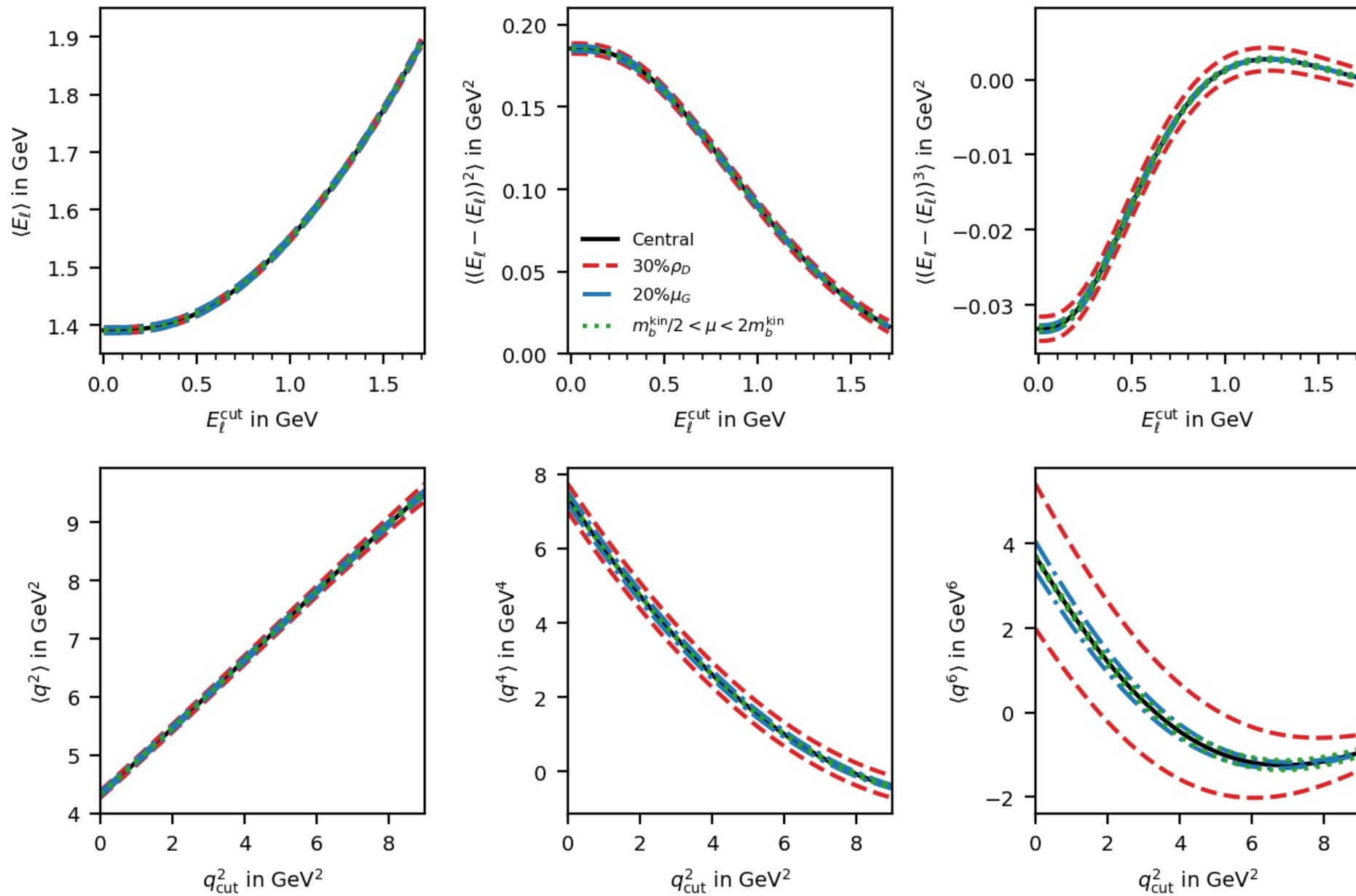
```
[10]: 10.64996041511315
```

Centralized Q2 moments

Q2 moments are evaluated with `Q2moments.moment_n_KIN_MS(q2cut, par, hqe, wc)`, for instance

```
[11]: q2cut = 8.0  
kolya.Q2moments.moment_1_KIN_MS(q2cut, par, hqe, wc)
```

```
[11]: 8.971188963587162
```



Conclusions and Outlook

- We plan to scrutinise data on inclusive $B \rightarrow X_c \ell \bar{\nu}_\ell$ for NP contributions.
 - Good place to look are moments of the **electron energy** spectrum and A_{FB} .
 - M_X^2 and q^2 moments more sensitive to $1/m_b$ corrections. More difficult to disentangle NP.
 - We developed open-source python package **Kolya**
-
- Ongoing global fit of available inclusive data from B -factories.
 - Future **use lattice results** for inclusive decays for non-pert. inputs.
 - Study interplay with exclusive decays $B \rightarrow D^{(*)} \ell \bar{\nu}_\ell$